DIRECTIONAL STUDIES IN COSMIC RAYS

WITH PARTICULAR REFERENCE TO PROCESSES INVOLVING MESONS.

THESIS SUBMITTED FOR EXAMINATION FOR THE DEGREE OF

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by

K.B. FENTON, B.Sc. (TAS.)

C O N T E N T S.

Introduction.

PART I — Variations with Zenith Angle of the Directional Intensity of Cosmic Rays.

PART II — Variation of the Directional Intensities.

PART III — The High Latitude East-West Asymmetry.

INTRODUCTION.

Cosmic ray investigations in Hobart were initiated by my brother Dr. A. G. Fenton in 1946. The problem chosen by him for investigation was the high latitude E-W asymmetry, since it seemed that, during the difficult period of building up a research programme in a field which has attracted many workers in other parts of the world, useful results might be obtained from the beginning. The first measurements carried out by my brother and Dr. D.W.P. Burbury indicated that an asymmetry existed at Hobart.

In 1948, I was able to take an active part in this work. It had become apparent that measurements in a higher geomagnetic latitude would be desirable. My brother and I therefore suggested to the Australian National Antarctic Research Expedition (A.N.A.R.E.) that measurements should be made at the Antarctic Scientific Station on Macquarie Island, which had just been established. We were able to secure a grant from A.N.A.R.E. to construct the necessary equipment for this work.

Construction was commenced early in 1949 and was completed in time to send it to the Island in March, 1950. Since I was to instal and operate the equipment on the Island for the following fourteen months, it was necessary for me to pay close attention to all details of design. The actual work was divided among Dr. A.G. Fenton, Dr. Burbury and myself. My brother undertook the design and construction of the Geiger-Muller counters, Dr. Burbury was responsible for the design and much of the construction of the mechanical features, switching mechanism for rotating the apparatus and the 35 mm recording camera. I was responsible for the design of the telescopes, for the design and construction of the electronic circuits

and vibrator power supply and of course, for a great deal of the organizing of spare components, and packing in a manner to withstand the hazardous landing operations etc.

Upon arrival at the Island, it was necessary for the other cosmic ray physicist, Mr. N.R. Parsons (who was to be engaged on another problem) and myself to erect and equip a Physics Hut, lay underground power cables from the Diesel generator (about 200 yards away), construct a cement block for the gear, build a hut for the accumulators etc. This work was completed in about six weeks and the equipment began operation on June 1, 1950.

During its operation on the Island certain features which required modification became apparent. These were made using materials at hand. The most important of these was the change of the telescopes from 2-fold to 3-fold by the addition of a third tray of counters, and some extra circuitry in order to reduce the accidental rate. The addition of the third tray was satisfactorily accomplished, but the circuit changes were make-shift. Fortunately, a ship called at the Island in February, 1951 and I was able to send back to Hobart sufficient details for new circuits to be constructed and sent down with the relief party in May, 1951.

Mr. R.M. Jacklyn took over the operation of the equipment in May and installed the new circuitry. Upon my return to Hobart, I kept in touch with Jacklyn by radio once or twice each week during his period there. I also supervised the handling of the detailed results which Jacklyn despatched by telegram each day.

During my fourteen months on the Island, detailed results were not transmitted to Hobart. Using a hand

calculating machine, I was able to perform sufficient computations to know when significant answers on any run had been obtained. Upon my return, more detailed computations using faster machines were begun.

Mr. F. Jacka of A.N.A.R.E. had developed the punched card method of obtaining sums of squares and sums of products required in correlation analyses. Therefore, in the year during which Jacklyn was on the Island, the results were punched on Hollerith cards in Hobart and sent to Jacka (in Melbourne) who used Hollerith sorting and tabulating machines belonging to the Commonwealth Meteorological Bureau. The sums of squares and products were sent back to Hobart enabling us to work out correlation and regression coefficients etc. Incidentally, even with the aid of the Hollerith machines, these computations have occupied a full-time computor for more than a year, and some calculations are still not complete.

As a rule, the aspects of the work for which I have been responsible will be clear. We may briefly mention the more important of these.

Firstly, in the Appendix to Part I, a table is presented giving the range of μ-mesons in air as a function of momentum. The calculations on which this is based were performed during my stay at Macquarie Island. These tables have proved very useful for the determination of the expected E-W asymmetry, of expected barometer coefficients, and of differential momentum spectra at various zenith angles when the vertical spectrum is known. Also during my period on the Island, I realised the need for modifications to Johnson's theory of the E-W effect, and some of the necessary computations

were made. Upon my return to Hobart, it was found that Dr. Burbury had been working along similar lines and as a result, a joint paper was published (ref. III.5)). Since then I have examined the problem still more closely and the discussion in Part III is a result of the examination.

The interpretation of the results obtained at Macquarie Island is largely due to me, although of course, there have been numerous discussions with other members of the Laboratory.

The soft component telescope, being used for the investigation of the zenith angle variation of the electronic component (discussed in Part I, section C) is of my design and construction.

It will be clear that the success of a programme of this nature largely depends upon the co-operation and enthusiasm of many people. To Professor A.L. McAulay, Professor of Physics, thanks are due for the support he has given throughout, but particularly during the period when the equipment for the Macquarie Island experiments was being constructed. I have had several valuable discussions with Professor E.J.G. Pitman, Professor of Mathematics, on the statistical portion of this study. I am also indebted to Mr. N.R. Parsons and Mr. F. Jacka for discussions on statistics. My thanks are due, of course, to all members of the party stationed at Macquarie Island, particularly to Mr. Parsons whose assistance at all times was greatly appreciated. To Mr. R.M. Jacklyn, credit is due for operating the equipment so successfully at Macquarie Island during its second year there. Thanks are due to Dr. D.W.P. Burbury for the part he played in the construction of the Macquarie Island equipment, and for the

collaboration during the preparation of our joint publication on the E.W. asymmetry. Finally I must thank my brother, Dr. A.G. Fenton, leader of the cosmic ray group at Hobart, who has taken a very keen and direct interest in all aspects of the work and with whom I have had many valuable discussions on the interpretation of results.

PART I.

VARIATIONS WITH ZENITH ANGLE OF THE DIRECTIONAL INTENSITY OF COSMIC RAYS.

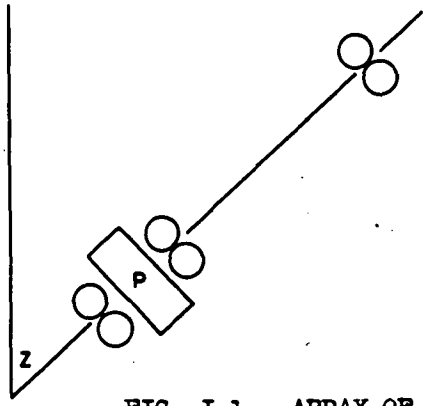
A. The variation with zenith angle of the penetrating component at low altitudes.

1. Although many experiments have been performed to investigate the variation of the directional intensity with zenith angle, most of these have dealt with the total intensity, with little absorber used to remove the soft component. In this section discussion will be restricted to those experiments performed at low altitudes (less than 300 m) and high latitudes (higher than 45° geomagnetic latitude) in which at least 10 cm Pb absorber have been used.

2. Johnson (1), Skobelzyn (2) and others have pointed out that the total intensity varies as $\cos^{\lambda} Z$ with zenith angle, Z, where λ is approximately 2. In reviewing the experiments under consideration, the chief aim is to see whether a $\cos^{\lambda} Z$ law is satisfactory for the penetrating component and to determine the best value of the exponent, λ. The analyses of the data have been carried out in the same way for each experiment by the method set out below.

3. If the counting rate N(Z) of a telescope set with its axis at any zenith angle Z (from the vertical) is related to the counting rate N(0) when the axis is vertical by the relation

$$N(Z) = N(0) \cos^{\lambda} Z$$

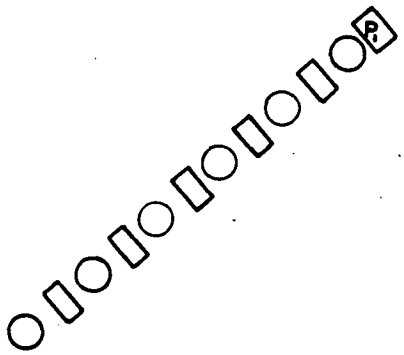


COUNTER DIAMETER 4.3 cm

COUNTER LENGTH 30 cm

ABSORBER P 10 cm Pb

FIG. I.1 ARRAY OF COCCONI AND TONGIORGI (3).



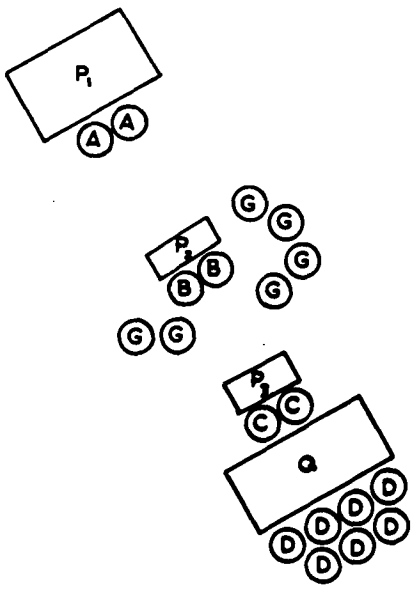
COUNTER DIAMETER 4.24 cm

COUNTER LENGTH 20 cm

HALF ANGLE $6.7^{\circ} \times 28.8^{\circ}$

ABSORBERS P_1 3 cm Pb
Others 2 cm Pb

FIG. I.2 ARRAY OF GREISEN (4).



COUNTER DIAMETER 2.5 cm

HALF ANGLE $13^{\circ} \times$?

ABSORBERS P_1 up to 18.5 cm Pb
P_2 2 cm Pb
P_3 2 cm Pb
Q 8 cm Pb

FIG. I.3 ARRAY OF ROGOZINSKI AND VOISIN (5,6)

then the exponent, λ, may be determined from the relation

$$\lambda = \frac{\log R}{\log \cos Z_2 - \log \cos Z_1}$$

where $R = N(Z_2)/N(Z_1)$, the ratio of the counting rates at zenith angles Z_2 and Z_1.

If δR is the error (probable error, or standard deviation or other measure of the uncertainty) of R, then the corresponding error of λ is

$$\delta\lambda = \frac{\delta R}{R(\log_e \cos Z_2 - \log_e \cos Z_1)}$$

If a set of such values of λ and $\delta\lambda$ is available, the weighted mean may be computed, the weight assigned to each value of λ being proportional to the inverse square of the corresponding $\delta\lambda$. The standard deviation quoted with the weighted means have been calculated from the relation

$$\sigma^2 = \frac{\Sigma(\lambda - \bar{\lambda})^2/(\delta\lambda)^2}{\Sigma\ 1/(\delta\lambda)^2}$$

σ is thus the standard deviation of a single estimate and not that of the weighted mean (which would be $\sigma/\sqrt{n-1}$, where n is the number of the estimates of λ).

4. Cocconi and Tongiorgi (3), working at Milan (altitude 120 m, geomagnetic latitude 46.5°N), used the counter array shown in Fig. I.1 which gives all the available details. The experiments were conducted firstly in front of a large window in one of the main buildings and later in a wooden cabin in the garden. The authors state that the measurements taken in these different surroundings gave the same results from the two stations. Table I.1 gives the counting rates obtained in the main building except that for $Z = 0°$, which was obtained in the wooden building in which

there would have been less absorbing material overhead. These rates have been recalculated from the total number of counts and the time intervals given and have been corrected for showers using the authors' observed shower rates for each setting (slight errors are present in the authors' calculated rates). It is not stated whether the results were corrected for the barometer effect.

TABLE I.1.

Directional intensities with 10 cm Pb. Cocconi and Tongiorgi (3) (The errors are standard deviations).

Z	N(Z), counts per min.	N(Z)/N(0)	λ
0°	4.623 ± 0.057		
20°	4.518 ± 0.044	0.977 ± 0.015	0.373 ± 0.255
30°	3.849 ± 0.038	0.833 ± 0.013	1.273 ± 0.110
45°	2.316 ± 0.034	0.501 ± 0.010	1.994 ± 0.055
60°	1.130 ± 0.021	0.244 ± 0.005	2.033 ± 0.027
75.5°	0.287 ± 0.008	0.062 ± 0.002	2.007 ± 0.022

Table I.1 also gives the values of λ calculated from the ratios N(Z)/N(0). It will be noted that the first two values differ markedly from the remaining three, and it would appear that a simple $\cos^{\lambda} Z$ law does not apply. However, in view of the fact that the counting rate N(0) for the vertical rays was not measured in the same place as the others, it has seemed worth while omitting the value N(0) altogether and noting whether the $\cos^{\lambda} Z$ law applies to the remainder of the data. The results are given in Table I.2 from which it may be seen that the values of λ are fairly close to one another, sufficiently close to warrant the belief that a $\cos^{\lambda} Z$ law is at least roughly followed, provided we are justified in neglecting the counting rate at $Z = 0^\circ$ because of the different surroundings.

The weighted mean of the values of λ given in Table I.2 is

$$\bar{\lambda} = 2.135 \pm 0.081$$

TABLE I.2

Values of λ obtained by omitting N(0) from the data of Cocconi and Tongiorgi (3).

Z	$N(Z)/N(20^\circ)$	λ
30°	0.852 ± 0.012	1.947 ± 0.168
45°	0.513 ± 0.009	2.344 ± 0.062
60°	0.250 ± 0.005	2.194 ± 0.033
75.5°	0.064 ± 0.002	2.083 ± 0.022

5. Greisen (4) using the telescope shown diagrammatically in Fig.I.2, examined the zenith angle variation at Ithaca (altitude 259 m, geomagnetic latitude 54°N). The axis of the telescope was tilted towards the South. The results of a run, in which all the Pb was in position (13 cm, total absorber equivalent to 167 g cm^{-2}Pb) and in which six-fold coincidences (1,2,3,4,5,6,) were registered, are given in Table I.3. It is not stated whether the results were corrected for barometric fluctuations.

The values of λ given in this table are reasonably close to one another. The weighted mean is

$$\bar{\lambda} = 2.123 \pm 0.069$$

TABLE I.3

Directional intensities with 13 cm Pb. Greisen (4).
(The errors are standard deviations).

