

Robust temporal optimisation for a crop planning problem under climate change uncertainty

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ARTICLE INFO

Keywords:

Robust optimisation
Climate change
Crop planning
Deep uncertainty

ABSTRACT

Considering a temporal dimension allows for the delivery of rolling solutions to complex real-world problems. Moving forward in time brings uncertainty, and large margins for potential error in solutions. For the multi-year crop planning problem, the largest uncertainty is how the climate will change over coming decades. The innovation this paper presents are novel methods that allow the solver to produce feasible solutions under all climate models tested, simultaneously. Three new measures of robustness are introduced and evaluated. The highly robust solutions are shown to vary little across different climate change projections, maintaining consistent net revenue and environmental flow deficits.

1. Introduction

Time is an essential part of all human activities, yet its treatment in optimisation problems has been fairly limited and unstructured. The recent work of Randall, Montgomery and Lewis [1] has shown that using an evolutionary algorithm (EA) to optimise over discrete and connected time units gives the ability to do long-term planning in an automated way. The issue, though, becomes a question of the value of the solutions that have been produced. As planning is necessarily forward in time, uncertainty increases the further the temporal margin is pushed, as there may be many valid ways in which necessary values could be predicted. Therefore, measures must be taken to reduce the risk that uncertainty brings. A means to address this comes via *robust optimisation* [2]. Though not specifically designed with time in mind, it is demonstrated here that it can be adapted and is eminently suitable for this purpose. Therefore, in this paper, robust and temporal optimisation methods are combined to address this issue.

There is a great need for techniques of the type previously described. A large sector of many economies that requires extended planning horizons, and which is subject to a great deal of variability due to climate, is agriculture [3]. The problem that is used in this paper, which aims to sensibly conserve water in a drier future, can indeed be classified as a “wicked problem” [4]. It is used as the basis of the development of the robust temporal framework and its formulation is outlined in Randall et al. [1]. It concerns the planting of mixed crops in a given area over time. Cropping, in terms of hectare allocations,

is optimised so that overall net revenue is maximised while the subsequent water environmental flow deficit is minimised. In this problem, the optimisation is carried out over a number of years (a decade for the test problem instances).

In that previous work, only one climate projection scenario could be used. The work in this paper, however, presents a new form of innovative robust optimisation that allows the solver system to be robust against multiple data sets with respect to an element of change across those sets. This then allows the effects of multiple climate change scenarios to be evaluated simultaneously and feasibly using novel multi-objective functions. To the knowledge of the authors, this has not been achieved before. This new robust framework is also generalisable to other problems.

The remainder of this paper is organised as follows. Section 2 describes a crop planning decision problem that will underpin the novel temporal robust framework, while Section 3 examines the area of robustness from an operations research and evolutionary algorithm perspective. Section 4 expands the definition of robustness to suit a broader range of real-world problems. Section 5 explains a robust temporal framework that includes new and adapted robust measures. Section 6 implements these ideas on two decade-long problem instances, one near-term and another far-future, that have known climate change models factored into them. Section 7 elaborates on some of the issues raised with the new form of robustness introduced in this paper. Section 8 gives the conclusions and outlines the next stages of the research.

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2. A crop planning optimisation problem

Agricultural production is one of the most important activities that humanity must undertake on a consistent basis. With growing populations, human induced and natural climate change, it is essential that analytic tools, such as the techniques available to optimisation and data analytics, be used to ensure ongoing food security. Implicit in this is the fact that long term planning horizons for crop planning and harvesting must be considered and factored into the techniques that provide decision support. For reviews of work in the area of optimisation of agriculture and water use the reader is referred to Oyeboode, Babatunde, Monyei and Babatunde [5], Khadem, Rougé and Harou [6], Gebre, Cattrysse, and Van Orshoven [7], and Doorn [8].

A good example of representing agriculture as an optimisation problem was first introduced by Xevi and Khan [9]. They describe and model a problem which aims to return a selection of crops for an area (in their case, the Murrumbidgee Irrigation Area (MIA) in New South Wales, Australia), that maximises a combined net revenue across the chosen crops, while minimising crop variable costs and groundwater pumping expenses. The latter were considered under three separate scenarios of wet, dry and average annualised conditions. The decision variable that their goal programming approach considered was to determine the number of hectares of land that should be devoted to a particular crop. They used a selection of fifteen crops that are commonly grown in the case study region of the MIA. Their model only considered a single year.

As noted by Lewis and Randall [10], there were certain issues with the previous work that could be addressed. Specifically, in Xevi and Khan's model the relationship between water requirements and groundwater pumping needed was not defined. Another issue was that the goal programming approach was not able to examine trade-off solutions between attainable net revenue and water usage. As such, Lewis and Randall [10] extended the previous work by addressing these two concerns in a revised model. They precisely modelled the allocation of surface water and ground water as well as sensibly incorporating variable costs into the costs objective. The important aspect of this work was to add an environmental objective to ensure sufficient downstream flows for environmental purposes. Other reforms included limiting the output of certain crops, based on economic viability. The results, particularly in the dry (minimal rainfall scenario) suggested that crops that society has depended on, like rice and cotton, will not be viable if desired environmental flows are to be maintained. While these results were interesting, it was difficult to use this for long term planning purposes.

To adequately address the crop planning problem above, a temporal component needed to be added. According to Randall et al. [1] a temporal optimisation problem is defined as: "an optimisation problem in which all relevant temporal data is considered, as well as the interactions and cumulative effects of these data" (p. 2). In effect, these problems are a set of joined problems in which each member of the set represents one time unit (such as a year). These time units are sequential, and every time unit is represented in a defined range (such as a decade). These problems recognise that there may be dependencies between the members that will affect the objective value(s) and the feasibility of constraints.

The temporal aspect was introduced by extending the original "annual" expression of the problem across a number of years.¹ A decision vector is constructed, partitioned into several, successive years. Each year consists of two components: (a) a vector of all of the possible crops (16 in work to date) with each element containing the number

of hectares of land assigned to the crop and (b) a vector of all of the months in that year with each element containing the amount of water (in gegalitres) that will be released for environmental purposes. For a 10-year planning horizon and 16 crops this leads to a 280-element decision vector. Each crop has a total crop income that it can achieve for each hectare planted, and the amount of water it will require to grow, given the projected climate data for that year. These values are summed together, i.e., the revenues from year 1 to year 10, are totalled and represent that net revenue objective value. Similarly, the environmental flow deficits for each year are summed to produce the second objective value. This temporal expression of the problem is described in Equations 8–14 (p. 4) of Randall et al. [1].

A trade-off surface generated by the solver will typically consist of solutions having properties from high revenues with large water usage, to lower revenues, but far more sustainable in terms of the water required. Temporal components of the problem were defined as conditions that need to have interactions amongst sequential years. In this context, they related to the perennial plants.² The two temporal components were:

- *Maturity*—As the crop matures over a period of years, the yield percentage will change. For example, after five years, grapes may have a yield of 40% of the total potential crop, whereas this may be 80% at seven years. This value is multiplied by the total crop income for a unit of land that has been planted continuously with the same crop for that period of time. This was complicated by the fact that each new planting from a time period would begin its own maturity cycle. When less area is allocated to a crop in one year than previously, the most recently planted area is assumed to be removed.
- *Establishment and removal costs*—If units of land (hectares) are added to a crop, or removed from it, from year to year, appropriate establishment and removal costs will be subtracted from that year's total revenue. For example, if citrus trees are to be removed this will require a significant amount of work and hence expenditure.

It is important to note that temporal optimisation, for this problem, could only accommodate one set of climate data. As there are valid alternative climate models, the development of a robust version of this concept was necessary, which is defined next.

3. Robust optimisation

Robust optimisation has been applied for both exact and heuristic approaches. Given the nature of the problem in this paper, and the fact that heuristic algorithms will be used, the focus will be placed on the latter. However, in terms of the former, some well-known works are described here.

