# What do they know? A comparison of pre-service teachers' and in-service teachers' decimal mathematical content knowledge

by

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Abstract

The main aim of this paper is to report on an investigation into primary pre-service and in-service teachers' content knowledge of decimals. The participants were asked to complete four decimal tasks including the ordering of decimals, operating with decimals and converting a fraction to a decimal. The results from both sets of participants are compared and incorrect answers are described and discussed. The findings indicated a reliance on formal procedures and that incorrect responses indicated a fundamental lack of understanding of place value and the presence of misconceptions that are commonly held by their student counterparts. The study contributes to the limited field of research that uses comparative studies to look specifically at pre-service and in-service teachers' knowledge of decimals.

## Introduction

It seems reasonable to suggest that effective mathematics teachers possess a sound understanding of the mathematics they teach, and teachers' mathematical knowledge has continued to be a much-discussed issue in contemporary debates about improving mathematics teaching and learning (Ball, Lubienski & Mewborn, 2001). While it is generally accepted that teacher knowledge encompasses more than just content, it is this subject matter knowledge that impacts upon other types of knowledge as identified by researchers such as Shulman (1987) and Ball and colleagues (e.g., Ball, Thames & Phelps, 2008). Specifically, this paper uses the responses obtained from a selection of primary pre-service teachers and in-service teachers to highlight the common issues from the two groups in terms of what their responses revealed about their ability to order decimals and carry out operations with decimals.

## Review of the literature

### Mathematical content knowledge

The seminal work of Shulman (1987) and his colleagues highlighted the importance of considering the different types of knowledge required for teaching. Shulman identified that a teacher's knowledge base was comprised of seven different aspects, including pedagogical content knowledge (PCK) which he defined as "an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8) and content knowledge. Content knowledge, on the other hand, is concerned with expertise in the particular discipline being taught and it is this mathematical content knowledge (MCK) that is the focus of this paper.

According to Ma (1999) teachers are expected to understand what they teach, and require a certain depth of understanding in order to provide sound explanations of mathematical ideas (Ma, 1999). She used the term Profound Understanding of Fundamental Mathematics (PUFM) to refer to MCK which has breadth, depth, connectedness and thoroughness. Schoenfeld and Kilpatrick (2008) also referred to the importance of teachers knowing school mathematics in depth and breadth, with the general consensus being that this knowledge in turn impacts upon PCK and therefore upon the effectiveness of instruction. The literature reveals, however, that many elementary teachers lack conceptual understanding of mathematics (e.g., Mewborn, 2001), and that both in-service and pre-service teachers' limited mathematical content knowledge and confidence with doing mathematics is of particular concern (e.g., Ball, 1990; Lange & Meaney, 2011; Ryan & Williams, 2007).

Ball, et al. (2008) building on the work of Shulman, (1987) proposed a model for distinguishing the different types of knowledge required for teaching (see Figure 1). Of particular relevance to this paper is the first domain, Common Content Knowledge (CCK) which is defined as "the mathematical knowledge and skill used in settings other than teaching" (p. 399). This knowledge, which is considered to include the mathematics knowledge and skills that others would know, is not 'unique' to teaching as such, but essential for enabling teachers to know the material they teach, recognise when students give wrong answers or for recognising when the textbook gives an inaccurate definition. Ball et al. (2008) found that this knowledge was critical in terms of planning and carrying out instruction. While the other domains are not discussed here, it is important to acknowledge that all contribute to effective teaching, and that general mathematical ability does not fully account for the knowledge and skills in teaching mathematics (Hill, Ball & Schilling, 2004).

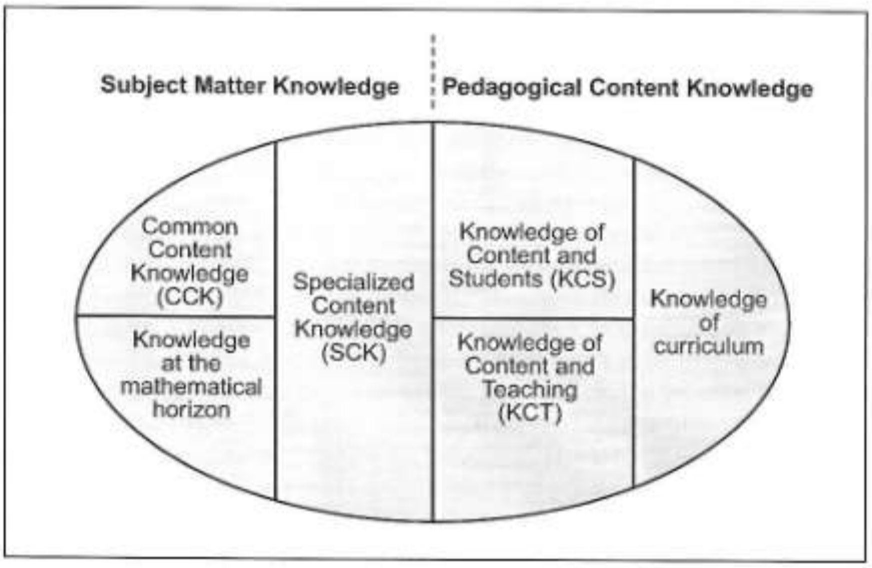


*Figure 1.* Domain map for mathematical knowledge for teaching.

