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ORIGINAL ARTICLE

A marriage of continuance: professional development for mathematics lecturers

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Abstract In a 2-year project, we developed and trialled a mode of lecturing professional development amongst staff in our department of mathematics. Theoretically grounded in Schoenfeld's resources, orientations, and goals (ROG) model of teacher action, a group met regularly to discuss both the video excerpts of themselves lecturing along with written pre- and post-lecture statements of their "ROGs". We found evidence of improved teaching performance but more interestingly, identified key aspects of our practice and of undergraduate mathematics that received repeated attention and developed further theoretical insight into lecturer behaviour in mathematics. The trial has been successful enough to be expanded into further groups that now constitute a professional development culture within our department.

Keywords Professional development · Mathematics · Lecturing · Undergraduate · Orientations · Goals

It was inevitable. The moment at the beginning of the 1990s when the professors in the Department of Mathematics decided to appoint mathematics educators into the department, you could have predicted that, eventually, thoughts would turn to pedagogical professional development for mathematics lecturers.[1] Twenty years is elephantine as a courtship, conception, and gestation period but the baby has eventually been born healthy and ready to yell. We have three aims in writing: to trace, briefly, the way professional development for lecturers came to life in a department of mathematics; to describe a research project; and to report on the research evidence which, we believe, demonstrate the effectiveness of a group discussion of video lecturing guided by theoretical analyses, all taking place in an atmosphere of trust.

[1] We note that what we refer to as lecturers are more often called instructors in the USA.

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Courtship and conception

The first mathematics education appointments were ex-secondary teachers with strong mathematics degrees; that is, experienced teachers but emerging researchers. Bringing mathematics educators into a mathematics department did not meet with universal approval. The competition for resources and a desire for appointments in other areas added to lecturers' anxiousness about being judged in the pedagogical field. It took about 5 years for the Mathematics Education Unit (MEU) to be recognised as pulling their weight in attracting resources into the department and also as adequate teachers of mathematics at first- and second-year levels. Only then did a few mathematicians become interested in the educational research and development activities.

In the first decade, three main factors contributed to the success of the MEU. The most important was that the Unit served the functions for which it was proposed, that is, creating stronger links with schools in order to attract students into the mathematical sciences. Furthermore, members of the Unit were successful on standard university criteria: winning external research grants, performing service functions in the wider community, being promoted, publishing and developing international research recognition, as well as achieving high teaching evaluations.

The second factor was the unwavering support of senior staff. It was the professors who argued for the MEU's expansion, and it was the Deans of both Science and Education, and the Vice Chancellor, who requested that the Unit consider delivering preservice secondary teacher education in the mid-1990s. Our successful foray into this area established the reputation of the Unit both internally and externally.

A third factor was a collegial one. As ex-school teachers, members of the MEU were more socially active than had been the norm in the department, and mathematicians were drawn into a more interactive (if noisy) environment. A number of new appointments in the department contributed to the growing interest in pedagogical issues.

It was inevitable that MEU members' interest in tertiary education would grow as we were predisposed to being reflective about our own lecturing. The Mathematics Department tearoom had always been a place where students' performance, assessment, and curriculum issues were discussed but the cynical one-liners moved aside for longer, and sometimes heated, debates about technology, student behaviour, the changing student demographic, and the demands of other departments.

In addition, as mathematics education researchers, we had grown in our research practice, our understanding of theoretical issues, and our confidence to tackle larger projects. This turn of our attention to the tertiary environment coincided with an increasing university demand for all staff to produce evidence of their pedagogical effectiveness. Thus, by 2008, the MEU was firmly established as an entity and recognised as a component part of the department. We argue that this courtship was significant for subsequent events.

A wider context for the study was growing attention to tertiary education. During the 1980s, for example, Halloun and Hestenes (1985) found that traditional instruction resulted in very little conceptual change for physics students. Others took up the challenge and began collecting research evidence for more interactive modes, for example, Hestenes himself (1987), Richard Hake at Illinois (1985), and Eric Mazur in Harvard (2009). The subsequent search for pedagogical alternatives included team-based learning (Michaelsen et al. 2002), the Moore method (Cohen 1982), collaborative group work (Grouws and Cebulla 2000), and the utilisation of web-based tools (Markauskaite et al. 2006). The awareness of the need for improved pedagogic practices meant that university employers began to ask for evidence of teaching effectiveness.

Tertiary institutions then began to support activities that evaluate and improve teaching. Student evaluations are now pervasive, and many universities have diplomas in tertiary teaching. Most development is generic across disciplines, which may be why some lecturers perceive such courses as irrelevant. As professionals, university staff do not embrace their own professional development in a comprehensive way.

The birth of the project

The acceptance of mathematics educators within a mathematics department, the external pressure to address teaching issues in the university, together with a visit from Alan Schoenfeld from UC Berkeley, and a research grant opportunity led to the development of DATUM: a project for the Development and Analysis of Teaching in Undergraduate Mathematics. The project involved a group of eight staff (four mathematicians and four mathematics educators) making videos of their lecturing and using these in-group discussions enhanced by pre- and post-lecture statements of intent and reflection. We have audiotapes of all discussions and interviews of all those video-taped.

We begin with a discussion of the theoretical foundations of the project and then describe how the project developed and the manner in which data was collected.

Theoretical underpinnings

Compared with school-based research, investigations into undergraduate mathematics education had been relatively modest until the last 10 years (Selden and Selden 2001). Many papers are by lecturers writing about their teaching methods or descriptions of innovative courses. However, “very little research has focused directly on teaching practice—what teachers do and think daily, in class and out, as they perform their teaching work” (Speer et al. 2010: 99). One reason for this gap has been described as the strained relations between professional mathematicians and their mathematics education colleagues, leading to a lack of productive dialogue on pedagogy and hence few collaborative research efforts (Artigue 2001; Nardi et al. 2005). We believe that the process followed in our department has found a possible way through this potential impasse.

