Hydraulics of Broad-Crested Weirs with Varying Side Slopes

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Abstract: The flow of water over a trapezoidal, broad-crested, or embankment weir with varying upstream and downstream slopes has been investigated. Data are presented comparing the effect of slopes of $2H:1V$, $1H:1V$ and vertical in various combinations on the upstream and downstream faces of the weir. Pressure and surface profiles were self-similar for all cases tested. Increasing the upstream slope to the vertical decreased the height of the surface profile and, hence, the static pressure of the crest. It also reduced the discharge coefficient. The variation in downstream static pressures was negligible though. Varying the downstream slope had a negligible effect on the surface and pressure profiles over the weir. Changes in flow were constrained to the region downstream of the crest. Cavitation could occur at the downstream corner of the weir if the upstream head was sufficiently high and a sloped face was used. This paper presents data that will be of use in the design of hydraulic structures for flow control and measurement.

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Introduction

Broad-crested weirs and embankment weirs are common engineering structures in irrigation systems, hydroelectric schemes, and highways. The distinction between these is generally the end-face angle, with the embankment weir having sloped end faces, and the typical broad-crested weir having vertical end faces. A number of authors have reviewed the performance of broad-crested weirs with different designs, all with the requirement that the weir crest be sufficiently long to ensure horizontal streamlines. Studies typically investigated symmetrical weirs, with upstream and downstream faces that were identical in slope. Hager and Schwalt (1994) outlined performance and design guidelines for a sharp-cornered, vertical-faced, broad-crested weir. Fritz and Hager (1998) provided similar discharge coefficient and flow data for an embankment weir with faces sloped at $2H:1V$. They demonstrated the improved flow rates that can be achieved when the upstream corner separation is reduced. Gogus et al. (2006) provided extensive data outlining the performance of broad-crested weirs with compound cross section and different step heights. Similarly Johnson (2000) outlined design discharge coefficient data for the broad-crested and sharp-crested weirs. He found that the discharge coefficient collapsed to a single curve when plotted against H_1/w, for values of $H_1/w<2.00$.

Fritz and Hager (1998) and Hager and Schwalt (1994) define C_D as shown in Eq. (1). For their own data and that of Bazin (1898) and Kindsvater (1964), they have show this to collapse to a single curve defined by Eq. (2), when C_D is plotted against relative crest length $\varepsilon=H_1/(H_1+w)$, with $0<\varepsilon<1$. This correlation relates to flow over an embankment weir with both the upstream and downstream faces sloped at $1V:2H$. The discharge coefficient for a vertical-sided, standard, broad-crested weir was approximately 10% less than this value over the range of relative crest length $0<\varepsilon<1$

$$Q = C_D b\sqrt{2gH_1^3} \tag{1}$$

$$C_D = 0.43 + 0.06\sin[\pi(\varepsilon - 0.55)] \tag{2}$$

In particular situations, the structural design of the weir may provide some flexibility, to modify the upstream and downstream face slopes to provide the best hydraulic characteristics. One critical condition that needs to be addressed is the possibility of cavitation, particularly under extreme overflow conditions that can cause extensive damage and failure of hydraulic structures (Fortner 2003; Inozemtsev 1969). The weir should also be capable of passing the required flow rate in extreme flow situations to prevent failure or flooding over other structures. Many studies, including those referenced above, have considered aspects of the design and discharge coefficient of particular weir designs, but there is limited data in the literature directly comparing weirs of different slopes under the same experimental conditions.

Experimental Facility

The experimental measurements were conducted in a horizontal (zero slope) research flume with cross section 200 mm wide and 400 mm high; the total length of the flume was 5.4 m. The tailwater was maintained at a low level, which did not affect the oncoming flow for any of the cases presented.

The weir was constructed from three parts: the upstream face, the downstream face, and the rectangular crest. The upstream and downstream faces could be interchanged in order to produce different combinations of upstream and downstream slopes, with a constant crest length. The slopes tested were $2H:1V$, $1H:1V$ and

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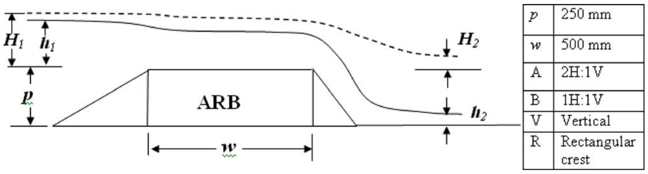

Fig. 1. Weir model component dimensions and naming convention

V (vertical) all with effectively sharp edges at the joins. The dimensions, naming conventions and configurations for the model components, are summarized in Fig. 1.

Volumetric flow rate was measured downstream of the test weir using a 90° V-notch weir to ±0.01 L/s. The surface profile was measured vertically and horizontally to ±0.5 mm using a point gauge on a traverse apparatus. The static pressure on the weir surface was measured at a series of static pressure tappings, spaced between 1.8 and 100 mm apart. The pressure could be measured to a precision of ±0.5 mm of water head.