Another framework derived from Shulman's (1987) work was developed by Chick, Pham, Baker and Cheng (2006) and was divided into three categories: *Clearly PCK, Content Knowledge in a Pedagogical Context and Pedagogical Knowledge in a Content Context*. The second category summarised and focused on mathematics content knowledge in a mathematics teaching context, listing five descriptors of MCK teachers should demonstrate (see Table 1). 'Mathematical structure and connections', for example, refers to the teacher making connections between concepts and topics, while 'Profound understanding of fundamental mathematics' links with Ma's (1999) study and refers to the teacher having a thorough conceptual understanding of identified aspects of mathematics. Chick and colleagues used their framework to analyse data collected from teachers about teaching and content, using observations, discussions and interviews (e.g., Baker & Chick, 2006). Due to the nature of the data collected in this paper, the categories of 'procedural knowledge' and 'methods of solution' proved useful in the analysis. In-depth interviews or observation may have gathered additional data for coding correct responses using the four remaining PCK categories.

Table 1. *Content Knowledge in a Pedagogical Context (Chick, Baker, Pham, & Cheng, 2006 p. 299)*

| **PCK Category** | **Evident when the teacher…** |
|---|---|
| *Content Knowledge in a Pedagogical Context* | |
| Profound Understanding of Fundamental Mathematics | Exhibits deep and thorough conceptual understanding of identified aspects of mathematics |
| Deconstructing Content to Key Components | Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept |
| Mathematical Structure and Connections | Makes connections between concepts and topics, including interdependence of concepts |
| Procedural Knowledge | Displays skills for solving mathematical problems (conceptual understanding need not be evident) |
| Methods of Solution | Demonstrates a method for solving a mathematical problem |

## Difficulties with decimals

The concept of decimals is recognised as a significant source of learning difficulties with students and pre-service teachers (Stacey et al., 2001; Steinle & Stacey, 1998; Ubuz & Yayan, 2010) and it is reasonable to suggest that similar issues exist for in-service teachers. In one of a few studies that have documented in-service teachers' subject matter of decimals, Ubuz and Yayan (2010) found that primary teachers experienced difficulties in scale reading, operating with decimals and ordering decimals. Stacey and colleagues' research has focused on students and pre-service teachers, with findings indicating that similar misconceptions occur in both populations.

Steinle and Stacey (1998) identified that there were a number of ways to describe incorrect thinking about decimal notation. Some students generally believed that a longer decimal is larger than a shorter decimal, and would select, for example, 4.63 as being larger than 4.8 because it is longer. This is a common misconception, particularly in younger children, and involves an over-generalisation of whole number thinking, whereby 463 would be larger than

48. Another misconception, termed ‘shorter-is-larger’ refers to the generalisation that any number of tenths is greater than any number of hundredths because one tenth is larger than one hundredth (denominator thinking). The shorter-is-larger category also includes reciprocal thinking, whereby inappropriate conclusions are made such as 0.3 is larger than 0.4 because $\frac{1}{3}$ is larger than $\frac{1}{4}$, and negative thinking where decimals are associated with negative numbers, and 0.6 may be seen as smaller than 0 in the same way that negative 6 is smaller than 0 (Steinle & Stacey, 1998). Studies have found that ‘shorter-is-larger’ misconceptions can persist through to high school, with an average of 11% of Year 10 students demonstrating one or more misconceptions of this kind (Steinle & Stacey, 1998), indicating that this belief may well continue into adulthood. A recent study by Vamvakoussi, Van Dooren and Verschaffel (2012) confirmed this, with a likely explanation being attributable to an intrusion of knowledge about fractions. Steinle and Stacey (1998) also identified instances of ‘truncated thinking’ and ‘rounding thinking’ in which the contexts of money or length were used in order to make sense of decimal notation. The limitations of these approaches include confusion when numbers involve more than two decimal places and the application of a ‘rule’ with little understanding of why it works.

Steinle and Stacey’s (1998) identification of incidences of incorrect decimal thinking was largely derived from a comparison test in which students were required to name the larger number. Similar tests have been used, with similar results obtained, by other researchers (e.g, Roche, 2005), indicating that the types of incorrect thinking are consistent across a range of age groups and contexts. In their work with primary teachers, Ubuz and Yayan (2010) found examples of ‘shorter-is-larger’ thinking, with 11% of the teachers considering that 0.1 was larger than both 0.23 and 0.07, and 32% incorrectly ordering the decimals as 0.248, 0.85, 0.63 and 0.4 from the smallest to the biggest. Similarly, Maher & Muir (2011) found evidence of ‘shorter-is-larger’ thinking in a selection of pre-service teachers, with some identifying 8.245 as being larger than 8.24563 and 0.3 as being larger than 0.426. Their study, which involved a mathematical items test and one-on-one interviews, found that the pre-service teachers primarily relied on procedural approaches to solve problems and that the domain of decimals proved especially challenging for them.

