MATHEMATICS IN SCHOOL AND LIFE: IMPLICATIONS FOR SECONDARY MATHEMATICS

By

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DECLARATION

This thesis contains no material that has been accepted for a degree or diploma by the University or any other institution except by way of background information and duly acknowledged in the thesis

To the best of my knowledge and belief this thesis contains no material previously published or written by another person except where due acknowledgement is made in the text of the thesis

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ABSTRACT

Concerns have been expressed by employers that many young people coming into the workforce do not have the mathematical skills that are desired. Concerns have also been expressed about the increasing departure of school mathematics from the demands of daily life. These concerns are not confined to Australia and it is suggested that something maybe amiss with the secondary mathematics curriculum. This study is concerned with identifying and closing any gaps that may exist between mathematics that is taught in secondary school and that practiced in daily life.

This study opens with an investigation of the development of mathematics and mathematics education in order that the reasons for the mathematics that is currently taught in secondary schools can be best understood. Previous research regarding the mathematics that is used in everyday life is extended and compared with the mainstream syllabus of today as exhibited in text books used in the classroom. The lessons that have been learned from observations of past and present practices are finally drawn together to propose a new secondary mathematics course. In addition to improving the numeracy of students, it is expected that additional benefits will accrue in the reduction of the common anxiety in students that is associated with mathematics, and an easing of the ever-increasing shortage of specialist mathematics teachers.

The solution that is offered is to approach secondary mathematics from a lateral point of view with the design of a bipartite course. The core or compulsory syllabus of the proposed course concentrates on the mathematics found in daily life so that all students leave school familiar and confident in their ability to perform simple mathematical tasks with a high degree of

accuracy time after time. Those students who have a deeper interest in mathematics would be able to choose an elective mathematics syllabus where they can study with others of a like mind.

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CHAPTER 1

INTRODUCTION

1.1 Preamble

The reasoning behind the author's desire to consider a reform of secondary mathematics is embedded in the history of his working life. The two major influences have come from years at sea as a deck officer in the merchant navy and then in secondary education as a teacher of mathematics and science. More than three score years of varied experiences bring bias as well as knowledge to the table of study, where hopefully both can be balanced against the findings of others.

The period of time when the author was at sea in the 1960's spelled the end of the carriage of general cargo in individual packages and the accompanying complex stability calculations with the introduction of the containerization (the carriage of individual packages in 40 tonne containers). In addition, within a further 10 years, dedicated calculators had taken much of the manual calculations out of daily sailing and celestial navigation problems. When pencil and paper, sextant and chronometer ruled the day in 1963, courses and examinations were brief and fierce. It was customary to be ashore for a mere 6 weeks to prepare for one's Second Mate's certificate and examination pass marks were as high as 95% with a degree of accuracy in navigation problems to the nearest 0.1 nautical mile. This demand for accuracy was essential in all tasks aboard ship whether securing a rope or calculating the closing vector of another vessel on a radar plot. Mistakes could be very costly either in monetary or human terms.

Following emigration to Australia, the author took up teaching secondary mathematics and science in the Tasmanian state school system. This period in the late 1960s and early 1970s was one of immense change in mathematics education. Text books were designed to introduce both teacher and student to the language of the American "New Math" of Bourbaki thinking, and the slide rule gave way to electronic calculators. The author found the formal language and patterns of thinking required by the new mathematics a stark contrast to the specific application of mathematics in his previous life at sea.

Fifteen years of teaching in both private and state secondary schools were replaced by a return to an association with the sea when the author was invited to take up a three year contract at the Australian Maritime College (AMC) in 1983. By this time electronic navigation systems were used rather than personal calculations and onboard computers handled stability problems. There were still requirements that future officers had to know how to do such calculations, but in practice it was not only not required, but not allowed. Courses for maritime qualifications were now three years rather than a few weeks and the mathematics component of a much more sophisticated nature than 20 years previously, despite the fact that there were fewer calculations to do aboard ship than there had been in days of yore.

Upon returning to secondary education in 1989, there followed a decade of innovative teaching including climate modelling with year 10 students (Faulkner, 1992), before the author moved on to his final appointment at a post compulsory college. The freedom of the previous years was replaced by the need to help students attain the best possible results in their final two years of schooling. The nature of advanced mathematics remained firmly fixed to university requirements as might be expected, but the most rapidly expanding courses in fundamental and applied mathematics were in a state of constant flux. The author became involved in course design, helping to write the text for the year 11 applied mathematic course and developing the first computer-based learning programme for it in Tasmania. It was during these latter years that the author began to question the validity of the mathematics that was taught.

The decision was made to embark upon a formal study of mathematics both inside and outside the classroom to establish whether the perceived differences between the two actually existed, and if so, to what degree. Outside the classroom is taken to mean the mathematics used in the everyday life of an adult. Everyday life implies situations inside the workplace from unskilled labour to senior management and the professional, and outside the workplace including situations of either a domestic or recreational nature. The natural programme for such research seemed to open with a study the history of mathematics and mathematics education to find the reasons for what is and has been taught. Fieldwork would be needed to define the nature of mathematics used currently both inside and outside the workplace and a study of texts used in schools to define mathematics taught inside the classroom. If a marked difference was shown to exist between mathematics inside and outside the classroom, and bearing in mind the philosophy that one should not criticize without offering a solution, the final chapter, in addition to drawing together the findings of the study, should suggest such an answer in the way of syllabus changes.

1.2 Aims and objectives

The hypothesis is put, that there is a difference between mathematics that is taught inside schools and that practiced outside schools. The difference is divided into two parts that form the first two objectives of the study. Based upon the initial premise, the outcomes of the first two objectives inform the third objective, which relates to a suggested curriculum reform.

1. Establish if the mathematics that is taught in schools is adequate or more complex than is needed by the average person in everyday life.
2. Establish if the degree of accuracy required in mathematics that is taught in schools does or does not reflect the need of the majority of people in everyday life.
3. If objectives 1 and 2 are found in the negative, design a junior (grade/year 8, 9 and 10) secondary mathematics curriculum that would enable all students to have the ability to apply mathematics in the context of daily life, in other words, be numerate. At the same time provision must be made for the needs of those students who wish to study mathematics beyond secondary schooling.

In order that the objectives can be achieved, there are a series of aims that form the framework for associated arguments. These are to:

- Trace the development of mathematics
- Examine the development of mathematics education

- Investigate the mathematics that is used in everyday life
- Establish the mathematics that is taught in Australian schools at the time of the study
- Summarise issues that have arisen during the study
- If necessary, offer a new secondary mathematics curriculum based upon issues that have arisen during the study
- Note any additional benefits that might accrue from the revised curriculum

1.3 Methodology

This research requires the use of multiple methodologies to investigate both qualitative and quantitative data and if necessary, apply the findings to the structure of a new curriculum. The initial methodology applied in Chapters 2, 3 and 4, arises from the suggestions of Goodson (1985) that qualitative data arising from study of relevant histories enables the description of constraints under which the current curriculum is practiced. The literature search is embedded in each section of the study rather than being confined to one specific chapter. Analysis of the qualitative data alone is not thought to be a sufficient foundation upon which to base any changes that might be made to the curriculum. Support for the qualitative findings is hence offered by pursuing quantitative data in the way of a field study. This type of mixed methodology is called an *embedded design* by Cresswell (2002).

Information arising from a field study as reported in Chapter 5 draws on quantitative data derived from questionnaires distributed to a survey population.

This method of collecting information is described as a valid survey method in educational research by Berends (2006). Further qualitative data are drawn together in Chapter 6, where the mathematics that has been taught in schools during the last 120 years is analysed through inspection of the texts that have been used in the classroom (Good, 1972).

The final Chapter (7) compares the mathematics that is taught in schools, with that practiced in everyday life and places the findings in the context of the development of mathematics and mathematics education. The methodology of applying qualitative and quantative data in the resolution of a practical problem, which is in this case the mathematics curriculum, has been described by Wiersing and Jurs (2000). A new curriculum is suggested that is in harmony with the current national and state policies in Australia and is expected to deliver greater benefits than current practices allow. This thesis, in questioning particular educational practices and suggesting more suitable alternatives, lays claim to have a philosophic component in the methodologies that are applied (Burbuler & Warnick, 2006). For example, should the core secondary mathematics curriculum build a pure mathematical foundation for tertiary studies, or pursue a utilitarian form that will prepare students for the demands of life that most will face as adults? A more detailed description of the structure of each chapter of the study follows in section 1.4.

1.4 Structure of the study

It is necessary to show how any difference between mathematics that is taught in schools and the mathematics that is used outside schools has arisen. The pathway that has been chosen starts with a brief history of mathematics before tracing the development of mathematics education. Mathematics that is used outside schools is investigated by way a combination of literature search and field survey before the mathematics that is taught in schools is determined through use of school texts. The issues that are raised in this research are drawn together to form the basis for the proposal for a new curriculum and the possible benefits eventuating from the implementation of the curriculum.

1.4.1 Chapter 2

This study concerns itself with mathematics education and as such requires some understanding of the debate surrounding the question, “What is mathematics?” Chapter 2 opens with this discussion, which ranges from the ideas of Plato (428-348 B.C.) (trans. 1952) to those of Restivo (2007), before moving on to the history of mathematics.

The history of mathematics has been well documented from the arithmetic of the ancient Babylonians and Egyptians to the explosion of knowledge during recent times by authors such as Bell (1937), Boyer (1968), and Kline (1972). For the purposes of this study the salient points are judged to be centred upon the emergence of the pure mathematics of the ancient Greeks and the twin developments in pure and applied mathematics since that time. Mathematics, as a discipline and education merge when the knowledge that one person has is passed to another. The expression and transmission of mathematical knowledge has assumed greater importance as more people have needed basic mathematical skills. In addition there is an expanding need for more sophisticated forms of mathematics by specialists in various fields as society becomes increasingly reliant on modern technologies.

The growth of mathematical knowledge has accelerated to such a degree that the last 200 years have seen as much invention in mathematical fields as can be found in the entire history of mathematics up to that point. The combination of the expansion of mathematical knowledge and increasing need for everyone to have some basic mathematical knowledge present a challenging task for curriculum designers.

1.4.2 Chapter 3

Reasons for the design of the current mathematics curriculum can be better understood, if the development of mathematics education is traced from the earliest records to the present day. There are two parts to this development with the first being the syllabus, which is discussed in Chapter 3 (see endnote 1), and the second being the reasons for the implementation of the syllabus that form the body of Chapter 4.

The syllabus has changed considerably since the earliest records of schooling in the times of Classical Greece (Lawrence, 1970, chapter 1). Arithmetic that was once the province of tertiary education is now part of primary education, and calculus was only passed down from universities to secondary schools in the mid 19th century. Inexorably, the content of secondary mathematics education has become more complex and the syllabus more crowded as the new discoveries of yesterday are passed on the students of today (Howson, 1982).

1.4.3 Chapter 4

The contents of mathematical syllabi have been shaped by the future role that the recipient has been expected to play in society. In ancient times, all menial work, including the rudimentary arithmetic essential to the operation of a state, was done by slaves. Secondary and tertiary education in mathematics was offered to the citizen or landed gentry with the sole objective of improving the mind and utilitarian thought was deemed undesirable in such studies. This attitude persisted in England up until the 1850s and echoes are still found amongst those involved in mathematics education today.

Arguments concerning whether mathematics should be taught as an art or a utility, to whom it should be taught and when, ideally fall into philosophic discussions concerning mathematics education. Prior to the advent of compulsory education and the involvement of the state, such thoughts were left to the individual philosophers and educators of the time from Plato to Dewey (1859-1952), and the writings of such people still play an important part in government policies that shape educational practices of today. In 2008, there exist both national and state policy documents concerning mathematics education in Australia. These reflect the increasing recognition of the importance of mathematics for the economic health of the country. The relationship between mathematics education and the workforce is clearly spelt out in the document of the Department of Education, Tasmania (2007), *Tasmania's Literacy and Numeracy Plan* (TLNP), which was part of the Human Capital Agenda of the Council of Australian Governments. It was found desirable to ensure that (p. 3), "…young people are provided with the education and training they require to be a productive member of the workforce." This final statement is quite different from those found in *The Republic* (Plato, trans. 1966), where the objective was the education of the future ruling classes.

1.4.4 Chapter 5

The use of mathematics outside the classroom had been investigated in countries as far afield as Britain (Cockroft, 1982), the United States of America (Lave, Murtaugh, & de la Rocha, 1984), Brazil (Carraher, 1991), and Australia (FitzSimons, 2005); and this list is by no means exhaustive. The general consensus was that everyday mathematics calls for simple skills accompanied by a high level of accuracy. The difficulty lay in chosing the appropriate mathematical technique and using it correctly in the context of the situation.

All these points assisted in the design of a survey (Appendix B) to find out if the mathematics used outside the classroom today (2006) is as simple and with the same requirements for accuracy that previous researchers had established. It extends the research of others by more closely questioning the mathematics that is used and the expectations of accuracy from others and by oneself. The survey is designed to include people using mathematics both inside and outside the workplace. Inside the workplace every attempt is made to obtain as wide a variety of workers as possible including those people who run their own businesses. The research area (Appendix A) includes a provincial city and the surrounding towns and villages, typical of life outside a major city in Australia. The results support the literature and further define the mathematical skills and applications that are most used, as well as least used, in the community.

1.4.5 Chapter 6

Having established a pattern of the mathematics that is used outside schools, the next step is to determine what mathematics is taught inside schools and to this end school texts that span more than a century of mathematics teaching are summarised. The texts of the authors that were most widely used both in England and Australia, including Godfrey and Siddons (1903, 1912), Siddons and Snell (1945) and Durell (1936, 1957), are still available as far back as Pendlebury (1896). The proof of use of these old texts, is found in the inclusion of students' names, odd calculations and comments in the books. In addition, the texts that had been used by the author while both learning and teaching over a period more than 50 years, are amongst those referenced. Colleagues who are still teaching assisted in making an informed choice of texts in current use, from the many that are available.

This Chapter (6) focuses on grades 8 and 10, when comparing the development of the syllabus, as these grades determine a beginning and end of compulsory secondary education that are common throughout Australia. To assist in the brevity of the discussion in this chapter, problems that are cited in the texts are listed in Appendix E as are the contents of selected texts.

It is found that the mathematics of today has a broader base than that of 100 years ago. There is a continued growth in chance and data in addition to studies in transformation and analytic geometry. The approach to deductive geometry is such that a student is introduced gently to the reasoning behind proofs, rather than being thrust into the memorisation of them. Many of the secondary arithmetic techniques of Pendlebury's day are now the province of primary schools, reflecting the continued pressure to learn more mathematics at an earlier age. A notable reduction in monetary problems that were common to all secondary arithmetic texts prior to 1980, is found in the texts of the 21st century.

1.4.6 Chapter 7

The differences that arise between the observations of mathematics used in daily life in Chapter 5 and the mathematics that is taught in schools (Chapter 6) are summarised and the reasons for such differences arising discussed in the light of the developments in mathematics and mathematics education mentioned in Chapters 2, 3 and 4. A final analysis is made of the most recent (2007, 2008) documents regarding Australian national and Tasmanian state mathematics education. For example in the *National Report on Schooling in Australia* (Ministerial Council for Education, Early Childhood Development and Youth Affairs (MCEECDYA), 2006 p. 2), the 1998 aim to ensure that all students were numerate was restated, yet the results of the *National Assessment Programme for Literacy and Numeracy* (2008) showed that this benchmark had not been attained and the *National Numeracy Review Report* (2008), suggests that there remains much to be done. The combined issues are drawn together to form the foundation on which the suggested new curriculum is built.

The suggested new curriculum has a bipartite approach with two syllabi being put forward. Traditionally, the core syllabus that all students must study has been designed to offer those students who have the desire and capability every opportunity to study mathematics at a higher level, from grade 11 to university.

Students who do not wish to follow this path may be enrolled in streams of mathematics that are still within the core but pitched at a different academic level. It is proposed that the core syllabus that is followed by all students in the junior secondary sector be constrained to those skills that are identified as being used by the majority (greater than 80%) of people in their daily lives. These skills should be exercised in the contexts found in day to day situations and the contexts made as realistic as possible, which includes the sort of accuracy that is found in everyday life.

Those students who have reason or purely a passing fancy to study mathematics that is more complex than the everyday requirements would be offered an elective with every advantage that only an elective subject can offer in the way of specialist teachers and a diverse programme. Such a programme could satisfy all the requirements of the current mainstream core syllabus if that is thought desirable, and mathematics series such as *Maths Quest* (Stambulic, Iampolski, Phillips, & Watson, 2002) or *New Signpost* (McSeveny, Conway, & Wilkes, 2004) offer a guide as to the content of such a course.

The core syllabus is set against the prevailing outcomes and standards as published in the *Learning, Teaching and Assessment Guide* (LTAG) (Department of Education, Tasmania, 2008a), and common ground is found between the core and elective syllabi in *The Tasmanian Curriculum: Mathematics–numeracy: K – 10 syllabus and support materials* (TCMN) (Department of Education, Tasmania, 2008b). There is no national statement regarding mathematics beyond grade 10, but Matters and Masters (2007) published a report of those topics most commonly studied in grades 11 and 12. This information is allied to existing criteria statements found in the *Tasmanian Certificate of Education* statements for *Senior Secondary Mathematics* (Tasmanian Qualification Authority, 2008).

It is thought that the two additional benefits sufficiently important to be specifically mentioned would be a reduction in the anxiety associated with mathematics (Goldin, 2002) and an easing of the demand for specialised mathematics teachers of which there was an increasing shortage in 2006 (Harris & Jensz, 2006).

The summary suggests that such a bipartite approach to the secondary mathematic curriculum enables the preservation of mathematics as an art and at the same times serves the utilitarian needs of the majority. The statements of Ernest (2000) regarding the basic needs of the majority and Restivo (1992) in reference to the different needs of the crowd and lovers of wisdom close the thesis.

1.5 Limitations

There are always constraints on any endeavour. In the case of this study the normal issues of space, time, money and logistics all prevailed to some degree.

1.5.1 Literature search

Records of mathematics education were extracted from the 15 schools listed in Table 1.1. These ranged from primary schools to post compulsory colleges, including both private and state schools in Australia and England. Some records had been lost from educational archives as time had passed, but there were schools with mathematics texts that could trace the syllabus back for the better part of a century, such as Launceston Church Grammar School, and teachers who started their careers in the 1950s. The oldest schools from which records were sought were Colchester Royal Grammar School (1206) and Wisbech Grammar School (1379), and the newest Lake Tuggeranong College (1990).

Table 1.1
Schools from which texts were read for this study

School	Location
Christies Beach High School	Chrities Beach, S.A., Australia
Colchester Royal Grammar School	Colchester, Essex, England
Exeter High School	Exeter, Tas., Australia
H.M.S.Worcester (Training Ship)	Greenhithe, Kent, England
Kings Meadows High School	Launceston, Tas., Australia
Lake Tuggeranong College	Canberra, A.C.T., Australia
Launceston Church Grammar School	Launceston, Tas., Australia
Launceston College	Launceston, Tas., Australia
McKinnon High School	Melbourne, Vic., Australia
Neutral Junction Station (Aboriginal School)	Neutral Junction, N.T., Australia
Queechy High School	Launceston, Tas., Australia
Riverside Primary School	Launceston, Tas., Australia
Skippers Hill Preparatory School	Five Ashes, Sussex, England
St. Peter's Collegiate School	Adelaide, S.A., Australia
Wisbech Grammar School	Wisbech, Cambridgeshire, England

Note. A.C.T. – Australian Capital Territory, N.T. – Northern Territory, S.A. – South Australia, Tas. – Tasmania, Vic. – Victoria.

Overseas travel to England and Europe, North America and different parts of Australia to view original documents and talk to people in various schools was necessarily limited, and places were chosen where the author had some personal contact. Text books and other records pertaining to mathematics education were interspersed with occasional accounts written more than 100 years ago about people who started schools. Some of these were found on the shelves of libraries in remote towns with a rich history in education, as far apart as England, Australia, and Canada. Of the cities and towns that are listed in Table 1.2, the most rewarding researches were carried out in Launceston and Port Alberni, with records extant of the educational efforts of the first white settlers. Amongst these were found personal accounts of hardships, failures and successes. In the case of Port Alberni, detail was given of the first schooling of the children of employees of the Hudson Bay Company as well as the native tribes of the West Coast of Vancouver Island.

Table 1.2
Cities or towns in which libraries were accessed

City/Town	Country	City/Town	Country
Adelaide	Australia	Perth	Australia
Alice Springs	Australia	Kamloops	Canada
Brisbane	Australia	Ladnor	Canada
Canberra	Australia	Port Alberni	Canada
Hobart	Australia	Shipston on Stour	England
Launceston	Australia	Warwick	England
Melbourne	Australia	Wisbech	England

It was found that there had been more written about the history and development of mathematics and mathematics education than could possibly be read and only a small part of what was read could be included if the thesis were to kept to a reasonable length. The structure of knowledge that was built was guided by a desire to show the temporal and spatial developments in mathematics and mathematics education that have led to the construction of the mathematics curriculum practiced in Australia at the time of this study. A lack of ability to read Greek, Latin and Arabic fluently meant that translations of all the older works had to suffice, and it was appreciated that, particularly with the philosophers, subtle nuances were inevitably lost in translation. The library and research facilities offered by the University of Tasmania greatly assisted the literature searches, which included online access to electronic resources and the archives of other universities.

1.5.2 Field survey

The field survey reported in Chapter 5 was limited by cost and logistics. It had to be designed so that the researcher could manage the survey and the cost was within budget. For these reasons, the survey area was within 50 kilometres of where the researcher lived and only 600 questionnaires printed off. The needs of the respondents had to be considered and the original list of 200 questions was reduced to around 100 on advice from colleagues and friends who trialled the questionnaire. A final factor in the questionnaire design was the estimation that before the final results could be obtained, more than 10,000 answers would have to be analysed if the response to the handout was in the ratio of one in six.

1.5.3 School text books

Upon making the decision to use school text books as a guide for the description of the mathematics taught in schools, it remained to select the texts and decide how far back to go in time to trace the development of the syllabus of today (2007). Mathematics education in Australia and England remained parallel until the 1960s and the same texts had largely been used in both countries since the settlement of Australia. It was decided to go as far back as the turn of the 20^{th} century when English secondary schools started to break away from the dominance of Euclidean geometry, and authors were chosen in arithmetic, algebra and geometry whose books sometimes remained in print for more than half a century. A singular progression was thus made possible in each of the three mathematical disciplines, but texts written specifically for innovative schemes such as "New Math" or for individual topics such as matrices were not included.

For nearly 50 years, Australia has designed a mathematics curriculum that although similar to England, has sufficient individual features to be considered unique. The series of texts that serviced secondary school courses sometimes contained up to a dozen books, and more recently, additional compact discs. In order that the volume of reading could be reduced to a manageable size, and bearing in mind the limits of compulsory secondary education, two texts were chosen to outline the curriculum, one for grade/year 8 and the other, grade/year 10. For example *Secondary Mathematics Series (SMS) 1* and *3* written by Clapp, Close, Hamann, Lang, and McDonald (1972) for grades 8 and 10, remained in use for at least 10 years and typified the Australian approach to the inclusion of "New Math" in mathematics. There were minor changes to the curriculum during the ensuing 30 years, which culminated in the texts that were in use in 2007. At this time there were more than 10 series of mathematics texts available in Australia and from these two series were chosen where first hand evidence was gained of their use in Tasmania, Victoria and New South Wales. There was no judgement that one series was any better than another.

1.5.4 Curriculum reform

Reform of the secondary mathematics curriculum is the end product of the final chapter. In this study, the description of reform is limited to changing the priorities and rearranging the content of the existing syllabus, splitting it into two distinct parts. The relationship between the syllabi, the existing National (NNR, 2008) and state (LTAG, 2008) statements of mathematics and numeracy are noted and this marks the boundary of the discussion.

1.6 Ethics

Permission was sought from the Ethics Committee of Social Sciences of the University of Tasmania to undertake the survey reported in Chapter 5. The Minimum Risk Application was approved in April, 2006, Reference Number H9076 (Appendix G).

CHAPTER 2

HISTORICAL ASPECTS OF MATHEMATICS AND SOCIETY

Introduction

In the introduction it is stated that the final chapter of this thesis puts formal arguments for changes to the secondary mathematics curriculum. Further, it is indicated that there is a need for a study of the history of a subject before change is made. Chapter 2 begins such a study with a view of the history of mathematics that is particularly relevant to this thesis.

The discussion opens with an abbreviated definition of mathematics from the thoughts of Plato and Aristotle to those of Bell and Restivo. Ideas relating to mathematics as an art or utility are put forward as well as the differences between mathematics being part of the universe or an invention.

The ancient mathematics of Mesopotamia and Egypt are then discussed, where written evidence indicated a wide use of arithmetic for state, engineering and individual matters including accounting, mensuration and flood prognostication. Finally the blossoming of mathematics during Greek times is discussed.

In the next section, trigonometry is used as an example of a branch of mathematics with its roots in Greek times, and which has been applied by simple seafarers over the last five hundred years in discovering the more remote parts of the world. Trigonometry is a sophisticated mathematical tool to the average person, even today and the key that enables people to use such tools is developing simple expression to replace the arcane language of the mathematicians who invented the formulae.

Finally the division of mathematics into pure and applied fields is investigated, from the applied arithmetic of the ancient peoples and deductive thinking of Greek times, to mathematicians of the last half millennia who were exemplars of both fields.

2.1 What is mathematics?

This thesis is concerned with mathematics education and it is reasonable to ask, “What is mathematics?” Questions regarding the nature of mathematics have been asked since the time of the ancient Greeks and could well remain one of the permanent philosophical problems for those interested in articulation of one’s thoughts on the topic. Mathematics may be classed as a science or a creative art, and is closely allied with physics and music, but it is of a more illusive nature than either, changing in nature according to one’s frame of reference.

There are two aspects to consider when seeking answers to the question, “What is mathematics?” The first is concerned with the problem of mathematics being discovered or invented by the human mind, and the second is whether the content of mathematics is simply about number, or something altogether more complex.

The first philosopher to consider the nature of mathematics whose writings are extant, was Plato. In *The Republic,* Plato (trans. 1952c, p. 393) pointed out “…all arithmetic and calculation have to do with number…” and then stated that number is abstract, it has no tangible body, and therefore the study compels the soul to reason. In Plato's letter *Meno* (trans. 1952b, pp. 180-183), the implications of the discussion between Socrates and Meno regarding the slave of Meno and mathematics education are that each person possess all the mathematical knowledge that exists, it has always been there, and it simply requires the right question at the right time to unlock it. In other words mathematics exists in some transcendental realm to which everyone has access. In considering the philosophy of Plato, it must be remembered that he believed in many gods and the immortal soul of man, thus when a person was born, the soul carried all knowledge that had accrued since the beginning of time. Four hundred years later, Nichomacus (c. 100 B.C.) (trans. 1952, p. 813) wrote, "Arithmetic was created in the mind of God before the universe, earth or man." This could be construed as the Nichomacus's Christian interpretation of the ideas of Plato. On a more pragmatic note in *Gorgias*, Plato (trans. 1952a, p. 254) had Socrates remark, "…arithmetic is one those arts which take effect through words – words about odd numbers and even numbers and how many there are of each." The manner of describing mathematics as an art was supported in *Metaphysics, book 1,* by Aristotle (384-322 B.C.) (trans. 1952a, 981b [15-25]) (endnote 2) who noted, "…mathematical art was founded in Egypt for recreation of the priestly caste in their leisure." Aristotle constructed his philosophical arguments with more rigor than Plato and without his poetic phrasing. In *Metaphysics, book VIII*, Aristotle (trans. 1952a, 1076a [1-4]) drew together his previous arguments with the statement, "…it is impossible for mathematical objects to exist in sensible things." He then argued further (1077b [15-20]), "…either mathematical objects do not exist at all, or they exist in a special way." Finally Aristotle wrote (1077b [30-35]), "…it is true to say without qualification – that the objects of mathematics exist with the characters ascribed to them by mathematicians." Summing the first and last statements together, it is possible to conclude that mathematics is a product of the minds of mathematicians. A summary of these ancient ideas is that mathematics exists, it is not part of our sensible world but whether it has always existed or is an invention of man is open to debate.

Nearly 2000 years later, Descartes (1596-1650) in *Rules for Direction of the Mind* (trans. 1952, p. 31) asked the question, "For is there a single Arithmetician who does not believe that numbers with which he deals are not merely held in abstraction for any subject matter by understanding, but are really distinct objects of the imagination?" In doing so he held open the door for the further definition of mathematics by Kant (1724-1804) (*The Critique of Pure Reason*, trans.1952, pp. 17-18) who wrote, "…mathematical judgements are always synthetic" and "…proper mathematical propositions are always judgements a priori…" The idea that mathematics had always existed in some transcendental realm was no longer mentioned and as Bell (1945, p. 11) suggested, mathematics was an invention rather than a discovery of man. A more modern idea of mathematics, which carried forward the heart of earlier philosophies was espoused by Tarp, Valero, and Skovsmose (2003) and Restivo (2007), in suggesting that mathematics was a social construction, marrying the concepts of mental construction and social interaction.

The content of mathematics for thousands of years was arithmetic from counting to state accountancy and measurement that made the building of the pyramids possible. The quadrivium (White, 1981, p. 162) could be used as a guide as to the content of mathematics from Greek times until Frege (1848-1925) seriously questioned the relationship between logic and mathematics and put the proposition that arithmetic is simply a development of logic (*The Foundations of Arithmetic*, trans. 1974, p. 99). Although his theory was fatally flawed, a relationship between arithmetic and logic was accepted and put on a firmer footing by Russell (1872-1970) (Kline, 1972, p. 1192). From ancient times until Gödel (1906-1978) published his incompleteness theorem in 1931 (Van Heijenoort, 1970), the idea that mathematics and the truth were synonymous was held dear by mathematicians. The association of a philosophic concept of truth with mathematics was not so strange, particularly when one considered mathematics as a creative art as did Hardy (1877-1947) (1967, p. 115). A broader and more central view of mathematics was espoused by Courant and Robbins (1958, p. 1) who put number as the basis for modern mathematics. Scruton (1994, p. 394) thought that mathematics was a practice that got its sense from counting, measuring and calculating, a stance clearly identified by students (Gordon & Nicholas, 1992), parents (Pritchard, 2004) and engineers (personal communication, J. Vickers, March 1, 2008). The perception of mathematics has changed from being an intrinsic part of the human mind to becoming an invention.

2.2 Origins

The tally marks found on paleolithic bones of a wolf in Vestonice by Karl Absolon in 1937, dated at around 30,000 B.C. (Barrow, 1992, p. 31), are but one example of the early efforts to count. The reasons for their existence can only be conjectured; perhaps they represented the members of a tribe, days since someone left, or sheep in a flock. Such marks are quite distinct from ancient paintings and other artworks found in caves from Australia to France and beyond.

The invention of number and the development of methods of counting to involve the wider aspects of arithmetic have been positively traced back over the last 4000 years, although it seems probable from evidence found in China (Grosier, 1972), Mesopotamia and Egypt (Bell, 1937), that at least the basic operations of addition, subtraction, multiplication and division were known before that period. In the Indus valley the oral history of the Hindu way of life also recorded early uses of arithmetic and geometry. An example for such use was the plan, including orientation, for specific construction of fire altars, down to the exact number of bricks in each layer (Fuerstein, Kak, & Frawley, 1995, p. 201).

The most abundant evidence of early mathematical works was found in the clay tablets of the Akkadian period of Mesopotamia, around 2000 B.C. These tablets were still being discovered in the early 20th century and in such quantity that much information remained to be translated from the original cuneiform by the time of the second world war (Neugebauer, Sachs, & Goetze,1945). The Egyptian papyri including that written by the scribe A'hmosé around 1650 B.C., offer further insights into the development of mathematics during these times. In the translation of the Rhind papyrus of A'hmosé, Chase (1927-9, p. 24) noted that as well as posing problems of a strictly practical nature, the Egyptians studied mathematics and other subjects for their own sake. Another ancient Egyptian document *The Leather Roll*, also described the many uses Egyptian civilizations had for mathematics in all walks of everyday life (Bunt, Jones, & Bedient, 1976). It is almost possible to see the problems that beset the overseer in a large building project, from ordering the bricks to feeding the labourers. Lest it be thought that the invention of arithmetic was confined to Asia and Europe, Alden-Mason (1957) reported on evidence of mnemonic devices for counting found in Peru, dating back to between 9000 and 1250 B.C.

The image that emerged from the clay tablets of Mesopotamia and the Rhind papyrus, is of a more complex society than may first be thought. One may conjecture that it was the need for various forms of arithmetic in the society of the time that provided the impetus for their invention. At least 4000 years ago, there was already a separation between the type of mathematics needed by society at large, and that needed by the individual. The administrators in those ancient times were already implementing forward planning with the requisite mathematical tools being invented and used. Economics of the time were centred upon agriculture, and agriculture depended, as it does today, on the supply of water. Knowledge of the times of flood and along the coasts, of tide, was the key to foreseeing the times when sowing and reaping should be carried out. The division and payment of labour necessary for building irrigation schemes, as well as the supply of materials, were essential in making the best use of waters available for extending pastures necessary for agriculture, especially grain (Bell, 1945).

The expansion of the Egyptian society required knowledge of weights and measures, of grain storage and exchange rates. Taxes had to be exacted in order that giant systems of irrigation could be built and the land divided fairly amongst the people. Hand in hand with these inventions and developments were the uses of number by religious orders of the day. Places of worship had to be built. These were often complex engineering projects that spanned lifetimes. The arithmetic and geometry necessary to foresee the demand for materials and cost in labour, had and still do have, deep economic and social significance (Erman, trans. 1971, p. 364). The results of adverse planning arising from poor mathematics can be seen through the ages with structures collapsing, mines closing, and people's livelihoods, and sometimes lives, being lost. Public holidays in observation of religious occasions were rigidly observed and were woven into the fabric of the society, as they are today, requiring knowledge of astronomy, time and an accurate calendar. An effort at foreseeing what the future had in store through astrology with a corresponding study of astronomy was commonly associated with religious orders.

The question of time seems to be present in every age and earlier than that for which there are any records; and time is synonymous with number. The measurement of time is inextricably woven into astronomy, as days pass into seasons, and years into lunar cycles. Evidence of the importance of time for earnest observation and to achieve the extraordinary accuracy shown in the Egyptian calendar year, which was calculated as being 365¼ days, existed from the first Pharoahs in upper and lower Egypt (Gillings, 1972, p. 235). This achievement was possibly one of the major factors in settlement along the Nile being such a success, as it was the keystone to prognosticate flooding and associated agricultural practices.

The question has been raised as to why Egyptian mathematics did not progress further (Boyer, 1968, p. 23). The answer often given, was that life along the Nile was about as good as one could wish for at the time, and the mathematics of the day was sufficient for the needs of that civilization.

The heart of mathematics has often been said to lie in the Greek civilization that flourished between 600 B.C. and A.D. 400. It was during this period that the idea of proof came to be developed and with it the arguments so important to mathematicians concerning truth and logical thought. If the tree of mathematics developed as a young plant under the care of Mesopotamians and Egyptians, it first blossomed in the 6th century B.C. when the Greek Thales (640-546 B.C.) was "the only one whose wisdom stepped into speculation beyond the bounds of practical utility" (Heath, 1921a, p. 128). Pythagoras was reputed to have developed the best known theorem of them all in stating with regard to a right angled triangle, that the square on the hypotenuse is equal to the sum of the squares on the other two sides. As with what is known of Thales, there are no written records extant, and all that is left is a trail of discovery, the evidence for which lies in the books of later observers such as Eudemus of Rhodes (4th c. B.C.).

Allen (1997) thought that Eudoxus (408-355 B.C.) was second only to Archimedes (287-212 B.C.) in standing as a mathematician, "…having a profound influence on the establishment of deductive organization of proof on the basis of explicit theorems." The model he developed illustrating the movement of the sun, moon and stars, as well as the retrograde motion of the planets serves as ample testament to his outstanding mathematical ability. The greatest contribution that were made by Eudoxus to original mathematics were his statements concerning proportion and magnitude that were reported in books four and five of the monumental work of Euclid (300 B.C.). *The Elements* (trans. 1952) drew together all the geometry known to that time, extended it and gave it structure that had been hitherto unseen.

Aristotle (384-322 B.C.) was the first philosopher to apply a mathematical structure to logic as illustrated in his discussions concerning syllogisms in his work entitled *Prior Analytics* (trans. 1952b, pp. 53, 118). His work was of the greatest contrast to that of Archimedes who was concerned primarily with the mathematics of physics, establishing the entire science of hydrostatics in such a way that the direct applications from his works *On Floating Bodies* (trans. 1952) could be made to the stability courses for seafarers two thousand years later (Kemp & Young, 1959). The last Greek mathematician to be mentioned here is Apollonius of Perga (262-190 B.C.). The exercises found today in upper secondary school mathematics in parabolas, hyperbolas, ellipses and circles all had their birthplace in the elegant curves and inventions of Apollonius' work on conics (trans. 1952). There are many famous Greeks mathematicians and much has been written by eminent scholars such as Sir Thomas Heath (1861-1940). However, the purpose of this chapter is to lay the first part of the groundwork for the possibility of reform within the secondary mathematical curriculum.

2.3 Trigonometry

One of the branches of mathematics that has been invented and developed in direct response to the practical need for more powerful mathematical tools has been trigonometry. Today every electrical tradesman in training and practice, is asked to recognize the role of a sine wave in understanding the behaviour of alternating current. More than 2000 years ago, accurate calculation in geography and astronomy called for measurement of angles and arcs in spherical triangles, and this was beyond the knowledge of the day. The Greek mathematician Hipparchus (190-120 B.C.) is accorded the mantle of being the founder of trigonometry but the *Sphaerica* of Menelaus (c. A.D. 98) and the thirteen books of the *Almagest* by Ptolemy (A.D. 100-168) (trans. 1952) all played equally important roles in the foundations of the discipline and its direct application to astronomy (Kline, 1972, p. 120). Indeed it is to Ptolemy that a great debt is to be acknowledged, for without his comprehensive writings, much of the original work would have been lost.

It is difficult to say how mathematical thought passed from one geographic location to another, and the study of trigonometry is no exception. The need to have a better understanding of astronomy provided strong incentive for continued development in trigonometry in places far from the Mediterranean civilizations, as shown in India by the work of Áryabahta (5th c.) in developing the sine rule (Datta & Singh, 1962, p. 125). This rule was stated a second time by Abu'l-Wafã (940-998) in Baghdad, some five centuries later (Berggren, 1986, p. 174). For nearly 1000 years, trigonometry was rarely given any attention unless it was associated with astronomy until the discovery of America by Columbus in 1492, changed the face of mercantile shipping. Prior to this time, maritime trading was essentially a coastal affair, and voyages out of sight of land were relatively short. Now the need to find a solution to the courses and distances to sail on the longer voyages (May, 1972) became important to those who wished to reap the rewards from trading in any riches that might exist in the new lands to the west. The riches became apparent as Spanish adventurers returned from their western expeditions with reports of more gold and silver than the merchants and princes who had backed them had dreamed existed. Driven by a mixture of empire expansion, commerce and simple adventure, navigation became the catalyst for moving trigonometry from the cloisters of the mathematicians to the everyday world.

A few names stand out when discussing the trials of making trigonometry a skill useable by many. Boyer (1968, p. 279) reported that Leonardo of Pisa or "Fibonacci" (1180-1250) brought Hindu-Arabic numerals to Europe.

The Hindu-Arabic number system provided a much simpler means of doing arithmetic than the Roman numerals used at the time and this enabled Fibonacci to bring a new degree of accuracy to surveying by applying trigonometry to the calculations. Johannes Müller (1436-76), otherwise known as Regiomontanus gathered all the knowledge concerning trigonometry together, and although there is some debate, Kline (1972, p. 239), ascribes to Regiomontanus the invention of the cosine formula. The formula is still used by navigators today for many problems, from compass error and great circle sailing to intercept methods for position fixing. The other mathematician who shared in this achievement was the great algebraist, Vieta (1540-1603) who applied algebra to trigonometry and expanded the theory of right-angled spherical triangles (Rouse Ball, 1960, p. 233). The problems of multiplication for those less skilled in using the cosine formula were finally overcome with the invention of log tables by Napier (1550-1617) and the cunning use of the ancient Hindu idea of versines [$(1-\cos\theta)$], so that the final form of the cosine formula for the use of seafarers became the haversine [$\frac{1}{2}(1-\cos\theta)$] formula. This had the advantage of eliminating negative numbers in the solution (Williams, 1997). The task of taking these brilliant mathematical ideas and making them of use to the seafarer to solve particular problems fell to the likes of Nunez (1492-1577) and Norwood (1590-1676) (Taylor, 1956). In more recent times, the task was taken on by such inventive and gifted navigators as Cook (1728-1779) and Bowditch (1773-1838) (Hewson, 1983). Bowditch's classical book *The Practical Navigator* is still available and in use today.

Seafarers in general, including navigators, were men of limited education and with a few exceptions, the mathematics of mathematicians was quite useless to them. Trigonometry was a description of something that was unknown, using symbols and words that were effectively a foreign language. Some of the difficulties faced in expressing a formula in manner that could be used by seafarers, can be observed by looking at the cosine formula for calculating the side of a spherical triangle *ABC*:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

Before the advent of calculators the long multiplication required a considerable arithmetic dexterity, beyond the reach of many high school students today, and certainly too difficult for the unlearned seafarers of days gone by. Even the use of logs for multiplication was only one part of the jigsaw. When this formula was directly applied to finding the distance to travel in a great circle sailing problem it became

$$\cos(\text{dist}) = \cos(PA)\cos(PB) + \sin(PA)\sin(PB)\cos(APB),$$

where: In the spherical triangle *PAB*,

P – Pole,	*A* – position A,
B – position B,	*PA* – co. latitude (initial),
PB – co. latitude (final),	*AB* – distance,
APB – difference of longitude,	*PAB* – course A,
PBA – course B and	co – complement.

Errors arising when one had to multiply negatives as well as positives and decide what that outcome might be, were reduced by the invention of haversines and the great circle formula for distance still taught to professional navigators in 1983 (Frost, 1985) was

$$\text{hav (dist)} = \text{hav}\,(APB)\,\sin\,(PA)\ \sin\,(PB) + \text{hav}\,(PA \sim PB),$$

where: hav – haversine and

$\sim$ represents the difference between the two variables.