Any investigation of teaching practice has to include the thinking, judgments, and decision-making of lecturers as they prepare for and teach undergraduate mathematics classes (Speer et al. 2010), and this requires a suitable theoretical framework for analysis. Over a number of years, Schoenfeld (2008, 2010) has worked to refine a theoretical framework for goal-oriented decision-making in school teaching. His framework employs the knowledge, orientations, and goals (since renamed resources, orientations, and goals or ROG) that teachers call upon to describe how these are linked to in-the-moment decisions made in the classroom.

The framework helps to analyse, and reflect on, how decisions are made. In this context, a teacher's goals may be large or small and either pre-conceived or arise during the lesson. The goals may also be immediate or long term and conscious or unconscious. An individual's orientations, "…an inclusive term encompassing a group of related terms such as dispositions, beliefs, values, tastes and preferences." (Schoenfeld 2010: 29) not only shape the goals set but also "…the prioritization of the goals … and the prioritization of the knowledge that is used in the service of those goals." (*ibid.*, p. 29). Once the teacher has oriented himself and set goals for the current situation, he decides on the direction to take to achieve the goals and call on resources, which, according to Schoenfeld (2010), primarily comprise teacher procedural and conceptual knowledge, along with heuristics or problem-solving strategies. However, resources also include available physical entities, such as pens and whiteboards, textbooks, models, digital technology, etc., as well as attributes including time and energy. Thus, decision-making involves complex interactions of an individual's ROG for a given situation, and it can be seen as a dynamic, changing entity that is updated as a result of teaching interactions. The quality of the decision-making affects how successful a teacher is in attaining his goals. The metacognitive processes of self-monitoring and self-regulation are also important in deciding how well things are progressing (Schoenfeld 2010). Orientations are crucial since "What people perceive, how they interpret it, and how they prioritize the ways they might respond to what they see are all shaped in fundamental ways by their orientations." (Schoenfeld 2010: 44). Teachers with different sets of beliefs and values regarding what counts in learning mathematics will make different choices. Knowledge, similarly, is critically important in order to understand what students say and to provide tools to react to it.

Schoenfeld's framework has been used to investigate how teacher beliefs about the nature of teaching, learning, and mathematics can influence practice, including goal setting (Aguirre and Speer 2000), as well as to describe how subject matter goals and beliefs can dominate pedagogical content goals and beliefs (Törner et al. 2010). While the framework had previously been applied only to school teaching, the principles appeared highly relevant to the lecture situation. Other research by Kane et al. (2004) had concluded that while good teaching is not innate, it can be learned. Further, they suggest that the key for new teachers at the tertiary level to do so is the development of the skills of reflective practice. Our aim was to use the framework to encourage this.

Having established a theoretical framework, there was still a need to attend to the group construction since an individual's development of mathematics teaching practice "is most effective when it takes place in a supportive community through which knowledge can develop and be evaluated critically" (Jaworski 2003: 252). We were aware of the principles of effective communities of practice (Lave and Wenger 1991) and the culture of enquiring conversation for professional development (Rowland 2000). Our intention was to build a co-learning group (Jaworski 2001; Wagner 1997), where both mathematics educators and mathematicians are learners, engaged in action and reflection. In our case, all the mathematics educators were also lecturing university mathematics and so we were what Jaworski (2003) calls insider researchers and hence developed our own teaching. One role of such a community of inquiry is to reflect critically on the process of teaching practice (Jaworski 2003; Wells 1999). In this kind of environment, new norms can emerge that influence cultures of practice

(Jaworski 2001). All members of our community were co-learners who took part in the development of a language and an attitude that enabled us to be critical about each others' practices in an acceptable way—referred to in the literature as critical alignment (Jaworski 2006; Wenger 1998).

A key component of developing expertise in teaching practice involves decisions on where to focus one's attention (Russ et al. 2011), which may involve advance preparation in order to attend to specific events (Mason 1989, 2002, 2008). One aspect of Mason's discipline of noticing is 'noticing-in-the-moment', which can lead us to informed action-in-the-moment, and is enhanced by structuring our attention. One example of an area that repays attention is noticing the use of questions in teaching practice. As Mason (2000) observes

> the style and format of the questions used by lecturers and tutors profoundly influence students' conceptions of what mathematics is about and how it is conducted. By looking at reasons for asking questions, and becoming aware of different types of questions which mathematicians typically ask themselves, we can enrich students' experience of mathematics. (p. 97).

We therefore set out to examine university mathematics teaching practice using a team approach (Keynes and Olson 2001) with a supportive community of inquiry in which lecturer ROGs could come to the fore in discussions and reflection on practice. One hypothesis was that if lecturers had a desire to improve their teaching practice then an explicit awareness of the ROG might be a step to improved practice.

The engagement: How the project developed

In late 2008, we received a visit from Alan Schoenfeld and spent some time discussing how to apply his ROG framework. We turned his ideas on ourselves as lecturers, and in 2009, four of us in the MEU, with a visiting teacher fellow, began, as part of our own professional development, to video ourselves, to be explicit about our "ROGs", and to discuss video excerpts in a small group, with Alan contributing on Skype. Then came the opportunity to apply for a large grant. We invited those of our colleagues who we thought would be interested to join the research and were delighted with a 100 % positive response. We wrote an extended proposal for a major project we called DATUM. The DATUM research questions addressed in this paper are

- How can Schoenfeld's resources, orientation, and goals (ROG) framework be adapted to support lecturer professional development?
- Can an effective lecturing professional development strategy be built around peer discussion of lecturer ROGs matched against videoed lecturing practice?

When we won the funding, we began construction of a supportive community of inquiry with the aim of collaborating in an examination and discussion of lecturer actions and practices as seen through video recordings (Kazemi et al. 2009; Prushiek et al. 2001).