Other research has also identified difficulties when operating with decimals (e.g., Bell, Fischbein & Greer, 1984; Irwin, 2001; Ryan & Williams, 2007; Okazaki & Koyama, 2005) and converting decimal fractions (Ryan & Williams, 2007). Ryan and Williams (2007) found, for example, that 24% of a large cohort of pre-service teachers could not correctly write 912 + 4/100 in decimal form, with the most common error being 912.004. Other evidences of difficulties included 9% who identified 0.3 as being one-third, 8% who thought that 0.125 was 1/125 and 31% who could not give the correct answer to 300.62 divided by 100 (Ryan & Williams, 2007). Common incorrect responses to ‘0.3 x 0.24’ included 0.72 and 7.2, indicating “a procedural bug in attach-and-reattach decimal points” (Ryan & Williams, 2007, p. 142) and/or the tendency to treat decimal numbers as whole numbers. Irwin (2001) also found that a common misconception was to think of decimals as a ‘decorative dot’ and that “when you do something to one side of the dot, you also do it to another” (p. 402).

## Methodology

This study used quantitative methods to analyse a selection of primary pre-service teachers and in-service teachers' responses to four decimal items in a Mathematical Competency, Skills and Knowledge Test.

During the first-year of their course, 279 primary pre-service teachers completed a Mathematical Competency, Skills and Knowledge Test as part of a hurdle requirement for their four-year Bachelor of Education Course. This test assessed pre-service teachers' MCK of number, fractions, decimals, percentage, ratio, space, area, volume, measurement, statistics and probability. It consisted of 49 items. All items required short answers using words or symbols (numbers) and working out was encouraged. No calculators were permitted and pre-service teachers were given two hours to complete the test.

Four test items relating to knowledge of decimals were selected in order to identify pre-service teachers' understanding, errors and misconceptions of this topic. For this paper a sample of 28% of the total cohort of pre-service teachers' responses (N=279) was randomly selected. This sample size was determined after a total of 80 (28%) responses had been tallied and the answers analysed showed a likely pattern of responses had emerged. It was originally intended to select the same number of in-service teachers to complete the test and 80 was a more realistic sample size for this group of participants (in the end, a total of 58 in-service teachers volunteered to participate). The decimal items included questions that were deemed appropriate for students from Years 5-8. Table 2 lists the four decimal items and matches them to the sub-strands of Number and Algebra within the Australian Curriculum: Mathematics V3.0. (ACARA, 2012).

For Item 30 the pre-service teachers had to order four decimal fractions from least value to greatest. The correct order was 3.03, 3.033, 3.303, 3.33. This is a relevant understanding at Year 5 as students are expected to order decimals, however the Australian curriculum does not state to how many decimal places students should know this. Of the four items this should have been the easiest one to answer correctly.

Item 34 is expected knowledge at Year 6. At this level students make connections between equivalent fractions and decimals. To solve this item the pre-service teachers needed to write 3/7 as a decimal number, recording four decimal places. The response to five decimal places is 0.42857 therefore the 57 thousandths needs to be rounded to 60 thousandths. The correct response to four decimal places is 0.4286.

Item 32 and Item 33 would be considered the most difficult items as this knowledge is targeted at students in Year 7. For Item 32 the pre-service teachers had to place the decimal point in the correct position for 12.68 multiplied by 0.9. The correct response was 19.654. This is expected understanding at Year 7 as students multiply decimals using effective written strategies. For Item 33 the pre-service teachers had to calculate the value of 6.3 divided by 0.9. The correct response was 5.67. At Year 7 students are expected to divide decimals using effective written strategies.

Table 2. *Decimal test items and sub-strands of Number and Algebra (ACARA,2012).*

| Item number | Item as appeared on test | Australian Curriculum. Number and Algebra, Sub strand |
|---|---|---|
| 30 | **Ordering of decimals** List these decimal fractions in order from least value to greatest value 3.303,3.03,3.33,3.033 | Compare, order and represent decimals (Year 5). |
| 32 | **Multiplying decimals** The decimal point on the calculator is not working. It shows the product of 12.68 x 1.55 is 19654. Show what the correct answer should be. | Multiply and divide fractions and decimals using effective written strategies and digital technologies (Year 7) |
| 33 | **Division of decimals** Find the value of 6.3 divided by 0.9 | Multiply and divide fractions and decimals using effective written strategies and digital technologies (Year 7) |
| 34 | **Converting a fraction to a decimal** Write 3/7 as a decimal. Show four decimal places | Make connections between equivalent fractions, decimals and percentages (Year 6) |