Sine and cosine were reversed for sides *PA* and *PB* to allow for the fact that they were the complement of the latitudes of the initial and final positions. The formula then became the more recognizable navigational equation

$$\text{hav (dist)} = \text{hav (d.long) cos (lat A) cos (lat B)} + \text{hav (d.lat.)},$$

where: dist – distance, lat A. – latitude A,
lat B – latitude B, d.long – difference of longitude and
d.lat – difference of latitude.

Consider the group of problems besetting the navigator embarking on a passage if a vessel is making a great circle course from an initial position (A) Port Elizabeth 34^0 15'S 25^0 30'E to final position (B) Fremantle 31^0 50'S 115^0 30'E. The four basic calculations required are the initial and final courses, the distance and the most southerly latitude (vertex). In this case, just the distance is calculated using log tables (Burton, 1963) as was still done in practice in 1965 (Faulkner, 1967).

Lat (A) = 34^0 15'S,
lat (B) = 31^0 50'S,
d.long = long (A) – long (B),
d.long (APB) = 25^0 30'E - 115^0 30'E = 90^0,
d.lat = lat (A) $\sim$ lat (B),
d.lat = 34^0 15'S $\sim$ 31^0 50'S = 2^0 25'N,

log hav	d.long	90^0 00'	9.69897	
log cos	lat A	34^0 15'S	9.91729	
log cos	lat B	31^0 50'S	9.92921	+
log hav			9.54547	
nat hav			0.35113	
nat hav	d.lat	2^0 25'N	0.00051	+
nat hav dist			0.35164	

Dist = 72^0 44.43' = 4364.4 nautical miles [1'arc = 1 nautical mile]

It is as well to remember the conditions under which these calculations had to be undertaken. When both natural log and log log tables were used in a complex operation such as this where terms have to be both multiplied and added, interpolation is required when moving from one to another as well as when determining the final angular solution. In the brief daylight of high latitudes in the northern and southern oceans, the work often had to be done by candlelight and performing calculations on a vessel smaller than a modern maxi-racing yacht was and is a task often exacerbated by seasickness brought on by the incessant and violent motion.

Bringing the calculations into the most recent times and using the electronic calculator, the cosine formula may be directly and quickly applied using

$$\cos(\text{dist}) = \sin(\text{lat A})\sin(\text{lat B}) + \cos(\text{lat A})\cos(\text{lat B})\cos(\text{d.long}).$$

The solution from the above formula is in angular form, whereas distance at sea is usually expressed in nautical miles. Since one minute of arc of a great circle on the surface of the Earth is one nautical mile, the formula may be more nicely expressed as

$$\text{dist.} = 60 \cos^{-1}[\sin(\text{lat A}) \sin(\text{lat B}) + \cos(\text{lat A}) \cos(\text{lat B}) \cos(\text{d.long})].$$

Applying this formula to the problem of finding the distance between Port Elizabeth and Fremantle,

$$\text{Dist} = 60 \cos^{-1}[\sin(34^0 15') \sin(31^0 50') + \cos(34^0 15') \cos(31^0 50') \cos(90^0)]$$

$$\text{dist} = 4363.9 \text{ nm} \qquad [\text{nm – nautical miles, 1nm} = 1852\text{m}].$$

The ideas associated with explaining complex mathematical formula so they can be applied by navigators, engineers and others on a daily basis, are among the most important yet least known parts of the history of applied mathematics.

2.4 Expression

The manner in which the thoughts of the leading mathematicians are expressed is not as concise or clear as the everyday user might wish, even if the user has the intellectual background to understand what is being written. The difficulties faced with describing trigonometry so that it was available to many, are part of an ongoing process repeated time and time again with mathematical inventions, from division to calculus and beyond. When one considers the poor educational background of most people even a century ago (Barcan, 1966, p. 9), the task of making trigonometry useable to the seafarer or surveyor was considerable. It must be remembered that trigonometry was beyond the university courses taught in Newton's day. A more lucid form of expression was needed to allow the mathematics of the elite astronomer to be available to others perhaps not as well educated but in need of the mathematics that had been developed. Possibly the most important step in simplifying mathematical symbolism in Europe occurred when Fibonacci published the Hindu Muslim number system including the idea of zero (Seile, 2000). A great debt is owed to the Hindu Muslim world for its integer notation, which allowed lengthy calculations to be made with far greater ease than was the case with any other form of notation (Boyer, 1968, p. 237).

The clarity of the writings of Regiomontanus and the ease of expression brought to algebra by Vieta paved the way for the ideas of Widman (1462-1498) in the application of the symbols of + for addition and – for subtraction in 1489, and Recorde (1510-1558) for the use of = for equality in 1557 (Smith, 1958). Although the language of mathematics was constantly being simplified, the common use of these symbols was not apparent for a further 200 years. Looking at the proliferation in mathematical invention over the last 500 years using all the power of modern symbolism, it is not inconceivable that one of the reasons for the lack of progress in mathematics in Roman times was simply the cumbersome number system. One can only admire the efforts of the Egyptians, the Hindus and Arabs who largely wrote out their problems without any of the shorthand symbols we take for granted today. There has always been some form of learning and teaching in mathematics, but the dispersion of mathematical knowledge today, amongst the general population as opposed to the favoured few, is only possible through the use of symbols (Weaver, 2004) that allow the thoughts of the great minds to become accessible to all.

2.5 Transmission

Recalling the evidence of the 2000 years that elapsed between the writings of A'hmosé and Ptolemy, highlights the fact that brilliant thought like brass must be remembered and polished, or it dims with time. The key to accurate memory has been for millennia the written word, and it is to those who have had the patience and diligence to encapsulate the works of those who have gone before in a lucid manner that attention is now turned.

A'hmosé was the earliest writer or scribe to be identified in the transmission of mathematics. A copy of the papyrus he wrote (The Rhind Papyrus), was found by Henry Rhind (1833-1863) in 1858 and later translated by Chase (1927-9). It proved a treasure trove of information concerning the mathematics known at the time and the use that was made of it. Although the arithmetic and geometry used was of a practical mien, some of the problems were undoubtedly made up to test the user, as they would have been quite impractical for the shepherd or builder. One example was problem 67 concerning the herdsman and his stock (Chase, 1927-9, vol. 1, p. 103). "The herdsman came to the stocktaking with 70 cattle. The accountant said to the herdsman. 'Very few tribute-cattle art thou bringing; pray where are all thy tribute-cattle?' The herdsman replied to him, 'what I have brought is ⅔ of ⅓ of the cattle that thou hast committed to me. Count and thou wilt find that I have brought the full number.' "

It brings to mind similar problems found in school-books today.

Most of the original mathematics of Euclid would not have survived the ravages of time, but for the scholarship of Pappus (A.D. 380). He brought together the comments of Hero of Alexandria (b. 2nd c. B.C.) and Theon of Alexandria (A.D. 335-405) regarding the great works of Euclid and without his efforts there might have been no record available (Cuomo, 2001). The last of the great Greek mathematicians, Ptolemy was best known as an astronomer through his collection of 13 books known by the Arabs as *Almagest* or "*The Greatest*" (Ptolemy, trans.1952). This work stood as a monument to early Greek brilliance, being used as a reference for a millennium and being finally brought to Europe through the diligence of Regiomontanus.

As with Euclid, it is doubtful if the work of Ptolemy would have survived if it had not been for the diligence and brilliance of the mathematicians of the Arab world, and in particular Al Khowârismî (9th c.), sometimes known as the father of algebra, in copying and expanding the works of the Greek empire (Berggren, 1986, p. 6). In addition the Arabs used the marvelous works of Hindu mathematicians, such as the sine rule of Árya-Bhatta (5th c.) and of Brahmegupta (7th c.), who was perhaps the greatest of all the early Hindu mathematicians, known through his works *Brahme-Sphuta-Sidd'hánta* (Colebrook, 1918).

The story of mathematics could now be seen to stretch from India to the Mediterranean. In China a parallel story could be traced through ancient writings on pottery and then on paper. History records that the first mathematician of note in China was Liu Hui (3rd c. B.C.), who collected all mathematical knowledge of his time, and expanded on it in his comments in the ancient document *The Nine Chapters on the Mathematical Art.* Nearly 1000 years later, Master

Sūn's *Mathematical Manual* again encapsulated all that was known. Lest it be thought that this was some primitive arithmetic, it was noted by Yăn and Shírán (1987, p. 110) that the algebra of China was 400 years ahead of the algebra of Europe in the 13th century.

Some 200 years after Fibonacci brought the mathematics of the Hindu-Muslim world to Europe, Regiomontanus set about encapsulating the geometry and arithmetic of previous years, suggesting improvements and developing a new way of expressing formulae. He was sufficiently concerned about others being able to read of his work, that he set up his own printing press. Regiomontanus's notes indicated (Boyer, 1968, p. 301) that many of the books he planned were never written, due to his untimely death. Within 200 years, by the time Descartes (1596-1650) and Newton (1642-1727) had made their marks, it was becoming too great a task for one person to be expert in all fields of mathematics, although it must have seemed that the intellects of Euler (1707-1783), Gauss (1777-1817) or Poincaré (1854-1912) encompassed everything (James, 2002). A new type of mathematician was taking a place in the field however, that of the mathematical author, one who had a gift for making clear the complicated thought of other great minds. Recorde was the first to write a mathematics text in English in 1543, and two centuries later, Hutton (1727-1833), an applied mathematician in his own right, wrote with such clarity, that his text-books were used in America as well as England (Howson, 1982, p. 22).

Today the phrase, "publish or perish" has a nasty sting to it when applied to those employed in research and lecturing, but there is the point to bear in mind, that unless knowledge is passed on, there can be no growth. In times when there was little learning available, word of mouth was probably sufficient, but during the last 2000 years, there has been an increasing expansion of knowledge. In close association with this, a spiraling number of books have been written, read, translated and updated to try and encapsulate the knowledge of the past. There must be many a scientist who has read with wonder the discoveries of others in distant lands, and many students have been grateful for authors such as Stroud (1987), who had an amazing ability to make the complex mathematics of the engineer clear enough to learn without a teacher. The written word is the key to ensuring that the next generation can learn from previous achievements and mistakes.

Newton was considered by many to be the greatest mind of all time. He was not interested in the opinions of others and was reluctant to publish, although he kept copious notes. Many of these papers first came to light in the early 20th century (Gleick, 2003, p. 193). These had been held by the Earls of Macclesfield since the second earl, who was also the president of the Royal Society between 1752 and 1762. Those thought to be of scientific significance were passed on to the Cambridge University library and the balance put up for auction at Sotheby's in 1936, to be bought by the economist, John Maynard Keynes (1883-1946). Realizing the significance of Newton's papers, Keynes made them available to scholars who wished to study them, thus adding another chapter in the transmission of mathematical thought.

2.6 Pure and Applied Mathematics

Commentators such as Kline (1972, p. 23) have indicated that the mathematics of ancient Egypt and Mesopotamia was crude when

compared with that of the Greeks. In ancient Egypt and Mesopotamia, various techniques were invented to make counting easier for practical purposes and at that stage the development of mathematics ceased. Thales has been called the father of pure mathematics and for five hundred years Greek mathematicians took mathematics from being a tool for affairs of state from building to taxes, and turned into a mental exercise for its own sake. From this time on, pure mathematical thought has expanded spasmodically, until the present time sees thousands of new ideas presented each year through journals and conferences.
Throughout the Classical and Hellenistic periods of Greek history, the study of number transcended the practical to assume mystical properties. These properties were pursued by the brotherhood in the Pythagorean school and associated with the paradoxes of Zeno (4^{th} c. B.C.). It was thought that if the universe could be described in terms of the number, then so could the philosophies and logical arguments of men. Because of this, concepts such as zero and negative numbers could not exist, and the concept of an irrational number raised serious philosophic difficulties. The idea that a mathematical concept could not be allowed because it did not support the philosophic relationship with nature that existed at the time, carried mathematics far away from the mundanity of the everyday world, state pension schemes, taxes, vast bridges and the distant stars, to the esoteric arguments about logic pursued 2000 years later by Descartes, Frege (1848-1952) and Russell (1872-1970). The birth of logic in mathematics was found in *The Elements* of Euclid and the heart of this work that has drawn the admiration of mathematicians through the ages, was the development of deductive proof.
In contrast to Euclid, Archimedes, Newton and Gauss all directed their minds to practical applications. The use of the Archimedes screw to draw water

from the Nile was still being used 2000 years after he invented it, but that is not the reason Archimedes is remembered as one of the greatest mathematicians of all time as well as the father of physics (Boyer, 1968, p. 136). The mathematical reasoning behind the screw found in the works of Archimedes *On Spirals* (trans. 1953a), and mathematical arguments for using levers found in book 1 of *On the Equilibrium* (trans. 1953b), formed part of the vast array of mathematical knowledge Archimedes assembled, invented, and applied to physics.

Newton was the second of the great mathematicians who also had an ability to link pure mathematical thought with immediate application, making his mark as an astronomer and physicist as well as a mathematician. Like Archimedes he used all of the power of mathematics that others had devised in order to advance his understanding of the laws of nature, inventing as well the fluxions or calculus to be able to describe motion at any point in time in precise mathematical terms (Newton, trans.1952). Newton's time as Master of the Mint in England enabled the country to operate with a single currency for the first time (Gleick, 2003, p. 165); and the economic benefits were immeasurable.

The third mathematician was Gauss. He was a more worldly man than either of the others, bound in the affairs of his state and driven by curiosity into geodetics, magnetism, optics and astronomy, as well as by his mathematical work in several areas including combinatorial theory. Bühler (1944, p. 140) stated that Gauss believed that the extent to which nature could be described in mathematical terms was indicative of the depth of our understanding. His diversity and brilliance of thought overwhelmed his contemporaries in the tumult and development that accompanied Napoleonic times. It took the genius of Gauss (Cooke, 1997) to sort out the problems of the state pension scheme and to rectify it for his patron the Duke of Brunswick (1717-1815). This may not have been at the highest pinnacle of mathematical endeavour, but it served to show the intellectual power needed to serve the state and subsequently advantage the people.

By the mid 19th century, separate chairs of pure and applied mathematics had been established in France. Barrow (1992, p. 13) suggested that the divide between pure and applied mathematics resulted from: The invention of non-Euclidean geometry (Riemann, 1826-1866) and the development of logic to suit these non-Euclidean forms. Certainly by the turn of the 20th century, Hilbert (1862-1944), the leading mathematician of his era, was moved to state publicly his concern regarding the shift towards the abstract and away from the study of concrete problems (Kline, 1972, p. 1038). The explosion of mathematical discovery during the 20th century was naturally accompanied by earnest debate, as the solution of one problem raised the spectre of several other problems to be solved. The literature of Gödel (1906-1937) highlighting the frailties in mathematical truth, was seen as a challenge to describe more completely the mind of man and taken up by Penrose (1942-) in his book *Shadows of the Mind* (1995).

In closing this section it is pertinent to look further at mathematicians who personify pure mathematics, having noted the contributions of Archimedes, Newton and Gauss. George Boole (1815-1864) is best known for his development of symbolism to enable logic to be explained with rigour in algebraic terms (Boyer, 1968, p. 326). He has been called the purest of mathematicians, dealing only with the symbolism of mathematics. There are however many contenders for this title, including Ramanajun (1887-1920) who developed an interest in mathematics simply because it was there. His intellect shone like a beacon, and even without formal education he achieved at the highest levels. Finally it would be churlish to omit the peripatetic Erdös (1913-1996). He has been described as the purest abstract mind since Wittgenstein (1889-1951), travelling the world posing and answering innumerable problems and publishing more than 1400 papers (Hoffman, 1998). The expansion of pure mathematics was reflected in educational philosophies holding sway in universities (Hodgkin, 1981) and the schools from which they drew their students until Napoleonic times in Europe and the 19th century in England .

Summary

The opening discussion in Chapter 2 argued that mathematics is an invention, a social construction with number as a foundation. It was also shown that, depending upon the individual use or view of a person, mathematics could be seen as a utility or an art, or both.

For more than 2000 years mathematics was essentially arithmetic. Various techniques were invented to perform more complex tasks as society demanded. Hand in hand with the practical, were the less worldly problems that those who had the knowledge set down for those who wished to learn.

For a brief period during the times of the Greek civilization, ideas of mathematics were looked at in the light of nature, man and philosophy. Questions were posed and answers constructed to show that for something to be true mathematically, a statement must hold for all conditions. Deductive proof became the mark of mathematical truth.

Trigonometry was used as an example of one branch of mathematics that has developed since Greek times and has moved from the domain of the mathematician to that of the seafarer with limited education. Such moves have been shown to be aided by gifted writers who have translated the difficult explanations of the original mathematics, into a form that anyone who wishes may use. Such authors have at times invented their own symbols making a particular operation easier to understand.

The aspects of mathematics that constantly appealed to the rulers of nations and merchant princes of the times were the ways that mathematics could be applied to problems of state. As has been noted, all three of the greatest mathematicians possessed this ability to use mathematical invention in some practical way. There is obviously a place for both pure and applied forms of mathematics and it has been finding a balance between the two that has raised spirited debate in mathematics education for more than a century. Mathematics education is a basic component of the development of mathematics and is part of the core curriculum in schools today. The development of mathematics education is investigated in the next two chapters.

CHAPTER 3

MATHEMATICS, EDUCATION AND SCHOOLS

Introduction

The previous chapter describes some details of the ever increasing body of mathematics. The exponential growth of published knowledge has been due to several factors apart from the proliferation of mathematical brilliance. These factors include the developments in the print media, the simplification of mathematical expression and the expansion of education. It is to the latter that this chapter is addressed.

Mathematics education is followed from the earliest records in the time of Classical Greece to the turn of the 20th century. The upper boundary has been chosen because it marks the period when primary education became compulsory in many countries, and in England it became the period of change away from the dominance of Euclidean proofs in secondary education.

The influence of Greek and Roman education faded in Europe as the Dark Ages gripped country after country and the candle of knowledge was kept alight by the Moorish Empire, with centres in Baghdad and Cordova. The Middle Ages were marked by the brilliance of the Renaissance and moves away from the Roman church. Although schooling was not a common expectation, wherever possible, arithmetic was included in the curriculum.

The establishment of new countries such as the United States of America and Australia added impetus for mathematics to be taught in the vernacular, and include matter that was relevant to those countries rather than ancient Greece.

3.1 Ancient times

The mathematics used 4000 years ago was arithmetic and geometry that lent itself to the agricultural economies of Egypt and Mesopotamia. These have been briefly described in Chapter 2 and predate any description of formal schooling. The first evidence of a group of people coming together to learn mathematics rests in Greek civilization.

State schooling for the common child first appeared in the records of Sparta, the strongest and most successful of the Classical Greek city-states in the 5th and 6th centuries B.C. The state directed elementary schools to prepare boys who were between 7 and 14 years of age, to become warriors to defend the state. The preparation included reading, writing, arithmetic, music and recitation but these were secondary to all aspects of physical education pertinent to the training of the warrior. The arithmetic was essentially finger counting and it seemed that the four operations of addition, subtraction, division and multiplication were too sophisticated for these ancient times (Marrou, 1956, p. 158). Boys from wealthier families continued their schooling through the gymnasia between the ages of 14 and 18 years, learning the arts associated with ruling the state. The third stage of education was military at the ephebic schools and this continued until young men were twenty. As early as Athenian times (4th c. B.C.) girls were included in elementary schooling alongside boys, although the life of young girls at home still contained much vigorous education and athleticism.

As the pendulum of power swung to Athens, so too did the heart of progress in education and schooling. It was from the Greek culture and beauty of Athens that Western education began (Castle, 1965, p. 11). The thoughts of such gifted people as Plato (428-348 B.C.), Isocrates (436-338 B.C.) and Aristotle still rebound in the corridors of education policy makers today. These great thinkers all had their own academies, where those of a like mind could seek knowledge at a higher level than the schooling available for children. As happens today the ideas of these men, and their academies, coloured the educational practices in the first two stages of education. The emphasis moved away from the militaristic support of the Spartan state and towards a more literate and cultured learning, mirroring the society of Athens.

The time structure for the three stages of education remained roughly the same throughout Greco-Roman education, and the subjects taught in elementary schools remained similar. It was in the second stage in particular that the pattern of education changed as the thoughts of the Pythagorean and Platonic Academies started to take effect, particularly in the mathematical sciences. At the gymnasia, one could expect to include the subjects of Spartan times, grammar, literature, Latin and Greek as well as more arithmetic and some geometry. There was still a strong emphasis on physical education as befitted a culture that esteemed athletic endeavour.

By the time the first professional teachers, the Sophists (Dewey, 1968, p.330) established themselves through the inspirations of Isocrates, mathematics in various forms had been taught in schools for 200 years and was deemed an essential part of elementary education. Other disciplines including astronomy and mathematical aspects of music increasingly formed part of the secondary curriculum, to develop the young mind in diverse ways.

The arguments and philosophies of Plato and Isocrates (described in Chapter 4) were many and varied, but it was Aristotle who brought together the best of both great men and offered a model for the secondary curriculum still used 2000 years later. Aristotle taught what came to be known as the seven liberal arts (Castle, 1965, p. 59) namely; the trivium, grammar, rhetoric and dialectic and the quadrivium, geometry, arithmetic, astronomy and music. The names trivium and quadrivium were ascribed to the subjects by Boethius (d. A.D. 524) around the end of the 5th century (White, 1981, p. 162). To Aristotle must also go the credit of girls learning alongside boys in the gymnasia.

Roman education essentially followed the pattern of the Greeks. A more pragmatic approach reflected the spirit of the people and was in turn particularly expressed in applying geometry away from the mind towards the utilitarian ends of engineering and surveying, and arithmetic towards bookkeeping and economics of state. Unlike Athens, Rome was the heart of the greatest empire the world had known. Romans believed that education was one of the keys to the maintenance of their empire and established their schools wherever they went, from Egypt to England.

The expression of Greek education that was felt in Europe was manifested through Rome and the most gifted writer of his day, Boethius (Smith, 1970, p. 51). The book for which Boethius was best known *De Institutione Arithmetica* (Caldwell, 1981, p. 137), became the standard for European education and was still prescribed reading at Cambridge University in the 17th century

(White, 1981, p. 185). This was mostly a translation of the works of Nichomacus of Gerasa, (c. A.D. 100). Nichomacus wrote on all four of the quadrivium in his book *Introduction to Arithmetic,* and such was the quality of his writing that it became an instant classic (Nichomacus, 1952). Nichomacus spared no effort in attempting to make the mathematical knowledge that was dear to him clear to others. He wrote out multiplication tables (Nichomacus, trans. 1952, p. 825) and used analogies to explain terms such as equal (p. 823). He pointed out the absolute necessity for studying arithmetic and the importance of number in the disciplines of geometry, astronomy or music (harmonics). For example how can one describe a triangle or a quadrangle, without the numbers three and four (p. 813)?

Throughout the 1000 years of education in antiquity during Greek and Roman times, mathematics has played an increasingly important role in the state and its citizens. In Greek times, the expansion was the result of their explorations into the relationship of man and nature, and in Roman times the role of man in sustaining a great and complex empire. This magnificent inheritance was nearly lost as Europe plunged into the Dark Ages with war and famine abroad in the land.

3.2 The Dark Age

The Early Middle Age, or Dark Age, lasted for some 800 years from 500 to 1300. The warlike tribes of North Eastern Europe, swept over all of Europe leaving only the Eastern Roman Empire intact with the centre of learning and culture in Byzantium. It was not until 1453 that the Ottoman Turks finally overran Byzantium and the last of the Roman Empire fell (Gord, 2005).

The devastation due to war particularly affected centres of learning, as the marauding Germanic tribes had no particular regard for either books or government. In the lawlessness that accompanied the ceaseless wars, first people, then books, and finally the spirit for learning decayed as centuries passed. England was to some extent protected being geographically isolated, but constant strife was abroad in the country after the Romans left (Bede, the Venerable, trans.1849).

Although learning was decaying in Europe, the years between 786-1100 were a time when the Arab Muslim Empire flourished, with centres of learning in Baghdad and Cordova. In Baghdad, the Abbasid Caliphs, particularly Hārūn al-Rashīd (786-809) gave great support for learning (Kimball, 2000). A great library was constructed in Baghdad called the House of Wisdom. Works from across the known world, from India to Greece, were brought here and translated. Scholars made comments on the works and improved on many, particularly in areas of practical use, for example, trigonometry and astronomy (Thornton, 2003). Possibly the best known of their mathematicians, Al-Khowârizmî, translated mathematical works from both Greek and Indian sources, which subsequently provided an invaluable source of knowledge for Europe in the late Middle Ages (1300-1660). Education was not just an affair for the few; there were 30 colleges built in Baghdad and in the 10th century al-Azhar was established as a university in Cairo. To all intents and purposes this is the oldest university in the world as it is still receiving students. Elementary schools were common and the scholar revered in this period that became known as the Arab Golden Age.

In the west, the Moors invaded Spain in the 8th century settling around Cordova and Toledo. They brought their sophisticated practices in agriculture, metallurgy and commerce and changed the region from a poor and backward farming region to a wealthy centre for commerce and culture. Both cities still exhibit the architectural wonders of a bygone age when London was still a small town. Once again schools and libraries abounded, but in isolation and seemingly unknown with regard to the rest of Europe. When the Moorish city of Toledo was overcome in 1105 (Derhak, 1995-2000), the Christian world of Europe finally became aware of the intellectual treasures residing in the great library there. There were 600,000 manuscripts found in the library containing the collected wisdom of the ages from Greece to the Indus and from Persia to Egypt. It seemed no small coincidence that in the ensuing 200 years the great universities of Europe bloomed, Oxford and Cambridge, Paris, Bologna, Padua and Salerno (Dickson, 1996). The Arab peoples had their own philosophies and did not appear to need those of Greece, although they were aware of them. Their approach to learning was a practical one and mathematics can be traced at every level, and found in every library. It should not be thought that utility meant a lack of appreciation for beauty and form. The exquisite architectural applications of geometry of the Palace of Alhambra in Granada have not been bettered to the present day.

The first glimmer of cultural light in Europe came with the Carolingian Renaissance. The court of Charlemagne the Great (768-814) brought scholars from all over Europe to enrich the Palace school and the court. Foremost amongst these was Alcuin (735-804), who was educated in the liberal arts at the cathedral school in York (Ritchie, 2001). Alcuin remained at the school in York as a teacher and later headmaster. He later established the great library in

Aachen (or Aix-le-Chapelle) for Charlemagne and set up a system of intellectual learning in the monasteries and cathedrals. He was involved in the development of Carolingian miniscule writing. Here for the first time words were separated from one and other, letter form was standardized and close attention was paid to the accuracy of copy. The benefits for all forms of learning remain incalculable. This brief period of learning was dashed when the Vikings swept across Europe and darkness spread once more.

England had never been a land of learning. The Romans had brought their schooling with them as was their practice (Lawrence, 1970, p. 48), and when they left in the 5th century, the country retreated into small warring tribes and the poverty associated with such an existence. However as well as the remnants of a school system, Christianity remained, and it was this that left the door open for a return to a more civilized way of life. Pope Gregory the Great (d.605), alarmed by the drift of England away from the Roman Church, sent Archbishop Augustine (Saint Augustine of Canterbury, d.604) as his emissary to England. At the time King Ethelbert (d.616) presided over English lands from Kent to the Humber and his wife Bertha was a staunch Catholic (Bede, the Venerable, trans. 1849, p. 37). It was with their support that Augustine established himself in Canterbury and in 598 and opened up a school attached to the church Ethelbert had given him (Leach, 1969, p. 3). The prime purpose of the school was to train the clergy for their duties. This training included the use of the arithmetic text of Boethius although mathematics only played a minor role in the educational pattern of the life of the clergy. The Venerable Bede, (trans. 1849, p.172) had direct evidence of the mathematics taught by Archbishop Theodore (666-690) and the Abbott Hadrian (668-709) through one of their pupils, Alden, who said that he had found the arithmetic difficult. Indeed the Venerable Bede (633-735) himself taught the seven liberal arts as well as other subjects associated with the church and the Catholic faith.

Across Europe, the only mathematician of note following the last of the Greeks was Leonardo of Pisa (Fibonacci) (Kline, 1972, p. 209; Boyer, 1968, p. 283). Boethius was not included in such illustrious company although his texts provided the backbone for mathematical education throughout the Dark Ages and beyond. It now seems of great significance that the church schools and the guilds with their apprenticeship schemes, kept the use of mathematics alive and people's minds in touch with the study of number for 900 years.

Europe slowly stabilized between the 9th and 12th century. The power of the church grew following Charlemagne being crowned Emperor of the Holy Roman Empire (800) and schooling followed the development of cathedrals, accompanied by the need to educate the clergy. Watson (1968, p. 27) reported that the trivium and quadrivium formed part of the curriculum of these schools which were the forerunners of the old grammar schools. Utilitarian forms of arithmetic, the tools of management were taught as well as the theoretical writings of Nichomacus as translated by Boethius. Pope Sylvester 11 was sufficiently fascinated to study and write his own version of *The Arithmetic* of Boethius around the turn of the 11th century (Pekonen, 2000) and the era was opening for further development.

The great educators of the 12th and 13th century include Sacrobosco (1195-1256) and Roger Bacon (1214-1292). They were among the first educators to travel the path of grammar school and university. Elementary schools had for many years provided for the basic need to read, write and to count. The grammar school was developed to provide for children who had to learn Latin, the language of the church and most books. The standard of education was still quite basic, equivalent to the middle years in today's English preparatory schools. However it formed the secondary education of the day. In addition although university was really the equivalent of today's secondary school in academic standard, it provided the atmosphere for the enquiring mind to flower. Sacrobosco (Mosley, 1999) was educated in Yorkshire and at Oxford, known as John of Holywood or Johannes de Sacrobosco. He went to the University of Paris to study and teach, eventually becoming a professor of mathematics. He wrote several books including a treatise on the workings of the quadrant (for measuring angles of heavenly bodies). His best known work was a translation of Ptolemy's *Almagest* in *Tractatus de Sphaera.* In addition he promoted the use of Arabic number and algebra in *De Algorismi*. These translations were probably from Arabic to Latin, the latter being the teaching language of the day, and for the next 400 years. Roger Bacon (O'Connor & Robertson, 2003) was one of the outstanding scholars of his day. As with many university graduates his future lay in the church and teaching. He was a prolific author and the first to attempt to prove the case for a knowledge of mathematics in order to understand the *Bible*. In the fourth part of Bacon's *Opus Maius* (trans. 1996, p. 180), he states "If, now, we may deduce what subjects are rightly associated with the study of theology, we will discover that mathematics and its parts are imperative…..." These were years when it was of prime importance to have the blessings of the church for what one wrote or else fear its retribution. To address a subject so close to the heart of the church was daring and indicative of Bacon's powers of scholarship. It was the church that provided the continuity of education and schooling at the time and any argument for mathematics to remain part of the curriculum was welcome. The church had absolute power over what was taught in their schools, and only included material that supported the Catholic view of Christianity and the physical world that reflected such viewpoint.

3.3 The Late Middle Age

The 14th century is best remembered for the Black Death or Bubonic Plague, which swept across Europe reducing the population by a third. The history of the Plague is perhaps one of the most compelling arguments for education for all. People where more inclined to believe in superstition and magic as preventatives, lacking the ability to understand the basic arguments for cleanliness. It is also remembered as the time of the Renaissance. This brilliant time of learning and culture was marked by such figures as Regiomontanus, Copernicus (1473-1543) and Galileo (1550-1600). Among the people who wielded great influence during this period were the Medici family, who ruled Florence and Tuscany (1434-1737) and their most famous protégé, Michelangelo (1475-1564). Petrarch (1304-1374) was perhaps the most important figure from the point of view of philosophy and education. Sadlon (1999) called Petrarch the father of humanism and the Renaissance. He was a prolific author and inspired firstly the people of Padua and then northern Italy to embrace a new age. The spirit of humanism is still alive in education today.

The first English figure of educational note in the 14th century was William of Wykeham, Bishop of Winchester (1324-1404), known in England as the Euclid of his time for his power in geometry. As The Exchequer, he was an illustrious figure in England and one of the richest men in the country. In 1387 he founded Winchester (Public) School (Watson, 1968, p. 207), so that those who were poor and needed learning could be served. This was the first of the nine public schools endowed with private funds from wealthy philanthropists or the crown. Winchester School was quite separate from the cathedral and marked a new age in education to complement the elementary schools already spreading across the land, in which reading, writing and arithmetic were taught by the local priest or his appointee. The schools of the time where teachers were available to teach mathematics, were few and far between. The standard text for these teachers was Boethius, but mostly mathematics was taught by the Guilds (Lucas, 1972, p. 217) through apprenticeship schemes. In the large part, teaching under the Guilds was directly from master to apprentice, but some schools were later established, including the Merchant Taylors' school (1620). Mathematics taught by the Guilds was necessarily utilitarian, driven by the demands and funding of commerce and industry, from cobbler to architect.

Probably the greatest invention of the millennia, the printing press by Johannes Gutenburg (1400-1468) in 1450, meant that books could become available to other scholars with much greater ease than being copied by hand. The Italian mathematician de Borgo San Sepolcro (1445-1509) wrote the first great work on mathematics that was printed (Smith, 1970, p. 54). In Germany, Widman gave the first lecture on algebra and wrote the first book on arithmetic in 1488 (O'Connor & Robertson, 1996a). As well as employing + and – signs for addition and subtraction he managed to explain division, although this operation was almost unknown at the time. However it was the teaching and books of Reis

(1492-1559) that were best remembered in German mathematics education.

Recorde, teacher and author, was the first and possibly the best known of those who made mathematics available to all those who wished to learn in England. He wrote clearly and in English, a marked departure from the Latin of Tonstall (1474-1459) and others (Howson, 1982, p. 26). The most popular of his five books was *The Groundes of Artes* first published sometime between 1540 and 1542 in London (Smith, 1970, p. 213). The book was designed as a teach-yourself arithmetic text by way of examples and description (exercises were not included in such text books until much later on). It is difficult to imagine the impact such a book must have had on mathematics education. This was the first and for a time, the only, arithmetic text book available in English, and written for the market place rather than academia.

The 16th century was marked by the gathering pace of the reformation. Germany was in the vanguard of the movement where the leading figures of Luther (1483-1546) and Sturm (1507-1589) carried great influence with their opinions. The Protestants understood the powerful role of education in changing the opinions of people in the revolt against Catholic dominance. Luther spoke of the importance of mathematics in education and Sturm put it into practice (Lucas, 1972, p. 251). Sturm and Erasmus (1466-1536) had been educated by the Brethren of the Common People. This was a charitable order that operated 150 schools in the Netherlands and had been teaching since the 12th century (Mulhern, 1959, p. 386). The Huguenots in France were no less forward in their educational policies and provoked the formation of the Catholic teaching orders of the Jesuits in 1550 and Ursulines. Both of these groups saw mathematics as important and included it in the various curriculae that were put forward. The mid 16th century saw the revitalization of the College Royale, an academy or university for the elite. An additional eight chairs were funded including one for mathematics. The ensuing 100 years was to be a great age for higher education in France, and with it a statement concerning the fundamental position of mathematics (Bowen, 1981, p. 52). Perhaps the most important aspect of the freeing of education in the northern part of Europe from Catholic dominance, was the move towards teaching in the vernacular.

The King of England, Henry VIII (r.1509-1547) has been called the most educated monarch of his age (Leach, 1969, p. 277). When he took England away from the Roman Church and formed the Church of England, it paved the way for a form of schooling more suited to the needs of the English peoples. The Dissolution of the Monasteries (1536) and their replacement by church schools was accompanied by the establishment of more grammar schools so that over 200 were in existence by the end of his reign. This expansion was continued and accelerated by Elizabeth 1 (r. 1558-1603). In each major town or city, there was now the pathway for most who were earnest enough to pursue learning to the highest level, although barriers of fate, finance and circumstance excluded most from following their chosen path. There were both elementary and grammar schools operated privately or by the town councils, and arithmetic was considered a necessary part of a child's education. Oughtred (1574-1660) was a prime example of the classical scholar. Schooled at Eton and Cambridge he later took private students who lived with him free of charge. Included amongst these were the great architect Wren (1632-1723) and Wallis (1616-1703) who became the foremost mathematician in England. The greatest written work of Oughtred was *Clavis Mathematicae*, which included Hindu-Arabic numerals and decimal fractions, as well as the use of (×) for multiplication. The device for which engineers and schoolchildren were grateful for until the 1970s, was Oughtred's invention in 1630 of the slide rule (O'Connor & Robertson, 1996b). He was in many ways the true renaissance mathematician, learning, teaching, writing and inventing new technology for calculating.

At the turn of the 17th century when the first settlers were establishing a new life in America, there were two strong and established groups, Catholics and Protestants, vying for places in the schooling of the child. In the far south in Mexico, a mathematics book had already been written by Freyle in 1556, driven by the concern for fair exchange between gold and currency (Jones & Coxford, 1970, p. 12). In the far northern settlement of Quebec, the French Catholic orders of Jesuits established a college for boys in 1635, teaching the seven liberal arts and in 1636, an Ursuline Convent was established which offered schooling for girls (Potvin, 1970, p. 235).

In the state of Massachusetts, the first school was established in 1635 by common vote. This became Harvard College in 1636, named after the benefactor who gifted a large sum of money to raise the standard of the school. The first ruling to ensure that education was in every town of 50 families was promulgated by the General Council of Massachusetts in 1642 (Mulhern, 1959, p. 377). In America, arithmetic was an accepted part of any elementary curriculum, but the future of the quadrivium in higher education was increasingly insecure as more people wanted to learn mathematics and people with the ability to teach at this level were few indeed.

In English and Scottish universities, the first chairs of mathematics were established at Aberdeen in 1613 and Oxford in 1619. Concern over the increasing distance between the mathematics learnt at university and that required by society, led to the formation of Gresham College in the 1650s. Here the thrust was towards a practical schooling and the curriculum included geometry, astronomy, arithmetic, logarithms, trigonometry and navigation (O'Connor & Robertson, 2000). This was a trend that was to increase over the centuries.

3.4 1660 - 1800

The civil war and Cromwellian rule in England (1642-1669) was over and the monarchy had been restored. Charles II (1660-1685) was inclined to support education, although he kept tight control, with the Church of England as to the content of the curriculum. Each monarch jealously guarded and zealously fought to hold the religion of his or her persuasion in the dominant position, and education played a key role in this. With the Act of Uniformity (1662) Charles II parliament ensured that teachers, particularly in the endowed schools and universities, had sworn an oath to the Church of England. When dissenters opened their own schools, Charles II used the Five Mile Act (1665) to prevent them teaching within five miles of a town These acts forced the hand of many educators and the result was to the benefit of mathematics and the sciences. They were now freed of many of the constraints of church and state and could serve the local people with an education that was better suited to their needs.

England's defence and trade relied upon its maritime ability and it was realized that the only way to increase the navigational competence of seafaring officers was through education, primarily in applying mathematics to navigation. Pepys (1633-1700), through many years as a secretary to the navy (in charge of the shipyards) and later Secretary to the Admiralty (Latham, 2003), understood from his own experiences the difference that a knowledge of mathematics made to the economic and efficient operation of shipping concerns. In 1662, Pepys had employed the mate, a Mr. Cooper, from the vessel “Royall Charles” to teach him the arithmetic he needed to better understand the operation of dockyards and the recquisitioning of ships (Pepys, trans. 1919, p. 272). Pepys was instrumental in setting up the first specialist school of mathematics for mariners The Royal Mathematics School, created within Christ’s Hospital (School) in 1673. His endeavours were only made possible through the support of Charles II (Watson, 1968, p. 130) and the role of such patrons cannot be overestimated in the development of mathematics in schools throughout the ages.

In 1663, Cambridge established the Lucasian chair of mathematics, the first professor being Barrow (1630-1677). Upon realizing the brilliance of Newton, Barrow stepped down after only seven years in favour of him (Gleick, 2003, p. 70). When one considers the prestige and pecuniary aspects of the position, it was an extraordinary step to take. One of the highest recognitions of achievement in one’s discipline lay in acquiring such a chair and the path was long and arduous. This was an age when even elementary mathematical knowledge was not commonplace, and higher mathematical knowledge had to be aggressively sought after. The role of the university of the time, becomes once again highlighted, not so much as a place where one could sup from the bowl of wisdom, but where one might meet like minds and be enthused to continue the search. In contrast, the most popular mathematics book of the time, which ran to 60 editions, was the *Arithmetic* of Cocker (1632-1676), published in 1677 (De Morgan, 1970, p. 56). This followed in the pattern set by Recorde and was written in the vernacular with a lucid hand, and offered the opportunity for one to learn the practicalities of arithmetic by oneself. It was of inestimable benefit to teachers, the majority of whom had few mathematics skills.

In 1698, Savery (1680-1716) patented the first steam engine and Watt (1736-1819) brought it to a practical stage for use in a mill in 1782. This period encompassed the start of the industrial revolution that took England to the forefront as a wealthy trading nation (Kreis, 2001). It changed the way that people lived their lives, moving from rural to urban lifestyles, from a working day ruled by the sun to one ruled by the clock. The industrial revolution enabled anyone to aspire to wealth through work, rather than inheritance, royal favour or war. The implications for education became apparent as the state, town councils and industrialists began to understand that competence in reading writing and arithmetic was one of the keys to affluence. Academies were established to cater for the needs of industry by dissenters, of which the most famous was Warrington (est. 1757). It was here that the famous chemist Priestley (1733-1804) taught in 1761. The standard and variety of the curriculum of the academy were such that it was known as the Athens of the north (Howson, 1982, p. 56). Another example of the change in public attitude towards mathematics was the publication of the first mathematics text by a woman, Margaret Bryan (b. c. 1760) in 1790. Mechanics Institutes made their appearance (Rogers, 1981) and it was in these that most of the mathematical learning of the working man took place.