Z	N(Z), counts per min.	N(Z)/N(0)	λ
0°	2.27 ± 0.021		
29°	1.67 ± 0.024	0.736 ± 0.013	2.291 ± 0.128
46°	1.08 ± 0.020	0.476 ± 0.010	2.039 ± 0.057
56°	0.65 ± 0.015	0.286 ± 0.007	2.152 ± 0.043

6. Rogozinski and Voisin (5, 6, 7) have conducted a series of experiments at Meudon (altitude 148 m, geomagnetic latitude 51.4°N) using the counter array illustrated in Fig.I.3. This same array was used for an investigation of the variation with zenith angle of the differential spectrum. Therefore, although the differential results will not be discussed till later, a few remarks on the array will be made. Coincidences (ABC), (ABCD), and (ABCG) were recorded simultaneously by steel pens engraving a uniformly moving aluminium tape. From this record the rates of anticoincidences (ABC-G) and (ABC-(D+G)) were deduced, representing single non-shower-producing particles (assumed to be mesons). The counting rate of events (ABC-(D+G)) represents mesons stopping in the absorber Q and hence gives the rate required for the differential investigations. The purpose of P_2 (2 cm Pb) is to increase the chance of showers being produced by any electrons which may emerge from P_1. P_3 (2 cm Pb) is to absorb any low energy shower electrons which may be missed by the G counters. The presence of P_2 and P_3 also reduces the number of coincidences, which, in their absence, would be produced by decay electrons from mesons not belonging to the beam defined by the telescope.

TABLE I.4

Directional intensity with 14.5 cm Pb. Rogozinski and Voisin (5,6). (The errors are the standard deviations).

Z	N(Z), counts per hour	N(Z)/N(0)	λ
0°	145.2 ± 1.7		-
30°	103.5 ± 1.6	0.713 ± 0.014	2.349 ± 0.135
60°	31.7 ± 0.5	0.218 ± 0.004	2.196 ± 0.028
73°	10.2 ± 0.5	0.070 ± 0.004	2.160 ± 0.041

Tables I.4 and I.5 give the results of measurements (ABC-G) obtained with $P_1+P_2+P_3$ equal to 14.5 cm and 22.5 cm Pb respectively. For these measurements the telescope was inclined to the East. The authors state that their results have been corrected for the finite solid angle of the telescope and for barometric changes. No indication is given of what value was taken for the barometer coefficient.

TABLE I.5

Directional intensity with 22.5 cm Pb. Rogozinski and Voisin (5,6). (The errors are the standard deviations).

Z	N(Z), counts per hour	N(Z)/N(0)	λ
0°	133.7 ± 1.5		-
30°	97.0 ± 1.5	0.726 ± 0.014	2.231 ± 0.133
60°	28.1 ± 0.4	0.210 ± 0.004	2.250 ± 0.026
73°	9.5 ± 0.5	0.071 ± 0.004	2.150 ± 0.044

The weighted mean for the 14.5 cm results is

$$\bar{\lambda}_{14.5} = 2.189 \pm 0.032.$$

None of the individual values differs significantly from this mean.

The weighted mean for the 22.5 cm results is

$$\bar{\lambda}_{22.5} = 2.224 \pm 0.043.$$

Again none of the individual values differ significantly from the mean. The means for these two thicknesses are statistically equal. There is no evidence from these results for a change of λ for different cut-off momenta.

Voisin (8) carried the measurements for $Z = 0^\circ$, 30°, 60° and 73° to a higher degree of accuracy using 14 cm Pb, and a little later Rogozinski and Voisin (9) published results for $Z = 15^\circ$ and 45° as well. The calculations based on these results are given in Table I.6.

TABLE I.6

Directional intensity with 14 cm Pb. Rogozinski and Voisin (9). (The errors are the standard deviations)

Z	N(Z), counts per hour	N(Z)/N(0)	λ
0°	143.3 ± 0.6		
15°	134.9 ± 1.0	0.941 ± 0.007	1.735 ± 0.216
30°	104.5 ± 0.6	0.729 ± 0.005	2.194 ± 0.049
45°	74.6 ± 0.6	0.521 ± 0.005	1.884 ± 0.026
60°	32.8 ± 0.3	0.229 ± 0.002	2.128 ± 0.015
73°	10.0 ± 0.14	0.070 ± 0.001	2.165 ± 0.012

The weighted mean of these results is

$$\bar{\lambda}_{14} = 2.122 \pm 0.086$$

which is not significantly different from $\bar{\lambda}_{14.5}$. The values of λ for $Z = 15^\circ$ and 45° are a good deal less than the values for the other three zenith angles. However λ_{15°, which has a large standard

deviation, differs by less than twice its standard deviation from the weighted mean and therefore cannot be regarded as a significantly small value. On the other hand, λ_{45^o} differs by about ten times its standard deviation from the weighted mean. The chance of this happening in random sampling from a normal population is negligible.* The authors make no comment about this result. But there do not appear to be sufficiently strong grounds for supposing that the $\cos^{\lambda} Z$ law does not apply. Although the results were corrected for pressure, a number of other factors may have operated to produce a higher counting rate at 45° than would be expected from a $\cos^{2.122} Z$ law. Furthermore, it should be borne in mind that the errors assigned to the values of λ are the statistical errors only. An error made, for instance in measuring the angle of inclination of the telescope could produce quite a marked effect, although in this case it would need to be of about a degree to bring the result into proper harmony with the $\cos^{\lambda} Z$ law. We recall that the value of λ obtained by Greisen for $Z = 46^o$, although lower than the values for the other two angles, does not differ significantly from the weighted mean of his results (which is identical with that of Rogozinski and Voisin).

* The sampling distribution for λ has not been worked out. One therefore has no justification for merely assuming that it would be normal and performing tests of significance as if it were. However, in the absence of a knowledge of the sampling distribution, the best one can do is to regard it as normal for the limited number of tests needed here.

7. Earlier (8) these authors had remarked that if the correction due to side showers, and shower producing particles in the beam defined by the telescope were not made, the value obtained for λ was nearly 2. Similar calculations based on the uncorrected results of Voisin (9) for 14 cm Pb have therefore been made. These are presented in Table I.7. The weighted mean is

$$\bar{\lambda} = 2.057 \pm 0.008.$$

TABLE I.7

Directional intensity with 14 cm Pb without correction for showers. Voisin (9).
(The errors are the standard deviations).

Z	N(Z), counts per hour	N(Z)/N(0)	λ
0°	149.3 ± 0.6		
30°	110.3 ± 0.6	0.739 ± 0.005	$2.102. \pm 0.047$
60°	36.0 ± 0.3	0.241 ± 0.002	2.053 ± 0.013
73°	11.9 ± 0.14	0.080 ± 0.001	2.057 ± 0.010

Since the uncorrected results are not given for $Z = 15^\circ$ and 45°, this value should be compared with the weighted mean of the results in Table I.6 with values $Z = 15^\circ$ and 45° omitted. This is

$$\bar{\lambda} = 2.151 \pm 0.035.$$

Regarding this last value as the mean of a normal population of standard deviation 0.035, the probability of finding a value of λ less than or equal to 2.057 (the value for the weighted mean of the uncorrected results) is 0.004. If the uncertainty in the value 2.057 be taken into account, the probability of so large a deviation would be a little greater but still highly unlikely. The significance tests here are not subject

to the same uncertainty as in considering the value λ_{45° above, because the measurements in the present case were made simultaneously. Hence it is probable that the removal of the side showers and shower producing particles has increased the value of λ.

8. Discussion. It will be seen that there is no strong evidence from the experiments considered that a $\cos^{\lambda} Z$ law is not followed for the penetrating component for angles as great as 73°. Furthermore, it appears that the law holds equally well for thicknesses of Pb between 10 and 22.5 cm. The weighted mean of Greisen's results with 13 cm Pb (given in Table I.3), of Rogozinski and Voisin with 22.5 cm (given in Table I.5) and with 14 cm (in Table I.6) is

$$\bar{\lambda} = 2.135 \pm 0.088.$$

However, in spite of this, there does not seem to be justification in assuming that the law holds rigidly when measurements have been made at only a few angles and when the precision of the values of λ is low at the small zenith angles. Further measurements would be highly desirable. This is particularly so in view of the fact that many American workers, e.g. Schremp, Banos and Ribner (10, 11, 12, 13), have reported observations on the total radiation which indicate that there are deviations from a smooth $\cos^{\lambda} Z$ law in geomagnetic latitudes ranging from 29°N to 59°N. Although this considerable body of work will not be reviewed here, it should be mentioned that Cocconi and Tongiorgi (14) at Passo Sella (altitude 2200 m, geomagnetic latitude 49°N) in Italy, find no evidence for a fine structure using 13 cm Pb.

Some results are available from a short experiment performed at Hobart in which Z was changed in 5° steps from 20° to 60°.

9. Experiments at Hobart. These measurements were made with equipment described in Part III, after its return from Macquarie Island. Briefly, there are two 3-fold telescopes each of counting area 400 cm^2 and half angles 15° in each direction. In this experiment, the one telescope used was inclined to the East, and 12 cm Pb were placed in it.

The object of this investigation was to obtain values of λ with a statistical accuracy of about 5 percent at each of a number of zenith angles. It was therefore, possible to plan before-hand approximately how long the telescope would have to run at any angle. The application of formulae given earlier (par. 3) shows that a much greater length of time must be spent at the small zenith angles than at the large angles to achieve a given statistical accuracy. With this equipment, to obtain values of λ for angles less than 20° with a precision of only 5 percent requires a much longer time than could be spared - many months. (It is presumably for this reason that no measurements of high accuracy have ever been made in the low zenith angle region).

The experiment was not conducted in such a way that the average pressure at each angle was the same. Therefore, pressure corrections were necessary. As we shall see in Part II, there is no strong evidence for a change of pressure coefficient with zenith angle. Hence the coefficient derived from the Macquarie Island results for a zenith angle of 45°, viz., -5.945 percent per in. Hg, was used.

TABLE I.8

Results of Hobart experiments with 12 cm Pb.

Z	No. of hours	Av. Press. (in. Hg)	Rate corrected to 30 in. Hg (counts per hour)	λ
0°	90	29.458	840.42 ± 3.11	
19°54'	121	29.914	758.78 ± 2.51	1.663 ± 0.081
25°10'	51	29.810	678.69 ± 3.67	2.143 ± 0.066
29°57'	20	29.992	610.91 ± 5.53	2.228 ± 0.068
35° 4'	6	29.849	568.68 ± 9.78	1.949 ± 0.088
40° 6'	5	29.317	511.36 ± 10.32	1.853 ± 0.077
45° 1'	12	29.382	412.44 ± 5.97	2.053 ± 0.043
50° 1'	2	29.483	327.61 ± 13.00	2.130 ± 0.098
55° 2'	9	30.103	273.33 ± 5.49	2.017 ± 0.037
60°18'	17	29.824	194.94 ± 3.40	2.080 ± 0.025

The results are listed in Table I.8. The weighted mean value of λ is

$$\bar{\lambda} = 2.046 \pm 0.105.$$

We note that the value of λ obtained for Z = 20° differs from the weighted mean by many times its standard deviation. However one does not feel inclined to attach much importance to this result because the value of λ is fairly sensitive to the pressure correction in the low angle region. If the uncorrected results are used, the value for Z = 20° is 2.112.

These results do little more than confirm the view expressed above that further measurements are desirable, particularly in the small zenith angle region.

10. The expected zenith angle variation. For the calculation of the expected barometer effect at various zenith angles (to be dealt with in Part II) and of the expected East-West asymmetry (Part III), certain numerical computations were made. These are of such a nature that it is possible to use them for the determination of the expected zenith angle dependence of the hard component. We shall therefore consider this numerical work now, from the point of view of present needs.

To determine the intensity of the hard component at any zenith angle Z, we require the integral

$$\int_{p_1}^{\infty} N(p)\, dp$$

where $N(p)\, dp$ is the number of particles of momentum in the range p, p+dp reaching the observer at zenith angle Z, and where p_1 is the minimum momentum which the observer's equipment can record.

Hence, we require to know the differential momentum spectrum at the zenith angle considered. No experimental determination of this seems to have been made for inclined directions. However, the vertical momentum spectrum has been investigated by a large number of workers. Their results may be used in the following way.

If we consider a beam of μ-mesons* arriving vertically at sea level in a certain narrow momentum range, it is possible to work out the intensity of this beam at any altitude by taking into account the spontaneous decay, assuming that no production has taken place between sea level and the altitude considered.

* There is evidence that most of the penetrating particles reaching sea level are μ-mesons. Mylroi and Wilson (15) find that in the vertical sea level beam approximately 1 percent of the particles are protons.

Whilst there is no doubt that production of μ-mesons does take place in the lower atmosphere, the work of Duperier (16) on the variations of the vertical intensity suggests that the bulk of the mesons reaching sea level are produced at about the 100 mb level. We can make the assumption that the mesons are produced at this level (i.e. after the primaries have traversed approximately 100 g cm^{-2} of atmosphere) and calculate the intensity at that altitude. This can be done for a number of small momentum ranges so that the differential production spectrum can be worked out from the observed sea level spectrum.

The next step is to assume that the radiation is isotropic at the production level. This appears to be permissable, to a first approximation at least, partly because of direct experiments by Winckler and Stroud (17) and partly because of the absence of a latitude effect in high latitudes. Thus we assume that the production spectrum determined from the vertical sea level spectrum will give the initial intensity in any other direction. Taking decay into account, it is then possible to work out the differential momentum spectrum at any angle at sea level.

It will be seen that this procedure involves a few uncertainties. The aim here is merely to see how well calculations based on these assumptions agree with observations. The method outlined is a way of testing the hypotheses and is not regarded as more than this.

We now proceed with the method in detail.

If we consider μ-mesons of rest mass μ and proper mean lifetime τ, it is readily shown that the fraction surviving after travelling a distance s_1 is $\exp\left(-\frac{1}{\tau c}\int_0^{s_1}\frac{ds}{p/\mu c}\right)$ where p is the momentum over the element of path ds.

Therefore, the problem which confronts us is the determination of the integral in this expression. Integrals of this form have been determined numerically for a number of zenith angles and for a number of final momenta. The method used was as follows.

Consider μ-mesons with a certain value of $p/\mu c$ at sea level. Values of $p/\mu c$ were determined for these mesons at a number of equally spaced points along the path from sea level to the point beyond which 100 g cm^{-2} of atmosphere remain in the direction of the path. The integral was then computed using Gregory's method (Whittaker and Robinson (18) p.143).

The value of $p/\mu c$ at each of these points was determined from a table of $p/\mu c$ against range, specially constructed for this purpose. This table is given and discussed in the Appendix at the end of this part. To use the table for this purpose, it is necessary to know the mass of air between sea level and each of the equally spaced points along the trajectory. These have been worked out, using Eq.(viii) of the Appendix, for the exponential approximation to the Macquarie Island atmosphere discussed in the Appendix.

The equal increments of path length for these calculations were usually taken as a kilometre. However, in working out the integrals for some of the low final momentum mesons, 0.5 km steps were used in the lower part of the atmosphere and for some of the higher momentum cases at large zenith angles larger steps in the higher atmosphere were used.

TABLE I. 9.

Integrals $\int_0^s \frac{ds}{p/uc}$ for various zenith angles and final momenta. Computed for the Macquarie Island exponential atmosphere. s is in km.

