Stigmergy-based Influence Maximization in Social Networks

Weihua Li[1], Quan Bai[1], Chang Jiang[1] and Minjie Zhang[2]

[1] Auckland University of Technology, New Zealand,
weihua.li@aut.ac.nz; quan.bai@aut.ac.nz; rky9795@autuni.ac.nz
[2] University of Wollongong, Australia
minjie@uow.edu.au

Abstract. Influence maximization is an important research topic which has been extensively studied in various fields. In this paper, a stigmergy-based approach has been proposed to tackle the influence maximization problem. We modelled the influence propagation process as ant's crawling behaviours, and their communications rely on a kind of biological chemicals, i.e., pheromone. The amount of the pheromone allocation is concerning the factors of influence propagation in the social network. The model is capable of analysing influential relationships in a social network in decentralized manners and identifying the influential users more efficiently than traditional seed selection algorithms.

Keywords: Influence maximization, ant algorithm, stigmergy

1 Introduction

With the development of social networks, on-line marketing has developed in an unprecedented scale. One of the typical on-line sales strategies is viral marketing, which propagates influence through 'word-of-mouth' effect [2]. It is capable of increasing brand awareness and achieving marketing objectives effectively. One of the critical tasks is to understand how to select a set of influential users from the network to propagate influence as much as possible with limited resources, namely, influence maximization, and the solution is NP-hard [3][7]. Thus, approximation approaches are considered as a replacement. In general, if a set of influential users, i.e., seed set, can be selected properly and completely, we regard that the influence spread has been achieved. Most researchers seek solutions for influence maximization problem based on the centralized influence diffusion models, such as, the classic Independent Cascade (IC) model and Linear Threshold (LT) model [7]. However, these centralized approaches are normally not efficient, especially when the network is large-scale and dynamic. Specifically, these approaches require a central component to complete all tasks alone. Furthermore, the seed selection algorithms under the traditional influence diffusion models are time-consuming. By contrast, decentralized approaches tend to share the workload by distributing the computational tasks to individuals.

There are two kinds of decentralized approaches in terms of communications. One relies on the direct communications among the individuals, such as cellular automata [9], where each cell in the grid adapts its state by looking at the adjacent neighbours based on a set of rules. While, the other focuses on the indirect communications by reading or analysing the messages left by the peers. One of the typical approaches is ant and stigmergy algorithm [4]. The French Entomologist, Pierre-Paul Grasse defines stigmergy as "stimulation of workers by the performance they have achieved", which is associated with two major features of ants [1]. First, the communication among the ants is indirect. To be more specific, stigmergy is a particular indirect communication mechanism that ants exploited to harmonize their daily tasks with each other. Their indirect communication is conducted through leaving 'pheromone' on the trails, which is a kind of chemical substance and evaporates over time. Second, ants' activities are self-organized. They can complete a complicated task independently without any control. With the development of stigmergy, it has been applied for communication network routing, exploratory data analysis, and diagram drawing etc.

In this paper, we exploit a novel decentralized approach, the Stigmergy-based Influence Maximization approach (SIM), to tackle the influence maximization problem. In SIM, influence propagation process is modelled as ants crawling across the network topology. Furthermore, the ant's key behaviours, including path selection and pheromone allocation, have been modelled for selecting suitable nodes to achieve influence maximization. The former aims to identify the next node to walk when an ant faces multiple options. While, the objective of the latter is to allocate pheromone on the specific nodes based on the heuristics when an ant explores a possible influence-diffusion path. Experiments have been conducted to evaluate the performance of SIM by comparing with the traditional seed selection algorithms, such as greedy selection, degree-based selection and random selection. The results demonstrate that the proposed model is more advanced by considering both efficiency and effectiveness, and can dramatically reduce computational overhead compared with centralized approaches.

The rest of this paper is organized as follows. Section 2 reviews the literatures related to this research work. Section 3 systematically elaborates the SIM approach, including problem description, formal definitions, path selection and pheromone operations. In Section 4, experiments are conducted to evaluate the performance of SIM. Finally, the paper is concluded in Section 5.

2 Related Work

In on-line marking, it is critical to investigate how to propagate influence in a social network with limited budgets. Motivated by this background, influence maximization aims to select a set of influential users from the network to diffuse influence as much as possible with finite resources [7]. Many studies on influence maximization problem are conducted on the basis of two fundamental influence propagation models, i.e., IC and LT [7]. Both models have two key properties, i.e., propagation and attenuation. The influence initiates from the seed set, i.e., activated nodes. They transfer their influence through the correlation graph, whereas the power of this effect decreases when hopping further and further away from the activated nodes.