Newcastle was a good example of an industrial centre having a thriving ship building industry and coal mines. The teacher, author, scientist and mathematician, Hutton, grew up in this climate of hard work and promise, being the son of a coal miner. As he was unfit for work in the mine, his parents had him taught to read and write. He made his introduction to mathematics (which was to be his passport to success) in evening classes with a Mr. James. He proved a more than able pupil and taught at the main secondary school in the area. Here the mathematics curriculum was quite pure and had little application for the local people. As a consequence, Hutton started his own mathematical school in 1760, teaching now at two schools. In 1764 Hutton published *The Schoolmasters Guide or a Complete System of Practical Arithmetic,* a book that was widely used and led Hutton into further work in mathematics teacher education (Howson, 1982, p. 64). The Royal Military Academy at Woolwich, London had been opened in 1741, and with the death of Cowley, Hutton successfully applied for the position teaching mathematics there in 1773. It was at this time that Hutton became the editor of the Ladies Diary, a general publication that put mathematical problems into the community engendering lively discussion. By now Hutton was moving in the most elite circles; he was the foreign secretary to the Royal Society (founded, 1662), and received the Copley medal in 1778 for his paper on the force of gunpowder and the velocity of cannonballs (O' Connor & Robertson, 2002, p. 2). The book that Hutton wrote between 1798 and 1801, *A course of mathematics for cadets of the Royal Military Academy* was also adopted by The United States Military Academy at West Point, in 1802. Hutton's life typified the dreams of the industrial revolution. Through hard work and application one could rise from poverty to wealth and influence.

In the new lands of America, one of the most exciting social experiments in western history was taking place. William Penn (1644-1718) settled in Pennsylvania, establishing Philadelphia in 1682 with his band of Quaker brethren (Powell, 2005, p. 4). The first elementary school was set up in the following year, a school that included reading, writing and keeping of accounts and was also free of the religious bigotry that was to be found in many of the new settlements. The prodigious Benjamin Franklin (1706-1790) established a subscriber library in Philadelphia in 1731 (Lemay, 1997), which played a pivotal role in the town becoming the cultural centre of America. Numerous small private schools were established including the school of Grew (d. 1759), specializing in mathematics. In 1751 another dream of Franklin's was realized when the Charity School was opened (McConaghy, 2004a, p. 1), catering for boys from 8 to 18 years of age. The curriculum was centred on reading, writing and arithmetic as well as the principles of Christianity. Franklin believed in education for girls as well as boys and the girls school was opened in the following year. Grew had been appointed as the mathematics teacher at the Charity School, and when it was chartered as the College of Philadelphia in 1776 became the first professor of mathematics.

With this appointment, the College of Philadelphia was following the pattern of Harvard College where Greenwood (1702-1745) had been appointed as the first Hillis professor of mathematics in 1727. Greenwood also wrote the first American mathematics book for schools, *Arithmetic Vulgar, (common and decimal)* in 1729 (Good, 1956, p. 35). The state took over the College of Philadelphia in 1779 and renamed it the University of the State of Pennsylvania. This was the first university in what was to become the United States of America (McConaghy, 2004b), although there were now 8 colleges with chairs of mathematics established and an infrastructure of elementary and grammar schools to support them. These educational advances were made all the more remarkable in that they happened during the social unrest created by the War of

Independence (1775-1783), which was as much civil as against England. The tone of the mathematics that was taught became increasingly utilitarian as the influence of English classical education weakened and the need for a wider competence in trades based mathematics increased (Butts & Cremin, 1965, p. 124). This change was accelerated by the needs of the War of Independence and the natural hostility felt towards the England of George III (r. 1760-1820).

French mathematicians and scientists entered a golden age of genius and discovery in the 18th century. Early reflections of the age were to be seen in the creation of a school for girls by a remarkable woman, the Marquise de Maintenon (1635-1719), who was the morganatic wife of Louis XIV (r. 1643-1715)(Goyau, 2004, p. 2). The school had started in Rueil and transferred to Noisy-le-Sec before becoming established at Saint Cyr and called the Institut de Saint-Louis in 1686. The philosophies of Fénelon (1651-1715) were pursued so that the education of the girls included mathematics and the sciences, the aim was to develop for the state, educated young women rather than nuns, as was the custom for teaching orders such as the Ursulines.

Eighteenth century mathematics in France was coloured by names such as Lagrange (1736-1813), Condorcet (1743-1794), Laplace (1749-1827), Legendre (1752-1833), Carnot (1753-1823) and Monge (1746-1818). These men were not only renowned mathematicians, but also played vital roles in the future of their country at the time of the French Revolution (1789-1799) and wrote books that influenced education elsewhere. Monge and Carnot established the L'École Polytechnique in 1794 (Armytage, 1974, p. 88). The Polytechnique aimed at training engineers and set the tone for Europe. The mathematics that was taught included the latest thinking in the descriptive geometry of Monge and the analysis of Lagrange. L'École Normale Supérieuse opened it's doors in the same year with the aim of producing teachers with sufficient skills to service the increasing number of secondary schools. The staff of these two schools included many of the mathematicians mentioned above, some teaching at both establishments. .

Further to the east, Poland was establishing the first national education scheme that has been recorded with the formation of the Commission for National Education in 1773. This was largely a political body and the practice of education was left to the Society for Elementary Text Books, which was formed by a group of scientists, educationalists and teachers in 1777. Education was to be in the vernacular and necessitated new texts. To this end the Society advertised for authors across Europe. The position for writing mathematics texts was taken up by L'Huilliere (1750-1840) a mathematician of note from Geneva who was tutoring in Poland (Szreter, 1974, p. 54). Although this attempt at nationalization of education was short lived, it presaged things to come.

3.5 1800 – 1900

Just as the 18th century had seen a new country added to those involved in the history of western education, so did the 19th century. The continent that was to become Australia received the initial fleet of settlers in 1788. This far outpost of Britain was to be used as a colony for those felons who were to be transported from the homeland; men, women and children. In 1800 there were less than 100 children in the area now known as Sydney. Secondary education was available to the boys of the middle and upper classes at the Tull's academy in

Paramatta where English, writing, mathematics, book-keeping and French might be learnt. In 1809 a girls' school was also opened by Mrs. Marchant (Barcan, 1966, p. 5).

The development of primary and secondary education continued as the number of children quickly grew. Church, state and private enterprise all played their part in the education of the fledgling colony and in 1851 Sydney University was established. According to Grimison (1990, p. 310), the senior public exams for secondary schools in 1867 upon which the university selected students included a broad selection of subjects among which were arithmetic, algebra, geometry, mechanics and applied mechanics. As might be expected, the aspirations of the university, and therefore secondary schools that prepared students for university, were modelled along similar lines to those in Britain. However there were some variations that arose due to the conditions and isolation of the country from the motherland. A fine example may be taken from the history of St Peter's College in Adelaide. South Australia was settled as an independent colony in 1836, and elementary schools were set up immediately. The Church of England had already decided upon a church or choir school and this developed into the incorporated Church of England School of St. Peter's, Adelaide in 1849. The school was founded to produce Christian gentlemen and a curriculum designed accordingly. By 1857, the governors decided that any boy of 13 could choose whether to study Latin or Greek or otherwise. This was quite a departure from their English counterparts. Commercial subjects were being taught as well as geography, mathematics, science (including chemistry) and the scriptures. Outside examiners reports were to the effect that 'the work accomplished was on the whole, very favourable' (Price, 1947, p. 18).

The development of Launceston in northern Van Diemen's Land (the island of Tasmania) is interesting to observe during these years. Van Diemen's Land was initially settled as a penal colony in 1803, with habitation and administration centred in the south of the island in Hobart. Launceston was established in 1820 and it took several days to travel either by land or sea from one town to another. It was not until 1817 that free settlers were accepted into the colony and transportation continued for a further 36 years. Nevertheless, those freed convicts, servants of the crown and sundry adventurous souls took little time to create a centre of culture in the wilderness. John Pascoe Fawkner (1792-1869) merchant adventurer and eventual founder of Melbourne opened a reading room for the public (Reynolds, 1969, p. 39) in Launceston in 1825 at the same time that an elementary school was established. In 1839, the government prepared a report detailing the standards that students in the elementary schools could be expected to achieve (Phillips, 1985, p. 18). In addition to reading and writing, this included the first four rules of arithmetic for boys, and proficiency in needlework and knitting for the girls.

A group of Launceston businessmen decided that an establishment was needed to supply the present and future workforce and offer the general community the opportunity to engage in appropriate forms of mathematical and scientific education. In Britain, mechanics institutes performed such tasks as well as being a platform for lectures, talks, local libraries and exhibitions. To these ends the Launceston Mechanics Institute was brought into being in 1842 (Petrow, 1998, p. 1). Following the institute was the formation of the Launceston Library Society in 1845, a private library with membership by donation. Some 9 years later when government funding became available, this became the public library.

1846 proved a momentous year for education in Australia with the formation of what is now the oldest private school in the country, Launceston Church Grammar School and a few months later, Hutchins School in Hobart. There were already several one room private educational establishments where one teacher taught a few students. These 'businesses' came and went as the years passed, as did several larger schools. As the population of Launceston grew and became wealthier, the need for a proper secondary school had become obvious. The first teacher Rev. H. P. Kane (1825-1893), proved an inspired choice (Alexander, 1996, p. 6), despite his lack of qualifications. He was more than competent to teach Latin, Greek, mathematics, navigation, mensuration, English language, comprehension and elocution as well as geography, the use of globes and French. In addition to this range of subjects, the boys were also offered drawing and commercial subjects. Kane was by all accounts a popular and respected teacher who went a long way towards setting the school on a firm footing. There were entry scholarships available in reading, writing and arithmetic for boys between 13 and 16 years of age. As in England, this practice was to ensure that the opportunity for a grammar school education was available for any boy of ability regardless of financial status.

The Mechanics Institute did not offer regular courses of instruction, although what it did offer was seen as most valuable and must have been of considerable academic standard as one of the people who gave lectures there was Kane of grammar school fame. In order that regular educational courses of a utilitarian nature to service the needs of industry and commerce could operate, the Launceston Technical College was established in 1888, with Ockleton appointed to teach mathematics the following year (Proverbs, 1988). The remaining educational and cultural establishment, the Queen Victoria Museum and Art Gallery, took over the collection of the mechanics institute and opened its doors in 1891. In the brief space of 80 years, a town had been established in the wilderness, 12 000 miles from the homeland of its people, and equipped itself with the means to educate its citizens both in and out of school to a standard acceptable in any town of similar size in Britain. This was even more meritorious when one considers the lawlessness and general poverty in Van Diemen's Land at the time (Melville, 1865, p. 40). This sort of entrepeneural effort was repeated across the developing face of Australia. It enabled the difficulties that governments and the people faced, in creating an educated society scattered over vast distances, to be overcome.

The political situation in North America during the 19th century was one of high drama and great change. The boundaries between Canada and the United States of America were defined; there was war and revolution in Europe and the century was divided by the Civil War between the northern and southern states of the U.S.A (1861-1865). Against this backdrop there was a stark contrast between the culture of the eastern states personified in Philadelphia and the raw struggle to establish an existence remotely approaching civilization on the north west coast of British Columbia. The Hudson's Bay Company established a trading post at Comasak or Fort Victoria on Vancouver Island in 1843 (Iredale, 2005) and took little time to arrange for schooling. Dr. J. McLoughlin (1784-1857) who headed operations in the north-west for the Company was a man of vision who had a history of being concerned for the welfare of workers. The first school was established in 1849 (Johnson, 1968, p. 62) as soon as the Rev. Staines and his wife arrived and by 1852 three elementary schools were available for the new settlers. Each of these schools offered reading, writing and arithmetic, although the latter was often poorly taught and sometimes neglected altogether. Within 50 years of establishment, the colony of Vancouver Island had appointed Cridge (1817-1913) as the first Superintendent of Education and the first government portfolio for education had been created (1892) following the creation of British Columbia in 1871.

In eastern Canada the two names that dominated education (Crawford, 1970, p. 373) were the Rev. Dr. Strachan (1778-1867) and Ryerson (1803-1882). Strachan was of Scottish descent. He was a churchman and teacher who put his heart and soul into these twin callings. A man of principle and ability, he created schools and wrote his own mathematics text, *A concise introduction to the use of arithmetic for the use of schools.* In 1816 his suggested curriculum for grammar schools included in year three, for children aged from 11 to 13 years, algebra as well as arithmetic and geometry, and for the following year a continuation of the algebra and an introduction to Euclid. The fifth year expanded the mathematics curriculum considerably continuing the study of algebra, Euclid and introducing trigonometry and its applications, surveying, navigation and the elements of astronomy.

Ryerson, an administrator and politician, had a vision for education. Although of limited formal education, Ryerson understood the need for education opportunity to be offered to the many children growing up deep in the backwoods who were illiterate and innumerate (Putman, 1912, p. 24). He reported to the legislature advocating education for all in 1844 and upon being appointed Assistant Superintendent for Education for Upper Canada in 1846 set about bringing this to reality. Ryerson was finally appointed Superintendent of Education in 1876, in effect becoming the Provincial Secretary. He organized every facet of education as long as his political power remained.

The development of education continued in the U.S.A. with the move away from the churches and an increasing recognition of the role of government in schooling continued. Mathematics was increasingly influenced by the advanced thinking of the French and evidenced by the use of such publications as Legendre's *Elements of Geometry* at West Point Military Academy. The Academy opened in 1802 and followed the example of l'École Polytechnique. It stood as the standard for mathematics, and outshone the universities still bound by the mathematics of Oxford and Cambridge. The concern for development and expertise in applied mathematics was further shown when the Massachusetts Institute of Technology was opened in 1876 (Jones & Coxford, 1970, p. 228). The American institutes continued to follow the French pattern rather than that of England and the academic standard was of the highest order. A high school for girls was set up in Boston in 1828 and within 30 years the state of Massachusetts had enabled legislation creating compulsory elementary education for all children. By 1855 there were 239 colleges, 6185 academies and 80 978 elementary schools spread across the face of the developing U.S.A. according to Bowen (1981, p. 275), each teaching some form of mathematics.

It is difficult to choose those that influenced mathematical education most from the illustrious list of mathematicians that graced the European scene in the nineteenth century. Certainly the Frenchman Cauchy (1789-1857) is notable not only as a mathematician of the highest caliber (Young, 1998, p. 103) but also as one who gave a good deal of his life to educating others. He started his teaching career in 1815 at L'École Polytechnique and over the next 10 years taught at the Académie des Sciences and Collège de France. He had considerable influence on the content and method of higher mathematics and his book *Cours d'Analyse*, designed for students at the Polytechnique and published in 1821, was a classic in its development of basic theorems in calculus. In the latter half of the century Poincaré (James, 2002, p. 81) thought to be one of the most brilliant mathematical minds of all time, showed to any who aspired to the highest level of mathematics what was possible using the existing framework of mathematics education.

Education across France continued to be wracked by the battles between the Catholic and Protestant church, and church and state for the hearts and minds of the children. The Falloux law of the 15th of March 1850, went some way to solve the problems. Falloux (1811-1855) was the first of the French Ministers of Education to plan his educational policies with industrial and commercial considerations to the forefront (Lynch, 1974, p. 9), with all its implications for mathematics. He opened the door for the immense changes that were to be brought about by Ferry (1832-1893) in 1880 as the Premier. It was Ferry who finally routed the influence of the church and set up a national system of education that was free, lay and compulsory for elementary schools (Power, 2005) and he is regarded as the founder of the modern French educational system.

The best of the opportunities that Prussian education offered may be seen by looking at the history of the great Gauss. He grew up in the town and duchy of Brunswick, attending elementary school and secondary school there (Bühler, 1944, p. 7). His teachers were astute enough to see his talent and with the patronage of the Duke of Brunswick, he attended the academy in

Brunswick, finally entering the University of Gŏttingen in 1795. There was an excellent library and opportunity for him to teach either there or at the newly opened University of Berlin. The latter was the brainchild of Wilhelm von Humboldt (1767-1835) who was in charge of the section for education in the Prussian Ministry of the Interior following the Prussian defeat by Napoleon in 1806. In addition von Humboldt was responsible for the concept of the gymnasium with its classical secondary curriculum of Latin, Greek, German, mathematics and sundry other subjects (Rowlinson, 1974, p. 26).

The establishment of a Ministry of Education in 1817 also saw the creation of divisions within the ministry for each type of schooling. The volksschule that predated the gymnasium was established for the general educational purposes of children beyond elementary school. The large difference between the trade oriented education of the one and the classical curriculum of the other was filled by another educational system, the realschule. The breadth of mathematics was now complete from the arithmetic required for everyday life, through the mathematical needs driven by the industrial revolution to the classical requirements of the purist. As the pace of the development of industry, had increased, so had the demand for schools and teachers. This demand provided increasing employment for mathematicians in addition to the research pursued in university employment (Schneider, 1981, p. 117).

Britain was deep in the grip of the industrial revolution and imperial expansion. Buoyed by the development of the steam engine and victory over Napoleon 1 (1769-1855) at Waterloo (1815), the British people had every right to look forward with confidence to the coming years. A new age of applied mathematics was given additional impetus with the invention of the steam locomotive by the great engineer, Stephenson (1781-1848) in 1814. Within 11 years this new invention was brought into use by industry. In an age of engineers, the greatest in Britain was I. K. Brunel (1806-1859). He was the first one to calculate accurately the stresses that had to be overcome in large structures. He successfully designed and oversaw the building of hundreds of miles of railway track, with bridges and tunnels as required. The greatest of these bridges is the Royal Albert Bridge built in 1859 although this accomplishment is historically overshadowed by his finest maritime creation, the vessel "Great Eastern" (Gardner, 1998). It is hardly suprising that given such examples of the power of applied mathematics, engineers and teachers clamoured for change in the Euclid based mathematical ideas that still held sway in grammar schools and universities.

Despite the industrial advances of Britain, the Newcastle Report (1858) (Dyson & Lovelock, 1975) showed that there was little mathematics taught in the majority of elementary schools. Grammar schools also remained aloof from the needs of industry, indeed the first mathematics teacher to be appointed to a public (endowed) school was not until 1834. The two leading universities Oxford and Cambridge were eventually forced through public dissatisfaction to acknowledge mathematics and science to be on equal footing with Latin and Greek (Bowen, 1981, p. 304). In 1860, London was the first university to admit women, although the education offered by Cheltenham Ladies College which opened in 1858, was first class and included mathematics (Howson, 1982, p. 172).

A realization of the importance of mathematics for the workforce was given weight when the Clarendon Commission, which sat between 1861 and 1864, recommended that every boy be taught mathematics. It was also at this time that the universities moved their first two years of mathematics into the secondary sector and instigated public exams. This came in the middle of a period when there had been increasing political pressure for the government to play a greater role in education. In 1870, the Forster Act compelled all children who were between 5 and 11 years of age, to attend school. This combined with the Clarendon Commission and the universities needs, had many ramifications for the constant demand for more and better trained mathematics teachers.

The growth of the number of properly trained teachers, and the increasing number of articulate engineers offered a platform for a body to be formed that would put pressure on the universities to teach mathematics. More specifically a form of geometry that was more appropriate to the day and age than the memorization of Euclid that was still the current mode of learning and examination. In 1871 the Association for the Improvement of Geometry Teaching was formed. Despite repeated lobbying over many years, the strength of the universities and particularly Cambridge proved too strong. Some change had occurred however and 1880 saw the first calculus questions in the entry exams for Oxford and Cambridge, implying a study of the subject in the senior years of secondary education.

Astonishingly, in the early 20th century, there was still a broad school of thought that the female brain was not suited to the academic rigors despite such illustrious mathematicians as Mary Somerville (1780-1872), Ada Byron (Countess of Lovelace) (1815-1852) and Sofia Kovaleskya (1850-1891). A further example was provided in 1890 when, in the rigorous atmosphere of the examinations upon which the best mathematicians at Cambridge are decided, Phillipa Fawcett (d. 1938) was ranked ahead of them all (Griffiths &

Howson, 1974, p. 15). Despite her obvious mathematical superiority, no formal recognition was granted. But outside the cloistered world of the universities it was quite a different matter. The age had arrived when man or woman, rich or poor, it was ability that counted rather than birth.

Summary

During the 2500 years that have elapsed since the schools of Ancient Greece, students have been asked to learn mathematics at increasing levels of sophistication, with the result that at least half of the secondary mathematics curriculum at the turn of the 20th century had been thought the province of the university 100 years beforehand.

By the year 1900, Britain, Western Europe, North America and Australasia regarded education as an essential part of society, making it compulsory for all children to attend elementary school. In all these countries arithmetic was included as part of the curriculum. In addition, secondary and tertiary forms of education had been established with accompanying courses in mathematics.

The state of mathematical education at the beginning of the 20th century seems an appropriate moment to summarize the history of development to date and pause before embarking upon the philosophic discussions that drove changes to the curriculum from this time on and the raising of school leaving age beyond 15 years accompanied by compulsory secondary education.

CHAPTER 4

IDEAS IN MATHEMATICS EDUCATION

Introduction

This chapter adds depth to, and extends, the description of mathematics education given in the previous chapter by looking at the ideas that have shaped mathematics curricula. The earliest writings that specifically relate to education are from Greek and Roman times and the philosophers of those ages all included mathematics in their ideas concerning what should be taught. The mathematician who has had most influence on secondary mathematics has been Euclid, and for this reason some discussion of his writing is included.

Following the fall of the Roman Empire and the ensuing Dark Ages, education awoke with the Renaissance and developed in fits and starts until the implementation of compulsory education with the accompaniment of state funded schools. Schooling that had been at the mercy of wealthy patronage now became a constant resource found everywhere. This latter development has had serious consequences for mathematics as the curriculum designed for gentlemen in the 19th century was inappropriate for all now enrolled in secondary education.

A major factor concerning teaching practice and curriculum design during the last 50 years has been the advances in technologies employed in the mathematics classroom. With advances in electronic aids, the manipulation of numbers and equations have been made far easier, with ensuing consequences for learning. The chapter closes with an overview of the trends in research in the 21st century that reveal commonalities across many countries with regard to teacher supply, student drop-out rates and other socio-cultural issues.

4.1 Greeks and Romans

There is ample evidence of scribes passing on their skills in mathematics so that younger people could take their place recording and calculating. This was found in the clay tablets of Babylonia (Leick, 1951, p. 92) and the papyri of Egypt (Gillings, 1972, p. 91) a thousand years before Greek philosophers espoused their educational thoughts. Yet Kline (1972. p. 11) suggested that there was only a crude form of mathematics before the Greeks, just utilitarian forms of arithmetic and geometry. Kline preferred to think of mathematics in a form where each statement was reasoned and proved to hold ground against similar statements. His attitude to the role of the Greeks of Classical and Hellenic times may be summed up in his quote (Kline, 1972, p. 24) "…that in the history of mathematics, the Greeks are the supreme event." The early classical writings that most encapsulated these ideals were those of Euclid. The proof contained in proposition 47 of *Elements 1* (Euclid, trans. 1952, p. 28), "In right angled triangles the square on the side subtending the right angle is equal to the squares in the sides containing the right angle," was still used in schools in Australia in the 1970s (Clapp, Hamann, & Lang, 1970, p. 52), albeit in different forms and ascribed to Pythagoras. By way of example of Euclid's work, the complete proof chosen here, proposition 15 from *Elements 1* (Euclid, trans. 1952, p. 10), is seldom studied in schools today, but the geometric knowledge is in constant use.

Proposition 15

If two straight lines cut one another, they make the vertical angles equal to one another.

For let the straight lines *AB, CD* cut one another at the point *E*;

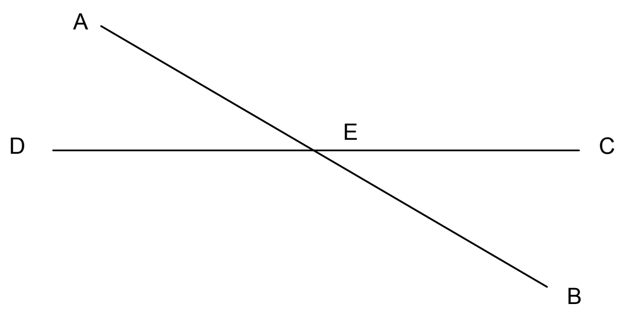
I say that the angle *AEC* is equal to the angle *DEB*,

and the angle *CEB* to the angle *AED*.

For since the straight line *AE* stands on the straight line *CD*, making the angles *CEA*, *AED*,

The angles *CEA*, *AED* are equal to two right angles.

[1, 13]



Again, since the straight line *DE* stands on the straight line *AB*, making the angles *AED*, *DEB*,

the angles *DEB*, *AED* are also equal to two right angles.

[1, 13]

But the angles *CEA*, *AED* were also proved equal to two right angles;

therefore the angles *CEA*, *AED* are equal to the angles *AED*, *DEB*.

[post. 4 and C.N. 1]

Similarly it can be proved that the angles *CEB*, *DEA* are also equal.

Therefore etc.

Q.E.D.

[PORISM. From this it is manifest that, if two straight lines cut one another, they will make the angles at the point of section equal to four right angles.]

Greek mathematicians and educational philosophers eschewed utilitarian mathematics (Russell, 1979, p. 221) yet Euclid wrote on optics, Eratosthenes (276-190 B.C.) was the geographer of the age whilst Hipparchus, Apollonius and Menelaus (A.D. 70-140) all applied their mathematics to the study of astronomy.

The three Greek names that stand out as forming the basis for many of our educational ideas are Plato, Isocrates and Aristotle. The finely detailed writings, particularly of Plato and Aristotle enable one to form one's own view on their philosophies rather than depending on the interpretation of others or the myths, for example, which surround Pythagoras and his followers where no written words of theirs survive (Rouse Ball, 1960, p. 20). The lives of these great men were not lived in academic isolation and it is probable that Plato and Isocrates influenced each other in theory and practice (Marrou, 1956, p. 134).

Plato put into words many of his philosophies, which were expressed as those of Socrates in dialogues contained in *The Republic* (Plato, trans.1966). In book 3 (Plato, trans. 1966, p. 58), Plato stated his ideas using Socrates speaking to Aideimantus thus: "The aim of education is to produce people who live in harmony and rhythm with each other and nature. To appreciate the good and the beautiful." Plato took this further in book 4 of *The Republic* (Plato, trans.1966, p. 71) where Socrates suggested that "Good education makes good citizens, helped by good education become better than they were." Taking the discussion a step further in book 7 of *The Republic* (Plato, trans.1966, p. 131), Plato expressed through Socrates, the thought that number and arithmetic be compulsory for those who aspire to take part in the higher work of the state. Having made his statement concerning compulsory curricula, Plato stated a few pages further on (Plato, trans.1966, p. 138) “Nothing learned through force from without takes root rightly in the mind.”

Plato went into much finer detail in his description of what was desirable in the education of the good citizen in *The Laws* (Plato, trans.1970). The style of expression followed that of a dialogue between the Athenian and Clenias. Plato, through the Athenian, opened up the discussion (Plato, trans.1970, p. 73) with “What we have in mind is education from childhood in virtue, a training that produces a keen desire to become a perfect citizen.” Not content with this lofty aim he spoke specifically of mathematics (Plato, trans.1970, p. 218). “For domestic and public purposes and all professional skills, no single branch of a child’s education has such an enormous range of applications as mathematics, but its greatest advantage is that it wakes up the sleepy ignoramus and make him quick to understand.” Having extolled the virtues of mathematics, he went on to state a word of caution (p. 311): “None of these subjects (arithmetic, geometry and astronomy) must be studied in minute detail by the general public, but only by the chosen few.” He further pointed out that one cannot dispense with the basic necessities (of mathematics) for the man in the street. It should be noted that Plato championed the cause of education for both girls and boys on equal footing.

It is the latter comments of Plato’s that are often overlooked. He saw clearly that a detailed study of mathematics was not for everyone, but a basic form of mathematics was. In our present world of rapidly expanding knowledge, there is immense pressure to cram as much knowledge as one possibly can into the child, forgetting perhaps that many do not have the capacity to absorb what one might like to teach. Plato also discussed the different types of knowledge in the dialogue between Protarchus and Socrates. In *Philebus* (trans.1952d, pp. 633-634), Socrates pointed out that there were two kinds of knowledge, productive and educational, and further, that there were two kinds of arithmetic, popular and philosophical. By popular, Socrates gave an example of that used by the carpenter in ship or house building. In the language of today, the arithmetic of the carpenter belongs to applied mathematics and the arithmetic of the philosopher forms pure mathematics. He then went on to suggest that where the art of arithmetic is driven by philosophical thought, it is "infinitely superior in accuracy and truth." The effect of this statement upon mathematics education has been profound and the influence of Plato is still found in the schools of today.

Whereas Plato was the philosopher of the day, Isocrates was the outstanding teacher and educator. He took what he thought was the best of Plato and applied his many years of professional teaching experience to his wisdom as evidenced in the few writings that remain of his. In his speech *Against the Sophists* (1969, p. 212), he stated bluntly, “If all who are engaged in the profession of education were willing to state the facts instead of making greater promises than they can possibly fulfil, they would not be in such bad repute with the lay public.” This point is as valid today as it was when Isocrates was criticizing the professional teachers of the Sophists. Today there is much talk about excellence, and once again the words from long ago state simply what is known but sometimes forgotten. Isocrates was 82 when he wrote *The Antidosis* and amongst many other gems wrote (Isocrates, trans. 1969, p. 227), “To stand head and shoulders above one’s fellow man, natural ability and hard work need to be brought hand in hand.”

One cannot leave Isocrates without bringing to mind two of the phrases from his *Speech to Demonicus* (Isocrates, trans.1980) where he stated, "If you love knowledge, you will be the master of knowledge." Further in keeping with his recognition of the effort required, he exhorted Demonicus, "Do not hesitate to travel the long road to those who profess to offer some useful instruction;…"

The lives of Aristotle, Plato and Isocrates overlapped, with Aristotle taking over the mantle of the ageing Plato. Aristotle had the great gift of being able to bring together the best of the writings of Plato and Isocrates, applying his insight, and expressing them with a lucidity and detail that has made him revered through the ages. Russell (1979, p. 242) called him the last optimistic Greek philosopher. In *The Politics*, Aristotle explained his ideas of education for citizen and state. He stated clearly that the state depended upon education to bring about a common unity (Aristotle, trans. 1983, p. 116). As such it was a public concern and should be the same for everyone (p. 452). Today one may argue that if the state wishes to be numerate, then it is the business of the state to provide education such that all citizens are numerate.

Like Plato and Isocrates, Aristotle emphasised the development of the good man (Aristotle, trans. 1983, p. 433) and the details of each term used in defining the virtues are found in another of Aristotle's publications, *The Nicomachean Ethics*. Amongst his arguments, Aristotle divided the soul into the rational and irrational (Aristotle, trans. 1980, p. 138) and further suggested that the rational soul stemmed from variable or invariable sources, where the variable was calculative and the invariable scientific. The reason behind this was that to deliberate or calculate is the same thing, and one does not deliberate an invariable. He brought to mind that the proper object of calculation, as contemplation, was to seek truth, and the search should correspond with the right desire. A nice point was raised concerning development of the young mind. It was expected that the young may become wise in arithmetic and geometry, but that the practical wisdom required to make good everyday decisions only came with experience (Aristotle, trans. 1980, p. 148). Plato had made a similar point in *The Republic,* although it is often forgotten today, that one cannot put an old head on young shoulders.

Dewey (1968, p. 229) pointed out that the values that were common in society in the days of Plato and Aristotle generated attitudes that seem strange today. The idea that pure knowledge was of greater worth than applied knowledge did not seem so strange in a society where all manual and clerical work was done by slaves. It was natural for Plato to look down on the learning of arithmetic and geometry for practical purposes, as there were few applications to be found in that society. Dewey (1968, p. 260) stated, "Of the segregation of educational values, that between culture and utility is probably the most fundamental."

The Romans were not a philosophic people (Castle, 1965, p. 108), but understood the mechanisms required for an ordered society where the family unit held the central place. To this end education was centred upon the idea of self restraint and filial submission rather than intellectual pursuits (Gwynn, 1926, p. 17). The search for truth so prized amongst the Greeks was set aside in preference to the acquisition of knowledge and skills necessary to build the ordered way of life so desired by the Romans (Lawrence, 1970, p. 26).

Greek culture had an increasing influence upon the traditional education of Roman youth. By the 2nd century B.C., evidence of Hellenic culture was everywhere. Marrou (1956, p. 391) suggested that the greatness in Roman education lay in carrying Hellenic culture to the frontiers of the Roman Empire. Even if one were to look simply at the influence through space and time of the books of Boethius, the debt that is owed to both Romans and Greeks, particularly in mathematics education is clear.

The two greatest figures in Roman education were Cicero (d. 59 B.C.) and Quintilian (c. A.D. 30 – 95) (Castle, 1965, p. 133). Cicero used his power as a statesman to influence Roman education, setting out his general ideas of education in *Artes liberales* and a theory for higher education including a utilitarian use of mathematics in *Politio humanitas* (Gwynn, 1926, p. 85). These curriculae included the subjects contained within the trivium and quadrivium. In his concern for the role of oratory in teaching, Cicero pointed out that if you wish to explain the intricacies of mathematics, physics or other subjects, then you must learn to orate clearly, especially with regard to the subject (Cicero, trans. 1904, p. 25), an issue that is relevant today. Hooker (1999, p. 5) regarded Cicero as the most important author overall regarding the ethos of humanitas. Within this ethos, the prime concern in raising the young was to develop a sense of human dignity and human sympathy, to behave with courtesy and kindliness (Gwynn, 1926, p. 85). Mathematics was a necessary part of the structure of the ordered society of Cicero, but took second place to the ethos of humanitas.

Quintilian took the part of Isocrates as the teacher of his age. Illich (1950, p. 51) suggested that Quintilian's gift lay in applying Greek education to the Roman Empire, but he offered more than a single gift to the people of his time and those following. In his publication *Institutio Oratia* (trans. 1967, p. 101) Quintilian argued that if each child were taught according to his or her own ability, then "those of feeble intellect would achieve greater success."

Quintilian was primarily a teacher of oratory and stated in the *Institutio Oratoria* the need to learn geometry to improve the way an argument might be structured (p. 99). Like all good teachers, any tool that might be brought to bear to assist the achievement of his overall goal was investigated and included if thought desirable. In a final analysis, perhaps the tool that best served western mathematical education for the millennium following the fall of the Roman Empire was the arithmetic text of Boethius.

4.2 The Dark Ages and beyond

A thousand years followed the demise of the Roman Empire when education and the pursuit of knowledge in the west fell into disrepair. The church kept the text of Boethius in print for the clergy, appreciating the utilitarian use of his arithmetic in managing a diocese, and to a small extent passed this arithmetic knowledge onto the children of the day. There was little thought given as to what was suitable for the child to learn or the loftier aspirations espoused in the ethos of the humanitas of Cicero.

There were great minds at work however, grappling with the problems of the how, what and why of education, and struggling to convince others of the benefits of these concerns. The Spaniard Vives (1492-1540), wrote widely on education, encapsulating the wisdom of Greek and Roman times and his own teacher Erasmus, and drawing on his own experiences in Europe and England. He understood that not everyone had a talent for mathematics and thought students should study at their own pace and to their own level. He further pointed out that no part of life is devoid of numbers (Vives, trans.1971, pp. 202-203). The man who described education most completely was the Moravian (The Czech Republic) Comenius (1592-1670). In his best known work *The Great Didactic*, there was no branch of education he did not consider, from advising teachers to be "pleasant, thorough, of gentle morals and deep piety" (Comenius, trans.1967, p. 5) to agreeing with Vives in the unique ability of each child (p. 116). He looked at the curricula advising contentment with small progress, not to teach those things that are useless and suggesting that only things that can be easily demonstrated should be taught (pp. 131-144). The regard with which Comenius was held cannot be over-emphasised. In 1641, Hartlib presented such an effective case to the English Houses of Parliament that an invitation was issued for Comenius to come to England to advise on schooling and education (Bowen, 1981, p. 91). More than 300 years later, Lawrence (1970, p. 96) suggested that *The Great Didactic* was one of the great works on education, and lamented that the ideas that had been put forward were not more broadly applied.

The philosopher whose writings had possibly the broadest influence on education was Rousseau (1712-1778). His little series of five books describing the education of Émile was revered from the west coast of America to the eastern borders of Europe. The beautiful expression of Rousseau allowed the reader to gloss over any naïve shortcomings and share the profound idea of man as the noble savage, essentially good and to be treated with respect. Affected greatly by *Émile* (Rousseau, trans. 1956), Pestalozzi (1746-1827) published widely himself. He was a professional educator who dedicated his life to schooling. In his discourse *The Method* (1800), he asked teachers "What would you do, if you wished to give a single child all the knowledge and practical skill he needs, so that by wise care of his best opportunities he might reach inner content?" (Pestalozzi, trans. 1973a, p. 199). He strove to simplify knowledge and in his book *How Gertude teaches her children* (1801), drew attention to the importance of arithmetic for both sharpening the mind and for its practical uses (Pestalozzi, trans.1973b, p. 132). Mathematics was becoming available for the general public, in a language they could understand, and in a form that was conducive to application to everyday needs. This was considered important in North America and shown in the address Franklin gave to the trustees of the Academy of Philadelphia in 1751 (Franklin, 1973). He used this occasion to stress the importance of teaching mathematics in English rather than the traditional Latin, and in a practical manner that was suitable for the American society of the day.

Although the quadrivium still held sway as the mathematics curriculum in secondary education in England until the mid 19th century, a new mathematical thinking was abroad and had already been brought into use in Europe and the United States of America in the shape of the calculus of Newton and Leibniz and analytic geometry of Descartes.

The deductive geometry of Euclid, still very much part of the English mathematics curriculum in the 1950s (Faulkner, 1959), had one Postulate or Axiom, that raised questions among mathematicians. Postulate 5 appeared too detailed compared with the others and Euclid seemed reluctant to use it in the 48 Propositions (theorems) of his first book of *Elements* (Euclid, trans. 1952). In fact the first 28 Propositions were proved without the use of Postulate 5, as was Proposition 31. The questions that arose upon investigation of Postulate 5 led to the development of new forms of geometry. Postulate 5 states, "That if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines if produced indefinitely meet on that side on which are the angles less than the two right angles" (Euclid, trans. 1952, p. 2). The four mathematicians who shared the credit for non-Euclidean geometry were Saccheri in 1773, Lobashevsky in 1830, Bolyai in 1831 and Gauss. Gauss did not publish specifically but shared his thoughts albeit indirectly, with Lobachevsky and Bolyai (Trudeau, 1987, pp. 157-159). The definitive name associated with non-Euclidean geometry is that of Riemann who published his famous paper in 1854 (Dubbey, 1970, p. 85) describing what has become known as Riemannian space. The effects on mathematical thinking were profound. The Euclidean geometry that had been the cornerstone of deductive mathematics for so long, was not the only form that could be studied and some of the original propositions could not apply to the new geometries. What was so important about this was that mathematical "truths" could now be questioned.

The problems of curriculum designers became rapidly more acute in the latter years of the 19th and early years of the 20th century. An increasing number of major mathematical inventions came to light including group theory, the algebra of complex numbers and quaternions of Hamilton (1805-1865), algebraic logic of Boole (1815-1864) and the matrices of Cayley (1821-1895) (Ashurst, 1982, pp. 24-51). Finally, Maxwell (1831-1879) brought us vector analysis (Dubbey, 1970, pp. 91-112) and Cantor (1845-1918) the language of set theory. These five mathematicians have been singled out as reflecting the various topics one may find in the modern grade 12 (student age, 17-18 years) mathematics curriculum adding set theory, complex numbers, vector analysis and matrix algebra, to conics, trigonometry, calculus and probability. The logic of Boole is found in all areas where computing may be found.

It seemed natural for a university such as Cambridge to move what it saw as older, more familiar and thus easier topics such as elementary calculus and trigonometry into the secondary sector, so as to make room to study these new and fascinating topics. This process first officially noted in 1880, is still in operation today, the secondary curriculum content being driven by the demands of the universities. Mathematicians in universities wanted to learn and teach new and exciting things that held a challenge. This is the sort of attitude that is expected regarding research, but as far as teaching is concerned the rationale seemed to have become not one of humanitas, but of pleasing one's self. By 1900, students and teachers in secondary schools in England were involved with the sort of mathematics that enabled the best students a chance pass their entrance exams and study at university. A century later this is still largely the case, but changes have been made.

The dominant questions of secondary school mathematics teachers at the turn of the 20th century were, (a) why should one be only allowed one form of proof to a Euclidean proposition, and more importantly, (b) why should Euclidean geometry still dominate mathematics exams for universities when there was so much more to learn that was in desperate demand by engineers and other professional applied mathematicians (Perry, 1900, pp. 319-320). There were still echoes of arguments for learning mathematics to improve the mind rather than for more utilitarian purposes, but these were being forced backwards by the insatiable demand of society for more people with the appropriate mathematical skills necessary to develop the technology and management strategies of the day and the foreseeable future. Academic mathematicians such as Hardy (1877-1947) made it quite clear that for people such as himself there was only pure mathematics, for pure mathematics was real and "real mathematics has permanent aesthetic value" (Hardy, 1967, p. 131). People such as Hardy wielded great influence over the thinking of university mathematicians of his day and were not going to abdicate from the fight to preserve pure mathematics from the inroads of the applied mathematics required by engineers such as Perry (1850-1920). Schools to this day are dominated by the ethos of pure mathematics, even though mastery at even a basic level is for the gifted few.

The American philosopher and educationalist Dewey brought together many of the ideas of earlier educationalists and expressed them in terms more appropriate to the times. He was held in the highest esteem as far abroad as France and Australia, and his writings used in teacher training for the best part of the 20th century. In his *Pedagogic Creed,* which he wrote in 1897, he supported the idea and stressed the importance of the child as an individual (Dewey, 1959a, p. 22), and reiterated the caution not to be too ambitious in teaching complex topics (p. 25). Expanding his thoughts two years later in *The School and Society*, he damned current educational practices with such remarks as "Our present education ... is one dominated almost entirely by the mediaevel conception of learning in something which appeals for the most part simply to the intellectual aspect of our nature." (Dewey, 1959b, p. 47). He pointed out that it is useless to bemoan the good old days, and that there should be a natural connection of the everyday life of the child with the business environment about him (p. 78). With particular regard to arithmetic, Dewey illustrated cases where the problems that were offered to the student no longer existed in society (Dewey, 1959b, p. 79).

He was scathing in his comments such as "They were kept after they had ceased to have practical utility, for the sake of mental discipline, 'they were such hard problems, you know.'" Dewey was very much concerned with the practicalities of education but revealed in *The Child and the Curriculum* beautifully expressed thoughts that transcend the ordinary such as "To the growth of the child, all studies are subservient;..." (Dewey, 1959c, p. 95), and finally, "Let the child's nature fulfill its own destiny, revealed to you in whatever of science and art and industry the world now holds as its own." (p. 111). It is upon thoughts such as these that the structures of mathematical education in the 20th century were shaped.