In this discussion, we draw on data collected over the following 2 years. Six members agreed to videotape at least two lectures per year, including all the mathematicians. All of them, before and after the lectures, wrote notes based on their ROG for the lecture. An example of a ROG is given in Fig. 1. It was not considered necessary to ask all the lecturers to be recorded, and one of the other lecturers taught a mathematics education course that was not suitable for the project anyway. Where possible, the same lecturer was videotaped again at a later date. At each videoed lecture, another person observed and took notes. The lecturer selected a 2- to 4-min excerpt from the videotape to be watched by the group. Counterintuitively, lecturers chose parts in which they felt less comfortable. For example, mathematics educators chose parts where their mathematical knowledge might be questioned, and mathematics researchers chose moments of pedagogical concern.

The group had three 1-hour meetings each semester to discuss lecturing, prompted by viewing one or two excerpts and the ROGs. A total of 19 lectures were captured on video, and 15 full meetings were held. Data included the videotapes themselves, voice recordings, and transcripts of the meetings, analysed against a code representing features identified as recurring or of interest. Also, each of the mathematicians was interviewed about their experience. One of the mathematics educators describes his experience in detail in Barton (2011). Finally, as a check on reliability and validity, independent evaluators examined lecture recordings for evidence of changes in practice (see below).

We used the ROG framework to analyse our data and present evidence that lecturers' resources, both mathematical and pedagogical, were enhanced; that we developed increased awareness of our orientations; and that the process led to shifts in goals that were seen as productive.

A feature of the community was its supportive nonjudgmental nature. [In the following, Sandy, Simon, Abi, and Mark are the mathematicians' aliases]. As Sandy said "I would like to.. record that I have been very happy with the supportive atmosphere, it's been very.. I haven't felt nervous about having people in my lecture or watching the video". This was despite his admission that "It's pretty revealing watching yourself being videoed isn't it?" Without the level of trust that was developed, none of the interactions would have occurred.

Lecturers frequently chose to focus the group's attention on interludes in the lecture when unexpected decisions were made. These discussions revealed a balance between the pedagogical and mathematical demands of teaching situations (Paterson et al. 2011a). The 'mixed' nature of the community was seen as an asset. Mark commented that he "gained a mathematics education perspective of the [video] clip, which clarified in my own mind what I do when I teach…It's good to have some of the theory behind it". The reverse was also true.

In an earlier paper, we have explored the role of the internal disciplinarian (Paterson et al. 2011b) where we highlighted the tension in balancing pedagogical and mathematical demands in lecturing. This tension was the subject of numerous debates. Jaworski et al. (2009) also speak of the tensions experienced by lecturers arising from a desire to satisfy both student needs and mathematical values.

In the following section, we examine the discussions that occurred, the pedagogical and mathematical issues that arose, the goals that were articulated and altered, and the role awareness of orientations plays in decision-making and promoting reflection.

KOG
Lecturer: [Name]
Course: [Number] "Number Theory"
Date: Wednesday August 18, 2010
Lecture: Introduction to continued fractions

Knowledge
The students' knowledge: This is a [4^{th} year] course. I assume mathematical maturity and independence. I assume the students have strong background in algebra and are able to work out details and read proofs in their own time. I also assume an interest in (and curiosity about) numbers. I assume the students welcome a challenge.
Specific knowledge relevant for this lecture: Good ability with algebraic manipulation, fractions and real numbers. I assume they know that $\sqrt{2} \approx 1.414$.
Lecturer's technical knowledge: Number theory, both theoretical and computational. Experience with developing analogies which allow clear motivation for the problems being considered. Experience in difficulties that students sometimes have with learning this material.
Orientations
I believe the class to be attentive and serious. The majority of the students are very talented at mathematics and pick up new ideas quickly. There are 1 or 2 students who are not as quick as the others (but are still good and able students), and it is important for me not to go too quickly for them. The class are very reluctant to speak or ask questions in class (even when it is clear they know the answers), which is a pity. This is possibly due to the presence of a few MSc and PhD students who are sitting in.
I hand out detailed lecture notes so I assume most students are not writing down very much during the lecture. I assume they are following the details closely and learning the content in real time. I always have at least one exercise in the lecture, where I give the students several minutes to work something out for themselves. In this lecture it will be a computational exercise, but often it is a proof.
This is a theoretical course and I place a lot of emphasis on being able to understand the theory and do proofs. I also use practical applications as motivation, and there is plenty of computational content as well.
For the specific lecture, I want continued fractions to appear as a natural concept (hence the spiel about representing numbers if you had an infinite number of fingers). The applications of continued fractions will come in the following lectures, so it is important that this lecture maintains interest just for its own sake.
Goals
The goals of the course are to survey those topics in number theory which are most traditionally taught at masters level (with a slight bias towards my own areas of interest). The course is designed to be good general preparation for post-grad study in number theory. The course will also increase the students' mathematical maturity, in particular it will give exposure to different proof techniques and with many abstract concepts. The course also shows how the structure of mathematics grows and how initially mysterious problems can be tamed by making appropriate definitions and developing the right theoretical tools.
The goals of the lecture are: To define continued fractions, to show how to compute them, and to prove some basic theoretical results. The most important theoretical part is to state and prove correctness of the recurrence formulae for computing the convergents. This proof works by studying a more general problem (real coefficients). I hope to give a flavour for just how neat this proof is -- it is an example of how in mathematics it is sometimes easier to prove a more general result. This fact relates to one of the themes of the course: emphasizing to the students how the right theoretical tools and proof techniques can tame a mathematical problem.

Fig. 1 An example of a ROG written by a mathematics lecturer

Resources–pedagogical

A wide range of pedagogical questions were discussed: how to encourage students to ask and answer questions, what to do when a student gives an incorrect answer to a question, how to promote discussion and student response in lectures, the role of lecture notes, and when to ensure the mathematics is completely correct. It was a feature of the project that, once raised, concerns were frequently revisited in subsequent meetings.