There are a couple of popular seed selection approaches, such as greedy selection, degree-based selection and random selection. Many research works have been conducted to improve the efficiency and effectiveness of seed selection algorithms on influence maximization. Chen et al. study the efficient influence maximization by improving the original greedy selection and proposing a novel seed selection approach, namely, degree discount heuristics for the uniform IC model, where all edge probabilities are the same [2]. Goyal et al. design and propose a novel CELF algorithm, i.e., CELF++, to reduce running time [6]. Zhang et al. research the least Cost Influence Problem (CIP) in multiplex network, and the CIP is alleviated by mapping a set of networks into a single one via lossless and lossy coupling schemes [12]. However, all these approaches only can be applied in a static network and the network topology must be discovered. Specifically, they cannot handle the dynamics of social networks. Meanwhile, the traditional approaches are not applicable when the global view is unavailable.

Ant and stigmergy-based algorithms do not rely on the network typology, and the computation is decentralized. Stigmergy consists in the main body of ant colony knowledge, as it is a particular mechanism exploited for indirect communication among ants to control and coordinate their tasks. In natural environments, stigmergy-based systems have been demonstrated that they can be utilized for generating complicated and robust behaviours in the systems even if each ant has limited or no intelligence. Nest building is the representative example of stigmergy. Some researchers has applied stigmergy for computer science fields. Dorigo et al. introduce how to solve the Travel Salesman Problem (TSP) [11] by leveraging ant and stigmergy-based algorithms, where the pheromone allocation is concerning the distances among the cities [4]. Ahmed et al. propose a stigmergy-based approach for modelling dynamic interactions among web service agents in decentralized environments [8]. Takahashi et al. proposed anticipatory stigmergy model with allocation strategy for sharing near future traffic information related to traffic congestion management in a decentralized environment [10].

3 Stigmergy-based Influence Maximization Modelling

SIM tends to select appropriate influential candidates by considering both influence strengths among users and the assembled influential effect. In this model, numerous ants walk simultaneously and update the shared environment by distributing pheromone, and the influence propagation process is simulated as crawling behaviours of ants. The influential users can be identified when the pheromone distribution in the network starts to converge, and the seed selection is based on the pheromone amount of each node. The SIM will be elaborated in the following subsections.

3.1 Problem Description

Suppose an organization plans to promote a particular product in a large-scale on-line social network. Due to limited budgets and insufficient time, the organization needs to select k initial candidates as influential users to experience the product as soon as possible, hoping that these users can recommend it in their social circle. Ideally, the k influential users can produce maximum influence in the social network.

3.2 Formal Definitions

Definition 1: A **Social network** is defined as a weighted graph $G = (V, E)$ with a clear topological structure, where $V = \{v_1, v_2, ..., v_n\}$ stands for the nodes (users) in the network, $E = \{e_{ij} | v_i \in V \wedge v_j \in V, v_i \neq v_j\}$ denotes the edges (relationships) among nodes. A particular edge can be represented as a three-tuple, i.e., $e_{ij} = (v_i, v_j, w_{ij})$, where w_{ij} is the weight of e_{ij} which represents the influence strength. Each node v_i has a set of neighbours $\{v_j | v_j \in \Gamma(v_i), e_{ij} \in E\}$. While, $v_i.q$ indicates the pheromone amount (see Definition 4) accumulated on corresponding node v_i, which can be regarded as an attribute of v_i. Similarly, since w_{ij} represents the weight of edge e_{ij}, it is denoted by using the notation $e_{ij}.w$ in this paper.

Definition 2: An **Ant** a_m is defined as an autonomous agent in the network G, which crawls across G based on the network topology. An ant can be represented as a three-tuple, i.e., $a_m = (m, q_m^n, T_m^n)$, which means ant a_m carries q_m^n pheromone in tour T_m^n (see Definition 3). There exist a number of ants, $A = \{a_1, a_2, ..., a_n\}$, in the social network, and they keep crawling in the network. Moreover, they are capable of discovering and evaluating the amount of pheromone on the current node and the ones nearby. However, the ants cannot communicate directly with each other.

