INTERNATIONAL JOURNAL OF MATHEMATICAL EDUCATION IN SCIENCE AND TECHNOLOGY
https://doi.org/10.1080/0020739X.2022.2119176

The influences of social, cognitive, and teaching presence on pre-service teachers' online engagement in productive mathematical discourse

Tracey Muir

Faculty of Education, University of Tasmania, Launceston, Australia

ABSTRACT

The increase in online education has implications for mathematics teacher educators who are tasked with teaching mathematics content and pedagogy to pre-service teachers (PSTs) who do not attend on-campus. Concerns about PSTs' content knowledge continues to be an ongoing issue, and ways to address this, especially in an online environment, require further investigation. While online courses offer accessibility and flexibility, it can be challenging to translate synchronous on campus teaching and experiences into the online environment. The importance of productive mathematical discourse has been identified as an appropriate pedagogy, yet the opportunity to engage in such discourse can be limited in the online environment. This paper provides the results from a case study of an online forum, which highlights how pre-service teachers can be engaged in productive online mathematical discussions, particularly when facilitated by the instructor's and other learners' interactions and presence. The findings indicated that social, cognitive, and teaching presence influenced the nature and frequency of online mathematical discourse. The study contributes to existing research through conceptualizing how productive mathematical conversations can occur outside of the classroom setting, with implications for mathematics teacher educators who are tasked with teaching content, pedagogy and practices to PSTs.

ARTICLE HISTORY
Received 23 January 2022

KEYWORDS
Preservice teachers; teacher knowledge; mathematical knowledge for teaching; student work; online learning

1. Introduction

While the increase in online education offers accessibility and flexibility in the higher education sector, it can be challenging to transfer subjects traditionally delivered on-campus to the online environment. The pedagogy of mathematics education taught to pre-service teachers (PSTs) is typically characterized using interactive materials and productive discussions delivered on-campus where interaction is synchronous. It is arguably more challenging to enact these practices in an asynchronous manner, yet many PSTs are being educated using blended or fully online models of instruction. For online teacher educators, the lack of face-to-face interactions with PSTs can make it challenging to monitor and encourage engagement, ensure active participation, and prepare students for the

CONTACT Tracey Muir tracey.muir@utas.edu.au

realities of classroom teaching. This task may be even more challenging for mathematics teacher educators (MTEs), who are required to teach mathematics pedagogy to cohorts of PSTs who are often not confident with the subject matter and have limited mathematical content knowledge (Anthony et al., 2016). There continues to be widespread concern about the depth of mathematical content knowledge as demonstrated by PSTs (e.g. Anthony et al., 2016), with research in this area largely focused on how this is monitored and addressed in traditional face-to-face tutorial classes (e.g. Hurst & Cooke, 2014), or through practicum experiences (e.g. Livy et al., 2016). The challenges of connecting PSTs' course theory with practice are also well documented in the literature (e.g. Anthony et al., 2018; Gronow et al., 2019; Livy et al., 2017; Wright, 2017), with most accounts highlighting the occurrence of this in on-campus situations. On-campus tutorials offer opportunities for MTEs to model and unpack effective mathematics teaching practices, such as those recommended by the National Council of Teachers of Mathematics (NCTM) (2014), including using and connecting mathematical representations and facilitating meaningful mathematical discourse. In an online environment, however, opportunities for discourse are typically provided through writing responses in discussion board forums whose effectiveness is debatable (Douglas et al., 2020; Thomas & Thorpe, 2019), and has not been closely examined in a mathematics education context. Accounts of how PSTs' can learn to become mathematics teachers through a fully online environment in general are limited, although recent interest in the work of MTEs is beginning to address this (e.g. Beswick & Goos, 2018). This paper adds to the limited literature through addressing the following research questions:

- In what ways (if any) can online discussions be characterized as being productive mathematical discourse?
- What affordances do discussion boards offer that promote PSTs engagement in productive mathematical discourse?

2. Review of the literature

2.1. Productive discourse

A meaningful mathematical discourse engages students with important mathematical ideas, and can focus on concepts, procedures, problem-solving strategies, representations, and reasoning (Staples & King, 2017). One of the seven effective mathematics teaching practices, identified by NCTM (2014), is the facilitation of meaningful mathematical discourse attempting to build a shared understanding of mathematical ideas by analysing and comparing students' approaches and arguments. Similarly, Anthony and Walshaw (2009) identified mathematical communication using facilitation of classroom dialogue focused toward mathematical argumentation as one of ten principles of effective mathematics pedagogy. In the primary classroom, specific pedagogical actions used to facilitate discourse include encouraging students to provide conceptually focused justification for mathematical actions, questioning in sustained exchanges, and facilitating student examination of similarities and differences across multiple strategies (Kazemi, 1998). The teacher plays a key role in the facilitation of such discourse through participation and orchestration (McCrone, 2005) and ensures that it is conceptually focused and reflective (Hunter, 2009). Staples and King (2017) identified three key functions of the teacher's role: eliciting student thinking, supporting student-student exchanges about mathematical ideas, and guiding the mathematics. Pedagogical practices such as the five productive talk moves (Chapin et al., 2009), which include revoicing, can be used by teachers to position students in the interactive dialogue and to clarify reasoning, highlight specific aspects of the mathematical thinking, or extend, rephrase, and further develop it (Hunter, 2009).

Stein et al. (2008) also point to mathematical discussions being a key part of effective mathematics teaching. For them, mathematical discussions support student learning through using mathematical discourse practices, making student's thinking visible, and encouraging students to construct and evaluate their own and each other's mathematical ideas. Their identification of five practices that orchestrate productive mathematical discussions demonstrate that productive interactions must go beyond 'show and tell', and instead engage students in sense-making through connecting the mathematics and drawing together students' important and worthwhile ideas.

In summary, productive discourse is characterized by classroom dialogue that is conceptually focused on mathematics, makes students' thinking visible and encourages explanation, reasoning, and justification. The descriptions of productive discourse are based on an underlying tenet that they occur in real time, usually within the setting of a classroom. In order to prepare PSTs for the facilitation of productive mathematical discourse with their future students, on campus MTEs can model what productive discourse looks like in their synchronous classes. For online MTEs, this can be difficult to achieve, yet it is an important aspect of pedagogical practice that needs to be addressed if PSTs are to be appropriately prepared for facilitating productive mathematical discourse.