4.3 The twentieth century

The mathematics of senior school students in Europe (and England) during the 19th century was remote from everyday life (Restivo, 1992, p. 141). This was understandable from the viewpoint of preparing gentlemen to take their place in the leisured and ruling classes and such an approach to education has been described by Ernest (1991, p. 168.), as Old Humanist. The growing dissatisfaction with the sort of mathematics, imposed on schools by the entry requirements of Oxford and Cambridge Universities, was led by members of the Association for the Improvement of Geometry Teaching (AIGT). Although they were not able to bring the universities to accommodate a new approach, they did create the climate that enabled Perry to exert sufficient pressure to force change early in the 20th century. Perry had shown the sort of mathematics required by the new technological age in sore need of skilled engineers and scientists, in the course he devised at Finsbury College (Howson, 1982, p. 147). Perry went to great lengths to ensure that what he considered an effective mathematics syllabus for the average student at the turn of the century was known by others and published in *Nature* (Perry, 1900, pp. 319-320), details including the following points.

Arithmetic: Decimals, scientific notation, logs, the use of formulae, slide-rule, square roots, ratio and variation.

Algebra: Formulae, simple transformation and manipulation, quadratic factors.

Mensuration: Area and volume of regular figures as well as density problems.

Squared paper: Statistical display, time, tide, temperature information, plotting a point, functions ($y = ax^n$, $y = ae^n$), solutions and roots of an equation, speed, distance and time.

Geometry: Accurate drawing including the use of a protractor included with all facets from properties of triangles and circles to truths concerning propositions.

Trigonometry: Sine, cosine and tangent ratios with respect to right triangles, their ratios and areas.

Perry included a knowledge of scalars and vectors as well as differential calculus regarding slopes and curves. This syllabus did not extend to his advanced mathematics course.

Perry had sufficient status to question publicly the educational value of Euclid in 1901 (Fujita, 2001a) and be listened to with respect by the powers that existed in the leading universities of the day. With Forsyth (1858-1942), the

Sadlerian Professor of Mathematics at Cambridge chairing the reform committee for entry requirements for Oxford and Cambridge, and Perry's clear idea of the needs of 20th century society, the way was open to request Godfrey (1873-1924) and Siddons (1876-1959) to write a text book for schools that could replace Euclid's *Elements*. *Elementary Geometry* (1903) was the first of a series of texts written by Godfrey and Siddons that sold over a 1 000 000 copies, providing the backbone for mathematical education in England for 50 years. This embodied the ethos that Godfrey and Siddons shared with Perry that mathematics should be taught as a tool, rather than some esoteric exercise to improve the mind of a gentleman. It should not be taken for granted that this implied a text lacking formality. The book was beautifully written containing more problems requiring proofs in any one chapter than might be found in the whole of a grade 12 course today. Another great name of the time was Nunn. His treatise *The Teaching of Algebra* (1914) was a fine early example of the concerns that were taken to educate teachers into the mathematics of the day. Australia closely followed the educational practices of the mother country England until the 1950's (Clements, 1989, pp. 1-24), and this included using the texts of Godfrey and Siddons who were regarded as two of the most important people involved in early 20th century reform in English mathematical education (Fujita, 2001b).

The acceptance of the mathematical requirements of the 20th century allowed the inclusion of new topics in the curricula being designed for the new age such as matrices, vectors, complex numbers, set theory, analytic geometry and calculus. The expansion of the secondary mathematics curriculum coincided with a rapid increase in numbers of students continuing their education beyond elementary school (which usually ended around 13 years of age). There were several factors at work here including the government accepting responsibility for state education in the *Educational Act* of 1902 and the need for an ever increasing number of people in the workforce with higher mathematical skills. With an initial and dramatic reform achieved, and with the deaths of Perry and Godfrey in the 1920's, a period of consolidation ensued, lasting for more than a quarter of a century. Godfrey had expressed his opinion that the human brain does not alter, but as complex ideas are better understood and expressed in simple terms, they come within the ambit of understanding of the common man. This was published after Godfrey's death by his writing partner Siddons under their joint names (Godfrey & Siddons, 1937, pp. 41-42). Both Hadow (1926) and the Great Britain Board of Education Consultative Committee on Secondary Education (GBBECCSE) (1939) had expressed concern that the curriculum really only suited the few who went on to university, for scientists and engineers; and indeed it was, for the thinking of Perry still prevailed, and the books of Godfrey and Siddons were still used. The GBBECCSE (1939, p. 161) stated that mathematics should be a practical affair and expressed the radical notion that mathematics for the majority of pupils could be taught in less time than was the current practice (1939, p. 238). At the same time, Siddons had bemoaned the restrictions that examinations placed on the teacher (Siddons, 1936), echoing the thoughts of his illustrious teacher Rawdon Levett (d. 1923), and no doubt those of many teachers today.

The period of the Second World War (1939-1945) brought its own revelations concerning education and what could be done when the need was strong enough. An explosion in numbers of students in secondary education followed the *British Education Act* of 1944, which raised the school leaving age to

15 years. People involved in mathematics education were now clearly aware that the mathematical requirements for university entrance that had driven the secondary curriculum were not suitable for the majority of secondary students and new curricula were devised. The tripartite secondary system in England and Australia (another facet of the *Education Act* of 1944) allowed the grammar schools to cater for the academic elite and technical colleges to develop mathematical courses suited to trades and engineering. The largest sector however, was in the secondary modern schools and it was here that the most original thinking probably took place. There were no experts and no history to help with the design of mathematics courses for these students who were not yet able to decide what sort of education was suitable for their ensuing adult lives. The widest variety of practically based courses were implemented (Cheshire Education Committee, 1958) and with the replacement of the tripartite system with comprehensive secondary schools, it was these courses that spelled the most significant shift in thinking in the courses that were built in the second half of the 20^{th} century. It became increasingly clear that everyone used mathematics as a tool, not just engineers and scientists, and as was pointed out by Restivo (1992, p. 109), "…the mathematics of the crowd is different from the mathematics of the lovers of wisdom."

Three men who greatly influenced post 1950 educational reform in the western world were Piaget (1896-1980), Skinner (1904-1990) and Bruner (1915-). Piaget believed that the growth of knowledge was a progressive construction of logically embedded structures superseding one another by a process of inclusion of lower less powerful logical means into higher more powerful ones up to adulthood (Smith, 2000). Piaget thought that each person had a unique mathematical ability; some were brilliant but others would experience extreme difficulty with the simplest sum (Piaget, trans.1970, p. 44). He also cautioned that children did not think like adults (p. 101). Skinner is possibly the best known of the three outside academic circles. His famous experiments with rats reinforced the hypothesis that a behaviour followed by a reinforcing stimulus resulted in an increased probability of that behaviour occurring in the future (Boeree, 1998). An American, like Skinner, Bruner "...has had a profound influence on education – and upon those researchers and students he has worked with" according to Smith (2002). His book *The Process of Education* (Bruner, 1966), helped shape policies in American schooling and subsequently across the western world. Bruner developed a theory that looked to environmental and experiential factors in cognitive growth, and suggested that intellectual ability developed in stages. He cautioned that one can only process so much information at any one time (Bruner, 1971, p. 126). Bruner differed from Piaget in thinking that, "any subject can be taught effectively in some intellectual form to any child at any stage of development." (p. 33). This structural approach was adopted by the University of Illinois Committee on School Mathematics (UICSM) when it set in motion a workable model for what became known as "New Math" (Howson, Keitel, & Kilpatrick, 1981, p. 134).

"New Math" came into being through the dissatisfaction of American universities with the type of mathematics that was being taught in the secondary sector in the 1950s. The approach then was one of utility and application to service the needs of a rapidly growing technological society. In the terms of Ernest (1991, p. 152), the mathematics was following the philosophy of the Technological Pragmatist. Academic mathematicians felt that the mathematics taught in the secondary sector was piecemeal and lacked explanation. A possible answer was to take up the thinking of the Bourbaki group in France. Nicolas Bourbaki is a pseudonym used by a group of mathematicians throughout the 20^{th} century and to the present day (Boyer, 1968, p. 674). The main thrust of their ideas was to create a structure within which all mathematics could be explained with rigour using the same language. The Bourbaki thinking served to widen the gap between university and high school mathematics ideals but New Math was nevertheless implemented. Although the original concept had been to develop a new mathematics for the gifted student, the project was applied across the board in the 1960s and 1970s, under the guise of the School Mathematics Project (SMP) in England and then Africa (Thwaites, 1972). To the teacher this meant learning virtually a new language from sets to functions, and explaining the mysteries of the commutative and associative laws to those in junior high school. New Math proved much too formal for the average student and was found to be a handy scapegoat for industrialists who blamed it for the lack of mathematical ability in their young employees, suggesting that the project was too elitist (Howson, Keitel, & Kilpatrick, 1981, p. 139). Much of the structure has remained in senior mathematics courses as may be evidenced in the texts, e.g., *A Course in 12MT Mathematics* (Walsh, 1997).

The pattern of education that one sees today in the western world, with both primary and secondary schooling available even in small towns, has only developed since the 1960s. This has brought its own problems in the wider variety of students enrolled in mathematics and the ever-present shortage of teachers trained to teach mathematics at the secondary level, and particularly at the senior secondary level (Howson, 1982, p. 199).

It was also in the 1960's that the debate regarding curriculum design in mathematics education took on an international flavour. The problems of one country were often found to be shared with another, and in addition, the needs of emerging nations were increasingly being felt. The International Association for the Evaluation of Educational Achievement (IEA, founded 1959) set in motion the *First International Mathematics Study* (FIMS) during the period 1963-1967. This was followed by the second study (SIMS) which was completed in 1981, and a third study, this time including science (TIMSS). The third study commenced in 1997 and was still under way in 2005 (National Centre for Educational Studies (NCES), 2005). The second study is of particular interest as it deals with the analysis of mathematics curricula during the same period as the landmark report of Cockroft, *Mathematics Counts* (1982), to the British government. The Cockroft report was the first in depth look at the mathematics that was in everyday use, from the home through shopping and domestic finance, to industry and commerce. The report took these findings and made recommendations to be borne in mind in any future mathematical curriculum review. In essence the report found that everyday mathematics was simple, around grade 4 level, but the calculations had to correct all the time. In addition people performed their daily calculations with sometimes bewildering speed. These findings were substantiated on the opposite side of the world in Australia a quarter of a century later (Northcote & McIntosh, 1999). A syllabus was put forward as appropriate for most students attending secondary school, a précis of which follows (Cockroft, 1982, pp. 136-138).

♦ Arithmetic: Perform small calculations without a calculator such as five times four pounds, ten shillings, and a quarter of eight pounds, sixteen shillings.

♦ Perform mental arithmetic calculations up to ten times ten and simple division like nine divided by three.

♦ Understand and calculate percentage with relation to money problems.

♦ Use a calculator to calculate wages, overtime and costing.

♦ Recognize the relationship between simple fractions and decimal equivalents and change from one to another using a calculator.

♦ Spatial concepts: Familiarity with common terms of length, weight and capacity. Recognition common shapes such as circle, rectangle and triangle. The use and description of the general terms, side, diagonal, angle, radius, etc. In addition calculate and roughly estimate the capacity of a cylinder or rectangular tank. It was thought important that accurate drawings should be able to be made of all the above shapes. Maps, bearings, position and scale were also included, as were ratio and proportion with for example, gears.

♦ Other skills thought useful included familiarity with statistics in graph form, terms such as average and a knowledge of simple probability. There was also a requirement to understand graphs that convey information such as temperature, usage or costs.

In contrast, the 1981 SIMS report (Travers & Westbury, 1989) dealt with the topic content of mathematics curricula experienced by two populations spread over 20 countries. Population A was drawn from the first year of secondary education or thirteen year olds and population B the final year where students went on to the sixth year of mathematics, age eighteen. Of the countries participating in the population A survey, four declined to report on the population

B survey. Table 4.1 is compiled by the author with data extracted from Tables 2.4.1 and Figures 5.4.1, 5.4.2, 5.4.4, 5.4.6 and 5.4.8 (Travers & Westbury, 1989, pp. 29-30, 123-134) analysing population A. The parameters for E (Essential), V (Very important), I (Important), N (Not important), and M (Most distribution) with L (Least distribution) are explained in greater detail at the end of Table 4.1.

Table 4.1
Population A: The understanding and distribution of content topics (part 1)

Content topics	Understanding	Distribution
Arithmetic		
Natural numbers and whole numbers	E	M
Common fractions	V	M
Decimal fractions	E	M
Ratio, proportion, percentage	V	M
Number theory	N	M
Powers and exponents	N	M
Other numeration systems	N	-
Square roots	N	L
Dimensional analysis	N	M
Measurement		
Standard units of measure	E	M
Estimation	I	M
Approximation	I	M
Determination of measure: areas, volumes, etc	N	M
Algebra		
Integers	V	M
Rationals	I	M
Integer exponents	N	L
Formulas and algebraic expressions	N	M
Polynomials and rational expressions	I	M

Table 4.1
Population A: The understanding and distribution of content topics continued (part 2)

Content topics	Understanding	Distribution
Equations and inequations (linear only)	I	M
Relations and functions	I	L
Systems of linear equations	N	-
Finite systems	N	-
Finite sets	I	L
Flowcharts and programming	N	-
Real numbers	N	-
Geometry		
Classification of plane figures	I	M
Properties of plane figures	I	L
Congruence of plane figures	I	L
Similarity of plane figures	I	L
Geometric constructions	I	L
Pythagorian triangles	I	L
Coordinates	I	L
Simple deduction	I	L
Informal transformations in geometry	I	L
Relationships between lines and planes in space	N	-
Solids (symmetry proportions)	I	-
Spatial visulization and representation	N	L
Orientation (spatial)	N	-
Decomposition of figures	N	-
Transformational geometry	I	-
Statistics		
Data collection	I	L
Organization of data	I	L
Representation of data	I	L
Interpretation of data (mean, median, mode)	I	L

Table 4.1
Population A: The understanding and distribution of content topics continued (part 3)

Content topics	Understanding	Distribution
Combinatoric	N	-
Outcomes, sample space and events	N	-
Counting of sets P(A B), P(A B), independent events	N	-
Mutually exclusive events	N	-
complementary events	N	-

Notes. Rating scale: Understanding was extrapolated from Table 2.4.1, where
E = essential and included topics that were designated V (very important) for computation, comprehension and application,
V = very important was designated to those topics which included one V and two I (important ratings),
I = important represented the topics where all three topics were rated I and
N = not important was given to those topics where less than three I ratings were noted.
Distribution was extrapolated from tables 5.4.1, 5.4.2, 5.4.4, 5.4.6 and 5.4.8 where
M = most distribution and included topics rated with a coverage of greater than 80% in more than 80% of the countries surveyed,
A = average distribution including topics with a coverage greater than 60% in 50% to 80% of the countries
L = least distribution including topics with a coverage of less than 61% in less than 50% of the countries and
- = topics not included in tables in chapter 5 of Travers and Westbury.

A brief glance enables one to observe that of the 49 topics thought worthy of inclusion, only 6 fell into the essential (E) or very important categories that were implemented by an average number of countries or better (A or M). Topics covered in international mathematics education were found to be taught over a range of hours per year from 67 to 202 (Travers & Westbury, 1989, p. 68), the number of hours not necessarily reflecting the range of topics implemented. The curriculum designer was left with a bewildering variety of options even to design the simplest syllabus, following this report.

In a similar fashion to Table 4.1, Table 4.2 is compiled from data extracted from Tables 2.5.1, 5.7.1, 5.7.2, 5.7.4, 5.7.6, 5.7.8 and 5.7.9 (Travers & Westbury, 1989) analyzing population B.

Table 4.2
Population B: The understanding and distribution of content topics (part 1)

Content topics	Understanding	Distribution
Sets and relations		
Set notation	N	-
Set operations (e.g., union, inclusion)	N	M
Relations	N	-
Functions	E	L
Infinite sets and cardinal algebra (rationals and reals)	N	-
Number systems		
Common laws for number systems	I	M
Natural numbers	I	L
Decimals	I	L
Real numbers	I	L
Complex numbers	V	L
Algebra		
Polynomials (over R)	E	L
Quotients of polynomials	I	M
Roots and radicals	V	M
Equations and inequalities	E	L
Systems of equations and inequalities	E	L

Table 4.2
Population B: The understanding and distribution of content topics continued (part 2)

Content topics	Understanding	Distribution
Algebra		
Matrices	I	L
Groups, rings and fields	N	-
Geometry		
Euclidean (synthetic) geometry	N	L
Affine and projective geometry in the plane	N	-
Analytic (coordinate) geometry in the plane	V	L
Three dimensional coordinate geometry	N	-
Vector methods	I	L
Trigonometry	E	L
Finite geometries	N	-
Elements of topology	N	-
Transformational geometry	I	L
Elementary functions and calculus		
Elementary functions and calculus	E	L
Properties of functions	E	L
Limits and continuity	I	L
Differentiation*	E	L
Applications of the derivative	E	L
Integration*	E	L
Techniques of integration	V	L
Applications of integration	E	L
Differential equations	I	-
Sequences and series of functions	N	-
Probability and statistics		
Probability	V	L
Statistics	I	L
Distributions	I	L
Statistical inference	N	-
Bivariate statistics	N	-

Table 4.2
Population B: The understanding and distribution of content topics continued (part 3)

Content topics	Understanding	Distribution
Finite mathematics		
Combinatorics	I	-
Computer science	I	-
Logic	N	-

Notes. Rating scale: Understanding was extrapolated from T able 2.5.1, where
E = essential and included topics that were designated V (very important) for computation, comprehension and application,
V = very important was designated to those topics which included one V and two I (important ratings),
I = important represented the topics where all three topics were rated I and
N = not important was given to those topics where less than three I ratings were noted.
Distribution was extrapolated from tables 5.7.1, 5.7.2, 5.7.4, 5.7.6, 5.7.8 and 5.7.9 where
M = most distribution and included topics rated with a coverage of greater than 80% in more than 80% of the countries surveyed,
A = average distribution including topics with a coverage greater than 60% in 50% to 80% of the countries
L = least distribution including topics with a coverage of less than 61% in less than 50% of the countries and
* = topics not included in tables in chapter 5.

There was a more concise choice of mathematical topics (43) in the final year of schooling and a greater consensus regarding understanding and distribution, with 15 topics being ranked as very important or essential being distributed across an average number of countries or better. It would have appeared a simpler matter to streamline the curriculum for population B when compared to population A except for the fact that the curriculum for A did not have the pressure from tertiary institutions and the parents of students aspiring to enter these establishments, which was ever present for students in their sixth and final year (grade 12) of mathematics. The difference in emphasis between the IEA studies and *Mathematics Counts,* was indicative of the breadth of concerns researched in mathematics education, and the problem of offering mathematics for all and delivering the appropriate mathematics now fully realized.

The most rapidly developing areas in the ever-expanding curriculum of mathematics are chance and data or as they are also called, probability and statistics. Statistics in secondary education includes, data collection, organization, display and interpretation. Although the history of statistics may be traced back to the 1660s (Shaughnessy, Garfield, & Greer, 1996, p. 205), it is only in the last half century that it has become part of the general curriculum. There was no mention of statistics in the 1934 elementary mathematics exams (O level equivalent) set by Oxford and Cambridge for 16 year olds for example (Howson, 1982, p. 237) and yet by 1994 it was included in all 10 levels (grades 1-10) in the document *Mathematics – a curriculum profile for Australian schools* (Australian Education Council, 1994). Statistics was included in many general textbooks for Australian lower secondary mathematics in the 1970s, one example being *Arithmetic 1* (Clapp, Close, Hamann, Lang, & McDonald, 1970). However the inclusion of statistics in Australia between 1964 and 1978 was a variable affair as pointed out by Rosier (1980, pp. 22, 37, 39) and this was reflected in international studies carried out in the 1980s as shown in Tables 4.1 and 4.2 on the previous pages. In more recent years, Pfannkuch (2000) noted that thinking in statistics during the previous 30 years had moved from mathematical to the empirical. This possibly reflected the acknowledgement of the practical nature of statistics (Cockroft, 1982, p. 234). Further investigations on pedagogic thought in statistics have been followed by Pfannkuch and Watson (2004) who indicated a shift (since 1996), away from cognitive studies and towards educational statistics and research. By 2002, statistics was studied in most Australian schools, both primary and secondary, and included in exams for both pure and applied mathematics in grade 12 in Tasmania (Faulkner, 2002).

4.4 Technology and mathematics education

Mathematics in the classroom properly moved away from the tedious calculations of long division and multiplication with the introduction of *Four Figure Tables* by Godfrey and Siddons in 1913 and still in print in 1961. This little book was easily their best seller and enabled the senior student to work with considerable accuracy in trigonometry as well as quickly calculating the roots and powers of numbers. This was not to say that such calculations were learnt in their laborious entirety, but they could now be embedded in more complex problems with the aid of four figure tables. Such mathematical tables were used in classrooms and industry until the 1980s when the electronic revolution finally replaced them. For the best part of the 20th century the slide rule was also available for secondary students and the use of it was still taught in the 1960s. This was a most useful aid for answers to three significant figures and handled trigonometry ratios in addition to multiplication and division. It was the most used tool of applied mathematicians, enabling a quick solution to a problem without the need of pencil and paper or tables. These two aids removed much of the drudgery from the classroom, but time had to be spent learning to use them, and fluency in use meant constant practice.

Many teachers thought, and still hold the view that mathematics is about the mental exercise and discipline required to use and to manipulate the symbols of mathematics from natural to complex numbers, from addition to matrices. Their view that any aid that is used to make this easier, detracts from both the mental exercise and the discipline, was sorely tested with the advent of the hand held electronic calculator. The first generation of these to be used in the classroom, typified by the Sanyo CX-8020, performed the four arithmetic functions with a percentage operation. The latter was to answer the needs of industry, banks and retail houses. For a decade during the 1970s, four-figure log tables and the electronic calculator existed side-by-side in the classroom. Following the entry of electronic calculators into secondary classrooms, the arithmetic calculator was found in primary schools with increasing frequency. The use of the calculator and the implications for curriculum reform was naturally questioned by researchers with positive aspects outweighing the negative ones (Sims, 1989; Stacey & McGregor, 1999).

By the end of the 1970s, the scientific calculator was available for secondary school students. These instruments were not cheap when new, but year-by-year more second-hand models were passed from one student to another so that within ten years it ‘was expected’ that every child would have one. These devices completely replaced four-figure log tables with facilities to perform all the operations of the tables, added to which there were memory banks and bracketing operations available to enable more difficult orders of operations to be performed. Their designs varied considerably and one function that made a calculator more preferable to a secondary student involved in trigonometry was the ability to change degrees decimal to degrees and minutes and vice versa such as found on the Canon F-58. This singular function on its own highlighted the subtle sophistication of exercises and problems being tackled by students. A student might expect to triple the number of problems tackled in a term's work as well as encompassing a far greater breadth of application than was possible when log tables were used. Yet in their day, log tables opened up similarly new vistas to the students using them. The use of the scientific calculator was not confined to arithmetic and trigonometry; standard deviation and regression for statistical operations were also common features. These latter features were embedded for industrial use although even in 2003, students were required to calculate these functions in time consuming tables rather than using technology to apply the processes in practical exercises. These practices were reflected in the texts written to service the needs of the State Schools Board of Tasmania (Launceston College, 2002). Calculators were used to check answers but details of working out still had to be shown.

A new generation of calculators with abilities to produce displays of graphs and interpret the functions was found in the classrooms and examination halls of senior secondary students in the 1990s. These graphic calculators (GC) had been in experimental use for nearly ten years before authorities such as the Victorian Board of Education considered their use in the Victorian Certificate of Education exams (VCE). Graphic calculators and Calculator Algebra Systems (CAS) implied new and deeper changes to the mathematics curriculum for senior secondary students. The study of functions with its calculation of turning points and intercepts followed by the sketch of the function, so long the bedrock of an introduction to calculus, was suddenly available at the press of a few buttons. The use of such technology would trivialize over 60% of the existing assessment mechanisms (Asp & McCrae, 1999, p. 150). Adding to this burden for curriculum writers and designers of today, is the question of the use of programmable calculators that have been used by industry for more than twenty years. CAS stands on the edge of this technology, supplying a variety of algebraic systems for the user that virtually does away with the need to manipulate any algebraic equation.

The final quarter of a century of education has been greatly influenced by the invention of the desktop computer and its more modern counterpart, the laptop computer. In practice it has been used as an extension to the calculator, so rather than being used simply to find the square root of a number, a spreadsheet might embed this calculation in a much larger field of information. Sometimes this is crudely called number crunching. It is at this junction, where equations may be designed in the language of the computer and placed in a spreadsheet, that students can, often for the first time, see the power of mathematics and technology. Each part is dependent on the other to produce results that might otherwise not even have been envisaged, simply due to time constraints (Björk & Brolin, 1999). All of the facilities of GCs and CAS may be replicated by computers, but not with the same ease of handling or cost. Authorities in mathematics education have been inconsistent in their approach to the coupling of the computer age and school mathematics. Although the use of modern electronic aids such as the programmable calculator and laptop computer in the classroom has been encouraged for more than a decade, the examination hall is guided by a different approach. The idea of a student having access to memory banks of their own choosing has always hinted of cheating in an examination environment and in 2006, the situation remained that electronic devices with programmable memories were not allowed. As at the turn of the 20th century, 100 years later the tight reins of an examination oriented system of anachronistic design have a backwash effect on the flow of development in mathematics learning (Smith, 2004, p. 94; Conway & Sloane, 2005, p. 28).

With the sophistication of modern technology arises the question of affordability. Today with the problem of providing classrooms and teachers for children across the world, the introduction of any further expensive aids to learning must surely widen the gap between more affluent countries and others, as highlighted in the SIMS and TIMSS studies.

4.5 Twenty-first century

The main discourse in mathematics education has swung away from a focus on the arguments of Perry and Cayley concerning the pure mathematics of Euclid for the education of gentlemen and the applied mathematics required by the engineers and scientists of the 20th century. The SIMS study showed that the senior secondary modern mathematics curriculum world-wide included all the tools thought important by Perry and more. The most discussed issue of the new century centred upon the use of the new powerful calculators. If one could devise a calculator that would do even the most complex tasks in algebra, what was there left to teach (Arnold, 2004, p. 21)?

The two examples that follow offer an insight into the extension of mathematics teaching that can be gained through judicious use of modern technology. Tanner (2009) applied the power of computing to help students understand circle theorems. All other facets of teaching were employed in this exercise with paper folding preceeding the traditional pencil and paper processes before the computer laboratory was visited. In the second example, a version of the classic written problem of the "Golden Apples" was solved by generating the relevant data on a CAS and then creating the appropriate equation (Özgün-Koca & Edwards, 2009). These two diverse problems in geometry and algebra serve to illustrate an important aspect of the application of modern technology to the mathematics classroom. It is always difficult to find the time that is necessary to do enough problems for the student to understand fully the processes involved in finding a solution to the problem. In both of the cases above, the speed of calculation using the appropriate technology allows the student access to this further learning.

Further trends in research were reported by Walshaw and Anthony (2004, pp. 3-28) upon conducting a review of papers published concerning mathematics education between 2001 and 2003. The papers that appeared with the greatest frequency were centred upon number/computation (13%) with those in geometry/measurement including 8% (Fig. 1, p. 7). Of these papers the division between primary and secondary was fairly even with 18% being left for the tertiary sector (Fig. 3, p. 12). Most of this research (60%) was taken up with concern over students (Fig. 4, p. 13). The 2004 report included 43% of papers looking at cognitive/ learning issues and those concerning socio-cultural aspects of mathematics education (Fig. 2, p. 8).

In the study of Atweh, Meaney, McMurchy, Pilkington, Nayland, and Trinick (2004, pp. 30-33), socio-economic status was seen as a dominant variable affecting achievement in mathematics, with indigenous students being amongst the most disadvantaged. These findings correspond with the ideas that belief and attitude play an important role in learning. However, the major affective issue found involved in the learning of mathematics was identified as anxiety (Schuck & Grootenboer, 2004, p. 63). Gender was no longer seen as a major point in participation in the final years of secondary schooling, although more boys than girls chose mathematics oriented careers (Vale, Forgasz, & Horne, 2004, pp. 75-81). An interesting point was brought to mind in that although boys seemed to handle the operations of graphics calculators with greater ease than girls, the latter were superior in algebraic manipulation and analytic reasoning (Vale, Forgasz, & Horne, 2004, p. 91). The impact of modern technology has included the interactive effects of the internet and multimedia. The beneficial effects of the computer were observed both as a tutor and/or a tool and by way of contrast, concerns were expressed regarding the implications of CAS for the curriculum and teaching practices (Goos & Cretchley, 2004, p. 152). Stacey, McRae, Chick, Asp, and Leigh-Lancaster (2000, p. 572) suggested that "the enveloping tide of hand held technology has profound implications for the curriculum and learning practices". In another paper, a series of desirable goals in CAS-active mathematics for debate was shown by Stacey, Asp, and McCrae (2000, p. 246, Fig. 2) and these are:

1. To make students better users of mathematics
2. To increase congruence between real maths and school maths
3. To achieve deeper learning by students
4. To promote a less procedural view of mathematics and
5. To introduce new topics into the curriculum.

The increasing availability of this powerful technology is such as to bring into question even the most basic learning operations solving for example, polynomials. Students have expressed disquiet with expressions such as "real learning is done with pen and paper" (Warren & Pierce, 2004, p. 302), and have felt the use of CAS is illegitimate (p. 305). Certainly the role of hand-held and portable electronic aids to calculation in assessment has been discussed since the 1970s. One issue considered in the use of programmable calculators and laptop computers is that of memory. It has been thought undesirable that students should have access to a personal bank of information in electronic storage for use in an examination (Board of Studies, New South Wales, 2008) and yet the use of powerful calculators is seen as essential to new age learning and assessment (Stacey, McRae, Chick, Asp, & Leigh-Lancaster, 2000, p. 578). For example one could put in all the trigonometrical identities and binomial theorem proof or even call up information from the internet or local web using wireless technology. This argument had not been fully resolved half a decade later. In contrast to the debate surrounding CAS and learning in the senior grades of secondary education, Forster, Flynn, Frid, and Sparrow (2004, p. 314), found that only 62% of primary children used their arithmetic calculators efficiently.

A perennial concern in schooling is the question of student enrolment. In the United Kingdom, there has been much disquiet about the falling numbers of students enrolled in the years of post compulsory mathematics. In the United Kingdom, there has been much disquiet about the falling numbers of students enrolled in the years of post compulsory mathematics. These are the final two years of secondary education and beyond. The report by Smith (2004), *Making Mathematics Count*, raised a number of points that have implications for all curriculum designers. As a direct result of changes to the curriculum and assessment procedures for General School Certificate (GCE) advanced (A) level exams in 2000, there was a 20% downturn in enrolment the following year and this trend had not been reversed by 2004 (Smith, 2004, p. 92). First year university enrolments for mathematics units were taken from this cohort, and formed only 1% of the student population that started secondary mathematics. This small elite group formed the great majority of the workforce available to advance technology, medicine and science needed by society. The concern of authorities was so great as to consider financial incentives for students to enrol in the GCE 'A' level courses (p. 93). Financial incentives were not new to Australia where means tested federal government financial aid has been available (for more than 20 years) to students if they remain in school for their post compulsory years in grades 11 and 12 and through their first degrees at university. In 2005 in Tasmania, additional state government financial incentives were awarded to those students enrolling in education degrees to attempt to address the increasingly serious shortage of teachers.

This study reflects the concerns raised in the report *International trends in post-primary mathematics education* brought together by Conway and Sloane for the National Council for Curriculum and Assessment in 2005, which spoke of the poor understanding most students had of mathematics (Conway & Sloane, 2005, p. 8), a matter that has been raised throughout history. They suggested that in the 21^{st} millennium, this was of greater concern than ever before and the disconnectedness that existed between the curriculum and society should be addressed (p. 9). The idea of a basic mathematics education should include those skills a society cannot do without (p. 11). Basic mathematics should not be seen as a course for slow learners, but rather in this other light. In 2005, authorities world-wide were aware of the issues of basic education, the needs of society and

1the disconnectedness of the curriculum. In this light, the most significant current trends in curriculum design and assessment were found to be towards a more real-life approach to mathematics (Conway & Sloane, 2005, p. 223). This has included problems couched in practical terms and using appropriate language, less time spent in solving and remembering classical proofs, as well as on-going assessment that tests the weekly progress of the student rather than relying on end-of-year examinations.

Future trends in mathematics education research were seen by Jones (2004, p. 371) to be towards socio-cultural issues. How does one overcome the problems of overcrowding in the curriculum and provide access to the same quality of learning for students from diverse backgrounds? Begg (2000, p. 20) had already indicated the desirability of providing the type of mathematics that was suitable for individual ethnic groups. One approach is to follow the comment of Conway and Sloane (2005, p. 36) in suggesting that mathematical literacy should reflect "a student's capacity to use their mathematical knowledge for informed citizenship", and set about the task of deciding what mathematics an informed citizen might need.

Summary

For more than 2000 years, the classical education for boys comprised the Greek seven liberal arts that make up the trivium and quadrivium of the Roman Boethius. Even at the turn of the nineteenth century in England a grammar school education included a similar content. Within the next 200 years, the only subjects that remained were mathematical, including arithmetic and geometry. Latin and

Greek were still commonly taught in the 1950s, but became rarely offered within 50 years. In mathematics education, the content had moved away from memorizing the proof-laden books of Euclid and towards a mastery of the technology including the calculators and computers that are available to the mathematician of today. Perhaps the greatest change has been the increase in standard of mathematics that students face. Upper primary classes today handle the sort of mathematics that was the province of university students 800 years ago, and upper secondary classes are required to master the university problems of only 100 years ago. The amount of knowledge the human brain can absorb is finite however, limited by ability and time. It is possible that education systems are already attempting to teach students mathematical techniques that will always remain too difficult for them to master completely in the time available.

Mathematics education has changed dramatically even during the last century. Secondary education has become an accepted, indeed compulsory part of learning, with mathematics in many countries from Japan across Asia and Europe to Finland and from Canada through the Americas to Chile. Mathematics always remains in the core of subjects deemed essential for these diverse societies including the African and Pacific nations. Australian research now centers upon refining ways to teach and precisely what to teach to various parts of each community. One of the arguments regarding what to teach is related to the mathematics that is used in the daily life of an adult, and this is the subject for the next chapter.

CHAPTER 5

AN INVESTIGATION INTO THE MATHEMATICS USED IN EVERYDAY LIFE

Introduction

The first four chapters of this thesis have laid the foundations for a thorough investigation into the role that mathematics plays in the education of the young. In Chapter 2 it was observed that mathematics had developed from simply counting one's possessions, for example sheep, to a discipline that could be held to account for most things we wish to describe in the world around us, from the way we make decisions to the origins of the Cosmos. Chapters 3 and 4 looked at the expansion of mathematics education from Greek times to the present day. It is apparent that the content of the current mathematics curriculum naturally relies on developments in mathematics that occurred a century ago or more. The philosophies of what should be taught and to whom will probably never yield a final solution, but will always vacillate between pure and applied forms of mathematics that are either simpler or more complex than that taught at present. A final part of the jigsaw that must be put together before the role of mathematics in education for the present and future can be properly analysed is that of the use of mathematics by people after they have left school. It is to this end that this chapter is addressed.

5.1 Preparation for the current study of the mathematics used in everyday life

Adults use mathematics regularly in their daily lives both at work and in the home (Pritchard, 2004, p. 482) and in most school texts examples have been found (Nay, 1970, p. 126) that reflect a concern with the application of mathematics outside the classroom. Mack (1990, p. 89) suggested that meeting the needs of the everyday life of the individual should be included in the list of goals for mathematics education although this call was largely unheeded in the 1990s according to Teese (2000, p. 193). It has been suggested (White & Michelmore, 2004, p. 593) that when one looks at the history of mathematics education, the curriculum "…has become increasingly independent of experience as more systems and structures have been invented." This was Perry's cry in 1899 when he delivered a series of lectures in practical mathematics to working men in London (Perry, 1910), and was expressed in government reports in England by Hadow (1926), Great Britain Board of Education Consultative Committee on Secondary Education (1939) and Cockroft (1982), as mentioned in Chapter 4. Howson (2002, p. 81) stated, "Clearly the prime aim of school maths must be to provide all students with that most required by today's thinking citizen." Some indication of the direction to take in establishing what was most required was suggested by Ernest (2000, p. 4) when he wrote "More math skills beyond the basics are not needed amongst the general populace in industrialized society."

There has been public concern expressed for more than 100 years regarding the relationship between the mathematics that is taught in schools and that which is used everyday by the great majority of people for either trivial or important tasks. Both individuals (Northcote & McIntosh, 1999) and governments (National Numeracy Review Report, 2008) have suggested that the everyday mathematics that adults use should play a leading role in shaping the core curriculum in mathematics that is part of the compulsory education system from kindergarten to grade 10. In addressing this problem today, it is necessary to outline briefly the work that has been done, particularly in the last quarter century and investigate the current role that mathematics has to play in everyday life both inside and outside the workplace.

The seminal work investigating mathematics education and mathematics used in the workplace was carried out in the United Kingdom in the early 1980's. The Cockroft report as it came to be known has the official title of *Mathematics Counts* and was published in 1982. This government sponsored work was massive in the breadth and depth of the fieldwork that was pursued. Part 1, section 3 was devoted specifically to mathematics in the workplace. Data derived from the workforce were commissioned by the University of Bath (1981) and this report is more closely investigated here. The study concentrated upon those jobs that could be entered at 16-18 years of age and sought to classify the subjects into those who used no, some or considerable mathematics, rather than being in skilled, semi-skilled or unskilled employment. The latter was and still is the traditional division of labour. People were interviewed and their work observed (p. 4) although it was noted that it was "all too easy to read more into the situation than was really there" (p. 141).

Mathematics in the workplace can be implicit or explicit. Explicit mathematics was included in the "category for specific tasks incorporating mathematics" (STIM) (University of Bath, 1981, p. 7). It was employed for example, in the work of clerks who constantly used calculators and tables.

Implicit mathematics was defined as "mathematics incorporating specific tasks" (MIST) (University of Bath, 1981, p. 7). The University of Bath researchers reported "vast armies" of people (p. 25) who were found to use no explicit mathematics at all but did apply implicit mathematics (MIST) in measurement, codes, size, length, volume, mass, location, counting (tallying), copying, weighing, numeric instructions, measuring instruments and machines with gauges (pp. 26-8). It was found that a trend existed to develop machines that reduced the need for the operatives to use calculations (p. 33). People who made use of these machines, either lathes or calculators were not expected to know the processes of achieving a result, but through experience, to realise what was acceptable and what was not (University of Bath, 1981, p. 115). The skills of simple mathematics were used as tools in the workplace (pp. 139-140), and any devices that could reduce the use of mathematics were welcomed as time saving although it was understood they were not free from error.

In more specific terms and at the upper end of mathematical skills found in the workplace, trigonometry was confined to the engineering industry and limited to right angled ratios of sine, cosine and tangent where tables or calculators were used to find the required ratios (University of Bath, 1981, p. 144). Formulae were written out and looked upon as a set of instructions to find the answer to a problem which was invariably common to the worker in a particular situation. Manipulation of a formula was not required and equations such as quadratic or simultaneous, were noticeable by their absence (p. 156). Long division and multiplication were rarely used and always with a calculator as were operations of addition and subtraction where decimals were applied (p. 168). There was little manipulation of fractions apart from measurements in length (p. 228) where parts of an inch such as sixteenths or sixty-fourths were used. The point was made that both metric and imperial measure were found in industry and this was still the case in 2006, for example in aviation (E. Reid, personal communication, July 4, 2006). With regard to the use of statistics, data collection, and reading and entering data on time graphs were commonplace as were the uses of averages, means and the reading of bar graphs. Standard deviation was not generally expected to be interpreted where it was stated, although the boundaries might be used to divide relevant information (p. 230).

Employers generally required a higher standard of school mathematics from their prospective employees than was found in the workplace and senior secondary or college (grades 11, 12 & 13) mathematics reflected this by teaching well beyond the requirements of most future employment in Britain (University of Bath, 1981, p. 213). Employers wanted their employees to be quick and 100% accurate in the mathematics they used in their employment (p. 214), which was a further factor in bringing into question the majority of the mathematics curriculum from percentages to calculus. This last point was made separately by one of the investigators. Fitzgerald (1983a, p. 15) suggested that the school curriculum included much that was not used by young adults in the workplace.

In contrast to the University of Bath report, Lave, Murtaugh and de la Rocha (1984) looked at the way arithmetic was used in shopping in the United States of America. The speed at which shoppers completed their calculations and the accuracy of their results (98%) was startling (p. 82) when put into the context of the environment with all its distractions, the continual balancing of size against cost, and the added necessity to change units of measurement, for example ounces to pounds, constantly. They found that shoppers would repeat a calculation two or three times to check that they were correct. One may consider the matter of mathematics in shopping trivial when compared with calculations for tolerance required in machining an engine part, but as Lave (1988, p. 156) pointed out, shoppers have to make decisions to buy the food that is essential to feed the family. These decisions cannot be avoided and the constraints of budget and taste may well vary from week to week.

Overlapping the work of the Cockroft committee, Harris (1991) pursued the nature of mathematics used in the workplace in London in her project, *Maths in Work*. The aim was to compare jobs on a measure of mathematical skill frequency. The investigation (p. 134) was carried out by way of questionnaires delivered by skilled researchers and responses were elicited for both particular mathematical skills and contexts in which they were used. The four responses offered to the respondents in their use of certain mathematical skills were: very often, fairly often, hardly ever and never, the last being the only negative option. The results were viewed as minimalistic bearing in mind that many people did not recognize the skills that they were using or did not realize they were using a skill when doing a particular task (STIM). The skill results were an aggregate of the three positive options (Harris, 1991, p. 135) and only one skill was used by more than 80% of the population surveyed; this was reading and writing numbers. Addition and subtraction of whole numbers without aid, were the other two skills recorded as in use by more than 50% of the respondents. At the other end of the scale, of the 39 skills investigated (p. 136), the least used were in order: quadratics, log tables, solving formulae with more than one unknown, roots of numbers, slide rule use, trigonometry, powers of numbers and solving formulae with one unknown. Of these, the first four were used by only 1% and the others by 2% of the population surveyed. In 2006 one may discount the slide rule and log tables and look instead at the use of calculating devices which Harris found were used by 46% of respondents.