Definition 3: A **tour** $T_m^n =< v_1, v_2, ..., v_n >$ is defined as the path that ant a_m walks through in the n round. Specifically, ant a_m randomly selects a starting point. Next, it crawls from one node to the adjacent neighbours and eventually ceases when reaches the end point v_e, where $\Gamma(v_e) \subset T_m^n \cup |\Gamma(v_e)| = 1$.

Definition 4: Pheromone represents the information and heuristics passed by an ant to the peers based on its experience. q_m^n denotes the total amount of artificial pheromone carried by ant a_m in the n round, which will be distributed to each node of T_m^n after a_m completes the tour.

3.3 Path Selection

In this context, path selection is one of the ant's basic behaviours, which describes how a particular ant a_m selects the next node to walk when standing at v_i and facing multiple choices $V_c = \{v_j | v_j \in \Gamma(v_i) \wedge e_{ij} \in E\}$.

Basically, the path selection decision is based on two aspects, which are the pheromone amount of v_j, i.e., $v_j.q$, and the weight of corresponding edge, i.e., $e_{ij}.w$. The path selection behaviour has been modelled as a probabilistic event by using Equation 1, where p_{ij} denotes the probability that an ant walks from node v_i to v_j.

$$p_{ij} = \begin{cases} \frac{e_{ij}.w \cdot v_j.q}{\sum_{v_x \in \Gamma(v_i)} e_{ix}.w \cdot v_x.q}, & e_{ij} \in E \\ 0, & e_{ij} \notin E \end{cases} \tag{1}$$

Here, we demonstrate the path selection by giving two concrete examples. In Figure 1, ant a_i starts from node v_i and confronts three options, i.e., v_k, v_j and v_n. The decision is made by considering both targeting nodes' pheromone amount and the influence strength / weight of the corresponding edges. In this diagram, the probability of choosing node v_j is calculated as: $p_{ij} = e_{ij}.w \cdot v_j.q/(e_{ij}.w \cdot v_j.q + e_{ik}.w \cdot v_k.q + e_{in}.w \cdot v_n.q) = 0.8 \times 0.5/(0.4 \times 0.6 + 0.8 \times 0.5 + 1.0 \times 0.7) = 29.85\%$

Figure 2 demonstrates another example, where two ants, i.e., a_i and a_j, walk in the same network. Based on the path selection principles, they cannot choose the nodes which have been walked through within the same tour, but they can choose the ones that other ants have passed before in the either current or previous iterations.

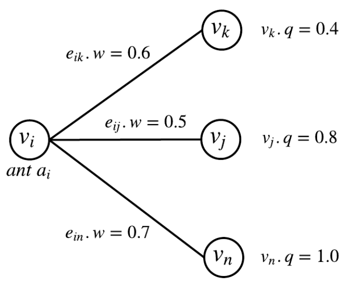

Fig. 1. Path selection of an ant

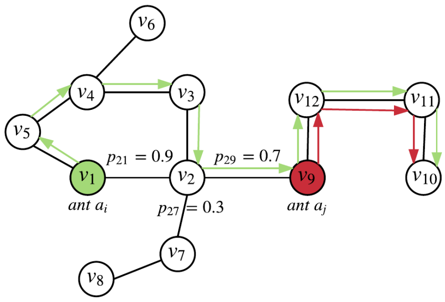

Fig. 2. Path selection of multiple ants

Each ant keeps performing an iterative process: walking and selecting path, whereas, the action stops when the ant reaches the end point. In other words, the iterative process triggered by ant m in round n produces a path vector, i.e., tour T^n_m. The tour formation is described in Algorithm 1.

Algorithm 1 presents the process of how a particular ant completes a tour. The input of this algorithm includes ant a_m and the round index n, while the output is tour T^n_m. Line 3 shows the criteria of walking to the next node. Lines 5-10 demonstrate the targeting candidates selection, where σ is a predefined threshold to filter out those candidates with low probability. Lines 11-17 indicate the path selection process. The iterative walking process ends when all of the current node v_s's neighbours reside in the tour list T^n_m.