The approach overflow head, h_1, was varied between 60 and 145 mm (a ratio of overflow to weir height of 0.24–0.58). The lower limit was based on a level below which surface tension effects would become significant; the upper limit on the flume's capacity.

Total head was measured to ±0.5 mm of water head using a pitot tube with an internal diameter of 2 mm. Water temperature was monitored to ±0.1°C with a digital thermometer upstream of the weir. All uncertainties were calculated based on a 95% confidence probability.

Weir scaling was based on Froude number similarity; the small size of the flume precluded both Reynolds and Froude number similarity. The Reynolds number was sufficiently high to have fully turbulent flow, and the Froude number is generally accepted as the key parameter in these studies.

Results

Table 1 outlines the key parameters for the series of test data produced. The range of relative crest lengths, ε, was limited to 0.11–0.23 due to the size of the flume used for testing. The Froude number was calculated at $4h_1$ upstream of the weir's upstream toe.

Discharge Coefficient

The discharge coefficients calculated using Eq. (1) are shown in Fig. 2, with the curve fit for configuration *ARA* proposed by Fritz and Hager (1998) [Eq. (2)] indicated for comparison. Fig. 2 shows that the slope of the upstream face has the most significant impact on the discharge coefficient of the weir. The measurements presented show that the discharge coefficient was identical over the range of flow rates tested for both *BRA* and *BRV* (1:1 upstream slope with $1V{:}2H$ or V downstream slope, respectively). *ARB* had the highest value of C_D, but exhibited some difference from the *ARA* [Eq. (2)] data, demonstrating a lesser dependence of C_D on the downstream slope. The vertical upstream face (*VRB*) gave a 10–15% lower C_D due to the significantly larger separation zone at the upstream face.

Trends lines included in Fig. 2 were calculated using a correlation [Eq. (3)] based on the data collected, which corrects Eq. (2) for the influence of upstream face angle, θ (rad)

$$C_D = 0.43 + 0.06\sin[\pi(\varepsilon - 0.55)] - 0.0396\theta + 0.0029 \quad (3)$$

Table 1. Flow Parameters and Discharge Coefficient for Weir Configurations Tested

Weir	Test number	Q (m^3/s)	h_1 (m)	H_1 (m)	h_2[a] (m)	C_D [Eq. (2)]	ε $H_1/(H_1+w)$	F
ARB	1	0.0181	0.147	0.150	0.038	0.353	0.230	0.12
	2	0.0152	0.130	0.133	0.034	0.356	0.209	0.10
	3	0.0109	0.105	0.106	0.019	0.355	0.175	0.08
	4	0.0079	0.086	0.087	0.017	0.352	0.147	0.07
	5	0.0055	0.065	0.066	0.011	0.366	0.116	0.05
BRA	1	0.0164	0.140	0.142	0.036	0.346	0.221	0.11
	2	0.0133	0.122	0.124	0.029	0.343	0.199	0.09
	3	0.0102	0.103	0.104	0.022	0.344	0.172	0.08
	4	0.0073	0.083	0.084	0.022	0.341	0.144	0.06
	5	0.0048	0.065	0.066	0.012	0.319	0.116	0.04
VRB	1	0.0149	0.140	0.142	0.024	0.314	0.221	0.10
	2	0.0119	0.122	0.123	0.019	0.312	0.198	0.08
	3	0.0091	0.103	0.104	0.015	0.308	0.172	0.07
	4	0.0064	0.082	0.083	0.011	0.307	0.142	0.05
	5	0.0040	0.062	0.062	0.008	0.294	0.111	0.04
BRV	1	0.0164	0.140	0.142	0.035	0.346	0.221	0.11
	2	0.0130	0.121	0.122	0.030	0.342	0.197	0.09
	3	0.0100	0.103	0.104	0.020	0.337	0.172	0.08
	4	0.0071	0.082	0.083	0.016	0.336	0.142	0.06
	5	0.0048	0.065	0.065	0.015	0.323	0.115	0.04

[a]Indicates head measured above base of flume.