In an early lecture, Abi focused our attention on an incident in which a student gave an incorrect response to a question. There was a spirited debate around possible paths of action, informed both by people's experiences and research on lecture interactions, for example, Initiation–Response–Evaluation (IRE) questioning sequences (Mehan 1979). Subsequent discussion raised the question of engaging students through questioning. At the end of the semester, Mark ran an innovative and effective revision lecture, asking students to send in email questions and preparing answers to these.

Writing ROGs made people more aware of their knowledge of the students. We saw in our analysis of Simon's decisions (Paterson et al. 2011b) that when he saw the students in the class as talented, he paid more attention to "inducting them into being mathematicians". He later said "Before a lecture you should … remind yourself who is in the class, what they know, and it's astonishing to think that that is not automatically done…It forced me to think much more clearly about what my expectations of the students really are…I think I probably should write a ROG every time five minutes before my lecture just for the pure reason to remind myself that there are students out there and they have a position".

Resources–mathematical

The importance to lecturers of the mathematics (Keynes and Olson 2001) and ensuring that the mathematics they taught was correct was noted above. The mathematicians' fine-grained exploration of mathematical ideas was, for us mathematics educators, very exciting. We briefly entered, with our colleagues as guides, the worlds of mathematics they inhabit. We learnt in some depth about continued fractions, stochastic differential equations, logistic equations, phase portraits, subgroups, differential equations, and the finer points of continuity.

The mathematicians in the group conduct research in a range of mathematical disciplines: algebra, number theory, and applied mathematics. They enjoyed the diversity of reactions to their 'slices of teaching'. For example, in a discussion about sequences and series, there was a spirited discussion about why at school we tend to ask students to find a general equation linking the value of a term to its position rather than working with the recursive form of the relationship between terms. In contrast, they argued that a recursive formulation is more useful; it is more commonly utilised in research that seeks to predict a future scenario on the basis of current modelling of data.

A question that falls in the grey area between pedagogy and mathematics is the role of other resources, technology (in the form of MATLAB), and lecture notes—possibly from a previous lecturer. Becoming more aware of their role in the teaching and learning process has proved interesting. We saw how Sandy interacted with

MATLAB and Simon's 'argument' with his notes that led to his spending time 'making the mathematics right'.

The discussions about content items described above frequently contained references to the perceived value of the content, such as which aspects of it were critical to the current lecture and which elements may be needed for future learning. While not always framed within this context, these discussions in many ways reflect the content–value categories described in Oates (2009) and Barton (2011). The apparent differences in the ways colleagues perceived the relative *pragmatic*, *epistemic*[2], and *heuristic* (referred to as *pedagogical* in Oates 2009) values of specific topics provoked considerable discussion and evidence of the role our *orientations* provide in the organisation of our lectures and the decisions we make about how we teach a certain topic. It became clear that in writing their ROGs, lecturers are often able to articulate their goals more clearly than their orientations. Consequently, we address goals next and discuss the role of orientations at the end of the section.

Goals–and shifts

In writing their ROGs, most lecturers were clear about their goals for both the course and the particular lecture (see Fig. 1). For example:

- The goals of the course are to survey those topics in number theory that are most traditionally taught at masters level.
- The goals of the lecture are to define continued fractions, to show how to compute them, and to prove some basic theoretical results.

They were also aware when these are not met or deviated from. For example, Simon stated in the ROG he wrote before a lecture that, "I assume the students have strong background in algebra and are able to work out details and read proofs in their own time. The 'slice' of the lecture he chose for the group to watch showed that he did not do that for reasons that became clear during the discussion. We have discussed this decision and its connection to his orientations in depth in Paterson et al. (2011c). Instances of deviating from stated goals often gave rise to discussions that revealed underlying orientations. It may well be that the surfacing and awareness of orientations is a major mechanism for change, leading to the professional development identified in this project.

Awareness of orientations

The issue of awareness of orientations appears to be a crucial aspect in the decision-making that leads to changes in practice. In support of this, Paterson et al. (2011a: 993) conclude

[2] Here we follow Artigue (2002) in considering pragmatic value as "productive potential (efficiency, cost, field of validity)" (p. 248) or how much can be efficiently accomplished using something and the epistemic value as a contribution "to the understanding of the objects they involve" (p. 248).

> Creating a forum to discuss the decisions involved in lecturing situations often leads to an awareness of unarticulated, taken as given, orientations and their consequent impact on teaching. Awareness of any inner tension and the need to resolve it (Speer 2008) is an important part of reflecting on our role as tertiary teachers. Encouraging this engagement can result in effective incremental professional growth (Speer 2008).

It was clear that the lecturers were much less able to articulate their orientations than their goals and resources. Frequently, they only emerged during the discussions. For example, Abi said in her ROG that "I see this whole course partially as an exercise in 'public understanding of mathematics', and so try to treat the lectures as such—rarely going into much depth mathematically". However, it became clear during the viewing and discussion that there were certain mathematical ideas that she felt had to be articulated, even if only to herself.

> So you know that f_{n+1} is bigger than f_n so this is going to be a number that is bigger than 1.
>
> Right? [sounds as if she hears herself and adds this] Or equal to 1.

In a later discussion with Simon, she asks "So should you just ignore that corner [of important mathematics] and just hope that it's not noticed? But then is that bad because you've somehow told them something incorrect?"

In an interview with Sandy, it became clear that, having satisfied his primary goal of ensuring that the students were aware of the periodic nature of the solution to the logistic equation, his teacher orientation took control and he chose not to pursue a problem the MATLAB solution had raised. By contrast, Simon in a similar situation chose to follow the mathematics and 'make it right'. We referred back to these moments frequently and used them to amplify subsequent decisions and call orientations into question. When discussing a later video of Sandy's on stochastic differential equations, one of the authors was able to ask, without needing to elaborate, "So who is in charge here, Sandy, the teacher or the mathematician?" To which Sandy replied "I think both!"