	Final Momentum, Mev/c	355	849	1335	2691	3859	5814	7467
$Z = 0^\circ$	Initial Momentum, Mev/c	2266	2855	3416	4921	6179	8248	9977
	$\int_0^{16.5} \frac{ds}{p/uc}$, unit 10^5 cm.	1.3463	0.9081	0.7036	0.4430	0.3394	0.2448	0.1988
	Final Momentum,	245.3	693.8	1335	2248	4016	5814	8007
$Z = 30^\circ$	Initial Momentum,	2492	3031	3787	4818	6744	8657	10966
	$\int_0^{20} \frac{ds}{p/uc}$, unit 10^5 cm.	1.6264	1.1017	0.7861	0.5717	0.3806	0.2863	0.2205
	Final Momentum,	245.3	849	1335	2691	3859	5814	7467
$Z = 45^\circ$	Initial Momentum,	3114	3858	4440	5988	7272	9375	11118
	$\int_0^{27} \frac{ds}{p/uc}$, unit 10^5 cm.	1.7962	1.1573	0.9304	0.6204	0.4884	0.3626	0.2991
	Final Momentum,	245.3	693.8	1335	2284	4016	5814	
$Z = 60^\circ$	Initial Momentum,	4545	5123	5927	7013	8196	10991	
	$\int_0^{43} \frac{ds}{p/uc}$, unit 10^5 cm.	2.0393	1.5086	1.1625	0.9039	0.6483	0.5092	

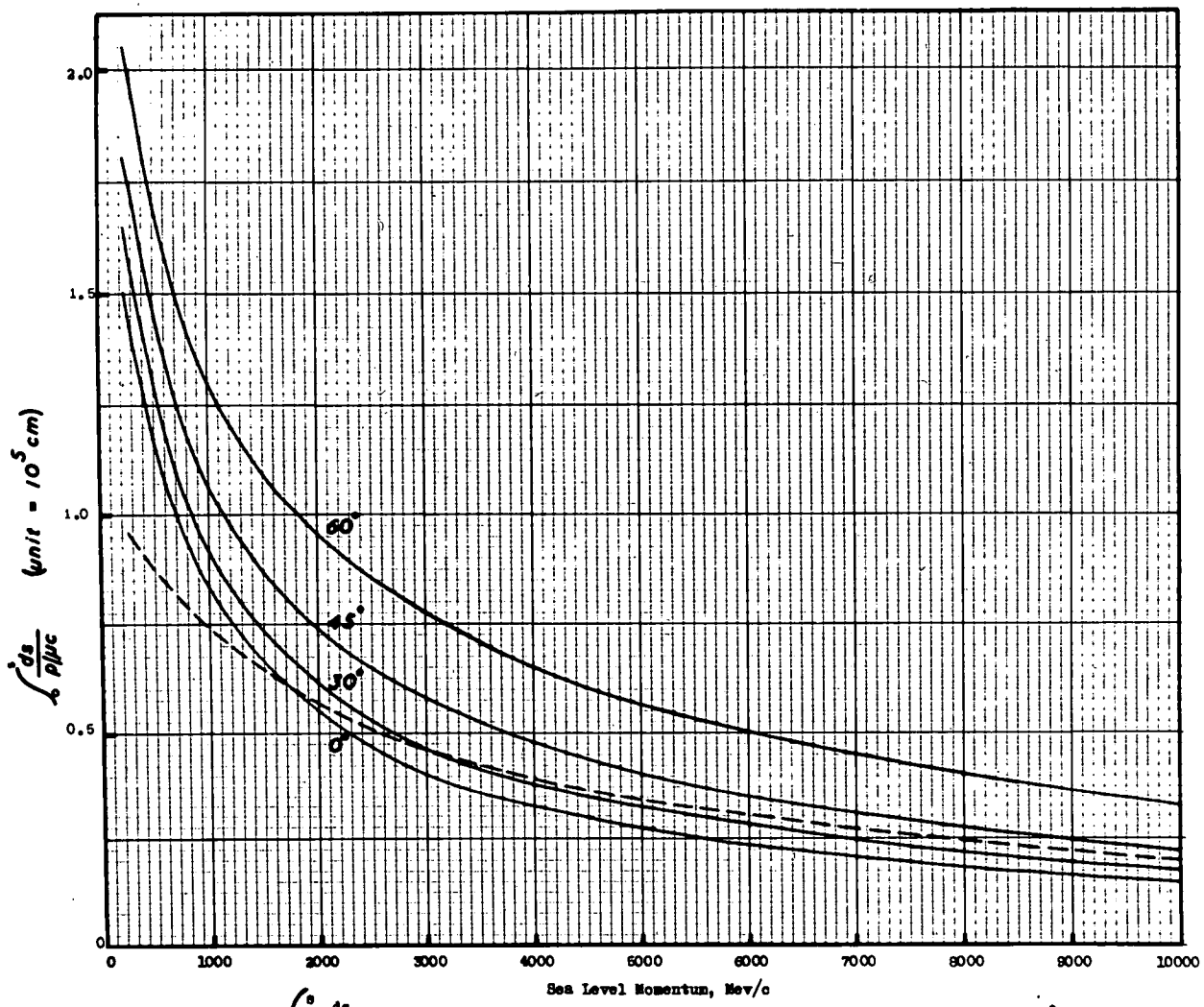


FIG. I.4 $\int_0^s \frac{ds}{p/\mu c}$ as a function of zenith angle and sea level momentum. s = 16.5 km for 0°, s = 20 km for 30°, s = 27 km for 45°, and s = 43 km for 60°. The broken curve gives $\frac{s}{p/\mu c}$ for 45° (see Part III, par. 6).

The integrals so determined are thought to be fairly precise since no approximations about the rate of energy loss are involved. The values which have been determined are given in Table I.9 and are shown plotted against the final momenta in Fig. I.4. Integrals have not been worked out for initial momenta much in excess of 10,000 Mev/c. This is because the range-momentum table for the μ-mesons has not been computed in the region where radiative collisions become important.

The sea level differential momentum spectrum upon which all the calculations considered in this Part and in Parts II and III are based, is that given by Rossi (19) in his Fig.4, which is redrawn in Fig.I.5 here. Rossi considered this to be the most reasonable estimate of the spectrum at the time. Subsequent work, such as that of Caro, Parry and Rathgeber (20,21) does not seem to give any reason to modify it in the region below 10^{10} ev/c. The work of Caro et al. suggests that the spectrum falls off more rapidly beyond 10^{10} ev/c than Rossi's.

The spectrum at the 100 g cm^{-2} level obtained by using Rossi's spectrum and the integrals for $Z = 0^{\circ}$ is given in Fig.I.6. (The value of the proper mean lifetime of the μ-meson for this and subsequent work has been taken as 2.15 μsec). It will be seen that the spectrum may be represented quite well by

$$N(p)\,dp = kp^{-\gamma}dp,$$

where γ is very close to 3 (the value worked out from the slope of the log-log plot is 2.96).

By regarding the spectrum plotted in Fig.I.6 as the production spectrum and using the integrals for the other zenith angles, the sea level differential momentum spectra are as shown by the approximately labelled curves in Fig.I.5. (As stated before, we have assumed that the radiation is isotropic at production).

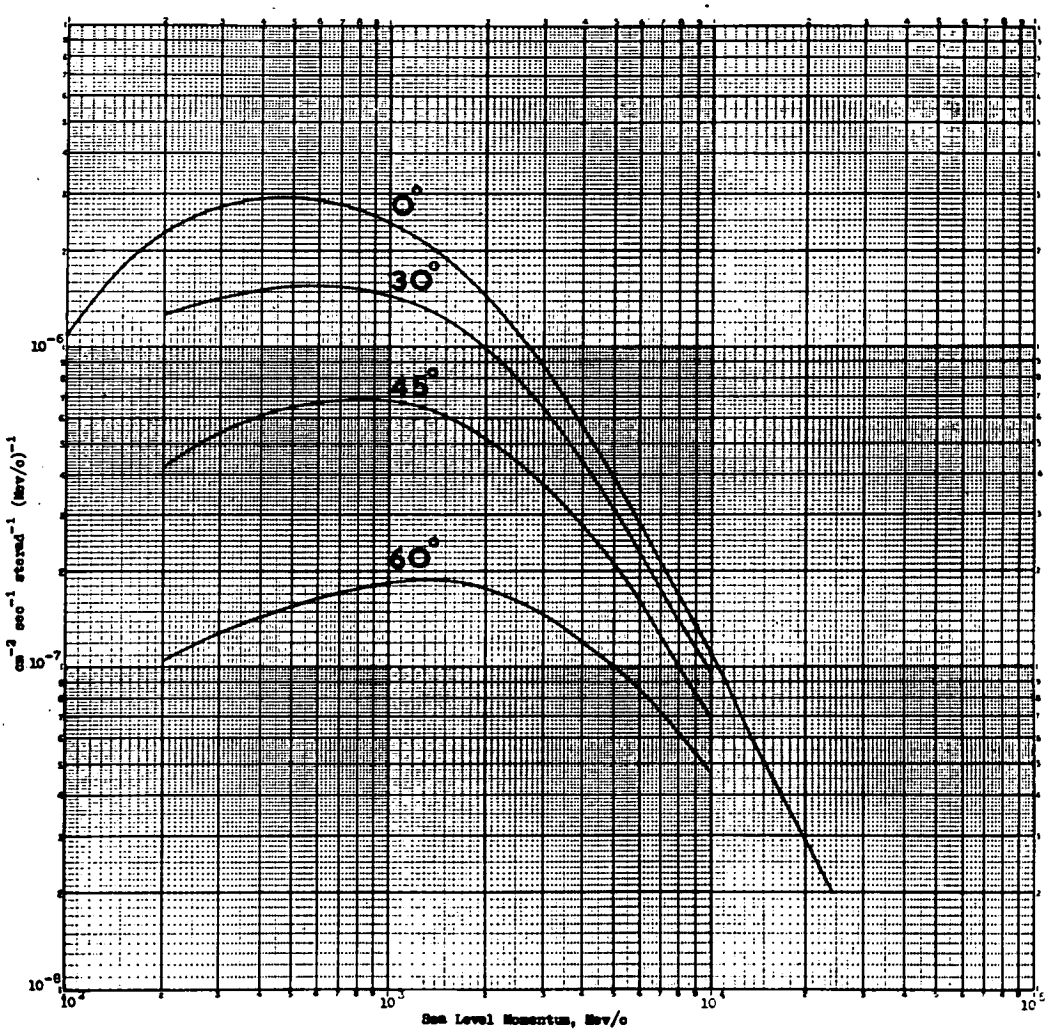


FIG. I.5 Differential momentum spectra of mesons at sea level for various zenith angles. The 0° spectrum is due to Rossi (19). The others are based on this spectrum.

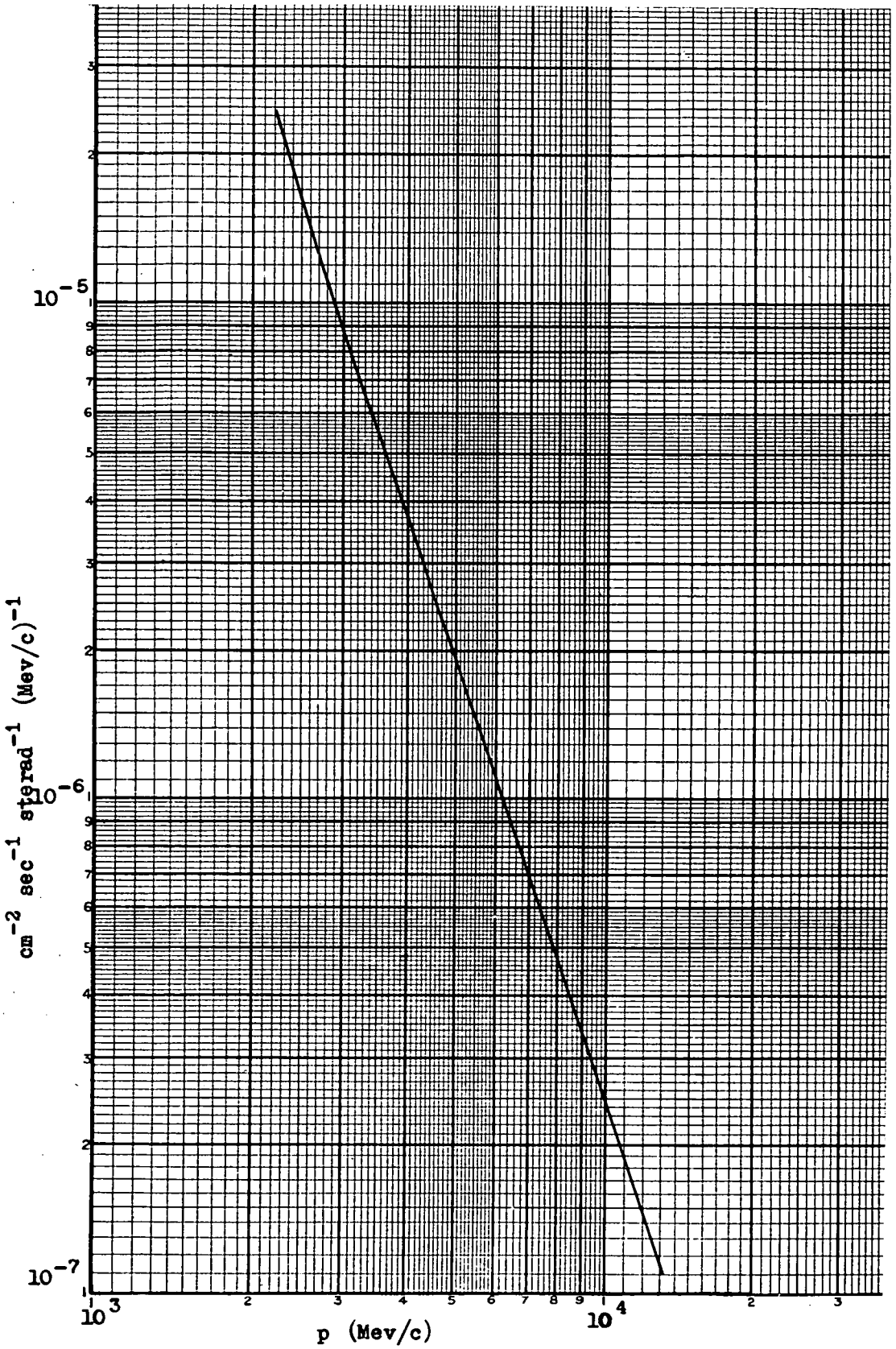


FIG. I.6 Differential momentum spectrum of μ-mesons at the 100 g cm^{-2} level of the atmosphere, based on the vertical differential spectrum at sea level given by Rossi (19).