An extraordinary number of occupations and situations have been investigated. Children with little or no schooling who eke out a living as street vendors in Brazil were investigated by Carraher (1991) and at the other end of the employment spectrum, engineers in factories were studied by Wilkinson (1991). Both investigators found there was a need to think mathematically and make decisions under pressure in adverse conditions. The key to continual success from highly accurate conclusions was to be found in using mathematics that was well within the capability of the individual on a daily basis. A particularly interesting piece of research was reported by Zevenbergen (2003) arising from her work with pool builders. This study of a single work situation unearthed the same difficulty experienced by Harris inasmuch as many of the workers learned their mathematics intuitively on the job but could not articulate what they knew (Zevenbergen, 2003, p. 176). Sub contractors removing large volumes of soil estimated the time and trucks required by experience (p. 177) and this time factor was also important for the box and framers who had to complete the job within a contractual time frame as well as complying with building regulations (p. 180). It was a matter of high priority for all operators to be cost effective because without this constraint they could not earn enough to stay in business (p. 184).

The mathematics used by nurses has been and still is a matter of great concern (Gillies, 2004, p. 255). This group of people with work goals that are far removed from formal mathematics faced with hourly, rather than daily decisions involving number that can have the most serious consequences if errors are made.

Pozzi, Noss, and Hoyles (1998) gave an example of a simple drug calculation that highlighted the importance of context in calculation. "Draw up 85mg of an antibiotic, given a vial containing 100mg of the drug in 2ml" (p. 108). Even given an instruction such as this, a nurse would be expected to know if a drug dosage was appropriate for a particular patient with the many constraints that must be considered. A dosage that was often used might be worked out by rule of thumb, otherwise the general procedure was found to be

$$\frac{\textit{Amount you want}}{\textit{Amount you have}} \quad x \quad \textit{Volume it is in.}$$

The most common use of mathematics lay in filling in the Fluid Balance Chart of a patient. These charts had eleven columns detailing such things as drug intake and urine output, with entries for each hour in the day (p. 111) with the hourly balances being either positive or negative. This process was not just a matter of blindly filling in details. Each entry had to be seen in the context associated with the best possible nursing practice or the summoning of further medical help.

A return to a more general study of mathematics used in the community was made in West Australia by Northcote and McIntosh (1999). They recruited 200 volunteers to record the mathematics that they used in a 24 hour period. Seven of these volunteers were then interviewed further. The volunteers ranged from school leavers to university lecturers and included people in homes, shops and workplaces. The vast majority (84.6%) of the calculations involved mental arithmetic, with the most common calculation involving time (24.9%) and the second most common shopping (22.9%). In all, over two thirds of all calculations were reported as being within the range of a grade 4 child (p. 20) with addition (45.7%) and subtraction (42.5%) being the operations used most.

The pharmaceutical industry straddles the gap between industry and medical care being at once responsible to shareholders for a profit, and simultaneously delivering the meticulous care demanded by medicine in both research, production and delivery. Workers in the production of pharmaceuticals vary in their roles from warehouse operators to those employed in producing the chemicals themselves. The mathematics needed in the duties of these functions, quite separate from the research part of the industry, was reported by FitzSimons (2000). As was expected the accuracy of measurement in compounding the chemicals for production was of paramount importance (p. 176), and accuracy was required in the warehouse as well. Goods stored for delivery required a location system catering for area and height. An additional factor was time because many products had limited shelf life and/or delivery was required within a certain time. All this meant a constant application to numerical detail in four dimensions (p. 177) for those employed in the warehouse. In all phases of the pharmaceutical industry computers were used to control stock production, movement and sales, and although software was dedicated to the job, it only worked if data entered were always correct.

The use of mathematics in the daily lives of two further groups has lent greater breadth to this area of research in mathematics education. There has always been a large number of unemployed which amongst younger people could be as high as one in four. Baynham and Johnston (2001) showed that the need to budget to an extreme degree (p. 113) was an essential part of a successful and sometimes lifelong existence for the unemployed, added to which was the need to interpret the mathematics of the bureaucracy that dispensed financial support. The second group included, were those involved in the special services that could be called upon when unexpected situations in society arose. The Moura coal mining disaster of 1994 in which 11 people were killed was investigated by Lukin (2001) to find out what part mathematics played in the mining itself and in the recovery phase of operations. As might be expected simple statistics were applied everywhere by the miners themselves, as well as by rescuers and investigators. In this disaster as elsewhere, the education of the workforce was brought to the fore, and understanding the mechanics of the combustible gases found in coal mines with the associated safe working practices was not possible without the defining boundaries set by number (p. 169). A final point was that the logistics of moving resources quickly and to the best effect in a disaster were only possible through a grasp of the mathematics of the situation that had to be applied quickly and under stress.

More recent research has continued the investigation into different occupations, including chemical spraying and handling, building, graphic designing, fund raising, playgroup management, hair-dressing, wholesale warehousing, aged care management and post office management, which have all been part of the work done by FitzSimons, who has taken into particular consideration the contexts in which the mathematics were applied. FitzSimons stated (2005, p. 27) concerning numeracy in the workplace, "…when mapped onto school mathematics curricula the actual levels appear to be relatively low at first, but these need to be understood as situated within often pressured situations in relation to time, money and safety, and almost always involve dealing with other people in the process." Two further comments that highlighted FizSimons' understanding of mathematics in the workplace (p. 37) were "…the priority is to get the job done as efficiently as possible – not to practise and refine mathematical skills." And, "Unlike school, the results really matter in terms of public and personal safety, environmental protection and maintaining one's job."

It seems clear from the literature that significant differences have developed over many years, between the mathematics that is taught in schools and that used in the everyday life of adults both inside and outside the workplace. The study reported here arose from questions about whether such differences still exist, how wide spread they might be, and if differences do exist, what are they? The first step was to define as clearly as possible the specific mathematical skills used in a wide variety of situations and occupations, as well people's expectations of accuracy for the applications of mathematics in our society, and to this end a survey of the local population was devised and implemented.

5.2 Survey of mathematics used in the daily lives of adults.

5.2.1 Survey design

It was decided to research the mathematics that is used in the home and the workplace in Australia. The two largest surveys of this type carried out in either Great Britain or Australia have been those by the University of Bath (1981) and Harris (1991), both in Great Britain. The methodology used for the surveys in these cases, were quite different, with the University of Bath choosing interviews and Harris questionnaires. In a more recent study seeking general information about mathematics that adults use in the Australian community, Northcote and

McIntosh (1999) asked respondents to record all the mathematics that they had used in a 24 hour period. The classes of results of Harris (1991), more closely resembled those sought in this survey than those of the University of Bath (1981) and the temporal constraint applied by Northcote and McIntosh (1999) was not desired. For these reasons, the chosen instrument for the survey was that of the questionnaire.

In designing the survey the parameters suggested by Lesh (2000, p. 184) were born in mind. Lesh suggested that if the kind of mathematical abilities needed for success in real life situations are to be investigated, then four points need to be considered. These are:

(a) where should the investigator look?

(b) whom should the investigator observe?

(c) when should the investigator observe them? and

(d) what counts as a mathematics activity?

The design of the survey is now described in relation to each of these factors.

(a) Where should the investigator look?

A locale was chosen that included a provincial city and its outlying regions (Appendix A). The city of Launceston is in the state of Tasmania, Australia, and the outlying regions include most of the Tamar Valley. This has an area of some 60km by 20km and includes a population of approximately 100,000. The city is diverse in its industries and commerce although the surrounding areas are predominantly rural with several small towns settled along the Tamar river, which provides access for merchant shipping.

(b) Whom should the investigator observe?

Initially the population that had left school was the target, but this was split into two groups, those people who were outside the workplace, and those who were inside. It was not going to be possible to reach every type of workplace but it was thought essential that professions, trades, semi-skilled and unskilled workers were included in as wide a range of occupations as possible.

(c) When should the investigations take place?

The method of investigation chosen was by questionnaire, therefore a particular interval of time such as spring or summer, was not essential, although it was thought desirable to avoid the end of the financial year and the Christmas/New Year break. Participants were given a week before replies were collected and the whole study extended over a period of two months at which point the analysis of the data commenced

(d) What counts as a mathematics activity?

It was left to the respondents to decide what counted as a mathematical activity. Questions were so constructed as to elicit responses from those situations where mathematical activity might be implicit (STIM) as well as those where there was an explicit (MIST) use of mathematical skills.

Sampling methodology

Part 1: The general approach

With the exception of the University of Bath (1981) survey and Harris (1991), samples taken by other researchers in this field have been less than 200 in size. The size of the samples in this survey was limited by cost to a print run of 600 questionnaires, with the aim of achieving a minimum sample size of 30 for each of the two types of questionnaires. Cluster combined with multi stage sampling (Hannagan, 1982) with details as mentioned in previously, were chosen as a combined method suited for sampling people outside the workplace, and simple random sampling (Walpole, 1982) was applied to select workplaces from the telephone directory. From the viewpoint of complete reliability, the validity of the results from these smaller samples may be called into question, a point that could also be leveled at this survey. However it is felt that these surveys have fulfilled the prime purpose of drawing the reader's attention to differences that have been shown to exist between mathematics inside and outside the classroom. It is the intention of this survey to define further any differences that previous researchers have indicated. The results cannot be taken as absolute, but offer a data set that is statistically acceptable Koopmans (1987).

Part 2: Further details

There were three methods of distribution of the questionnaires. Clubs as well as businesses were approached in addition to random door knocking of domestic dwellings. With regard to the clubs and businesses, a principal was approached and further distribution and collection was left as an internal matter unless an external investigator was specifically requested. Complete anonymity was assured by the absence of any personal details in the returned forms. The questionnaires were designed for people who had left school and included people who were retired as well those inside and outside the workplace. The suburbs and streets of Launceston that were visited were chosen by random number generation and random spatial selection using the Tasmanian Towns Street Atlas (Tasmap, 2005) as a guide. Outside the city, the townships of Georgetown, Beauty Point, Exeter and districts of Legana, Rosevears and Riverside in the West Tamar Municipality were covered. These are towns and villages in the Tamar Valley spread along the length of the 60 km length of the valley from the coast to the city of Launceston and including both sides of the Tamar river.

Constructing the questionnaires

Questionnaire for people outside the workplace.

An initial guide to the content of the questions that should be included was found in extracts from Tables 4.1 and 4.2 (Chapter 4, section 3). For example, in Table 4.1, arithmetic was indicated as essential and taught in most countries studied, whereas integer exponents was not considered important and taught in less than half the countries. Combining this with other investigations mentioned above, the first choices of what to ask in the sample seemed obvious: the basic operations addition, subtraction, multiplication and division as well as percentage, fractions, decimals, powers and ratios (Appendix B, Table B.1). A variety of school texts including Lynch and Parr (1986) were consulted and examples from grade 7 work (McSeveny, 2003) were used to set a framework of questions attempting to find out specifically what type of problems were in everyday use. In geometry, questions were included relating to areas and volumes, from lawns to boxes and balls. In addition, questions regarding the recognition of size and the ratio of one size to another were also asked. Practical applications of basic skills were put forward in the regions of taxation, insurance, and general household problems such as shopping and mixtures for cakes, as well as investment. Technology questions regarding measurement, calculators and computer use seemed appropriate to today's society and also formed part of the initial design for the questionnaire. Comments were made by Lave, Murtaugh and de la Rocha (1984) regarding accuracy as reported previously in this chapter. Although it seemed self evident that one expects the correct change all the time from the supermarket, or the correct medicine from the chemist, this was not reflected in the level of accuracy expected from school leavers in any mathematics course they pursued in Tasmania following Tasmanian Qualification Authority guidelines (2005). These guidelines were not dissimilar to those published by other States in Australia in that year. A class (group) of questions was therefore included in the questionnaires regarding the accuracy expected and used by adults in mathematics. A class of questions was also constructed to investigate the use of statistics and probability both at a basic and more advanced level, the more advanced level being more closely associated with workplace requirements.

Questionnaire for people in the workplace

There was a core of questions that were common for people in and outside the workplace and these made up around half of the questions asked of people in the workplace. The selection of further topics for investigation of mathematics in the workplace was more difficult than those for outside the workplace because of the bewildering number that might be chosen. An initial list was made from school leaving texts such as Stewart (1995), and advice sought from friends and acquaintances as well as referring to Table 4.2 (Chapter 4, section 3). A considerable part of advanced mathematics in upper secondary schools is given up to the study of calculus and a separate class was made up to investigate the various skills involved that might be used in the workplace. A second class sought to find out other higher mathematical skills taught in senior secondary schools that might be used such as conics, vectors, matrices and complex numbers. It was understood that only a small part of the workforce might use such mathematical tools, but the snapshot of this elite community was thought important as part of the overall picture of mathematics in the workplace. Applied mathematics courses in senior secondary mathematics have included such topics as linear programming, break-even analysis and annuity calculations as well as use of computer spreadsheets for multiple calculations, and this seemed a good opportunity to find out how many of these skills were used outside school. A class concerning the mathematics applicable to those who run their own businesses was put together including questions regarding profit margins, superannuation, the costs of materials and loan interest calculations. The matter of technology in use in the workplace was approached with considerable interest as it extended matters investigated in the more basic section.

Refining the questionnaire designs

The first draft included nearly 200 questions and was thought too long. Although many questions were common to work and home, there were many that were unique to one or the other. Two questionnaires were then devised, one for the workplace and one for outside the workplace. The result was that each handout had around 100 questions split into a variety of classes such as Arithmetic (class 1) or Geometry (class 3). Each question offered three alternative responses. They were often or always, sometimes and never or approximate (Appendix B, Table B.2). The responses were similar to those used by Harris (1991, p. 137) in her British survey.

The responses always, sometimes and approximate were used in the Accuracy classes (13 & 39) and the responses often, sometimes and never were used in all other classes except 14. The three questions in class 14 simply required a yes or no response. A pilot survey of the questionnaires was conducted amongst friends and colleagues and with their comments in mind the final drafts included 91 questions divided into 20 classes for the person outside the workplace and 115 questions divided into 24 classes for those inside the workplace. Copies of the two questionnaires used in the study are found in Appendix B.

5.2.2 Data processing

To facilitate ranking, ‘often/always’, ‘sometimes’ and ‘never’ responses were assigned a numerical loading. The problem that arose was how to assign an objective interpretation to the subjective meaning of the words used in the responses. If the response ‘never’ could be allocated a value of zero percent, and always a value of one hundred percent, then it seemed reasonable to suggest that ‘sometimes’ implied responding half the time or fifty percent. These were subjective choices and any value could be applied as long as an increase in frequency was maintained between ‘never’, ‘sometimes’, and ‘often/always’.

The initial approach was to assign a loading of 2:1 between the responses always/often and sometimes. Taking sometimes as fifty percent would imply always/often was one hundred percent. It was thought that this was too absolute a figure for the interpretation of the response ‘often’ that occurred in fourteen of the sixteen sections (Appendix B). Hence ‘always/often’ responses were assigned a loading of 8, ‘sometimes’ responses a loading of 5, and ‘never/approximate’ responses a loading of 0. Although the ‘never/approximate’ responses were thus reduced to zero in the loading calculations they were included in the sum of questions replied to**.** The number of replies (60) to each question were added together and weighted so that each question had the same value. Weighting reduced bias that might have been evident where some questions had fewer replies than others. Using question 20 as an example, the weighting was 60/59, there being 59 replies out of a possible 60. This weighting was applied to the total of the results arising from the 3 responses;

often –	11 replies	x	loading 8	=	88
sometimes –	39 replies	x	loading 5	=	195
never –	9 replies	x	loading 0	=	0
		sum of results		=	283
sum of results - 283		x	weighting 60/59	=	287.7966
Maximum weighted result			60 x 8 x 1	=	480

The weighted result was then expressed as a percentile and rounded to 2 decimal places:

Total	287.7966	/	4.8	=	59.96

Questions with the highest percentile points indicated the highest usage of a mathematical skill or application by the respondents and these data are found in Tables C.1 and C.2. Each question was also numbered so that questions in each processed table could be related to the original questionnaires in Appendix B.

5.2.3 Sampling details

The two Tables C.1 and C.2 were combined and the questions ranked in order of usage of a mathematical skill or application by the respondents resulting in four Tables 5.1, 5.2, 5.3, and 5.4. Questions that were common to both questionnaires were not repeated and further data processing details regarding these questions are found in Appendix C. Finally, details of the three classes of questions omitted from the overall ranking are described in section 5.2.4.

Table 5.0

Sample distribution inside the workplace (Part 1)

Occupation	Number	Own business	Trade/profession
administration accounting	3		professional
administration reception	3		
administrative manager	2		
baker	1	own business	trade
bank officer	1		
café manager	2	own business	
carpenter	2	own business	trade
chemist	1	own business	professional
chemist's assistant	1		
developer	1	own business	builder
doctor	1	own business	professional
editor	1		
electrician	1	own business	trade
engineer	1	own business	professional
garage owner	2	own business	trade
grocer	1	own business	
hairdresser	1		trade
journalist	4		
machinist/fitter	3		trade
mechanic	4		trade
newsagent	2	own business	
nurse	2		professional
petrol station manager	1		
photographer	1		
real estate agent	1		

Table 5.0

Sample distribution inside the workplace continued (Part 2)

Occupation	Number	Own business	Trade/profession
repair technician	2	own business	trade
restaurant manager	1	own business	
sales assistant	5		
teacher	1		professional
undefined	8		

Sixty people responded to the questionnaire for people inside the workplace and a further 60 people responded to the questionnaire for people outside the workplace. Workplace sampling included 9 professionals and 16 trades-people as well as 17 who ran their own businesses. The response from the factory floor was disappointing however and the positive skew that such a collection base has on these data in that usages are seen to be higher than they otherwise might be, is borne in mind

5.2.4 Classes separated from general ranking

Two classes (26, 27) dealing with calculus and other higher mathematics (Table B.3) were only pertinent to a few respondents and if included would negatively skew the overall results with usage percentiles being depressed. Although the number of people who gave positive responses to these questions was small, there was still one mathematical skill that was used far more frequently than any other in these classes. Operating with given differentials (question 232) with a 10.53 percentile ranking (Table C.2) had a considerably higher usage score than all other skills named in questions 230-243. The next most used skill in this class concerned using differentials that you have to derive (question 233) and the commonality of use was in the approximate ratio of 1:2 when set against question 232. Question 238 was excluded from this discussion because of aberrant data. Some people thought (personal comments, this survey, November, 2006) that complex numbers meant difficult calculations. Questions from class 35 (Table B.3) were particularly aimed at people who were operating their own businesses. The 17 respondents to this class gave information that was additional to rather than part of the general body of data. All scores in this class fell within three percentile points and would have been included in Table 5.3.

5.2.5 Comments

There was provision in the questionnaires for comments to be made by the participants and although there were not many of these, they made interesting reading. Comments in the outside the workplace questionnaire were:

1. *Most of the maths I learnt at school was useless.*
2. *Even though I'd like the above (section about accuracy needed in calculation) to be correct, it isn't always the case!*
3. *Information taught in schools regarding banking was most useful.*
4. *The older I get the more I realize the importance of maths and the more the world has maths at the centre of events/how it works.*
5. *Without an appreciation of numbers many people have no idea if an answer to a calculation is correct. The outcome is that people accept answers that are wrong. As an example, Launceston city council rate*

> *increase for 2005/6 has under-stated for what was actually applied. Yet senior councillors admitted they did not know how the increase was calculated. The outcome is that you can tell people anything unless they have the ability to determine the correct number then they must accept what they are told. Frightening!!*

The comments made by those in the workplace, especially those made by people who ran small businesses and relied heavily upon their small staff, were illuminating. They were:

1. *Schools should teach children the value of money and how to work out their cost of living.*
2. *Also understand what their earning capacity is depending on the field of work or career choice.*
3. *They should be taught how to use credit and the pitfalls.*
4. *Algebra should only be taught to those who think they will need it in their professions.*
5. *Mental arithmetic we use often, even with technology available, particularly addition, subtraction and to work out percentage for return on interest rates etc – this really speeds up the service and helps with accuracy making us more efficient.*

5.2.6 Ranked data

The four Tables (5.1, 5.2, 5.3, 5.4) that follow are drawn from Tables C.1 and C.2. The four divisions have been named as essential, very important,

important or less important according to the commonality of their use in adult mathematics either inside or outside the workplace and follow similar categories adopted in Table 4.1 in Chapter 4. The boundaries for inclusion in Tables 5.1, 5.2, 5.3, and 5.4 were calculated by using one standard deviation above and below the mean, dividing data into four parts.

Table 5.1
Mathematics deemed essential according to commonality of use (Part 1).

Rank	Number	Questions
1	84	How often would you like the government to be correct in the amount of tax you have to pay?
2	314	How often must you be correct when sending out an invoice to a customer?
3	315	How often must you be correct when calculating wages?
4	89	How often do you expect a chemist to put the correct number of pills of the correct weight in your prescription?
5	308	How often would you like the amount of goods you ordered to be correct?
6	81	How often do you expect your bank statement to be correct?
7	2	How often do you use addition?
8	3	How often do you use subtraction?
9	1	How often do you read numbers (in the newspaper, prices, wages etc)?
10	82	How often do you expect the amount of car fuel you ordered to be correct?
11	88	How often must you be correct when putting oil and petrol together to make 2 stroke fuel?
12	33	How often do you find one thing is bigger than another?
13	83	How often do you expect a discount advertised for a product to be correct?

Table 5.1
Mathematics deemed essential according to commonality of use continued (Part 2).

Rank	Number	Questions
14	90	How often should the check-out person count out the correct change from the till for your purchases?
15	4	How often do you use multiplication?
16	91	How often do you expect the correct quantity of heating fuel that you have paid for to be left (wood, oil, gas, coal)?
17	312	How often must you choose the right mathematical technique to apply when using a calculator?
18	80	How often do you expect your shopping bill to be correct?
19	5	How often do you use division?
20	86	How often should your answer be correct if you are checking your shopping bill?
21	313	How often must applying a formula in your trade or profession be correct?
22	85	How often do you expect the government to be correct in the amount of money you receive for child endowment, a pension or other payment?
23	84	How often do you expect the government to be correct in the amount of tax you have to pay?
24	311	How often must you be correct when putting ingredients together to make something (e.g. cement, glue, paint, fuel)?
25	36	How often do you find something is heavier than something else?
26	35	How often do you find that one thing is a smaller version of another?
27	34	How often do you find there are a few of these things but many of those?
28	87	How often would you like to be correct when putting ingredients together to make a cake or other dish?
29	7	How often do you use percentages?

Notes.
Essential – this ranking implies a usage score that is in excess of 82.96 percentile points.
The following information applies to all tables in chapter 5.
Rank – the order of usage in relation to others in Tables 5.1, 5.2, 5.3 and 5.4.
Number – Referencing to questions in Tables B.2, B.3, C.1 and C.2
Questions – some of the questions in Tables 5.1, 5.2, 5.3 and 5.4 are not as originally phrased, prefaces such as "How often." have been added. The original questions are contained in Tables B.2 and B.3 in Appendix B.

Skills deemed essential because of their common use in daily life were dominated by those concerning accuracy. Fowler and Fowler (1967) defined accuracy as something that is precise, exact or correct. In this study accuracy, implied that an answer was correct or incorrect, as opposed to the precision determined by the number of significant figures. Questions were posed to investigate both the accuracy expected in the mathematics that was applied by others such as "How often do you expect a chemist to put the correct number of pills of the correct weight in your prescription?", ranked at number 4, and the accuracy expected in one's own calculations as shown in "How often must you be correct when sending out an invoice to a customer?" which was ranked number 2. It was shown that people appreciated the importance of applying the correct mathematical technique when using a calculator (rank, no. 17) and the importance of choosing the right formula for application in a trade or profession (rank, no. 21). The mathematical skills listed as essential included the four basic operations of addition (rank, no. 7), subtraction (rank, no. 8), multiplication (rank, no. 15) and division (rank, no. 19). Percentage operations also fell into the essential category although ranked last of the 29 in this grouping. These basic operations have been shown here in the context of daily life with the additional expectations of accuracy and the use of electronic aids. In addition to basic operations, adults frequently used their ability to gauge if objects were larger or heavier than others, or if there were a few of one thing and many of another.

Table 5.2
Mathematics deemed very important according to commonality of use (Part 1).

Rank	Number	Questions
1	12	How often do you use a calculator to work out mathematical problems?
2	51	How often do you calculate if the discount stated by a shop is correct?
3	11	How often do you read a scale to measure a weight, length or temperature?
4	48	How often do you calculate if your wages are correct?
5	8	How often do you use decimals?
6	49	How often do you calculate if any amount drawn from your wages is correct?
7	52	How often do you calculate if your shopping bill is correct?
8	44	How often do you find the total value of your house, car, boat etc.?
9	45	How often do you calculate the total value of the contents of your house, car, boat, etc.?
10	50	How often do you calculate if your tax return is correct?
11	60	How often do you calculate a problem like 158.74 + 176.24 + 12.87 + 18.66 + 4.25 + 1.95?
12	39	How often do you calculate the total tax paid for the year?
13	64	How often do you calculate half of 72 centimetres?
14	248	How often do you use a till or dedicated calculator?
15	46	How often do you calculate the value of specific items at the time of insurance?

Table 5.2
Mathematics deemed very important according to commonality of use continued (Part 2).

Rank	Number	Questions
16	61	How often do you calculate a problem like 12,764.87 – 4,321.64?
17	6	How often do you use fractions?
18	10	How often do you use ratios (for example when mixing fuels or cooking)?
19	24	How often do you understand a graph in a newspaper?
20	249	How often do you use a computer with dedicated software?
21	38	How often do you calculate your total earnings for the year?
22	62	How often do you calculate a problem like 2 times 36 grams?
23	63	How often do you calculate a problem like 48 millilitres divided by 4?
24	37	How often do you find if lots of objects will fit into a container?
25	55	How often do you calculate if your power bill is correct?
26	43	How often do you calculate tax owing or owed?
27	66	How often do you calculate a problem like 0.75 of 124?
28	68	How often do you calculate a problem like a mixture of 50:1?
29	42	How often do you work out medicare deductions?
30	20	How often do you use an average or mean (for example, temperature or wage)?
31	40	How often do you work out business deductions?
32	65	How often do you calculate a problem like 23% x $24,864.00
33	19	How often do you use angles such as 30, 45 or 90 degrees?
34	270	How often do you find the cost of replacing equipment/clothing?
35	268	How often do you work out deductions for travel?

Note. Very important - this ranking implies a usage score that is between 54.20 and 82.96 percentile points.

Table 5.2 enveloping all questions with scores falling into the upper standard deviation of the distribution, revealed the use of additional arithmetic skills including decimals, fractions and ratios. The basic skills mentioned in Table 5.1 were further defined in the survey with multiple steps of addition and subtraction involving figures in the ten thousands. Both operations included figures to two decimal places as did multiplication with percentages. The four basic operations were also recognized using a variety of words as might be expected with mathematics found in out-of-school contexts.

The most commonly used geometrical skill was included in this category with the use of common angles such as 30, 45 and 60 degrees (rank, no. 33). A further mathematical discipline mentioned here was statistics, specifically being able to understand a graph in a newspaper (rank, no. 19) and the use of averages or means (rank, no. 30). Using a calculator to assist in mathematical operations and reading scales for weight, length or temperature both were ranked with usages in excess of the 80th percentile (Tables C.1 and C.2). Two other technological skills found in Table 5.2 were the use of a till or dedicated calculator (rank, no. 14) and computers with dedicated software (rank, no. 20). This latter description implied that you enter the data and the computer does all the calculations.

Table 5.3 is the largest one of the four tables, 5.1, 5.2, 5.3, and 5.4. It has been split into two parts with the first part including all questions with usages between 39.85 and 54.2 percentile points and the second all questions with usages between 25.44 and 39.84 percentile points. This division is the arithmetic mean of the upper and lower boundaries which are 25.44 and 54.2 percentile. The ranking of questions is continuous for all of Table 5.3 as if there were no division between the two parts.

Table 5.3

Mathematics deemed important according to commonality of use (Part 1)

Rank	Number	Questions
1	14	How often do you calculate area, say for material, painting a wall, lawn seed etc.?
2	26	How often do you calculate the selling price if you want to make a 10% profit?
3	41	How often do you calculate dependants' deductions?
4	272	How often do you calculate the amount you are paid per actual hour of work, rather than attendance at a workplace? This may be more or less than your stated wage/salary or package.
5	53	How often do you calculate the interest charged on a hire purchase agreement is correct?
6	21	How often do you use the median (such as house price or tax)?
7	74	How often do you estimate the time it will be say in London or Auckland, when it is 7am here?
8	69	How often do you calculate your average speed if you had to walk 15km in 6 hours?
9	251	How often do you use a calculator without dedicated software?
10	54	How often do you calculate if the amount owing on your mortgage is correct?
11	47	How often do you calculate the cost of insurance as a percent of the value of what is being insured?
12	22	How often do you calculate odds (typically on a horse race)?
13	227	How often do you calculate standard deviation (e.g. sales of a product)?
14	271	How often do you calculate the cost of attending a conference involving travel and accommodation?
15	250	How often do you use a computer without dedicated software?

Table 5.3
Mathematics deemed important according to commonality of use continued (Part 2)

Rank	Number	Questions
16	228	How often do you use regression analysis (e.g. profits over a period of years)?
17	13	How often do you use a computer spreadsheet to work out mathematics problems?
18	9	How often do you use exponents or powers such as 10 cubed?
19	220	How often do you use formulas as they are written (for example; area = length x breadth)?
20	269	How often do you calculate depreciation on equipment/clothing?
21	15	How often do you calculate the surface area or volume of a box?
22	25	How often do you calculate the cost of a marketable parcel of shares, say 500 at $4.55?
23	67	How often do you do problems like 10 x 10 x 10 x 10 x 10 x 10, or 10 to the power of 6?
24	28	How often do you calculate the dividend you expect to be paid?
25	23	How often do you do problems using chance (could you survive, win the lottery, calculate the fall of a card)?
26	17	How often do you calculate the diameter, radius, area, or perimeter of a circle?
27	72	How often do you make an accurate scale drawing of the house/flat you are living in?
28	267	How often do you use depreciation for tax purposes using a depreciation formula?
29	27	How often do you calculate a brokerage charge?
30	273	How often do you calculate the cost of changing jobs?

Table 5.3
Mathematics deemed important according to commonality of use continued (Part 3)

Rank	Number	Questions
31	276	How often do you use break even analysis (for finding out when a business will make a profit)?
32	30	How often do you calculate a price earning ratio?
33	274	How often do you calculate the cost of insurance as a percent of the value of what is being insured?
34	56	How often do you use travel graphs (for a picture of distance, time and speed of a journey)?
35	18	How often do you calculate the diameter, radius, surface area or volume of a cylinder?
36	76	How often do you find where you are on a map if you had travelled 320km from your home in a direction that is due east?
37	58	How often do you use a depreciation formula (to calculate depreciation for tax purposes)?
38	16	How often do you calculate the diameter, radius, surface area or volume of a sphere?
39	221	How often do you use formulas that have to be rearranged (for example, if the formula you are given is area = length x breadth, and you want to find the length, the rearranged formula will be; length = area / breadth?
40	29	How often do you calculate dividend yield?

Note. Important - this ranking implies a usage score that is between and 25.44 and 54.20 percentile points.

Further geometrical skills fell into Table 5.3 with the calculation of two dimensional areas (rank, no. 1) being far the most common in use, having a 52.1 percentile. Against this, other types of calculations included the area of circles, scale drawing, the area and volume of cylinders and lastly the area and volume of a sphere. These latter two fell below the 30th percentile. Calculations in probability and statistics associated with chance, odds and median, and the use of regression analysis and standard deviation were all included in the upper half of this table as were computers that did not have dedicated software and the use of computer spreadsheets with mathematical calculations ranked at numbers 17 and 19 respectively. Table 5.3 included many applications of mathematics to money related matters associated with everyday life such as taxation and insurance.

Of the more sophisticated topics found in the final two years of applied mathematics curricula, modelling in "break even" analysis and annuities was ranked number 31. Some of the items in Table 5.3 included here were unexpected in that it was thought that more common use would be made them and they would fall into the essential or very important groups. For example, question 9 dealt with exponents, an arithmetic skill introduced in junior secondary curricula. The ranking of this particular skill below the 40th percentile (Tables C.1 and C.2) marked it as quite distinct from all others in class 1 (Arithmetic) which lay above the 69th percentile. The first question (220) regarding an algebraic skill (class 23) to appear in this table was the question regarding the use of a formula that did not have to be transformed ranked 19th in Table 5.3. This ranking implies a usage of 37.7 percentile points as compared to the next most used algebraic skill (rank, no. 39) with 27.12 percentile points.

The most surprising information gleaned from Table 5.4 was that no one used a graphics calculator (rank, no. 25), considering their wide-spread use in senior secondary grades. All of class 37 dealing with basic algebra skills were included in the lowest part of Table 5.4 and were below the 13th percentile in usage. The least used of these skills were simultaneous equations (rank, no. 24) and graphing a simple function such as $3y = 4x$-7 (rank, no. 23) scoring less than 10 percentile points (Table C.1).

The plane triangle trigonometry skills investigated in the survey are also found in Table 5.4 with spherical trigonometry being included in the advanced mathematics section. The classical question dealing with right angled trigonometry concerning ladders (rank, no. 13) was reported with the least usage at 16.59%.

Table 5.4
Mathematics deemed less important according to commonality of use (Part 1)

Rank	Number	Questions
1	57	How often do you use an annuity formula (to find out how much you have to put into a savings account at regular intervals to have a holiday or pay cash for a car, perhaps in two years time)?
2	31	How often do you find the selling or buying price from a price graph?
3	73	How often do you calculate how many litres of water a cylindrical tank would hold if it has a diameter of 2.8m and a height of 2.6m?
4	32	How often do you find selling or buying price from moving averages?
5	75	How often do you calculate the latitude and longitude if you had travelled 320km from your home in a direction due east?
6	229	How often do you use plane triangle formulae for estimating length, area or angle?
7	245	How often do you find the likelyhood that profits will increase from a graph of consumer demand?
8	260	How often do you create moving averages for selling or buying prices?

Table 5.4
Mathematics deemed less important according to commonality of use continued (Part 2)

Rank	Number	Questions
9	275	How often do you calculate insurance as a percentage of income?
10	71	How often do you find the angles needed at the corners of a shade cloth that has sides of 3.2m, 3.7m and 2.6m?
11	59	How often do you use probability to work out the odds of drawing a card to make a pair if you hold the 2 of clubs and there are 34 cards left in the pack including the 2 of hearts and the 2 of diamonds?
12	244	How often do you work out the chance of a faulty product occurring in an assembly line?
13	70	How often do you work out the length of a ladder needed if the foot is to make an angle of 85 degrees to the ground, and the head reach the top of a wall that is 4.2m in height?
14	247	How often do you work out a standard deviation (to describe the spread of information you have about something such as the number of people who might buy your product)?
15	222	How often do you use proofs to show that the formula you are using will work under the circumstances where it is applied?
16	246	How often do you calculate a line of best fit to interpret the information in a scatter graph?
17	300	How often do you do problems like: solve for x, 8 = 3x + 8y?
18	296	How often do you do problems like simplify 0.5b + 0.5h?
19	297	How often do you do problems like expand 9.8(h + t)?
20	298	How often do you do problems like factorise 16a + 12o + 8b?
21	223	How often do you use formulas you have had to construct (such as applied in engineering or scientific research)?
22	278	How often do you use linear programming (to find out which of two options you should choose)?

Table 5.4
Mathematics deemed less important according to commonality of use continued (Part 3)

Rank	Number	Questions
23	299	How often do you do problems like graph 3y = 4x – 7?
24	301	How often do you do problems like find the intersection when 4x + 7 = y and 6 - 3x = y?
25	252	How often do you use a graphics calculator or calculator assisted algebra system?

Note. Less important - this ranking implies a usage score that is below 25.44 percentile points.

There were three questions in class 14 of Table B.2 that stood apart from all others in that they only required a yes/no answer and these are now reviewed separately. Respondents outside the workplace were asked about the mathematics that they used in their hobbies, sport recreation or other interests. Only one in three replied that the skills they used were not listed on the questionnaire and again one in three replied that they had learnt these skills after they had left school. Three out of four responded that the skills they used that they had learnt at school had to be revised before they could be used again.

5.3 Discussion in relation to earlier findings with reference to mathematics used in the workplace.

If the findings of the research reported in this chapter and in the appendices are compared with the literature, some interesting images are formed. Some researchers reported their results more than 25 years ago and technology in the form of dedicated calculators and computers have forced immense changes in the way that work is carried out both in the office and on the factory floor. Even the procedures followed in domestic finances have been affected by the electronic revolution. Another factor that has to be added to the mix is that of increasing affluence, with the accompaniment of easier loans and increased spending power by company and individual alike. It might be expected that a survey of mathematics used by adults in 2006 as opposed to 1981 would illustrate these differences. The dominant factor reflecting and extending the work of FitzSimons (2005, p. 33) and Gillies (2004, p. 259) is that concerning the high degree of accuracy expected of the individual and by the individual (Table 5.1). This is accompanied by the four basic arithmetic operations of addition, subtraction, multiplication and division and is in accord with the University of Bath report (1981, p. 229). The concerns expressed by Fitzgerald (1983b, p. 18) that there was little reason to include algebra in secondary syllabi, is upheld. Two further points may be made in this area, the first being that if a formula is used, it is used most commonly without manipulation, as a 'pro forma' into which numbers can be put to yield the desired solution (University of Bath, 1981, p. 151). The second is with regard to the graphing of functions. The lowly ranking of this suggests that the comments of Fitzgerald (1983b, p. 18) regarding the paucity of the use of this skill are relevant today.

The mathematical skills used most commonly (Tables 5.1 and 5.2) may be found in primary texts such as those by Vincent (2001) or McSeveny, Parker and Collard (2004), and the study has shown that a knowledge of these skills is essential. To assume that adult mathematics is the same as that of the primary school classroom is quite incorrect. It is only by putting the simple mathematical skills used everyday in the proper context, that the full appreciation of the applied mathematics in daily and common use can be gained. This study showed basic operations being used in countless ways from shopping to running one's own business. Earlier workers (Harris, 1991, p. 143; Northcote & McIntosh, 1999, p. 19) had commented on the importance of context which include the pressures exerted on an operator whilst doing calculations including noise, personal issues, staying economically afloat and the constant demand to be right.

Cockroft (1982, p. 140) recommended a child should learn the geometry of simple figures including area and volume, and the results of Table 5.3 continues to support the importance of learning these skills. A caution may be sounded as to what is included in simple figures, for even the common calculations of the circle, sphere and cylinder were not found to be in general use. Geometrical and technical drawing both require the reading of scales, be they simply millimetres for the measuring the length of a line or degrees for measuring an angle. Scales in their myriad forms were found to be one of the most often used applications of number (Table 5.2) and the importance of this skill in the context of the workplace, for example in nursing, was highlighted by Pozzi, Noss, and Hoyles (1998, p. 112).

Trigonometry was rarely used although it showed in the meta-analysis of Table 4.2 as being taught in more than 80% of the countries surveyed in the SIMS report. The use of statistical skills by the community in the study, were widespread although mostly confined to a knowledge of what the term average means, and the ability to understand a graph as found by Noss, Hoyles, and Pozzi (2000, p. 12).

Summary

The results shown in Tables 5.1, 5.2, 5.3, and 5.4, are in accord with other researchers mentioned in this chapter and illustrate the ongoing dominance of the use of arithmetic, in comparison with other forms of mathematics, by adults who have left school. There is no evidence to suggest that as technology in the workforce has become more sophisticated, the demands of mathematics have become more complex. The contrary appears to be the case, with more difficult calculations, from long division to trigonometry problems, being completed with the aid of a calculator or even dedicated computer. In all areas of mathematics the demands of society are for basic skills that are overwhelmingly arithmetic. The operator must be able to apply these skills on a daily basis with a high degree of accuracy. This pattern is repeated in the home, on the factory floor and in the manager's office. Serious consideration in mathematics curriculum design should be given to the low demands of society for the use of exponents, algebraic skills, trigonometry and the absence of use of graphics calculators. The use of mathematics by adults outside schools has now been investigated and the next step is to determine the mathematics that has been and is taught in schools, before going on in the final chapter to argue for changes to the secondary mathematics curriculum.

CHAPTER 6

MATHEMATICS TAUGHT IN SECONDARY EDUCATION

Introduction

The mathematics used in the daily lives of adults is described in Chapter 5. This chapter concentrates on the mathematics that has been taught in secondary schools. To this end the mathematics text books that have been used in the classroom are taken as a guide to teaching content as opposed to curriculum guidelines.

To understand better the reasons for the content of the modern text, some of the more commonly used books of the past century are included in the study. Evidence of use in the classroom was provided by student notations within a text, or comments made by teachers. Further details of all problems referred to in this chapter are to be found in Appendix E.4.

The chapter opens with an account of the relationship in mathematics education between Australia and England before addressing each of the three major disciplines, arithmetic, geometry and algebra in turn. Each section follows the development of mathematics texts with many examples of problems and comments on the texts under discussion.

6.1 Australia and England

Mathematics education in Australia prior to the 1960s followed the practices of English schools (Clements, Grimison, & Ellerton, 1989, p. 50) and shared the common texts of the day such as those by Durell (1936, 1957), Godfrey and Siddons (1912) and Siddons and Hughes (1929). These books were found in both state and private schools and reflected the common desire to offer all students the opportunity to sit for university entrance examinations either in Australia or England. When the school leaving age was raised to 15 years in the 1940's, new texts began to appear to cater for those students who were studying towards a trade, and for those students who did not wish to go on to further education. There were no experts in these areas of mathematics education (Clements, Grimison, & Ellerton, 1989, p. 68), and the syllabi showed new aspects of secondary mathematics. Australian mathematics texts began to be in more common use by the late 1960's, and over the intervening years have developed to suit the particular requirements of Australian schools. The publication of these was driven as much by the need to explain the New Math of the Bourbaki school, as for a desire for independence (K. Bolger, personal communication, March 18, 2007).