Algorithm 1 Tour Formation Algorithm

Input: a_m, n
Output: $T_m^n, T_m^n \subseteq V$

Initialize a_m and random select a starting point $v_s, v_s \in V$
Initialize a tour list $T_m^n := \emptyset$
while $\exists \Gamma(v_s) \wedge \Gamma(v_s) \not\subset T_m^n$ **do**
  Initialize candidate list $V_c := \emptyset$
  for $\forall v_i \in \Gamma(v_s) \wedge v_i \notin T_m^n$ **do**
    Compute the probability p_{si} using Equation 1.
    if $p_{si} > \sigma$ **then**
      $V_c := V_c \cup \{v_i\}$
    end if
  end for
  if $V_c \neq \emptyset$ **then**
    Determine the next node $v_n \in V_c$ using Equation 1.
    $T_m^n := T_m^n \cup \{v_n\}$
    $v_s := v_n$
  else
    $v_s := null$
  end if
end while

3.4 Pheromone Operations

Sub-network Generation Sub-network generation is the preliminary step of pheromone operations. After ant a_m completes a tour T_m^n, a corresponding sub-network $G_m^n = (V_m^n, E_m^n)$ will be generated based on the path that a_m walked through. V_m^n incorporates all nodes in tour T_m^n and their valid first-layer neighbours $\Gamma(T_m^n)$, thus, $V_m^n = T_m^n \cup \Gamma(T_m^n)$. While, the edge set E_m^n includes all the links among V_m^n.

The total amount of pheromone q_m^n carried by ant a_m for tour T_m^n depends on the total number of nodes in the sub-network, i.e., $|V_m^n|$. Each node in the sub-network contributes one unit of pheromone. Figure 3 presents an example of a generated sub-network. An ant walked from node v_a to node v_e sequentially. By walking pass each of them, the ant searches for the valid first-layer neighbours. In this way, a sub-network is generated.

Pheromone Allocation Pheromone allocation in general refers to how ants leave the biological information on the nodes that they have walked through. The distribution of pheromone plays an important role in the stigmergy algorithms, since it updates the context by considering the relevant impact factors. Therefore, the solution is continuously being optimized.

In the current setting, the pheromone distribution is based on size of the sub-network. The shorter length path and larger sub-network size, the more pheromone will be allocated on each node of the tour. Equation 2 aims to compute the number of connected neighbours of node v_i in the sub-network G_m^n.

Equation 3 describes the pheromone accumulation of node v_m in tour T_m^n, which is calculated by adding up all the pheromone contributions given by the direct neighbours $\Gamma(v_m)$.

$$v_i.N = |\{v_i | v_i \in V_m^n \wedge \Gamma(v_i) \in T_i^n\}| \tag{2}$$

$$v_m.\Delta q = \begin{cases} \sum_{v_i \in \Gamma(v_m)} \frac{1}{v_i.N}, & v_m \in T_m^n, v_i.N \neq 0 \\ 0, & v_m \in T_m^n, v_i.N = 0 \end{cases} \tag{3}$$

Figure 3 shows an example of a specific sub-network $G_m^n = (V_m^n, E_m^n)$, where the tour travelled by ant a_m is represented as $T_m^n =< v_a, v_b, v_c, v_d, v_e >$, $V_m^n = \{v_a, v_b, v_c, v_d, v_e, v_f, v_k, v_h, v_i\}$ and E_m^n includes all the edges among the nodes in V_m^n, $|E_m^n| = 12$ in this diagram. Node v_f is the direct neighbour of two nodes in tour T_m^n, hence both v_a and v_b obtain half of a unit pheromone from v_f. Meanwhile, node v_b contributes 0.5 unit pheromone to v_a and v_c, but v_f and v_k are not considered in this scope. Therefore, we can derive that the pheromone gain for node v_a and v_b are 1.5 and 2.0 respectively.

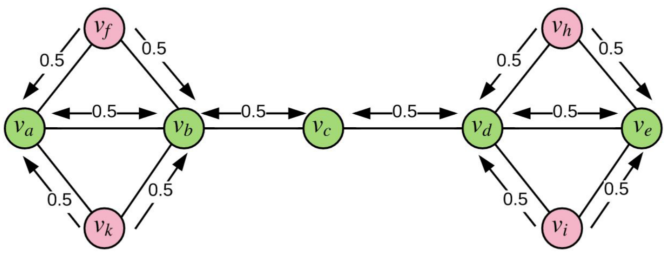

Fig. 3. Pheromone allocation in a tour with five nodes

Algorithm 2 shows the pheromone allocation process initiated by ant a_m in tour T_m^n. The distribution is based on the explored sub-network G_m^n's topology. The input is a specific tour T_m^n. Whereas, the output is pheromone amount update. Specifically, this algorithm aims to change the context by updating the pheromone amount located in each node of the tour path. Lines 1-9 initialize and construct the sub-network G_m^n. The objective of Lines 10-12 is to obtain the denominator for each node which supposes to contribute pheromone to the nodes in tour path. Lines 13-14 show the variations of pheromone.

