On the decision support model for the patient admission scheduling problem with random arrivals and departures: A solution approach.

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Abstract

The focus of this work is the numerical application of a stochastic decision support model for the patient admission scheduling problem with random arrivals and departures. Here, we discuss the methodology for applying our model to real-world problems. We outline a solution approach for efficient computation, provide a numerical analysis of the model, and illustrate the methodology with examples.

A key component of the model is an integer linear program which formulates the patient admission scheduling problem as an optimisation of the total expected cost accumulated over a finite planning horizon. We rewrite some of the components of this integer linear program in order to improve numerical efficiency. We use discrete phase-type distributions to model the random arrivals and departures. We argue that this stochastic component is essential for an accurate treatment of real-world problems which are stochastic in nature. We support our claim with simple numerical examples, show that the optimal solutions obtained from deterministic models are inadequate when compared with the solutions of our stochastic model. We also construct more complex numerical examples for large-scale problems using heuristics that approximate the objective function, in order to demonstrate that our model can be efficiently applied in real-world problems, which typically involve large sets of data.

Keywords: patient admission scheduling, stochastic programming, integer programming, phase-type distribution.

1 Introduction

In the patient admission scheduling (PAS) problem, patients arriving to the hospital system need to be allocated to rooms in relevant wards in an optimal manner, so as to minimise the total cost accumulated over some planning horizon, subject to the availability of resources as well as the needs of the patients. Since the arrivals of the patients to the system, their departures, their needs and preferences, as well as

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the availability of resources are all random, suitable stochastic decision support tools are required that could be used by the managers in their decision making and planning. The key goal is to improve the outcomes for the patients, as well as to reduce the costs, and improve the overall efficiency of the system, which consists of a number of interconnected smaller parts which can affect the flow of one another.

Bilgin et al. [2] solved the PAS problem in a static context by assigning a fixed number of patients with a fixed length of stay as well to a given set of beds. To solve the problem they used a hyper-heuristic approach, and concluded that it is not possible to see an improvement in the patient to bed allocation by considering the PAS problem in a static context. Vancroonenburg et al. [8] developed a model using ILP to solve the PAS problem. The delay and overcrowding costs were not considered in the objective function. They used a sensitivity analysis based on the bed occupancy and length of stay to evaluate the performance of the model. They concluded that the model which considered more information of the real-world scenario gives a better solution. Ceschia and Schaerf [3] used a meta-heuristic approach with a local search method, whereas Lusby et al. [6] used a simulated annealing technique with an adoptive large neighbourhood search to solve the PAS problem. Ceschia and Schaerf [3] developed a model which included patient to room assignment cost, gender violation cost, overcrowding cost, and delay cost, but excluding a transfer cost. Following to Ceschia and Schaerf [3], Lusby et al. [6] developed a model with five cost components including transfer. Both Ceschia and Schaerf [3] and Lusby et al. [6] were able to obtain solutions for large sized data instances. The Lusby et al. [6] model contained more information with regard to the full mathematical formulation of the PAS problem. Considering the dynamic nature of the problem, their model still did not capture the possibility of early departures due to death or getting better treatment before the departure date since they used a fixed value for the length of stay.

Abera et al. [1] proposed a stochastic model for the PAS problem, where they assumed that arrivals and departures are random, and suggested an integer linear program (ILP) formulation with stochastic parameters and a stochastic cost function. The model built upon earlier work by Lusby et al. [6], in which the objective function is a deterministic total cost and the length of stay of each patient is constant. In order to handle the random nature of the process, they considered the PAS problem in a dynamic context, in which the current information about the patients and the system is updated regularly, and then the problem was solved using a deterministic model. Abera et al. [1] also assumed a dynamic context, but with the addition of stochastic parameters to capture the random behaviour of system more closely.

In this paper, we explore the numerical application of the proposed model by Abera et al. [1]. The rest of the paper is structured as follows. In Section 2, we describe the model proposed in Abera et al. [1]. In Section 3 we discuss techniques for an efficient application of the model for large-scale problems, which need to be handled in real-world hospital systems. In Section 4 we construct numerical examples to demonstrate that our stochastic model performs better than Lusby et al. [6] deterministic model. We also show that our model can be applied efficiently for large-scale problems. We conclude with section 5 where we also provide some directions for future research.

2 The Model

We now describe the model in Abera et al. [1].

2.1 Parameters and variables

Let $\mathcal{P} = \{1, \dots, P\}$ be the set of patients, $\mathcal{R} = \{1, \dots, R\}$ be the set of rooms, and suppose that each patient $p \in \mathcal{P}$ needs to be assigned to some room $r \in \mathcal{R}$ on some day $d \in \mathcal{D}$, over the planning horizon $\mathcal{D} = \{1, 2, \dots, D\}$. For each patient, we have information about their age, gender, required treatment, and room preference. The goal is to find an assignment σ that is optimal.

The capacity κ_r of room r is the total number of beds in room r. Let $\widehat{\kappa}_r$ be the maximum allowed total number of patients in room r, after taking into account an overstay risk. We denote by d_p^{plan} the pre-determined admission date of patient p, and by d_p^{reg} their registration date, which is the date at which they become known to the system. Patients are classified into three groups, admitted patients, planned patients, and emergency patients. For emergency patients we have $d_p^{plan} = d_p^{reg}$. Let $d_p \in \mathcal{D}_p$ be the admission day of patient p, where $\mathcal{D}_p = \{d_p^{plan}, \cdots, d_p^{max}\}$ where $d_p^{max} \leq D$ is the maximum acceptable day for patient p to be admitted to the hospital. The length of stay of patient p is the random variable L_p, which records the number of days patient p will stay in the hospital until they get discharged. We assume that L_p takes values $\ell_p = 0, 1, \ldots, \ell_p^{max}$, for some $\ell_p^{max} > 0$. A hospital consists of different wards $\mathcal{W}_i$, $i = 1, \ldots, W$. Wards support specialisms S_u, $u = 1, \ldots, S$. Specialisms correspond to treating some kind of pathology such as cardiovascular disease, oncology, or dermatology. Ward $\mathcal{W}_i$ accepts patients between some minimum age $a(\mathcal{W}_i)$ and maximum age $A(\mathcal{W}_i)$. Let $\mathcal{M} \subset \mathcal{P}$ be the set of all male patients, and $\mathcal{F} \subset \mathcal{P}$ be the set of all female patients. Each room has a specified gender policy, male only M, female only F, the gender of the first patient SG (same-gender policy), or all genders are allowed N. We denote by $\mathcal{R}^M, \mathcal{R}^F, \mathcal{R}^{SG}, \mathcal{R}^N \subset \mathcal{R}$ the sets of all rooms with policies M, F, SG, N, respectively.