Online discussion typically occurs through the use of asynchronous discussion boards where students post responses to questions and engage in discussion about nominated topics, yet active engagement in online discussions can be difficult to achieve (Hew et al., 2010). While discussion boards can support active learning and peer interaction, particularly when facilitated by a knowledgeable teacher/instructor (Martin et al., 2018), it has not been widely demonstrated that they can facilitate the type of productive mathematical discourse as described by Staples and King (2017) and others (e.g. NCTM, 2014; Stein et al., 2008).

2.2. Discussion boards

The use of interactive discussion boards in online learning suggests they can be important tools for fostering student engagement (Sabry & Baldwin, 2003). In terms of affordances, discussion boards can provide more reflection time and potentially less stressful opportunities for students and facilitators to share their thoughts and opinions than face-to-face interactions (Douglas et al., 2020). The discussion board format also provides opportunities for more sustained interaction than might be achieved in a face-to-face tutorial, with educators estimating that their interactions with students can be up to three times longer than interaction with face-to-face students (Martin, 2014). Instructors' use of facilitation strategies, such as presence in discussion forums, can enhance engagement, with evidence showing that instructor presence enhances students' motivation to learn, increases the depth and quality of students' interactions and discussions, and can reduce a sense of loneliness (Martin et al., 2018).

Instructor or teaching presence is theorized to consist of three components: instructional design, facilitation, and direct instruction (Anderson et al., 2001). Research findings have indicated that instructional design and clearly defined roles of instructors are critical in facilitating cognitive presence, particularly in online discussions (e.g. Garrison & Cleveland-Innes, 2005; Gasovic et al., 2015). Instructor presence in discussion forums was identified as a key facilitation strategy that influenced student engagement and learning in the online environment (Martin et al., 2018), although there is little agreement about what constitutes instructor presence in terms of quality of interaction and content (Thomas & Thorpe, 2019). While the research shows that a sense of online instructor presence enables positive learner-instructor interaction (Shea & Bidjerano, 2010), thick descriptions of what this constitutes is missing in the literature, including with reference to online mathematical discussions. Recommendations for instructors have included practices such as starting major discussion threads, narrowing down topics, and responding promptly to students' posts (Martin et al., 2018). These recommendations were considered by the instructor in this study when selecting discussion topics and responding to students' posts.

2.3. Online mathematics teacher education

The integration of technology has enabled educators, including MTEs, to create learning experiences that actively and meaningfully engage students in course content (Engelbrecht et al., 2020). While this creates possibilities for reconceptualizing delivery of mathematics education courses, it seems the digitally networked world is only just starting to penetrate the typical classroom (Engelbrecht et al., 2020). Online mathematics teacher education is an emerging field of research, focusing on identifying and analysing the affordances of the online environment and the challenges faced by teachers and students when learning and teaching without face-to-face interaction (Borba & Llinares, 2012). While online collaboration has been investigated in relation to practicing teachers' professional development (e.g. Guerudet et al., 2012), less attention has been paid to PSTs' experiences. Other studies have focused on the role of technology and participants' interaction with multi-media (e.g. Borba, 2012; Gadanidis & Geiger, 2010), including the use of virtual manipulatives (Rodriguez, 2015, as cited in Hoyos et al., 2018), particularly in hybrid learning environments (Hoyos et al., 2018). The purpose of this article, however, is not to demonstrate how technology can be applied in the online environment, but rather discusses a case study that illustrates how technology, along with other pedagogical practices, can be used to encourage productive online discourse. While not focused on online delivery, a recent review of teacher education pedagogies in the Australasian context, identified three categories of approaches to pre-service teacher learning: creating approximations of practice or authentic experiences in ITE settings; building PST's identities as teachers of mathematics; using digital technologies to support PST learning (Way et al., 2020). As more ITE courses are being delivered in fully online or hybrid modes, it is timely to examine in what ways identified approaches to PST learning can be replicated or achieved in the online environment. This paper investigates whether the online instructor can successfully prepare and teach PSTs the required Mathematical Content Knowledge (MCK) and Pedagogical Content Knowledge (PCK) in an online environment where the main source of interaction is through synchronous discussion boards.

3. Theoretical framework

The reviewed literature characterized what productive mathematical discourse involves and highlighted the role of discussion boards in higher education online environments. The key role of the teacher was also highlighted as being influential in the facilitation of productive mathematical discourse and in online discussion boards. Garrison et al. (2001) devised a community of inquiry model which incorporates discourse and teaching presence as elements of a worthwhile educational experience.

The model assumes that learning occurs within the Community of Inquiry through the interaction of cognitive presence, social presence, and teaching presence. Cognitive presence, a vital element in critical thinking, refers to the extent to which the participants can construct meaning through sustained communication (Garrison et al., 2001). Social presence refers to the ability of participants to present themselves to others as 'real people' (Garrison et al., 2001, p. 89). Social presence can be seen as a direct contributor to the success of the educational experience, particularly when valuing affective goals, such as willingness to contribute to class discussions. Teaching presence is the responsibility of the instructor and includes the design of the educational experience and the facilitation of students' learning and interactions. In the online context, it encompasses the three characteristics of teaching presence identified by Anderson et al. (2001): design and administration, facilitating discourse, and direct instruction.

Garrison et al.'s (2001) model is relevant for understanding that MTEs are responsible for selecting the mathematical content and appropriate pedagogies of the course. They set the climate for respectful exchanges to occur, and support discourse that is focused on productive mathematical conversations. It is, however, equally applicable to a range of generic teaching situations and disciplines, so aspects of the model have been extended to demonstrate its applicability to the work of the MTE. Figure 1 depicts an adapted version of Garrison et al.'s (2001) model. This adapted model incorporates aspects of MTEs' work and practice that recognize the unique elements involved in teaching mathematics education to PSTs. Discourse in the context of mathematics education is often characterized as being 'productive' or purposeful, and as highlighted previously, should include elements such as justification and reasoning, with a focus on eliciting student thinking. Figure 1 shows that Supporting Discourse involves eliciting student thinking and providing opportunities for the exchange of mathematical ideas to occur. The reference to talk moves, justification and questioning highlight strategies that are characteristic of productive classroom discourse. In terms of content, MTEs are tasked with increasing PSTs' own mathematical content knowledge (MCK), together with the mathematics knowledge required for teaching and how to teach it (PCK), along with connecting this knowledge to the professional practice that occurs in classrooms.