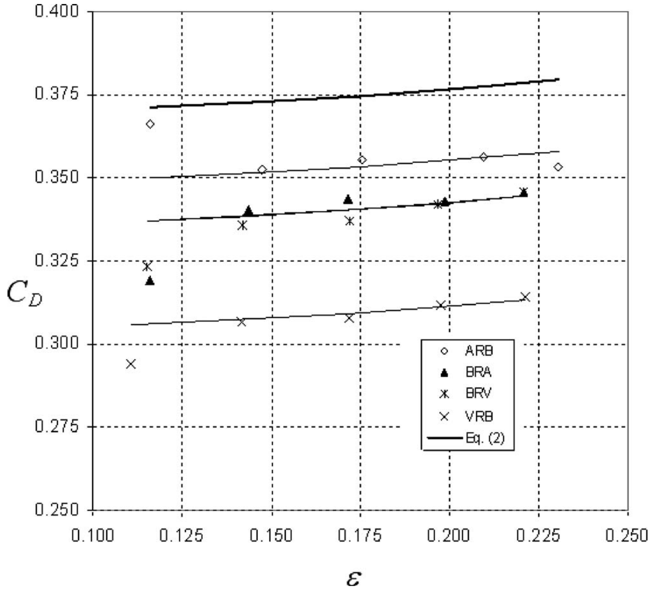

Fig. 2. Discharge coefficient (CD) (ε)

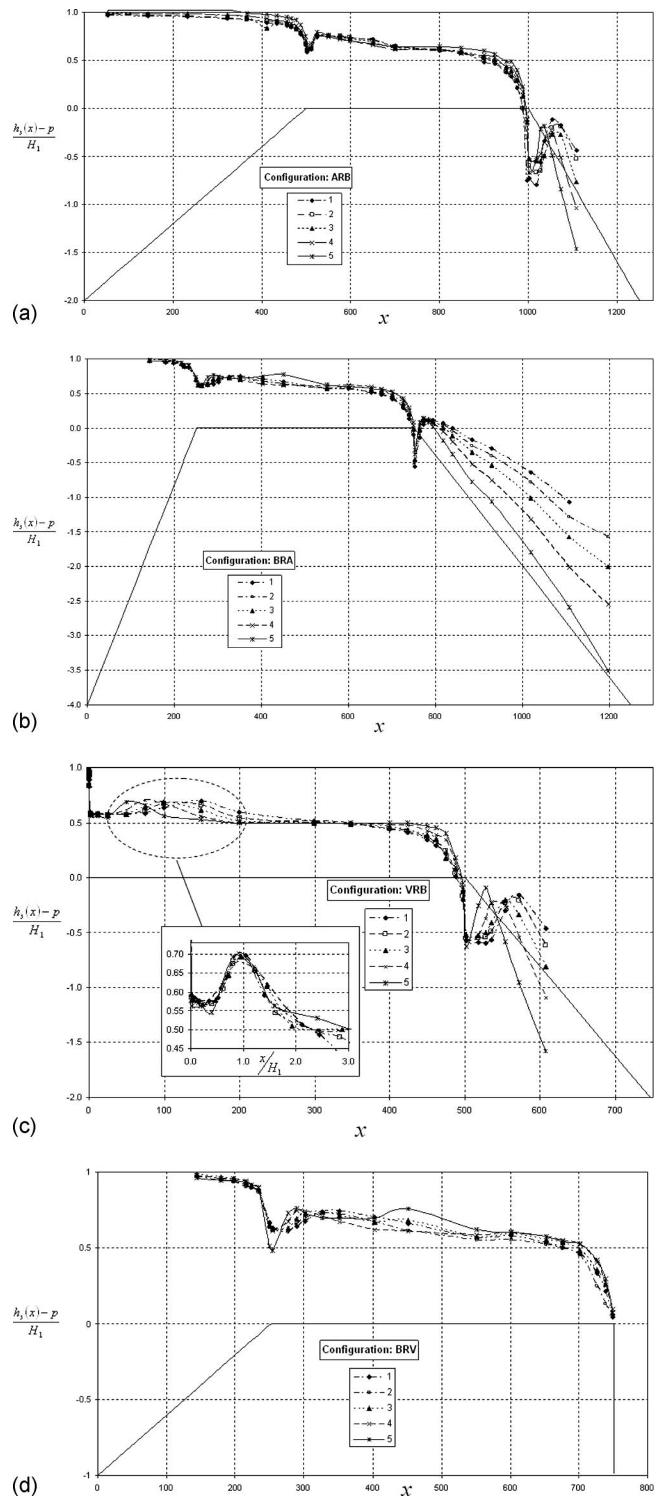

Fig. 3. Dimensionless static pressure distribution for configurations: (a) *ARB*; (b) *BRA*; (c) *VRB*; and (d) *BRV*

Hydraulic Grade Line

The dimensionless static pressure profiles, or hydraulic grade line $[(h_s(x)-p)/H_1]$ in Figs. 3(a–d) have been plotted with ordinate x measured from the upstream toe of the weir. The static pressure, h_s, is measured at pressure tappings located on the surface of the weir, as previously described. The static pressure profiles show self-similarity over the upstream face and crest, but similarity is not maintained for flow over the downstream face. The static pressure exhibits two distinct dips at the upstream and downstream corners of the weir, where the corner is sharp, the radius of curvature is near zero, and hence the curvature is very high and convex. At the upstream corner, the hydraulic grade lines (when plotted in dimensional form) do not cross, and the lowest static head at this location occurs at the lowest flow rate. In the limit of zero flow, the static pressure at this location drops to zero or atmospheric. Conversely, at the downstream corner, the hydraulic grade lines (in dimensional form, h_s) do cross, with the lowest static pressure occurring for the highest flow rate. Hence this could potentially be a site for cavitation to occur if the upstream head, or flowrate, is sufficiently high.