This interesting phenomenon of an internal debate between the lecturer as teacher and the lecturer as mathematician (Paterson et al. 2011a; Paterson et al. 2011c) was observed on a number of occasions. It appears that lecturer decisions can at times be explained in terms of the outcome of this debate. On some occasions, it is the mathematician that 'wins the day' and pursues the mathematics, seeking to 'make it right' despite the students being unworried about the 'notational conundrum' he has seen looming up ahead. On other occasions, the teacher prevails, conscious that, for these students at this time, the important pedagogical goals have been attained and the deeper understanding of mathematics needs to be left for another day or another course. We have already seen evidence of Abi assuring herself in an aside about the finer points of an argument.

We have found it useful (Paterson et al. 2011a) to explain in-the-moment decisions by teachers in terms of the relationships between clusters of orientations influencing current goals, in the manner of Thagard (2000) 'raft' of beliefs. The way in which the orientations are clustered, and the relative importance of these clusters, seem to play a part in establishing the priority of the goals. We reiterate that, in terms of professional development, it is our orientations that we often have the most difficulty identifying and hence these may best be revealed through a discussion of decisions. Given our argument that such decisions can be explained in terms of clusters of orientations, increasing awareness of these may begin to explain the effectiveness of this mode of professional development. We contend that promoting an awareness of such inner tensions is an important part of reflecting on our role as tertiary teachers, which in turn will enable an effective, incremental approach to growth in professional development (Speer 2008).

Promotion of lecturer self-reflection

DATUM provided an opportunity for individual lecturers to observe themselves. Did this promote self-reflection? The unequivocal answer is "Yes". For example, Simon commented that he is now much more aware of the importance of articulating his thinking to the students. One of the mathematics educators made use of the opportunity of having five recordings of the same lecture over a 2-year period and his colleagues' comments. He used the data to try out a framework that was designed to capture what is important about undergraduate course delivery, asking, Can I design a framework that will grow my understanding by shedding new light on an old practice? What framework will make it easy to see how I should change my courses or lecture more effectively? Simultaneously, he reflected on both his practice and the way he developed his practice (Barton 2011). The lecture that was captured, and recaptured, follows an introduction to vector spaces. Students meet the concept of an eigenvector—a concept they will use not only in applications but also in further mathematics as the course develops. The framework focuses on the interplay between aspects of the "mathematical essence" of the lecture and aspects of the "learning culture" in which it is embedded, observing the interplay from each of three perspectives: the immediate practical value of what is going on (the pragmatic contribution), the way the lesson contributes to broader understanding (the epistemic contribution), and the future value of the work (the heuristic contribution). Four episodes in the lecture were analysed in depth—two are briefly described to demonstrate the level of reflection achieved.

The opening 2 min of the lecture was spent differently in each recording: electing a class representative, explaining the usefulness of eigenvectors, establishing desired norms for lecture behaviour, and describing how the Google search engine works as an example of an application of eigenvectors. Contemplating this variation made the lecturer aware that in the interaction between mathematical essence and learning culture, there are many competing priorities. But the first few minutes are the moments of most attention and should be used carefully. Using that valuable time to elect a class representative was not efficient, and the Google example is probably interesting enough in itself to be placed elsewhere with more effect. However, both setting learning culture norms or giving a wider overview of where this material is heading are good uses of that time. As a result of this analysis, the lecturer is now motivated to plan this time in a deliberate way, rather than let serendipity determine his actions.

In the second videoed lecture, a student asked whether every matrix has an eigenvector. The lecturer latched onto the student question with delight, praising its inquisitiveness, and built it into subsequent lectures. The sequence of videos shows, first, the lecturer welcoming the question but being surprised by it and unsure of its nuances. On the next occasion, he asked it deliberately but answered with a poorly disguised pretence of ignorance. In the following iteration, he used the moment to highlight the nature of the characteristic equation, and finally, he strategically posed the question as a production of further mathematics and a preview of the concept of complex eigenvalues.

Reflecting on the sequence, the lecturer reported a certain amount of satisfaction. He had correctly identified an important teaching moment, had made the correct decision in the moment, and had subsequently developed this moment to have pragmatic, epistemic, and heuristic value. But during group discussion, a colleague questioned the lecturer's espoused desire to model mathematical behaviour during lectures. She noted that the effect of the evolution of the student question into a programmed teaching moment had eliminated an opportunity to model mathematical behaviour. The lecturer acknowledged the contradiction and is now thinking how to retain the pedagogical value of the question in a way that retains mathematical processing.

These two examples highlight how having videos of (repeated) lectures can contribute not only to reflection and changes in practice but also the benefit of both collegial insight and of theoretical frameworks as a reflective aid. Even those members of the group whose lecturing was not videoed report a heightened awareness of how they ask questions and develop a range of interactions with students.

Finally, we discuss the outcomes of the project and summarise the key principles for implementing this mode of professional development.

Outcomes

Seeking objective evidence

In order to obtain an objective perspective on our contention that there had been a positive shift in lecturer practice, four experienced university mathematics educators from the USA (3) and the UK (1), who were not involved in the project, were given videos of three or four pairs of lectures from each of the three lecturers. The first lecture in each pair was taken from early in the project and the second towards the end. Using the evidence in the videos, they were asked to order them on the basis of perceived professional development and provide reasons for their order. The result of the blind assessment of the pairs of lectures was 70 % correct, with evaluators correctly assessing which lecture was post-development in 3/4 cases for lecturer A, 3/3 cases for lecturer B, and 1/3 cases for lecturer C. Indicative characteristics reported by the assessors included confidence, student interaction, motivating examples, and content approaches. Thus, we argue that there is some objective evidence that this professional development process is effective in terms of improved lecturing performance, even in the short time span of the project. In addition, data from the project has been used by lecturers in a number of successful promotion applications to substantiate claims of professional development in teaching practice.