Bertsimas and Sim [11] present a robust framework and show that the degree of robust conservatism for potential constraint violation can be incorporated into altered integer linear programs. Using a portfolio and knapsack problem instances, they were able to adequately determine the probability of constraint violation and robust cost. Ben-Tal and Nemirovsky [12] distinguish between hard and soft constraints for robust problems expressed as linear programs. Using a robust counterpart, they ensure that the former can be satisfied, even under uncertain conditions. Gorrisen, Yanıkoğlu and Hertog [13] provide a practical guide to using robust optimisation from an operations research oriented perspective. In it, amongst other topics, they describe multi-stage and adjustable robust optimisation, how to choose the uncertainty set and what is truly meant by the worst case in a robust setting. Additionally,

¹ As a temporal measure, years were used for this problem, given the planning horizons that farmers use. Temporal optimisation applied to other problems will use different units. For example, in scheduling or planning problems, days may be a more appropriate unit.

² These are plants such as grapes and citrus which can be planted for two years or more, but can exist and be productive potentially for decades.

the different aspects of robust solutions (such as probability distributions and sampling methods) are set forth giving a basis for ways in which the quality of different robust solutions can be compared.

Heuristic algorithms have often been used in solving large and intractable problems, to provide approximate solutions in relatively short periods of computational time [14]. Metaheuristic approaches, such as evolutionary algorithms, are appropriate for real-world problems that do not often have a single, clearly specified objective (they are multi- or many-objective in nature), or are difficult (NP-hard) with moderate size. The crop selection component of the problem considered here can be considered a variant of the knapsack problem [15] in which each crop (in each planning year) represents an item that may be selected, but for which the quantity (1–120 k) must also be chosen, subject to land area and water availability constraints. The additional 120 decision variables relating to environmental water flows further add to the problem's complexity.

While the concept of multi-criteria optima was first espoused in the 19th century [16] and Pareto optimality was formally defined at the start of the 20th century, it was not until the publication of Pareto's work in English in 1971 [17] that its use began to grow rapidly in the computational sciences. Now its theory is well established and resources for its practical application are readily available [18]. The outcome from the use of metaheuristic evolutionary algorithms for multi-objective optimisation problems is a set of solutions that approximate a Pareto-optimal set. As such, they are close to the achievable envelope of feasible solutions.

In the context of real-world problems, uncertainties are inevitable: they may arise in input parameters, environmental or operating conditions, or the outputs generated (for example, from numerical modelling.) Two main approaches to dealing with uncertainty are stochastic optimisation [19] and robust optimisation [20].

To apply stochastic optimisation one requires some probabilistic knowledge of the distribution of uncertainties [13], which is often unavailable for real-world problems. As a case in point, the application area of the work described in this paper does not provide this sort of information.

Robust optimisation refers to finding optimal solutions for a particular problem that have least variability in response to probable uncertainties. It does not require any probabilistic knowledge but instead assumes that uncertain parameters arise from an "uncertainty set". The instances of this set are denoted as *scenarios* [21].

Although these uncertainties are usually small perturbations, they can have significant impacts on outcomes. Beyer and Sendoff [20] identify that for certain optima in a search space, small perturbations in solution values would lead to large changes in objective values. In the context of real-world problems (such as engineering problems) that have certain tolerances, this may lead to unacceptably large variations in performance. Thus robust optimisation is concerned with finding solutions that, when uncertain quantities are varied, display only small amounts of change to the value of the objective function or functions. While these solutions may not represent the global optima, they will fare better under uncertainty as objective variation is minimised.

In their seminal work, Deb and Gupta [22] develop two definitions for robustness for multi-objective optimisation and approaches to their solution that they label Type I and Type II:

- For Type I, the problem objective functions are replaced by their means over a given neighbourhood, giving "expectation values", which are then minimised (assuming, without loss of generality, that the objective functions are to be minimised.)
- For Type II, objective functions are constrained below a given "variance" threshold.

Using a number of standard multi-objective test functions and the real-world welded beam problem running under Non-dominated Sorting Genetic Algorithm II (NSGA-II) [23], they found that Type II was a more practicable approach. It was determined that more work

needed to be done to reduce the computational time required, particularly for calculating neighbourhood values. Section 4 expands on these mechanisms to support their application to a broader range of real-world problems. Robust applications since have largely followed these perturbation-based approach models.

Central to these methods is an implicit assumption that problems have some underlying "correct" solutions that have been perturbed by features of the problem environment. This can be expressed as a belief that the feasible solution set remains unchanged in different scenarios [21]. If this does not hold for a given problem, the feasible set is defined as the set of solutions feasible for every scenario. There is another implicit assumption in this definition that, for practical purposes, the perturbations are relatively minor.

There have been recent works that have moved beyond tolerances, to allowing robust methods to consider changes in data and multiple data sets. These data sets (scenarios) are either generated by random perturbations of an original data set or are drawn from real-world observations. Typically, the solver will consider each data instance separately and generate different solutions that satisfy these. After the solver has completed, post processing, by comparative metrics and discussion, then determines how robust the solutions are. Good examples of this type of approach are by Toklu, Gambardella and Montemanni [24] and Toklu, Yanik, and Montemanni [25] when solving the travelling salesman problem and vehicle routing problem, respectively. Beh, Zheng, Dandy, Maier, and Kapelan [26] also take the novel approach of using artificial neural networks to assess objective function values and robust measures for each of the scenario solutions.

Ide and Schöbel [21] present a survey paper of robust applications up to the year 2016. One of the most important findings is the different types of solutions that robust applications produce. They refer to solutions as being "flimsily robust" if they are only robust efficient in a *single* scenario, or "highly robust efficient" if they are efficient for *all* scenarios. They admit that many real-world problems are not likely to have many highly robust efficient solutions. A further class of "light robustness" is defined in reference to a "nominal" scenario, chosen as "most likely" or "most important".

For the problem addressed in this work, there are a set of chaotically different, equally probable scenarios. No single scenario is considered, *a priori*, more likely or important than others. Furthermore, the actual likelihood of a particular scenario cannot be determined [27], i.e., the problem is subject to "deep uncertainty" [28–31]. Perturbations are sufficiently large that standard (non-robust oriented) algorithmic mechanisms are inadequate to produce anything other than "flimsily robust" solutions. It should be noted that the uncertainty affects the *feasibility* of the solution. However, as there are no *nominal* parameter values, alternative algorithmic approaches focused on achieving minimal deviation from such (c.f., Gabriel, Murat and Thiele [32]) are thus impractical.

In fact, there is still "no clear methodology on how to address robust problems" [33] (p. 8). For the target problem, exceeding constraints imposed by limited resources is not possible, and the consequences of decisions that lead to infeasible solutions can be catastrophic in financial terms, endangering the future viability of agricultural enterprises. In these circumstances, useful solutions must be highly robust efficient, sometimes termed "strictly robust" [33]. Some computational approaches to achieve useful solutions to the problem are described in Section 4. The novel form of robust temporal optimisation (RTO) described there aims to produce highly robust efficient solutions.

4. An expanded set of robust optimisation components

As outlined in Section 3, in the literature to date for both single- and multi-objective optimisation, quite a few robust implementations relate to solution tolerances. This means minimising the variation across a sample of neighbouring points in state space. While this is clearly useful, this paper takes a broader approach. In general terms, a "robust" solution is robust *with respect to* some aspect of the problem that varies,

Table 1
An expanded set of robust optimisation components.