In terms of content, MTEs are tasked with increasing PSTs' own mathematical content knowledge (MCK), together with the mathematics knowledge required for teaching and how to teach it (PCK), along with connecting this knowledge to the professional practice that occurs in classrooms. The instructor ultimately selects the content, with the model depicting that this includes aspects of professional practice such as representations and approximations of practice, which have been identified elsewhere in the literature as core components of ITE courses (e.g. Way et al., 2020). Finally, within the social presence aspect

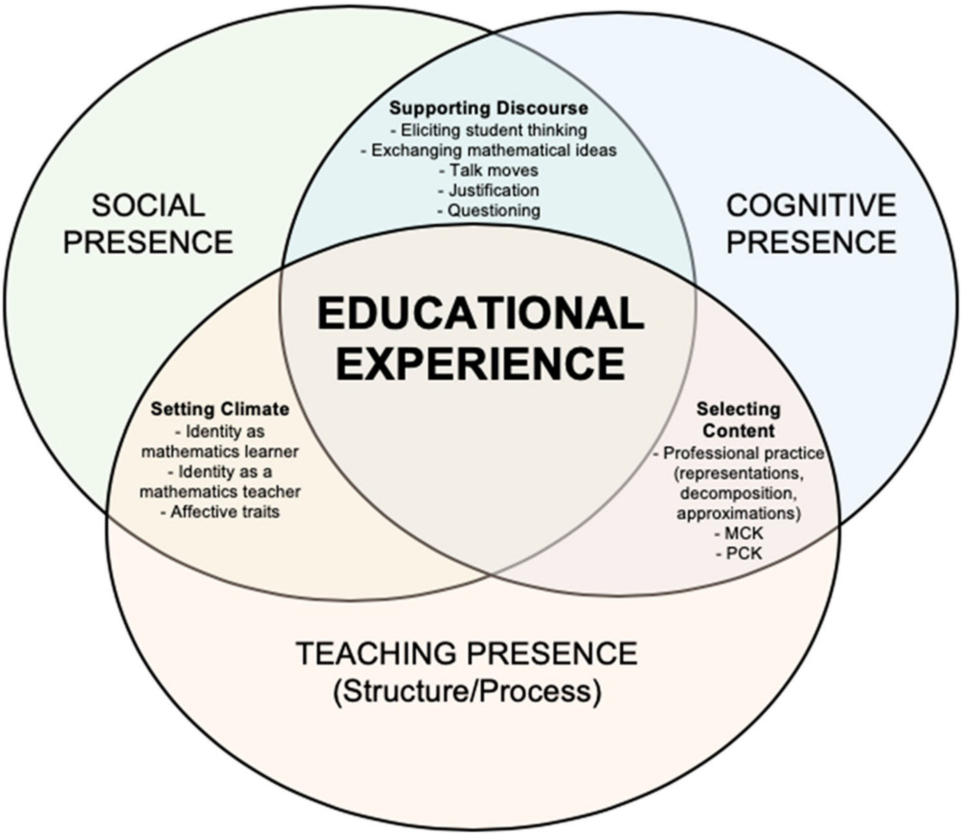

Figure 1. Adapted model showing Community of Inquiry in a MTE context.

of the model, an important component of MTEs' work involves recognizing the tendency for PSTs to experience maths anxiety (Hembree, 1990) and to facilitate their developing identity as a mathematics learner and teacher. Setting the climate includes recognition that a Community of Inquiry in a mathematics context involves addressing students' identity as a mathematics learner and teacher, which are influenced by affective traits. In summary, this adapted model incorporates and makes explicit the aspects of the work of an MTE that go beyond the generic educational experience as described by Garrison et al.'s (2001) original Community of Inquiry model. The adopted model was used for interpreting the results discussed in this study and paper.

4. Methodology

The study adopted a constructivist perspective, characterized by the beliefs that knowledge is constructed rather than discovered, and that there are multiple perspectives or interpretations (Stake, 1995) of that knowledge. This approach was appropriate given that the research occurred in a natural setting, whereby the researcher, as a MTE, was positioned within the research (Creswell & Plano Clark, 2013). Furthermore, a case study approach was adopted to investigate whether or not online discussions could be characterized as being productive mathematical discourse and if so, how these discussions were facilitated by the MTE.

The case took place in a regional Australian university and involved a cohort of 55 online PSTs enrolled in TPM1 (Teaching Primary Mathematics 1) in Semester 1, 2019. This unit was the first of two mathematics pedagogy units studied by PSTs in a two-year Masters of Teaching Degree. The primary means of interaction in the unit was through dedicated weekly discussion boards, which provided a space for the PSTs to contribute to questions or topics related to each week's content. The instructor's role in the unit was to provide different stimulus questions for the PSTs to respond to each week and to interact with the PSTs through discussion posts and replies. While the instructor was active in facilitating the discussion and thus provided favourable conditions for discussion to occur, the results still demonstrate what can be achieved in such an environment. The posts from those dedicated weekly discussion boards, form the basis of the results discussed in this paper. Full ethical approval was granted to access and report on PSTs' discussion posts and the PSTs provided consent for this.

The discussion board posts were analysed using both inductive and deductive thematic analysis (Boyatzis, 1998; Crabtree & Miller, 1999). Initially, open coding was used to analyse discussion board exchanges. Early discussion posts were characterized by a series of separate posts, rather than contributions to an ongoing thread. These were excluded from analysis as they were not deemed to be discourse, but rather a series of disconnected statements. Threads of four posts or more where exchanges occurred between two or more participants were considered examples of discourse and subsequently coded. Codes were then clustered into three broad categories, related to the elements identified in Figure 1: setting climate, supporting discourse and selecting content. As an example, the following post was initially given 5 inductive codes: mathematical thinking; mathematical content knowledge; mathematical vocabulary; fractions; mathematical explanation. It was deductively coded as supporting discourse as it elicited student thinking, showed an exchange of a mathematical idea, and contained a mathematical explanation. It was also coded as an example of MCK as the student's explanation was focused on the mathematical content knowledge associated with how to divide fractions (this post was also accompanied by a diagram, shown in Figure 3).

> If we have a chocolate bar of 4 pieces and we want to divide it by 1/3 then each piece (or whole) is cut into 3 pieces - so we can start to see that we multiply the numerator of our first original fraction (4) by the denominator of the second original fraction (3) to find out how many 1/3 segments we have. Then we divide this by the number of thirds we are interested in. In the first case this is 1. In the second case this is 2.