An assignment σ of patients to rooms is a collection of decisions $x_{p,r,d}(\sigma)$ and $y_{p,d}(\sigma)$,

$$\begin{aligned} x_{p,r,d}(\sigma) &= \mathbf{1}\{\text{patient } p \text{ is assigned to room } r \text{ on day } d\} \qquad (1) \\ y_{p,d}(\sigma) &= \mathbf{1}\{\text{patient } p \text{ is admitted on day } d\}, \qquad (2) \end{aligned}$$

where $\mathbf{1}\{\cdot\}$ is an indicator function which takes the value 1 if the statement is true, and 0 otherwise. To model the violation of a gender policy in room r on day d, we use variables

$$\begin{aligned} m_{r,d}(\sigma) &= \mathbf{1}\{\text{there is at least one male patient in room } r \text{ on day } d\}, \qquad (3) \\ f_{r,d}(\sigma) &= \mathbf{1}\{\text{there is at least one female patient in room } r \text{ on day } d\}, \qquad (4) \\ b_{r,d}(\sigma) &= \mathbf{1}\{\text{both genders are present in room } r \text{ on day } d\}, \qquad (5) \end{aligned}$$

To model the room features needed by a patient p, we define

$$\begin{aligned} NRF_j(p,r)(\sigma) &= \mathbf{1}\{\text{feature } j \text{ needed by patient } p \text{ is provided in room } r\}, \qquad (6) \\ PRF_j(p,r)(\sigma) &= \mathbf{1}\{\text{feature } j \text{ prefered by patient } p \text{ is provided in room } r\}, \qquad (7) \end{aligned}$$

To model the transfer of patient p from room r to another room r^* on day d, we use the variable

$$t_{p,r,r^*,d}(\sigma) = \mathbf{1}\{x_{p,r,d-1}(\sigma) = 1, x_{p,r^*,d}(\sigma) = 1, r^* \neq r\}. \qquad (8)$$

To model the gender conflict in room r on day d we use the variable

$$b_{r,d}(\sigma) = \mathbf{1}\{Q_{r,d}(\sigma)\}. \qquad (9)$$

where $Q_{r,d}(\sigma)$ is the event that a gender conflict is observed in room r on day d, given assignment σ.

From Abera et al. [1], we have $E(b_{r,d}(\sigma)) = P_r(Q_{r,d}(\sigma))$, and

$$\begin{aligned} 1 - P_r(Q_{r,d}(\sigma)) &= \prod_{\mathcal{M}_{r,d}} x_{p,r,d}(\sigma) P_r(L_p < d - d_p(\sigma)) + \prod_{\mathcal{F}_{r,d}} x_{p,r,d}(\sigma) P_r(L_p < d - d_p(\sigma)) \\ &\quad - \prod_{\mathcal{M}_{r,d} \cup \mathcal{F}_{r,d}} x_{p,r,d}(\sigma) P_r(L_p < d - d_p(\sigma)), \qquad (10) \end{aligned}$$

where $\mathcal{M}_{r,d} = \{p \in \mathcal{M} : x_{p,r,d}(\sigma) = 1\}$ and $\mathcal{F}_{r,d} = \{p \in \mathcal{F} : x_{p,r,d}(\sigma) = 1\}$ is the set of males and the set of females assigned to room r on day d, respectively.

Let $Y_{r,d}(\sigma)$ be a random variable recording the number of patients in room r on day d, given assignment σ. Then $(E(Y_{r,d}(\sigma)) - \kappa_r)$ is the expected excess in room r on day d. Then from Abera et al. [1],

$$E(Y_{r,d}(\sigma)) \;=\; \sum_{p\in\mathcal{P}} x_{p,r,d}(\sigma) P_r(L_p \geq d - d_p(\sigma)). \tag{11}$$

To model the cost of assignment σ, we use the following parameters: the cost $c_{p,r,d}$ of assigning patient p to room r on day d, the cost $c^{(T)}_{p,r,r^*,d}$ of transferring patient p from room r to room r^* on day d, the penalty $c^{(G)}_{r,d}$ incurred for violating the gender policy in room r on day d, the penalty $c^{(O)}_{r,d}$ incurred when the capacity κ_r of room r is exceeded on day d, and the penalty $c^{(De)}_{p,d}$ incurred for delaying the admission of patient p on day d.

2.2 Stochastic ILP

In order to solve the PAS problem, Abera et al [1] formulated the following stochastic ILP

$$\begin{aligned}
\min_{\sigma} \Bigg\{ \quad & \sum_{p\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c_{p,r} \times x_{p,r,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)) \\
& + \sum_{p\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}}\sum_{r^*\in\mathcal{R}} c^{(T)}_{p,r,r^*} \times t_{p,r,r^*,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)) \\
& + \sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c^{(G)}_{r,d} \times P_r(Q_{r,d}(\sigma)) + \sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c^{(O)}_{r,d} \times \left(\frac{\max\{0,\; E(Y_{r,d}(\sigma)) - \kappa_r\}}{\widehat{\kappa}_r - \kappa_r} \right) \\
& + \sum_{p\in\mathcal{P}} c^{(De)}_{p,d} \times \sum_{d\in\mathcal{D}} \left(\frac{d - d_p^{plan}}{d_p^{max} - d_p^{plan}} \right) \times y_{p,d}(\sigma) \quad \Bigg\}
\end{aligned} \tag{12}$$