Pheromone Evaporation Pheromone evaporation is a common phenomenon, where the amount of allocated pheromone decreases over time. In ant and stigmergy algorithms, it helps to avoid the convergence to a locally optimal solution. Pheromone evaporates from each node within the scope of the whole network at

Algorithm 2 Pheromone Allocation Algorithm

Input: T_m^n

Output: *pheromone changes for all the nodes in* T_m^n

Initialize sub-network graph $G_m^n := (V_m^n, E_m^n), V_m^n := \emptyset, E_m^n := \emptyset$
for $\forall v_i \in T_m^n$ **do**
    for $\forall v_j \in (\mathit{\Gamma}(v_i) \cup v_i)$ **do**
        $V_m^n := V_m^n \cup \{v_j\}$
        if $p_{ij} > 0 \wedge i \neq j$ **then**
            $E_m^n := E_m^n \cup \{e_{ij}\}$
        end if
    end for
end for
for $\forall v_n \in V_m^n$ **do**
    Compute $v_n.N$ using Equation 2
end for
for $\forall v_m \in T_m^n$ **do**
    $v_m.q := v_m.q + v_m.\mathit{\Delta} q$, using Equation 3
end for

the same time. At a justified time, all of the nodes in the network will evaporate a predefined unit of pheromone. The pheromone evaporation is quantified by using Equation 4, where the amount of pheromone evaporated from each node is associated with the time difference $\mathit{\Delta} t$ and the evaporation speed λ.

$$EQ = e^{\frac{\mathit{\Delta} t}{\lambda}}, \lambda \neq 0 \tag{4}$$

3.5 Seed Selection

Seed selection aims to select a set of influential users from a specific network, so that they can propagate influence to others. There are quite a few classic seed selection approaches. More specifically, degree-based seed selection tends to select the nodes with high node degree. Intuitively, the users with large friend circle can influence more users in the social network. However, this does not hold in general, e.g., two connected users with very high degree may have a lot of common friends, in other words, the impact generated by both may be pretty much close to choosing either of them. Another well-known approach is greedy selection, which aims to obtain the maximum influence marginal gain in selecting each seed. However, this approach is not applicable in large-scale networks due to the computational overhead. Random selection is also applied in some cases, but its performance is normally the worst since it is not based on any heuristics.

The seed selection in stigmergy-based algorithm relies on the amount of pheromone allocated on each node. The selection is similar to degree-based approach, but it identifies the influential users by ranking the pheromone degree of each node.

In Algorithm 3, the input includes the number of ants n, seed set size k, evaporation speed λ, time difference $\mathit{\Delta} t$ and the network $G = (V, E)$. Lines

Algorithm 3 Seed Selection Algorithm

Input: $n, k, \lambda, \Delta t, G = (V, E)$
Output: V_s

1: Initialize ant set $A := \{a_1, a_2, ..., a_n\}$ which contains n ants.
2: Initialize seed set $V_s := \emptyset$
3: All the n ants start to crawl in network G in the distributed servers.
4: **while** !convergence **do**
5:     Compute EQ using Equation 4.
6:     **for** $v_i \in V$ **do**
7:         $v_i.q := v_i.q - EQ$
8:     **end for**
9:     Sleep for Δt
10: **end while**
11: Sort V order by q descend
12: **for** $\forall v_i \in V$ **do**
13:     **if** $|V_s| < k$ **then**
14:         $V_s := V_s \cup \{v_i\}$
15:     **end if**
16: **end for**

1-2 initialize the ants and seed set. Line 3 indicates the ants' autonomous behaviours in the network by using Algorithms 1 and 2. Lines 4-10 show the global pheromone evaporation process. Lines 11-16 indicate the seed selection from the updated environment.