As in the above example, most responses were initially assigned multiple codes. While the researcher had extensive experience with coding, in order to gauge inter-reliability, two weeks of discussion posts were also coded by a colleague of the researcher, who was not involved with the study. The colleague was provided with a codebook and discussion occurred until agreement was reached for coding to the three broad categories: Setting Climate, Supporting Discourse and Selecting Content Each response was also coded as 'student' or 'instructor'. Figure 2 outlines the final themes and associated sub-themes identified.

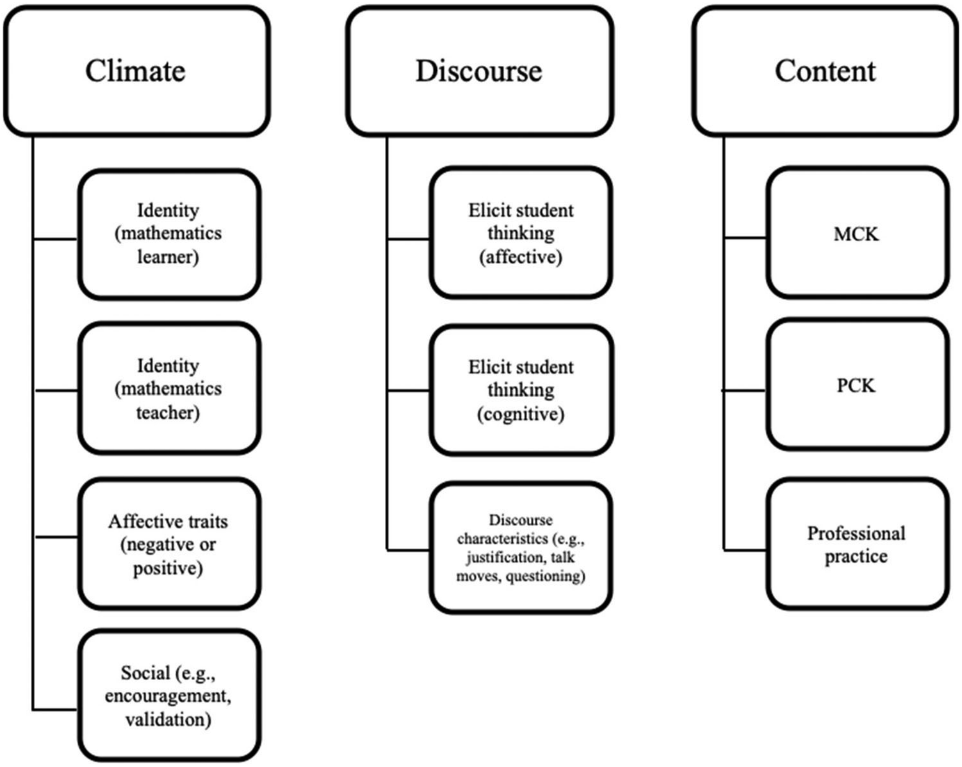

Figure 2. Themes and sub-themes.

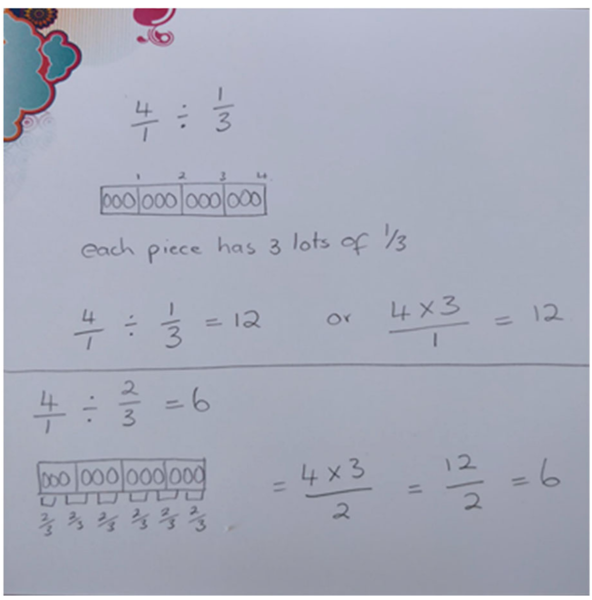

Figure 3. Diagrammatic representation showing why we invert and multiply when dividing fractions.

5. Results from analysing discussion threads

The results are presented in the form of two vignettes that illustrate aspects of the educational experience identified in Figure 1. While all exchanges contained elements from each part of the model, the two selected respectively differ in their intentions. The first vignette provides an example of supporting discourse in relation to eliciting student thinking and exchanging mathematical ideas. The instructor's intention with the topic in the second vignette is to focus more on PSTs' PCK and to connect the discussion with past classroom practice and implications for their future teaching. While there were examples of posts related to establishing a climate for the PSTs to share aspects of their identity, it was beyond the scope of this paper to include those exchanges.

5.1. Vignette 1: Bethany wants to know …

To provide a stimulus to encourage the PSTs to engage with the learning content, the instructor posted a short video excerpt in the learning content for week 6 in which 'Bethany', a school student, asked the question: "Why do we invert and multiply when dividing fractions?" The PSTs were asked to post responses to the discussion board demonstrating how they would address Bethany's question. When selecting this content, the instructor was looking to develop cognitive presence, particularly in relation to PSTs' MCK, and their ability to use appropriate representations to address Bethany's question. The instructor's role was to support the discourse that occurred and to guide PSTs to post conceptual, rather than procedural, responses. There was a total of 23 replies and a total of 89 views of these posts. View metrics were recorded for each individual post, with the total indicating that at least some of the 55 PSTs repeatedly viewed some of the posts. with the following excerpt illustrating the nature of the exchanges that occurred:

> Karen: It's funny that I have also asked this question and my teacher's answer was because "it is the way it is". We invert it because dividing fractions is difficult. Since dividing is the opposite of multiplying, you invert the fraction to make it the opposite of the original fraction … [31/3]
>
> Chris: Hi Karen. I liked how you made it simple for Bethany. I would also explain to Bethany that we invert the dividing fraction and turn it into a multiplication equation because it is a shortcut – who doesn't like shortcuts? [31/3]
>
> Sam: Hi Chris – I agree – and I also think it's definitely important to use visual representations to ensure Bethany understands WHY the rule works – otherwise it's just 'another' mathematical rule. [1/4]
>
> Instructor: Yes I'm with you Sam – saying it's a shortcut is just like a rule – so I'm interested in your responses as to why it works and if you could explain with a visual, go for it. [4/4]

