The Numerical Dysfunction

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Introduction

The opinions of Anatole Beck in his article *The Decimal Dysfunction* [1] were refreshing and interesting, and his discussion of enumeration and mensuration was surely important and provocative. A learned and detailed argument devoted to showing as "folly" the SI metric system adopted by so much of the world, and soon to be adopted by the U.S.A. [2], deserves some serious response. If the SI system is indeed folly, then mathematicians everywhere have a duty make this as widely known as possible. If it is not, then a rebuttal should be published.

That the only published reaction [3] should be jocular, however witty, therefore deserves comment. Jerry King, in his splendid *The Art of Mathematics*, writes "The applied mathematician emphasizes the application; the pure mathematician reveres the analysis." [4 p.110] Perhaps then neither kind of mathematician sees simple enumeration and mensuration as worthy of consideration, so that it remains for a technologist to cry shame on those mathematicians who ignored Professor Beck's argument, and who thus show themselves apathetic about the innumeracy that "plagues far too many otherwise knowledgeable citizens" [5 p.3] and about "the declining mathematical abilities of American [and other] students" [4 p.176].

This article is intended, therefore, to give views on enumeration and mensuration from a technologist, one with a background in engineering and cognitive science, with three decades of experience in the computing industry, and with a lifelong interest and a decade of experience in education. This article argues that the SI metric system is indeed flawed, though not in the way Professor Beck sees it; that the way we have come to represent numerical values is even more flawed; and that the general public would be best served by a reduced SI metric system supported by an improved (SI numeric ?) system for representing numeric values. If these arguments are valid, then mathematicians everywhere have a professional duty actively to promote reforms, of the kind described here, as a necessary basis for efforts to reverse present trends in public innumeracy.

Enumeration

A major theme of Professor Beck's article is, as its name proclaims, that decimal enumeration is not the best enumeration system. The reason ? Ten "appears essentially not at all in mathematics, where the natural system of numeration is binary. .... One might blasphemously take the importance of 2 in mathematics as a sign that God does His arithmetic in binary". By definition God is omniscient, and it is blasphemous to imply that She has to do any arithmetic at all! But 1 is much more important in mathematics than 2 is, so wouldn't unary numbers—tallies, which indeed have a long tradition [6]—be better still? This is the analytical argument, appropriate to a pure mathematician.

However, although binary enumeration might be analytically ideal, it is not after all practical—an applied mathematician's point of view, now. "Binary numbers are too long to read conveniently and too confusing to the eye. The clear compromise is a crypto-binary system, such as octal or hexidecimal." In what way, then, are these systems a clear improvement on the decimal? To a society now used to decimal enumeration, any non-decimal system will be confusing.

Would octal or hexidecimal be more convenient than decimal, for example in being more accurate or brief? Octal integers are a little longer than decimal, but hexidecimal are somewhat shorter. All are exact. Not much to justify change in that. But integers are not the only kind of numbers. Numbers with a decimal fraction component, as for example 5.141, are also commonly used. Here the three systems give the same length for expressing a half, but decimal is longer than either of the others for the quarters and the eighths. However, the decimal system can express fifths exactly, which neither the octal nor the hexidecimal can do.

Along this line, it is significant to observe that the smaller its denominator the more used a fraction is likely to be. This observation is behind the benefits so often argued for the duodecimal system of enumeration, which can express halves, thirds, quarters, and sixths exactly and succinctly. The most personal of the old Imperial measures were conveniently used duodecimally—twelve inches to the foot, twelve pennies to the shilling. We still have twelve hours to the o'clock, and twelve months to the year. The movement for a thorough dozenal system is quite old—Isaac Pitman tried to introduce it with his first shorthand system [7]. Full and lucid arguments for the system can be found in texts [8], and in the publications of the various Dozenal Societies. (The Dozenal Society of America advertises from time to time in *The Journal of Recreational Mathematics* and, going by the World Wide Web, has its headquarters on the Nassau campus of SUNY.) However, the dozenal cause now seems a hopeless one, simply because of the practical difficulties facing any scheme of conversion, given that decimal enumeration has taken such a global hold since the First World War.