6.2 Arithmetic

Pendlebury (1854-1941) wrote the first of several arithmetic texts in 1886 and they remained in print for over half a century. In considering the work of Pendlebury one must remember the advances that had been made in the standard of school mathematics courses during the 19th century. In 1801 Hutton published his texts for the Royal Military Academy as mentioned earlier in this thesis (Chapter 3, section 4), and there was nothing that was found in them that could not be handled by a secondary student a century later. In the 11th edition printed in 1836, the arithmetic section included problems of an applied nature such as might be useful for training a future army officer of the day. The algebra progressed as far as quadratic equations and the binomial theorem but it was in the geometry that innovations were made. Hutton found a balance between the standard Euclidean practices of the day and the practical requirements of the army officer. He went as far in trigonometry as problems in right-angled triangles and also covered in great detail the use of logarithms. Seven figure log tables were still fairly new in their general application and took a great deal of repeated use before one could guarantee accurate results when using them for problems such as those found in trigonometry or compound division. The first volume ended with samples of problems that might be used by artificers. Perhaps Hutton followed Pepys' philosophy (Chapter 3, section 4), in believing that it was a good idea to be able to calculate how the moneys were spent in an army or a navy. Lest one think that such problems were simple, consider the work of the tiler (Hutton & Gregory, 1836, p. 458) who had to account for the angle of the roof, overhang and weight of the tiles. When calculating how many tiles would be required to cover the roof, the tradesman also had to include the overlap of tiles, both lateral and longitudinal as well as where the tiles were doubled. Finally the number of laths (thin battens of wood) required to support the tiles had to be estimated before the final costing for the materials could be made.

In the preface of *Arithmetic* (p. vi) published in 1896, Pendlebury wrote, "I have attempted to satisfy as far as possible the wants of those who have no oral teacher to aid them." and on the next page (p. vii), "In the following pages I have considered with such fullness as the necessary limits allow me, so much of the science of Arithmetic as is needful for school use and for the Civil Services and also examinations." As this text was in use for such a lengthy period and the initial publication was in the first quarter century of compulsory education, the contents are listed in Appendix E.1.

There were no logarithm operations in this text, which forced the student to pursue what would seem today to be arduous tasks of long multiplication and division, particularly with regard to the applied problems that occupied two thirds of the book. The arithmetic operations were confined to the simple and compound operations of the four basic rules, addition, subtraction, multiplication and division, with their extension over fractions and decimals as well as their application to raising numbers to powers and finding the roots of numbers. All mensuration was included in the text, from problems involving the area of squares to those concerned with circles and cylinders. The application of various arithmetic skills to situations where money was involved, extended from a simple description of the English monetary system through simple and compound interest as far as stocks and shares and proportionalism in partnerships. The sphere of application was extended to include problems regarding weight, in the several systems that operated at the turn of the century, and temperature, both Celsius and Fahrenheit.

In Pendlebury's text, exercises were referred to as examples and catalogued in Roman numerals. Apart from exercises in the pure skills of arithmetic where numerical descriptions were used, most problems were written as text, as were the extensive descriptions of the various techniques involved in each section of the book. This latter point was important. There were very few qualified teachers around and fewer mathematics teachers. Classes were large and monitors who fulfilled a role that was similar to today's teacher's aides, were often the only regular help available to the student. It was of the utmost importance that descriptions of procedures were as full as possible. This meant of course that students had to be able to read before they could embark on learning mathematics.

The most immediate difference in the arithmetic of pre-calculator days and now was in the sheer size of some of the operations; consider the problem of dividing ₤130264. 9*s*. 6*d* by 9416 (Pendlebury, 1896, p. 71). Long multiplication was taken for granted but even when operating with decimals it could be a lengthy process of concentrated care as required in question 19 (p. 147). Amongst the features that helped make this book so popular were the basic explanations of things of which knowledge was often taken for granted. Examples were found in Pendlebury's descriptions of angles, number and time (pp. 169-171) including the Gregorian Correction in the calender-year. The introduction to simple interest (p. 216) gave a description of what interest is and why one might pay it. No formula was used in this approach; rather a fully detailed description of the multiplication process involved.

The sort of question that has been asked since ancient times and is still asked today was exemplified in question 5 (Pendlebury, 1896, p. 265) concerning men digging a trench. Here an arithmetic process had to be chosen before even the first number could be written down. The process was not difficult, but the choice required some proper mathematical thought. A further example was found among the 234 additional written problems towards the end of the book where question 220 (p. 379), involved costing the digging of a well. One could imagine a Mesopotamian scribe 4000 years ago, setting similar problems for his pupil. The most difficult arithmetic problems in the book concerned square roots, where the simplest questions involving perfect squares and primes could take a dozen lines of calculation, such as question 8 (p. 286). This calculation was a long and sometimes complicated problem to solve without the use of logarithms or a calculator.

Half a century later, Durell (1882-1968) set a new standard for arithmetic texts. The overall format was similar to that used by Pendlebury inasmuch as a single text encompassed the three years of secondary schooling, which became compulsory following the *British Education Act* of 1944 and the ensuing division of primary and secondary education. In the *General Arithmetic for Schools* Durell (1936, p. vi) stated in the opening words of his preface, "The character of this book has been determined by the belief that the primary object in the teaching of elementary arithmetic is to secure accuracy." The book was still in print in 1957 and used both in England and Australia.

The advancement in subject matter for students is apparent when the texts of Pendlebury and Durell are put side by side. Durell's text omitted the four basic operations both in simple and compound terms and opened with discussions concerning prime factors and indices. The description of the operations of indices for multiplication and division was encompassed in the two formulae:

$$x^p \times x^q = x^{p+q} \text{ and } \quad x^p \div x^q = x^{p-q} \quad .$$

These formulae were applied in an algebraic exercise with indices (Durell, 1936, p. 287) by way of an introduction to the use of logarithms tables. Godfrey and Siddons's four figure tables referred to in this book were only published in 1913 (Chapter 4, section 4), and were simpler to use and less expensive to buy than the previous seven figure tables available to Pendlebury. The power that the use of four figure tables gave to students may be gleaned from question 11 (p. 300) which looked at calculations in entertainment tax. Fomulae were further used in the descriptions of simple interest (p. 210),

$$I = \frac{P \times R \times T}{100},$$

and of compound interest in parallel with iterative multiplication procedures (Durell, 1936, p. 340),

$$£P(1 + r/100)^n$$

This use of formulae was a departure from the approach used by Pendlebury and and formed part of Durell's idea of abbreviating written descriptions and using more numerical expressions. Mensuration was extended to include cones (Durell, 1936, p. 320) and spheres (p. 321) with both specific gravity and Archimedes law being described in detail before the associated mathematical problems were investigated in exercises 164 (p. 331) and 165 (p. 334).

A new feature was added to the arithmetic text of the day when Durell included a chapter on graphs. As a rationale for inclusion, the opening sentence bears repeating (Durell, 1936, p. 93): "The object in representing facts graphically, is to convey information *rapidly*." Both bar graphs and line graphs were utilised and problems extended from interpreting graphs and drawing them from tables, to using travel graphs for solving problems such as question 7 (p. 107).

The common features of basic arithmetic, that is the four basic rules, fractions, decimals, percentages, mensuration and applied problems concerning money, were also found elsewhere, for example in France, Spain and the United States of America. In Spain Vallès also included book keeping and the manipulation of symbols, which would be today associated more with algebra. Six figure log tables were included (Vallès, 1918, p. 122) with use of the tables explained at length. A quarter of Moore's American text was given up to basic manipulations, which were accompanied by extensive use of tables for multiplication (Moore, 1907, p. 37) and for compound interest. The latter point on compound interest is an immensely practical one, for even today bank managers refer to tables to establish interest and payments due on mortgages rather than apply a formula. As appropriate to a commercial arithmetic, Moore's text extended well beyond the normal boundaries of secondary school arithmetic in its application of the basic skills.

The French school arithmetic of Bourgaux mixed practical and pure problems with illustrations (Bourgaux, 1942, p. 132) to assist the student. A section on geometry was included with practical methods of establishing perpendicular lines and right angles (p. 236), both essential to the building trades. Once again all of these texts required the student to be able to read before arithmetic learning could commence.

The third quarter of the 20th century saw the firm establishment of Australian mathematics texts which the majority of students used. The new books appearing in the 1960s were written so that a matched series covered all the work needed to fulfil the secondary mathematics syllabus to the end of grade 10. One such series was the *SMS* set produced for South Australia between 1968 and 1972. Clapp, Close, Hamann, Lang and McDonald (1972) split the series to produce a slim book for each discipline for each year. Because arithmetic, algebra and geometry each had a dedicated text for each year, the changes that accrued in arithmetic for example, from the Bourbaki school of thought could be clearly seen. These changes were not as easy to observe where arithmetic, algebra and geometry were all included in one book for each year. This was the approach taken by McMullen and Williams (1970) in their popular *On Course Mathematics* series.

Arithmetic 1 of the *Secondary Mathematics Series (SMS)* by Clapp, Close, Hamann, Lang and McDonald (1970) was an updated part of the *Pathfinder* series written in the early 1960s. The most dramatic departures from the syllabus covered by the earlier authors Pendlebury and Durell were the introduction of Venn diagrams and the use of set language. Typical use of the new language of sets was illustrated in question 1 (Clapp, Close, Hamann, Lang, & McDonald, 1970, p. 22).

Express the relations $n(A \cup B) = nA + nB - n(A \cap B)$ in words.

Apart from the description of sets, the book reflected the desire to teach mathematics in the language of numbers rather than words and as a result the descriptions were simple and brief. The problems in those chapters dealing with basic operations were nearly all pure arithmetic rather than of an applied nature. There was a separate chapter on statistics and statistical graphs, illustrating the growing recognition of the use of this branch of mathematics in many walks of life.

The growing importance of statistics could be further seen in *Arithmetic 3* of the series (Clapp, Close, Hamann, Lang, & McDonald, 1972). The study of statistics in secondary mathematics was extended to involve the student in processing raw data as opposed to drawing a graph from a data table. Skills such as recognizing class scores, boundaries and mid-points, had to be allied to calculating frequencies and medians to solve such problems.

A further new feature at this time was the introduction of the slide-rule to the classroom. In previous years, use of the slide-rule had been confined to technical colleges and engineering practices and perhaps this was the first true application of technology to learning mathematics in the secondary classroom if one discounts four-figure tables. The slide-rule was first introduced to the students when they were in grade 9 and using *Arithmetic 2* (Clapp, Hamann, Lang, & McDonald, 1971). The Aristo slide-rule referred to in this book (p. 6), was an inexpensive instrument and allowed an accuracy of three significant figures in an answer.

In 2007, there were at least 10 complete sets of secondary school mathematics texts published in Australia. Two texts are selected that have been commonly in use in Tasmania where many schools select the most appropriate text for a grade rather than the most appropriate series for all grades. In this case the grade 8 text is chosen from the *New Signpost* series and the grade 10 from the

Maths Quest series. Both series combined the various mathematical disciplines into a single text for each year of learning. These books were used across Australia, with the authors adjusting the texts to the specific needs of the major states such as Victoria and New South Wales. This was particularly necessary in the senior secondary grades of 10, 11, and 12.

McSeveny, Conway, and Wilkes (2004) accompanied their *New Signpost* grade 8 text with an interactive compact disc, enabling teachers and students to enrich the work outlined in the text. In keeping with access to modern technology, work was included with computer spreadsheets as well as the scientific calculator that had been in classroom use for a quarter of a century. As with Lynch, Parr, Picking, and Keating (1980), prior knowledge of the basic operations in arithmetic was assumed. The book reflected a return to a greater use of applied problems as evidenced in question 3 (McSeveny, Conway, & Wilkes, 2004, p. 62) which looked at the costs in putting on a show, and question 7 (p. 126) involving sport with probability:

Venn diagrams and set language were still used, but more in keeping with the context rather than as a separate skill. Some of the mathematical language was quite formal; for example the description of a simple event in probability was $P(E) = n(E)/n(S)$, echoing the philosophy of the Bourbaki school. In the main however, the move away from the formal descriptions of the 1980s was evident. Statistics continued to expand with analysis of data including stem and leaf plots and in graphing, the application of scatter diagrams and dot plots.

The majority of state high schools and private schools in Launceston, Tasmania, chose *Math Quest 10,* written for the state of Victoria as their preferred mathematics text for the last year (grade 10) of compulsory education in 2007, the contents of which can be found in Appendix E.2. Stambulic, Iampolsky, Phillips and Watson (2002) in their *Maths Quest 10*, extended the student from the basic arithmetic skill of using compound fractions to the use of standard deviation in statistical analysis, including computer applications. In total, 10 of the 15 chapters of *Maths Quest 10* were concerned with arithmetic skills and their application. Of these 10 chapters, 3 were combined with algebra, for example one dealing with linear equations, two with probability and statistics, and two with graphing techniques. There were no chapters on social arithmetic dealing with monetary problems such as simple or compound interest.

6.3 Geometry

Geometry taught in secondary schools changed dramatically away from the study of strictly Euclidean proofs through the efforts of such men as Forsyth and Perry at the turn of the 20th century (Chapter 4, section 4). At the time the First World War broke out (1914), two books covered the entire course of secondary geometry. Godfrey and Siddons' *Modern Geometry* (1912) served as the text for senior secondary students and was designed to prepare students for the relatively new entry requirements set by the Cambridge examination board, and remained in print until *A New Geometry* by Siddons and Snell (1945) took over from it. It is to the second of the books, written to serve the needs of students up to grade 10 that attention is focused. Hall and Stevens (1903) wrote *A School Geometry*, which remained in use in England and Australia until the 1950s. The copy used for this study was the 28th print of the second edition.

In the preface of the book, Hall and Stevens (1931, p. vii) wrote, "…a pupil should gain his first geometrical ideas from a short preliminary course of a practical and experimental character." Although most of the book was taken up with the reasoning that lay behind the geometrical shapes that were observed, more than a quarter was given over to construction and problems that overlapped those found in the mensuration parts of Pendlebury. One such problem involving area, was question 14 (Hall & Stevens, 1931, p. 102).

More commonly exercises were set around one or two particular theorems. Nine out of ten questions were of a nature similar to question 6 (p. 45).

> In triangle *ABC*, the angles at *B* and *C* are 74^0 and 62^0; if *AB* and *AC* are produced, deduce the angle between the bisectors of the exterior angles. Check your results graphically.

The tenth question would require a proof to be written as in question 10 (p. 45).

> In any *regular* polygon of *n* sides, each angle contains $2(n\text{-}2)/n$ right angles.
> (i) Deduce this result from the Enunciation of Corollary 1.
> (ii) Prove it independently by joining one vertex *A* to each of the others (except the two immediately adjacent to *A*) thus dividing the polygon into *n*-2 triangles.

In the 1960s and 1970s deductive geometry was still very much to the fore in senior secondary courses as evidenced by McMullen and Williams' (1970), *A Short Deductive Geometry,* which was part of the *On Course Mathematics* series. In years 8 to 10 the influence of the Bourbaki school of thought saw the introduction of transformation geometry and cartesian frameworks, with the ensuing reduction in the study of Euclidean theorems and the application of deductive proofs. Along with the introduction of set language and Venn diagrams in arithmetic courses, the changes in geometry for lower secondary years were the most dramatic in mathematics education for over half a century. For this reason, the contents of the geometry course in the *SMS* series are included in Appendix E.3.

In *Geometry 1* of the *SMS* series written by Clapp, Hamann, and Lang (1969a), set language was used to describe basic concepts such as two straight lines in a plane will (a) either intersect or (b) be parallel. Statement (a) was explained in set parlance by $l_1 \cap l_2 = \{P\}$ and statement (b) $l_1 \cap l_2 = \{\ \}$ (p. 3). A new unit in transformation geometry included rotation, translation and reflection. A good idea of the level of understanding that was required was gained from question 8 (p. 65).

Transformation geometry was then used to assist the student to understand the concept of symmetry for which an example was found in question 11 (Clapp, Hamann, & Lang, 1969a, p. 85). The ability to transform a figure was also used in congruence, and the old exercises of showing that two triangles were congruent, took on a new light. The text was a mixture of the old and the new, with most units leading students to what had become an established part of basic geometry in the study of angles, triangles, polygons, and polyhedra. All of these units used a practical approach where the student was asked variously to draw, measure, cut out and construct various figures and shapes, and theory was only lightly touched upon.

Geometry 3 (Clapp, Hamann, & Lang, 1972a), designed for grade 10 students, saw the inclusion of vectors and trigonometry, previously reserved for grades 11 and 12 as well as coordinate geometry which had been traditionally linked with introductory calculus studies. Although Euclidean theorems were not introduced, there was a complete introduction to reasoning and proof. The student was expected to gain an idea of how a proof was structured, and why a particular approach to a proof might be taken. This was quite different from the notion of memorising many proofs without questioning their structure as had been the case in earlier texts.

Basic operations in addition, subtraction and multiplication of vectors were covered and an additional use of vectors was made to introduce the student to the language of matrices and complex numbers, without specific mention being made of either (Clapp, Hamann, & Lang, 1972a, pp. 130-131). Coordinate geometry was extended as far as the equation of a straight line (p. 115), as well as resolving parallel and perpendicular lines and trigonometry was taken beyond the problems of right-angled triangles to the CAST diagram and an introduction to trigonometry graphs. It should be remembered that tables still had to be used to complete any trigonometry calculation in the 1970s.

Another concept that was introduced to this new geometry was that of groups. Groups were used to describe such abstract constructs as the associative and commutative rules. In the examples given (Clapp, Hamann, & Lang, 1972a, p. 106) regarding the set of all integers and the operation of addition (+), the first axiom for the group was stated in the form:

(a) For all a, $b \in$ J, ($a + b$) $\in$ J. (True).

In looking at the tables of contents of the new geometry espoused in the *SMS* series in Appendix E.3 for grades 8, 9 and 10, it may be observed that of the 36 units that the student was asked to study, and leaving aside the 3 that were purely revision, of the remaining 33, 12 were devoted to the new geometrical concepts arising from the French Bourbaki school and the American application of it as mentioned earlier in this thesis (Chapter 4, section 3), and 4 included material that was previously, the province of more senior years of learning.

McSeveny, Conway, and Wilkes, *New Signposts* (2004), preserved all the major facets of basic geometry that had become a familiar part of grade 8 mathematics. Their work took the student through the construction of angles and bisection of lines as well as the reasoning behind geometrical solutions. For example the student was asked to give a reason for the each answer, where one of the questions (1a, p. 153) concerned the base angles of an isosceles triangle. Studies in area and volume extended from triangles and rectangles to circles, cylinders and pipes, the latter three using the π button on the calculator. Coordinate geometry was applied in describing the straight line and transformation geometry used in congruence exercises. In the latter case, students were asked to draw congruent figures rather than prove that the given figures were congruent. An understanding of congruence ratios such as side, angle, side was thus approached from a practical point of view rather than that of pure reason. This method was thoroughly in keeping with the philosophy of Hall and Stevens (1931) as mentioned earlier in this chapter. Dilation or enlargement was introduced by way of scale drawing and used to show similarity between figures.

The modern approach to grade 10 geometry was expressed by Stambulic, Iampolski, Phillips and Watson (2002). The book included outlines of careers such as nursing (p. 302), hydrology (p. 342) and notes on famous mathematicians of the past for example, Eratosthenes (p. 386), and Thales of Miletus (p. 420). There was also an excursion into the use of the graphics calculator, the latest form of learning technology in the analysis of intercepts, and turning points in parabolas (p. 164).

The geometry in this book had great breadth to it, with each topic being extended from the mathematical theory into a practical situation. When using coordinate geometry to investigate linear graphs, exercise 3A (Stambulic, Iampolski, Phillips, & Watson, p. 75) asked the student to "Plot the linear graph defined by the following rules for the given range of x – values." The language was much more informal than that used in the previous two decades, a pattern that was followed throughout the geometry chapters. A few pages further on the student was asked to solve a more practical problem in cleaning a cafeteria (Stambulic, Iampolski, Phillips, & Watson, 2002, question 9, p. 91) and it was expected a graphical analysis would be used.

Graphical analysis was also employed to investigate quadratic equations and inequations as far as turning points and intercepts and the resolution of simultaneous equations was tackled in a similar manner. A further example of the language used in phrasing problems was observed in exercise 7A (Stambulic, Iampolski, Phillips, & Watson, 2002, p. 225) where the student was asked to solve pairs of simultaneous equations using graphical methods.

The breadth of the syllabus was best observed in the two chapters concerning mensuration and geometry of the circle. The former dealt with the traditional problems of measurement, some of which had a history stretching back to Mesopotamia. The importance of teaching students about calculations involving volume, area, perimeter and surface hence had been maintained more than 4000 years. Properties of the circle had their origins in the classic mathematics of Greece and question 8, (Stambulic, Iampolski, Phillips & Watson, 2002, p. 367) was typical. The latter part of this chapter investigated practical aspects of geometry of the circle and in particular Earth geometry and the great circle. A similar error was made here (p. 381) to that found in Lynch and Parr (1986, p. 279) when calculating distances, e.g., from Capetown to Rome, or San Francisco to Honolulu. The authors assumed that Earth is a sphere when it is in fact an oblate spheroid. With the exception of the Equator, the incorrect answer will ensue if segments of an arc of a great circle are used to calculate distances on the surface of the planet. This error in school texts may be traced back at least 100 years and is still perpetuated today. The correct approach using the spherical trigonometry of standard navigational practice, was outlined by Faulkner (2004).

Further geometry included conic sections (Stambulic, Iampolski, Phillips, & Watson, 2002, p. 410) as well as nets (p. 429), and used enlargement through transformation geometry to explain how maps were made followed by an exercise in map measurement (p. 450).

Trigonometry was introduced by way of problems associated with right angled triangles, and went on to include the unit circle, circular functions and graphs of sin θ and cos θ. There was no manipulation of trigonometry functions in graphs, e.g. from $y = \sin\theta$ to $y = 2\sin\theta + 4$, and trigonometry identity manipulation that had been to the fore in previous texts was omitted.

6.4 Algebra

For 70 years, two books dominated the teaching of algebra in secondary schools in England and Australia. Hall and Knight first published *Elementary Algebra* in 1885 and it was still being used in the 1950's. In the 1946 edition used for this study, the first eight chapters were pure algebra starting with the substitution of numbers into expressions and moving through all facets of manipulating symbols. Question 4 (Hall & Knight, 1946, p. 248) gave an idea of the standard of problem that asked the student to prove the identity:

$$(a + b + c)(ab + bc + ca) - abc = (a + b)(b + c)(c + a).$$

The text included graphing functions of the straight line, quadratics, cubes and inverses with intersections of each function with another and in addition, practical problems were found throughout. Many topics found in arithmetic were given an algebraic treatment, such as surds, ratio, proportion and variation as well as arithmetic, geometric and harmonic progressions. Problems in interest and annuities were included as exercises in the use of formulae and their manipulations to solve practical problems.

A New Algebra for Schools Parts 1 and II (Durell, 1930) was published with the intention of providing a text that was suitable for all secondary students. Additional texts were published for advanced students and Part 1 was also published as a separate text. Durell's text overtook that of Hall and Knight in general use and remained the text of choice until itself was overtaken by local publications as already mentioned in this chapter. Part 1 of the 1957 edition opened with how, when and why one used symbols, with exercises for the student in exchanging word sentences for algebraic sentences and vice versa. Question 63 (Durell, 1957, p. 7) was typical of these problems.

Having learnt the idea of using symbols instead of numbers, the student was then led directly into the application of formulae. No manipulation was required at this stage as shown in question 6 (Durell, 1957, p. 22). The natural sequence in this learning seemed to be the investigation of equations and their manipulation that was applied in chapter 3, to help find the answers to problems such as question 2(i) (Durell, 1957, p. 34), where the student had to find the value of the unknown.

With these skills acquired, the student was then tested by problems applied outside the classroom, one of which was question 17 (Durell, 1957, p. 37) which dealt with the cost of travel. Having assumed that the student could handle the everyday applications of algebra in the way of formulae and equations, the text returned to the elementary processes of using symbols in addition, subtraction, multiplication and division, with bracketed operations being treated as a separate and more advanced skill later in the book.

Graphs which conveyed information of a statistical nature were examined, with students asked to extract information from a graph, and construct graphs from tabular information and from raw data. Although the type of graph was not described formally, bar and line graphs were used as well as dot plots. Travel graphs provided a source for investigation as evidenced by question 15 (Durell, 1957, p. 80) where the student was asked to draw a rough travel graph.

In the study of directed numbers, the word integer was not used but the four basic operations were covered, as well as square roots and bracketed procedures. Part 1 closed with a second look at simple equations and problems that included an initial observation of formulae manipulation using the simple interest formula $PRT/100$ as an example (Durell, 1957, p. 124).

Part 2 of Durell's text was clearly a step higher in learning difficulty with the opening chapter involving the student with simultaneous equations of the first degree and including fractions. Question 7 (p. 152) coaxed the student towards applying this new knowledge in estimating the cost of cutlery.

Graphs were used to illustrate functions such as quadratics and as an alternative method for finding the solution of simultaneous equations. This was the purpose of question 5 (Durell, 1957, p. 167), where the student was asked to find the graphical solution and compare the answer with the algebraic solution given. Exercises in algebraic operations associated with products, factors and quotients were pursued, with the several processes for finding the solutions to quadratic equations including the general statement $ax^2 + bx + c = 0$, graphing and completing the square being covered. Operations in algebraic fractions were undertaken before methods for establishing the roots of a quadratic by use of formula were covered in concert with further work on equations. In the final chapter, simultaneous equations were extended to involve three unknowns, with graphical solutions sought as well as algebraic.

Algebra taught in secondary schools arising from the mathematics influenced by the Bourbaki school of thought was similar in content to that detailed by Durell and others a quarter of a century earlier, although the language that was used and the accent on certain parts of the syllabus changed considerably.

Clapp, Hamann, and Lang (1969b) repeated Part 1 of Durell's text in their *SMS* text *Algebra 1,* designed for grade 8 students. The use of symbols, formulae, equations and graphs was all included, as well as an introduction to inductive and deductive reasoning. Set language was used in both explanation and problem solving as shown in question 18(8) (Clapp, Hamann, & Lang, 1969b, p. 12) where the student was asked to find the solution for:

$\{n, n \in \mathrm{Q}: 22 < 3n - 7 \leq 53 \}$.

Graphs were introduced as a way of expressing information in several ways including picture, circle, bar, histogram and frequency polygon, using statistical information set out in tables. In keeping with the further weight given to graphing, ordered pairs were discussed and through them the associated ideas of relations, domain and range. Typical of the knowledge sought from the student was question 7(1) (Clapp, Hamann, & Lang, 1969b, p. 50).

The accent upon graphing was very much to the fore in *Algebra 3*, the *SMS* text for grade 10 students by Clapp, Hamann, and Lang (1972b). The opening chapter plunged straight into the argument explaining when a relationship was a function, and when it was not. The parabolas associated with quadratic equations were investigated at length, with the latter including the dilation, translation and reflection of a parabola by way of explaining the effect of *a, b* and *c* on the equation $y = ax^2 + bx + c$. Where the study of algebra might end, and coordinate geometry begin, remains a moot point and was highlighted by the inclusion of a chapter dedicated to coordinate geometry.

The two approaches used to finding the solutions to problems involving equations of the straight line and the parabola were by graphing and algebraic means. In the second case, algebraic approaches included completing the square and use of the quadratic formula as well as factorisation. With these skills covered, the student was asked to find the solution or intersection of two linear equations, and further, a linear equation and a quadratic. An example of the former was given (Clapp, Hamann, & Lang, 1972b, p. 85) where the student was asked to find the elements of the sets formed by the intersections. With the algebra that was included in Durell (Parts 1 and 2) covered, Clapp, Hamann, and Lang set out to extend the student's knowledge further.

The basic idea behind building a computer programme was introduced, using an iterative approach to finding square roots as an example of a problem to be solved (Clapp, Hamann, & Lang, 1972b, p. 97). This was the first of several topics, the next being a return to graphing with intersections of the plane being investigated and linear programming. A practical application of this was found in question 2 (Clapp, Hamann, & Lang, 1972b, p. 111), where money had to be distributed between two brothers.

There was a preliminary approach to sequence and series, and proof by induction was used in conjunction with series in exercise 8E (Clapp, Hamann, & Lang, 1972b, p. 123). The final chapter was taken up with algebraic treatments of exponents and logs with graphs being used again for explanation and exercise. The chapter ended with a challenging exercise in growth and decay for which question 4 (p. 140) is one example.

The most modern grade 8 text that was read in this study was from the *Signpost* series. McSeveny, Conway, and Wilkes (2004) reinforced the order of learning in algebra that had been recognized for the best part of a century. Junior secondary years concentrated on becoming familiar with the basic operations involving pronumerals and used these procedures to solve equations as well as applying formulae. Indices were introduced, assuming no knowledge (p. 98) before students were led slowly through the intricacies of using brackets for grouping numbers to factorisation (question 5l, p. 103). As with all preceding texts, algebraic fractions were thoroughly investigated, and the student's knowledge strengthened with problems similar to question 10b (p. 10).

Equations, inequations and formulae were approached with a variety of procedures being pursued. The student's powers of manipulation were initially probed in questions such as 3u (McSeveny, Conway, & Wilkes, 2004, p. 249) where the aim was to find the value of an unknown. Question 6c (p. 255), took this procedure a long step further, and applied it to the familiar formula associated with Pythagoras. The power to create and manipulate equations was further applied to problems such as question 2a (McSeveny, Conway, & Wilkes, 2004, p. 256). In addition, linear equations were described graphically and formulae applied to mensuration problems.

The final grade 10 text book from which information was gleaned for algebra was written by Stambulic, Iampolski, Phillips, and Watson (2002). There was nothing new in *Maths Quest 10,* but all the features that had become the accepted standard of algebra at this level of learning were explored. There was far less use of set language and there were more applied problems than had been seen since Durell's (1957) text. Even the revision sections contain the familiar Centigrade/Fahrenheit equation in question 2b (p. 41).

Students were thoroughly versed in both linear and quadratic equations. The study of quadratics stepped from the expansion of binomial expressions, through factorisation, to the use of the quadratic formula as in question 2c (Stambulic, Iampolski, Phillips, & Watson, 2002, p. 126) where the student was requested to give an exact answer. Graphs were also used as the alternate approach to algebra in the solution of simultaneous equations, going as far as the combination of quadratic and linear equations. Question 6 (p. 253) was indicative of the level of understanding required of a grade 10 student when dealing with simultaneous equations or as in this case, inequations.

Further evidence of the standard of learning that was expected came to light in problems concerning exponentials. This part of mathematics can seem quite removed from the things a student might need to know until it is realized that the natural world is explained in terms of growth and decay, and to describe this even in the simplest terms, use has to be made of exponential equations. Question 7 (Stambulic, Iampolski, Phillips, & Watson, 2002, p. 290) drew together skills in algebra and arithmetic in posing a problem in population growth.

Summary

The major changes that occurred in the arithmetic associated with secondary school learning since 1886, were the move of compound operations in the basic skills into mid primary years and the growth of statistics. The language and techniques used to present mathematical information to the student naturally changed, but in essence all of the arithmetic taught a century ago was still taught in the 10 years of compulsory education in 2007, with the exception of arithmetic applied to monetary problems from tax and investment to simple and compound interest.

The most recent geometry text was found to be of a more practical nature than that of 100 years ago and possibly asked more of the student as regards breadth of learning and less in depth. The emphasis on memorising Euclidean proofs so dominant in English mathematics education in the 19th century has given way to an emphasis on coordinate geometry that was used as a bridge between arithmetic, algebra and geometry.

There have been three basic disciplines taught in secondary mathematics education, arithmetic, algebra and geometry. In recent times statistics and probability have been taught separately from arithmetic, and trigonometry regarded as separate from geometry. Algebra has changed least of all as time has gone by. There is only so much one can learn in the years available. If the starting point is concerned with explaining the replacement of a numeral with a pronumeral and the pathway then leads towards toward understanding linear and quadratic equations, the simultaneous solution of linear and quadratic equations form a rough limit upon what a student can learn in the time available.

The final chapter will put forward arguments for changes that should be made to the secondary mathematics curriculum if it is to provide a more effective mathematics education for the majority whilst maintaining the mathematics required for entry to tertiary institutions.

CHAPTER 7

ARGUMENTS FOR A BIPARTITE MATHEMATICS CURRICULUM

Introduction

. Chapters 2, 3 and 4 demonstrate that the secondary mathematics curriculum practiced today in many countries is largely the result of the developments that have occurred in mathematics, which have been echoed, sometimes centuries later, in the classroom. Chapters 5 and 6 lay the foundations for the contention that a change is required to the secondary mathematics curriculum if the greatest good is to be done for the majority of students. This chapter consolidates the argument that the current secondary mathematics curriculum does not address the basic needs of the large majority of adults in their use of mathematics in everyday life and suggests a solution.

The skills and applications of mathematics that are most commonly used outside the classroom are extracted from the survey results introduced in Chapter 5, section 5.2.6, accompanied by implications for the school curriculum. The secondary school mathematics curriculum that has been taught over the last century as reported in Chapter 6 is analysed by way of bringing together the most common features of the Second International Mathematics Study (SIMS) and two texts used in 2007. In addition, accuracy as applied to mathematics in this thesis is defined and the differences between the accuracy demanded by schools and that expected in mathematics beyond school are outlined.

The differences that have arisen between mathematics that is taught in the classroom and the mathematics that is practiced outside the classroom are discussed in the light of the developments that have occurred over two millennia in mathematics and mathematics education, as articulated in Chapters 2, 3 and 4. Attention is then turned to concerns expressed in recent (2004 - 2008) national and state reports in Australia, and these are combined with issues that have arisen from this study to set the foundations for a new curriculum.

The tentative solution that is suggested is a bipartite curriculum, which offers the basic mathematics demanded by everyday life as the core syllabus. An elective syllabus is then proposed to serve the needs of those who have goals or desires in learning mathematics beyond the ordinary. Finally there is a brief discussion of further benefits that might accrue in a reduction of anxiety in students and an easing of the demand for specialised mathematics teachers.

7.1 Mathematics outside and inside the classroom

7.1.1 Mathematics outside the classroom

There has been an increasing interest during the last 25 years, in describing the exact nature of mathematics used by adults in their daily lives, particularly but not exclusively in the workplace. The need for communal mathematical knowledge was recognized in the Clarendon Commission, which sat in England between 1861 and 1864, and the nature of this knowledge has been increasingly defined as time has progressed. Lave, Murtaugh, and de la Rocha (1984, p. 82), Carraher (1991, p. 174), FitzSimons (2005, p. 36), and others found that the mathematics that was used in everyday life was of a simple nature, applied with a high degree of accuracy, time and time again.

The survey carried out in the course of this study concerning mathematics used in the daily life of adults both inside and outside the workplace found similar results, which were condensed in Tables 5.1, 5.2, 5.3 and 5.4 from the initial data recorded in Appendix C. Questions regarding accuracy, both expected and required in calculations, dominated Table 5.1, which described the mathematics deemed essential according to the commonality of use, where essential implied a usage score of more than 82.96% of the population surveyed. Questions with the highest percentile points indicate the highest usage of a mathematical skill or application by the respondents, and the mathematical model used to express raw survey data as percentiles is given in Appendix C. Clearly there are high expectations in the community for accuracy from others as well as accuracy from oneself in the application of mathematics outside the classroom.

Mathematics skills and applications most commonly used by the majority of people (above the mean response of 54.20%) are extracted from Tables 5.1 and 5.2 and the percentage use taken from Tables C.1 and C.2 (Appendix C) to construct Tables 7.1, 7.2, and 7.3. In the survey (Appendices B and C), there were several questions posed regarding each skill, but only the response with the highest percentage use is included in Table 7.1. For example there were four questions put regarding proportion, with responses ranging from the least used, which was concerning the number of objects fitting into a container (64%), to the top ranked question involving the respondent with how large an object was (93%). This procedure established the listing in Table 7.1.

Table 7.1
Mathematical skills used by the majority of people in everyday life

Mathematical Skill	Use (%)
How often do you use addition?	97
How often do you use subtraction?	97
How often do you read numbers (in the newspapers, prices, wages etc.)?	97
How often do you find proportion (bigger/smaller, heavier/lighter, many/few)?	97
How often do you use multiplication?	93
How often do you use division?	91
How often do you use percentages?	85
How often do you use decimals?	80
How often do you use fractions?	70
How often do you use ratios (for example when mixing fuels or cooking)?	70
How often do you understand a graph in a newspaper?	69
How often do you use an average or mean?	60
How often do you use angles such as 30, 45 or 90 degrees?	56

Note. Use is a score derived from the number of respondents who used a skill (Appendix C), set against the total score available representing all respondents (Ch. 5, section 2.1), and is given as a percentage rounded up to the nearest whole number.

The basic nature of mathematics that is in daily use has been noted by other researchers including the University of Bath report (1981, p. 229), Harris (1991, p. 135), and FitzSimons (2005, p. 27), but the simplicity of operation has to be put into the context of the pressures exerted by the demands of daily life before it can be properly appreciated (Pozzi, Noss, & Hoyles, 1998, p. 108). Part of the context depends upon the way a mathematical skill is applied and a summary of the most commonly used applications is encompassed in Table 7.2.

Table 7.2
Mathematical applications used by the majority of people in everyday life

Mathematical Application	Use (%)
How often do you calculate if the discount stated by a shop is correct?	82
How often do you calculate if your wages are correct?	81
How often do you calculate if your shopping bill is correct?	79
How often do you calculate if your tax return is correct?	75
How often do you calculate half of 72 centimetres?	72

Note. Use is a score derived from the number of respondents who used an application, set against the total score available representing all respondents (Ch. 5, section 2.1), and is given as a percentage rounded up to the nearest whole number.

Using the same approach as Table 7.1, each question regarding wages, bills or taxes in Table 7.2 represents several questions posed with only the top ranked question being listed. Tax questions for example, ranged from "How often do you work out deductions for travel?" with a 55% response, to the fourth question listed in Table 7.2, which indicated a usage of 75% . The last question listed in Table 7.2 concerns measurement and was again the top ranked of the three that appeared in the upper half of the results, with others in relation to measurements of weight and volume scoring 66% and 64% respectively. The results in Table 7.2 indicate that most problems to which the majority of people apply mathematics in their daily lives, involve arithmetic and concern money.

There has been lively debate concerning the application of technology in the form of calculators and computers in mathematics education as detailed in Chapter 4 (section 4). The details for the Table 7.3 are garnered from Tables 5.1, 5.2, 5.3, and 5.4, as well as Tables C.1 and C.2.

Table 7.3
The use of technology in the mathematics used by people in everyday life

Technology applied	Use (%)
How often do you use a calculator to work out mathematical problems?	82
How often do you use a till or dedicated calculator?	71
How often do you use a computer with dedicated software?	69
How often do you use a computer without dedicated software?	48
How often do you use a computer spread sheet to work out problems?	39
How often do you use a graphics calculator or calculator assisted algebra system?	0

Note. Use is a score derived from the number of respondents who used some form of technology set against the total score available representing all respondents (Ch. 5, section 2.1), and is given as a percentage rounded up to the nearest whole number.

Table 7.3 suggests there are serious implications for curriculum designers. On the one hand the type of calculator that will perform arithmetic operations is less than half the price of a scientific calculator capable of performing all the functions required by higher mathematics courses. On the other hand the dedicated software used in computers is highly expensive, in addition to which each piece of software is tailored to a specific situation and constantly updated, resulting in professional training usually being provided by the employer. The usage figures in Table 7.3 indicate that there is a case to answer for the way computers and the more sophisticated calculators are applied in the teaching of mathematics, particularly with regard to the use of graphics calculators. Less than half the population used a computer that did not have dedicated software and no one used a graphics calculator outside schools. Computers and graphics calculators are expensive for students and schools to buy which is an additional factor in the debate concerning their use in the classroom and examination halls (Ch. 4, section 4).

7.1.2 Mathematics inside the classroom

An international snapshot (SIMS) of the mathematics taught in secondary schools was reported in Tables 4.1 and 4.2 (Chapter 4, section 3). Table 7.4 is an extract from Table 4.1 relating to a population of 13-year-old students from 20 countries. All of the content topics listed in Table 7.4 were taught in more than 80% of the countries that took part in the survey, and hence fell into the most widely taught topics in Table 4.1. There appears to be an implicit assumption that the four basic arithmetic operations of addition, subtraction, multiplication and division have already been learnt, as nowhere in the 49 subject topics listed in Table 4.1 is there any mention of them. The two topics in Table 7.4 that do not correspond to those listed in Table 7.1 are units of measurement and integers. It is difficult to know if the approach to integers is arithmetic or algebraic, as such details are lacking.

Table 7.4
Subject topics deemed essential (E) or very important (V) for 13-year-olds

Content topics	Understanding
Natural numbers and whole numbers	E
Common fractions	V
Decimal fractions	E
Ratio, proportion, percentage	E
Standard units of measure	E
Integers (algebra)	V

In an appropriate Australian text in use at the time (Lynch, Parr, Picking, & Keating, 1980, p. 82), the section relating to integers was an algebraic exercise in multiplication following a chapter on directed numbers.

In Australia all students in schools pursue secondary education between grade 8 and grade 10; beyond these points there is variation as each state has independent education legislation. For this reason, texts for grades 8 and 10 were chosen for closer study in Chapters 6 and 7. In Table 7.5, an outline of the grade 8 syllabus is given by listing the chapters from the appropriate *Signpost* series text, which is described in detail in Chapter 6.

Table 7.5
Chapters in McSeveny, Conway, and Wilkes (2004) text for grade 8

No.	Chapter heading
1	Review of last year's work
2	Working mathematically
3	Investigating Pythagoras' theorem
4	Percentages
5	Patterns and algebra
6	Probability
7	Reasoning in geometry
8	Graphs and tables
9	Areas and volumes
10	Equations and formulae
11	Ratio, rates and scale drawing
12	The number plane
13	Geometric constructions and congruent figures
14	Statistics
15	Circles and cylinders
16	Similarity
17	Using calculators and spreadsheets

Of the 17 chapters listed in Table 7.5, four correspond to the information in Tables 7.1, 7.2, 7.3, and 7.4. Operations in the arithmetic revision contained in chapter 1 go as far as powers of 4 and cube roots, and also include such aspects as highest common factor and lowest common denominator, implying that the arithmetic in grade 7 is already in advance of the everyday needs of most people. The student's knowledge in percentage operations, reading graphs, and ratios, which are all skills mentioned in Table 7.1, is extended in chapters 4, 8 and 11. The remaining 13 chapters in algebra, geometry, probability, statistics and technology are all part of the longitudinal structure of mainstream mathematics preparation for possible university entrance.