The profiles with a vertical face (*VRB* and *BRV*)—typical of the square-ended broad-crested weirs—exhibit some different features from the sloped-faced weirs *ARB* and *BRA*. The static pressure profile for *VRB* [Fig. 3(c)] exhibits a peak in static pressure as the streamlines become horizontal near the start of the crest. The insert in this figure uses the different horizontal ordinate, x/H_l to demonstrate that both location and magnitude of the static pressure peak are similar when the horizontal ordinate is made dimensionless with the total overflow head H_1. The location and magnitude of this peak at 0.95 H_1 and 0.69 H_1 respectively, are in good agreement with the measurement of the same feature by Hager and Schwalt (1994) for a broad-crested weir with vertical end faces at 1.05 H_1 and 0.73 H_1, respectively.

Profile *BRV* [Fig. 3(d)] exhibits leaping flow at the weir end, with flow separating from the end of the weir. The area under the leaping flow is aspirated and the weir face experiences atmospheric, or zero, static pressure; hence, cavitation would not be an issue for the vertical-ended weir. There may be a critical downstream slope between 1:1 and vertical that allows leaping flow to be spontaneously generated, and cavitation to be prevented. It was found that if a pressure tapping near the upper edge of the 1:1 downstream slope was left open (thus aspirating the flow near the corner) the nappe could be aerated. This forced the flow to leap from the downstream corner. This is similar to the aeration technique now implemented in many modern dams (Fortner 2003).

Free-Surface Profile

The water surface profile was also measured for the configurations previously described. The profiles exhibit many characteristics typical of broad-crested weir flow. These include the following:

1. A reduction in surface height near the upstream corner of the crest;
2. Horizontal streamlines over the weir crest;
3. A reduction in profile near the downstream end of the crest to follow the downstream slope (except for the vertical downstream face which exhibits leaping flow); and
4. Flow that departs the toe of the weir with a low head and high velocity.

The surface profiles were made dimensionless by dividing by the upstream overflow head H_1 The profiles were similar both upstream and over the crest of the weir but exhibited differences over the downstream face.

The lowest flow rate for the V and B configurations exhibited an undular standing wave pattern. This wave flow pattern was not observed for the ARB configuration, with a $2H:1V$ slope on the upstream face. The alternative relative weir length, $\zeta = H_1/w$, for these cases was between 0.12 and 0.13. This is in agreement with Govinda et al. (1963) who found that undular flow tended to occur for flows with $\zeta = 0.1$, where the weir is typically classified as long crested rather than broad crested. The Weber number for these flows was $\mathsf{W} \approx 37 > 1$; therefore, surface tension was not significant and the undular flow was generated by viscosity.

Conclusions

This paper has provided data and some insight into the effect of variable slope on the flow over a broad-crested or embankment weir. The slope of the upstream and downstream faces was varied between vertical and $2H:1V$ slopes to determine the impact on the weir's performance.

The use of a more gradual $(2H:1V)$ slope on the upstream weir face was demonstrated to provide a higher discharge coefficient than a 1:1 or vertical slope. This may be more useful for earth or embankment weirs.

The performance of the weir can be optimized by varying the downstream slope. For example, providing a vertical face downstream will produce leaping flow, preventing cavitation at high flow rates. The use of a vertical instead of a sloped downstream face has been shown to have negligible effect on the discharge coefficient or flow over the weir up to the downstream corner.

Notation

The following symbols are used in this technical note:

b = Weir (flume) width transverse to flow direction (m);
C_D = discharge coefficient;
d = total flow depth above floor of flume (m);
F = Froude number, $F = V/\sqrt{gd}$;
g = acceleration due to gravity (m^2/s);
H_1 = upstream overflow total energy head (m), $H_1 = h_1 + [Q^2/2gb^2(h_1+p)^2]$;
h_s = static, or piezometric pressure head (m);
h_1 = overflow piezometric head upstream of weir (m);
h_2 = piezometric head downstream of weir (relative to base of flume) (m);
p = weir height (m);
Q = flow rate (m^3/s);
V = flow velocity (m/s);
w = length of weir crest (from upstream to downstream corner of weir) (m);
x = horizontal distance from upstream toe of weir (m);
y = vertical height of surface profile above flume base (m);
ε = relative crest length, $\varepsilon = H_1/(H_1+w)$; and
ζ = alternative relative crest length, $\zeta = H_1/w$.

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