Even if a way could be found to convert to a dozenal system, doing so would not compel the abandonment of the metric system. The SI metric standards are not inherently decimal because the basic and secondary units of measurement could as well be used with a dozenal system of enumeration as with a decimal system. Mensuration combines an enumerated value with a unit of measure, and a good metric system will provide practical and useful units of measure.

Mensuration

The many old metric systems were practical and useful in respect of how quantities could then be measured (often by some human action giving units like paces or bowshots) and how standards could be administered [9]. There were different units of length for different ways of measuring them, different units of quantity for different things being measured, and different units for different towns and villages. However, the old measures were prone to being used by the powerful to exploit the weak, as implied by various admonitions in both the Bible (for example, Deuteronomy 25 : 13–16) and the Quran (Sura 83 : 1–17).

The worldwide metric system defines as few basic units as possible, and secondary units such as for areas and volumes are derived from the basic ones. Though it might spring rationally or irrationally from the French Revolution, the system is overwhelmingly superior to the old humanistic systems if only because its arithmetic simplicity and world-wide acceptance make it less subject to cheating and misunderstanding. The difficulties for adoption of the metric system now are much fewer and more transient than for the illiterate and innumerate society of Revolutionary France, where the changeover lasted for two generations [9 p.264].

Of course, there is something intrinsically valuable about a culture having a characteristic way of doing things, and this is true of household measures as well as of say cuisine. But the basic vehicle of culture is language, and anyone truly concerned about the preservation of cultural richness and variety (as surely we all should be) would be much better employed combatting the present oligolingual rush than opposing metrication. Languages are dying off at a much faster rate than species !

The strange thing about the metric system, though, is that, while the basic units (and some of the secondary ones) are widely and consistently applied, each of these is the basis of a bewildering collection of pseudounits defined through a somewhat arbitrary system of scaling prefixes. Not only are the prefixes weird in themselves, but they also have inconvenient abbreviations, including highly confusable upper and lower cases of the same letter (Y is *yotta*, y is *yocto* [2]), and even a letter (μ) from the Greek alphabet. Although it is averred that the prefixes are easy to learn and use, in practice their spelling, their pronunciation, and their meanings are all confused and confusing in popular use. And it is popular use that's important.

These prefixes are really only suited for use in private among consenting adults. It took a physicist, the famous Richard Feynman, to advocate that the prefixes be abandoned because they actually express scalings of the measurements being made, and because they are "really only necessitated by the cumbersome way we name numbers." [10]

What does the experience of Australia, a country converted to SI metrics only a few decades ago, have to tell about the popular use of the prefixes ?

The most important measurements in everyday life are those of length, weight, volume, and temperature. Here in Australia, the discouragement of the prefixes which are not multiples of one thousand seems to have had good effect, the faulty early publicity notwithstanding [an example in 11]. This is a very good thing, because now there are fewer ways to express any measurement and this must greatly reduce the potential for confusion. Centimetres are occasionally used (so are, for the moment, feet and inches when giving anyone's height), and the hectare seems to have replaced the acre for people of large property. But centi- and hecto- are otherwise never seen, and the confusing deci- and deka- have disappeared altogether.

Celsius, the new temperature unit, took over straight away, possibly because the old scale was plainly silly and the cultural value slim. Oddly enough the unit is almost always spoken of simply as *degrees*. For lengths, people seem comfortable with millimetres and metres and kilometres, though in casual speech the abbreviations *mil* and *kay* are more often used, particularly the latter. Grams and kilograms are comfortably used for weights, though the abbreviation *kilo* (pronounced *killo*) is preferred to the full name. For volumes, the use of millilitres and litres has completely taken over, though again the abbreviation *mills* (not *mil* as for lengths !) is often heard. The use of a secondary unit, litres, for volume is justified by its relative brevity in speech, so that no abbreviation is needed. The only problem is that the litre has become somewhat divorced from the cubic metre, and people are not always able to compare volumes in the two units swiftly and reliably.