	Description
Quality measures ($f(x)$)	
Original objective(s)	In cases where robustness is sought with respect to implementation tolerance, the solution's original value(s) may be used; on its own this offers no guarantee of robustness
Average	Type I as defined by Deb and Gupta [22]
Median	An alternative to average that may better suit problems where the variability is intrinsic to the problem and there is no original, 'central' solution
Variability measures ($var(x)$)	
Range	Type II as defined by Deb and Gupta [22]
Standard deviation	Combines features of Type I and II by capturing both variability and central tendency
Maximum variation per component	Where a solution has components for which variation (range, standard deviation, etc.) can be measured independently, taking the maximum variation across components may be suitable. (For the multi-year crop planning problem each year is a component.)
Controls	
None	No direct control over robustness is employed. If average or median is used as a quality measure this will implicitly respond to <i>some</i> variation in solution quality
Constraint	Solutions are considered infeasible if the variability measure exceeds a user-defined threshold: $var(x) > tol$ (part of Deb and Gupta's Type II)
Penalty	A weighted penalty is applied to the selected quality measure based on the degree of variability: $f(x) - w \times var(x)$
Objective	The variability measure is treated as an additional objective, to be minimised

with implementation tolerance being just one example. This idea, as seen in the previous section has been emergent in the literature and examples include changing data files, as has been covered in Section 3. A robust optimisation algorithm must select which aspects of the problem it seeks to be robust to. In the example agricultural planning problem solved in the present work, solutions need to be robust to variations in climatic conditions over multiple years, represented by the variability in predictions from different climate models. Each of these models can be considered as a separate scenario.

The range of mechanisms for measuring and ensuring robustness is also broader than the Type I and II definitions proposed by Deb and Gupta [22]. Type I is not a measure of robustness, but of the central tendency of a solution's objective values when implemented, while Type II is *one* measure of robustness – the range of observed objective values – conflated with a particular, constraint-based mechanism for ensuring robust solutions. Considered more broadly, this implies the existence of various components to a robust optimisation algorithm, a subset of which is summarised in Table 1.

Consequently, and in consideration of the above, discovering robust solutions for a problem requires at least four attributes:

1. How is robustness defined? Robustness is, in fact, a relative measure and must be defined relative to a particular quantity. The work in this field to date has had an emphasis on *implementation robustness* whereby additional sample solutions nearby in the design space are generated so as to determine the level of variation. The present work, however, focuses on robustness with respect to different climate scenarios with each solution being re-evaluated under each scenario.
2. What is the measure of a solution's quality? This could be the original solution's value (possible when considering implementation robustness, but inappropriate for the current problem of crop planning), or the average or median over sampled objective values.
3. How is variability (a lack of robustness) measured? While range has been a commonly used measure in past work, standard deviation can be a more sophisticated measure of spread that does not need a central point (and hence is suitable in situations where a single 'original' solution does not exist or when a single solution can be evaluated under different, equally likely scenarios). Sample standard deviation should be used. Approaches

for implementation robustness only ever sample a subset of neighbouring solutions. In the present crop planning problem, there are many potential climate models that may exist and only a subset are being used. Additionally, if a problem has multiple components for which objective values may be measured separately, variability may be measured for each and a summary measure (like maximum) taken of that.

4. How can robustness be enforced? The aim is to encourage or enforce robust solutions throughout the search process. Options include using no active control (allowing the summary quality measure to account for any lack of robustness), treating robustness as a constraint, using variability as a penalty, or adding robustness as an additional objective to be minimised.

It may be noted that, consistent with the approach of Mirjalili, Lewis et al. [34,35], all the robustness methods implemented make use of *existing* solution samples. No additional solutions are constructed or sampled in order to assess the degree of robustness.

5. A robust temporal framework

Referring to the working definition of temporal optimisation in Section 2, an inevitable part of it is projecting conditions into the future. For the problem under consideration, these are the climate conditions which affect a crop's water needs. Any such projections will have a degree of uncertainty attached to them. Hence the need for a set of tools and concepts that are in the realm of robust optimisation [2,36].

As previously discussed, Beyer and Sendoff [20] describe the chief characteristics of robust optimisation as being that it is immune to parameter and model sensitivity. The temporal version of the water management problem has a number of variables/parameters that can change with time. These are any of those that are now augmented by a y (year) index. The major form of uncertainty inherent to this model comes from climate, in particular the prediction of rainfall and temperature over the coming years and decades. In the temporal model of this problem, these values are used to calculate the water requirement ($WREQ$) values for each crop.

Randall et al. [1] based their temperature and rainfall predictions on a large study conducted into climate ensemble methods [37], which focused on New South Wales and the Australian Capital Territory. This is referred to as NARCLiM (NSW/ACT Regional Climate Modelling

Project) and covers the area of interest, namely the Murrumbidgee Irrigation Area (MIA). Twelve different climate models are developed for the region in the timeframes of 2020–2040 and 2060–2080. The work herein will concentrate on the first decade of each of these ranges. The 12 models are all combinations of four global models downscaled by three regional models. Each of these models predicts monthly rainfall and temperature in a defined sub-region. The output of each model can be considered as a separate scenario. The temperature and rainfall values are used to calculate the *WREQ* values, expected river flows and cost of groundwater pumping necessary for the problem. The main question is how to sensibly include these climate change models as part of a generalisable robust approach. Three initial candidate approaches have been identified below.

For this problem, and others like it, solutions in the local neighbourhood in decision space (defined by smooth changes in crop allocations or water usage) are very similar to each other. Variation in solution performance comes from the actual climatic conditions encountered when it is implemented in the field, which is captured by using different data sets to predict those values. As introduced in Section 4, this approach will be robust with respect to climate change models. Accordingly, each solution generated by the solver is evaluated over all possible scenarios (in this case climate models) simultaneously rather than generating separate solutions for each scenario. These climate models define the relevant neighbourhood (which has a size of 12).

The main aims of this form of robust optimisation is to (a) find solutions that can satisfy all models (data sets) and (b) minimise the variation for the two objective values across these models. Therefore, in terms of the latter, rather than use difference from a central point to characterise dispersion, range or sample standard deviation are used instead. This makes intuitive sense for the problem under consideration as solutions are sought that, across the various climate models, are very close to one another in terms of net revenue (*NR*) and environmental flow deficit (*EFD*). While these may not necessarily produce the best solution in either objective, at least farm and regional planning can be undertaken with greater certainty. The normal robust dominance rules will then apply to determine if the solution will be eligible to be part of the attainment surface. If the solution is not feasible under the other climate models, then it simply needs to be reported to be infeasible (as it would fail any robustness test).

The three approaches examined here are a subset of the many possible defined by the components in Table 1. All three use *average* across the climate models' objective values as the quality measure for solutions (all models are considered equally likely), with two applying a penalty-based approach to control the degree of robustness. This gives three approaches, illustrated below and in Algorithm 1:

1. **Average:** Solutions are assigned the average of the objective measures across climate models, with no direct control over robustness. This is the baseline, control setting, similar to Deb and Gupta's Type I.
2. **Range:** Each solution's objectives are penalised by a weighted range across models, hence $NR = avg(NR) - w \cdot range(NR)$ and the equivalent for *EFD*, where *w* is the weight (a parameter).
3. **Maximum annual variation:** Variation (in this case, standard deviation) is measured across models for each year in the planning horizon, and the maximum of these is used as a weighted penalty.

Note that given the robust optimisation components identified in Table 1 many other possibilities exist to construct a robust technique for this problem. The above three examples avoid the need to pre-select an acceptance threshold (as would be used in a constraint-based approach) and are good as an initial investigation. Integrating these into the overall solver framework yields Algorithm 2.