We are now able to work out the integral

$$\int_{p_1}^{\infty} N(p)\, dp$$

for these four directions by numerically integrating over the spectra in Fig. I.5. The value of p_1 taken here is 250 Mev/c, which, for μ-mesons, is approximately the cut-off momentum for 12 cm Pb. Because the spectra have not been determined beyond 10^{10} ev/c, the numerical integration has been carried only this far. The determination of the contribution beyond this momentum has been worked out by analytical integration, assuming the sea-level spectrum to vary as $p^{-\gamma}$ in the high momentum region. The slope of Rossi's curve beyond 10^{10} ev/c corresponds to $\gamma = 1.9$. One would expect that in the very high momentum region, where decay of the μ-mesons is negligible, the sea level spectrum would be similar to the production spectrum. Evidence that γ does increase with momentum comes from the work of Caro et al. (20), who find $\gamma = 3.0 \pm 0.2$, and from experiments underground (George (22)). Therefore, it would seem that to determine the integral

$$\int_{10^{10}\,ev/c}^{\infty} kp^{-\gamma}\, dp$$

we should take $\gamma = 3$. If this is done, we find that the vertical intensity, integrated from 250 Mev/c to ∞, is $0.79 \times 10^{-2} cm^{-2} sec^{-1}$ $sterad^{-1}$, compared with the observed intensity 0.83×10^{-2} quoted by Rossi (19), for the component at sea level which can penetrate 167 g cm^{-2} Pb (corresponding to 300 Mev/c μ-mesons). The difference probably arises through taking a p^{-3} spectrum beyond 10^{10} ev/c. If we assume a p^{-2} spectrum beyond 10^{10} ev/c, the vertical intensity is 0.846×10^{-2}, or, if the cut-off momentum is taken as 300 Mev/c instead of 250 Mev/c, $0.833 \times 10^{-2} cm^{-2} sec^{-1}$ $sterad^{-1}$ which agrees well with the observed value. It therefore seems probable that the bulk of the μ-mesons of momentum beyond

10^{10}ev/c can be taken as following a p^{-2} law without serious error. This has been done for all four zenith angles, giving the results listed in Table I.10.

TABLE I.10

Calculated directional intensities of μ-mesons at sea level and comparison with a $\cos^2 Z$ law.

Z	$\int_{250}^{\infty} N(p)\,dp$ $cm^{-2}sec^{-1}sterad^{-1}$	N(Z)/N(0)	$\cos^2 Z$
0°	0.846×10^{-2}		
30°	0.592×10^{-2}	0.70	0.75
45°	0.338×10^{-2}	0.40	0.50
60°	0.158×10^{-2}	0.19	0.25

It will be seen from this Table that the intensities calculated for the three inclined directions are lower than those observed. We note also that the ratio $N(45^\circ)/N(30^\circ)$ is 0.57 compared with 0.67 according to the $\cos^2 Z$ law, and $N(60^\circ)/N(45^\circ)$ is 0.47 compared with 0.50. This shows that for large zenith angles the calculated intensities vary more nearly as $\cos^2 Z$ than for small angles.

This study allows us to gain some idea of the limitations of the methods that have been used and which will be used again in Parts II and III. Because all the intensities are underestimated, it is reasonable to suppose that mesons produced in the lower atmosphere make an important contribution to the sea level intensity. But, because the ratio of the 60° and 45° intensities is not far from the observed value, it seems likely that production of radiation arriving at large angles does take place at considerable distances from the observer (in the inclined directions).

Work of this kind has been done previously by others. M.E. Rose (23) used numerical methods to determine the sea level directional spectra, basing his work on a $p^{-2.9}$ production spectrum and assuming that production takes place at the 100 g cm^{-2} level. To facilitate the integrations, Rose used an analytical representation of the range-momentum relationship. He found the directional distribution so obtained conformed quite closely to the $\cos^2 Z$ law for intermediate angles (30°-40°), but at smaller angles it fell off less rapidly than $\cos^2 Z$ and at larger zenith angles more rapidly. Trumpy and Ubisch (24) using an approximate method, found much closer agreement with the observed intensities than the method used here gives. Kraushaar (25) claims to have obtained good agreement by assuming that the production does not take place at the 100 g cm^{-2} level, but occurs throughout the atmosphere according to a production spectrum derived by Sands (26) from the study of low momentum mesons.

B. The variation with Zenith Angle of the Intensity of low energy mesons at low altitudes.

11. Several experiments have recently been made to determine how the intensity of mesons in a narrow momentum band at the low momentum end of the spectrum varies with zenith angle. No consistent picture so far seems to have emerged from these investigations.

12. Rogozinski and Voisin (5,6), using the array described in par. 6 (Fig. I.3) and working at Meudon, measured differential intensities for $Z = 0^\circ$, 30°, 60°, and 73° in the three bands 188 - 302 Mev/c, 302 - 410 Mev/c and 410 - 517 Mev/c. These bands were selected by using (see Fig. I.3) $P_1+P_2+P_3$ equal to 6.5, 14.5 and 22.5 cm Pb respectively, with Q equal to 8cm Pb throughout. The results are listed in Table I.11. The values of λ have been determined from them in the same way as in Section A.

TABLE I.11

Directional intensities of μ-mesons in narrow momentum bands. Rogozinski and Voisin (5,6)

Momentum Band	Z	N(Z), counts per hour	λ
	0°	4.55 ± 0.34	
188 - 302 Mev/c	30°	3.22 ± 0.21	2.40 ± 0.69
	60°	0.88 ± 0.10	2.37 ± 0.20
	73°	0.28 ± 0.12	2.27 ± 0.35
	0°	3.78 ± 0.30	
302 - 410 Mev/c	30°	2.28 ± 0.26	3.51 ± 0.97
	60°	1.09 ± 0.18	1.79 ± 0.26
	73°	0.44 ± 0.04	1.75 ± 0.10
	0°	3.01 ± 0.24	
410 - 517 Mev/c	30°	2.23 ± 0.26	2.08 ± 0.98
	60°	0.85 ± 0.08	1.82 ± 0.18
	73°	0.34 ± 0.10	1.77 ± 0.25

As the authors themselves remark, the statistical accuracy is not high enough to infer a directional distribution. It appears that in the intermediate range (302 - 410 Mev/c) the intensity does not follow a $\cos^{\lambda} Z$ law. In the case of the other two bands it may be possible to represent the distribution by this law.

13. These authors, continuing their work at Meudon, carried the measurements in the range 300 - 410 Mev/c to a higher degree of precision and extended the observations to $Z = 15^{\circ}$ and 45° as well (9). The results so obtained are given in Table I.12.

TABLE I.12

Directional intensities of μ-mesons in the momentum band 300-410 Mev/c. Rogozinski and Voisin (9).

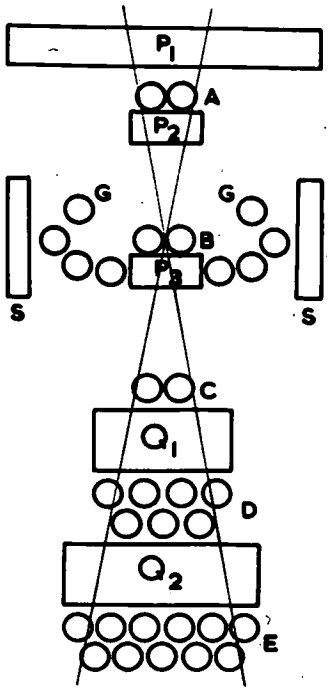
Z	N(Z), counts per hour	λ
0°	3.63 ± 0.10	-
15°	3.10 ± 0.16	4.54 ± 1.68
30°	2.30 ± 0.10	3.17 ± 0.36
45°	1.53 ± 0.08	2.49 ± 0.17
60°	0.82 ± 0.05	2.15 ± 0.17
73°	0.38 ± 0.04	1.84 ± 0.09

Although the standard deviation of the value of λ for $Z = 15^{\circ}$ is large, it seems very unlikely that a $\cos^{\lambda} Z$ law is followed by these results. However, Rogozinski and Voisin find that they closely fit the law

$$N(Z)/N(0) = 1 - a \sin^{b} Z \qquad -(i)$$

where $a = 0.95 \pm 0.02$

$b = 1.40 \pm 0.20.$

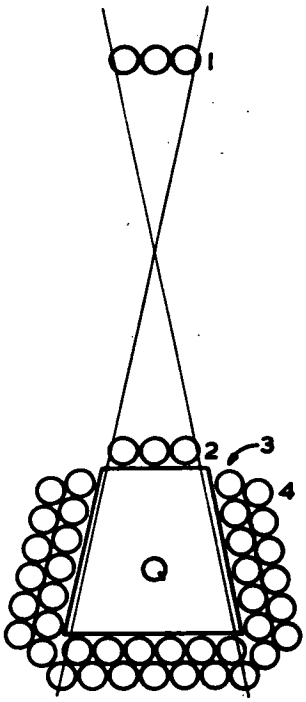


COUNTERS A, B, C 2.5 cm x 9 cm
D, E 2.5 cm x 36 cm
G 2.5 cm x ?

HALF ANGLE $10^{\circ} \times 20^{\circ}$

ABSORBERS $P_1 = P_2 = P_3 = 5$ cm Pb
$Q_1 = Q_2 = 7.5$ cm Pb
$S = 2$ cm Pb

FIG. I.7 ARRAY OF VOISIN (27).



COUNTERS 1, 3, 4 2.5 cm x 40 cm
2 2.5 cm x 20 cm

HALF ANGLE $12^{\circ} \times 40^{\circ}$

ABSORBER $Q = 25$ g cm^{-2} graphite

FIG. I.8 DELAYED COINCIDENCE ARRAY OF KRAUSHAAR (25).

14. Voisin (27), continuing these investigations at Ottawa (altitude 50 m, geomagnetic latitude 56.8°N) found further support for the above empirical law. Simultaneous measurements were made in two momentum bands (300 - 410 Mev/c and 410 - 510 Mev/c) at angles from 0° to 80°. The counter array used (shown in Fig. I.7) was not very different from the one used in the Meudon experiments. Anticoincidences (ABC-DG) and (ABCD-EG) gave the rate of mesons stopping in Q_1 and Q_2. The results are given in Table I.13. This time the values of λ given have been worked out by the author. This table shows that a $\cos^{\lambda} Z$ law is unsuitable for these results. Voisin finds that they may be fitted to a law of the form (i), the constants being

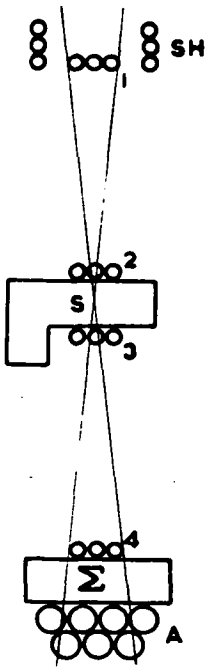
for the band 300 - 410 Mev/c $\quad a = 0.98 \pm 0.02$, $\quad b = 1.47 \pm 0.12$

for the band 410 - 510 Mev/c $\quad a = 1.03 \pm 0.03$, $\quad b = 1.61 \pm 0.15$

TABLE I.13

Directional distribution of low momentum mesons. Voisin (27).

Momentum Band	Z	N(Z), counts per hour	λ
300 - 410 Mev/c	0°	3.59 ± 0.10	
	30°	2.33 ± 0.18	3.00 ± 0.53
	60°	0.75 ± 0.03	2.26 ± 0.07
	75°	0.19 ± 0.02	2.17 ± 0.08
	80°	0.18 ± 0.02	1.71 ± 0.06
410 - 510 Mev/c	0°	3.62 ± 0.10	
	30°	2.36 ± 0.11	2.97 ± 0.34
	60°	0.60 ± 0.03	2.59 ± 0.08
	75°	0.13 ± 0.01	2.46 ± 0.06
	80°	0.09 ± 0.01	2.11 ± 0.06

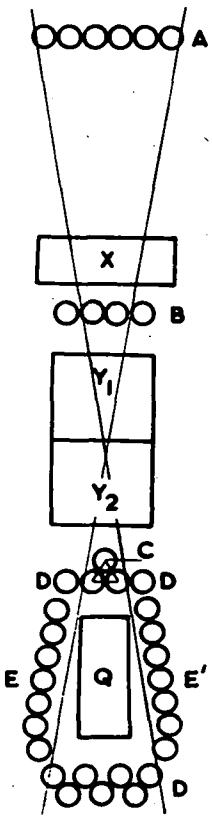


COUNTERS 1, 2, 3, 4, SH 2.5 cm x 40 cm
A 4 cm x 100 cm

HALF ANGLE $6^{\circ} \times 30^{\circ}$

ABSORBERS $S = \Sigma = 93.4$ g cm^{-2} Pb

FIG. I.9 ANTICOINCIDENCE ARRAY OF KRAUSHAAR (25).



COUNTERS A, B, D (top), E
2 in. x 20 in.
C 2 in. x 15 in.
D (bottom) 2 in. x 34 in.

HALF ANGLE $9.2^{\circ} \times 19.7^{\circ}$

ABSORBERS $X = 116$ g cm^{-2} Pb
$Y_1 = Y_2 = 200$ g cm^{-2} Fe
$Q = 53$ g cm^{-2} graphite

FIG. I.10 ARRAY OF ZAR (28).

15. Kraushaar (25), working at Ithaca (altitude 260 m, geomagnetic latitude 54°N) used two methods to determine the zenith angle variation of low mementum particles. In one experiment he used the delayed coincidence technique to identify the μ-mesons of very low momentum which stopped in 25 g cm^{-2} of graphite. Fig.I.8 shows the array used for this experiment. An event recorded as a delayed coincidence was such that a coincidence (1,2) at t = 0, accompanied by no count from trays 3 or 4 from t = -15 μsec to t = 1 μsec, was followed by a coincidence (3,4) in the interval t = 1 - 7 μsec. The results of these measurements, together with values of λ calculated from them, are given in Table I.14.

TABLE I.14

Rate of delayed coincidences from μ-mesons stopping in 25 g cm^{-2} of graphite. Kraushaar (25).

Z	N(Z), counts per hour	λ
0°	1.27 ± 0.08	
30°	0.74 ± 0.06	3.75 ± 0.65
60°	0.12 ± 0.03	3.40 ± 0.37

These results give a weighted mean for λ of 3.49 ± 0.15.

In the other experiment, Kraushaar used an anti-coincidence method using the array shown in Fig. I.9. Anti-coincidences (1,2,3,4 - SH - A) were recorded. S and Σ were each 93.4 g cm^{-2} Pb which would mean that, for μ-mesons, the momentum band selected was 215 - 320 Mev/c. Unfortunately, Kraushaar does not list his results from this experiment in a table. From his graph of relative counting rates it is possible to read off the values for Z = 0°, 30°, 45° and 60° at which angles the measurements were made. The three values of λ obtained from these are 3.21, 3.57 and 3.25 respectively. No attempt has been made to deduce their standard deviations from the graph. The value of λ by Kraushaar is 3.3.

Thus Kraushaar's results suggest that a $\cos^{\lambda} Z$ law does adequately represent the distribution for both the very low momentum band and the 215 - 320 Mev/c range, with the value of λ about 3.3 in each case.