The conclusion to be drawn from the Australian experience is that, while the common metric units of measure have been everywhere adopted, their names have been found difficult, and all the long ones have been abbreviated in common speech, typically by elision of the basic unit name. Measures, and numbers, must be simple to be popular.

Emancipation

The challenge is to free numbers generally from the thrall of technologists and mathematicians so that more of them become easy for people to use. A great way to start is to get rid of the metric prefixes along the lines suggested by Feynman, and to build on popular usage.

• Let any number to be interpreted as scaled UP by 1000 be suffixed by $_{\mathrm{k}}$, and let a number like 100_{k} be pronounced *one hundred kay*.
• Let any number to be interpreted as scaled DOWN by 1000 be suffixed by $_{\mathrm{m}}$, and let a number like 100_{m} be pronounced *one hundred mil*.
• Let any number to be scaled UP twice by 1000, that is up by 1000000, be suffixed by $_{\mathrm{k}}2$, and let a number like $100_{\mathrm{k}}2$ be pronounced *one hundred kay two*.
• Let any number to be scaled DOWN twice by 1000, that is down by 1000000, be suffixed by $_{\mathrm{m}}2$, and let a number like $100_{\mathrm{m}}2$ be pronounced *one hundred mil two*.

Adoption of these rules, and of the extensions to other scales that they imply, is in accord with, indeed would reinforce, both the intent of the SI metric standards, and the common sense of popular linguistic practice. Adoption of these rules would allow the metric prefixes and their upper case, lower case, and Greek abbreviations to be forgotten, would allow common talk of numbers to be as loose or precise as needed, and would deliver a wider range of numbers and quantities into common parlance and understanding. Measurements outside the scales of common usage would at least be recognised roughly for what they are, if not wholly understood.

These rules are simple enough to be accepted by the general public, and expressive enough to be used by scientists and engineers, and even by mathematicians. Indeed the notation is similar to the so-called engineering or e- notation, but better than it because there are fewer ways to represent any particular value. E-notation was adopted by technical people submitting to the limitations of the printers that were attached to early digital computers, and in it $100_{k}2$ might represented as 1E8 or 100E6 or 0.1E9.

Adoption of the notation for scaled numbers proposed here could allow dropping of quirky notations which disguise pure numbers as measurements, for example, under the pseudounit *decibel*, and eventually even the percentage, and its pseudosubunit the point or percentage point, might meet their Boojum and softly vanish away.

Most importantly, the notation would allow phasing out the present usage in mathematics and science which shows scaled numbers as expressions like 3×10^{10}. This style is particularly confusing for students. Is it a number, or is it an expression, or is it a calculation ? A mathematician or scientist may be able to see immediately past the calculation to the number it produces, but to ordinary mortals the expression hides the number. Mathematicians, or at least mathematics teachers, have in this ambiguity another very good reason for adopting a scheme like that suggested above for representing scaled numbers.

Representation

To confuse expressions like 3×10^{10} with numbers is bad enough, but at least elementary school children are not normally exposed to this particular ambiguity. However they are exposed to a very similar ambiguity early in their arithmetic education, an ambiguity that some would have costs the average pupil six months of schooling, and brings some pupils a lifetime of innumeracy. This is the ambiguity in notations such as -1 and -15 where the role of the hyphen is ambiguous [12]. Is it the sign for the property of negativity, or is it the symbol for the function of subtraction ? A conspicuous sign is needed to stand for the property of negativeness in a number, a sign quite distinct from the symbol for subtraction.

Because the present ambiguity is not overtly recognised in early schooling, few adults are even aware of it. Perhaps mathematicians consciously distinguish the two meanings given to the hyphen. “Unfortunately, what is clear to a mathematician is not always transparent to the rest of us.” [4 p.50] Particularly not to children. That this ambiguity is a real problem is shown by the many texts for teaching elementary mathematics that use temporary notational subterfuges in an attempt to overcome the ambiguity. However, the most popular method merely raises the hyphen to the superscript position, which doesn’t actually change the sign, and certainly doesn’t make the distinction obvious. Some texts even increase the problem by using a raised + to mark positivity [e.g. 13 p.153], thereby spreading the ambiguity to another basic symbol.