Algorithm 1 Robust Temporal Solution Evaluation

```

for  $s \in S$  do
   $(NR_s, EFD_s, feasible_s) \leftarrow$  Evaluate the solution using scenario  $s$ 
end for
 $feasible \leftarrow feasible_s \quad \forall s \in S$ 
 $(NR, EFD) \leftarrow$  Average  $(NR_s, EFD_s)$  across  $s \in S$ 
if robustness is Average then
   $(penalty_{NR}, penalty_{EFD}) \leftarrow (0, 0)$ 
else if robustness is Range then
   $penalty_{NR} \leftarrow w \cdot range(NR_{s \in S})$ 
   $penalty_{EFD} \leftarrow w \cdot range(EFD_{s \in S})$ 
else if robustness is Maximum annual variation then
   $penalty_{NR} \leftarrow w \cdot \arg \max \{stdev(NR_{s \in S}^y), y \in Years\}$ 
   $penalty_{EFD} \leftarrow w \cdot \arg \max \{stdev(EFD_{s \in S}^y), y \in Years\}$ 
end if
 $(NR, EFD) \leftarrow (NR, EFD) - (penalty_{NR}, penalty_{EFD})$ 

```

Algorithm 2 Robust Temporal Optimisation Framework

```

 $S \leftarrow$  Load problem scenarios
 $P \leftarrow$  Generate  $m$  solutions
while termination criteria not met do
   $P' \leftarrow$  Generate  $n$  solutions from  $P$  (using any suitable EA)
  for  $s \in P'$  do
    Evaluate robust objective value of  $s$  using Algorithm 1
  end for
   $P \leftarrow$  Non-dominated sorting of  $P \cup P'$ 
end while
Output  $P$ 

```

6. Computational experience

As mentioned above, the aim of this work is twofold. The first, and most important question is, can a robust model and implementation be produced such that solutions will be feasible across all of the climate models that are presented to it? The second question is, do the proposed methods minimise variations across the two objective values of *NR* and *EFD*?

In regard to the first question, initial experimentation was undertaken in which, for each decade data file, the 12 climate models were each run separately and the final archive of each of these 12 runs preserved. After this, each of the solutions in a particular archive was evaluated against the other 11 climate models. The aim was to determine if any of the solutions were feasible across more than one model. For the data files used here (described next), *none of the solutions were generally feasible*. In essence, there was *zero robustness* exhibited by the existing temporal optimisation approach as developed by Randall et al. [1].

This section applies the robust techniques described in Section 5 with two decade-long data files. As mentioned, the extent to which the generated solutions are now valid and feasible across the climate models will be tested, as well as the degree to which they vary. In a practical sense, the latter is very important as it will point to crop mixes that will give more certainty to farmers and regional planners, no matter which of the climate change models turns out to be correct.

6.1. Temporal decadal data

As previously indicated, the set of climate change models used in this study are from the NARCLIM Project 1.0 [37]. This is a research partnership between the New South Wales and Australian Capital Territory state governments in Australia and the Climate Change Research Centre at the University of New South Wales. It incorporates four

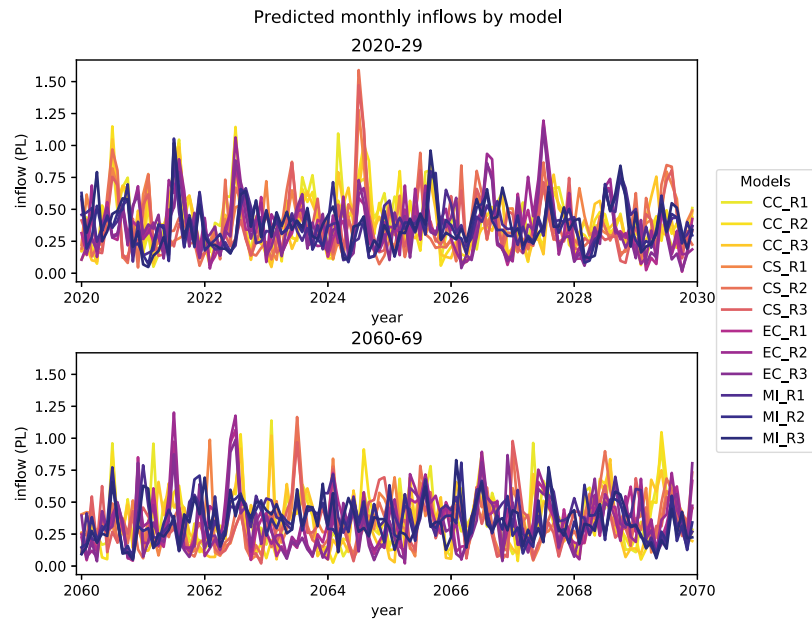


Fig. 1. Predicted monthly water inflows for the 12 climate models (four model families, each with three regional models). Seasonal patterns such as high winter rainfall are evident, but considerable disagreement exists between models about the timing of precipitation.

global climate models named CCCMA3.1, CSIRO-MK3, ESCHAM5 and MIROC3.2. Each of these is dynamically downscaled by three regional climate models, thus giving 12 models in total. The complete data sets include predictions for three time periods: 1990 to 2009 (base), 2020 to 2039 (near future), and 2060 to 2079 (far future). The predictions for each model consist of daily temperature and precipitation levels. Meteorological data are available on a 10 km grid across the NARCLIM domain, which covers most of South East Australia.

For this study, the predominantly future decadal periods of 2020–2029 and 2060–2069 are used. Note that in the previous work of Randall et al. [1], only the model CCCMA3.1_R1 (the CCCMA global model combined with the first local downscaled model) was used as it was deemed an average model. This is now simply one of the 12 alternative models that the robust temporal framework uses.

As in the precursor work [1], water requirements and projected income per hectare for 16 crops are used: rice, wheat, barley, maize, canola, oats, soybean, winter pasture, summer pasture, lucerne, vines, winter vegetables, summer vegetables, citrus, stone fruit, and cotton.

As mentioned above, solutions produced under one climate model are generally infeasible when re-evaluated under other climate models. This is due to disparities in the timing of rainfall and resultant monthly inflow totals predicted by each model (see Fig. 1), which causes solutions optimised for one model to fail a groundwater pumping constraint as they require more water than is (legally) available for one or more of the 120 months in the planning horizon. Fig. 1 presents a qualitative view of the disagreement between climate models' rainfall predictions, which concerns not just annual or decadal total inflow, but the months in which it is predicted to occur. As crops' water needs vary by month, the timing of inflows is important for which crops can be effectively grown.

Consequently, modified instances were created that used the minimum per month inflow across all 12 models (instances still differed in crop water requirements and anticipated cost of groundwater pumping based on temperature estimates). This is a form of *a priori* robustness, because solutions are *guaranteed* to be feasible with respect to water consumption and the groundwater pumping constraint. However, they will also be *highly conservative*—a point that will require future investigation.

Given that the modified instances all share a common set of inflows there will be no observable variation in the *EFD* objective in the

present study. Therefore, the remainder of the investigation focuses on the robustness of the net revenue *NR* objective.

6.2. Experimental design

The set of experiments in this paper set out to test the notion that the revised form of robustness produces valid solutions across a range of models (or data sets) and to characterise the degree of variability that each of the three measures produces (from Section 5). In particular, the experiments seek to explore the impacts of the choice of penalty-based robustness approach and associated penalty weights. Both range and standard deviation were investigated as measures of variability, with range used in combination with the final solution value and standard deviation used with the maximum per-year measure of variability across models. When using the *range* approach, the penalty weight $w \in \{0.1, 1, 10\}$, while when using the *maximum annual variation* approach $w \in \{1, 10, 100\}$, as the base penalty is an order of magnitude smaller than the range, being the variability for a single year instead of across the entire decade.

The solver used is multi-objective Differential Evolution (DE) applied to the temporal crop planning problem defined in Randall et al. [1]. This was effective at producing trade-off solutions to this problem for a single climate scenario. The solver uses the solution generation mechanics of DE/rand/1/bin [38] with newly generated and archive solutions filtered using non-dominated sorting at each iteration. The DE parameters were $Cr = 0.5$ and $F = 0.8$ (as per Randall et al. [1]). In this work, 11 randomised trials were performed for each combination of robustness approach and penalty weight (where applicable) for the two decadal instances 2020–29 and 2060–69.

Search progress in objective space was measured by comparing the *generational distance* (GD) [1,39] between the attainment surface defined by the average *NR* value per solution and an artificial Pareto front generated by selecting the best trade-off solutions produced across all approaches. The values reported are thus indicative of relative performance more than absolute performance. To ensure both objectives carry equal weight, objective values are normalised within the experimentally observed bounds for each decadal problem.