4 Experiments and Analysis

4.1 Experiment Setup

MovieLens[3] dataset has been used for the experiments. It is a stable benchmark dataset, which contains around one million ratings for 3,900 movies given by 6,040 users. To filter noise data, users whose number of ratings are less than 50 have been removed from the dataset. There are no explicit links among the users, but the implicit links can be generated according to the ratings to items. Moreover, in order to control the computing time, we select three sub-graphs of the network with different scales, i.e., size of 500, 750 and 1000 respectively, for the experiments.

The node degree distributions of three sub-graphs are represented as Figure 4, 5 and 6. All of them follows the power-law distribution pattern which is satisfied by most real networks [5].

4.2 Global Pheromone Distribution

As explained in Section 3.4, all the artificial ants crawl in the social network and allocate pheromone after completing tours, whereas the allocated pheromone

3 http://grouplens.org/datasets/movielens/

keeps evaporating over time. The total amount outstanding pheromone in the social network is regarded as the global pheromone.

The global pheromone distributions of three sub-graphs are demonstrated in Figures 7, 8, and 9. As we could observe from these three diagrams that the pheromone amount increases steadily and starts to oscillate when reaching a certain level. Thus, the pheromone allocation and evaluation almost achieve a balance. At this phase, it implies the network starts to converge, since the sequential pheromone ranking list does not vary a lot.

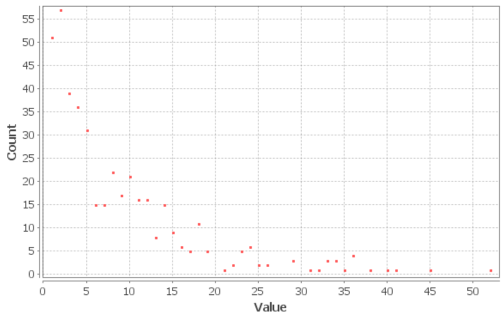

Fig. 4. Degree distribution (size = 500)

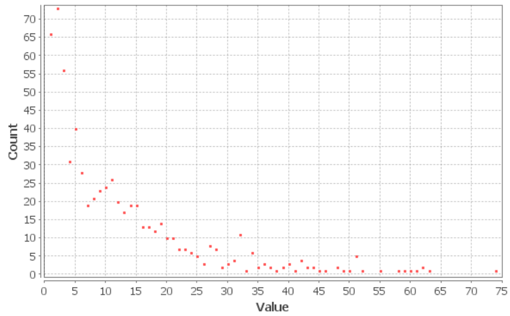

Fig. 5. Degree distribution (size = 750)

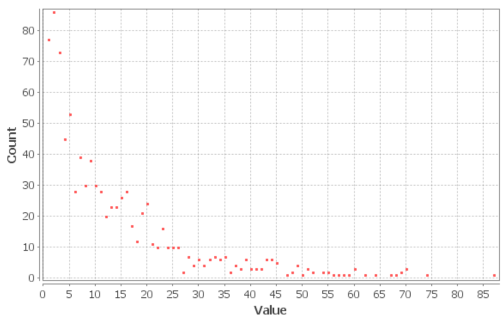

Fig. 6. Degree distribution (size = 1000)

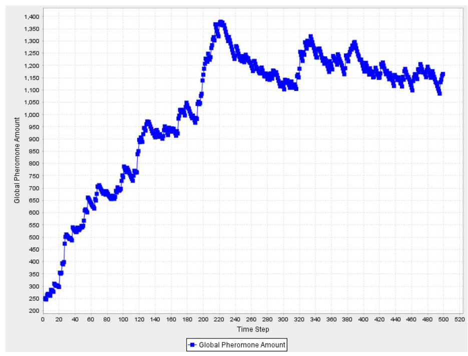

Fig. 7. Global pheromone distribution (size=500)

4.3 Experimental Results

We conducted two experiments by using the same social network of three different sizes, which are 500, 750 and 1000 respectively. The first experiment aims to evaluate the influence effectiveness of stigmergy-based algorithm, i.e., the total number of users activated by the seed set. While, the second tends to compare the efficiencies, i.e., the running time of selecting seed set. The counter parts of stigmergy-based algorithm include greedy selection, degree-based selection and random selection.

In the first experiment, seeds are selected from the proposed model, and input into the IC model to evaluate the influence effectiveness by comparing with the other classic algorithms. Figures 10, 11 and 12 demonstrate the influence effectiveness comparison among the four algorithms in three sub-graphs. The stigmergy-based algorithm performs better than both degree-based selection and random selection, and its performance is even closer to the greedy selection when the network size is 500. With the expansion of the graph, stigmergy-based selection's influence effectiveness drops a little bit but still outperforms the rest.