subject to

$$\begin{aligned}
\sum_{p\in P} x_{p,r,d}(\sigma) \leq \widehat{\kappa}_r, &\qquad \forall r \in R, \forall d \in D & (13)\\
\sum_{d\in D_p} y_{p,d}(\sigma) = 1, &\qquad \forall p \in P & (14)\\
\sum_{r\in R} x_{p,r,d}(\sigma) \geq y_{p,\bar{d}}(\sigma), &\qquad \forall p \in P, \bar{d} \in D_p, d = \bar{d}, \ldots, \bar{d} + l_p - 1 & (15)\\
f_{r,d}(\sigma) \geq x_{p,r,d}(\sigma), &\qquad \forall p \in \mathcal{F}, \forall r \in \mathcal{R} \cap R^{SG}, \forall d \in \mathcal{D} & (16)\\
m_{r,d}(\sigma) \geq x_{p,r,d}(\sigma), &\qquad \forall r \in \mathcal{R} \cap R^{SG}, \forall d \in \mathcal{D} & (17)\\
b_{r,d}(\sigma) \geq f_{r,d}(\sigma) + m_{r,d}(\sigma) - 1, &\qquad \forall r \in R^{SG}, \forall d \in D & (18)\\
t_{p,r,r^*,d}(\sigma) \geq x_{p,r,d}(\sigma) - x_{p,r,d-1}(\sigma), &\qquad \forall p \in \mathcal{P}, \forall r \in \mathcal{R}, d = 2, \ldots, \mathcal{D}. & (19)\\
x_{p,r,d}, \in \{0,1\}, &\qquad \forall p \in \mathcal{P}, \forall r \in \mathcal{R}, \forall d \in \mathcal{D} & (20)\\
y_{p,d} \in \{0,1\}, &\qquad \forall p \in \mathcal{P}, \forall d \in \mathcal{D} & (21)\\
f_{r,d}, m_{r,d}, b_{r,d} \in \{0,1\}, &\qquad \forall r \in \mathcal{R}, \forall d \in \mathcal{D}. & (22)
\end{aligned}$$

Here, (6) and (13)-(15) correspond to *hard* constraints which must be met, the remaining constraints (16)-(22) correspond to *soft* constraints which may be violated but are subject to penalties. The hard constraint in (6) is used in our code to identify the feasible rooms for each patient as well as in constructing the cost matrix $c_{p,r}$ of the objective function.

2.3 Modelling random arrivals and departures

To model the random departures we assume that the random variable L_p follows a discrete phase-type distribution (see Latouche and Ramaswami [5] or Neuts [7]) with parameters $\boldsymbol{\tau}$ and $\mathbf{P}$, we write

$$L_p \sim PH(\boldsymbol{\tau}, \mathbf{P}), \tag{23}$$

so that, for $\ell_p = 1, 2, \ldots$, we have,

$$\begin{aligned} Pr(L_p \leq \ell_p) &= 1 - \boldsymbol{\tau}\mathbf{P}^{\ell_p}\mathbf{1}, && (24) \\ Pr(L_p \geq \ell_p) &= \boldsymbol{\tau}\mathbf{P}^{\ell_p - 1}\mathbf{1}, && (25) \end{aligned}$$

where $\mathbf{1}$ is a (column) vector of ones of appropriate size. That is, we model L_p using an absorbing discrete-time Markov chain with state space $\mathcal{V} = \{1, \ldots, v, 0\}$, where 0 is the absorbing state, with the initial distribution vector $\boldsymbol{\tau} = [\tau_i]_{i=1,\ldots,v}$, and one-step transition probability matrix

$$\mathbf{P}^* = \begin{bmatrix} \mathbf{P} & \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}. \tag{26}$$

Here, states $1, \ldots, v$ may be interpreted as different stages of the medical treatment of a patient, and 0 as the departure of the patient from the system.

For a truncated distribution with $\ell_p = 1, \ldots, \ell_p^{max}$, we have

$$Pr(L_p \leq \ell_p) = 1 - \boldsymbol{\tau}\mathbf{P}^{\ell_p}\mathbf{1} \tag{27}$$

for $\ell_p = 1, 2, \ldots, \ell_p^{max} - 1$, and

$$Pr(L_p = \ell_p^{max}) = \boldsymbol{\tau}\mathbf{P}^{\ell_p^{max} - 1}\mathbf{1}, \tag{28}$$

so that $Pr(L_p \leq \ell_p^{max}) = 1$.

To incorporate the random arrivals that may occur during the planning horizon, we adopt an approach similar to Kumar et al. [4] using the following steps.

1. Generate the random arrivals of emergency patients in the time horizon $[0, D]$ using standard simulation methods for the required distribution (obtained from statistical analysis of the data).

2. Add the set of such generated patients to the problem, and solve it using the model in Section 2.2, treating these patients as registered patients, that is, patients that are known to the system.

3. Repeat Steps 1–2 multiple times then compare the different solutions by running simulations over some long time period, and choose the solution that gives the minimum of all the optimal objective function values.

3 Solution Approach

3.1 Modified stochastic ILP

In order to solve our stochastic model described in Section 2, we modify some of the cost components of the objective function of the stochastic ILP (equation (12)), to improve the efficiency of the calculations.

We note that the components

$$\sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} c_{r,d}^{(G)} \times P_r(Q_{r,d}(\sigma)) \tag{29}$$

and

$$\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c^{(O)}_{r,d} \times \left(\frac{\max\{0,\ E(Y_{r,d}(\sigma)) - \kappa_r\}}{\widehat{\kappa}_r - \kappa_r}\right) \tag{30}$$

are non-linear, and so we make the following changes in order to express the objective function as a linear function.

We introduce a new variable $\gamma_{r,d}(\sigma)$ to replace the max term in (30) and use it as a decision variable with constraints

$$\gamma_{r,d}(\sigma) \geq 0 \tag{31}$$

and

$$\gamma_{r,d}(\sigma) \geq E(Y_{r,d}(\sigma)) - \kappa_r. \tag{32}$$

We replace the right-hand sides of constraints (16) and (17) with

$$x_{pr,d}(\sigma)Pr(L_p \geq d - d_p) - \alpha_f, \tag{33}$$

$$x_{pr,d}(\sigma)Pr(L_p \geq d - d_p) - \alpha_m \tag{34}$$

respectively, where α_f is a small probability of having females in room r on day d, and α_m is a small probability of having males in room r on day d. Then we replace the $P_r(Q_{r,d}(\sigma))$ term in (29) with $b_{r,d,}(\sigma)$.