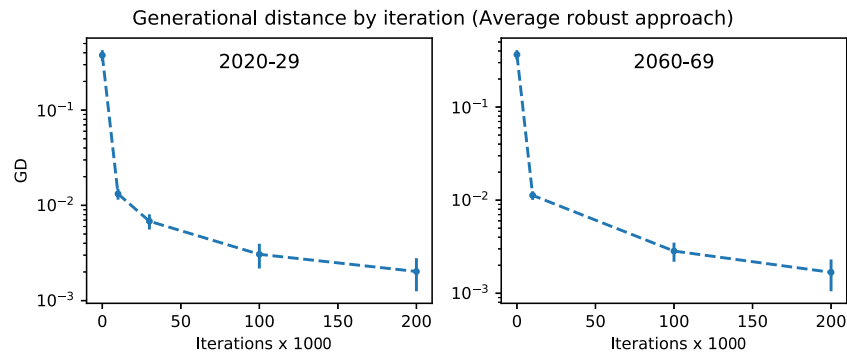


Fig. 2. Generational distance for the *Average* robustness approach over time. Error bars show the 95% confidence interval on the plotted averages (across 11 trials).

Table 2

95% confidence intervals on GD by robustness approach and approximate performance rank (bold indicates a statistically significant result).

Robustness approach	2020–2029	Rank	2060–69	Rank
<i>Average</i>	[0.0022, 0.0040]	3	[0.0022, 0.0035]	= 5
<i>Range</i> , $w = 0.1$	[0.0026, 0.0047]	= 4	[0.0019, 0.0035]	= 5
<i>Range</i> , $w = 1$	[0.0020, 0.0036]	= 1	[0.0017, 0.0032]	= 3
<i>Range</i> , $w = 10$	[0.0028, 0.0051]	= 4	[0.0016, 0.0027]	= 1
<i>Max annual variation</i> , $w = 1$	[0.0029, 0.0046]	= 4	[0.0017, 0.0033]	= 3
<i>Max annual variation</i> , $w = 10$	[0.0021, 0.0037]	= 1	[0.0013, 0.0029]	= 1
<i>Max annual variation</i> , $w = 100$	[0.0046, 0.0064]	7	[0.0021, 0.0034]	= 5

6.3. Results

Initial investigation found that, regardless of robustness approach used, the algorithm achieved most of its progress within 100,000 iterations (with 1,000,000 solutions being generated). Doubling the iterations only led to a further improvement of the order of 10^{-3} in the generational distance measure. Fig. 2 shows the average GD for solution sets produced by the *Average* robustness approach over time. While the algorithm continues improving beyond 100,000 iterations, the improvement is slight and in practice the relative performance of the different approaches (depicted in later figures) is relatively stable. This provides confidence that the attainment surfaces shown in the figures are close approximations to the Pareto-optimal set of solutions, the envelope of achievable, feasible solutions. Hence, all other experimental runs are stopped after 100,000 iterations.

Table 2 shows the 95% confidence intervals on the GD achieved by each robustness approach for the two problems. For 2020–29, both *Range* and *Max annual variation* with moderate penalty weights produce the best quality attainment surfaces, closely followed by *Average*, with *Max annual variation* with $w = 100$ (the maximum tested) producing the poorest attainment surfaces (a statistically significant result based on pairwise t-tests with $\alpha = 5\%$). There are no statistically significant differences in GD for the 2060–69 problem, although there the data suggest that *Range*, $w = 10$ (its maximum) and *Max annual variation*, $w = 10$ achieved superior results, while *Average* and *Max annual variation*, $w = 100$ performed relatively poorly.

Figs. 3 and 4 show the median attainment surfaces (in terms of GD) produced by each approach for 2020–29 and 2060–69, respectively. The top-left plot shows the surfaces defined by the *average(NR)* of each solution, while the remaining plots show the surfaces for each approach with error bars showing *range(NR)* for each solution. Note that the minimum achievable *EFD* is 936,539 for 2020–29 and 2,002,946 for 2060–69, and that every robustness approach had one or more (often all) trials discovering solutions with that minimum *EFD*. Illustrative objective values from different regions of the attainment surfaces for the best performing approaches (in terms of generational distance, not necessarily in terms of robustness) are presented in Table 3.

Firstly, it should be remembered that solutions are robust in *EFD* by design. Success in producing solutions robust in *NR* is measured

by examining the average per-solution *range(NR)*. As the error bars in Figs. 3 and 4 show, increasing the penalty weight for *Range* and *Max annual variation* can reduce the amount of variation in *NR*. This is further illustrated in Fig. 5, which shows the average per-solution *range(NR)* of solutions produced by each approach, within different *NR* bins. Note that the axes in the figure are scaled to improve readability, as the focus is on the relative differences between robustness control measures, not the absolute differences between approach or decadal problem instances. The observed *range(NR)* is proportional to *average(NR)* (and the amount of land allocated to cropping), so this *post hoc* measure of performance cannot be calculated across an entire solution set. Both *Range* and *Max annual variation* with their highest penalty weights produce clearly more robust solutions on the 2020–29 problem, which is also true for *Range*, $w = 10$ on the 2060–69 problem. It is not evident why *Max annual variation*, $w = 100$ did not produce more robust solutions (in terms of *range(NR)*) on the 2060–69 problem, but inspection of the solutions produced by the two penalty-based approaches suggest that *Max annual variation*, $w = 100$ reduced variability in the penalty values across the population of solutions, which consequently makes the penalty ineffective (all solutions' *NR* are reduced by similar amounts).

Given the lack of directly comparable robust techniques for this problem, additional experiments were conducted using the most effective robustness approach observed here with a strongly negative weight, *Range*, $w = -10$. This makes the algorithm seek less robust solutions, thereby providing an approximate upper bound on the degree of variability in the *NR* objective, which is plotted in black in Fig. 5. The degree of variability is $\sim 2.4\%$ (of a solution's *average(NR)*) when using no active robustness control, $\sim 1.7\%$ when using the best performing control (*Range*, $w = 10$) and $\sim 3\%$ when actively seeking less robust solutions. That the observed variability for techniques without active control over robustness (and those which penalise variability too weakly) lie approximately half way between the approximated upper bound and the best results observed suggests that the penalty-based techniques are able to find genuinely robust solutions for this problem.

It is interesting that while *Max annual variation*, $w = 100$ produces more robust solutions on 2020–29, its performance in terms of GD (a measure of the overall quality of the solutions produced) is the poorest, suggesting that there can be a trade-off between seeking robustness and search progress.

Table 3

Sample objective values for individual solutions from median trials for approaches achieving equal best GD performance (not best robustness). Objective region 'Best *EFD*' is a solution with equal-best *EFD* but highest *NR* within an attainment surface.