The same four items (Table 2) formed part of a similar test that was administered to a selection of primary school teachers. Convenience sampling was used to invite the teachers from two large primary schools to participate. It was anticipated that this would provide a cross-section of ages and years of teaching experiences. The teachers' test also included seven measurement and chance and data items and seven decimal items, with the intention that it would take the teachers only 20-30 minutes to complete. The tests were administered to the teachers who agreed to participate as part of an after school staff meeting. The teachers were asked to complete the tests individually, but it is unlikely that they would have had the same test conditions as the pre-service teachers. A spread sheet was used to record all responses, including incorrect responses. Frequency counts were then used to tally correct responses and common incorrect responses. These are presented in Tables 3, 4, 5, 6 and 7.

## Results and Discussion

Table 3 shows the number of correct responses received for all four decimal items from both cohorts of participants. Only 9% of pre-service teachers and 5% of in-service teachers correctly answered four items. Tables 4, 5, 6 and 7 show the common incorrect responses received and the relative percentages for each incorrect answer (‘n’ refers to the number of incorrect responses for that particular item and cohort). The results for Table 3 were analysed to identify content knowledge using the PCK framework of Chick, et al., (2006).

Table 3. Correct response rate for test items

| Item as appeared on test | % correct pre-service teachers (n=80) | % correct in-service teachers (n=58) |
|---|---|---|
| **Ordering of decimals**<br>List these decimal fractions in order from least value to greatest value<br>3.303, 3.03, 3.33, 3.033 | 60 (75%) | 29 (50%) |
| **Multiplying decimals**<br>The decimal point on the calculator is not working. It shows the product of 12.68 x 1.55 is 19654. Show what the correct answer should be. | 50 (63%) | 36 (62%) |
| **Division of decimals**<br>Find the value of 6.3 divided by 0.9 | 41 (51%) | 30 (52%) |
| **Converting a fraction to a decimal**<br>Write 3/7 as a decimal. Show four decimal places | 15 (19%) | 6 (10%) |

It should be noted that before completing the four decimal items the pre-service teachers had the opportunity to complete practice Mathematical, Skills and Knowledge Tests as well as revise their MCK as preparation for the assessment task in general. The pre-service teachers were required to pass their Mathematical Skills and Knowledge Test as part of their course requirements. The teachers completed the task without additional consequences. This may have influenced the results as pre-service teachers may have performed better when comparing the results of both cohorts. On the other hand because the teachers may be teaching this topic as part of their numeracy lessons with students it should be expected that they have this knowledge and may have completed a mathematical assessment test as part of their teacher education requirements recently or many years ago.

## Ordering of decimals

Table 4. Common incorrect responses and relative percentages for item 1 (ordering of decimals)

| Incorrect response | % pre-service teachers (n=19) | % in-service teachers (n=29) |
|---|---|---|
| 3.033, 3.03, 3.303, 3.33 | 4 (21%) | 11 (38%) |
| 3.03, 3.33, 3.303, 3.033<br>Or<br>3.03, 3.33, 3.033, 3.303<br>(2 decimal places, then 3 decimal places) | 2 (11%) | 5 (17%) |
| 3.03 as largest | 5 (26%) | 3 (10%) |
| Various other incorrect order | 7 (37%) | 10 (34%) |
| No response recorded | 1(5%) | 0 (0%) |

Table 4 shows the number and percentage of incorrect responses received for ordering decimals from both cohorts of participants (Item 1). The correct order from least to greatest value was 3.03, 3.033, 3.303, 3.33, and Table 3 shows that 76% of pre-service teachers and 50% of in-service teachers recorded the correct answer. This proved to be the easiest item for the pre-service teachers to answer. The pre-service teachers would have been exposed to this during their course tutorials, which may have accounted for the difference (26%) between both cohorts. Some of the teachers may have only taught in the early years (Prep to Year 4) and therefore may have forgotten the MCK for ordering decimal fractions or did not know how to answer this problem.

Some of the pre-service teachers (11%) and more teachers (17%) had difficulties similar to students. They ordered the decimals according to 2 decimal places then 3 decimal places. This is a common error with students who select the number with more digits after the decimal as larger and is described as an incorrect application of whole-number ideas (Van de Walle, Karp, & Bay-Williams, 2013).

Both cohorts provided a range of incorrect responses demonstrating a lack of understanding of place value and ordering decimal numbers. According to Booker, Bond, Briggs, Sparrow and Swan (2010), likely difficulties occur in decimals when they are spelled out rather than read using place value (e.g., 3.03 is not “three point zero three” it is three and three hundredths”); exposure to this type of teaching may have contributed to the errors involving ‘various other incorrect order’.