By including the above changes our stochastic model is now:

$$\begin{aligned}
\min_{\sigma}\{\ f(\sigma)\ \} \;=\; \min_{\sigma}\Bigg\{ & \sum_{p\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c_{p,r} \times x_{p,r,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)) \\
& + \sum_{p\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}}\sum_{r^*\in\mathcal{R}} c^{(T)}_{p,r,r^*} \times t_{p,r,r^*,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)) \\
& + \sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c^{(G)}_{r,d} \times b_{r,d}(\sigma) + \sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c^{(O)}_{r,d} \times \gamma_{r,d}(\sigma) \\
& + \sum_{p\in\mathcal{P}} c^{(De)}_{p,d} \times \sum_{d\in\mathcal{D}} \left(\frac{d - d^{plan}_p}{d^{max}_p - d^{plan}_p}\right) \times y_{p,d}(\sigma) \quad \Bigg\}
\end{aligned} \tag{35}$$

subject to

$$\sum_{p\in P} x_{p,r,d}(\sigma) \leq \widehat{\kappa}_r, \qquad \forall r \in R, \forall d \in D \tag{36}$$

$$\sum_{d\in D_p} y_{p,d}(\sigma) = 1, \qquad \forall p \in P \tag{37}$$

$$\sum_{r\in R} x_{p,r,d}(\sigma) \geq y_{p,\bar{d}}(\sigma), \qquad \forall p \in P, \bar{d} \in D_p, d = \bar{d}, \ldots, \bar{d} + l_p - 1 \tag{38}$$

$$f_{r,d}(\sigma) \geq x_{p,r,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)) - \alpha_f, \qquad \forall p \in \mathcal{F}, \forall r \in \mathcal{R} \cap R^{SG}, \forall d \in \mathcal{D} \tag{39}$$

$$m_{r,d}(\sigma) \geq x_{p,r,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)) - \alpha_m, \qquad \forall r \in \mathcal{R} \cap R^{SG}, \forall d \in \mathcal{D} \tag{40}$$

$$b_{r,d}(\sigma) \geq f_{r,d}(\sigma) + m_{r,d}(\sigma) - 1, \qquad \forall r \in R^{SG}, \forall d \in D \tag{41}$$

$$t_{p,r,r^*,d}(\sigma) \geq x_{p,r,d}(\sigma) - x_{p,r,d-1}(\sigma), \qquad \forall p \in \mathcal{P}, \forall r \in \mathcal{R}, d = 2, \ldots, \mathcal{D}. \tag{42}$$

$$\gamma_{r,d}(\sigma) \geq E(Y_{r,d}(\sigma)) - \kappa_r, \qquad \forall p \in \mathcal{P}, \forall r \in \mathcal{R}, \forall d \in \mathcal{D}. \tag{43}$$

$$x_{p,r,d}, \in \{0,1\}, \qquad \forall p \in \mathcal{P}, \forall r \in \mathcal{R}, \forall d \in \mathcal{D} \tag{44}$$

$$y_{p,d} \in \{0,1\}, \qquad \forall p \in \mathcal{P}, \forall d \in \mathcal{D} \tag{45}$$

$$f_{r,d}, m_{r,d}, b_{r,d} \in \{0,1\}, \qquad \forall r \in \mathcal{R}, \forall d \in \mathcal{D}. \tag{46}$$

$$\gamma_{r,d}(\sigma) \geq 0, \qquad \forall r \in \mathcal{R}, \forall d \in \mathcal{D}. \tag{47}$$

3.2 Heuristics for large-scale problems

Solving the large instances of the resulting mixed integer linear program is difficult, therefore we use a simulated annealing approach similar to that proposed by Ceschia and Schaerf [3]. Simulated annealing (SA) is a stochastic search algorithm which has been successfully applied for solving combinatorial optimisation problems. This algorithm consists of several components, and in this paper, we use the same structure and steps proposed by Ceschia and Schaerf [3] as follows:

- **Solution space**

  The solution space consists of solutions defined by two sets of vectors of equal sizes which is equal to the number of patients. The first vector defines the assignment of room r to patient p. The second vector represent the admission day of each patient p. In our search space, we exclude the room capacity constraints.

- **Initial solution**

  SA starts with an initial solution which plays an important role in convergence of the algorithm to a local optimum. The initial solution can be constructed or generated randomly. In our case, we solve the simplified deterministic ILP which consists of hard constraints (room capacity, patients length of stays, and admission day) and associated costs in the objective function. The optimal solution of this ILP is then improved through the steps of the SA.

- **Neighbourhood relations**

  We use the neighbourhood of the current solution to search the solution space. To obtain a neighbourhood we define three moves similar to that of Ceschia and Schaerf [3]. The first two moves (Move1 and Move2) are related to the room assignment, and the third move (Move3) is related to a possible admission day. For Move1 we first select a patient at random, and then we change the room number to another randomly selected room. For Move2 we swap the assignment of two randomly selected patients. For Move3 which is related to a patient's admission day, we increase (move forward) the admission day of a randomly selected patient. The complete neighbourhood is the union of the three moves. It should be noted that we exclude shifting the patient's admission day over the maximum admission date, as these moves create infeasible solutions.

- **Simulated annealing**

  Ceschia and Schaerf [3] and Lusby et al. [6] used SA to solve the PAS problem. We use the following variant of SA in our solver. As mentioned earlier, an initial solution is created using the simplified deterministic MILP model. We construct a loop which randomly generates a neighbour of the current solution at each iteration. Considering the move ϕ cost difference $\Delta\theta$ between the current solution and the new solution is evaluated. If $\Delta\theta \geq 0$, we take the new output as our current solution for the next move. Conversely, if $\Delta\theta<0$ the current solution is accepted as the new solution with probability $e^{-\Delta\theta/T}$, where T is the parameter denoting temperature. The initial temperature is denoted by T^{max}, which is also the maximum possible temperature. The temperature starts to fall with some cooling rate α. After a given number of iterations N, it leads to the minimum stopping temperature T^{min}. In each cooling step we have a temperature of $T_n = \alpha \times T_{n-1}$. The cooling rate α, the number of solutions sampled at each temperature T and the initial and stopping temperatures T^{max} and T^{min} are the controlling parameters. Regarding the neighbourhood used, we first select the neighbourhood followed by specifying the moves within the neighbourhood. In contrast to Ceschia and Schaerf [3], the neighbourhood is selected with some randomly generated probability for each of the three moves.