The ambiguity extends to the spoken word. The hyphen is read out as *minus* whether it is used as the negative sign or as the subtraction symbol. This is a severe problem because the natural word for the sign, *negative*, is three syllables long, one too many for it to be popular. Words like *off* and *short* can have the right kind of meaning, but might become ambiguous within sentences. Maybe the abbreviation *neg* could be adopted.

The negative sign should be used as a prefix, since it is spoken as an adjective, since the left end of any ordinary number is its most significant end, and since the negative sign is in some ways the most significant cipher as it completely reverses the significance of the value it prefixes. The wretchedly inadequate ASCII character set foisted on the world by the computing industry has no suitable symbol. Designing, or even producing a symbol is no problem nowadays, particularly for anyone using a facility like TEX. Selecting from what is already available in TEX, a suitable symbol might be a triangle, superscripted and reduced in size to be aesthetically and perceptually better. Compare $\bigtriangledown 72$ and $^{\triangledown}72$, though a superscript vee or cup could be used as an option for easier handwriting, as in $^{\vee}72$ or $^{\cup}72$. The problem is rather that of getting the symbol onto the everyday keyboard.

One new symbol is not enough. The fraction point needs a new symbol as much as the negative sign does. The dot used in much of the world for the fraction point is more inconspicuous than any other symbol apart from the blank space. Furthermore it is used as punctuation in ordinary text, leading to ambiguities in particular at the end of sentences ending in numbers. True, mathmaticians sometimes raise it a little to emphasise that it's not a period, but that is infrequently noticed even when it *is* done. An unambiguous *point* symbol is needed, and with TEX the point could be contrasted with but related to the negative sign, giving numbers like $7_{\vartriangle}2$ or $7_{\wedge}2$ or $7_{\cap}2$. That this inconspicuousness is recognised as a difficulty is demonstrated by the common precaution of protecting the dot from exposure by writing for example 01 rather than ·1, by the use of the comma instead of the dot for the fraction point in Continental Europe, and by the misbegotten attempt by the Australian Government to use the hyphen for the fraction point in monetary quantities when decimal currency was introduced.

An aside about monetary quantities. As they are imaginary values dependent only on social agreement, they could well be used more in mathematics. Imaginary numbers, as they are normally shown, suffer from a representational ambiguity much like that of scaled numbers, being almost always given as multiplicative expressions, like $5i$ and $16i$. These two imaginary numbers can be added and subtracted but not multiplied to give an imaginary result, and they can be divided to give a real result, behaviour generally reckoned to be too mysterious for ordinary understanding. But \$5 and \$16 behave in the same manner, a manner not at all mysterious even for many elementary school children. If small children can learn about imaginary values in the guise of money, it should be a minor extension to teach them about complex values provided they are written in the unambiguous style of 2\$3 and $_{\vartriangle}5\$^{\triangledown}1$ rather than as expressions like $2 + 3i$ and $0.5 - i$. And maybe it would be better in schools to talk of the two Cartesian parts of a complex number as *the amount* and *the money*, rather than as *the* *real part* (which is ambiguous) and *the imaginary part* (which is misleading).

Exactness

It is one thing to be able to express a value unambiguously as a value, and provision of a distinctive negative sign and fraction point allows this. It is another thing to be able to express how reliable or accurate a value is. A value can be completely reliable and accurate—in other words, exact—or it can be unreliable or inaccurate to some degree or other. To say whether a value is exact or not, and if not to say how much, is to tell the *whole* truth. A notation that allows this truth to be told would therefore be not only a public good, but a mathematical ideal—"mathematics is truth, truth mathematics" [4 p.177].

If a value, like a count or a fraction, is exact then its representation must show plainly that it is exact. A simple number like *one and two thirds* is exact and, moreover, commonly useful. Yet it is nowadays almost never represented exactly. Instead some approximation like 1.667 or 1.666667 is used. There are two quite different reasons for this.