16. Zar (28) has also performed some experiments using both the anticoincidence and the delayed coincidence techniques and he extended the portion of the momentum spectrum covered to over 1000 Mev/c. This work was performed in New York (approximately sea level, geomagnetic latitude 54°N). The apparatus used is illustrated in Fig.I.10. The absorber X consisted of 115 g cm^{-2} Pb and Y_1 and Y_2 each consisted of 200 g cm^{-2} Fe. Q consisted of 53 g cm^{-2} graphite. Thus three bands could be selected. Assuming the particles stopped in Q to be μ-mesons, the bands were 283-391 Mev/c with X, 615-730 Mev/c with $x + Y_1$, and 960-1080 Mev/c with $X + Y_1 + Y_2$. The circuitry consisted of a coincidence unit, an anticoincidence unit and a delay unit. The coincidence unit recorded (ABC). The anticoincidence unit received pulses from both the D counters and the coincidence unit and recorded (ABCD). The difference (ABC) - (ABCD) therefore gives the rate of single particles stopping in Q. The delay unit counted the decay electrons detected by the E or E' counters in the interval 1-9 μsecs after an anticoincidence. This unit received pulses from the coincidence unit, the anticoincidence unit and the E counters. An output pulse was generated whose height was proportional to the time between the coincidence and the delayed event, provided there was no anticoincidence at the same time. The output pulses from the delay circuit were sorted and counted in an 8-channel pulse height discriminator.

The data for each zenith angle and for each of three momentum bands investigated were divided into two parts. In one, the data were taken from the first channel of the delay recorder

which gave the number of particles which failed to set off the D counters, or the E counters within 1 μsec after a particle stopped in Q. By this means, most of the particles which were severely scattered in the graphite (and hence pass through the E counters) would be missed, as is desired. But mesons which came to rest and decayed within the first μsec were also missed. However these could be corrected for by extrapolation from the other channels of the delay unit. The results, corrected for this effect and also for fluctuations due to barometric changes etc., are given in Table I.15.

TABLE I.15

Differential spectrum of penetrating particles as a function of momentum and zenith angle. Zar (28).

Momentum band	Z	N(Z), counts per hour	λ
	0°	22.2 ± 0.8	
283-391 Mev/c	32°	12.7 ± 0.6	3.39 ± 0.36
	55°	4.7 ± 0.3	2.80 ± 0.13
	0°	16.2 ± 0.5	
615-730 Mev/c	32°	10.3 ± 0.4	2.75 ± 0.30
	55°	3.0 ± 0.3	3.03 ± 0.19
	0°	14.7 ± 0.4	
960-1080 Mev/c	32°	8.5 ± 0.5	3.32 ± 0.39
	55°	2.7 ± 0.3	3.04 ± 0.21

It will be seen that the values of λ are all concordant. The weighted mean is 2.95 ± 0.18.

The differential spectrum of particles which could be identified as μ-mesons with a high degree of certainty was investigated by using the data from the other channels of the delay unit. (The distribution of delays was consistent with a mean lifetime of 2.15 μsec.). The data, corrected for mesons which would have decayed after 9 μsecs, are given in Table I.16.

TABLE I.16

Differential spectrum of μ-mesons as a function of momentum and zenith angle. Zar (28).

Momentum Band	Z	N(Z), counts per hour	λ
	0°	1.98 ± 0.19	
283-391 Mev/c	32°	1.24 ± 0.14	2.84 ± 0.90
	55°	0.31 ± 0.06	3.34 ± 0.39
	0°	2.25 ± 0.18	
615-730 Mev/c	32°	1.32 ± 0.14	3.23 ± 0.81
	55°	0.38 ± 0.09	3.20 ± 0.45
	0°	2.22 ± 0.13	
960-1080 Mev/c	32°	1.52 ± 0.16	2.30 ± 0.73
	55°	0.48 ± 0.08	2.76 ± 0.32

Zar remarks that there is a suggestion that λ decreases with increasing momentum. However, in view of the large standard deviations, there is little to support this claim in these results. If one assumes that the same $\cos^{\lambda} Z$ law fits all three bands, one finds that the weighted mean is 2.99 ± 0.31.

17. George (22) mentions some experiments by Creamer which so far do not seem to have been published. Creamer studied the zenith angle distribution of slow μ-mesons which stopped in photographic emulsions. Detailed results are not presented by George, but a graph shows that the points for $Z = 0^\circ - 60^\circ$ (six of them for sea level investigation) fit a $\cos^{\lambda} Z$ law with $\lambda = 3.24 \pm 0.40$. A similar investigation underground also gave results consistent with a $\cos^{\lambda} Z$ law with $\lambda = 2.2 \pm 0.2$.

18. We have not so far considered the possible effects of albedo (radiation moving in an upward direction). In work with nuclear emulsions, where the direction of motion is known if the particle stops, or with the delayed coincidence technique,

where the sequence of events is known, albedo is unlikely to cause errors. However, it may do so in the anticoincidence experiments, particularly at large zenith angles.

Nuclear emulsion experiments by Camerini et al (29) at the Jungfraujoch (12,000 ft.) show that an appreciable μ-meson albedo exists, the intensity being about half the downward intensity of the μ-mesons capable of being detected in the plates used.

Counter experiments by Ritson (30), near sea level, using the delayed coincidence technique, suggest that the albedo is of low energy and that the ratio of the intensity of the backward to that of the forward mesons of low energy is 0.09 ± 0.02. Voisin (27), however, using the delayed coincidence method at Ottawa, claims to have found no appreciable albedo.

It does not seem likely that the intensity which Ritson observes is likely to produce an effect exceeding the statistical errors in the experiments so far performed.

<u>19. Discussion</u>. Although it might appear from this review that the weight of evidence favours a $\cos^{\lambda} Z$ law, with λ about 3, rather than Voisin's $1 - a \sin^{b} Z$ law, it must be noted that Voisin's measurements are the most precise so far made. It would be possible to see whether the results of Kraushaar and Zar are consistent with Voisin's law. However, in view of their large statistical errors and the small number of angles at which intensities were measured, it has not seemed worthwhile. It is clear that further measurements are required at a greater number of zenith angles and that the accuracy should be improved in the low angle region.

Using the differential momentum spectra given in Fig.I.5, we can determine the intensities assuming the mesons to have

originated at the 100 g cm^{-2} level. For the band centred at 355 Mev/c (a band studied by Voisin), the ratio N(Z)/N(0) for 30° is 0.51 compared with Voisin's observed value 0.65, and for 60° is 0.05 compared with Voisin's value 0.21. This indicates that a considerable part of the low momentum meson component originates in the lower atmosphere. However, even Kraushaar's theoretical spectra, which were determined by allowing for production throughout the atmosphere, also underestimate the intensities from inclined directions. He points out that this is probably due to his assumption that the mesons follow the direction of the producing component, and to the fact that the effect of scattering was neglected, so that his method of determining the differential momentum spectra is valid only for mesons of high momentum. Voisin observes that these effects would render the radiation more isotropic at large angles, as he finds it to be.

Another reason why the intensity of low momentum mesons does not vary as rapidly with zenith angle at large angles as at small angles may be suggested. Some low energy mesons are likely to be produced by energetic photons. Evidence to be presented in Section C, par.24, suggests that energetic electrons vary less rapidly than $\cos^2 Z$ with zenith angle. Presumably energetic photons will follow the same law. Thus, at large zenith angles, where an important fraction of the low momentum mesons may be produced by photons, the variation with zenith angle could be less rapid than at small angles.

This idea leads to the further suggestion that it would be of interest, in using the delayed coincidence technique, to observe the decay electrons from Pb (or other absorber of high atomic number) as well as from carbon, so that, in the former case only positive μ-mesons are studied and, in the latter, μ's of both signs. The reason for this is that it is known (Brueckner and Goldberger (31)) that mesons produced by photons

are predominantly negative. Therefore, if an appreciable number of low momentum mesons are produced by photons, there may be a difference in the zenith angle variation of the positives and negatives. Morewitz and Shamos (32) using substantially this technique, find a positive/negative ratio in the vertical beam at sea level of 1.06 ± 0.03 for μ-mesons of momentum about 250 Mev/c, compared with the mean ratio (over the spectrum) of 1.268 ± 0.023 found by Owen and Wilson (33). This low value may be partly due to mesons produced by photons.

It should be borne in mind that, due to the deflection of mesons in the earth's field, there is a greater positive excess in the West at an equal zenith angle in the East (where there is a negative excess at large angles, Groetzinger and McClure (34)). Zar's experiments were performed with the telescope inclined to the South, so that no effect due to changing positive excess from this cause would be present. Neither Kraushaar nor Voisin state what azimuth they used, although in the experiments of Rogozinski and Voisin (9) the telescope was inclined to the East. Thus at large zenith angles the particles observed would be predominantly negative and any effect due to production by photons would be enhanced.

C. Zenith Angle Variation of the Electronic Component.

20. It is sometimes stated (e.g. Kraushaar (25)) that the electronic component follows a $\cos^3 Z$ law. However, the evidence for this seems to be scanty, and it would appear that the zenith angle distribution law has not been satisfactorily established. Although an exhaustive search of the literature has not been undertaken on this subject, it appears that the only method so far used to investigate the variation is to make us of the difference between the counting rate of a telescope with no absorber and with about 10 cm Pb.

21. Cocconi and Tongiorgi (3), using the apparatus shown in Fig.I.1, measured the intensity with the absorber P removed as well as with P = 10 cm Pb (the results of the latter experiment have been discussed in par. 4). By taking the difference between these rates, values of λ may be determined for the component removed by 10 cm Pb. The results of these calculations are given in Table I.17. In par. 4, it was thought necessary to omit the 0° rate. If this is done here and values of λ determined for the remaining angles from the ratios $N(Z)/N(20^\circ)$, the values given in the column headed λ' are obtained. It will be seen that in neither case is there evidence for a $\cos^\lambda Z$ law. It should be noted also that the radiation becomes more isotropic at large angles.

TABLE I.17

Zenith angle variation of the soft component. Cocconi and Tongiorgi (3).

Z	N(Z), counts per min.	λ	λ'
0°	1.29 ± 0.094		
20°	0.88 ± 0.071	6.15 ± 1.75	
30°	0.59 ± 0.057	5.44 ± 0.84	4.89 ± 1.54
45°	0.64 ± 0.061	2.02 ± 0.35	1.12 ± 0.44
60°	0.29 ± 0.028	1.17 ± 0.17	1.76 ± 0.20
75.5°	0.11 ± 0.022	1.77 ± 0.15	1.57 ± 0.16

22. Greisen (4), using the array shown in Fig.I.2, measured the intensity at Ithaca with all the Pb absorber removed as well as with it in position. The difference rate in his experiment is for particles which can penetrate the counter walls and the wooden shelves which supported the Pb(12 g cm^{-2} brass + 1.7 g cm^{-2} wood) but which cannot penetrate 167 g cm^{-2} Pb. Greisen's data have been analysed in the usual way, giving the results set out in Table I.18.

TABLE I.18

Zenith angle variation of the soft component. Greisen (4)

Z	N(Z), counts per min.	λ
0°	0.59 ± 0.030	
29°	0.43 ± 0.035	2.36 ± 3.84
46°	0.16 ± 0.028	3.58 ± 1.48
56°	0.08 ± 0.021	3.43 ± 0.98

The errors are too large to justify inferring a $\cos^{\lambda} Z$ law. (It appears that it is this experiment which is usually thought to have established this law for the electronic component, with λ about 3).

23. In view of these results, it is clear that much more experimental evidence is needed. Since the origin of the soft component is not yet thoroughly understood, the problem is quite an important one. Kraushaar (25), for instance, states that the part of the electronic component which arises through processes involving μ-mesons (decay and knock-on events) should not depend on the zenith angle Z any more strongly than $\cos^2 Z$. Therefore, he infers, since the soft component follows a $\cos^3 Z$ law, the remaining part must arise from a source which varies very much more rapidly with zenith angle.

An experiment has, therefore, been begun at Hobart to investigate the zenith angle distribution of the electronic

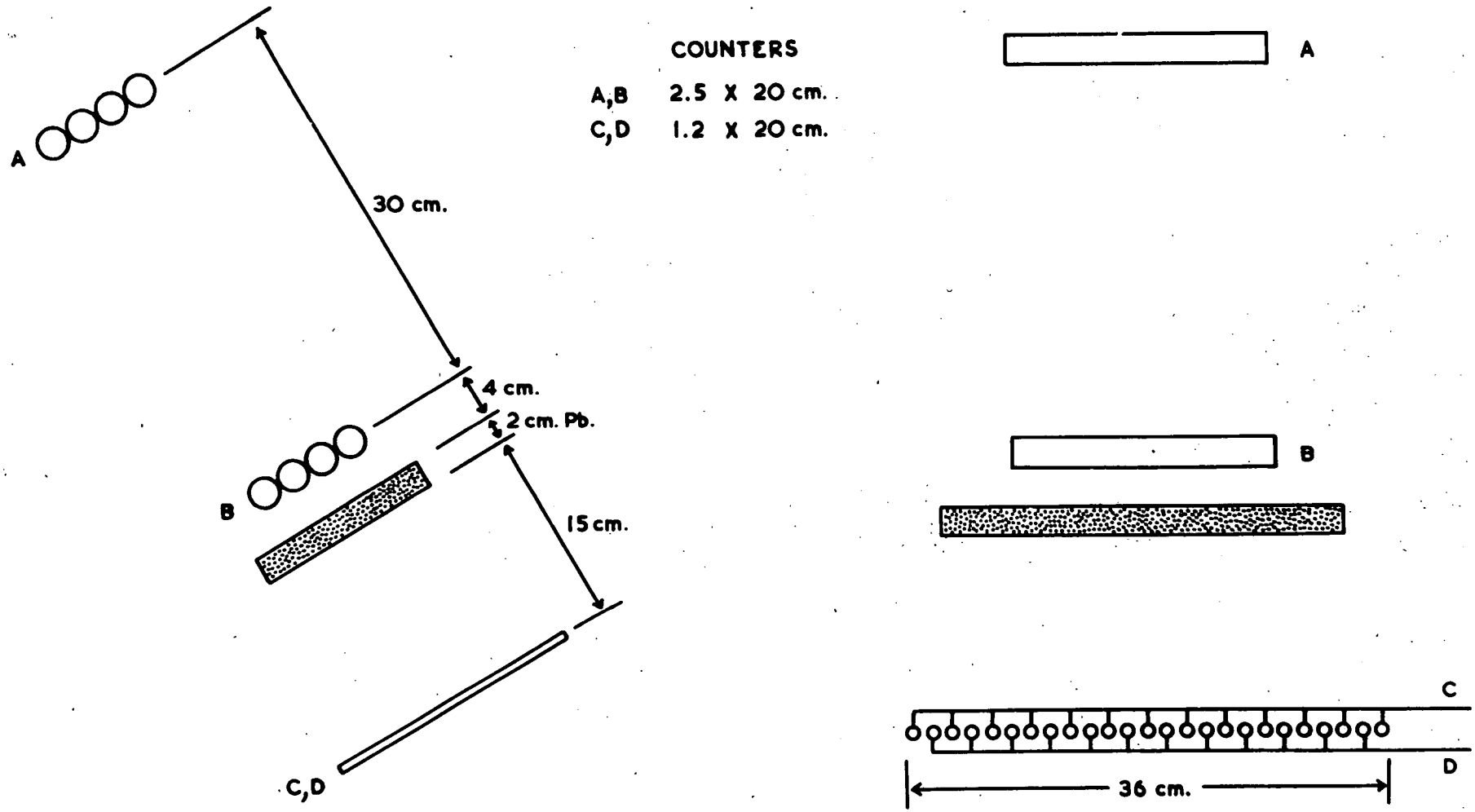


FIG. I.11 SOFT COMPONENT TELESCOPE

component. Results of measurements at only two angles are available at the present time. Even if these were of high accuracy, it would not be possible to infer the law. However, since the technique being employed appears to have been unused for this problem hitherto, the experiment will be discussed.