- **Parameter tuning for simulated annealing**

  We use similar configurable parameters to that of Ceschia and Schaerf [3]. The performance of the simulated annealing algorithm depends on the settings of these parameters. The simulated annealing solver is tuned using an automatic tool called iterated racing procedure, which allows exploration of large spaces on a large size instances effectively.

3.3 Deterministic ILP model

In order to test the appropriateness of our model for random environments, we compared its performance with the model of Lusby et al. [6] in the numerical examples which we will give in Section 4. Therefore, for completeness, in this section we describe the deterministic ILP in Lusby et al. [6], using the notation introduced earlier in Section 2.

Here, we assume that the length of stay of patient type p is a constant ℓ_p, and that patient p has an overstay risk $O_p \in \{0, 1\}$, where $O_p = 1$ means that the patient will stay $\ell_p + 1$ days in the system, and $O_p = 0$ means they do not. We use the following deterministic ILP, which is based on the model in Lusby et al. [6], with some slight modifications of the notation. Note that $Y_{r,d}(\sigma) - \kappa_r$ here is a constant, equivalent to $z_{r,d}$ in Lusby et al. [6]. Also note that we do not use the parameter $\widehat{\kappa}_r$ in the model below. Similar to Lusby et al. [6], we use the variable $\bar{x}_{p,r,d}(\sigma) \in \{0, 1\}$ to capture the possibility that patient p will stay in room $r \in \mathcal{R}$ for one more night following day d after the discharge due date, due to overstay risk O_p.

The resulting deterministic ILP is,

$$
\begin{aligned}
\min_{\sigma}\{\ g(\sigma)\ \} \quad = \quad \min_{\sigma} \Bigg\{ & \quad \sum_{p\in\mathcal{P}} \sum_{d\in\mathcal{D}} \sum_{r\in\mathcal{R}} c_{p,r} \times x_{p,r,d}(\sigma) \\
& + \sum_{p\in\mathcal{P}} \sum_{d\in\mathcal{D}} \sum_{r\in\mathcal{R}} \sum_{r^*\in\mathcal{R}} c^{(T)}_{p,r,r^*} \times t_{p,r,r^*,d}(\sigma) \\
& + \sum_{d\in\mathcal{D}} \sum_{r\in\mathcal{R}} c^{(G)}_{r,d} \times b_{r,d}(\sigma) + \sum_{d\in\mathcal{D}} \sum_{r\in\mathcal{R}} c^{(O)}_{r,d} \times max\{0,\ Y_{r,d}(\sigma) - \kappa_r\} \\
& + \sum_{p\in\mathcal{P}} c^{(De)}_{p,d} \times \sum_{d\in\mathcal{D}} \left(d - d^{plan}_p\right) \times y_{p,d}(\sigma) \qquad \Bigg\}
\end{aligned} \tag{48}
$$

subject to

$$\sum_{p\in P} x_{p,r,d}(\sigma) \leq \kappa_r, \qquad \forall r \in R, \forall d \in D \tag{49}$$

$$\sum_{d\in D_p} y_{p,d}(\sigma) = 1, \qquad \forall p \in P \tag{50}$$

$$\sum_{r\in R} x_{p,r,d}(\sigma) \geq y_{p,\bar{d}}(\sigma), \qquad \forall p \in P, \bar{d} \in D_p, d = \bar{d}, \ldots, \bar{d} + l_p - 1 \tag{51}$$

$$f_{r,d}(\sigma) \geq x_{p,r,d}(\sigma), \qquad \forall p \in \mathcal{F}, \forall r \in \mathcal{R} \cap R^{SG}, \forall d \in \mathcal{D} \tag{52}$$

$$m_{r,d}(\sigma) \geq x_{p,r,d}(\sigma), \qquad \forall r \in \mathcal{R} \cap R^{SG}, \forall d \in \mathcal{D} \tag{53}$$

$$b_{r,d}(\sigma) \geq f_{r,d}(\sigma) + m_{r,d}(\sigma) - 1, \qquad \forall r \in R^{SG}, \forall d \in D \tag{54}$$

$$t_{p,r,r^*,d}(\sigma) \geq x_{p,r,d}(\sigma) - x_{p,r,d-1}(\sigma) - y_{p,d}(\sigma), \qquad \forall p \in \mathcal{P}, \forall r \in \mathcal{R}, d = 2, \ldots, \mathcal{D}. \tag{55}$$

$$y_{p,d}(\sigma) + O_p \times x_{p,r,d+l_p-1}(\sigma) \leq 1 + \bar{x}_{p,r,d+l_p}(\sigma), \qquad \forall p \in \mathcal{P}, \forall r \in \mathcal{R}, \forall d \in \mathcal{D}. \tag{56}$$

$$\sum_{p\in P}(x_{p,r,d}(\sigma) + \bar{x}_{p,r,d}(\sigma)) \leq \kappa_r + z_{r,d}(\sigma), \qquad \forall r \in \mathcal{R}, \forall d \in \mathcal{D}. \tag{57}$$

$$x_{p,r,d}(\sigma), \bar{x}_{p,r,d}(\sigma) \in \{0, 1\}, \qquad \forall p \in \mathcal{P}, \forall r \in \mathcal{R}, \forall d \in \mathcal{D} \tag{58}$$

$$y_{p,d}(\sigma), t_{p,d}(\sigma) \in \{0, 1\}, \qquad \forall p \in \mathcal{P}, \forall d \in \mathcal{D} \tag{59}$$

$$f_{r,d}(\sigma), m_{r,d}(\sigma), b_{r,d}(\sigma) \in \{0, 1\}, \qquad \forall r \in \mathcal{R}, \forall d \in \mathcal{D} \tag{60}$$

$$z_{r,d}(\sigma) \geq 0, \qquad \forall r \in \mathcal{R}, \forall d \in \mathcal{D}. \tag{61}$$

3.4 Data structure

In our comparisons, similar to Lusby et al. [6], we use the data instances provided by Ceschia and Schaerf [3] see Table 1. The data set has nine families of instances which is labelled as small, medium, or large depending the number of departments, rooms, features, patients, and specialisms. These families were subdivide into short, medium, and long according to the length of planning horizon, which is 14, 28, and 56 days, respectively. For instance, the families with small family size turn into small-short, small-medium, and small-long. The nine families with 50 cases each are aimed at representing real world scenario.