After some similar postings from other PSTs, and following a period of no posts, the instructor then posted:

> Instructor: Hi all – I've checked with Bethany and she is still not convinced – she wants a diagram with pictures? Any ideas? [5/4]

Trudy then posted the following response and diagram:

> I've had a go at this with a bit of online help. I'm still having trouble putting it into simple words. If we have a chocolate bar of 4 pieces and we want to divide it by 1/3 then each piece (or whole) is cut into 3 pieces - so we can start to see that we multiply the numerator of our first

> original fraction (4) by the denominator of the second original fraction (3) to find out how many 1/3 segments we have. Then we divide this by the number of thirds we are interested in. In the first case this is 1. In the second case this is 2. [5/4]

Another PST, Liam, provided a link to a YouTube video and added:

> I like the idea of getting students to talk about what they notice and wonder about what is going on in the picture. I think that getting students to speculate on what end result might be before attempting to solve is also brilliant because it would help the teacher ascertain each student's understanding of the concept. [5/4]

This link was appreciated by the other PSTs, as Jill's response illustrates:

> Hi Liam - That was a really neat video, thanks for sharing! So many different examples of informal ways to approach the problem … I think I would have started by drawing cups too. [20/4]

This excerpt provides an example of discourse which elicited student thinking and enabled the PSTs to exchange mathematical ideas. Together with demonstrating an example of student discourse that focused on mathematical content, and students' MCK and PCK, this excerpt also demonstrates how the discourse was supported by the instructor and that PSTs were willing to participate in the discourse over an extended period. The first posting occurred on the 31st March, with the final post occurring on the 20th April, and while not all PSTs posted responses, there were 89 views of the thread, suggesting that PSTs had access to the content that formed the basis of the discourse.

5.2. Vignette 2: informal vs formal algorithms

The week 3 content examined the meaning of operations and required the PSTs to read and respond to an article that discussed written algorithms in the primary years (Clarke, 2005). Again, the instructor's role was pivotal in terms of selecting content, with the focus in this context being to get insights into PSTs' identities as a mathematics learner and teacher. PSTs had the opportunity to respond to one or more of four discussion prompts, including 'Response to Clarke article':

- After reading the Clarke (2005) article, respond to the following questions:
- What does Clarke identify as some elements to consider whether or not a student's invented algorithm is acceptable? Examine the informal algorithms you used to solve the equations earlier. To what extent do your informal algorithms meet his 'criteria'?
- There is consensus in the research literature that the teaching of formal algorithms should be delayed until at least fourth grade. How do you feel about this? Do you think children should be taught the formal algorithm for each operation?

This prompt resulted in 51 posts, across six different threads. The most prolific thread generated 44 posts, and 164 views and forms the basis of the discourse reported in this vignette.

The thread was initiated by one of the PSTs, Olivia, who after identifying acceptable elements of invented algorithms, posted:

> I wish I could have read the article earlier because I have already taught my kid using conventional written algorithms when he was in grade 1, as many parents in my country want their children to get a head start in education. I reflected that when he was struggling with 2 digits addition/subtractions, all of which were added by me to his homework, he was using fingers or drawing dots on paper, and it's very slow and inefficient, so my parents and I eagerly reminded him using the vertically written algorithms. How regretful I am now! I wish he could be untaught but… Now I realized that using written algorithm is nothing to be proud of and it is potentially very detrimental to student's natural mathematical thinking, given that it could largely weaken their mental strategies and number sense. [11th March]

A number of PSTs responded to this and acknowledged they shared similar experiences as Olivia. As intended, however, the article also provoked some differing perspectives about the teaching of formal algorithms, as the following posts illustrate:

> I really enjoyed Clarke's article and found that (at the moment at least) I mostly agree that formal algorithms should be delayed, especially when I look at how I tackle the problems we were given. I found when I was told to use the informal algorithms to solve the problems, it came a lot easier than when I have been trying to solve other problems we have been given. For example: 800–303, simply changed it to 800–300 = 500–3 = 497, rounding the numbers so they were easier to subtract was a lot easier than 'borrowing' from the next column/s, the formal algorithm I had been taught. I have found myself always trying to remember 'the correct way' or formal algorithm I was taught how to do something rather than just trying to figure it out based on my understanding of numbers, and I am coming un-stuck tackling the problems this way. However, just solving the problems based on my understanding of numbers, I am having a lot more success! I do believe there would be a time and a place for both in a primary classroom, but perhaps as he suggests, not until later primary to introduce formal algorithms. (Pam, 12th March)

> No matter what I always see the formal algorithm. I'm probably just being naive but if the students have a working informal algorithm, then they should not have to learn the formal. It will be too confusing and soon they will get calculators. (Marie, 14th March)

> Marie, I somewhat agree, in that a student should not be told their previously-okay informal algorithm is now incorrect; however, I don't think that formal algorithms should be 'all or nothing'. I like Pam's suggestion for considering them as alternative methods, with students then able to critically assess the pros/cons of both. (Brittany, 17th March)

> It might sound harsh, but the formal algorithm should be introduced at the start or not at all. If you introduce it in the 4th grade it will just confuse the students too much. This is from personal experience of being taught one way and then the next year's teacher said to do it another. (Marie, 14 March)

> I honestly don't know if formal algorithms should be introduced before or after Year 4. I'd imagine it depends on the child and the progress they had made. For some, the formal algorithm approach may be the key that opens their understanding to maths, in which case it would be a shame to have withheld it. Considering that many students will go on to learn more complex maths, learning the formal algorithms is going to be a necessary step in their study, but I can see the benefit of establishing a firm foundation in understanding basic maths concepts before moving on to things they are not developmentally ready for. (Emma, 15th March)

> Such an interesting article and an even MORE interesting discussion! I think this week's topic has raised a variety of emotions in me… most likely because it has caused me to question some of the things I have been doing at home to 'help' my children broaden their mathematical understanding, but really, I think it was probably undermining what may have been happening at school without me realising! It makes ALOT of sense to me to hold off on formal algorithms

> until grade 4 or so. Having your mind open to different ways of approaching mathematical problems helps to eliminate the very black/white, right/wrong nature of the maths I grew up learning. If nothing else, it breeds a better relationship with the subject when you aren't always getting things 'wrong'. (Sarah, 15th March)

The above posts were illustrative of the types of responses received and again provide insights into the PSTs' thinking, their identity as a mathematics learner and teacher, and affective traits. While the prompt also elicited discourse related to professional practice, the responses showed that the PSTs primarily engaged in discourse about mathematical ideas and their own beliefs.