### Multiplying decimals

Table 5. Common incorrect responses and relative percentages for item 2 (calculator place value)

| Incorrect response | % pre-service teachers (n=30) | % in-service teachers (n=22) |
|---|---|---|
| 196.54 | 6 (20%) | 7 (32%) |
| 1.9654 | 12 (40%) | 5 (23%) |
| 1965.4 | 6 (20%) | 3 (13%) |
| Other | 3 (10%) | 2 (9%) |
| No response | 3 (10%) | 5 (23%) |

The results show that the most common incorrect responses were 1.9654 for the pre-service teachers and 196.54 for the in-service teachers. The correct answer was 19.654 and as Table 3 shows, 63% of pre-service teachers and 62% of in-service teachers recorded the correct answer. It is likely that the 1.9654 response was derived from incorrectly applying a rule that 'moves' the decimal point in the answer to correspond with the numbers appearing after the decimal point in the multipliers. This rule corresponds, however, with the result obtained from doing a traditional long multiplication algorithm, which would have given the answer as 196540, and subsequently that approach would have resulted in the correct answer. As the given answer as displayed on the calculator did not include the zero then the respondents appeared to simply apply the rule, without considering the reasonableness of their answer. While 23% of the in-service teachers' incorrect responses showed that they also thought the answer was 1.9654, more (32%) thought the answer was 196.54. Again, this appears to be a place-value related error, and probably the result of over-generalising the 'rule' for adding and subtracting decimals, maintaining the two numbers after the decimal point in the answer, as consistent with the two numbers after the decimal point in the multipliers. It is interesting to note the differences between the two groups; it may be that the pre-service teachers may have been more likely to apply the rule for 'moving' the decimal point, due to recent schooling experiences. The in-service teachers, however, varied both in terms of years since formal schooling, and opportunity to teach operations with decimals, which may have accounted for the variation in their incorrect responses.

A total of 33% of incorrect answers indicated that the calculator would display 1965.4. This error seems to have been derived from ignoring the decimal point in either the first or second multiplier, and multiplying 1268 by 1.55 or 12.68 by 155 to obtain 1965.4. There were few 'other' responses received, with most being attributable to recording digits in the wrong order (e.g., 19.564, rather than 19.654).

The results share similarities with the results obtained for the pre-service teachers in Ryan and Williams' (2007) study. Both cohorts of participants showed a tendency to 'attach-and-reattach' decimal points, with the procedure even indicating a belief in the 'multiplication makes bigger' conception. The results also show a lack of conceptual understanding and number sense[1] (McIntosh, Reys, Reys, & Hope, 1997) about the place value of decimals, with given answers not evaluated for reasonableness and little evidence of estimating the answer as being approximately between 12 x 1 and 12 x 2.

### Division of decimals

Table 6. Common incorrect responses and relative percentages for item 3 (6.3 divided by 0.9)

| Incorrect response | % pre-service teachers (n=39) | % in-service teachers (n=28) |
|---|---|---|
| 0.7 | 6 (15%) | 11 (39%) |
| 0.07 | 4 (10%) | 4 (14%) |
| Various other | 16 (42%) | 3 (11%) |
| Blank | 13 (33%) | 10 (36%) |

The correct response to this item was 7 and as Table 3 shows, approximately 50% of both groups of participants recorded a correct response. Table 6 shows that common incorrect responses included 0.7 and 0.07. There were few written recordings of 'working out' on the test sheets with most participants simply recording an answer. The few examples that were recorded showed a tendency to use a division algorithm, sometimes accompanied by 'checking' through a multiplication algorithm; these examples all resulted in a correct answer. It is likely that the 0.7 response was a result of maintaining one-digit after the decimal point, in much the same way that 196.54 was recorded for item 2. This response was more common for the in-service teachers. A similar number of participants from both groups recorded 0.07 as the answer, which could indicate a strategy of 'attach-and-reattach' the decimal point. While a similar item used in Ryan and Williams (2007) study involved zeros (300.62 divided by 100), they found that 31% of pre-service teachers could not provide a correct answer, and that a common response was '3.62', indicating that the 'whole' and 'decimal' parts were treated as separate entities, rather than as parts of the one number. As with item 2, there was little evidence that either cohort used number sense or estimation to gauge the reasonableness of their answers.

### Converting a fraction to a decimal

Table 7. Common incorrect responses and relative percentages for item 4 (3 divided by 7)

| Incorrect response | % pre-service teachers (n=65) | % in-service teachers (n=52) |
|---|---|---|
| 0.4285 | 10 (15%) | 9 (17%) |
| 0.2333 | 3 (5%) | 1 (2%) |
| 0.4298/0.4299 | 5 (7%) | 5 (10%) |
| Whole number prefix (e.g., 42.09) | 7 (12%) | 8 (16%) |
| 2.333 (thinking 7/3) | 6 (9%) | 0 (0%) |
| Various | 19(29%) | 9 (17%) |
| Blank | 15 (23%) | 20 (38%) |

[1] Number sense in this context refers to a person's understanding of number concepts, operations, and applications of numbers and operations; it includes a person's inclination to use numbers in flexible ways and to assess the reasonableness of answers (McIntosh, Reys, Reys, & Hope, 1997).