Robustness approach	Objective	<i>EFD</i>	<i>NR</i> (\$B)			<i>NR</i>
	Region	(PL)	Average	Min	Max	Range
2020–29						
Range, $w = 1$	Best <i>EFD</i>	0.94	1.01	1.01	1.03	1.9%
	Midpoint	1.10	1.45	1.43	1.47	2.3%
	Max <i>NR</i>	1.92	1.88	1.86	1.91	2.4%
Max ann. var., $w = 10$	Best <i>EFD</i>	0.94	1.00	0.99	1.02	2.8%
	Midpoint	1.13	1.49	1.47	1.51	2.4%
	Max <i>NR</i>	2.08	1.97	1.95	2.00	2.5%
2060–69						
Range, $w = 10$	Best <i>EFD</i>	2.00	0.63	0.62	0.63	2.1%
	Midpoint	2.14	1.13	1.12	1.14	1.6%
	Max <i>NR</i>	3.47	1.64	1.63	1.65	1.6%
Max ann. var., $w = 10$	Best <i>EFD</i>	2.00	0.76	0.75	0.77	2.4%
	Midpoint	2.11	1.12	1.11	1.14	2.3%
	Max <i>NR</i>	2.91	1.49	1.46	1.50	2.5%

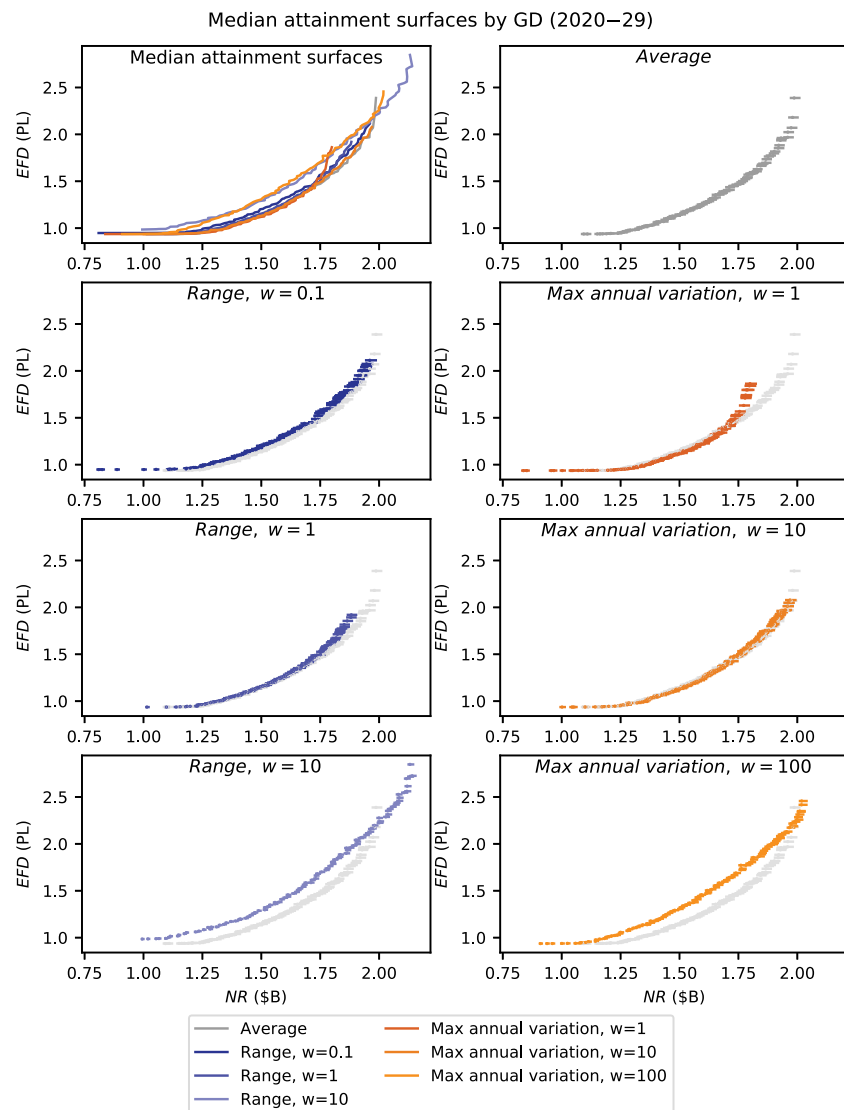


Fig. 3. Median attainment surfaces (selected by GD) for each robustness approach on 2020–29 problem, with *NR* objective range shown. The median attainment surface for *Average* is reproduced, faded, on each plot to provide a reference.

7. Discussion

The highly robust efficient solutions produced above required highly conservative estimates of water availability, a design decision that will

(unnecessarily) constrain crop production and revenues. At the same time, the results confirm that crop selections in the future will need to change in response to the deep uncertainty posed by climate change. Each of these issues is discussed below.

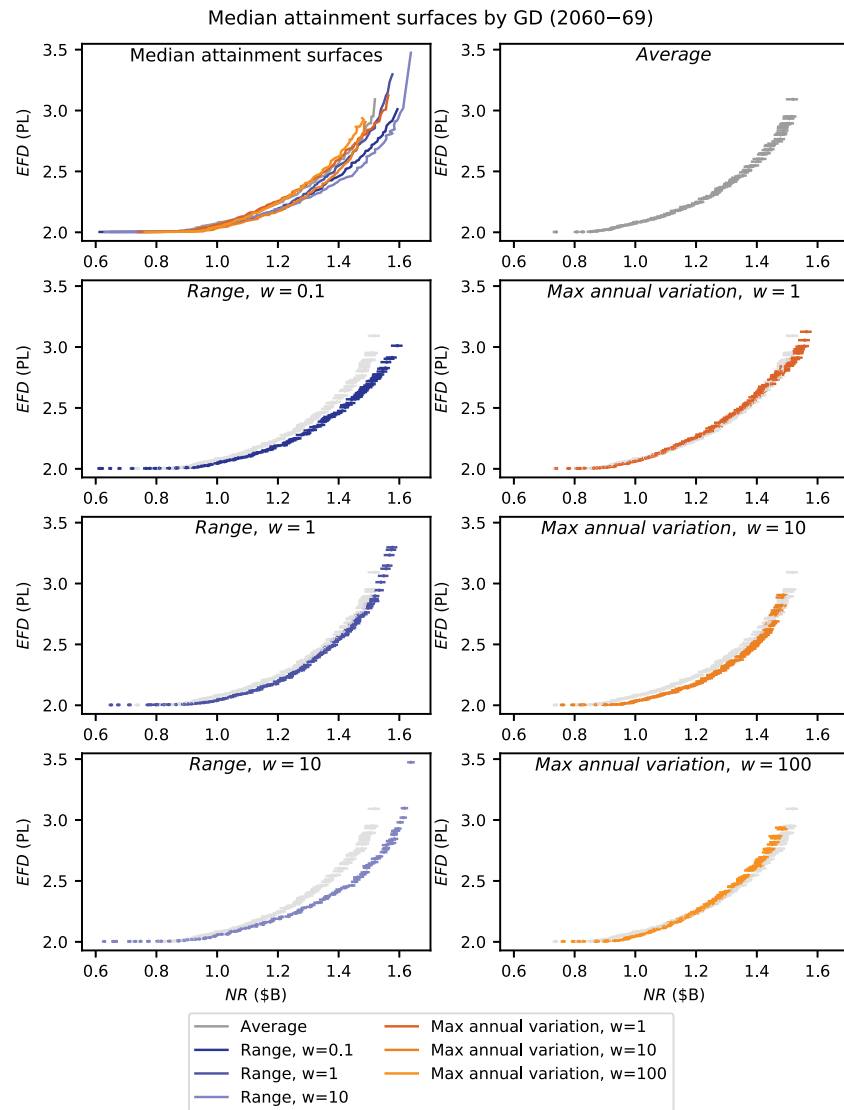


Fig. 4. Median attainment surfaces (selected by GD) for each robustness approach on 2060–69 problem, with NR objective range shown. The median attainment surface for *Average* is reproduced, faded, on each plot to provide a reference.

7.1. Achieving robustness at the expense of quality

Using the novel robust techniques developed in this paper, the solutions produced were “highly robust efficient”. However, to achieve this it was necessary to use the “worst case”, minimal inflows across the models, which adversely affects solutions achieving higher net revenues. As suggested by Ide and Schöbel [21], highly robust efficiency has come at the expense of solution quality. In contrast, while standard temporal optimisation can produce solutions that achieve much better outcomes in individual scenarios, they only produce flimsily robust solutions, where solutions are highly unlikely to be feasible under other climate models. The extent of this degradation in outcomes is illustrated in Fig. 6.