As the PSTs were willing to engage in discourse with each other, facilitating the development of social presence, the lecturer primarily took the role of an observer, while offering some regular commentary to indicate her ongoing teacher presence. There was also the opportunity to address aspects of the PSTs' MKT and PCK as they arose. For example, Marie asked 'how are we going to implement the formal algorithm in Grade 4? It would be like telling the students that the algorithm you are using now is rubbish and you must do it this way?' The lecturer took this opportunity to explain that 'The article is not meant to imply that you have to introduce the formal algorithm in Year 4 – it is just saying that it shouldn't be introduced earlier', thereby enabling meaning to be constructed through sustained communication (Garrison et al., 2001).

As with the other topics, the lecturer also included questions on relevant post to further elicit students' thinking and talk moves (Chapin et al., 2009) to generate extension of the discussion:

> There is a school of thought as to whether or not you should introduce the formal algorithm at all – what do others think about this?

In summary, the posts showed that this was a topic that generated a lot of responses, and that PSTs, such as Marie, were willing to engage with the discourse by posting more than one response.

6. Discussion

The following discussion is structured around answering the two research questions and focuses on the nature of the mathematical discussions and the role of the lecturer in facilitating these discussions.

6.1. Mathematical discussions

The two vignettes demonstrated how social, cognitive, and teaching presence was manifested in the online environment. The results show the mathematical discussions that occurred in the two vignettes included opportunities to engage in reasoning, justification and reflection, with the lecturer and PSTs supporting the discourse though the exchange of mathematical ideas and making student thinking visible. Teaching presence through the lecturer also facilitated the engagement with important mathematical ideas and concepts, including procedures, problem-solving strategies, and representations (Staples & King, 2017). Trudy's response, for example, provides evidence of moving beyond a focus on procedures and 'shortcuts' which was characteristic of earlier posts, to developing a conceptually based understanding of the procedure, using a diagram and accompanying explanation. The exchange also provided evidence of students' cognitive presence using representations that demonstrated an understanding of MKT and PCK. These opportunities were provided through the lecturers' deliberate selection of content designed to develop PSTs' understanding of professional practice, and knowledge required for teaching.

While classroom discourse is focused on mathematical conversations, the results show that mathematical discourse in the higher education space extends to the inclusion of professional practice such as PCK and MKT. The discourse that occurred particularly in vignette 2 did not necessarily require any discussion of mathematical concepts but was valuable in encouraging discussion about the learning and teaching of mathematics. The discourse that ensued was still considered productive in that connections were made between posts and opinions of others built on and discussed. Smith and Stein (2013) point to discourse that is essentially 'show and tell', whereby one student presentation would follow another with limited teacher (or student) commentary. Such exchanges are limited in that they do not capitalize on opportunities to draw connections among methods used, or tie them to widely shared disciplinary methods and concepts. This is limiting in that there is no mathematical or other reason for students to necessarily listen to and try to understand the methods of their classmates. The exchanges that occurred in the vignettes, however, were not isolated or disconnected posts, but demonstrated students' willingness to engage with others' ideas and consider them in relation to their own experiences and views. Brittany's post, for example, in Vignette 2, refers to earlier posts and acknowledged others' views. She acknowledged but respectfully disagreed with an earlier comment from Marie, and endorsed Pam's suggestion for considering alternative methods. The exchanges demonstrate the exchange of ideas as identified in Garrison et al.'s (2001) adapted model.

Concerns have been raised about online discussion boards in terms of their quality of interaction and content (Thomas & Thorpe, 2019) with in-class discussion often being portrayed as a better experience (e.g. Blackmon, 2012). Results from this study, however, point to some affordances that online discussion forums have that are not as easily replicable in the on-campus environment. All the discussions portrayed in the vignettes, for example, occurred over several days. In addition, different posts made on the same topic by the same PSTs demonstrated a willingness to be involved and participate in a sustained exchange. This characterization demonstrates that students, along with the instructor, played a role in supporting discourse through the exchanging of mathematical ideas (see Figure 1). Face-to-face tutorials are typically one to two hours in duration; hence class discussions are usually restricted to this duration and only accessible to those who attended the tutorial on that day. In contrast, the flexibility afforded to online learners, means that discussion topics and posts are able to be accessed at any time throughout the semester, and also re-read. While the results showed that not all PSTs contributed to the online discourses described, the metrics showed a high number of views (89 and 164 respectively for a cohort of 55). In this way, the discussion board forum encouraged the exchange of ideas and the interaction between peers in ways that went beyond the restricted exchange that is limited to a set tutorial period.

6.2. The role of the lecturer

The role of the lecturer in facilitating productive mathematical conversations was demonstrated through a teaching presence which attended to the aspects of selecting content, setting the climate, and supporting discourse as depicted in Figure 1. Purposeful selection of discussion content topics, with a goal to promote and even provoke contributions, allowed the lecturer to focus on different aspects of the PSTs' learning, such as their MCK, and PCK. Along with its focus on PCK, Vignette 2 also provided insights into the PSTs' beliefs, and their identity as a mathematics learner and teacher. Incorporating practices as recommended by Martin et al. (2018), such as limiting the topics to two or three per week and promoting the practice of extending threads rather than creating new ones, was helpful in motivating PSTs to post, and in supporting the discourse that occurred. In addition, the practice of the lecturer ending replies with a question also provided an example of supporting discourse, as the discussion was often then continued, highlighting the importance of lecturer presence (Martin et al., 2018; Muir et al., 2019). The lecturer also gave regular feedback on posts, thereby acknowledging the importance of maintaining instructor presence (Martin et al., 2018).

Vignette 1 provided an opportunity for PSTs to engage with both the mathematical content (cognitive presence) in terms of dividing fractions, and to demonstrate their understanding of professional practice through selecting appropriate representations and strategies that would be appropriate for teaching that concept. Initially for that vignette the lecturer's role was that of an observer, and when the discussion posts stopped, she asked a provocative question to stimulate discussion. This strategy provided an example of supporting discourse through eliciting student thinking and supporting the exchange of mathematical ideas (see Figure 1). This exchange, and others described in vignette 1, demonstrate how the role of the lecturer was influential in promoting purposeful discourse that made students' thinking visible and went beyond the tendency to 'show and tell' (Smith & Stein, 2013).