Family	Instances	Departments	Rooms	Features	Patients	Specialisms	Days
Small-Short	50	4	8	4	50	3	14
Small-Med	50	4	8	4	100	3	28
Small-Long	50	4	8	4	200	3	56
Med-Short	50	6	40	5	250	10	14
Med-Med	50	6	40	5	500	10	28
Med-Long	50	6	40	5	1000	10	56
Large-Short	50	8	160	6	1000	15	14
Large-Med	50	8	160	6	2000	15	28
Large-Long	50	8	160	6	4000	15	56

Table 1: Instance overview, source: Ceschia and Schaerf [3].

The data sets were created by an instance generator. The generator was designed to produce the nine families with 50 instances each. As input, the generator took the number of rooms, departments, patients, days, and room features to generate the random examples.

4 Numerical examples

In this section, we construct numerical examples to illustrate the application of our model. For each case, we first solve the deterministic problem as defined in Lusby et al. [6], and then our stochastic model using the ILP approach. We code the mathematical models presented in this paper in MS Visual C++ and use Concert Technology to build the model within C++. We then call CPLEX 12.8.0 MILP solver to solve the MILP parts of our models. We performed the experiments on an Intel(R) Core(TM) i5-7300U CPU @ 2.70 GHz computer with 16 gigabytes of installed memory running on Windows 10 Enterprise.

During the coding, we defined the set of patients, departments, and rooms. From the available set of departments, we considered the age policy and the major and minor specialisms to select the feasible departments for each patient in order to satisfy their requirements with minimum cost violation. Similar to Lusby et al. [6] and Ceschia and Schaerf [3], we set the cost of assignment to the infeasible room ($r \notin \mathcal{R}_f$) to be a huge number ($c_{p,r} = \infty$). We created a cost matrix which consists of all the possible costs of assignment of patient p to each feasible room r, considering all violation costs related to room gender policy, room preference, and room features.

4.1 Output based on small instances

Here, we evaluate the performance of our stochastic model and compare it to deterministic model in Lusby et al. [6]. We ran 20 tests each from a small family size with different planning horizons such as small-short, small-medium, and small-long families. Similar to Ceschia and Schaerf [3], we ran the code with a time limit of 840, 1,680, and 3,360 seconds for short, medium, and long planning horizons, respectively.

In order to test the performance of our model, and in particular to test the need for the use of stochastic parameters, we performed the following calculations for each of the instances.

- First, we ran Lusby et al. [6] to find the optimal solution σ^L.
- Next, we ran our stochastic model to find the optimal solution σ^*. Also, we used the same distribution for the length of stay L_p that Lusby et al. [6] and Ceschia and Schaerf [3] used.
- Further, using the stochastic objective function $f(\cdot)$ of our model in Section 3.1, we evaluated $f(\sigma^L)$, and $f(\sigma^*)$. In order to evaluate $f(\sigma^*)$, we used σ^L as input to the model. The results are summarised in Table 2.

Table 2 shows the results for the three families. Each row describes the outputs of one family using Lusby et al. [6] and our model. Comparing the outputs between $f(\sigma^L)$ and $f(\sigma^*)$, we report for each family the average solution, and the best value attained. The percentage gap between $f(\sigma^L)$ and $f(\sigma^*)$ is reported as $\triangle\ (\%) = \left[\frac{\text{Avg}_{f(\sigma^*)} - \text{Avg}_{f(\sigma^L)}}{\text{Avg}_{f(\sigma^L)}}\right] \times 100(\%)$, to see the assignment cost improvement. In the Instances column, we use parenthesis to show the number of feasible solutions obtained from each family instance. In the small-short family, each instance has 50 patients and a 14 day planning horizon as shown in Table 1. The average total cost of the assignment $f(\sigma^L)$ attained for the small-short family is 3,010. The value of $f(\sigma^L)$ increases to 5,517 in the small-medium, and 15,257 in small-long, as the family size and planning horizon increases. Similarly, the average total cost $f(\sigma^*)$ achieved using our model is 950, 1,922, and 5,538 in the small-short, small-medium, and small-long families, respectively. Therefore, there is a higher assignment cost improvement in $f(\sigma^*)$ compared to $f(\sigma^L)$ in each family, which is -68.43%, -65.17%, and -63.70% in the small-short, small-medium, and small-long families, respectively. These results clearly show that there is a considerable difference between our stochastic model and the model in Lusby et al. [6]. In particular, this indicates that applying the optimal solution based on a model with deterministic parameters to a process that has random arrivals and departures, results in a solution that is inferior, if the goal is to minimise the total expected cost.

Families	**Instances**	$f(\sigma^L)$		$f(\sigma^*)$		$\triangle$ (%)
		Avg	**Best**	**Avg**	**Best**	
Small-short	20 (20)	3,010	2,130	950	525	−68.43
Small-Medium	20 (20)	5,517	4,295	1,922	1,581	−65.17
Small-Long	20 (20)	15,257	12,956	5,538	4,832	−63.70

Table 2: Comparison between Lusby et al. [6] and our stochastic model when applied to the PAS problem with random arrivals and departures. Here, f is the stochastic objective function of our model (see Equation (35)) evaluated for σ^L, the optimal solution of the model in Lusby et al. [6], and σ^*, the optimal solution of our stochastic model, respectively.

4.2 Small instances, with phase-type distribution

Here, we consider the small instances shown in Table 1, but this time, instead of using the distribution of the length of stay L_p as used by Lusby et al. [6] and Ceschia and Schaerf [3], in which L_p for each patient p is drawn from the same log-normal distribution, we used discrete phase-type distributions which were defined in Section 2.3. We used a different distribution for each patient type.

Suppose that there are three types of patients arriving to the system at a frequent rate $\lambda_1 = 5$, a moderate rate $\lambda_2 = 3$, and a less frequent rate $\lambda_3 = 1$ per day, respectively. In order to generate a random number of arrivals of type-i patients, we generated a random number of arrivals that occurred during the time interval $[0, D]$ in the Poisson process with rate λ_i, and then, for each arrival, drew the random arrival time from a discrete uniform distribution on $\{1, \dots, D\}$.