24. Experiment at Hobart. In the experiments just discussed, the soft component was taken to be that portion of the cosmic ray beam which cannot penetrate 10 cm Pb (Cocconi and Tongiorgi) or 13 cm Pb (Greisen). It is usually assumed that the major part of this component consists of electrons. However, since some slow mesons are also removed, and since these vary with zenith angle in a manner not dissimilar to the soft component, it is desirable to determine the variation of the electron component itself.

In the present experiment the criterion used to select electrons is their ability to produce showers in 2 cm Pb. The counter array is shown in Fig. I.11. Fourfold coincidences (ABCD) are recorded. Such coincidences could arise in the following ways:

(a) A single electron (positive or negative) passing through A and B and producing a shower in P which is detected by the counters C, D.
Events of this type are the desired ones.

(b) A shower of any type from any direction.

(c) Accidental coincidences.

(d) A single meson or electron passing through A and B and producing a shower in one of the counter walls.

(e) A single meson passing through A and B and producing a knock-on in the 2 cm Pb.

(f) A single positive μ-meson from any direction coming to rest in the 2 cm Pb and its decay electron giving rise to a shower.

(g) A single electron from a direction not defined by the counters A and B, or a photon from any direction, giving rise to a shower in the 2 cm Pb.

(h) A nuclear explosion in the 2 cm Pb produced by a nucleon or π-meson.

By measuring the 4-fold coincidence rate without the 2 cm Pb, it is possible to obtain the background rate due to events of type (b), (c) and (d). However, the contribution of events of other types, which depend upon the presence of the Pb, is not determined by this means. Of these, the production of knock-on electrons by mesons is almost certainly the most important. So far, no attempt has been made to estimate the contribution of these events.

The geometry is not necessarily the best that could be devised. Only a little experimenting with separation of counters etc. was carried out before fixing on the present arrangement. The array could be improved by having some anti-counters to greatly reduce the effect of side showers. Some of the mesons producing knock-ons could be detected by having, say, 10 cm Pb below the shower counters with anticounters below this. However, the main object was to get some results with the cruder set-up and judge from these whether refinements would be worthwhile.

The counter array is mounted in a wooden frame which can be tilted to the East. The counters are mounted in such a way that the only absorber between A and C, D in the solid angle defined by the counters is the 2 cm Pb. A thin tin-plate cover placed over the wooden framework shields the counters electrically and from light. The equipment is operated in a building constructed of light weight fibro sheets in the walls and ceiling. The roof is of 1/2" wood covered with malthoid.

The counters are of the external cathode type and are operated from a bank of dry batteries. The 4-fold coincidence circuit has a resolving time of about 2 μsec. The mechanical register is photographed each hour.

The results to date are set out in Table I.19.

TABLE I.19

Zenith angle variation of the shower producing component at Hobart.

	2 cm Pb in			2 cm Pb out				
Z	No. of counts	No. of hours	Rate	No. of counts	No. of hours	Rate	Diff.	λ
0°	8042	264	30.462 ±0.340	2129	98	21.724 ±0.471	8.738 ± 0.581	
47°	1983	126	15.738 ±0.353	1109	114	9.728 ± 0.292	6.010 ±0.458	0.98 ± 0.26

Although we cannot infer the zenith angle distribution law from this single value of λ, we can state with reasonable confidence that a $\cos^3 Z$ law is not obeyed for the radiation which produces the showers detected by the array. It does not seem that the dependence is even as strong as $\cos^2 Z$.

It is desirable to know what energy electrons are responsible for the showers detected. A very rough idea of this may be formed in the following way. The counting rate of coincidences (AB) is about 1450 per hour with the array vertical. Of these, approximately 30 percent would be excluded by 10 cm Pb. Hence we may take it that about 430 per hour are soft. Now if we assume that about 8 per hour of these (corresponding to the difference rate for $Z = 0^{\circ}$ in Table I.19) produce showers, we infer that 2 percent of the soft component produces showers detected by the array. Janossy ((35), p. 251) gives an integral spectrum for the soft component. From this we find that about

2 percent of the electrons in cosmic rays at sea level are more energetic than 300 Mev. Thus, pending a more thorough investigation of the problem, we may take it that the electrons under investigations with this array have energies of at least 300 Mev.

Further discussion does not appear to be worthwhile until more results are available and until the energies are known with greater certainty.

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APPENDIX.

Determination of the integral $\int_{s_1}^{s_2} ds/p$

1. In considering the flight of mesons in the atmosphere it is often necessary to determine the integral $\int_{s_1}^{s_2} ds/p$, where ds is an element of path length and p is the momentum of the meson as it traverses this element. If p were a simple function of s, the determination would present no difficulty. For this reason, the assumption is often made that the rate of loss of momentum per unit mass of absorber traversed is constant, independent of momentum. However, where greater precision is required, this assumption cannot be made. An example will illustrate this point. A μ-meson reaching sea level with a momentum of 335 Mev/c has a momentum of 2250 Mev/c at the 100 mb level, and the initial rate of loss of momentum is about 1.2 times the final rate. A simply integrable function cannot be found which accurately represents the rate of loss of momentum. Hence numerical methods must be used.

What is required for the numerical integration for a particle of certain initial or final momentum is a series of values of the momentum at a number of points along its path. If a table or a chart giving range as a function of momentum is available, such a series of values may be readily obtained.

2. On Macquarie Island, where this work was commenced, the only charts available were those of Gross, published by Montgomery ((1), p. 349). If more than a few points are to be read off from these charts, the work is very tedious because they have been reduced to the small size necessary for inclusion in a book. Since there was no chance of obtaining larger copies, it appeared better to work out a completely new set of curves. This was done using the methods outlined below.

3. The range R (measured in g cm^{-2}) of a particle of momentum p is given by

$$R = \int_0^R dx = \int_p^0 \frac{dx}{dp}\, dp = \int_0^p \frac{dp}{-dp/dx}$$

In the present work we are not interested in particles with momenta less than about 200 Mev/c. For this reason only momenta above a certain lower limit p_1 (taken as 185.1 Mev/c) have been considered. Designating by R_1 the range of a particle of momentum p_1, we have

$$R = R_1 + \int_{p_1}^p \frac{dp}{-dp/dx} \qquad -(i)$$

To compute the integral we need to know the rate of loss of momentum, dp/dx. R_1 remains undetermined.

4. The rate of loss of energy per g cm^{-2}, dE/dx, for a fast particle which loses energy by ionization and excitation is given by the well known Bethe/Bloch relation (2, 3)

$$-\frac{dE}{dx} = \frac{4\pi e^4 N(Z/A)}{mc^2} \frac{1}{\beta^2} \left[\log \frac{2mc^2}{I} \frac{\beta^2}{1-\beta^2} - \beta^2\right]$$

where e is the charge of the electron

N is Avogadro's number

Z is the atomic number of the absorber

A is its atomic weight

m is the mass of the electron

c is the velocity of light

βc is the velocity of the fast particle

and I is a certain mean energy of excitation of the electrons of the absorber.

The validity of this relation has recently been questioned by Goodman, Nicholson and Rathgeber (4), who obtained results with

an argon filled proportional counter operated in their magnetic spectrometer, which indicated that the rate of ionization of particles of momenta exceeding 2.4×10^8 ev/c remains practically constant and does not increase with increasing momentum. However some more recent results, briefly reported by Kupperian and Palmatier (5), purport to show that the Bethe-Bloch relation is valid for argon. Their measurements were carried to only 1800 Mev. The likelihood of the Bethe-Bloch relation being correct is further increased by the fact that other recent experiments (e.g. those of Bowen and Roser (6)) show that for condensed substances the density correction (7,8,9) applied to the relation gives a good estimate of the rate of loss of energy. This suggests that in cases where the density correction is not necessary (in air, for energies below about 10^4 Mev (7)) the relation itself is satisfactory.

We proceed to determine the range from this relation. Following Wheeler and Ladenburg (10) we let

$$\beta = \tanh \Upsilon$$

If μ is the rest mass of the particle, the momentum, p, is

$$p = \mu\beta c/\sqrt{1 - \beta^2}$$

$$\text{Therefore} \quad p/\mu c = \beta/\sqrt{1 - \beta^2} = \sinh \Upsilon$$

To get dE in terms of Υ, we note that

$$E^2 = c^2(p^2 + \mu^2 c^2)$$

$$\text{Therefore} \quad E^2 = \mu^2 c^4 (p^2/\mu^2 c^2 + 1)$$

$$= \mu^2 c^4 \cosh^2 \Upsilon$$

$$\text{Hence} \quad dE = \mu c^2 \sinh \Upsilon \, d\Upsilon$$

Hence dE/dx becomes

$$-\frac{d\Upsilon}{dx} = \frac{4\pi e^4 N(Z/A)}{mc^2 \, \mu c^2} \left[\frac{a + 2 \log \sinh \Upsilon - \tanh^2 \Upsilon}{\sinh \Upsilon \tanh^2 \Upsilon}\right]; \quad -(ii)$$

$$\text{where} \quad a = \log 2mc^2/I$$

To evaluate the range from Eq.(i), it is simpler to work in terms of Υ. We have

$$R = R_1 + \int_{\Upsilon_1}^{\Upsilon} \frac{d\Upsilon}{-d\Upsilon/dx}$$

Substituting for $d\Upsilon/dx$ from Eq.(ii),

$$R = R_1 + \text{const.} \int_{\Upsilon_1}^{\Upsilon} \frac{\sinh \Upsilon \tanh^2 \Upsilon \, d\Upsilon}{a + 2 \log \sinh \Upsilon - \tanh^2 \Upsilon} \quad - \text{(iii)}$$

where const. $= mc^2 \mu c^2 / 4\pi e^4 N(Z/A)$.

The integral in Eq. (iii) may be computed for a number of values of Υ by straight-forward numerical methods. That used was Gregory's method (see, for instance, Whittaker and Robinson (11), p. 143), which is very simple to use when a table is drawn up giving values of the integrand for equal increments of the argument.

The values of the physical constants used for this computation were

$e = 4.803 \times 10^{-10}$ esu.

$m = 9.1 \times 10^{-28}$ g.

$\mu = 215$ m (Bishop (12) Retallack and Brode (13))

$c = 3 \times 10^{10}$ cm sec^{-1}

$N = 6.023 \times 10^{23}$ molecules per gram molecule

$Z/A = 0.5$ (average value for air)

$I = 80.5$ ev (Wilson (14))

Using these values of the physical constants in Eq. (iii) we have

$$R = R_1 + 7.1608 \int_{\Upsilon_1}^{\Upsilon} \frac{\sinh \Upsilon \tanh^2 \Upsilon \, d\Upsilon}{9.4510 + 2 \log \sinh \Upsilon - \tanh^2 \Upsilon} \quad -\text{(iv)}$$

The integral has been evaluated for Υ ranging from $\Upsilon_1 = 1.30$ (corresponding to $p = 185.1$ Mev/c) to $\Upsilon = 5.30$ (corresponding to $p = 10{,}922$ Mev/c) in steps of 0.05 from 1.30

to 3.70, in steps of 0.02 from 3.70 to 4.50, and in steps of 0.01 from 4.50 to 5.30. The table of values so obtained is given at the end of this Appendix. The values of the hyperbolic functions were taken from four-figure tables. Therefore, although computations were carried to six figures and then rounded off to four figures, it is not unlikely that some errors are present in the fourth figure. This may account for the slight irregularities which may be noted in some places.

Values of the dimensionless quantity $p/\mu c$ are also listed in the table. Most often the integral $\int_{s_1}^{s_2} \frac{ds}{p/\mu c}$ has been calculated instead of $\int_{s_1}^{s_2} ds/p$. For computing this integral the required values were at first obtained from a large graph of $p/\mu c$ against $R - R_1$. It has subsequently been found simpler to use the table itself. Since the steps in γ are so small, it is quite safe to employ linear interpolation for this purpose, making use of the ratio of differences $\frac{\delta(p/\mu c)}{\delta R}$ given in the fifth column of the table.

The integral of Eq. (iv) was not evaluated much beyond 10^{10} ev/c because the Bethe-Bloch relation, as quoted here, does not hold when radiative collisions become important. This implies that the energy of the particle should be small compared with $(\mu/m)\mu c^2$, which is 2.37×10^{10} ev for the μ-meson.

5. The representation of the atmosphere.

To obtain values of the momentum at various points along the trajectory from the table just discussed, it is necessary to know the mass of air between s_1 and each of the points along the path. A knowledge of the distribution of the air mass is required.

In the days before radiosonde data were available, it was usual to assume an exponential atmosphere, the pressure

at a height h above sea level being given by

$$p = p_o e^{-h/h_o},$$

where p_o is the sea level pressure and h_o is the height of the homogeneous atmosphere. The value used for h_o was somewhat arbitrarily taken as 8 km. The use of an exponential representation of course greatly simplifies any calculations, but it does not describe the atmosphere accurately. In recent years it has become more common to use aerological tables based on the radiosonde flights. Montgomery ((1), p. 347) gives such a table.

For calculations at Macquarie Island, two courses were open. Either a table could be drawn up based upon the daily radiosonde flights from that station, or an exponential representation, also based on these flights, could be derived. The latter course was adopted, partly because it was necessary to have the pressure-height relation for closer intervals of height than the Meteorological Staff were accustomed to work out. The following method was used to derive the best exponential representation.

6. The radiosonde gives the heights h_i of the various pressure levels p_i. If the atmosphere is exponential, we have

$$h = h_o \log p_o - h_o \log p_i. \qquad - (v)$$

If we use the principle of least squares to determine h_o, h_o will be of such a value that it renders the sum of the squares of the deviations of the actual heights (i.e., the h_i) from their estimates based on Eq.(v) a minimum. That is

$$\Sigma \left[h_i - (h_o \log p_o - h_o \log p_i)\right]^2$$

is to be a minimum. This occurs when the partial derivative of this expression with respect to h_o is zero.