For Item 4 the correct answer was 0.4286 and as Table 3 shows, 19% of pre-service teachers and 10% of in-service teachers recorded the correct answer. This was the most difficult item for both cohorts. This problem was a multi-step problem. To solve the answer correctly the first step was to complete the division of $\frac{3}{7}$. The next step was to round the decimal fraction 0.42857 (recurring) correctly to four decimal places changing 57 one-hundred thousandths to 6 ten thousandths.

One pre-service teacher used a rather unusual method to calculate the correct response for Item 4. Her solution focused on the multiplicative relationship, or unitary method, that would probably have been taught to her in secondary school (see Figure 2). As shown in Figure 2, she divided 100 by seven to find one seventh of one hundred (percent), then multiplied the answer by three to find three sevenths of one hundred (percent). This answer, 42.8571 would have then been divided by 100 to complete the steps and calculate the correct response, 0.428571, and then rounded to four decimal places, 0.4286. This is an example of Method of Solution (Chick, et al., 2006) as she demonstrated a method for solving a mathematical problem.

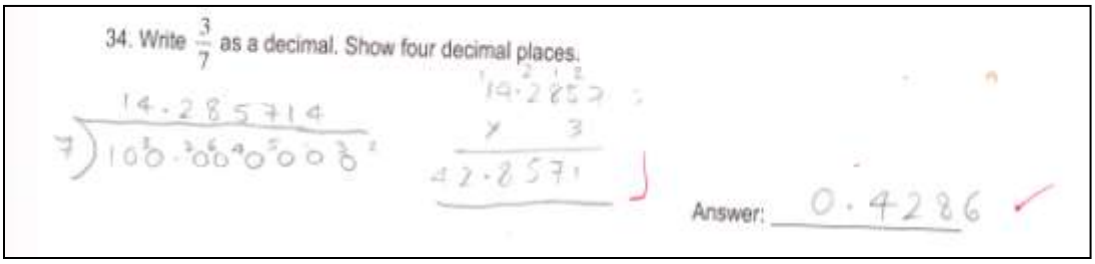


*Figure 2. Item 4 example of pre-service teachers' correct response using unitary method.*

Table 7 shows that the most common incorrect response for Item 4 was 0.4285 for both the pre-service teachers (15%) and the in-service teachers (17%). This error, 0.4285 may have resulted from not completing the rounding process or perhaps because the respondents did not know how to rename the decimal fraction to four decimal places. For another problem on the same test, the pre-service teachers had difficulty with a ratio scale item, with the results indicating that their answer was incomplete, rather than incorrect (Livy & Vale, 2011).

Six per cent of pre-service teachers divided 7 by 3, rather than 3 by 7, and recorded 2.333 as the answer. The responses indicate a lack of number sense and, as mentioned previously, no attempt to gauge the reasonableness of their answers. No in-service teachers provided the same incorrect response.

### Summary of results

The findings from the study show that the selected group of primary pre-service teachers and in-service teachers shared many similarities when it came to assessing their decimal knowledge, particularly in operating with decimals. Although more pre-service teachers were able to correctly order the decimals in Item 1, an analysis of incorrect responses across both cohorts showed the occurrence of similar errors. Patterns of error were also consistent with findings from the literature and showed both longer-is-larger and shorter-is-larger misconceptions (Steinle & Stacey, 1998). Similar numbers of pre-service teachers and in-service teachers (63% and 62% respectively) correctly found the answer when multiplying decimals, with the error patterns showing a tendency to incorrectly apply a rule, and with no 'checking' on the reasonableness of the answer. Division of decimals proved to be quite difficult for both pre-service and in-service teachers, with only 51% and 52% respectively obtaining the correct result. Again, errors indicated a reliance on rules, with the strategy of 'attach-and-reattach' the decimal point (Ryan & Williams, 2007) being a common approach. The most difficult item for both sets of participants was the final item whereby they were asked to write $\frac{3}{7}$ as a decimal. It seems that apart from 9% of pre-service teachers who incorrectly divided seven by three, most participants made errors that showed a limited understanding of 'rounding' conventions, ignoring the need to find the answer to five decimal places, then rounding appropriately to record the four decimal places. In terms of analysing the results with reference to Chick et al.'s (2006) framework, the responses showed evidence of procedural knowledge and demonstration of methods for solving mathematical problems. Although participants were encouraged to document any 'working out' used for solving the problems, many just provided an answer, and as previously explained, no additional verbal clarification was sought, which could be seen as a limitation to the study.