Here, in order to create the instances, we used the total number of arrivals N as shown in the *Patients* in Table 1, that is $N = 50, 100, 200$, and then, for patient type $i = 1, 2, 3$, we used the average number of arrivals $N_i = \left[\frac{N \times \lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}\right]$, rounded to the nearest integer (see Table 3).

Family	N	N_1	N_2	N_3
Small-Short	50	28	17	5
Small-Med	100	56	33	11
Small-Long	200	111	67	22

Table 3: Values of N_i, $i = 1, 2, 3$.

We assumed that patients with a small average length of stay $E(L_p)$ are those that arrive at the most frequent rate, and the less frequently arriving patients are those with longer average lengths of stay. The remaining patients with an average length of stay are considered as moderate arrivals. That is, for patient type $i = 1, 2, 3$ we assumed that the $\boldsymbol{\tau}$ and $\mathbf{P}_i$ for the distribution of $L_p \sim PH(\boldsymbol{\tau}, \mathbf{P}_i)$ were

$$\boldsymbol{\tau} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

and

$$\mathbf{P}_1 = \begin{bmatrix} 0.2 & 0.5 & 0 \\ 0 & 0.1 & 0.4 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} 0.3 & 0.6 & 0 \\ 0 & 0.2 & 0.5 \\ 0 & 0 & 0.4 \end{bmatrix}, \quad \mathbf{P}_3 = \begin{bmatrix} 0.4 & 0.5 & 0 \\ 0 & 0.4 & 0.5 \\ 0 & 0 & 0.7 \end{bmatrix}.$$

Then, to evaluate $Pr(L_p \leq \ell_p)$ for patient type-i we used, for $\ell_p = 1, \ldots, \ell_p^{max} - 1$,

$$Pr(L_p \geq \ell_p) \quad = \quad \boldsymbol{\tau}\mathbf{P}^{\ell_p - 1}\mathbf{1}, \tag{62}$$

and

$$Pr(L_p \geq \ell_p^{max}) = Pr(L_p = \ell_p^{max}) \quad = \quad \boldsymbol{\tau}\mathbf{P}^{\ell_p^{max} - 1}\mathbf{1}, \tag{63}$$

so that the distribution is truncated, with $\ell_p = 1, \ldots, \ell_p^{max}$. Here, we used $\ell_p^{max} = 9$ for each patient type, similar to the distribution in Lusby et al. [6] and Ceschia and Schaerf [3].

We evaluated probabilities $Pr(L_p = \ell_p)$ for patient type-i for all ℓ_p, and recorded as vectors

$$\mathbf{p}_i = [Pr(L_p = \ell_p)]_{\ell_p = 1, \ldots, 9},$$

with

$$\begin{aligned} \mathbf{p}_1 &= \begin{bmatrix} 0.3000 & 0.3100 & 0.1870 & 0.0999 & 0.0512 & 0.0259 & 0.0130 & 0.0065 & 0.0065 \end{bmatrix}, \\ \mathbf{p}_2 &= \begin{bmatrix} 0.1000 & 0.2100 & 0.2790 & 0.1989 & 0.1115 & 0.0553 & 0.0256 & 0.0113 & 0.0083 \end{bmatrix}, \\ \mathbf{p}_3 &= \begin{bmatrix} 0.1000 & 0.0900 & 0.1310 & 0.1429 & 0.1301 & 0.1069 & 0.0827 & 0.0617 & 0.1546 \end{bmatrix}. \end{aligned}$$

The corresponding means for patient type-i are $E_i(L_p)$, with

$$E_1(L_p) = 2.4935, \quad E_2(L_p) = 3.3869, \quad E_3(L_p) = 5.0010. \tag{64}$$

Incorporating the above three distributions and running 20 instances from each family, we evaluated the average objective function $f(\sigma^*)$ using our stochastic model (see Equation (35)). Table 4 indicates that, there is a -65.4%, -49.7%, and -27.2% reduction in the cost of assignment $f(\sigma^*)$ in small-short, small-medium, and small-long families, respectively. We used different probability distributions for different patient types in our stochastic model $f(\sigma^*)$, and compared to $f(\sigma^L)$. Table 4 supports the results in Table 2 even under the use of different probability distributions.

Families	**Instances**	$f(\sigma^L)$		$f(\sigma^*)$		$\triangle$ (%)
		Avg	**Best**	**Avg**	**Best**	
Small-short	20 (20)	2,754.8	2,262	952.71	809.4	−65.4
Small-Medium	20 (20)	6,282.6	4,638	3,161.4	2,665	−49.7
Small-Long	20 (20)	14,994	12,484	10,921.9	9,863.7	−27.2

Table 4: Comparison between Lusby et al. [6] and our stochastic model when applied to the PAS problem with random arrivals and departures, in which the length of stay of patient type-i follows phase type distribution $L_p \sim PH(\boldsymbol{\tau}, \mathbf{P}_i)$, $i = 1, 2, 3$. Here, f is the stochastic objective function of our model (see Equation (35)) evaluated for σ^L, the optimal solution of the model in Lusby et al. [6] where we used $\ell_p = E(L_p)$, and σ^*, the optimal solution of our stochastic model, respectively.

4.3 Large instances, with phase-type distribution

We now apply our model to the large instances. We considered that the length of stay of patient type-i follows discrete phase type distribution $L_p \sim PH(\boldsymbol{\tau}, \mathbf{P}_i)$, $i = 1, 2, 3$ which were defined in Section 2.3, using the same parameter as in Section 4.2.

In this case, due to the large size of the problem, we apply the heuristic method described in Section 3.2. In order to assess the performance of this approach, we compare the best value of $f(\sigma)$ in Equation (35) obtained by using heuristics, denoted $f(\sigma^{SA})$, with the best feasible value of $f(\sigma)$ in Equation (35) obtained by running our code for the ILP described in Section 3.1 in limited running time of 1680 seconds. The output is summarised in Table 5.