It can be argued that further objective evidence of the value of the process exists in the form of its voluntary uptake by other groups of lecturers, within the university, the country, and internationally. One of the extensions arising from the project was an application of the use of the ROG framework to examine the teaching practice of a mathematician at another New Zealand university. One of the authors formed a small community of inquiry with another mathematics educator and the mathematician. The research sought to determine the effectiveness of certain aspects of the lecturer's teaching of a second year linear algebra course. The project involved cycles of planning the relevant teaching material, implementing them and then reflecting on and evaluating the results. As part of this research, the mathematician spent up to an hour after each of 24 lectures writing a detailed diary reflecting on events in each lecture. He was also interviewed twice by the two mathematics educators at the beginning and end of all the lectures. Moreover, there were fortnightly Skype sessions after the lectures had finished as part of the community's discussion of issues arising. The interviews and the Skype discussions were audio-recorded and transcribed.

This project extension also proved valuable for the mathematician, and, as reported by Hannah et al. (2011: 983), he found participating in the community of practice an enriching experience.

> Sharing discussions with people who had taught similar courses gave him the opportunity to reflect on his teaching practice. As these people were also education researchers, the discussions were able to include ideas from recent research about teaching, and this led to an increased awareness of his own orientations and goals, which he hopes will lead to benefits for his future practice.

Further, those involved comment (Hannah et al. 2012: 1392) that "All the participants found the community of practice an enriching experience. Sharing discussions with fellow teachers gave them the opportunity to reflect on their teaching". This collaboration is continuing, via a university in the USA, with two recent publications, one of which considers the lecturing practice of another group member (Hannah et al. 2013a, 2013b).

It is now nearly 3 years since the conclusion of the project, and further evidence of its continuing effect is that the local marriage has also produced offspring in the form of three groups emerging from the project. These involve 10 lecturers and four mathematics educators, many of whom have been successfully employing the same group strategy for more than 2 years. In addition to the new members, all the previous participants still at the university wanted to continue. In one of the new groups, a lecturer, as the result of observing the 'cheerfulness and energy', a colleague evidenced in a video, revisited her own video to discover that she 'looked very serious and only smiled once!' As a result, she has changed her 'style' to present a more cheerful persona, which has also resulted in improved student evaluations! These positive outcomes were the subject of a paper published in the proceedings of a recent international conference (Paterson and Evans 2013). In another development, during 2012, two visitors of the mathematics department attended a series of our meetings. One is a topologist from Galway who has subsequently referenced our endeavour in a paper on professional development and notices involvement of five Irish mathematicians (Breen et al. 2012). Our experience of working with mathematicians has also impacted on our successful bid for a large grant supporting a research taking a holistic look at student learning in undergraduate mathematics. Without the network of relationships developed before and during this project, we could not have begun to countenance such a project.

These developments support the assertion that this project provides evidence that an effective professional development programme is one in which lecturers develop their identity as continuous learners as part of their teaching practice (Carpenter et al. 2004). Our experience would also suggest that while "teachers' inquiry does not survive well in isolation" (Carpenter et al. 2004: 8), it flourishes in a supportive community.

Key principles for future implementations

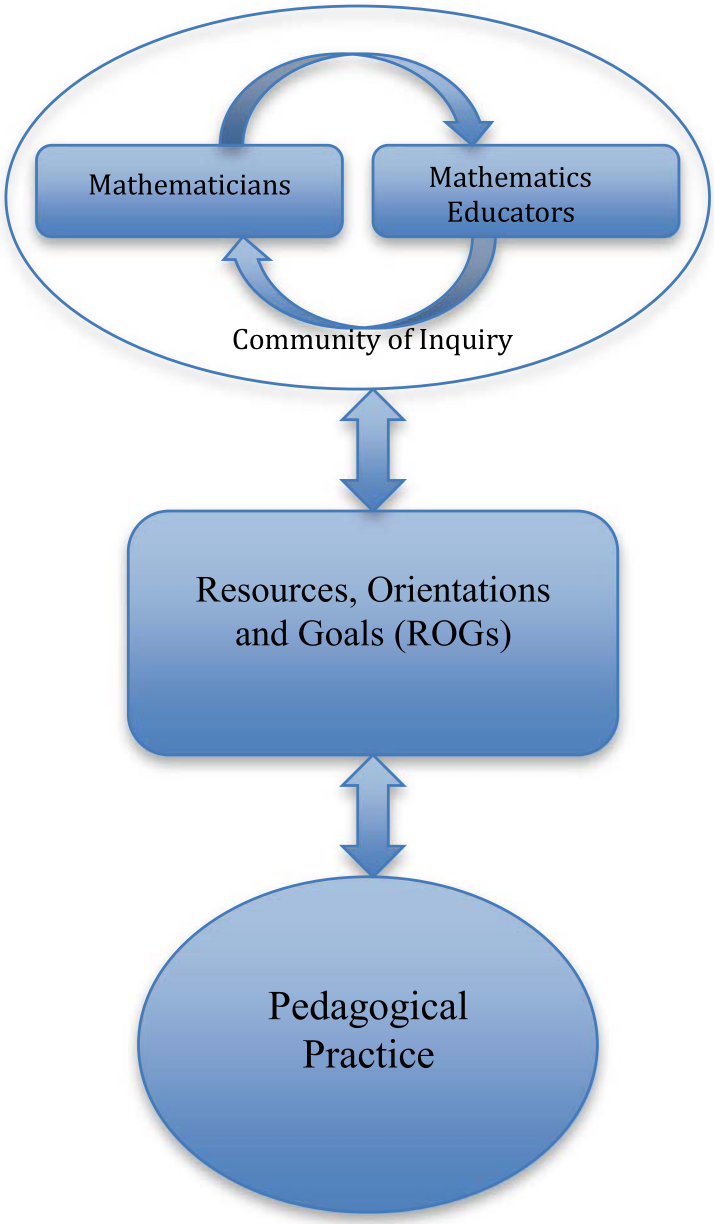
In our model of professional development activity, the community of inquiry members reflect on shared pedagogical practice through the lens of the ROG framework (see Fig. 2). The feedback from lecturers showed that the video/discussion process was very positively received, with the most value gained from the group meetings and discussions, although writing the ROGs and being able to watch oneself and others lecturing were also very positively regarded. Both pedagogical and mathematical issues were the subject of the community discussions, and this was reflected in improved lecturing. Thus, the professional development strategy is not only a source of improvement in both areas but is also a site where there is effective integration of pedagogy and content.