The role of the instructor extended beyond the teaching presence, influencing the social and cognitive presence of students as depicted in Figure 1. Selection of content and supporting discourse by eliciting students' thinking had the effect of encouraging the exchange of mathematical ideas between students. Practices such as the use of students' names and prompting questions were used to establish the lecturer/student relationship which has been shown to have a positive effect on engagement (Muir et al., 2019).

7. Conclusions and implications

The results and discussion demonstrate that the PSTs were willing to engage in online discussions about mathematical concepts, practices, pedagogy and their personal mathematical beliefs and identities. There was evidence of learner-learner interaction (Martin et al., 2018), including examples of exchanges between the same students, indicating a real investment with the content and with other learners, in that they did not just post once, but returned to the forum for further exchanges. The interaction between learner-instructor was evident, with the instructor establishing her presence through regular feedback, and provocative posts, particularly when a lack of activity was noted.

Although this paper provides limited examples of the online exchanges that occurred, it does demonstrate that productive mathematical discussions can occur online. Indeed, it could be argued that the online environment may have facilitated a richer exchange than might be achieved in an on-campus tutorial, in that the students had the opportunity to carefully consider and craft their responses, access other resources (e.g. Youtube clips), and continue the exchange over several days. Similarly, the online environment allows the lecturer to take more time to carefully consider and craft responses, rather than respond to 'in the moment' situations which occur in on-campus classes.

It is important to acknowledge the limitations of the study. While the results and discussion demonstrate the affordances of an online discussion forum, there were several favourable conditions that enabled this to occur. The role of the lecturer was vital, with instructor presence maintained through timely responses to posts, and prompting of students to engage in productive discussions through provocative comments and questions. At times, the PSTs also contributed to teaching presence through their posts in the discussion forums. The results demonstrate that the instructor's role extends beyond the provision of learning content, and that an active presence must be maintained for students to engage with content, each other and the instructor. Similar results may not have occurred if the content had not been suitably relevant, or the lecturer had not been particularly active or responsive. Even given the favourable conditions, there were several students who did not post responses or actively engage online through accessing course material or viewing others' posts. Future studies could further investigate students' motivations for posting and determine whether their engagement could be measured in different ways.

The study has implications for mathematical teacher educators who teach in either the on-campus or online space. The results indicate that it is possible to orchestrate productive mathematical discussions online, with a different experience than that produced in class. The study also demonstrates that discussion boards can be an effective means of communication, challenging findings that point to the limitations of such forums (e.g. Douglas et al., 2020). In addition, productive mathematical discussions in a higher education context can differ in nature from classroom discourse, extending to considerations of beliefs and pedagogical practices. The model that depicted elements of mathematical educational experience proved useful in considering the role of the instructor and the nature of online mathematical education, and hopefully will assist mathematical teacher educators with theorizing the work they do in pre-service teacher education courses. The model could also be used for MTEs to reflect upon how they might promote mathematical discourse, particularly in the online environment.

Disclosure statement

No potential conflict of interest was reported by the author.

ORCID

Tracey Muir http://orcid.org/0000-0001-6305-0679

References

Anderson, T., Rourke, L., Garrison, D. R., & Archer, W. (2001). Assessing teaching presence in a computer conferencing context. *Journal of Asynchronous Learning Networks*, *5*(2), 16–25. https://doi.org/10.24059/olj.v5i2.1875

Anthony, G., Averill, R., & Drake, M. (2018). Occasioning teacher-educators' learning through practice-based teacher education. *Mathematics Teacher Education and Development*, *20*(3), 4–19. https://eric.ed.gov/?id = EJ1195987.

Anthony, G., Cooke, A., & Muir, T. (2016). Challenges, reforms and learning in initial teacher education. In K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), *Research in mathematics education in australasia 2012–2015* (pp. 305–327). Springer.

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the west. *Journal of Mathematics Education*, *2*(2), 147–164. https://doi.org/10.12691/education-6-1-1

Beswick, K., & Goos, M. (2018). Mathematics teacher educator knowledge: What do we know and where to from here? *Journal of Mathematics Teacher Education*, *21*(5), 417–427. https://link.springer.com/article/10.1007s10857-018-9416-4. https://doi.org/10.1007/s10857-018-9416-4

Blackmon, S. (2012). Outcomes of chat and discussion board use in online learning: A research synthesis. *Journal of Educators Online*, *9*(2), n2. https://eric.ed.gov/?id = EJ985399. https://doi.org/10.9743/JEO.2012.2.4

Borba, M. C. (2012). Humans-with-media and continuing education for mathematics teachers in online environments. *ZDM–The International Journal on Mathematics Education*, *44*(6), 801–814. https://doi.org/10.1007/s11858-012-0436-8

Borba, M., & Llinares, S. (2012). Online mathematics teacher education: Overview of an emergent field of research. *ZDM*, *44*(6), 697–704. https://doi.org/10.1007/s11858-012-0457-3

Boyatzis, R. (1998). *Transforming qualitative information: Thematic analysis and code development*. Sage.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, grades k-6* (2nd ed.). Maths Solutions Publications.

Clarke, D. M. (2005). Written algorithms in the primary years: Undoing the good work? In M. Coupland, J. Anderson, & T. Spencer (Eds.), *Making mathematics vital* (pp. 93–98). Australian Association of Mathematics Teachers. https://www.aamt.edu.au/content/download/19063/252036/file/mm-vital.pdf.

Crabtree, B., & Miller, W. (1999). A template approach to text analysis: Developing and using codebooks. In B. Crabtree, & W. Miller (Eds.), *Doing qualitative research* (pp. 163–177). Sage.

Creswell, J., & Plano Clark, V. (2013). *Designing and conducting mixed methods research* (2nd ed.). Sage.