To ameliorate the effects of using the most conservative inflow, a variety of options are available. Inflows could be modelled on an annual basis as an aggregated/smoothed water availability. Fig. 7 demonstrates the effect of accumulating available water annually. Compared to the data in Fig. 1, the discrepancy between maximum water accumulated and minimum is of the order of a factor of 3, rather than a factor of 12. From within this “annual water budget” water could be released from upstream dams month-by-month as demanded for crop needs and desired environmental flow events, based on environmental

need analyses. The problem model itself can also be refined including the allowance for more realistic modelling of seasonal flows and the effects of dam storage [40]. The question of drought-proofing by use of dams and its feasibility over multi-year water shortages is a critical one, not only in Australia, but all arid and semi-arid environments across the world.

7.2. Implications for cropping in the future

The analysis of the solutions, in terms of which crops are being planted when, is highly instructive. It gives us a guide to the crops that are going to be suitable for production over the coming decades, and those which are not. The latter crops essentially clutter solutions with intermittent, small parcels of land, and do not contribute significantly to the net revenue or environmental flow deficit. Thus, these can be potentially eliminated with the revised data sets having only those crops that have been deemed significant. It is likely that clearer indications of how crop mixes need to change into the far future will be evident. It also has the advantage that the problem size will be significantly reduced, and further iterations of the algorithm will be possible, for the same expenditure of computational resources.

To facilitate this analysis, principles of visual analytics [41] have been applied. “Heatmaps” have been derived from the numerical data

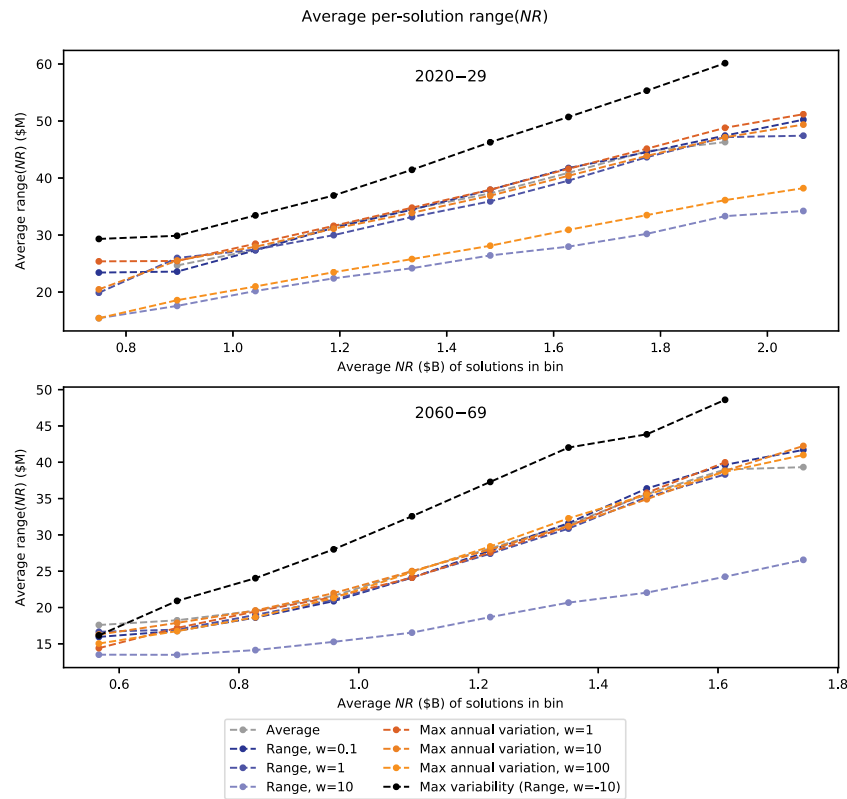


Fig. 5. Average $range(NR)$ of solutions within different NR bins by robustness approach and planning decade.

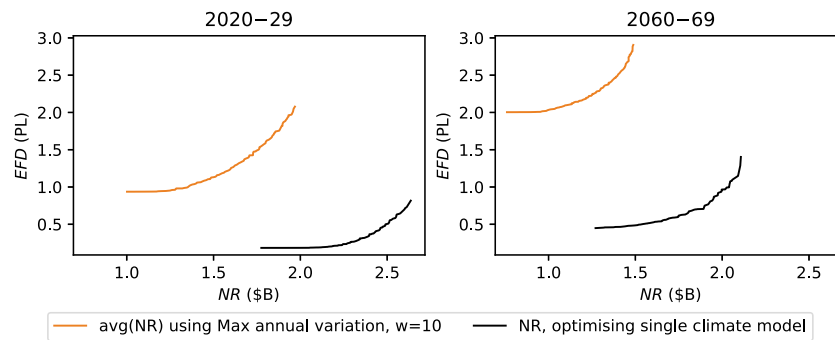


Fig. 6. Median attainment surfaces produced using robust approach (*Max annual variation*, $w=10$) and when optimising for a single climate model [1].

of the area of land devoted to individual crops in particular years, for the decade 2020–2029, and contrasted with the decade 2060–2069, displayed in Fig. 8. The heatmaps are divided into one showing broadacre crops and another for the perishable commodities that have been subject to constraints on maximum area of cultivation. It should be noted that colour intensity is thus quite different between these two classes, and they should be interpreted independently. In addition, two sets of heatmaps have been generated, for solutions maintaining equal best EFD (upper row) and those that attain maximum net revenue (lower row).

It can be seen immediately that, if water efficiency is considered important, the profitable cultivation of broadacre crops is significantly reduced by 2060. Even if more emphasis is placed on net revenue, as in the lower set of maps, the cultivation of cotton is still deprecated, with the exception of during the wettest years. Canola has become the major, surviving, broadacre crop.

The foundation of significant financial returns appears to be in horticulture. In the current models used in this work, these crops are subject to constraints on their areas of cultivation. Investigation needs

to be made into the implications of these preliminary findings and the relationships with market forces to ensure realistic recommendations can be derived for planning future profitable farming in the area considered in this case study. The significant contribution of the cultivation of stone fruit to future revenues also bears further consideration.

These results, in themselves not the intended primary contribution of the work due to their preliminary nature, give some indication of the utility of tools based on the methods and approaches proposed. While useful to assist decision making on the part of individual farmers, they can also provide guidance for regional policy-making in terms of allocation of scarce water resources. Inspection of Fig. 7 indicates the projected decline in available inflows, making this a policy issue of increasing importance. The possible changes required in agricultural enterprises may also be of significance in decisions of allocation of financial capital and fiscal policy regarding assistance with climate change adaptation.

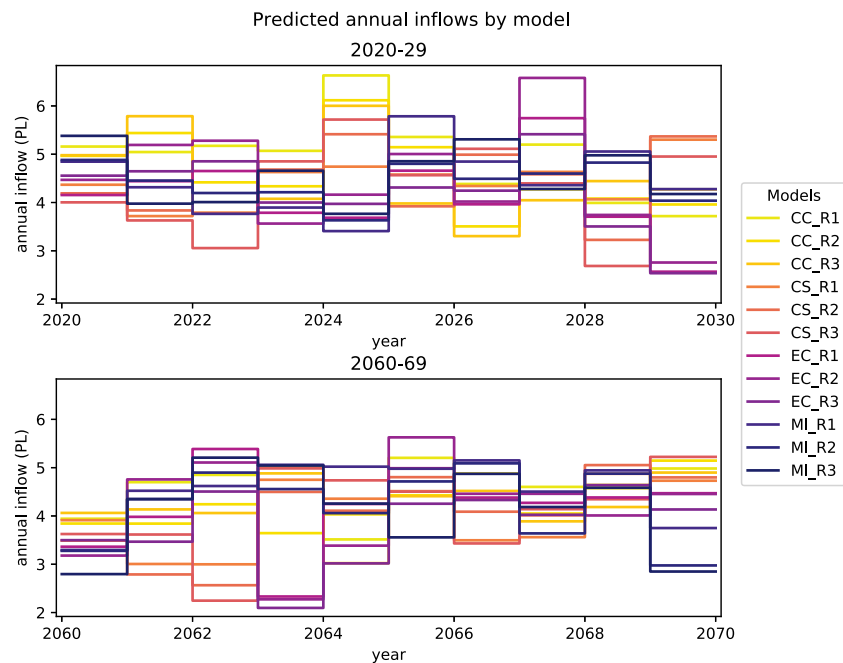


Fig. 7. Predicted annualised water inflows for the 12 climate models exhibits smaller variation between models than predicted monthly inflows (shown in Fig. 1).