In summary, this research study has addressed key issues previously raised (Keynes and Olson 2001) and provided evidence that the approach to professional development adopted here has produced benefits for all involved. These include perspectives on teaching improvements for all members of the community, along with insights into mathematical constructs and thinking for the educators and educational theory for the mathematicians. We conclude that there is good evidence that this form of professional development is viable, effective, and positively received. Hence, university mathematics departments can successfully undertake effective lecturing development in a way acceptable to staff, using a video/discussion group format with the ROG framework as a lens. Hence, we strongly recommend that university departments in the mathematical sciences consider implementation of a version of this programme. We recognise that significant distribution and adoption of any educational innovation is only possible if its use can be sustained in subsequent institutions (Coburn 2003). We hope that other groups will wish to avail themselves of the opportunity to build ownership of a similar professional development activity[3] and so we provide here a list of the key implementation principles as we perceive them.

- In order to focus group attention, only a short 3- to 4-min slice of the lecture video is required, providing a sufficient springboard for discussion.
- The lecturer should have control over the selection of the lecture to be videoed, the slice of video chosen, and whether any of it is viewed by others.
- The development of trust within the community of inquiry is essential. It is preferable to induct members to the group, one or at most two, at a time.
- Consideration should be given to the optimum size of the community; we recommend six members.
- Community composition should be homogeneous with respect to a focus on a common discipline of interest.

[3] A full bibliography of activity related to the project is available from the authors.

Fig. 2 An overview of the professional development framework



- Community composition should be heterogeneous across a number of variables, including research field (mathematicians and mathematics educators), mathematician background (pure and applied, different perspectives were in evidence on some issues), years of experience, and seniority (from tutors to full professors).
- The ROG framework is highly recommended since it was effective in developing and focusing the project and enhanced discussion, but may not be essential.
- Community members are encouraged to develop self-awareness of their teaching style and to use this as a catalyst for change.

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References

Aguirre, J., & Speer, N. M. (2000). Examining the relationship between beliefs and goals in teacher practice. *The Journal of Mathematical Behavior, 18*(3), 327–356.

Artigue, M. (2001). What can we learn from educational research at the university level? In D. A. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 207–220). Dordrecht: Kluwer.

Artigue, M. (2002). Learning mathematics in a CAS environment: the genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245–274. doi:10.1023/A:1022103903080.

Barton, B. (2011). Growing understanding of undergraduate mathematics: a good frame produces better tomatoes. *International Journal of Mathematical Education in Science and Technology, 42*(7), 963–974.

Breen, S., McCluskey, A., Meehan, M., O'Donovan, J., & O'Shea, A. (2012). Reflection on practice, in practice: the discipline of noticing. *Informal Proceedings of the British Society for Research into Learning Mathematics (BSRLM), 31*(3), 7–12.

Carpenter, T. P., Blanton, M. L., Cobb, P., Franke, M. L., Kaput, J., & McLain, K. (2004). *Scaling up innovative practices in mathematics and science. Research report from the national center for improving student learning and achievement in mathematics and science*. Madison: University of Wisconsin.

Coburn, C. E. (2003). Rethinking scale: moving beyond numbers to deep and lasting change. *Educational Researcher, 32*(6), 3–12.

Cohen, D. W. (1982). A modified Moore method for teaching undergraduate mathematics. *American Mathematical Monthly, 89*(7), 473–474. 487–490.

Grouws, D. A., & Cebulla, K. J. (2000). *Improving student achievement in mathematics*. Geneva: International Academy of Education.

Hake, R. R. (1985). Interactive-engagement versus traditional methods: a six thousand student survey of mechanics test data for introductory physics courses. *American Journal of Physics, 66*, 64–74.

Halloun, I., & Hestenes, D. (1985). The initial knowledge state of college physics students. *American Journal of Physics, 53*(11), 1043–55.

Hannah, J., Stewart, S., & Thomas, M. O. J. (2011). Analysing lecturer practice: the role of orientations and goals. *International Journal of Mathematical Education in Science and Technology, 42*(7), 975–984.

Hannah, J., Stewart, S., & Thomas, M. O. J. (2012). Student reactions to an approach to linear algebra emphasising embodiment and language. *Proceedings of the 12th International Congress on Mathematical Education (ICME-12) Topic study group 2*, 1386–1393, Seoul, Korea.

Hannah, J., Stewart, S., & Thomas, M. O. J. (2013a). Conflicting goals and decision making: the influences on a new lecturer. In A. M. Lindmeier & A. Heinze (Eds.) *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 425–432), Kiel, Germany.

Hannah, J., Stewart, S., & Thomas, M. O. J. (2013b). Emphasizing language and visualization in teaching linear algebra. *International Journal of Mathematical Education in Science and Technology, 44*(4), 475–489. doi:10.1080/0020739X.2012.756545.

Hestenes, D. (1987). Toward a modeling theory of physics instruction. *American Journal of Physics, 55*, 440.

Jaworski, B. (2001). Developing mathematics teaching: teachers, teacher-educators and researchers as co-learners. In F.-L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education*. Dordrecht: Kluwer.

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partnerships. *Educational Studies in Mathematics, 54*, 249–282.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*, 187–211.

Jaworski, B., Treffert, S., & Bartsch, T. (2009). Characterising the teaching of university mathematics: a case of linear algebra. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd conference of the international group for the psychology of mathematics education* (Vol. 3, pp. 249–256). Thessaloniki: IGPME.

Kane, R., Sandretto, S., & Heath, C. (2004). An investigation into excellent tertiary teaching: emphasising reflective practice. *Higher Education, 47*, 283–310.

Kazemi, E., Franke, M., & Lampert, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides (Proceedings of the 32nd annual conference of the mathematics education research group of Australasia)* (Vol. 1, pp. 11–29). Palmerston North: MERGA.