Thus we have

$$\sum (h_i - h_o \log p_o + h_o \log p_i) \log p_i = 0 \qquad -(vi)$$

Table I gives the average heights of the various pressure levels at Macquarie Island for the period April 1 to October 31, 1950. Using these, the value obtained from Eq.(vi) for h_o was 7.11 km. The mean sea level pressure was 1004.8 mb. We therefore represent the Macquarie Island atmosphere for this period by

$$p = 1005\ e^{-h/7.11} \text{ millibar.} \qquad -(vii)$$

The third column of Table I gives the heights calculated from Eq.(vii) and the fourth column the differences between the average and the calculated heights.

TABLE I.

Average pressure-heights values for the Macquarie Island atmosphere April 1 - October 31, 1950. Average sea level pressure 1004.8 mb h_{calc} obtained from $p = 1005 \exp(-h/7.11)$ mb.

p (millibar)	h_{obs} (metres)	h_{calc} (metres)	$\delta_{calc-obs}$
1000	44.9	347.0	+302.1
900	890.2	784.2	-106.0
850	1350.5	1190.2	-160.3
800	1822.2	1621.1	-201.1
700	2858.0	2571.7	-286.3
600	4029.1	3667.3	-361.8
500	5365.8	4963.5	-402.3
400	6935.3	6551.9	-383.4
300	8854.6	8595.3	-259.3
200	11429.3	11478.4	+ 49.1
100	15843.7	16718.5	+874.8
80	17268.7	17992.6	+723.9
60	19137.4	20038.8	+901.4
50	20299.3	21335.0	+1035.7
			$\Sigma\delta$ = +1726.5

Considering that the heights of the various levels change by amounts at least equal to these differences and often

by much more under different weather conditions, it does not seem that Eq.(vii) represents an atmosphere seriously different from the actual one.

7. It is necessary to calculate the mass of air between sea level and any point at distance s along an inclined trajectory. The mass of air vertically above the Macquarie Island station is 1025 g cm^{-2}. Hence for a path inclined at an angle Z to the vertical, the mass of air, x, between sea level and a point at distance s along the path is

$$x = \frac{1025}{\cos Z} \left[1 - e^{-s \cos Z/7.11}\right] \text{ g cm}^{-2} \qquad \text{(viii)}$$

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RANGE IN AIR OF μ-MESONS (Mass ≐ 215 m)

The numbers tabulated in the fourth column are the ranges in excess of that of a μ-meson of momentum 185.1 Mev/c.

Υ	$\sinh \Upsilon = p/\mu c$	p Mev/c	Range g cm^{-2}	$\delta(p/\mu c)/\delta R$
1.30	1.698	185.1		
1.35	1.799	196.1		
1.40	1.904	207.5	9.989	
				0.01979
1.45	2.014	219.5	15.55	
				0.01930
1.50	2.129	232.1	21.50	
				0.01901
1.55	2.250	245.3	27.87	
				0.01858
1.60	2.376	259.0	34.65	
				0.01817
1.65	2.507	273.3	41.86	
				0.01817
1.70	2.646	288.4	49.51	
				0.01776
1.75	2.790	304.1	57.62	
				0.01773
1.80	2.942	320.7	66.19	
				0.01755
1.85	3.101	338.0	75.25	
				0.01748
1.90	3.268	356.2	84.81	
				0.01738
1.95	3.443	375.3	94.87	
				0.01735
2.00	3.627	395.3	105.5	
				0.01730
2.05	3.820	416.4	116.6	
				0.01724
2.10	4.022	438.4	128.4	
				0.01722
2.15	4.234	461.5	140.7	
				0.01727
2.20	4.457	485.8	153.6	
				0.01728
2.25	4.691	511.3	167.1	
				0.01733
2.30	4.937	538.1	181.3	

γ	sinh γ = $p/\mu c$	p Mev/c	Range g cm^{-2}	$\frac{\delta(p/\mu c)}{\delta R}$
2.30	4.937	538.1	181.3	
				0.01735
2.35	5.195	566.3	196.2	
				0.01739
2.40	5.466	595.8	211.8	
				0.01748
2.45	5.751	626.9	228.1	
				0.01752
2.50	6.050	659.5	245.2	
				0.01764
2.55	6.365	693.8	263.0	
				0.01767
2.60	6.695	729.8	281.7	
				0.01777
2.65	7.042	767.6	301.2	
				0.01782
2.70	7.406	807.3	321.6	
				0.01794
2.75	7.789	849.0	343.0	
				0.01806
2.80	8.192	892.9	365.3	
				0.01814
2.85	8.615	939.0	388.6	
				0.01827
2.90	9.060	987.5	413.0	
				0.01835
2.95	9.527	1038	438.4	
				0.01854
3.00	10.02	1092	465.0	
				0.01835
3.05	10.53	1148	492.8	
				0.01896
3.10	11.08	1208	521.8	
				0.01881
3.15	11.65	1270	552.1	
				0.01896
3.20	12.25	1335	583.8	
				0.01905
3.25	12.88	1404	616.8	
				0.01912
3.30	13.54	1476	651.4	
				0.01914
3.35	14.23	1551	687.4	
				0.01966
3.40	14.97	1632	725.0	
				0.01933
3.45	15.73	1715	764.4	

γ	$\sinh \gamma = p/\mu c$	p Mev/c	Range g cm^{-2}	$\frac{\delta(p/\mu c)}{\delta R}$
3.45	15.73	1715	764.4	
				0.01973
3.50	16.54	1803	805.4	
				0.01983
3.55	17.39	1896	848.3	
				0.02010
3.60	18.29	1994	893.1	
				0.01988
3.65	19.22	2095	939.8	
				0.02028
3.70	20.21	2203	988.7	
				0.02035
3.72	20.62	2248	1009	
				0.02050
3.74	21.04	2293	1029	
				0.02015
3.76	21.46	2339	1050	
				0.02075
3.78	21.90	2387	1071	
				0.02038
3.80	22.34	2435	1093	
				0.02050
3.82	22.79	2484	1115	
				0.02058
3.84	23.25	2534	1137	
				0.02067
3.86	23.72	2585	1160	
				0.02075
3.88	24.20	2638	1183	
				0.02081
3.90	24.69	2691	1207	
				0.02087
3.92	25.19	2746	1231	
				0.02092
3.94	25.70	2801	1255	
				0.02097
3.96	26.22	2858	1280	
				0.02100
3.98	26.75	2916	1305	
				0.02103
4.00	27.29	2975	1331	
				0.02105
4.02	27.84	3035	1357	
				0.02106
4.04	28.40	3096	1383	
				0.02143
4.06	28.98	3159	1410	

γ	sinh γ = $p/\mu c$	p Mev/c	Range g cm^{-2}	$\frac{\delta(p/\mu c)}{\delta R}$
4.06	28.98	3159	1410	
				0.02104
4.08	29.56	3222	1438	
				0.02143
4.10	30.16	3287	1466	
				0.02138
4.12	30.77	3354	1495	
				0.02138
4.14	31.39	3422	1524	
				0.02168
4.16	32.03	3491	1553	
				0.02163
4.18	32.68	3562	1583	
				0.02158
4.20	33.34	3634	1614	
				0.02153
4.22	34.01	3707	1645	
				0.02179
4.24	34.70	3782	1677	
				0.02172
4.26	35.40	3859	1709	
				0.02166
4.28	36.11	3937	1742	
				0.02188
4.30	36.84	4016	1775	
				0.02209
4.32	37.59	4097	1809	
				0.02200
4.34	38.35	4180	1843	
				0.02190
4.36	39.12	4264	1879	
				0.02208
4.38	39.91	4350	1914	
				0.02224
4.40	40.72	4438	1951	
				0.02212
4.42	41.54	4528	1988	
				0.02227
4.44	42.38	4619	2026	
				0.02239
4.46	43.24	4713	2064	
				0.02229
4.48	44.11	4808	2103	
				0.02239
4.50	45.00	4905	2143	
				0.02281
4.51	45.46	4955	2163	

γ	sinh γ = p/μc	p Mev/c	Range g cm^{-2}	$\frac{\delta(p/\mu c)}{\delta R}$
4.51	45.46	4955	2163	
				0.02215
4.52	45.91	5004	2183	
				0.02245
4.53	46.37	5054	2204	
				0.02273
4.54	46.84	5106	2224	
				0.02253
4.55	47.31	5157	2245	
				0.02281
4.56	47.79	5209	2266	
				0.02261
4.57	48.27	5261	2288	
				0.02242
4.58	48.75	5314	2309	
				0.02268
4.59	49.24	5367	2331	
				0.02294
4.60	49.74	5422	2352	
				0.02274
4.61	5024	5476	2374	
				0.02254
4.62	50.74	5531	2396	
				0.02280
4.63	51.25	5586	2419	
				0.02304
4.64	51.77	5643	2441	
				0.02284
4.65	52.29	5700	2464	
				0.02264
4.66	52.81	5756	2487	
				0.02287
4.67	53.34	5814	2510	
				0.02310
4.68	53.88	5873	2534	
				0.02290
4.69	54.42	5932	2557	
				0.02312
4.70	54.97	5992	2581	
				0.02291
4.71	55.52	6052	2605	
				0.02313
4.72	56.08	6113	2629	
				0.02292
4.73	56.64	6174	2654	

γ	sinh γ = $p/\mu c$	p Mev/c	Range g cm^{-2}	$\frac{\delta(p/\mu c)}{\delta R}$
4.73	56.64	6174	2654	
				0.02313
4.74	57.21	6236	2678	
				0.02333
4.75	57.79	6299	2703	
				0.02312
4.76	58.37	6362	2728	
				0.02332
4.77	58.96	6427	2754	
				0.02311
4.78	59.55	6491	2779	
				0.02329
4.79	60.15	6556	2805	
				0.02309
4.80	60.75	6622	2831	
				0.02327
4.81	61.36	6688	2857	
				0.02344
4.82	61.98	6756	2884	
				0.02324
4.83	62.60	6823	2910	
				0.02341
4.84	63.23	6892	2937	
				0.02357
4.85	63.87	6962	2964	
				0.02336
4.86	64.51	7032	2992	
				0.02351
4.87	65.16	7102	3019	
				0.02332
4.88	65.81	7173	3047	
				0.02346
4.89	66.47	7245	3075	
				0.02361
4.90	67.14	7318	3104	
				0.02375
4.91	67.82	7392	3132	
				0.02354
4.92	68.50	7467	3161	
				0.02368
4.93	69.19	7542	3190	
				0.02347
4.94	69.88	7617	3220	
				0.02360
4.95	70.58	7693	3249	
				0.02373
4.96	71.29	7771	3279	
				0.02385
4.97	72.01	7849	3310	

γ	sinh γ = $p/\mu c$	p Mev/c	Range g cm^{-2}	$\frac{\delta(p/\mu c)}{\delta R}$
4.97	72.01	7849	3310	
				0.02364
4.98	72.73	7928	3340	
				0.02376
4.99	73.46	8007	3371	
				0.02387
5.00	74.20	8088	3402	
				0.02398
5.01	74.95	8170	3433	
				0.02535
5.02	75.75	8257	3465	
				0.02198
5.03	76.45	8333	3496	
				0.02491
5.04	77.25	8420	3529	
				0.02468
5.05	78.05	8507	3561	
				0.02294
5.06	78.80	8589	3594	
				0.02426
5.07	79.60	8676	3627	
				0.02404
5.08	80.40	8764	3660	
				0.02383
5.09	81.20	8851	3693	
				0.02362
5.10	82.00	8938	3727	
				0.02488
5.11	82.85	9031	3762	
				0.02321
5.12	83.65	9118	3796	
				0.02445
5.13	84.50	9211	3831	
				0.02423
5.14	85.35	9303	3866	
				0.02402
5.15	86.20	9396	3901	
				0.02520
5.16	87.10	9494	3937	
				0.02359
5.17	87.95	9587	3973	
				0.02476
5.18	88.85	9685	4009	
				0.02454
5.19	89.75	9783	4046	
				0.02432
5.20	90.65	9881	4083	

γ	$\sinh \gamma = p/\mu c$	p Mev/c	Range g cm^{-2}	$\frac{\delta(p/\mu c)}{\delta R}$
5.20	90.65	9881	4083	
				0.02411
5.21	91.55	9979	4120	
				0.02390
5.22	92.45	10077	4158	
				0.02501
5.23	93.40	10181	4196	
				0.02478
5.24	94.35	10284	4234	
				0.02327
5.25	95.25	10382	4273	
				0.02563
5.26	96.25	10491	4312	
				0.02540
5.27	97.25	10600	4351	
				0.02266
5.28	98.15	10698	4391	
				0.02621
5.29	99.20	10813	4431	
				0.02473
5.30	100.20	10922	4472	

PART II.

VARIATIONS OF THE DIRECTIONAL INTENSITIES.

1. In this Part we shall be concerned with the systematic changes in directional intensities which are related to variations in meteorological factors. The data which we shall analyse were obtained at Macquarie Island. The results will be compared with those obtained by other observers and with predictions based on processes of absorption and decay in the atmosphere.

Since the methods used in analyses of the experimental data are almost entirely statistical, we shall begin by giving an outline of these methods.

Statistical Methods

2. Different numbers of cosmic ray particles are recorded by a counter array in equal intervals of time. These fluctuations of counting rate are partly systematic, due to changes in meteorological conditions for instance, and partly random. It is necessary to be able to discover whether systematic variations are present, to measure their magnitude and to estimate what confidence can be placed in these measures.

3. In the absence of systematic changes (i.e. the counting rate is subject to random fluctuations only), the counting rate follows a Poisson distribution. This means that the probability of observing X counts in a given period is

$$P(X) = \frac{\overline{X}^{X} e^{-\overline{X}}}{X!} ,$$

where $\overline{X}$ is the average counting rate. An important property of the Poisson distribution is that the variance (which is a measure of the spread of the values, and is defined as

$\sigma^2 = \frac{1}{n} \Sigma(X - \bar{X})^2$, where n is the number of observations made) is equal to the mean, $\bar{X}$. Another important property is that for large values of $\bar{X}$ (10) the distribution very closely approximates the normal one in which the probability that a value lies in the range X,X+dX is

$$P(X)dX = \frac{1}{\sigma\sqrt{2\pi}} e^{-(X - \bar{X})^2/2\sigma^2} dX.$$

4. When systematic variations are present the distribution will no longer be a pure Poisson. This implies in general, that the variance, σ^2, is different from the mean, $\bar{X}$. Usually the systematic changes will increase the range of variation of the counting rate and hence increase the variance. In the search for systematic variations we need to know whether the observed variance is significantly different from the value expected if such variations are absent, i.e. the mean. A χ^2 test may be used for this purpose. χ^2 is defined as

$$\chi^2 = \Sigma \frac{(\text{observed value} - \text{expected value})^2}{\text{expected value}}$$

In this case, let us define a quantity χ_s^2 as

$$\chi_s^2 = \Sigma(X - \bar{X})^2 / \bar{X}$$

which, upon introducing the variance as defined above, becomes

$$\chi_s^2 = n\sigma^2 / \bar{X}.$$

Since for large values of $\bar{X}$ the distribution of X is approximately normal, we may expect χ_s^2 to follow the χ^2 distribution with (n - 2) degrees of freedom. Cochran (1) has shown that even for small values of $\bar{X}$ ($\sim$2) and for small values of n ($\sim$4) the approximation of χ_s^2 to χ^2 is not too bad, although the χ^2 distribution tends slightly to underestimate the probabilities.