In a similar fashion to Table 7.5, the chapters of Stambulic, Iampolski, Phillips, and Watson (2002) are used as a guide to the grade 10 syllabus, further detail of which are found in Chapter 6 and Appendix E.3. Chapters 1 and 2 contain material germane to Table 7.1, although at the upper end of operations that might be required on a daily basis. Most of the text is of the sort of rigor that is expected of a text to prepare 16-year-old students for the one or two years of post compulsory mathematics requirements for tertiary institutions.

Examination of the first five tables in this chapter shows that the mainstream or core mathematics that is taught in the classrooms of secondary schools is more complex than that required by the majority of people in their everyday lives, which supports Teese (2000, p. 190), who observed that mathematics taught in Victorian (Australian) schools did not reflect the mathematics of the workplace and served only theoretical studies. It has been suggested that this position has been brought about by the increasing separation of mathematics from experience as more mathematical structures have been invented (White & Michelmore, 2004, p. 293).

7.1.3 Accuracy required in mathematics in secondary schools today

As mentioned in Chapter 5, Fowler and Fowler (1967) defined accuracy as something that is precise, exact or correct, and this is the meaning that is accepted in this study. The word accuracy is applied equally to a group of problems or the results of a test or course. For example, if there are 10 questions in a test and 5 are answered correctly, then in the context of this thesis, the degree of accuracy achieved in the test is 50%.

The standard assessment pass mark has been between 40% and 50% for many years in Tasmania (Personal communication, W. Bounds, May 10, 2008) although it is difficult to say what is happening with the latest assessment instrument, which is in the second year of operation in 2008. The changing pass mark has been associated with the desire for a normal curve of distribution in student assessment (Personal communication, W. Hayward, May 8, 2008), and by accepting a pass mark of between 40 and 50% there is an implicit assumption in this study, that the accuracy achieved by students in solving mathematics problems also ranges between 40% and 50%. This is quite removed from the 90% (or greater) accuracy required in the mathematics of daily life as revealed in Chapter 5, sections 5.1, 5.2.6 and 5.3.

The three dominant issues relating to the mathematical ability of the student as reflected in a final award are the quality of teaching, the quality of learning and the syllabus. It is upon the latter issue that this study is focused.

Concerns have been raised as long as mathematics has been taught about the quality of teachers as mentioned in Chapters 3 and 4, but even the best teachers cannot enable every student to become an engineer, scientist or mathematician, even if the desire were there. All students could be awarded a pass in mathematics if the pass mark were lowered sufficiently; alternately all students could achieve a 90% pass mark if the syllabus were simple enough. This study suggests that the course of action that best suits the citizens of tomorrow, is to design a new mathematics curriculum that contains a simple core syllabus reflecting the mathematical needs of everyday life, and enabling students to achieve a high degree of accuracy.

7.1.4 Developments in mathematics and mathematics education.

In Chapter 2 it was argued that mathematics is an invention, a social construction of the human mind, rather than being part of the natural universe. For those who apply or study mathematics, it may be seen as a utility or an art or both, depending upon the interaction of a person with the structure.

Traditionally, the ancient Greeks turned utilitarian arithmetic into an art that exercised the mind in most abstract manner (Heath, 1921b), and this is still seen as the most desirable aspect of mathematics by many teachers and academics, even if it is admitted that this is not for everyone (Hardy, 1967, p. 131).

As mathematical knowledge expanded it became too much for one person to be expert in all that was known. Mathematicians started to be known by their expertise in pure or applied fields and the division between the two has been deepening ever since it was officially recognized with university chairs in the mid-18th century (Restivo, 1992, p. 142). Mathematical knowledge has increased at such a rate that the 19th and 20th centuries have seen as much expansion in both pure and applied mathematics as the entire history of mathematics before that time.

In concert with the expansion of mathematical knowledge, the pressure has increased on schools to implement curricula that are continually broadening and becoming more sophisticated. The arithmetic of primary schools has expanded from the finger counting of Classical Greece (Marrou, 1956, p. 73), so that by 1970, students were learning addition, subtraction, multiplication and division of fractions and decimals and more (Clapp, Close, Hamann, Lang, & McDonald, 1970). The secondary syllabus has moved from the quadrivium of arithmetic, geometry, music and astronomy to exclude the latter two and include a range of topics as described in Chapter 6, that would have amazed the mathematicians of even the 17th century.

When Perry (1900) set out his ideal syllabus for secondary students he was driven by the expanding need of an industrialised society for engineers and scientists. The secondary population was only 2% of those who attended elementary school and although the content was far more practical than had been practiced hitherto, there was sufficient rigor to satisfy the most discerning critic. A quarter of a century later the report of Hadow (1926) to the British government commenting on secondary mathematics education, which serviced 9% of the students who remained in education after elementary school, stated a much more basic position led by the statement that every student should have a grounding in mathematics that was practical, with an emphasis upon accuracy.

Immediately prior to the outbreak of the Second World War, the Great Britain Board of Education Consultative Committee on Secondary Education (1939), submitted a second report on education to the British government. The secondary school population had grown, so that around 1 in 4 students remained in school when their elementary schooling was complete, and again the necessity of mathematics in education was emphasised along with a description of it as a tool that was appropriately designed for the times. In more general terms the report stated (p. 161), “We accept fully the position that school studies should fit boys and girls for the practical affairs of life, and that if they do not do so they must be badly planned or badly conducted.”

Education in Australia reflected that of Britain and Andersen (1952) pointed out that the changes that had occurred to the population attending secondary schools had not been reflected in the mathematics syllabus. The syllabus he suggested was designed for the elite who made up the body of students half a century previously. The student population formed by the raising of school leaving age with its implications for compulsory secondary education, meant that the ability of the average student was below that of the elite group and the range of abilities was far greater than before. Andersen (1952, p. 8) went further in stating that, in the previous 30 years no field of educational endeavour had been as disappointing as arithmetic, accompanied by a steady decline in basic skills.

Research turned to the needs of society, and specifically the mathematical needs as reflected in the report to the British government by Cockroft (1982). All students now went on from primary schools to secondary, and comprehensive high schools had replaced the tripartite system that existed in the 1960s. In the tripartite system there were three types of secondary schools that students could attend, technical for students intending to pursue a trade, grammar for those with university in mind and the secondary modern for students who did not attend either technical or grammar schools. Entry to grammar schools from primary schools was restricted to those who gained a satisfactory score in the 11+ examinations, with a lesser score being required for entry to technical schools. There were no entry requirements for secondary modern schools. The overall view of the needs of society was essentially that of basic arithmetic, supplemented by a little knowledge of geometry, chance and data. In commenting upon the secondary mathematics curriculum of the time Cockroft wrote (1982, p. 449), "In our view, very many people in secondary schools are at present being required to follow mathematics syllabuses where the contents are too great and which are not suited to their level of attainment." In a plea for the welfare of the student he further indicated (p. 68), "…whatever their levels of attainment, pupils should not be allowed to experience repeated failure."

Much that has been written in reports such as those by Hadow, Spens, and Andersen is relevant today. Further research into the needs of society as evidenced in Chapter 5 of this study supports Howson (2002) in his call for all students to learn mathematics that is commonly useful, although research into the modern secondary mathematic syllabus as reported in Chapter 6, suggests that the findings of Cockroft (1982, p. 449) prevail to a large extent in Australia and elsewhere.

It is not possible to fit into a mathematics curriculum, all that might be deemed desirable by all interested parties and a choice must be made that will best suit the needs of most people for their adult life in the society of tomorrow. The gap that exists between the mathematics of the classroom and that of daily life must be reduced whilst maintaining a syllabus that will ensure a supply of people with sufficient skills in mathematics to attend to the specialist needs of a technology based society.

7.1.5 Concerns expressed in recent documents (2004 - 2008)

There are serious problems associated with the current curriculum as detailed in the *National Numeracy Review Report* (NNR) (2008). There is a conflict between the basic numeracy needs of society at large, and the desire by mathematics curriculum designers to provide the longitudinal framework that is necessary for mathematical studies beyond secondary schools (NNR, *Background Paper*, 2007, p. 17). In the NNR *Discussion Paper* (Australian Mathematical Sciences Institute, 2007, p. 7) it was stated that "… the numeracy needs of the adult in the workplace are not met by the school curriculum." It is thought therefore that any new design for a secondary core mathematics syllabus should bear in mind the findings enunciated in Chapter 5 and supported in the NNR *Background Paper* (2007, pp. 15-17) with regard to how mathematics is used in daily life, with particular emphasis upon simplicity and accuracy.

Tasmania, as a member of the Council of Australian Governments' (COAG) National Reform Agenda (NRA) published *Tasmania's Literacy and Numeracy Action Plan* (TLNAP) (Department of Education, Tasmania, 2007) in which it was stated (p. 3) that the state was committed to "…ensuring that young people are provided with the education and training they require to be a productive member of the workforce." It was also noted (p. 4) that "…core literacy and numeracy skills are fundamental requirements for learning and are essential for work and life opportunities beyond school," and further to this aim was the desire (p. 9) "…to increase the proportion of young people meeting basic literacy and numeracy standards and improve overall levels of achievement." There are many definitions of numeracy, which for this study are summed up by: A person who is numerate has the ability to cope with the mathematical demands met in daily life outside the classroom.

Further to the concerns mentioned in this section (7.1.5), there is a small group of people who possess mathematical abilities beyond the ordinary and who are essential to advances in society, from medicine to engineering and from economics to climate modelling (Smith, 2004, p. 4). These people are required to apply mathematics up to the highest levels, and schools must provide a pathway to prepare for further studies in mathematics at tertiary institutions in addition to ensuring the delivery of basic numeracy for all. The mathematical needs of this small group are quite different from those of most students and are best served by a syllabus that has goals other than basic numeracy. The solution that is proposed is to provide a bipartite curriculum that is split into core and elective syllabi.

7.2 Foundation issues for a bipartite curriculum arising from this study

The main issues that have arisen from this study are brought together in Table 7.6 and it is upon these that the structure of the suggested bipartite curriculum is based. It is felt important that there is a proper recognition of mathematics in each specific role as an art or a utility, but philosophic thoughts beyond this, in accompaniment with those ideas and beliefs that are part of teaching are recognized but are beyond the brief of this study. Points 2 and 3 appear obvious, but the continual crowding of the curriculum implied by point 4 fails to recognize them. Given the fact that mathematics forms part of the core curriculum that all students must study, it is of prime importance that the core secondary mathematics curriculum is designed to deliver the best possible outcome in terms of numeracy, with a syllabus that closely reflects the needs of students after they have left school. This study has supported the findings of others in that the needs of the community are best met by students being able to apply simple mathematical skills with great accuracy, time and time again.

Table 7.6

Foundation issues for a bipartite curriculum arising from this study

No	Issue
1	Mathematics is an invention of our minds, a social construction that may be seen as an art or as a utility.
2	Both pure and applied forms of mathematical knowledge are growing at an ever increasing rate.
3	No person can learn all that there is to know about mathematics.
4	Mathematical topics that were once the province of universities are now taught in secondary schools.
5	Current mainstream or core mathematics forms an integral part of compulsory secondary education for all people under 16 years of age in Australia.
6	The mainstream or core mathematics that is taught in secondary schools today is more complex than the daily requirements of most people.
7	The mainstream or core mathematics that is taught in secondary schools today does not reflect the degree of accuracy required by people in their everyday use of mathematics.
8	The mathematics used by most people in daily life is simple arithmetic applied with great accuracy in a consistent fashion.

7.3 A suggestion for a bipartite secondary mathematics curriculum

Preamble

The changes that are proposed to the secondary mathematics curriculum are more in the way of emphasis than content. The curriculum description concentrates on the rearrangement in content of the syllabi with the emphasis in the core curriculum shifting from method to accuracy. The changes proposed do not in general, conflict with statements concerning outcomes of educational policies already in place in Australia and particularly Tasmania. For this reason, fresh statements of desired outcomes have not been attempted and the proposed syllabi, in particular the core syllabus, are embedded in existing documents. The core syllabus reflects the issues brought together in Table 7.6 and is the centre of new curriculum, whereas the elective syllabus could if it were thought desirable, simply follow the present core syllabus as reflected by the texts of Stambulic, Iampolsky, Phillips, and Watson (2002), and McSeveny, Conway, and Wilkes (2004), both of which are discussed in Chapter 6.

7.3.1 The core syllabus

7.3.1.1 Objective and aims

The primary objective in the first part of the bipartite curriculum is to provide a core syllabus that allows the majority of students to be fully competent in the mathematical skills used in everyday life and confident of applying them to obtain a correct answer to a problem. To this end three dominant aims are considered.

a. The mathematics is to be constrained to those skills included in Tables 7.11, 7.12 and 7.13 with applications such as those listed in Tables 7.14.
b. The emphasis is to be on accuracy in results rather than method, and an acceptable degree of accuracy in such activities is at least 90%.
c. The technology used such as calculators and computers, should reflect the basic or applied nature of the technology applied outside the classroom.

7.3.1.2 Outcomes

With regard to outcomes, in 1994 the Australian Government commissioned the report *Mathematics – a curriculum profile for Australian schools,* which provided the groundwork upon which the policies of the various states (Council of Australian Governments) with regard to numeracy issues were based. The ensuing Tasmanian publication was *Key Intended Numeracy Outcomes* (Department of Education, Community and Cultural Development, Tasmania, 1997) and the outcomes stated in the *Learning, Teaching and Asssessment Guide* (Department of Education, Tasmania, 2008a) have not changed. A selection of these outcomes indicate the existing support for the core syllabus of secondary mathematics detailed in this study and are listed in Table F.1 in Appendix F. Four outcomes that have not been included in Table F.1, are listed in Table F.2. Outcome number 1 concerning testing conjectures, is thought too sophisticated a concept of mathematical thought, whereas number 8 involving chance, and numbers 12 and 13 concerned with space, are beyond the requirements of the core syllabus. A note of caution is made in the implementation of number 14 in Table F.1. Formulae are seldom manipulated and are given in written rather than symbolic form wherever possible in the mathematics of life. School practices should reflect this position as shown in the formula explaining the relationship between length, breadth and area where all three forms are given;

length x breadth = area, breadth = area / length, or length = area / breadth.

7.3.1.3 Content

In this section, the suggested core syllabus content is compared with *The Tasmanian Curriculum: Mathematics – numeracy* (TCMN), produced by the Department of Education, Tasmania (2008b). Table 7.7 relates the standards of attainment and stages of development in mathematics education, to the various years (grades) of the student that are described in the TCMN (2008b).

Table 7.7
Relationships between year (grade), standard, and stage of mathematics education

Year	Standard	Stage
5 - 6	2 - 4	6 - 10
7 - 8	3 - 4	8 - 12
9 - 10	4 - 5	10 - 15

Note. The contents are extracted from a similar table in the Department of Education, Tasmania publication, *The Tasmanian Curriculum: Mathematics – numeracy* (Department of Education, Tasmania, 2008b, pp. 10-11).

The suggested core syllabus in this thesis contains material that can be directly compared with the contents of number, measurement and chance and data in the TCMN (Department of Education, Tasmania, 2008b) document and Table 7.8 details the correspondence between the two. The complete body of mathematics contained in the TCMN is beyond the scope of the core syllabus, but could be included in the elective syllabus (Chapter 7, section 7.3.2.2).

The maximum requirements of the suggested core syllabus are such that Number (Table 7.8) excludes operations (Standard 5) in powers and roots as well as modelling. Measurement includes formulae, area and volume, but does not include circles or cylindrical tanks. Studies in chance and data are confined to the interpretation of statistical graphs and this is accompanied by expectations of knowledge of bias that could occur in the presentation of data in the graph.

Table 7.8
Relationship between the suggested core syllabus and standards and stages in the TCMN (2008) document

Topic	Standard	Stage
Number	5	13
Measurement	3	10
Chance and data	3	10

Note. The tabled data are the highest levels of association in the relationship.

Examples of prerequisite skills in mental arithmetic for the suggested core syllabus, are found in Table 7.9. These are derived from the first book in the series *Top Ten Mental* (Watson & Quinn, 1977a). This series was first published in

1959 and similar exercises were being used in 2008 (Personal communication, W. Bounds, 26 September, 2008).

Table 7.9
Examples of mental arithmetic prerequisites for the suggested core syllabus

No.	Question
1	Calculate 7 + 9 + 6
2	Calculate 13 – 4
3	Calculate 7 x 4
4	Calculate 36 ÷ 9
5	Calculate 1/10 + 3/10
6	Calculate 0.5 + 0.2
7	How many grams are there in ½ kg?
8	How many 5 litre drums can I fill from a 50 litre water barrel?
9	A TV show runs from 8.35 p.m. to 10.25 p.m. How long is this?
10	I had a $20 note and spent $16.30. How much money is left?

The examples of prerequisite mathematical skills for the suggested core syllabus listed in Table 7.10 are commensurate with years (grades) 5 and 6, associated with stages 6 and 7 (Table 7.7). It is assumed that these skills can be applied to simple situations either with or without the use of calculators. However, it is not assumed that all students will be able to achieve a high degree of accuracy with such calculations, or that students will be confident in the application of such skills to everyday situations. The term “prerequisite mathematics skill” implies that the student has been introduced to the skill rather than is competent in the use of it or in the application of it. It is considered that one of the functions of the suggested core secondary mathematics syllabus is to provide an environment within which a student may move from an introductory familiarity to being competent in the everyday application of the skill. An important enabling factor in this environment is the fostering of an understanding of the relationship of the various processes involved, for example, between fractions, decimals and percent.

Table 7.10
Examples of arithmetic prerequisites for the suggested core syllabus

No	Skill
1	1638 2716 8531 <u>3466 +</u>
2	5034 <u>2496 -</u>
3	2670 x 43
4	8646 ÷ 24
5	2/3 x 4.5
6	0.5 x 66%

It is thought important that the ability of the student to do mental arithmetic be encouraged and that this skill be practiced week by week throughout junior secondary mathematics education. A guide for Table 7.11 is taken from the fourth book in the series *Top Ten Mental* (Watson & Quinn, 1977b). Consultation with other subject teachers is considered of great benefit in constructing exercises that are pertinent to the current curriculum the student is learning in. If there is to be an emphasis upon the development of certain skills in mental arithmetic, it is suggested that the ability to find an approximate answer, particularly in the case of processes such as the addition and multiplication of monetary sums and the calculation of ratios with respect to the weight or volume of an item such as might be applied when shopping (Lave, Murtaugh, & de la Rocha, 1984).

Table 7.11
Examples of the level of mental arithmetic expected in the suggested grade 10 (Standard 5) core syllabus

No	Question
1	My shopping docket reads, 1.79 + 1.98 + 2.34 + 5.68 + 6.32. I have $20, can I pay the bill?
2	A magazine costs me $3.45. How much change can I expect from $10?
3	How much will it cost to buy 5 C.D.s at the bargain price of $4.99 each?
4	I have a litre of orange juice which I share equally with my three friends. How much do we get each (in millilitres)?
5	A darts player scores double 20, triple 1 and double 15. Adding all these together, what is the total score?
6	I buy a book for $5.00 and sell it for $6.00. What percent profit do I make?
7	The wall of a room measures 2500mm by 3000mm. What is the area of the wall in square metres?
8	A tank is 2m deep, 3m wide and 4m long. How many litres of honey will it hold if 1 cubic metre = 1000 litres?
9	It is 11.30 a.m. in Sydney. What time is it in London, if Sydney time is 10 hours ahead of London time?
10	A spa bath is 1/6th full and holds 30 litres of water. How much will it hold when it is half full?

The mathematical skills listed in Table 7.12 lie at the heart of the suggested core syllabus and are simple. The student is expected to be able to get the correct answer time after time without a calculator so that when the calculator is brought into use, the appropriate mathematical skills will be used quickly and effectively (Stacey, McCrae, Chick, Asp, & Leigh-Lancaster, 2000, p. 573).

Table 7.12
Examples of grade 10 (Standard 5) mathematics for the suggested core syllabus

No	Skill	No	Skill
1	12.65	3	568.24 x 52
	1.99		
	2.43	4	26,780.00 ÷ 26
	5.64		
	11.13	5	2 ½ x 19.53
	6.95		
	8.07	6	0.15 x 8,417.24
	2.14		
	5.23	7	5.5% x 16,417.00
	4.99+		
		8	Express 7.25% as a decimal
2	29,540.00		
	3,624.17-	9	Express 0.23 as a fraction

It is thought that the use of calculators should be confined to applying these skills in the context of everyday situations and that such applications will provide the bulk of the syllabus.

The second part of the mathematical skills that will be developed in the suggested core syllabus are found in Table 13. The skills include those elements of mensuration as well as chance and data that have been identified as essential or very important to the mathematical needs of people in their everyday lives. Measurement is an essential part of basic mathematics (Dowling, 1998), and includes the ability to convert common measurement units as well as read them on various instruments.

The success or failure of the suggested core syllabus will lie in the careful construction of contexts in which the skills in Tables 7.12 and 7.13 can be applied. The contextual exercises must be relevant and are best designed if they can be seen applicable to the needs of students either in the present time or the immediate future.

Table 7.13
Further skills in grade 10 (Standard 5) mathematics for the suggested core syllabus

No	Skill
1	Using an arithmetic calculator
2	Finding an approximate answer
3	Calculating the area of a rectangle
4	Calculating the volume of a box tank
5	Interpreting a statistical graph
6	Understanding the difference between mean and median
7	Using printed and electronic tables
8	Using measuring devices for time, length, mass and volume
9	Converting units such as grams to kilograms
10	Using 30, 45, 60 and 90 degree angles
11	Applying ratios such as 50:1 to mixtures
12	Using a word formula such as *length* x *breadth = area*
13	Awareness of tables to convert units such as pounds and ounces to grams and kilograms

Guides such as those published by SayGoodCredit.com (2008) from which Table 7.14 is derived offer a starting point but care should be used to eliminate those details that are unlikely to be of relevance to the young person. Each heading in this table implies a series of applied exercises that require decisions to be made as to which mathematical skill will be needed to solve the problem and some approximation of what the correct answer should be. In addition to the purely mathematical aspects of such exercises, there will always be discussion surrounding the desirability of spending money in various areas, the use of documents such as those for claiming social security and the interpretation of a bill or account, of which electricity is a prime example.

The range of situations to which the mathematical skills developed in the core syllabus can be applied is so large that even if a new context is used each week as a school exercise, there will always be some in reserve for those who are learning more quickly than others. For instance there were more than 20 different jobs advertised in *The Mercury* (Saturday, July 5, 2008) and 130 in the list of *Employment in Australia for trades people where there are shortages* published by the Australian Immigration (2008), in their *Migration Occupations in Demand List* Each of the jobs mentioned requires a specific application of the mathematics that is inherent in the core syllabus.

Table 7.14
Grade 10 (Standard 5) mathematics applications in personal budget management

Income	Housing	Personal	Transport	Obligations	Recreation
Employment	Cleaning	Clothing	Bicycle	Credit card	TV
Investments	Heating	Food	Bus	Personal Loan	Computer
Savings	Mortgage	Health	Car	Student loan	Dining out
Social security	Rates	Medical	Motorbike	Tax	Hobby
	Rental	Toiletry	Scooter		Holidays
	Repairs		Train		Music
	Telephone				Other
	Utilities				Sport

Note. After SayGoodCredit.com (2008), *Managing your credit – debt - budget*

7.3.2 Elective syllabus

7.3.2.1 Objective and aims

The primary objective in the second part of the bipartite curriculum is to provide a suggested elective syllabus that offers the student who might wish to go to university the mathematics environment that enables such a desire to be fulfilled. Three aims are directed at achieving the stated objective.

a. The course should provide a structure that prepares the student for further mathematical studies.

b. Method is more important than accuracy in this form of mathematics.

c. The technology used e.g., calculators and computers, should reflect the nature of the technology applied outside the secondary school classroom.

It is not seen as desirable to stipulate a specific mathematical syllabus for such an elective, rather this should be determined by the needs of the community and skills of the teachers who are available, so that it can be as relevant as possible and taught in the most professional manner.

7.3.2.2 Outcomes and content

If it is the desire of the school community to make this elective a springboard for university studies, then the following central theme is suggested. Firstly, the junior secondary grades of 8, 9 and 10 could follow the outcomes prescribed

(Department of Education, 2008b) up to stage 15, standard 5 for number, algebra, space, measurement and chance and data. A suitable content for such a course is found in such texts as have been discussed in Chapter 6, for example McSeveny, Conway, and Wilkes (2004) and Stambulic, Iampolsky, Phillips, and Watson (2002). The content of a suggested grade 8 text is listed earlier in this chapter in Table 7.5, and a possible grade 10 mathematics text is found in Appendix E.3. Table 7.15 lists the contents of an appropriate grade 9 text to complete a grade 8, 9 and 10 syllabus.

Table 7.15
A suggested elective syllabus for grade 9

Chapter	Topic
1	Basic number skills
2	Working mathematically
3	Algebraic expressions
4	Probability
5	Geometry
6	Indices
7	Perimeter, area and surface area
8	Equations and inequations
9	Consumer arithmetic
10	Coordinate geometry, graphing straight lines
11	Statistics
12	Formulae and problem solving
13	Coordinate geometry
14	Graphs of physical phenomena

Note. Chapters from McSeveny, Conway, and Wilson (2008). *New Signpost Mathematics 9: Stages 5.1 - 5.2.*

Currently (2008), there is no national statement with regard to senior mathematics courses in secondary education and each state provides individual syllabi that differ in content and assessment procedures to a considerable degree.

Extension studies for students that go beyond grade 10 mathematics, could bear in mind the findings of Matters and Masters (2007). A summary of the subjects that were found to be most commonly studied in grade 12 and deemed essential by teachers who were interviewed by Matters and Masters is contained in Table 7.16. Notable absences are sequence and series and conics, both of which fell below the parameters of the tabulated data used by Matters and Masters. This is not to suggest that other pathways in mathematics should not be pursued from deductive geometry to astronomy and from climate modelling to logic.

Table 7.16
Topics for extension studies in suggested electives

Topic	Topic
Calculus – differential	Calculus – integral
Calculus – rates of change	Functions and graphs
Functions and relations	Algebra of functions
Linear equations	Probability
Calculus – differential equations	Complex numbers
Statistics	Vectors
Matrices	

Note. Each topic was found to be in common use greater than 50% of the time and was rated as essential by Matters and Masters (2007, fig.1M, 2M, p. 61).

In completing this description of a bipartite approach to the mathematics curriculum, it is worthwhile to consider briefly the benefit to students who may change their minds about what they want to do after they leave school. If this included a change in the direction of their mathematical learning, then it is thought that the flexibility inherent in having an elective stream such as has been described, operating in tandem with the core curriculum, would enhance the ability of the students to alter their mathematics course at any time.

7.4 Additional benefits that might flow from such a curriculum

Three benefits are selected as the most important that might arise from the implementation of this curriculum. The stated goal of the Australian government is that every child shall have achieved a prescribed level of numeracy upon leaving school, and thus numeracy heads the list before anxiety. It is felt that in any new curriculum design, every effort should be made to reduce any anxiety that might arise in the student, and this is particularly so in the compulsory component of mathematics education. The third benefit is a duality. In secondary education there is a growing shortage of mathematics teachers and a reduction in the number of students studying the highest levels of mathematics in year 12. The literature suggests (section 7.4.3) that there is a couple between students studying higher mathematics courses in year 12 and the number of specialist mathematics teachers available to teach them. Any improvement in the coupling could be of mutual benefit for students and teacher supply. As a final point, it is realized that further research will be needed to show if and to what extent such benefits occur.

7.4.1 Improvement in numeracy

The Australian Department of Education, Employment and Workplace Relations (2008) stated in the *Literacy and Numeracy* document that "Literacy and numeracy are the cornerstones of all learning…," and this is in concert with the Australian Government's commitment to pursuing the goal that every child should have at least a minimum level of literacy and numeracy (*National Report on Schooling in Australia* (MCEECDYA, 2006, p. 2). As a method of assessing the progress of government reform in this matter, a series of benchmark tests have been constructed that are applied nationwide in both the public and private sector. The results of these tests in 2006 and 2007 showed that year 5 results hovered around 89% with year 7 results some 9% lower (MCEECDYA, 2007a). The tests were rewritten and a considerable improvement was shown in the 2008 results that extended to year 9 students. Results indicate that there was only 5.2% of the school population that had not achieved the minimum level of numeracy (year 5) in the year 9 benchmark test; on the other hand, 51.4% of students in the same test were below the year 7 levels, that is the final year in primary school (*National Assessment Programme: Literacy and Numeracy*, 2008). Extrapolating from MCEECDYA (2006, p. 6) school population figures, the total number of 2008 year 9 students is approximately 250,000 which means that 13,000 students have not achieved the minimum levels of numeracy, whereas 128,500 have not achieved the year 7 numeracy benchmark.

There are issues that should be taken into consideration when using the current benchmark testing system to quantify the degree of numeracy in the school community. The Australian Association of Mathematics Teachers (AAMT) (1997, p. 29) suggested that it was "naïve and dangerous" to use tests as an indicator of numeracy. Most people equate numeracy with the ability to perform mathematical skills, and the benchmark tests suggest that this is so: To be numerate is to be able to do arithmetic. In educational terms numeracy has a deeper meaning. Willis (1990) and the (AAMT) (1998) both defined numeracy as the ability to apply mathematics in the context of daily life, and this is the meaning accepted by the *National Numeracy Review Report* (2008). When business people complain that their employees cannot seem to do their mathematics (Howson, Keitel, & Kilpatrick, 1981, p. 139), and the survey in this study described in Chapter 5), they mean that they are innumerate. Zevenbergen (2003, p. 174) and FitzSimons (2005, p. 35) both indicated that numeracy was specific to a particular context. For example an experienced worker is likely to be numerically competent in a familiar workplace but could be seen as innumerate in another. Adults may have had good results in school mathematics tests, but lack the ability to apply their mathematical knowledge to situations that they are involved in.

If some assessment of the numeracy of the student is deemed necessary, particularly in the final years of schooling, then that assessment would be more applicable if it were of the application of mathematical skills in a particular context with the degree of accuracy, repetition and distractions that may be associated with that context. The emphasis of numeracy is upon result not method, and this is reflected in everyday life from engineers to car park attendants. These people are paid to be correct in the daily application of their mathematics no matter what pressures are brought to bear upon them. The method that is used so that a solution to a problem can be achieved is not in question as long as the correct answer is the result time and time again. The answer that is correct is often specific to that particular situation, for example blood pressure and age (Noss, Hoyles, & Pozzi, 2000, p. 14), and this implies an understanding of the meaning of the answer in the context to which it is applied.

The differences that exist between school mathematics and that applied day to day in or outside the workplace, have been discussed by Fitzgerald (1983b), Harris (1991), FitzSimons, Mlcek, Hull, and Wright (2005) and others. This study suggests that the core syllabus of the bipartite secondary mathematics curriculum will close the gap between school based mathematics and the mathematics required in daily life. This will be accomplished by strengthening basic skills, applying these skills in appropriate contexts and bolstering the confidence of the student as suggested in the *Statement of Learning for Mathematics* (MCEECDYA, 2007b, p. ii).

School-based mathematics in the core curriculum at secondary level should reflect the practices and philosophy of life beyond the classroom. Numeracy is the responsibility of mathematics teachers (AAMT, 1997, p.39) and is essential to the function of education in preparing, Tasmanians for example, to become healthier and better educated, thereby improving workforce participation and productivity (Department of Education, Tasmania, 2007, p. 1).

7.4.2 Reduction in anxiety in students

It is believed that designing a core syllabus that will enable most students to succeed most of the time will reduce the anxiety that has been identified with learning mathematics. In the early 1970s, it was believed that anxiety reduced a student’s ability to achieve in mathematics. More recently Sherman and Wither (2003, p. 142) tested this hypothesis and found it wanting. Their rigorous study rejected the hypothesis that mathematics anxiety causes a lack of mathematics achievement, and accepted the hypothesis that a lack of mathematics achievement causes mathematics anxiety.

Goldin (2002, p. 61) found that fear of mathematics was a common phenomenon whereas Schuck and Grootenboer (2003, p. 63) showed that anxiety was the most common emotion exhibited in the mathematics classroom. These observations reflects the work of Landbeck (1989, p. 223), in which three students out of four reported having negative feelings about mathematics. In addition Docking and Thornton (1979) stated that causes of high levels of anxiety found in mathematics were unique with respect to anxiety related to other subjects. It was suggested that mathematics anxiety "…may not be intrinsic to subject matter, but rather the need to achieve mastery of each level in order to make subsequent levels meaningful," a view supported by Taylor (2004).

In the collection of papers published by the Mathematical Association of Victoria under the title *Conflicts in Mathematics Education*, Leder (1984, p. 22) indicated that students are anxious to do well in mathematics, but have no real expectation of doing so. In addition, Newton (1984, p. 14) indicated that many students complain about the difficulty in mastering mathematics. He goes further to suggest that mathematics is about success in solving problems rather than improving the mind as far as children are concerned. Newton was passionate about what he termed the "disease of mathematics". When students fail at mathematics, the resultant anxiety affects others (p. 17). Expressing a more general view, Marshall and Watt (1999, p. 39, 129), stated that anxiety results in behaviour in a person that influences those around that person and for the student these include teachers, parents and schoolmates. This ripple aspect of anxiety engendered in the mathematics classroom has been observed in the detrimental effect it has on learning other subjects (Carroll, 1994, p. 140).

A preferable state is encapsulated in the statement by Sieber (1977, p. 12), "…obviously one of the aims of education is to create settings that do not engender anxiety in the first place." It is clear that any consideration of change in mathematics education should include the problems associated with anxiety, its identification, manifestations and consequences. Further research will be required to ascertain the benefits that are believed will accrue from the suggested core syllabus, with regard to a reduction in the anxiety in students that is associated with learning mathematics.

7.4.3 Easing of teacher shortage and increase in student retention

The Australian Education Union (2008) noted in a government submission that there was a serious shortage of qualified mathematics teachers in Australia with a worsening prognosis and to exacerbate matters, similar situations have been noted world wide (Harper, 2007). In 2005 half the schools in Australia had half the number of qualified mathematics teachers they required and three in four schools expressed difficulty in recruiting new mathematics teachers (Harris & Jensz, 2006). Teacher shortages have not been evenly distributed, with remote regions having experienced five times the difficulty that metropolitan areas had in recruiting qualified mathematics teachers (Lyons, Cooksey, Panizzon, Parnell, & Pegg, 2008).

The first step in increasing the supply of teachers requires an increase in the number of students who study mathematics at university. As pointed out by

Smith (2004), this implies increasing the number of year 12 students enrolled in "A" level mathematics courses as opposed to the 20% decrease between 1999 and 2003 experienced in Britain. A similar decrease has been observed in Australia, with for example, the enrolment figures for level II and level III (years 11 and 12) mathematics in Tasmania falling from 18.6% in 1996 to 11.8% in 2003 (Committee for the Review of Teaching and Teacher Education, 2003).

"The most important influence on how much children learn is the quality of teaching they receive." This statement by Buckingham (2005, p. 1) is enhanced by the recognition that a teacher can encourage a student to become a mathematics teacher (Gough, 2003). It was suggested by Kissane (2005), that one of the keys to maintaining and possibly to bolstering numbers in the higher mathematics courses of senior secondary schools was to provide appropriately qualified teachers, particularly in lower and middle high schools. As has been pointed out, such teachers are in short supply and this implies a negative influence upon the number of students who pursue mathematics at an intermediate or advanced level at least to year 12. The proposed curriculum offers a pathway to ease the demand for specialist mathematics teachers. It is believed that the core syllabus will require the services of a trained teacher, who fulfils the role of a facilitator and motivator rather than having a particular expertise in mathematics. This should remove the necessity to have specialist mathematics teachers for every student, and leave the way open for the mathematics teachers who are in the schools to teach homogenous classes of students who have a particular reason or interest in electing to learn mathematics. It is these students who will gain the greatest benefit from being taught by the constantly reducing supply of specialist mathematics teachers and it is amongst such students that future mathematics teachers will be most likely found.

Summary

Today most people use mathematics in one form or another in their daily lives. It is seen as desirable that schools educate students to a degree of numeracy that reflects the requirements of society and this point raises the question: What are the requirements of society? Traditionally, the secondary curriculum has offered a pathway to university mathematics as its prime function and in association with this mathematics has been seen as an art that assists the mind to create an argument in a logical fashion. This view of mathematics as an art is one that it is essential to preserve, but it is not the province for everyone. For the majority the necessity of mastering the essential utilitarian mathematical tools is of the greatest importance and sufficient for a constructive position in society.

In order that the secondary mathematics curriculum serves the needs of universities and society at large, it is suggested that a bipartite approach be adopted and two syllabi be written. The core syllabus to which all students should attend is simple, utilitarian in nature and bound by the requirements of society for a high and constant degree of accuracy. The elective syllabus provides the option for those who wish to do so, to build the mathematical basis that is required for further studies in mathematics beyond secondary level.

In closing the there are two comments that encapsulate the spirit of this study. Ernest (2000, p. 4) wrote "…more mathematical skills beyond the basic are not needed among the general population in an industrialised society." This understanding is complemented in mathematics education with the realization that, "…the mathematics of the crowd is different from the lovers of wisdom" Restivo (1992, p. 141).

ENDNOTES

1. In order to differentiate between chapters that are referred to in the literature and chapters that are referred to in this study, the chapters referred to in this study are capitalised, for example, Chapter 5.

2. Bekker referencing is used for *Metaphysics*, Aristotle (trans. 1952) in this chapter.

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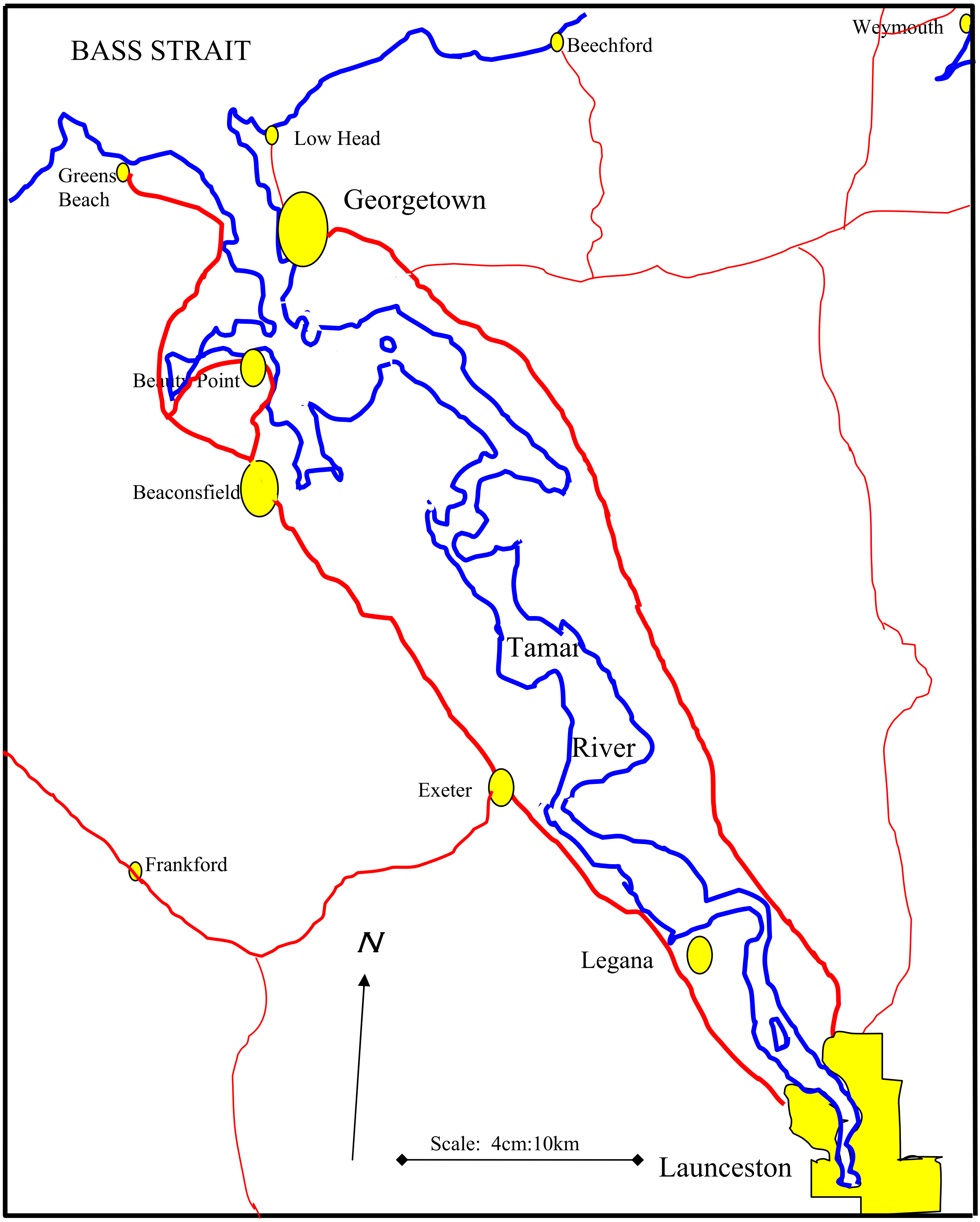
APPENDIX A

Area for the research on the mathematics used in the everyday life of adults

Fig. A.1 *Launceston and the Tamar Valley*



APPENDIX B

Questionnaires used to research the mathematics used in the everyday life of adults

There were two questionnaires put to the population that the survey encompassed. The first was designed for people outside the workplace and the second for people inside the workplace. Each questionnaire consisted of 4 and 5 pages of questions respectively, set out in 12 pt. New Times Roman, and with 3 exceptions, there are 3 possible answers to each question as can be seen in Table B.1. A large number of questions form a core that can be found in each questionnaire, with the balance of questions being appropriate to the particular environment. Examples of these include questions regarding shopping for those outside the workplace as opposed to using a till or dedicated calculator for those inside the workplace

Table B.1
Sample of questionnaire layout

No.	Please tick the column you think best describes your use of a mathematical skill since you left school	often	some times	never
	Arithmetic			
202	addition,			
203	subtraction,			

For brevity, the records of the questionnaires found in Appendix B (Tables B.2 to B.14) exclude the columns for the answers. The number column was not in the original questionnaire and is included so that questions can be referenced between the tables in Chapters, 5, 7 and Appendices B and C.