Keynes, H., & Olson, A. (2001). Professional development for changing undergraduate mathematics instruction. In D. Holton (Ed.), *The teaching and learning of mathematics at the university level: an ICMI study* (pp. 113–126). Dordrecht: Kluwer.

Lave, J., & Wenger, E. (1991). *Situated learning: legitimate peripheral participation*. Cambridge: Cambridge University Press.

Markauskaite, L., Goodyear, P., & Reimann, P. (Eds.) (2006). *Proceedings of the Australasian Society for Computers in Learning in Tertiary Education (ASCILITE)*. Available at http://www.ascilite.org.au/conferences/sydney06/proceeding/pdf_papers

Mason, J. (1989). Mathematical abstraction as the result of a delicate shift of attention. *For the Learning of Mathematics, 9*(2), 2–8.

Mason, J. (2000). Asking mathematical questions mathematically. *International Journal of Mathematical Education in Science and Technology, 31*(1), 97–111.

Mason, J. (2002). *Researching your own practice: the discipline of noticing*. London: Routledge Falmer.

Mason, J. (2008). Doing≠construing and doing+discussing≠learning: the importance of the structure of attention. *Proceedings of ICME-10, Copenhagen (CD version of proceedings)*. Available from http://www.icme10.dk/proceedings/pages/regular_pdf/RL_John_Mason.pdf.

Mazur, E. (2009). Farewell lecture. *Science, 2, 323*(5910), 50–51.

Mehan, H. (1979). *Learning lessons: social organization in the classroom*. Cambridge: Harvard University Press.

Michaelsen, L. K., Knight, A. B., & Fink, L. D. (2002). *Team-based learning: a transformative use of small groups*. Westport: Praeger.

Nardi, E., Jaworski, B., & Hegedus, S. (2005). A spectrum of pedagogical awareness for undergraduate mathematics: from 'tricks' to 'techniques'. *Journal for Research in Mathematics Education, 36*(4), 284–316.

Oates, G. (2009). Relative values of curriculum topics in undergraduate mathematics in an integrated technology environment. In B. Bicknell, R. Hunter, & T. Burgess (Eds.), *Crossing divides (Proceedings of the 32nd annual conference of the mathematics education research group of Australasia*, Vol. 2 pp. 419–426). Wellington, New Zealand.

Paterson, J., & Evans, T. (2013). Audience insights: feed forward in professional development. In D. King, B. Loch, & L. Rylands (Eds.), *Proceedings of Lighthouse Delta, the 9th Delta conference of teaching and learning of undergraduate mathematics and statistics through the fog* (pp. 132–140). Kiama: Delta.

Paterson, J., Thomas, M. O. J., Postlethwaite, C., & Taylor, S. (2011a). The internal disciplinarian: who is in control? In S. Brown, S. Larsen, K. Marrongelle, and M. Oehrtman (Eds.), *Proceedings of the 14th annual conference on research in undergraduate mathematics education* (Vol. 2, pp. 354–368). Portland, Oregon.

Paterson, J., Thomas, M. O. J., & Taylor, S. (2011b). Reaching decisions via internal dialogue: its role in a lecturer professional development model. In B. Ubuz (Ed.), *Proceedings of the 35th conference of the international group for the psychology of mathematics education* (Vol. 3, pp. 353–360). Ankara: IGPME.

Paterson, J., Thomas, M. O. J., & Taylor, S. (2011c). Decisions, decisions, decisions: what determines the path taken in lectures? *International Journal of Mathematical Education in Science and Technology, 42*(7), 985–996.

Prushiek, J., McCarty, B., & Mcintyre, S. (2001). Transforming professional development for preservice, inservice and university teachers through a collaborative capstone experience. *Education, 121*(4), 704–712.

Rowland, S. (2000). *The enquiring university teacher.* Philadelphia: OU Press.

Russ, R. S., Sherin, B., & Sherin, M. G. (2011). Images of expertise in mathematics teaching. *Expertise in Mathematics Instruction: An International Perspective*, 41–60.

Schoenfeld, A. H. (2008). On modeling teachers' in-the-moment decision-making. In A. H. Schoenfeld (Ed.), A study of teaching: multiple lenses, multiple views (*Journal for Research in Mathematics Education Monograph* 14, pp. 45–96). Reston, VA: NCTM.

Schoenfeld, A. H. (2010). *How we think. A theory of goal-oriented decision making and its educational applications*. New York: Routledge.

Selden, A., & Selden, J. (2001). Tertiary mathematics education research and its future. In D. Holton (Ed.), *The teaching and learning of mathematics at the university level: an ICMI study* (pp. 207–220). The Netherlands: Kluwer.

Speer, N. M. (2008). Connecting beliefs and practices: a fine-grained analysis of a college mathematics teacher's collections of beliefs and their relationship to his instructional practices. *Cognition and Instruction, 26*(2), 218–267.

Speer, N. M., Smith, J. P., & Horvath, A. (2010). Collegiate mathematics teaching: an unexamined practice. *The Journal of Mathematical Behavior, 29*, 99–114.

Thagard, P. (2000). *Coherence in thought and action*. Cambridge: MA: MIT Press.

Törner, G., Rolke, K., Rösken, B., & Sririman, B. (2010). Understanding a teacher's actions in the classroom by applying Schoenfeld's theory teaching-in-context: reflecting on goals and beliefs. In B. Sriraman & L. English (Eds.), *Theories of mathematics education, advances in mathematics education* (pp. 401–420). Berlin: Springer.

Wagner, J. (1997). The unavoidable intervention of educational research: a framework for reconsidering research-practitioner cooperation. *Educational Researcher, 26*(7), 13–22.

Wells, G. (1999). *Dialogic inquiry: towards a sociocultural practice and theory of education*. Cambridge: Cambridge University Press.

Wenger, E. (1998). *Communities of practice: learning, meaning and identity.* Cambridge: Cambridge University Press.