Douglas, T., James, A., Earwaker, L., Mather, C., & Murray, S. (2020). Online discussion boards: Improving practice and student engagement by harnessing facilitator perceptions. *Journal of University Teaching & Learning Practice*, *17*(3), 2020–2034. https://ro.uow.edu.au/jutlp/vol17/iss3/7. https://doi.org/10.53761/1.17.3.7

Engelbrecht, J., Llinares, S. l., & Borba, M. C. (2020). Transformation of the mathematics classroom with the internet. *ZDM*, *52*(5), 825–841. https://doi.org/10.1007/s11858-020-01176-4

Gadanidis, G., & Geiger, V. (2010). A social perspective on technology-enhanced mathematical learning: From collaboration to performance. *ZDM*, *42*(1), 91–104. https://doi.org/10.1007/s11858-009-0213-5

Garrison, D. R., Anderson, T., & Archer, W. (2001). Critical thinking, cognitive presence, and computer conferencing in distance education. *American Journal of Distance Education*, *15*(1), 7–23. https://doi.org/10.1080/08923640109527071

Garrison, D. R., & Cleveland-Innes, M. (2005). Facilitating cognitive presence in online learning: Interaction is not enough. *American Journal of Distance Education*, *19*(3), 133–148. https://doi.org/10.1207/s15389286ajde1903_2

Gasovic, D., Adescope, O., Joksimovic, S., & Kovanovic, V. (2015). Externally-facilitated regulation scaffolding and role assignment to develop cognitive presence in asynchronous online discussions. *Internet and Higher Education*, *24*, 53–65. https://doi.org/10.1016/j.iheduc.2014.09.006

Gronow, M., Cavanagh, M., & Mulligan, J. (2019). Primary pre-service teachers' noticing of structural thinking in mathematics. In G. Hine, S. Blackley, & A. Cooke (Eds.), *Proceedings of the 42nd annual conference of the mathematics education research group of australasia* (pp. 324–331). MERGA.

Guerudet, G., Sacristan, A. I., Soury-Lavergne, S., & Trouche, L. (2012). Online paths in mathematics teacher training: New resources and new skills for teacher educators. *ZDM*, *44*(6), 717–731. https://doi.org/10.1007/s11858-012-0424-z

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, *21*(1), 33–46. https://doi.org/10.2307/749455

Hew, K. F., Cheung, W. S., & Ng. C. (2010). Student contribution in asynchronous online discussion: A review of the research and empirical exploration. *Instructional Science 38*(6), 571–606. https://doi.org/10.1007/s11251-008-9087-0

Hoyos, V., Navarro, M. E., Raggi, V. J., & Rodriguez, G. (2018). Challenges and opportunities in distance and hybrid environments for technology-mediated mathematics teaching and learning. In J. Silverman, & V. Hoyos (Eds.), *Distance learning, e-learning and blended learning in mathematics education: International trends in research and development* (pp. 29–45). Springer.

Hunter, J. (2009). Developing a productive discourse community in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the mathematics education research group of australasia* (Vol. 1, pp. 249–256). MERGA.

Hurst, C., & Cooke, A. (2014). Seeking a balance: Helping pre-service teachers develop positive attitudes towards mathematics as they develop competency. *Open Journal of Social Sciences*, *2*(4), 210–216. https://doi.org/10.4236/jss.2014.24022

Kazemi, E. (1998). Discourse that promotes conceptual understanding. *Teaching Children Mathematics*, *4*(7), 410–414. https://eric.ed.gov/?id = EJ561776. https://doi.org/10.5951/TCM.4.7.0410

Livy, S., Downton, A., & Muir, T. (2017). Developing pre-service teachers' knowledge for teaching in the early years: Selecting and sequencing. *Mathematics Teacher Education and Development*, *19*(3), 17–25.

Livy, S., Vale, C., & Herbert, S. (2016). Developing primary pre-service teachers' mathematical content knowledge during practicum teaching. *Australian Journal of Teacher Education*, *41*(2), 152–173. https://search.informit.com.au/documentSummary;dn = 855891031724229;res = IELAPA. https://doi.org/10.14221/ajte.2016v41n2.10

Martin, A. (2014). Making online discussion more dynamic and engaging, 2019. Available: https://teachonline.asu.edu/2014/01/making-online-classroom-discussiondynamic-engaging/

Martin, F., Wang, C., & Sadaf, A. (2018). Student perception of helpfulness of facilitation strategies that enhance instructor presence, connectedness, engagement and learning in online courses. *The Internet and Higher Education*, *37*, 52–65. https://doi.org/10.1016/j.iheduc.2018.01.003

McCrone, S. (2005). The development of mathematical discussions: An investigation of a fifth-grade classroom. *Mathematical Thinking and Learning*, *7*(2), 111–133. https://doi.org/10.1207/s15327833mtl0702_2

Muir, T., Milthorpe, N., Stone, C., Dyment, J., Freeman, E., & Hopwood, B. (2019). Chronicling student engagement: Students' experience of online learning over time. *Distance Education*, *40*(2), 262–277. https://doi.org/10.1080/01587919.1600367

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.

Sabry, K., & Baldwin, L. (2003). Web-Based earning interaction and learning styles. *British Journal of Educational Technology*, *34*(4), 443–454. https://doi.org/10.1111/1467-8535.00341

Shea, P., & Bidjerano, T. (2010). Learning presence: Towards a theory of self-efficacy, self-regulation, and the development of a communities of inquiry in online and blended learning environments. *Computers & Education*, *55*(4), 1721–1731. https://doi.org/10.1016/j.compedu.2010.07.017

Smith, M. S., & Stein, M. K. (2013). *5 practices for orchestrating productive mathematical discussions*. NCTM.

Stake, R. E. (1995). *The art of case study research*. Sage Publications, Inc.

Staples, M., & King, S. (2017). Eliciting, supporting and guiding the math: Three key functions of the teacher's role in facilitating meaningful mathematical discourse. In D. A. Spangler & J. J. Wanko (Eds.), *Enhancing classroom practice with research behind principles to actions* (pp. 25–36). National Council of Teachers of Mathematics.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Thomas, G., & Thorpe, S. (2019). Enhancing the facilitation of online groups in higher education: A review of the literature on face-to-face and online group-facilitation. *Interactive Learning Environments*, *27*(1), 62–71. https://doi.org/10.1080/10494820.2018.1451897

Way, J., Cavanagh, M., Ell, F., Livy, S., & McMaster, H. (2020). Advancing our understanding of initial teacher education through research. In J. Way, C. Attard, J. Anderson, J. Bobis, H. McMaster, & K. Cartwright (Eds.), *Research in mathematics education in Australasia* (pp. 91–115). Springer.

Wright, V. (2017). Pre-service teachers' and tutors' perceptions about the value of talk moves. In A. Downton, S. Livy, & J. Hall (Eds.), *Proceedings of the 40th annual conference of the mathematics education research group of Australasia* (pp. 577–584). MERGA.