7.3. Dealing with uncertainty

The problem considered in this work can be characterised as having “deep uncertainty”. Approaches structured around Scenario-Focused Decision Analysis [29] have shown promise in dealing with problems of this nature, and may prove effective in this context. In addition, Roach et al. [42] have compared and contrasted Info-Gap Decision-Making (IGDM) methods with robust optimisation approaches, with particular application to the closely allied problem of water resource management. Subject to the ability to adequately formulate objective function risk thresholds for the problem considered in this paper, consideration may be given to IGDM.

When considering long-term planning, Walker et al. [30] emphasise the importance of plans being *adaptive*. As changing climate evolves, it may be possible to predict with increasing certainty which of several future climate scenarios are more likely, and adapt agricultural planning decisions accordingly. The potential outcomes of such an approach are worthy of further study.

8. Conclusions

Real-world problems are built around extensive forward planning and usually include elements of uncertainty. Robust temporal optimisation presents a set of techniques to address this systematically for uncertain environments. For the agricultural planning problem in this paper, varying crop mixes over extended timeframes were able to be derived that optimised overall revenue with minimal water deficit. Importantly, the amount of variation in net revenue due to the uncertainty of climatic conditions can be reduced by penalising less robust solutions. This was achieved by developing new robust temporal objective measures. The solutions that the solver produced, despite the particular robust method used, can all be classified as highly robust efficient. Of all of these, the *range*-based penalty with weights 1 and 10 seemed to show the most promise, able to encourage robustness without impeding overall search progress, but additional variations on robust mechanisms need to be explored.

There are a number of directions that this initial work can and should be taken. The framework for developing different robust techniques in Section 4 allows for many different types of robustness to

be defined. These need to be compared for robust performance on the data sets used in this paper. Additionally, the enhanced concept of robustness developed in this paper needs to be explored in problems beyond the domain of agriculture. Use of alternative EAs, such as NSGA-II [23], should also be investigated to explore the interaction between solver and robustness controls.

In terms of the problem itself, there are a number of significant directions for future work, some of which will be mentioned here. Initially, there needs to be improved/smoothed modelling of water inflows so that the discrete allocation to months does not create an unduly pessimistic prediction produced by taking the minimum across models, which may exclude novel and useful robust solutions. Additionally, climate change affects more than just temperature and rainfall levels. Some of the areas of particular concern are yield, quality and pest/disease load of the crops being produced. These factors influence price and hence have a direct bearing on the net revenue objective.

In addition, the structuring of the problem, as defined by Randall et al. [1], needs to be reformulated. At present, it simply allocates hectares of available land to crops each year considered in isolation. However, land, and in particular soil, has certain characteristics that yield better outcomes if certain crops are grown at certain times, and in certain sequences (to preserve nutritive value). Therefore, the focus needs to shift to decision values being based on land management units [43] instead. These will take into account physical characteristics of the land to better suit various crops and also allow for crop rotation sequences to be planned.

CRedit authorship contribution statement

M. Randall: Conceptualization, Investigation, Methodology, Software, Writing – original draft, Writing – review & editing, Validation, Project administration. **J. Montgomery:** Conceptualization, Writing – review & editing, Methodology, Software, Data curation, Validation, Visualization. **A. Lewis:** Conceptualization, Writing – original draft, Writing – review & editing, Methodology, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

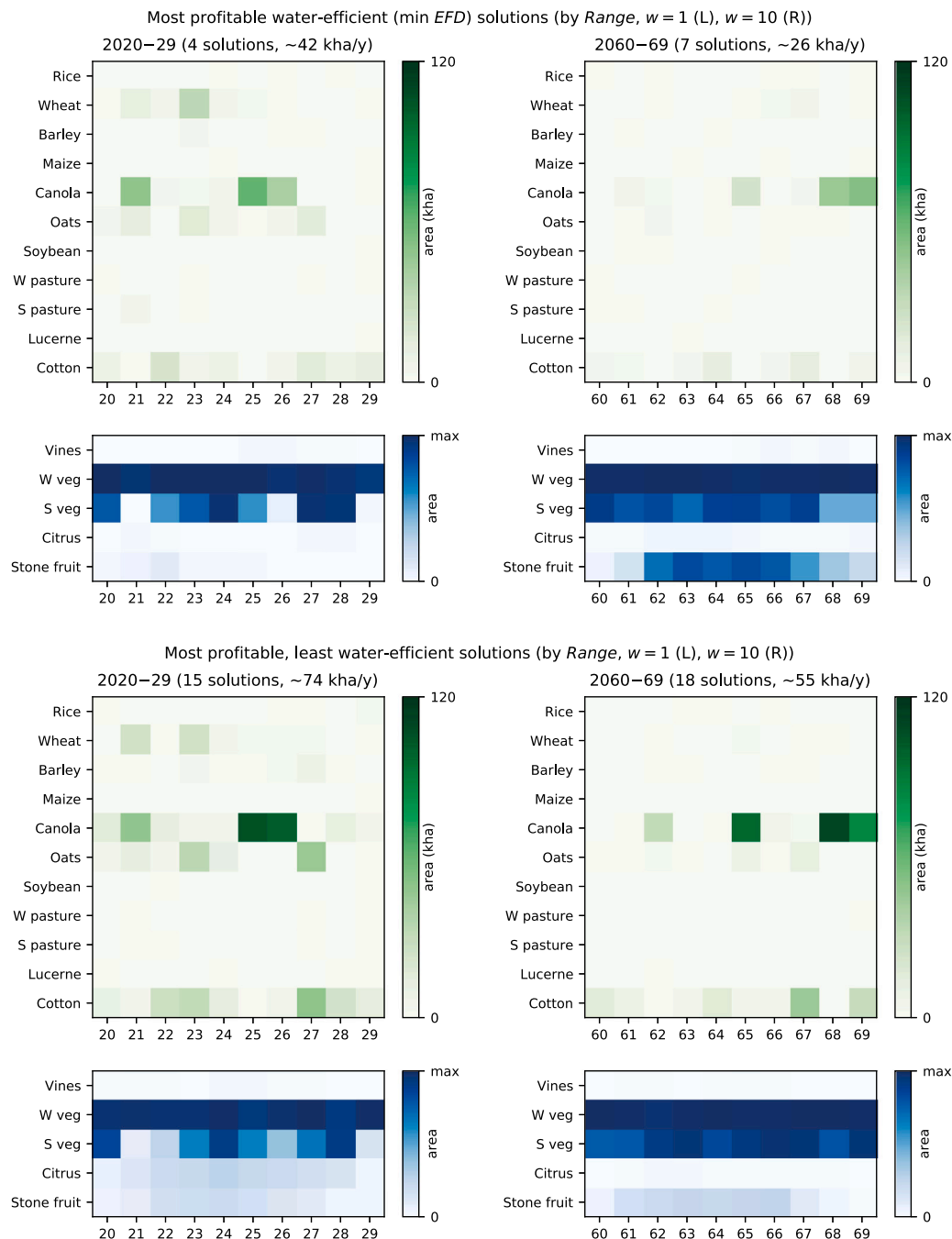


Fig. 8. Average crop area by year for selected solutions produced by the median trial with *Range*, $w = 1$ for 2020–29 (left column) and $w = 10$ for 2060–69 (right column). The top row are solutions with the highest *NR* but equal best *EFD*, bottom row are solutions from the upper extreme of those attainment surfaces (max *NR*, max *EFD*).

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