The second experiment analyses the efficiency of four seed selection algorithms by comparing the running time. Running time required by stigmergy-based algorithm includes ants initiation and pheromone operations. While, the other three algorithms are evaluated in the IC model. Figures 13, 14 and 15 show the efficiency comparison among the four algorithms in different sub-graphs. It is clear that the greedy selection is the most computational expensive of all, the running time increases dramatically when the seed set size enlarges. Both random and degree-based selection are very similar to each other in terms of efficiency. The stigmergy-based appear a little bit higher than degree-based selection, but it is much more efficient than the greedy selection and computational cost does not increase a lot with the expansion of the network.

In summary, observing from the experimental results, we can conclude that the stigmergy-based algorithm performs better than the traditional algorithms by considering both efficiency and effectiveness.

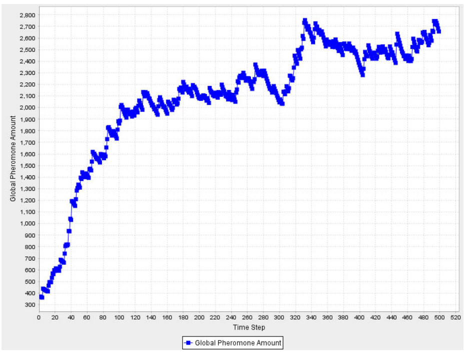

Fig. 8. Global pheromone distribution (size=750)

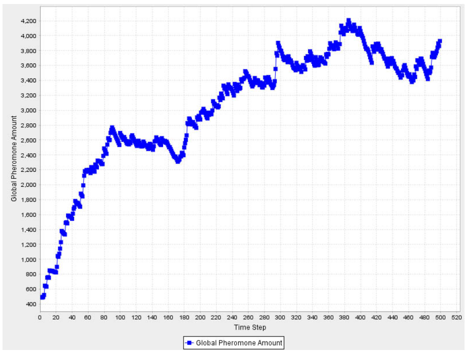

Fig. 9. Global pheromone distribution (size=1000)

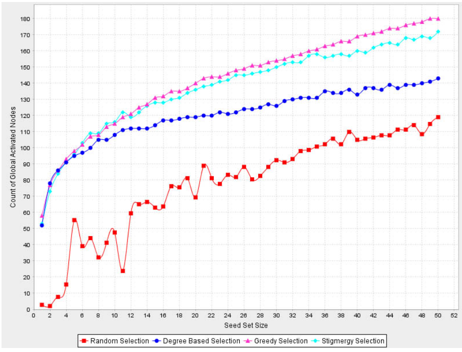

Fig. 10. Influence effectiveness comparison (size=500)

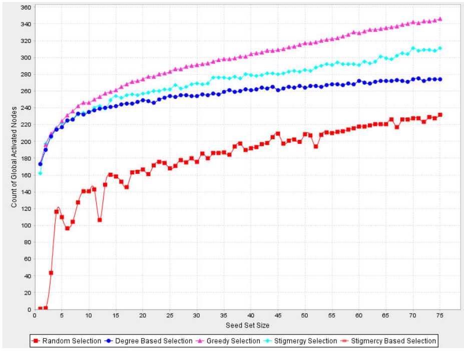

Fig. 11. Influence effectiveness comparison (size=750)

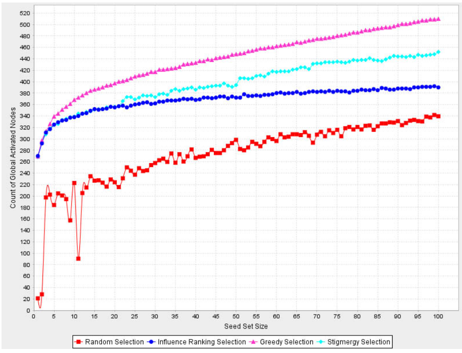

Fig. 12. Influence effectiveness comparison (size=1000)

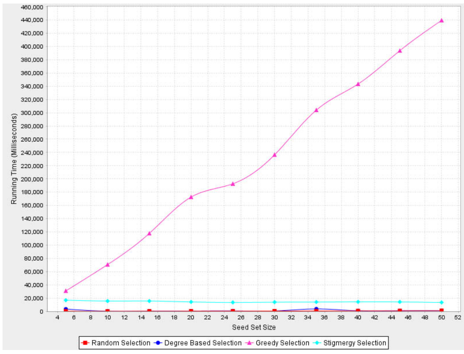

Fig. 13. Efficiency comparison (size=500)