## Conclusions and implications

Ma's (1999) seminal work highlighted concerns about some aspects of practicing teachers' content knowledge, and results from this study confirm that these concerns extend to knowledge of decimals. While further investigation did not occur into how this limited knowledge extended into practice, other studies have shown that limited decimal knowledge does affect pre-service teachers' abilities to identify errors in students' thinking and apply appropriate teaching approaches (e.g., Maher & Muir, 2011). With regard to the in-service teachers, the tests were anonymous and they did not receive feedback on their individual performance. They were given a copy of the correct responses, however, and many verbally expressed admission that their fraction and decimal knowledge was not strong. This is perhaps not surprising, as Ubuz and Yayan (2010) observed, many teachers study decimals as students themselves, then typically encounter decimals concepts only once more in their teacher education program before they are certified to teach. According to Anstey and Clarke (2010), teachers are more likely to want to learn about mathematical content if they are made aware of gaps in their current knowledge. It seems prudent, therefore, to provide teachers with the opportunity to confront their content knowledge in areas such as decimals, regardless of the grade level taught, in order to help them identify what they do not know, and then develop goals to further develop this knowledge. Similarly, Maher and Muir (2011) found that many of the pre-service teachers in their study were unaware that they had developed flawed understandings or misconceptions about decimals and were therefore unlikely to address this, unless these misconceptions were explicitly uncovered and addressed. Widjaja, Stacey and Steinle (2008) also highlight the danger of misconceptions being 'covered over rather than overcome' (p.1), while others have found that success with procedural fluency can sometimes hide underlying misconceptions (e.g., Ball, et al., 2008; Graeber & Tirosh, 1988, as cited in Okazaki & Koyama, 2005; Ryan & Williams, 2007).

One of the findings from the study suggested that both pre-service teachers and in-service teachers provided responses that showed a lack of number sense, particularly in terms of place value and the relative magnitude of numbers. The results indicated that, similar to the students in Irwin's (2001) study, both groups of participants showed an inclination to deal with fractions in a manner that suggested that they did not see decimals as having a meaning that might relate to size or quantity. According to Irwin (2001), strategies such as teaching decimals in context and incorporating cognitive conflict have been successful in addressing this tendency. For example, students who believed that one hundredth was written as 0.100 or that ¼ was equivalent to 0.4, were often taught knowledge out of context which did not encourage reflection on the incompatibility of these notions with principles such as place value. It may be necessary, therefore, to provide situations for pre-service and in-service teachers whereby they are given opportunities to engage in cognitive conflict, focusing on some of common decimal misconceptions. The cognitive conflict approach was found to be successful in assisting students' conceptual understanding of division of decimals (Okasaki & Koyama, 2005) and with addressing pre-service teachers' misconception that 'division always makes smaller' (Tirosh & Graber, 1990, as cited in Ubuz & Yayan, 2010). Ubuz and Yayan (2010) describe position-driven group discussion as being conducive to assisting in-service teachers to gain more robust understandings of decimals. Through providing them with discussion starters, such as 'can any fraction be turned into a decimal?' and 'can any decimal be converted to a fraction?' participants can be encouraged to grapple with these concepts and address their own underlying misconceptions.

Ball et al. (2008) point out that along with having good general mathematical ability, there are other skills and knowledge that are entailed in teaching mathematics. Many of the pre-service and in-service teachers who answered the decimal items in the tests considered themselves to be specialising or teaching in early childhood classes and may not have seen the need to possess such decimal knowledge or have had the opportunity to teach decimals to their students. This understandably may have contributed to the number of incorrect responses received. Correct answers and fluency with procedures, however, while necessary, are not sufficient for teaching (Ball et al., 2008). Ball et al., use the example of the subtraction algorithm to point out that teachers who teach subtraction must be able to perform the calculation themselves, but also need to know much more in order to be able to diagnose student errors and explain the meaning behind the algorithm. For the pre-service and in-service teachers in this study, the basis for future instruction would be shaky as many demonstrated they lacked the basic knowledge of being able to successfully carry out the algorithm required for the operation with decimals tasks.

Booker et al., (2010) suggested that some difficulties with decimals could be attributed to teaching methods that 'spell out' decimals rather than read them using place value. While the majority of pre-service teachers successfully ordered the four decimals in item 1, only 50% of in-service teachers could do this. As the in-service teachers varied in their years of teaching experience, with some having more than 20 years experience, it seems that, as Ubuz and Yayan, (2010) found, years of teaching did not seem a factor in improving decimal knowledge. Continuous professional learning therefore must be provided to address this. We would advocate the provision of focused professional learning that addresses their likely misconceptions and assists with overcoming them, such as providing opportunities for cognitive conflict as discussed earlier. Provision of manipulatives and pedagogical approaches that are used with students would also be appropriate. To assist with understanding place value, for example, a decimal number expander may be used to name and rename different decimal fractions. For example these decimal numbers can all be renamed as ten thousandths and ordered from least to greatest value: 3030 ten thousandths (3.03); 3033 ten thousandths (3.033); 3303 ten thousandths (3.303); 3330 ten thousandths (3.33).

