



REASONING ABOUT COVARIATION WITH TINKERPLOTS

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Declaration of Originality

This thesis contains no material which has been accepted for a degree or diploma by the University or any other institution, except by way of background information and duly acknowledged in the thesis, and to the best of the my knowledge and belief no material previously published or written by another person except where due acknowledgement is made in the text of the thesis, nor does the thesis contain any material that infringes copyright.

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Statement of Ethical Conduct

The research associated with this thesis abides by the international and Australian codes on human experimentation, as set out in the *National Statement of Ethical Conduct in Human Research* (2007) and interpreted by the Human Ethics Committee of the University.

Ethical approval for the research was gained from the *Southern Tasmania Social Sciences Human Research Ethics Committee* at the University of Tasmania in 2006 – Ethics Approval Number H8778. The committee adheres to the guidelines outlined in the *National Statement on Ethical Conduct in Human Research*. The research also had permission and approval from the Department of Education, Tasmania, and satisfied department criteria for *Conducting Research in Tasmanian Government Schools and Colleges* (2006).

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Noleine Fitzallen contributed the *Historical Developments in Graphing* and the *Current Graphing Curricula* sections. She also contributed to the refinement and presentation of the report. The report is independent of the thesis and does not include any data used in the thesis. The *Historical Developments in Graphing* section is reproduced in part in this thesis. The authorship of other information from the report included in the thesis is duly acknowledged.

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My life's journey has taken many twists and turns, providing me with the opportunity to experience a number of different careers and meet a diverse range of people. My partner once quipped "You haven't made up your mind what you want to be!" This may be the case but to me it does not matter. I want to explore new things, places and ideas. I want to continue to grow and develop and embrace new opportunities. This thesis is a reflection of my personal experiences and achievements and is influenced heavily by my family, people I have worked with and people I have not met, yet know through their research. Some people assisted greatly in the production of the actual artefact but many more had an impact on why I was in this place, at this time, to be able to embrace the opportunity of conducting research when it was offered. I would like to acknowledge that this thesis is a culmination of all these influences.

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development of innovative data analysis software that has these features imbedded in its design. Consequently, students using *TinkerPlots* are empowered to be creative and innovative in return.

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Abstract

Covariation is recognised as an important aspect of statistical thinking and reasoning and is used to explore the relationship between two attributes. Often, covariation is determined from the interpretation of scatterplots that display the correspondence of two numerical attributes and is described as a trend in the data. Scatterplots are utilised when conducting exploratory data analysis (EDA). EDA strategies are useful for interpreting the data as they allow the data to be manipulated in order to construct graphical representations that facilitate making sense of the data. The translation of EDA strategies into innovative software packages, such as *TinkerPlots: Dynamic Data Exploration*, has placed student learning about data analysis in technological environments and there is a need to investigate the way in which students learn in these contexts.

This inquiry had two objectives. The first objective was to further understanding of the factors that influence student learning when working with software packages. This is through the development of a conceptual framework for learning in EDA graphing environments that aligns with and extends current research about student understanding of graphing and data analysis. The second objective was to explore the intersection between the students' thinking and reasoning about covariation and the influence of *TinkerPlots* on that process, as students explore data sets to determine the relationship between variables and identify trends. To realise these objectives the following research questions are explored:

1. How can the learning behaviours of students as they engage with exploratory data analysis software be characterised through a framework that can then be used to explore and analyse students' understanding of covariation using *TinkerPlots*?
2. How do students interact with the exploratory data analysis software, *TinkerPlots*, to represent data in a variety of forms when exploring questions about relationships within a data set?

3. How do students demonstrate an understanding of covariation in the exploratory data analysis software environment afforded by *TinkerPlots* and use these understandings to provide informal justification for their conclusions about the relationships identified?

The inquiry employed an educational design research methodology within a pragmatist paradigm to facilitate the development of a systematic iterative study. The methodology was chosen to encapsulate the way students learn about the interpretation of graphical representations, more specifically related to covariation, in the technological software environment afforded by *TinkerPlots*.