Here, we first apply our model to find the best feasible assignment σ^F (where we limit the running time), and then use it as an initial solution for the heuristic method. This example illustrates that we are able to obtain results for large instances in a reasonable amount of computational time which is also practical for the purpose of decision making.

Families	**Instances**	$f(\sigma^F)$		$f(\sigma^{SA})$	
		Avg	**Best**	**Avg**	**Best**
Large-Medium	20 (20)	$53,971.5$	$51,132$	$35,755.4$	$34,035$

Table 5: The best value of $f(\sigma)$ obtained by applying heuristic, $f(\sigma^{SA})$, and the best feasible value of $f(\sigma)$ obtained by using our model, $f(\sigma^F)$ in limited running time. Here, we used the best feasible assignment σ^F as an initial solution for the heuristic method.

5 Conclusion and future-work

In this paper we have addressed the patient admission scheduling problem with a probabilistic approach. We developed a new stochastic model to capture the dynamic nature of the real world scenario. Our new stochastic formulation used a probabilistic length of stay instead of a fixed value because on a day-to-day basis, there is a probability that a patient can depart at any time from the hospital for different reasons.

This model resulted in good quality outputs for problems of different sizes and planning horizons. We evaluated the performance of our stochastic model and compared it to the deterministic model of Lusby et al. [6] for a small sized problem. To compare both models, first we found the optimal solutions σ^L using the Lusby et al. [6] model and σ^* using the Abera et al. [1] model, and we used those solutions to calculate the cost function $f(\sigma^L)$ and $f(\sigma^*)$ using our stochastic model. The solution obtained using $f(\sigma^*)$ indicated that there is an improvement by -68.43%, -65.17%, and -63% in the cost of assignment for all the three small-short, small-medium, and small-long families, respectively. Furthermore, we checked the performance of the Abera et al. [1] model by incorporating three different probability distributions of length of stay using discrete phase-type distribution as in Section 4.2. Instead of using one probabilistic distribution for the length of stay, as did Lusby et al. [6] we used these different distributions. We solved the Abera et al. [1] model and used the optimal solution σ^* to calculate the average cost function $f(\sigma^*)$. We compared $f(\sigma^*)$ with $f(\sigma^L)$ and there was an improvement by -65.4%, -49.7%, and -27.2% in the small-short, small-medium, and small-long families, respectively. Even though the use of different distributions for length of stay to obtain $f(\sigma^*)$ in Table 4 seems to worsen the result compared to the use of one distribution for length of stay $f(\sigma^*)$ in Table 2, the overall results of $f(\sigma^*)$ in both Table 2, and Table 4 compared to $f(\sigma^L)$ shows the use of stochastic model for PAS problems makes it more preferable compare to the deterministic models. Therefore, we conclude that we cannot use deterministic models to represent adequately a stochastic scenario.

In addition, we used a simulated annealing algorithm to solve the large size instances of the stochastic problem. The algorithm used the same configurations (temperatures, cooling scheme, and cost values in the objective function) of Ceschia and Schaerf [3], except that our new algorithm can handle probabilistic length of stay.

There are several directions for future research. First, we can design some experiments to tune the simulated annealing algorithm for stochastic problem. This is crucial as the performance of such heuristic depends heavily on its parameters. Second, we can design other heuristic methods to obtain solutions with good quality for the large scale instances of the stochastic problem. Further, we can explore the effect of phase-type distribution on the performance of our model. Finally, the model can be generalised. One such generalization is to include other decisions such as nurse rostering.

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7 Notation summary

Parameter	Description
p	A patient.
$\mathcal{P}$	Set of all patients.
$\mathcal{M}$	Set of all male patients.
$\mathcal{F}$	Set of all female patients.
r	A room.
R	Set of all rooms.
κ_p	Total number of beds in room r.
d	A day.
d_p	Admission date of patient $p \in \mathcal{P}$.
$\mathcal{D}$	Planning horizon.
ℓ_p^{max}	Maximum length of stay of patient $p \in \mathcal{P}$.
L_p	Length of stay of patient $p \in \mathcal{P}$.
W_i	Ward i, for $i = 1, 2, \ldots, W$.
S_u	Specialism u, for $u = 1, 2, \ldots, S$.
$x_{p,r,d}(\sigma)$	$x_{p,r,d}(\sigma) = 1$, if patient p is assigned to room r on day d. $x_{p,r,d}(\sigma) = 0$, otherwise.
$y_{p,d}(\sigma)$	$y_{p,d}(\sigma) = 1$, if patient p is admitted on day d. $y_{p,d}(\sigma) = 0$, otherwise.
$Y_{r,d}(\sigma)$	A random variable recording the number of patients in room r on day d.
$t_{p,r,r^*,d}(\sigma)$	$t_{p,r,r^*,d}(\sigma) = 1$ if patient p is transfered from room r to room r^* on day d. $t_{p,r,r^*,d}(\sigma) = 0$, otherwise.
$Q_{r,d}(\sigma)$	An event that a gender conflict is observed in room r on day d.
$b_{r,d}(\sigma)$	$b_{r,d}(\sigma) = 1$, if both genders are present in room r on day d, $b_{r,d}(\sigma) = 0$, otherwise.
$f_{r,d}(\sigma)$	$f_{r,d}(\sigma) = 1$, if there is at least one female patient in room r on day d. $f_{r,d}(\sigma) = 0$, otherwise.
$m_{r,d}(\sigma)$	$m_{r,d}(\sigma) = 1$, if there is at least one male patient in room r on day d. $m_{r,d}(\sigma) = 0$, otherwise.

Table 6: Parameters and variables of the model.

Parameter	Description
$c_{p,r,d}$	Cost for assigning patient p to room r on day d.
${c_{p,r,r^*}}^{(T)}$	Cost of transferring patient p from room r to room r^* on day d.
${c_{r,d}}^{(G)}$	Penalty incurred for the violation of gender policy in room r on day d.
${c_{r,d}}^{(O)}$	Penalty incurred when the capacity κ_r of room r is exceeded on day d.
${c_{p,d}}^{(De)}$	Penalty incurred for the admission delay of patient p on day d.

Table 7: Cost components of the objective function.

Parameter	**Description**
M	Male only policy.
F	Female only policy.
SG	Same-gender policy.
N	All genders policy.

Table 8: Gender policies for rooms.

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