Although a substantial body of research exists on the subject matter of students and pre-service teachers in the domain of decimals, studies investigating the subject matter knowledge of in-service teachers is not as prevalent (Ubuz & Yayan, 2010). It is hoped that the study reported in this paper contributes to the field of knowledge in this area and highlights the need to establish what knowledge is held and what common misconceptions are present. Once these aspects have been identified, then teaching and professional learning can occur to address these aspects. Future studies could involve the use of cognitive conflict situations to identify misconceptions, analyse contributing factors and devise appropriate teaching approaches. Our findings suggest that both pre-service teachers and in-service teachers possess similar decimal misconceptions as those held by their student counterparts, indicating that immediate action is required to avoid this continuing into the next generation of learners.

## References


ACARA. (2012). The Australian curriculum: Mathematics V3.0. Available: http://www.acara.edu.au/curriculum/curriculum.html

Anstey, L., & Clarke, B. (2010). Perceived professional learning needs of numeracy coaches. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia, Vol. 1* (pp. 45-52). Fremantle: MERGA.

Baker, M. & Chick, H. L. (2006). Pedagogical content knowledge for teaching mathematics: A case study of two teachers. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, Cultures and Learning Spaces* (Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia, pp. 60-67). Sydney: MERGA.

Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education, 21*(2), 132-144.

Ball, D., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed.), New York: Macmillan.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

Bell, A., Fischbein, E., & Greer, B. (1984). Choice of operation in verbal arithmetic problems: The effects of number size, problem structure and context. *Educational Studies in Mathematics, 15*(1984), 129-147.

Booker, G., Bond, D., Briggs, J., Sparrow, L., & Swan, P. (2010). *Teaching primary mathematics* (fourth ed.). Frenchs Forest, NSW: Pearson.

Chick, H., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical knowledge for decimals. In J. Novotna, H.Moranova, M. Kraka & N. Stehlikova (Eds.), *In Proceedings of the 30th annual conference of the International Group for the Psychology of Mathematics Education.* Prague: PME.

Hill, H., Ball, D., & Schilling, S. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105*(1), 11-30.

Irwin, K. (2001). Using everyday knowledge of decimals to enhance understanding. *Journal for Research in Mathematics Education, 32*(4), 399-420.

Lange, T., & Meaney, T. (2011). Pre-service teachers learning mathematics from the internet. In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds.), *Mathematics: Traditions and [new] practices* (Proceedings of the AAMT-MERGA Conference, pp. 438-445). Adelaide, SA: MERGA.

Livy, S., & Vale, C. (2011). First year pre-service teachers' mathematical content knowledge: methods of solution to a ratio question. *Mathematics Teacher Education and Development*, 13, (2), 22.

Ma, L. (1999). *Knowing and teaching elementary mathematics. Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Maher, N., & Muir, T. (2011). I think they think that zero point something is less than zero: Investigating pre-service teachers' responses to mathematical tasks. Proceedings of the annual Australian Association for Research in Education Conference, 27 November - 1 December 2011, Hobart, Tasmania, pp. 1-16.

McIntosh, A., Reys, B., Reys, R., & Hope, J. (1997). Number sense grades 4-6. Palo Alto, CA.: Dale Seymour Publications.

Mewborn, D. (2001). Teachers content knowledge, teacher education, and their effects on the preparation of elementary teachers in the United States. *Mathematics Teacher Education and Development* 3, 28-36.

Okazaki, M., & Koyama, M. (2005). Characteristics of 5th graders' logical development through learning division with decimals. *Educational Studies in Mathematics*, 60, 217-251.

Roche, A. (2005). Longer is larger or is it? *APMC 10*(3),11-16.

Ryan, J. & Williams, J. (2007). *Children's mathematics 4-15: Learning from errors and misconceptions.* New York. NY: Open University Press.

Schoenfeld, A. H., & Kilpatrick, J. (2008). Toward a theory of proficiency in teaching mathematics teaching. Reflection of practice with the knowledge quartet. London: SAGE Publications.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Stacey, K., Helme, S., Steinle, V., Baturo, A., Irwin, K. & Bana, J. (2001). Preservice teachers' knowledge of difficulties in decimal numeration. *Journal of Mathematics Teacher Education, 4,* 205-225.

Steinle, V. & Stacey, K. (1998). The incidence of misconceptions of decimal notation amongst students in Grades 5 to 10. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching Mathematics in New Times* (Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, pp. 548-555). Sydney: MERGA.

Ubuz, B., & Yayan, B. (2010). Primary teachers' subject matter knowledge: Decimals.*International Journal of Mathematics Education in Science and Technology 41*(6), 787-804.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics. Teaching developmentally. (eighth ed.)*. Boston, MA: Pearson Education.

Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. The Journal of Mathematical Behavior, 31(3), 344–355. doi:10.1016/j.jmathb.2012.02.001

Widjaja, W., Stacey, K., & Steinle, V. (2008). Misconceptions about density of decimals: Insights from pre-service teachers' work. Paper presented at Konferensi Nasional Matematika, Palambang, Indonesia, 24-26 July, 2008.