The first reason is that electronic calculators and computers, as they are almost all now designed, cannot do exact arithmetic except on a limited set of numbers. In particular, their rational arithmetic is approximate except for numbers whose denominator is a power of two. There is nothing necessary about this characteristic [14], which arose because the great limitations of early digital computers caused scientists to design an arithmetic based on semilogarithmic (wrongly called *floating point*) representation of numbers, an arithmetic now set in the concrete of an international standard always implemented directly in electronic circuitry.

The second reason is that, even if the arithmetic were exact for non-decimal fractions (sometimes called *common* or *vulgar* fractions), there is now no way in which such fractions can be either keyed directly and exactly into a calculation, nor shown exactly on a character display. Only decimal fractions can be keyed in directly and exactly, and only decimal fractions can be used to display usually approximate fractional results. While it is true that a number like *one and two thirds* has in the past been representable as $1^2\!/_3$ or as $1\frac{2}{3}$, the designers of electronic calculators and computers have not provided for this kind of representation either to be displayed or to be keyed in. More than ten years ago I was an observer at a meeting of senior mathematics teachers which agreed, without protest from any of the teachers, that common fractions should be dropped from the official syllabus for elementary schools of one of the states of Australia simply because electronic calculators don't provide for them.

It has often been remarked that the teaching of common fractions is not well done in elementary schools [15]. From this remark it is a short step to question whether common fractions should be taught at all. The mistake here is to suppose that decimal and common fractions should be distinctively taught. They should not. A fractional number is a fractional number, whether decimal or common. The fault is in the notation, which makes the numbers $2\frac{3}{4}$ and 2.75 so different in appearance. What is needed is a notational convention which makes it plain that a number like *one and two thirds* is a value for which an integral part, a numerator and a denominator can be specified, and which *as a special case* allows certain (decimal) denominators to be left out.

The problem with representations like $1^2/_3$ or $1\frac{2}{3}$ is that the numerator and denominator are distinguished from the integral part by typographical detail, and from each other by a symbol which implies that a calculation is to be carried out. These representations are neither perceptually sufficient, nor notationally unambiguous, nor electromechanically convenient. However, if a symbol like $^\circ$, distinctively pronounced say *dene* or *nom*, were adopted as a prefix to the denominator part of a fractional number, to follow the numerator part, then a very convenient and pedagogically salubrious notation is provided. The symbol / would not be suitable as it is now too often used to stand for the division function.

The number *two and three quarters* could be keyed into a calculator as $2_\triangle 3^\circ 4$ or $2_\triangle 75^\circ 100$ or $2_\triangle 75$, showing equivalences which should be easy for even the elementary school eye to see. Of course, a number like *two thirds* could be keyed in as $2^\circ 3$ or $0_\triangle 2^\circ 3$ or $4^\circ 6$, but there is no equivalent decimal fraction. Numbers with decimal fractions are distinguished from numbers with common fractions when they are displayed—a number that can be exactly represented more briefly with a decimal fraction than with a common fraction will be so represented. Otherwise there is no mysterious difference to confuse the young learner.

The extension of the mathematical symbol set by a third symbol here might with some little justice be considered more generous than is warranted. In this case, given that $^\triangledown$ and $^\circ$ are similar in size and placement, and that the negative sign must prefix a negative number while the denominator sign must separate a denominator from its numerator, the $^\triangledown$ could well be used as the symbol for both signs since the nature of the sign will be immediately distinguishable from its context.

Accuracy

It is one kind of truthfulness to provide for exact numbers all to be represented exactly. But there are two quite different kinds of numbers—exact and approximate—and these two kinds should be easily distinguished in their representation but are not. An approximate value can only be truthfully represented if its representation shows plainly, not only that it is approximate, but also how approximate. In other words, the representation of an inexact value should show how accurate that value is.

If 1·75 represents a fraction then its value is exact, if it represents a measurement then its value is approximate. In the technological world, or in the everyday world for that matter, that 1·75 is a measurement means that it is somewhere in the range 1·745 to 1·755, where the inaccuracy might spring from an unreliability in manufacture, from a limitation of a measuring tool, or from a perceived irrelevance for greater accuracy. That a number like 1·75 is a measurement will typically signify that it was taken from a decimally marked scale, or a digital readout, and is accurate to two decimal fraction places.