The inquiry was enacted across seven stages. Stage 0 involved the development of the research design. Stage 1 involved the development of a conceptual framework for learning in EDA software environments that incorporated four aspects of graphing and data analysis skills – *Generic knowledge*, *Being creative with data*, *Understanding data*, and *Thinking about data*. Stage 2 involved an evaluation of *TinkerPlots* to determine its usability as a teaching and learning tool. Stage 3 involved the development and evaluation of an assessment tool to determine the prior learning of students in relation to the interpretation of graphs, and select the participants for the data collection stage of the inquiry. Stage 4 involved the development and implementation of a sequence of learning experiences. The activities in the sequence of learning were based on recommendations from the research on the development of graphing and data analysis skills. The sequence of learning experiences was implemented with 12 students working in pairs, twice a week for 45 minutes, over a period of 6 weeks. In addition, the data generated from individual interviews with the 12 students conducted at the end of the sequence of learning were included in this stage. The data from the student interviews are presented as Student Profiles that encapsulate the way in which they used *TinkerPlots* to develop not only an understanding of covariation but also develop other data analysis skills and strategies. Stage 5 involved the presentation of the results for the Research Questions, with the discussion of the findings, implications of the inquiry, and recommendations for future research included in Stage 6. The presentation of the thesis follows this chronological order.

Through the evaluation of *TinkerPlots* and its subsequent implementation in the inquiry, it was identified that *TinkerPlots* provides a powerful learning environment for supporting students' understanding of covariation. In terms of student understanding of covariation, the inquiry identified that young students are able to reason about covariation and display three levels of reasoning. The results also suggest that students adopt three different strategies when accessing the features of *TinkerPlots* while creating and interpreting graphs. These strategies are: *Snatch and Grab*, *Proceed and Falter*, and *Explore and Complete*.

Outcomes of the inquiry are presented in relation to the thesis-developed *Model of Learning Behaviour in EDA Graphing Environments*. Within the framework of the model the students' development of covariation reasoning is revealed and discussed in terms of the potential of the results to inform the teaching and learning of covariation within EDA software environments and future curriculum development. Consideration was also given to the merits of the *Model of Learning Behaviour in EDA Graphing Environments* and its application throughout the inquiry process. Unexpected insights into the students' thinking and reasoning about association are also discussed to demonstrate the utility of the thesis-developed model and to highlight the need to further research in the area of student understanding of association.

Table of Contents

	Declaration of Originality	i
	Authority of Access	i
	Statement of Ethical Conduct	ii
	Statement of Co-authorship	iii
	Acknowledgements	v
	Abstract	vii
	Table of Contents	x
	List of Tables	xiv
	List of Figures	xv
Stage 0	Inquiry Commencement	1
	Inquiry Approach	3
	Inquiry Origins	5
	Inquiry Methodology	6
	Inquiry Aim	7
	Methodological and Theoretical Underpinnings	11
	Research Perspective	12
	Educational Design Research Methodology	13
	Inquiry Design	17
	Inquiry Stages	20
	Inquiry Overview	22
	Sampling Design	24
	Internal Validity	24
	Inquiry Significance	25
	Ethical Approval	26
	Structure of the Thesis	27
Stage 1	Development of a Model of Learning Behaviour	29
	Graphs and Graphing	30
	Historical Developments in Graphing	32
	Spatial Organisation for Data Analysis	33
	Discrete Quantitative Comparisons	33
	Continuous Distributions	34
	Multivariate Distributions and Correlation	35
	20 th Century and Future Developments	35
	Exploratory Data Analysis	36