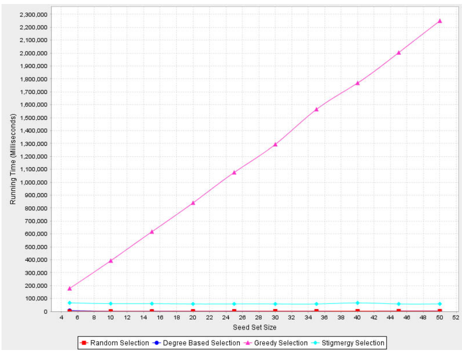

Fig. 14. Efficiency comparison (size=750)

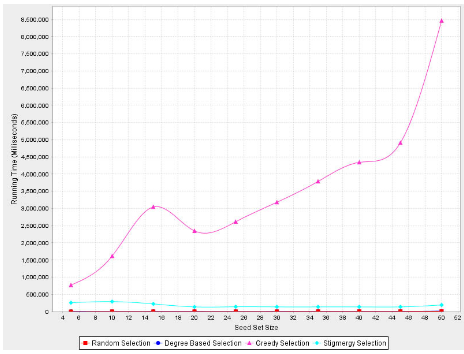

Fig. 15. Efficiency comparison (size=1000)

5 Conclusion and Future Work

In this research, we introduced a novel approach, i.e., stigmergy-based algorithm, to tackle the influence maximization problem in a decentralized environment. In the meanwhile, SIM model has been proposed and systematically elaborated. Experiments have been conducted to evaluate the performance of SIM. Experimental results reveal that SIM outperforms the traditional seed selection approaches, including greedy selection, degree-based selection and random selection, by considering both effectiveness and efficiency. Moreover, SIM is applicable for large-scale networks and even functions without a global view.

In the future, learning algorithms will be employed to improve the performance of the stigmergy-based algorithm in influence maximization problem. Meanwhile, we will consider a hybrid approach for developing a more practical model.

References

1. E. Bonabeau. Editor's introduction: stigmergy. *Artificial Life*, 5(2):95–96, 1999.
2. W. Chen, Y. Wang, and S. Yang. Efficient influence maximization in social networks. In *Proceedings of the 15th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 199–208. ACM, 2009.
3. P. Domingos and M. Richardson. Mining the network value of customers. In *Proceedings of the seventh ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 57–66. ACM, 2001.
4. M. Dorigo, E. Bonabeau, and G. Theraulaz. Ant algorithms and stigmergy. *Future Generation Computer Systems*, 16(8):851–871, 2000.
5. D. Easley and J. Kleinberg. *Networks, crowds, and markets: Reasoning about a highly connected world.* Cambridge University Press, 2010.
6. A. Goyal, W. Lu, and L. V. Lakshmanan. Celf++: optimizing the greedy algorithm for influence maximization in social networks. In *Proceedings of the 20th international conference companion on World wide web*, pages 47–48. ACM, 2011.
7. D. Kempe, J. Kleinberg, and É. Tardos. Maximizing the spread of influence through a social network. In *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 137–146. ACM, 2003.
8. A. Mostafa, M. Zhang, and Q. Bai. Trustworthy stigmergic service composition and adaptation in decentralized environments. *Service Computing, IEEE*, 2014.
9. D. Shiffman, S. Fry, and Z. Marsh. *The nature of code*, chapter 7 Cellular Automata, pages 323–330. D. Shiffman, 2012.
10. J. Takahashi, R. Kanamori, and T. Ito. A preliminary study on anticipatory stigmergy for traffic management. In *Web Intelligence and Intelligent Agent Technology (WI-IAT), 2012 /WIC/ACM International Conferences on*, volume 3, pages 399–405. IEEE, 2012.
11. K. Wang, L. Huang, C. Zhou, W. Pang, et al. Traveling salesman problem. In *Conference on Machine Learning and Cybernejcs*, volume 3, pages 1583–1585, 2003.
12. H. Zhang, D. T. Nguyen, H. Zhang, and M. T. Thai. Least cost influence maximization across multiple social networks. 2015.