The representation of such measurements should show them to be measurements. Suppose a number shown with both a fraction point $_\triangle$ and a scaling sign $_\mathrm{k}$ or $_\mathrm{m}$ but no denominator point $^\circ$ were treated as approximate beyond the last decimal place to a tolerance of plus or minus half that decimal place. Then $1_\mathrm{k}3$ would be treated as exact, while $1_\triangle 00_\mathrm{k}3$ would be treated as exact only to the last decimal place (in the range $995_\mathrm{k}2$ and $1005_\mathrm{k}2$), and would be a more accurate value than $1_\triangle 0_\mathrm{k}3$ (in the range $950_\mathrm{k}2$ to $1050_\mathrm{k}2$). This notational convention would provide a plain and simple means for decimally inexact values of this kind to be truthfully represented.

But not all inaccuracies are of this kind. The arithmetic difference between two exact numbers 2·75 and 1·75 is exactly 1. Between a measurement of 2·75 and an exact 1·75 it is somewhere in the range 0·95 to 1·05, which can be shown as $1_\triangle 0_\mathrm{k}0$. But between two measurements 2·75 and 1·75 the difference is somewhere in the range 0·9 to 1·1, which requires another notational rule to allow the value to be truthfully represented.

It would be convenient therefore to use the $^\circ$ symbol, when it appears between the fraction point and the scaling sign (where it is not needed as a denominator sign), as a tolerance sign. This rule would have the digits beyond the tolerance sign give the trailing decimal fraction digits which would give the upper limit of the range when they replace the corresponding trailing decimal fraction digits preceding the tolerance point and which specify the lower limit of the range. Then $2_\triangle 75_\mathrm{k}0 - 1_\triangle 75_\mathrm{k}0$ would give $_\triangle 9^\circ 1_\mathrm{k}0$. Also, $2_\triangle 34^\circ 5_\mathrm{k}$ would stand for a value in the range 2340 to 2350, and $2_\triangle 34^\circ 56_\mathrm{m}$ for a value in the range 0·00234 to 0·00256.

Better still would be a character set that provides italic digits as well as roman. Then $_\triangle 9^\circ 1_\mathrm{k}0$ and $2_\triangle 34^\circ 5_\mathrm{k}$ would appear as, or could be keyed in as, $_\triangle 9\mathit{1}_\mathrm{k}0$ and $2_\triangle 34\mathit{5}_\mathrm{k}$.

Only experience could show the level of arithmetic education at which these last notational conventions could be introduced. They would, however, be a valuable feature of any calculator and an enrichment for any talented students.

Conclusion

This article proposes, as steps necessary to reverse present trends towards popular innumeracy, that

- the adoption of SI metric basic and secondary units of measurement should be everywhere encouraged, being much better suited to popular use than the units traditionally used in the major English-speaking countries,
- the SI metric scaling system should be replaced by a simple system for representing scaled numbers, and
- traditional methods of representing numbers are otherwise unsatisfactory and warrant being replaced.

A primary source of good advice about reform in popular usage for numbers, and measurements, and calculations should be the mathematicians whose profession stands to gain most from wise reform, even if the choice and timing of those reforms are properly a matter for the public and its government to decide. Reforms of this kind would offer an opportunity to improve the aesthetics of mathematics generally, an aspect often considered fundamental for mathematicians [4 ch.5]. Mathematicians also have a natural responsibility for taking initiatives in promoting such reforms, and promptly introducing the teaching of them.

There is a very real and present danger that increasing and widening use of digital technology will prolong unthinking acceptance of a defective system for representing numbers, a system which supports if not causes innumeracy. The prospect is that the essential beauty of numbers and calculation will be forever hidden from the vast majority of people through persistence with notational conventions whose only justification is their traditional use, and whose ugliness and unwieldiness are obscured by the familiarity engendered through imposition in elementary schools.

The opportunity is for a much better notational convention to be agreed internationally, for better electronic measurement and calculation to be enabled by that convention, and for the technology to support better the promotion of public numeracy.

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