	Covariation	40
	Models of Graphing	47
	Summary of Identified Limitations of Existing Models of Graphing	51
	Development of a Model of Learning Behaviour in EDA Graphing Environments	51
	Concluding Remarks	56
Stage 2	Evaluation of <i>TinkerPlots</i>	57
	Evaluating EDA graphing software	58
	The Development of Criteria for Evaluating EDA Graphing Software	60
	Guidelines for Evaluating Educational Software	61
	Evaluating Visual Representations	62
	Summary of Evaluating Visual Representations	65
	Criteria for Exemplary Statistical Software	66
	Evaluation of <i>TinkerPlots</i> using criteria developed	69
	Criterion 1 – Ability to provide an accessible interface and software features that are easy to use	69
	Criterion 2 – Ability to assist with the recall of knowledge and represent data in different and multiple forms	71
	Criterion 3 – Ability to facilitate the process of translating between mathematical expression and natural language	74
	Criterion 4 – Ability to provide extended memory when organising and reorganising data	76
	Criterion 5 – Ability to provide an interactive environment that allows multiple entry points for abstraction of concepts	79
	Criterion 6 – Ability to construct and display representations used for both interpretive and expressive learning activities	80
	Summary of Evaluation of <i>TinkerPlots</i>	82
	Development of <i>TinkerPlots</i> : A History	82
	Application of <i>TinkerPlots</i>	87
	Summary of Application of <i>TinkerPlots</i>	92
	Research with TinkerPlots	92
	Summary of Research Using <i>TinkerPlots</i>	97
	Concluding Remark	97
Stage 3	Establishment of Student Prior Learning	99
	Students' Prior Learning	100
	Development of Student Survey	101
	Student Survey Analysis	102
	Student Survey Participants	104
	Student Survey Items and Results	104
	Item 1 – What is a Graph?	105
	Item 2 – Draw a Graph	109

	Item 3 – Use of Graphing Software	112
	Item 4 – Draw a Graph from Data	114
	Item 5 – Interpret a Pictograph	117
	Item 6 – Design Questions from Data Cards	123
	Item 7 – Interpret a Scatterplot	124
	Summary of Student Survey	128
	Selection of Participants for Stage 4	129
	Concluding Remarks	130
Stage 4	Sequence of Learning and Outcomes	131
	Developing a Sequence of Learning	132
	Structure of Learning	132
	Model of Statistical Learning	133
	Reforms in Statistics Education	135
	Content	135
	Pedagogy	143
	Technology	145
	Sequence of Learning Experiences	146
	Lesson Protocols	147
	Implementation of the Sequence of Learning Experiences	153
	Student Interviews	155
	Analysis of Student Interviews	159
	Student Profiles	160
	Blaire	161
	Kimberley	164
	James	170
	Jessica	176
	Jake	183
	Mitchell	187
	Natalie	191
	Rory	195
	Shaun	200
	Natasha	203
	Johnty	206
	William	210
	Concluding Remarks	213
Stage 5	The Findings	215
	Research Question 1 – Framework for Characterising Student Learning	216
	Generic Knowledge	217
	Being Creative	218
	Understanding Data	218
	Thinking About Data	220

Interconnections Among the Dimensions	221
Research Question 2 – Student Interaction with <i>TinkerPlots</i>	222
Snatch and Grab	222
Proceed and Falter	225
Explore and Complete	227
Discussion of Student Interaction with <i>TinkerPlots</i>	228
Research Question 3- Development of Understanding of Covariation	230
Student Reasoning about Covariation	231
Summary of Student Reasoning about Covariation	238
Discussion of Student Reasoning about Covariation	239
Postscript – Student Understanding of Association	240
Discussion of Student Understanding of Association	244
Concluding Remarks	246
Stage 6 Inquiry Conclusion	248
Summary of the Inquiry	249
Stage 0 – Inquiry Commencement	249
Stage 1 – Development of Model of Learning Behaviour	250
Stage 2 – Evaluation of <i>TinkerPlots</i>	251
Stage 3 – Establishment of Student Prior Learning	252
Stage 4 – Sequence of Learning and Outcomes	253
Stage 5 – The Findings	253
Implications of Inquiry	254
Implications for Curriculum Development – A Case for the Big Ideas of Statistics	255
Implications for Classroom Practice – A Case for TPCK	259
Implications for the Choice of Software – A Case for <i>TinkerPlots</i>	261
Directions for Future Research	264
Evaluation of the Inquiry	266
Scientific Principle 1 – Pose significant questions that can be investigated empirically	267
Scientific Principle 2 – Link research to relevant theory	268
Scientific Principle 3 – Use methods that permit the direct investigation of the question	269
Scientific Principle 4 – Provide coherent and explicit chain of reasoning	269
Scientific Principle 5 – Replicate and generalise across studies	270
Scientific Principle 6 – Disclose research to encourage professional scrutiny and critique	270
Concluding Remarks	272
References	274