Table B.2
Questionnaire for people outside the workplace (Part 1)

No	Please tick the column you think best describes your use of a mathematical skill since you left school
	Arithmetic
1	reading numbers (in the newspaper, prices, wages etc),
2	addition,
3	subtraction,
4	multiplication,
5	division,
6	fractions,
7	percentage,
8	decimals,
9	exponents or powers such as 10 cubed,
10	ratios (for example when mixing fuels or cooking).
	Technology
11	reading a scale to measure a weight, length or temperature,
12	using a calculator to work out mathematical problems,
13	using a computer spreadsheet to work out mathematics problems.
	Geometry
14	area, say for material, painting a wall, lawn seed etc.,
15	surface area or volume of a box,
16	diameter, radius, surface area or volume of a sphere,
17	diameter, radius, area, or perimeter of a circle,
18	diameter, radius, surface area or volume of a cylinder,
19	angles such as 30, 45 or 90 degrees.

Table B.2
Questionnaire for people outside the workplace continued (Part 2)

No	Please tick the column you think best describes your use of a mathematical skill since you left school
	Statistics and probability
20	average or mean (for example, temperature or wage),
21	median (such as house price or tax),
22	odds (typically on a horse race),
23	chance (could you survive, win the lottery, calculate the fall of a card),
24	understanding a graph in a newspaper.
	Investing money
25	cost of a marketable parcel of shares, say 500 at $4.55,
26	selling price if you want to make a 10% profit,
27	brokerage charge,
28	dividend you expect to be paid,
29	dividend yield,
30	price earning ratio,
31	selling or buying price from a price graph,
32	selling or buying price from moving averages.
	When you do things each day do you recognize when:
33	one thing is bigger than another,
34	there are a few of these things but many of those,
35	one thing is a smaller version of another,
36	something is heavier than something else,
37	lots of objects will fit into a container.

Table B.2
Questionnaire for people outside the workplace continued (Part 3)

No	**Please tick the column you think best describes your use of a mathematical skill since you left school**
	In completing a tax return have you worked out:
38	total earnings for the year,
39	total tax paid for the year,
40	business deductions,
41	dependants' deductions,
42	medicare deductions,
43	tax owing or owed?
	When looking at insurance cover on anything you own, have you worked out the:
44	total value of your house, car, boat etc.,
45	total value of the contents of any of the above,
46	value of specific items at the time of insurance,
47	the cost of insurance as a percent of the value of what is being insured?
	Domestic finance. Have you checked that:
48	your wages are correct,
49	any amount drawn from your wages is correct,
50	your tax return is correct,
51	the discount stated by a shop is correct,
52	your shopping bill is correct,
53	the interest charged on a hire purchase agreement is correct,
54	the amount owing on your mortgage is correct,
55	your power bill is correct?

Table B.2
Questionnaire for people outside the workplace continued (Part 4)

No	**Please tick the column you think best describes your use of a mathematical skill since you left school**
	Have you used:
56	travel graphs (for a picture of distance, time and speed of a journey),
57	an annuity formula (to find out how much you have to put into a savings account at regular intervals to have a holiday or pay cash for a car, perhaps in two years time),
58	a depreciation formula (to calculate depreciation for tax purposes),
59	probability to work out the odds of drawing a card to make a pair if you hold the 2 of clubs and there are 34 cards left in the pack including the 2 of hearts and the 2 of diamonds?
	Arithmetic. Have you done any calculations like:
60	158.74 + 176.24 + 12.87 + 18.66 + 4.25 + 1.95,
61	12,764.87 - 4,321.64,
62	2 times 36 grams,
63	48 millilitres divided by 4,
64	half of 72 centimetres,
65	23% x $24,864.00,
66	0.75 of 124,
67	10 x 10 x 10 x 10 x 10 x 10, or 10 to the power of 6,
68	A mixture of 50:1,
69	calculate your average speed if you had to walk 15km in 6 hours?

Table B.2
Questionnaire for people outside the workplace continued (Part 5)

No	Please tick the column you think best describes your use of a mathematical skill since you left school
	Geometry and trigonometry (space). Do you ever work out problems like these?
70	work out the length of a ladder needed if the foot is to make an angle of 85 degrees to the ground, and the head reach the top of a wall that is 4.2m in height,
71	find the angles needed at the corners of a shade cloth that has sides of 3.2m, 3.7m and 2.6m,
72	make an accurate scale drawing of the house/flat you are living in,
73	calculate how many litres of water a cylindrical tank would hold if it has a diameter of 2.8m and a height of 2.6m,
74	estimate the time it will be say in London or Auckland, when it is 7am here,
75	calculate the latitude and longitude if you had travelled 320km from your home in a direction due east,
76	find where you are on a map if you had travelled 320km from your home in a direction that is due east?
	In your hobby, sport, recreation or other interests:
77	Do you use any mathematical skills not listed here?
78	Did you learn these after you left school?
79	Were these skills you learnt at school but had to revise to use again?
	Are there any mathematical skills you have used that have been missed? List these below please.
	1
	2
	3

Table B.2
Questionnaire for people outside the workplace continued (Part 6)

No	Please tick the appropriate column for *always* if the answer must always be correct, *some-times* if the answers can be sometimes wrong, and *approx* if a rough answer will do
	This section is about the accuracy needed in calculations
80	How often do you expect your shopping bill to be correct?
81	How often do you expect your bank statement to be correct?
82	How often do you expect the amount of car fuel you ordered to be correct?
83	How often do you expect a discount advertised for a product to be correct?
84	How often do you expect the government to be correct in the amount of tax you have to pay?
85	How often do you expect the government to be correct in the amount of money you receive for child endowment, a pension or other payment?
86	How often should your answer be correct if you are checking your shopping bill?
87	How often would you like be correct when putting ingredients together to make a cake or other dish?
88	How often must you be correct when putting oil and petrol together to make 2 stroke fuel?
89	How often do you expect a chemist to put the correct number of pills of the correct weight in your prescription?
90	How often should the check-out person to count out the correct change from the till for your purchases?
91	How often do you expect the correct quantity of heating fuel that you have paid for to be left (wood, oil, gas, coal)?
	Are there any further comments you would like to make?
	1
	2
	3

Table B.3
Questionnaire for people inside the workplace (Part 1)

No.	Please tick the column you think best describes your use of a mathematical skill since you left school
	Arithmetic
201	reading numbers (in the newspaper, prices, wages etc),
202	addition,
203	subtraction,
204	multiplication,
205	division,
206	fractions,
207	percentage,
208	decimals,
209	exponents or powers such as 10 cubed,
210	ratios (for example when mixing fuels or cooking).
	Technology
211	reading a scale to measure a weight, length or temperature,
212	using a calculator to work out mathematical problems,
213	using a computer spreadsheet to work out mathematics problems.
	Geometry
214	area, say for material, painting a wall, lawn seed, bricking, etc.,
215	volume of a box or carton,
216	diameter, radius, surface area or volume of a sphere,
217	diameter, radius, area, or perimeter of a circle,
218	diameter, radius, surface area or volume of a cylinder,
219	angles such as 30, 45 or 90 degrees.

Table B.3
Questionnaire for people inside the workplace continued (Part 2)

No.	**Please tick the column you think best describes your use of a mathematical skill since you left school**
	Algebra
220	formulas as they are written (for example; area = length x breadth),
221	formulas that have to be rearranged (for example, if the formula you are given is area = length x breadth, and you want to find the length, the rearranged formula will be; length = area / breadth ,
222	proofs to show that the formula you are using will work under the circumstances where it is applied,
223	formulas you have had to construct (such as applied in engineering or scientific research).
	Statistics
224	average or mean (e.g. temperature or wage),
225	median (e.g. house price or tax),
226	graphs when they show how things (e.g. products) compare with each other,
227	standard deviation (e.g. sales of a product),
228	regression analysis (e.g. profits over a period of years) .
	Trigonometry
229	plane triangle formulae for estimating length, area or angle,
230	spherical triangle formula (e.g. Cosine Rule),
231	trigonometrical function graphs.
	Calculus
232	differentials that are already given to you,
233	differentials that you have to derive,
234	integrals that are already given to you,
235	integrals that you have to integrate,
236	proofs of differentials you have used,
237	proofs of integrals you have used.

Table B.3
Questionnaire for people inside the workplace continued (Part 3)

No.	Please tick the column you think best describes your use of a mathematical skill since you left school
	Other higher mathematical skills
238	complex numbers,
239	vectors,
240	matrices,
241	conics,
242	series,
243	set language and logic.
	Statistics and probability. Do you ever work out problems like these?
244	Work out the chance of a faulty product occurring in an assembly line.
245	Find the likelyhood that profits will increase from a graph of consumer demand.
246	Calculate a line of best fit to interpret the information in a scatter graph.
247	Work out a standard deviation (to describe the spread of information you have about something such as the number of people who might buy your product).
	Do you use any of the following technology at work?
	('Dedicated' implies you put in the numbers and the machine does the work).
248	A till or dedicated calculator,
249	a computer with dedicated software,
250	a computer without dedicated software,
251	a calculator without dedicated software,
252	a graphics calculator or calculator assisted algebra system.

Table B.3
Questionnaire for people inside the workplace continued (Part 4)

No.	**Please tick the column you think best describes your use of a mathematical skill since you left school**
	When investing money have you worked problems like these?
253	Finding the cost of a marketable parcel of shares, say 500 at $4.55,
254	the selling price if you want to make a net 10% profit,
255	finding the cost of a brokerage charge,
256	working out the dividend you expect to be paid,
257	establishing a dividend yield,
258	calculating a price earning ratio,
259	working out a selling or buying price from a price graph,
260	creating moving averages for selling or buying prices.
	In completing a tax return have you worked out any of the following?
261	Your total earnings for the year,
262	the total tax paid for the year,
263	your business deductions,
264	any dependants deductions,
265	all medicare deductions,
266	any tax owing or owed,
267	depreciation for tax purposes using a depreciation formula.
	In your work have you done any calculation like the following?
268	Working out deductions for travel,
269	depreciation on equipment/clothing,
270	the cost of replacing equipment/clothing,
271	the cost of attending a conference involving travel and accommodation,
272	the amount you are paid per actual hour of work, rather than attendance at a workplace. This may be more or less than your stated wage/salary or package,
273	the cost of changing jobs.

Table B.3
Questionnaire for people inside the workplace continued (Part 5)

No.	**Please tick the column you think best describes your use of a mathematical skill since you left school**
	When looking at insurance cover on anything you own, have you worked out the:
274	the cost of insurance as a percent of the value of what is being insured,
275	and/or insurance as a percentage of income?
	Have you used any of the following skills in your workplace?
276	Break even analysis (for finding out when a business will make a profit),
277	travel graphs (for a picture of distance, time and speed of a journey),
278	linear programming (to find out which of two options you should choose),
279	annuity formula (to find out how much you have to put into a savings account at regular intervals to buy a new machine or develop a new market say in five years time).
	If you run your own business do you ever have to work out:
280	your travel costs,
281	the cost of the materials you use,
282	a profit margin between buying something and selling it with all the business costs involved,
283	the total cost of a job including insurances, supernannuation etc.,
284	the interest charged on a hire purchase agreement is correct,
285	the amount owing on your mortgage is correct,
286	your power bill is correct?

Table B.3
Questionnaire for people inside the workplace continued (Part 6)

No.	Please tick the column you think best describes your use of a mathematical skill since you left school
	Arithmetic. Have you done any calculations like:
287	158.74 + 176.24 + 12.87 + 18.66 + 4.25 + 1.95,
288	12,764.87 – 4,321.64,
289	2 times 36 grammes,
290	48 millilitres divided by 4,
291	half of 72 centimetres,
292	23% x 24,864.00,
293	0.75 of 124,
294	10 x 10 x 10 x 10 x 10 x 10, or 10 to the power of 6,
295	A mixture of 50:1?
	Algebra. Have you done any calculations like:
296	simplify 0.5b + 0.5h,
297	expand 9.8(h + t),
298	factorise 16a + 12o + 8b,
299	graph 3y = 4x – 7,
300	solve for x, 8 = 3x + 8y,
301	find the intersection when 4x + 7 = y and 6 – 3x = y?
	Geometry and trigonometry (space). Do you ever work out problems like:
302	work out the height of a ladder needed if the foot is to make an angle of 85 degrees to the ground, and the head reach the top of a wall that is 4.2m in height,
303	find the angles needed at the corners of a shade cloth that has sides of 3.2m, 3.7m and 2.6m,
304	make an accurate scale drawing of the dwelling you are living in,
305	calculate how many litres of water a cylindrical tank would hold if it has a diameter of 2.8m and a height of 2.6m,
306	estimate time it will be say in London or Auckland, when it is 7am here?

Table B.3
Questionnaire for people inside the workplace continued (Part 7)

No.	Please tick the appropriate column for ***always*** if the answer must always be correct, ***some-times*** if the answers can be sometimes wrong, and ***approx*** if a rough answer will do
	This section is about the accuracy needed in calculations
307	How often do you expect your bank statement to be correct?
308	How often would you like the amount of goods you ordered to be correct?
309	How often do you expect a discount advertised for a product to be correct?
310	How often would you like the government to be correct in the amount of tax you have to pay?
311	How often must you be correct when putting ingredients together to make something (e.g. cement, glue, paint, fuel)?
312	How often must you choose the right mathematical technique to apply when using a calculator?
313	How often must applying a formula in your trade or profession be correct?
314	How often must you be correct when sending out an invoice to a customer?
315	How often must you be correct when calculating wages?
	Are there any mathematical skills you have used that have been missed?
	1
	2
	3
	4
	Are there any further comments you would like to make?
	1
	2
	3
	4
	What is your occupation please?

APPENDIX C

Questionnaires with unranked data

The first column in Tables C.1 and C.2 refer to the question numbers found in the first column (No.) in Tables B.2 and B.3 and the second column the final percentile of usage calculated for that question. Parts of section 5.2.2 are repeated here to facilitate the interpretation of Tables C.1 and C.2.

Data Processing

Often/always responses were assigned a loading of 8, sometimes responses a loading of 5, and never/approximate responses a loading of 0. The number of replies to each question were added together and weighted so that each question had the same value.

Using question 20 as an example, the weighting was 60/59, there being 59 replies out of a possible 60. This weighting was applied to the total of the results arising from the 3 responses;

often –	11 replies	x	loading 8	=	88
sometimes –	39 replies	x	loading 5	=	195
never –	9 replies	x	loading 0	=	0
sum of results - 283		x	weighting 60/59	=	287.7966

The maximum weighted result for a question was 60 x 8 = 480.

The weighted result was then changed to a percentile and rounded to 2 decimal places thus;

Total 287.7966 / 4.8 = 59.96

Numbers in the "Class" column refer to the class the question is grouped in, for example the Arithmetic class is number 1. Further details are found in Tables D.1 and D.2. The final column gives the number of the same question that recurs in Tables B.3 and C.2.

There were 53 questions that were common to the two questionnaires, the first of which was for people outside, and the second for people inside the workplace. Where a question was common, a mean result was calculated using the two weighted results expressed as a percentile, from each questionnaire. As an example using the question regarding the use of fractions, the following procedures were carried out. In the questionnaire for people outside the workplace the weighted result expressed as a percentile for this question (number 6) was 68.32 (Table C.1), and in the questionnaire for people inside the workplace the same question (number 206) had a weighted percentile result of 70.83 (Table C.2). The mean 69.58, was then allocated the question number from Table C.1 (6) and used for ranking (17^{th}) in Table 5.2.

Table C.1

Data arising from questions put to people outside the workplace (Part 1)

Table B.2 No.	Total	Often	Some times	Never	Class	Table B.3 No.
1	96.25	54	6	0	1	201
2	95.55	52	7	0	1	202
3	94.83	50	8	0	1	203
4	91.10	45	14	0	1	204
5	88.14	42	16	1	1	205
6	68.32	24	25	9	1	206
7	82.76	33	24	1	1	207
8	76.10	29	23	5	1	208
9	37.93	7	24	27	1	209
10	72.50	23	27	5	1	210
11	78.96	33	23	4	2	211
12	73.94	28	25	6	2	212
13	37.71	11	18	30	2	213
14	51.06	12	29	18	3	214
15	35.13	6	23	29	3	215
16	22.84	2	18	38	3	216
17	31.03	3	24	31	3	217
18	26.72	3	20	35	3	218
19	53.64	12	28	15	3	219
20	59.96	11	39	9	4	224
21	50.42	6	38	15	4	225
22	38.14	5	28	26	4	226
23	33.99	5	23	29	4	
24	68.97	15	40	3	4	

Table C.1
Data arising from questions put to people outside the workplace continued (Part 2)

Table B.2 No.	Total	Often	Some times	Never	Class	Table B.3 No.
25	39.62	9	23	27	5	253
26	52.12	12	30	17	5	254
27	33.19	8	18	32	5	255
28	35.81	8	21	30	5	256
29	31.36	6	20	33	5	257
30	30.51	8	16	35	5	258
31	29.24	6	18	35	5	259
32	21.98	4	14	40	5	
33	93.64	49	10	0	6	
34	87.71	43	14	2	6	
35	87.93	41	16	1	6	
36	88.35	44	13	2	6	
37	63.22	21	19	12	6	
38	76.91	36	15	8	7	261
39	76.91	36	15	8	7	262
40	58.62	24	16	18	7	263
41	54.61	23	13	21	7	264
42	66.10	29	16	14	7	265
43	66.53	28	18	13	7	266
44	77.54	32	22	5	8	
45	75.85	31	22	6	8	
46	70.34	29	20	10	8	
47	45.97	14	21	24	8	

Table C.1
Data arising from questions put to people outside the workplace continued (Part 3)

Table B.2 No.	Total	Often	Some times	Never	Class	Table B.3 No.
48	80.72	37	17	5	9	
49	78.60	37	15	7	9	
50	74.35	35	13	10	9	
51	81.78	37	18	4	9	
52	78.07	32	20	5	9	
53	50.44	20	14	23	9	
54	47.37	17	16	24	9	
55	62.93	24	20	14	9	
56	45.91	6	33	19	10	277
57	32.76	4	24	30	10	279
58	27.63	2	22	33	10	
59	16.81	1	14	43	10	
60	70.26	22	30	6	11	287
61	71.55	24	28	6	11	288
62	66.10	24	24	11	11	289
63	62.71	22	25	13	11	290
64	72.71	28	25	7	11	291
65	59.11	18	27	14	11	292
66	61.04	21	25	14	11	293
67	30.63	4	23	33	11	294
68	57.84	16	29	14	11	295
69	47.71	13	25	22	11	
70	18.96	2	15	43	12	302
71	19.28	2	15	42	12	303
72	35.81	3	29	27	12	304

Table C.1
Data arising from questions put to people outside the workplace continued (Part 4)

Table B.2 No.	Total	Often	Some times	Never	Class	Table B.3 No.
73	21.49	1	18	38	12	305
74	57.92	11	38	11	12	306
75	21.67	3	16	41	12	
76	28.96	3	23	34	12	
80	91.52	45	10	1	13	
81	97.32	52	4	0	13	307
82	95.31	49	7	0	13	
83	92.19	46	9	1	13	309
84	89.06	43	11	2	13	310
85	89.09	44	8	3	13	
86	90.18	43	12	1	13	
87	86.16	42	10	4	13	
88	93.75	50	4	2	13	
89	98.66	54	2	0	13	
90	93.08	49	5	2	13	
91	92.73	46	8	1	13	

Note. Questions 77, 78 and 79 only required yes/no responses and were excluded these tables.

Table C.2
Data arising from questions put to people inside the workplace (Part1)

Table B.3 No.	total	often	some times	never	class
201	96.25	54	6	0	20
202	98.13	57	3	0	20
203	98.13	57	3	0	20
204	95.00	52	8	0	20
205	93.13	49	11	0	20
206	70.83	25	28	7	20
207	86.44	41	16	2	20
208	81.99	39	15	5	20
209	37.92	9	22	29	20
210	66.35	17	28	7	20
211	83.33	40	16	4	21
212	90.04	45	13	1	21
213	40.25	10	22	27	21
214	53.13	10	35	15	22
215	38.98	8	24	27	22
216	31.67	4	24	32	22
217	36.65	6	25	28	22
218	31.88	6	21	33	22
219	57.11	15	29	14	22
220	37.71	11	18	30	23
221	27.12	6	16	37	23
222	13.13	1	11	48	23
223	9.60	1	7	48	23

Table C.2
Data arising from questions put to people inside the workplace continued (Part 2)

Table B.3 No.	total	often	some times	never	class
224	58.75	19	26	15	24
225	46.46	11	27	22	24
226	52.50	14	28	18	24
227	44.07	11	24	24	24
228	40.00	9	24	27	24
229	21.46	1	19	40	25
230	6.88	1	5	54	25
231	2.12	0	2	57	25
232	10.53	1	8	48	26
233	5.04	1	3	53	26
234	4.39	0	4	53	26
235	5.48	0	5	52	26
236	2.19	0	2	55	26
237	2.23	0	2	54	26
238	8.05	1	6	52	27
239	3.81	1	2	56	27
240	1.06	0	1	58	27
241	1.06	0	1	58	27
242	3.18	0	3	56	27
243	3.81	1	2	56	27
244	16.67	2	12	43	28
245	21.31	3	16	42	28
246	12.50	3	7	49	28
247	14.91	1	12	44	28

Table C.2
Data arising from questions put to people inside the workplace continued (Part 3)

Table B.3 No.	total	often	some times	never	class
248	70.34	34	12	13	29
249	68.54	33	13	14	29
250	42.92	17	14	29	29
251	47.46	23	8	28	29
252	0.00	0	0	59	29
253	33.41	10	15	33	30
254	51.08	19	17	22	30
255	31.78	10	14	35	30
256	33.90	10	16	33	30
257	21.98	4	14	40	30
258	30.51	8	16	35	30
259	20.13	5	11	43	30
260	20.76	6	10	43	30
261	67.58	23	27	9	31
262	66.53	23	26	10	31
263	59.32	20	24	15	31
264	48.31	16	20	23	31
265	53.81	18	22	19	31
266	55.30	17	25	17	31
267	33.19	8	18	32	31
268	54.45	14	29	16	32
269	37.29	12	16	31	32
270	54.74	13	30	15	32
271	43.53	9	26	23	32
272	51.04	15	25	20	32
273	32.31	4	21	28	32

Table C.2
Data arising from questions put to people inside the workplace continued (Part 4)

Table B.3 No.	total	often	some times	never	class
274	30.32	7	15	32	33
275	19.91	2	14	38	33
276	30.72	10	13	36	34
277	13.56	3	8	48	34
278	9.27	1	7	50	34
279	18.86	3	13	43	34
280	55.45	16	9	14	35
281	55.77	18	6	15	35
282	60.20	16	11	11	35
283	47.70	10	13	15	35
284	44.59	9	12	16	35
285	49.01	8	17	13	35
286	47.92	6	18	12	35
287	74.34	33	15	9	36
288	68.75	31	12	13	36
289	65.73	25	21	12	36
290	64.66	25	20	13	36
291	69.61	26	23	9	36
292	58.62	24	16	18	36
293	60.09	23	18	16	36
294	39.87	10	21	27	36
295	62.50	20	24	12	36

Table C.2
Data arising from questions put to people inside the workplace continued (Part 5)

Table B.3 No.	**total**	**often**	**some times**	**never**	**class**
296	11.18	2	7	48	37
297	11.18	2	7	48	37
298	10.09	2	6	49	37
299	8.04	2	4	50	37
300	12.05	3	6	47	37
301	6.92	2	3	51	37
302	14.22	2	10	46	38
303	14.47	2	10	45	38
304	31.47	7	18	33	38
305	25.65	3	19	36	38
306	38.58	8	23	27	38
307	96.77	53	5	0	39
308	97.41	54	4	0	39
309	94.83	50	8	0	39
310	100.00	58	0	0	39
311	88.36	45	10	3	39
312	92.46	48	9	1	39
313	89.66	47	8	3	39
314	100.00	58	0	0	39
315	100.00	58	0	0	39

APPENDIX D

Classes of questions

Table D.1
Classes of questions in the questionnaire for people outside the workplace

Number	Name
1	Arithmetic
2	Technology
3	Geometry
4	Statistics and probability
5	Investing money
6	Relative size or weight
7	Completing a tax return
8	Looking at insurance cover
9	Domestic finance
10	Have you used:
11	Arithmetic problems
12	Geometry and trigonometry (space) problems
13	Accuracy
14	Hobby, sport, recreation or other interests

Table D.2
Classes of questions in the questionnaire for people inside the workplace

Number	Name
20	Arithmetic
21	Technology
22	Geometry
23	algebra
24	Statistics
25	Trigonometry
26	Calculus
27	Other higher mathematical skills
28	Statistics and probability problems
29	Technology use
30	Investing money problems
31	Tax return problems
32	Work related cost calculations
33	Insurance cover problems
34	Use of higher applied mathematics skills
35	Own business problems
36	Arithmetic problems
37	Algebra problems
38	Geometry and trigonometry (space) problems
39	Accuracy

APPENDIX E

Secondary mathematics texts

E.1 Contents

Pendlebury, C. (1896).
Arithmetic.
Ninth edition, first edition 1886.
London: George Bell and Sons.

Part 1

Part 2

Pendlebury, C. (1896). Part 2 (cont.)

Appendices

Tables

Note. In the original contents, chapters 1-49 were numbered using Roman numerals.
The tables were found on page xi, preceding the Introductory chapter.

Clapp, E. K., Hamann, K. M., and Lang, I. P. (1969)
Geometry 1
In the *Secondary Mathematic Series (SMS)*
Second edition, first published 1967.
Adelaide: Rigby Limited.

Unit

1	Points, Lines and Surfaces
2	Plane Figures or Polygons
3	Angles
4	Simple Bearings
5	Angle Properties
6	The Triangle
7	More about Polygons
8	Polyhedra
9	Transformation Geometry
10	Symmetry
11	Congruence
12	Congruence Transformations Combined
13	General Revision Questions

Clapp, E. K., Hamann, K. M., and Lang, I. P. (1972)
Geometry 3.
Third edition, first published 1968.
Adelaide: Rigby Limited.

Unit

1	Review exercises
2	Locus
3	Geometry in Action
4	Transformation and Coordinates
5	Trigonometry
6	The Circle
7	Symmetry and Groups
8	Coordinate Geometry
9	Vectors
10	Reasoning and Proof

Stambulic, S., Iampolski, E., Phillips, D., & Watson, J. (2002).
Maths Quest 10 for Victoria.
Melbourne: John Wiley & Sons, Ltd.

Chapter 1	Rational and irrational numbers
	Operations with fractions
	Finite and recurring decimal numbers
	Irrational numbers
	Simpifying surds
	Addition and subtration of surds
	Multiplication and division of surds
	Writing surd fractions with a rational denominator
Chapter 2	Algebra and equations
	Operations with pronumerals
	Subsituting into expressions
	Expanding
	Factoring using common factors
	Adding and subtracting algebraic fractions
	Multiplying and dividing algebraic fractions
	Solving basic equations
	Solving more complex equations
	Solving inequations
Chapter 3	Linear graphs
	Plotting linear graphs
	Sketching linear graphs
	Finding linear equations
	Linear modelling
	Sketching linear equations
Chapter 4	Quadratic equations
	Expanding algebraic expressions
	Factorising expressions with two or four terms
	Factorising expressions with three terms
	Factorising by completing the square
	Mixed factorisation
	Solving quadratic equations
	Using the quadratic formula
	Using the discriminant
Chapter 5	Quadratic graphs
	Plotting parabolas
	Sketching parabolas using the basic graph of $y=x^2$
	Sketching parabolas in turning point form
	Sketching parabolas in the form $y=ax^2+bx+c$
	Solving quadratic inequations using sketch graphs

Stambulic, S., Iampolski, E., Phillips, D., & Watson, J. (2002), (cont.).

Chapter 6	Variation
	Direct variation
	Direct variation – the constant of variation
	Direct variation and ratio
	Partial variation
	Inverse variation
	Other forms of direct and inverse variation
	Identifying the type of variation
	Joint variation
Chapter 7	Simultaneous equations
	Graphical solution of simultaneous equations
	Algebraic solution of simultaneous equations
	Algebraic solution of simultaneous equations – substitution method
	Problems using simultaneous equations
	Solving a quadratic equation and a linear equation simultaneously
	Solving simultaneous inequations
Chapter 8	Exponential functions
	Index laws
	Negative indices
	Fractional indices
	Further use of index laws
	Exponential functions and their graphs
	Modelling exponential growth and decay
Chapter 9	Measurement
	Accuracy of measurement
	Estimation and approximation in measurement
	Error
	Perimeter
	Area
	Total surface area
	Volume
	Time, speed, density and concentration
Chapter 10	Circle geometry
	Intersecting chords, secants and tangents
	Angles in a circle
	Cyclic quadrilaterals
	Great circles
	Locus
Chapter 11	Further geometry
	Review of 2-dimensional and 3-dimensional drawing
	Cross sectional view of objects
	Similarity
	Congruence
	Nets, polyhedra construction and Euler's rule

Stambulic, S., Iampolski, E., Phillips, D., & Watson, J. (2002), (cont.).

Chapter 12	Trigonometry
	Using a calculator
	Using trigonometric ratios to find side lengths
	Using trigonometrical ratios to find angles
	Applications
	Bearings
	The unit circle – quadrant 1
	Circular functions
	Graphs of $y = \sin \theta$ and $y = \cos \theta$
Chapter 13	Probability
	Probability revision
	Complementary events
	Mutually exclusive events
	Lattice diagrams and tree diagrams
	Independent and dependent events
	Footy card collecting
	Subjective probability
	Maths Quest challenge
Chapter 14	Statistics
	Collecting data
	Presenting categorical and discrete data
	Representing data grouped into class intervals
	Measures of central tendency
	Measures of spread
	Bivariate data
	Lines of best fit
	Time series

E.2 Details of problems referred to in Chapter 5.

Note. Each problem is referenced to the author, date of publication and text, followed by the problem number (if there is one) and page in the text.

Pendlebury (1886). *Arithmetic.*

Example (p. 71).

£130264. 9s. 6d by 9416. This was a straightforward long division calculation with money involved, as opposed to applied problems in compound interest. In this example, the first step was to divide 130264 by 9416. As the answer would be more than 10, this process was split into 2 parts thus:

9416)130264	
94160	[9416 x 10]
36104	[130264 – 94160]
28248	[9416 x 3]
7856	[36104 – 28248]

This remainder was changed from pounds to shillings by multiplying by 20.

The following process was detailed in the text:

£.	s.	d.
9416) 130264	9	6
94160		
36104		
28248		
. 7856		
20		
157120 shillings		
94160		
62960		
56496		
6474		
12		
77688 pence		
75328		
2360		
4		
9440 farthings		
9416		
24		

<u>The quotient = £13. 16s. 8¼d</u>.

Pendlebury (1886), (cont.).

Question 19 (p. 147).

Find the value of 2.8125 of a cwt.
The student was given, 1 cwt (hundredweight) = 112lb (pounds) and 1lb = 16oz (ounces) in Avoirdupois Weight tables found in the opening pages of the text (p. xi).

Question 5, p. 265.

A can dig a trench in 3 days, B in 4 and C in 5 days. How long will it take them all together to dig the same length of ditch, and what proportion of the work will be done by each?

Question 220 (p.379).

A contractor agrees to sink an artesian well on the following terms: £5 per fathom for the first 30 fathoms consisting of gravel only, £35 per fathom through sand, £20 per fathom through clay, and £25 per fathom through chalk. He found the thickness of the chalk to be equal to half the depth of the well, and that the sand beds were 3 times as thick as the clay. At the end of the work he got £2700 for his trouble. What was the depth of the well?

Question 8 (p. 286).

Find by factor the square root of 63504.
- dividing by the lowest prime divisor –
2) 63504
2) 31752
2) 15876
2) 7938
3) 3969
3) 1323
3) 441
3) 147
7) 49
7) 7
1

$2^4 \times 3^4 \times 7^2 = 63504$
$\sqrt{(2^4 \times 3^4 \times 7^2)} = \sqrt{63504}$
$2^2 \times 3^2 \times 7 = \sqrt{63504} = 252$

Durell (1936). *General Arithmetic for Schools.*

Question 11 (p. 300).

Entertainment tax yielded £6,952,088 in 1931 and £7,868,908 in 1932. Find the increase percent.

Question 7 (p. 107).

To travel from one house to another, I can either motor from door to door at an average speed of 20 m.p.h., or I can go by train at an average speed of 45 m.p.h.; but if I travel by train it is necessary to allow 20 min. at each end for the journeys between the houses and the stations. Find graphically the shortest distance for which it is quicker to go by train, assuming that the distance between the stations is the same as that between the houses.

Clapp, Close, Hamann, Lang & McDonald (1970). *Secondary Mathematics Series (SMS) Arithmetic 1.*

Question 1 (p. 22).

Express the relation $n(A \cup B) = nA + nB - n(A \cap B)$ in words.

McSeveny, Conway and Wilkes (2004), *New Signpost Mathematics 8: Stage 4*.

Question 3 (p. 62).

Advertising for the show Nicholas Nickleby accounted for 7% of the production cost of \$1 500 000. How much was spent on advertising?

Question 7 (p. 126).

In our next game of soccer, the probability that we will win is 0.45 and that we will play a draw 0.23. What is the probability we will lose?

Hall and Stevens (1931). *A School Geometry.*

Question 14 (p. 102).

In a plan of a rectangular garden, the length and breadth are 3.6″ and 2.5″, one inch standing for 10 yards. Find the area of the garden.
If the area is increased by 300 sq. yds., the breadth remaining the same, what will the new length be? And how many inches will represent it on your plan?

Hall and Stevens (1931), (cont.).

Question 6 (p. 45).

In triangle ABC, the angles at B and C are 74^0 and 62^0; if *AB* and *AC* are produced, deduce the angle between the bisectors of the exterior angles. Check your results graphically.

Question 10 (p. 45).

In any *regular* polygon of *n* sides, each angle contains 2*(n*-2*)/n* right angles.
(i) Deduce this result from the Enunciation of Corollary 1.
Prove it independently by joining one vertex A to each of the others (except the two immediately adjacent to A) thus dividing the polygon into *n*-2 triangles.

Clapp, Hamann, and Lang (1969). *SMS Geometry 1.*

Question 8 (p. 65).

(1) Are there any points moved by reflection? What points?
(2) Are there any lines whose directions are not changed by a reflection? What lines?
(3) Reflection usually changes direction. Does it change length of a segment, straightness, size of angle, area?
(4) Can you slide a figure (without turning it over) on to its image under reflection?
(5) The operation of reflection is referred to as a "one to one operation". Can you give a reason for this name?

McSeveny, Conway, and Wilkes (2004). *New Signpost Mathematics 8: Stage 4*

Question 1a, (p. 153).

Given the figure of a triangle with the two sides equal and one base angle being 33^0 - find the other base angle named 'b'.

Stambulic, Iampolski, Phillips and Watson (2002). *Maths Quest 10 for Victoria*.

Question 9 (p. 91).

[It was expected that a graphical analysis would be used to solve this problem.]
Five students can clean a cafeteria in 15 minutes, three students would take 27 minutes. How long should it take one student to complete the task?

Stambulic, Iampolski, Phillips and Watson (2002), (cont.).

Question 8 (p. 367).

Prove the result: 'If a radius bisects a chord, then the radius meets the chord at right angles.'

Hall & Knight (1946). *Elementary Algebra.*

Question 4 (p. 248).

Prove the identity:
$(a + b + c)(ab + bc + ca) - abc = (a + b)(b + c)(c + a)$

Durell (1930). *A New Algebra for Schools Parts 1 and II.*

Question 63 (p. 7).

Multiply together b, a, c and double the result

Question 6 (p. 22).

With summer time the middle of the day may be taken as 1 o'clock; and so, if the Sun rises at t o'clock a.m., it sets at $14-t$ o'clock p.m. On May 20, in London, the Sun rises at 5 a.m., when does it set? What is the length of the day?

Question 2(i) (p. 34).

$a - 4 = 7$.

Question 17 (p. 37).

An excursion ticket is one-quarter of the ordinary fare. I save 5s. 6d. by taking an excursion ticket. What is the ordinary fare?

Question 15 (p. 80).

A boy walks for 5 minutes, runs for 2 minutes and stands still for 3 minutes, and then returns to his starting-point in a car.

Question 7 (p. 152).

2 knives and 4 forks cost £1. 4s.; 5 knives and 6 forks cost £2. Find the price of a knife and fork.

Hall & Knight (1946), (cont.).

Question 5 (p. 167).

$y = 2x - 1$,
$y = 9 - 2x$.

Clapp, Hamann, & Lang (1969). *SMS Algebra 1*.

Question 18(8) (p. 12).

$\{n, n \in \mathrm{Q}: 22 < 3n - 7 \leq 53\}$.

Question 7(1) (p. 50).

Draw the graph of the following relation by listing and plotting a set of ordered pairs that is the solution of the following sentence.
Take D = $\{x, x \in \mathrm{Q}: x \leq 6\}$. D = $\{(x, y) \mid y = x\}$.

Clapp, Hamann, and Lang (1972). *SMS Algebra 3.*

Question 2 (p. 111).

Two brothers each have a number of \$5 notes and together they have no more than \$90.
The older brother has at least twice as much money as the younger brother, who has at least \$20.
Write the solution of inequalities that describe this set of conditions.
Graph the system and find in how many ways it is possible for all conditions to be true.
What is the minimum amount the older brother can have?
What is the maximum amount the younger brother can have?

Question 4 (p. 140).

The number N_t of bacteria present in a second culture t seconds after it is established is given by $N_t = 500e^t$.
How many are present at (a) 1 second, (b) 2 seconds, (c) 3 seconds, (d) 6 seconds?

McSeveny, Conway and Wilkes (2004). *New Signpost Mathematics 8: Stage 4*.

Example (p. 98).

$x\ x\ x\ x = x^4$.

McSeveny, Conway and Wilkes (2004), (cont.).

Question 5l (p. 103).

Factorise:
$8pq - 4q^2$.

Question 10b (p. 106).

Simplify:
$\frac{ab}{3} \div \frac{ac}{2}$.

Question 3u (p. 249).

$2m - 3 = 5m + 1$. find m.

Question 6c (p. 255).

$c^2 = a^2 + b^2$, find a if $c = 17$ and $b = 15$.

Question 2a (p. 256).

If a number is multiplied by 3 and 5 is added to the product, the result is 17. What is the number?

Stambulic, Iampolski, Phillips & Watson (2002). *Maths Quest 10 for Victoria*.

Question 2b (p. 41).

$F = \frac{9C}{5} - 32$, calculate F when $C = 20$.

Question 2c (p. 126).

$x^2 - 5x + 2 = 0$.

Question 6 (p. 253).

Natsuko is starting to plan a monthly budget by classifying expenditures as rent and other expenses (r) and savings (x). Her total net income is \$2000 per month. She can spend no more than 30 percent of her income on rent.

(a) Write an inequation to express the fact that Natsuko can spend no more than \$2000 per month.
(b) Write an inequation to express the 30 percent rent limitation.
(c) Do any other inequations apply to this situation? Explain.
(d) Sketch a graph of the region that applies for all your inequations.
(e) State three possible solutions of allocating rent and other expenses/savings.

Stambulic, Iampolski, Phillips & Watson (2002), (cont.).

Question 7 (p. 290).

The population of a certain country is shown in the table below.

Year	1960	1965	1970	1975	1980
Population (in millions)	118	130	144	160	178

Assume that relationship between the population, P, and the year, n, can be modeled by the formula $P = ka^n$, where n is the number of years since 1960.

a. State the value of k.
b. Use the middle point of the data set to find the value of, a, rounded to two decimal places. Hence, write the formula that connects the two variables, P and n.
c. From the years given in the table, find the size of the population, using your formula. Compare the numbers obtained with the actual size of the population.
d. Predict the population of the country in the year 2005.

APPENDIX F

Outcomes

Table F.1

Extracts from Key Intended Numeracy Outcomes (1997) for grade 8 (Part 1)

No	Outcomes
2	Reflects on the processes and solution to problems and checks that solutions make sense in the original context
3	Counts, orders, estimates and describes with common fractions, decimals and percentages in a variety of contexts
4	Makes a suitable choice of operations involving whole and fractional numbers and percentages including those where more than one operation is needed
5	Estimates and calculates mentally with whole numbers , money and simple fractions including multiplying and dividing some two-digit numbers by one digit-numbers
6	Uses understood written methods to add, subtract, multiply and divide whole numbers, money and measures (two decimal places, whole number multipliers and divisors to 10)
7	Uses a calculator efficiently for operating on decimal numbers and commonly used percentages including where more than one operation is needed and interprets displays for division
9	Reads and describes information in tables (including some grouping of data), diagrams, line, bar graphs and pie graphs, fractions and means
10	Measures using conventional units and measuring equipment for length, mass, capacity, time, and angle and reading scales to the nearest marked graduation
11	Uses personal benchmarks to make sensible estimates of length, area, mass, time and capacity in common standard units and identifies unreasonable estimates of things
14	Follows and uses rules (formulae and sequenced steps) and describes relationships between related quantities expressed in word or symbols

Note. Outcomes pertinent to the core syllabus in this study

Table F.2
Extracts from Key Intended Numeracy Outcomes (1997) for grade 8 (Part 2)

Number	Outcome
1	Answering questions, testing conjectures and solving problems
8	Understanding and describing situations involving chance
12	Using spatial ideas, tools and techniques to interpret, draw and make
13	Visualising, analysing and representing arrangements and locations

Note. These are outcomes that are not used in the core syllabus in this study

APPENDIX G

Ethics

G.1 Ethics approval communication

Peter.Faulkner
From: "Marilyn Knott" <Marilyn.Knott@utas.edu.au>
To: <Jane. Watson@utas.edu.au>
Cc:<Kim. Beswick@utas.edu.au>; <peth@bigpond.net.au>
Sent: Wednesday, August 30,20062:59 PM
Subject: Ethics Application Approved: H9076 The role of applied mathematics in senior secondary education.

Dear Professor Watson

Ethics RefNo: H9076
Project title: The role of applied mathematics in senior secondary education.

This Ethics Minimal Risk application has been approved.

A signed copy of the formal approval letter will be sent to the Chief Investigator/Supervisor by mail in the next few days.

The Committee wish you all the best with the project.

Kind regards

Marilyn Knott

Marilyn Knott
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Web: http :www.research.utas.edu.au/

No virus found in this incoming message.
Checked by AVG Free Edition.
Version: 7.1.405 /VirusDatabase: 268.11.7/432 -Release Date: 8/29/06