In estimating the significance of differences, it is

usual to regard deviations which occur with a probability of 5 percent or less as significant.

5. Assuming that this test has established that systematic variations are present in the data, we must proceed to discover whether these fluctuations are related to changes in any other factors, such as meteorological conditions, and, if so, to measure the magnitude of the relationship between the variables. This is accomplished by means of a correlation analysis which we shall now consider briefly. It must be stated at the outset, however, that these statistical methods do not allow us to conclude that, say, pressure variations <u>cause</u> changes in cosmic ray intensity. They merely allow us the say that the changes in pressure and intensity are related.

<u>6. Correlation analysis.</u> The cosmic ray intensity will be denoted by the variable X_1 and the other variables (pressure, temperature, etc.) by X_2, X_3, and X_4. To simplify our expressions, we consider the deviations of the values of these variables from their means $\overline{X}_1$, $\overline{X}_2$, $\overline{X}_3$, and $\overline{X}_4$, denoting the deviations by x_1, x_2, x_3 and x_4.

In seeking the relationship between these deviations, we assume that they are linearly related. Sometimes we will be concerned with the correlation of x_1 with only one other variable, sometimes with more than one. We there-fore consider the relationships

$$x_1 = a_o + b_{12}x_2 \qquad \text{-(i)}$$

$$x_1 = a_1 + b_{12.3}x_2 + b_{13.2}x_3 \qquad \text{-(ii)}$$

$$x_1 = a_2 + b_{12.34}x_2 + b_{13.24}x_3 + b_{14.23}x_4. \qquad \text{-(iii)}$$

In these regression equations, the coefficients, b, are called the regression coefficients. Thus in Eq. (i) in which it is assumed that all the systematic variations of X_1 are related to X_2, b_{12} (the total regression coefficient) measures this relationship. The partial regression coefficients of the type $b_{12.3}$ and $b_{12.34}$ measure the relationship between X_1 and X_2, with X_3 (in (ii)) and X_3 and X_4 (in (iii)) held constant.

The regression coefficients are determined by applying the principle of least squares to the regression equations. That is, the values of the coefficients are determined in such a way that the sum of the squares of the deviations of the observed values of x_1 from their estimates calculated from the regression equations is a minimum. No attempt will be made to present the detailed theory here. The methods and notations are set out by Weatherburn (2). The required definitions and formulae will be quoted with sufficient explanation.

It turns out that the constants a_o, a_1 and a_2 in Eqs. (i)-(iii) are zero.

The total regression coefficients b_{ij}, measuring the slope of the line of regression of X_i on X_j, is found to be

$$b_{ij} = \frac{\Sigma x_i x_j}{\sigma_j^2}$$

The strength of the correlation between X_i and X_j is measured by the total correlation coefficient, r_{ij}, defined as

$$r_{ij} = \frac{\Sigma x_i x_j}{\sigma_i \sigma_j}$$

Thus $r_{ij} = \sqrt{b_{ij} b_{ji}}$.

Clearly, $r_{ij} = r_{ji}$. If the variables are independent, r_{ij} is not sufficiently different from zero. If they are strongly

correlated, r_{ij} approaches plus or minus 1, according as X_i increases or decreases as X_j increases.

The partial regression coefficients are related to the total correlation coefficients, and this fact is made use of in the computational scheme. The value of r_{ij} for each pair of variables is first worked out.

In correlation with N variables, it is convenient to define a quantity ω as

$$\omega = \begin{vmatrix} 1 & r_{12} & \cdots\cdots & r_{1N} \\ r_{21} & 1 & \cdots\cdots & r_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ r_{N1} & r_{N2} & \cdots\cdots & 1 \end{vmatrix}$$

Let ω_{ij} be the cofactor of the element in the ith row and j^{th} column of ω. Then the partial regression coefficients, abbreviated to the general form $b_{1i.(k)}$ turn out to be given by

$$b_{1i.(k)} = -\frac{\sigma_1 \omega_{1i}}{\sigma_i \omega_{11}} .$$

The <u>partial correlation coefficients</u>, $r_{1i.(k)}$, which measure the strength of the correlation between X_1 and X_i when the effects of other variables are eliminated, are given by

$$r_{1i.(k)} = -\frac{\omega_{1i}}{\sqrt{\omega_{11}\omega_{ii}}}$$

The <u>multiple correlation coefficient</u>, which measures how well the observed values agree with the ones predicted from the regression equation, is defined as

$$R_{1.(k)} = \sqrt{1 - \omega/\omega_{11}} .$$

$R_{1.(k)}$ is always positive, lying between 0 and 1.

It is also possible to calculate how much the variance of X_1 is reduced by taking into account the various systematic variations. The <u>residual variance</u>, $\sigma^2_{1.i}$, obtained by removing the effect related with the variable X_i (no account being taken of the other variables) is

$$\sigma^2_{1.i} = \sigma^2_1(1 - r^2_{1i}) .$$

The residual variance $\sigma^2_{1.(k)}$ obtained by removing the effects related to all the other variables is

$$\sigma^2_{1.(k)} = \sigma^2_1 \omega / \omega_{11} .$$

We can now summarize the expressions required for the computation of the various quantities. These are given in Table II.1.

7. We can apply the χ^2 test mentioned in par. 4 to see whether, by taking into account the effects associated with the other variables, the variance has been reduced to an amount which could occur in random sampling from a normal population whose variance and mean are equal. Our new χ^2_s will be determined from the residual variance as

$$\chi^2_s = n\, \sigma^2_{1.(k)} / \overline{X}_1 .$$

If we find that $\sigma^2_{1.(k)}$ still differs significantly from the mean, it is likely that we have overlooked the effect from some other factors whose variations produce systematic changes in the intensity. The converse does not apply, in general. That is, we cannot conclude that there are no other factors influencing the intensity if those we have considered give a residual variance insignificantly difference from the mean.

TABLE II. 1.

Expressions required in correlation analysis.

X_1 is the cosmic ray intensity.

X_2, X_3, X_4 are the other variables.

n is the number of sets of values.

Covariance	$\frac{1}{n} \Sigma\, x_i x_j$	=	$\frac{1}{n} \Sigma\, X_i X_j - \bar{X}_i \bar{X}_j$
Variance	σ_i^2	=	$\frac{1}{n} \Sigma\, x_i x_i$
Total Regression Coefficient	b_{ij}	=	$\frac{\Sigma\, x_i x_j}{\Sigma\, x_i x_i}$
Total Correlation Coefficient	r_{ij}	=	$\frac{\Sigma\, x_i x_j}{\sigma_i\, \sigma_j} = \sqrt{b_{ij}\, b_{ji}}$
Residual Variance of 1st Order	$\sigma_{1.i}^2$	=	$\sigma_1^2\, (1 - r_{1i}^2)$
Partial Regression Coefficient	$b_{1i.(k)}$	=	$-\frac{\sigma_1}{\sigma_i}\, \frac{\omega_{1i}}{\omega_{11}}$
Partial Correlation Coefficient	$r_{1i.(k)}$	=	$-\frac{\omega_{1i}}{\sqrt{\omega_{11}\, \omega_{ii}}}$
Residual Variance	$\sigma_{1.(k)}^2$	=	$\sigma_1^2\, \omega/\omega_{11}$
Multiple Correlation Coefficient	$R_{1.(k)}$	=	$\sqrt{1 - \omega/\omega_{11}}$

TABLE II. 1 (Contd).

Where for 3-fold analysis

$$\omega_{12} = r_{13}r_{23} - r_{12}$$

$$\omega_{13} = r_{12}r_{23} - r_{13}$$

$$\omega_{11} = 1 - r_{23}^2$$

$$\omega_{22} = 1 - r_{13}^2$$

$$\omega_{33} = 1 - r_{12}^2$$

$$\omega = \omega_{11} + r_{12}\omega_{12} + r_{13}\omega_{13}$$

and for 4-fold analysis

$$\omega_{12} = - r_{12} + r_{13}r_{23} + r_{14}r_{24} + r_{12}r_{34}^2 - r_{23}r_{34}r_{14} - r_{24}r_{34}r_{13}$$

$$\omega_{13} = - r_{13} + r_{12}r_{23} + r_{14}r_{34} + r_{13}r_{24}^2 - r_{23}r_{24}r_{14} - r_{24}r_{34}r_{12}$$

$$\omega_{14} = - r_{14} + r_{12}r_{24} + r_{13}r_{34} + r_{14}r_{23}^2 - r_{23}r_{24}r_{13} - r_{23}r_{34}r_{12}$$

$$\omega_{11} = 1 - r_{23}^2 - r_{24}^2 - r_{34}^2 + 2r_{23}r_{24}r_{34}$$

$$\omega_{22} = 1 - r_{13}^2 - r_{14}^2 - r_{34}^2 + 2r_{13}r_{14}r_{34}$$

$$\omega 33 = 1 - r_{12}^2 - r_{14}^2 - r_{24}^2 + 2r_{12}r_{14}r_{24}$$

$$\omega_{44} = 1 - r_{12}^2 - r_{13}^2 - r_{23}^2 + 2r_{12}r_{13}r_{23}$$

$$\omega = \omega_{11} + r_{12}\omega_{12} + r_{13}\omega_{13} + r_{14}\omega_{14}$$

8. Significance and fiducial limits of an observed regression coefficient.

It is essential to know whether our regression coefficients differ significantly from zero or from some hypothetical value, and to know in what range the true, or population, regression coefficient is likely to lie. This will be of importance, for instance, when we come to consider whether or not the pressure coefficients change with zenith angle.

This problem is discussed by Weatherburn (2, p.194) for the bivariate normal population. His relations have been modified slightly for computational purposes and extended to cover the multivariate normal population.

If $\beta_{1i.(k)}$ be the population regression coefficient (which we would like to know), p be the number of the independent variables, and $-\delta_{1i}$ be the cofactor of r_{1i} in ω_{1i}, then the statistic, t, defined as

$$t = \left[b_{1i.(k)} - \beta_{1i,(k)}\right] \sqrt{\frac{(n - p - 1)\sigma_i^2 \omega_{11}}{\delta_{1i} \sigma_{1.(k)}^2}}$$

follows Student's distribution.

To find the fiducial limits for $\beta_{1i.(k)}$ we calculate

$$\beta_{1i.(k)} = b_{1i.(k)} \pm t \sqrt{\frac{\delta_{1i} \sigma_{1.(k)}^2}{(n - p - 1)\sigma_i^2 \omega_{11}}}$$

by using the value of t (taken from a t-table) appropriate to the fiducial range required. As in most tests of significance, we are interested in the range in which $\beta_{1i.(k)}$ is likely to lie with a probability of 0.95. When $(n - p - 1)$ exceeds about 30, as it does in the work we shall be considering, t becomes a standard normal variable (mean zero and standard deviation unity). Thus the value we shall usually take is $t = 1.96$. In 2-fold and 3-fold correlation, $\delta_{1i} = 1$. In 4-fold correlation, $\delta_{12} = 1 - r_{34}^2$, $\delta_{13} = 1 - r_{24}^2$ and $\delta_{14} = 1 - r_{23}^2$.

9. Significance and fiducial limits of an observed correlation coefficient.

We also need to know whether an observed correlation coefficient is significantly different from zero, or whether two coefficients are significantly different from one another.

This problem is treated by Weatherburn (2, p. 192). It is shown for a bivariate normal population, that if the hypothesis is made that two variables X_i and X_j are uncorrelated, the statistic t, defined as

$$t = r_{ij}\sqrt{n - 2} \,/\, \sqrt{1 - r_{ij}^2}$$

follows the t-distribution for (n - 2) degrees of freedom. If the value of t calculated from this expression is improbable (i.e. there is a probability of 0.05 or less of such a value being found in random sampling), then this hypothesis (viz. that $r_{ij} = 0$) is discredited and we can conclude that the variables may be correlated.

For the partial regression coefficients in a multivariate normal distribution, the statistic

$$t = r_{ij.(k)}\sqrt{n - k - 2} \,/\, \sqrt{1 - r_{ij.(k)}^2}$$

follows the t-distribution with (n - k - 2) degrees of freedom, k being the number of secondary subscripts (Weatherburn (2) p.256).

Obtaining the fiducial range for a correlation coefficient is more difficult than for a regression coefficient because the sampling distribution of the correlation coefficient is far from normal. However, Fisher has shown that the variable $z_{ij.(k)}$ defined as

$$z_{ij.(k)} = \frac{1}{2}\log_e \frac{1 + r_{ij.(k)}}{1 - r_{ij.(k)}}$$

is distributed almost normally and has variance $1/(n - k - 3)$ and mean $\zeta_{ij.(k)}$ given by

$$\zeta_{ij.(k)} = \frac{1}{2}\log_e \frac{1 + \rho_{ij.(k)}}{1 - \rho_{ij.(k)}}$$

where n is the number of sets of values drawn from the multivariate normal population in which the correlation coefficients are $\rho_{ij.(k)}$. To determine the 95 percent fiducial ranges for the $\rho_{ij.(k)}$ it is necessary to transform the observed values of the $r_{ij.(k)}$ into the corresponding z value, using tables (which are given by Weatherburn (2, p.201) and Fisher (3, p.203)), find the 95 percent limits for $\zeta_{ij.(k)}$ (using tables of the normal variable), and transform back again to find these limits for $\rho_{ij.(k)}$. However, it is more convenient to consider the fiducial ranges of the values of the $\zeta_{ij.(k)}$ because the variance is independent of $\rho_{ij.(k)}$ and is the same for samples of the same size.

The sampling distribution for the multiple correlation coefficient, R, tends to normality when $R \neq 0$ and when n, the sample number, is large, the variance being $4R^2(1 - R^2)^2/n$ (see Kendall (4) p. 382). Hence the fiducial limits may be readily assigned when the conditions specified hold.

To test whether R is significantly different from zero (Kendall (4) p. 382), Fisher's z-distribution may be used, where

$$z = 1/2 \log_e \frac{R^2}{1 - R^2} \quad \frac{n - p}{p - 1}$$

p is the number of variables (i.e. $p = k + 2$), and the appropriate degrees of freedom , ν_1 and ν_2 are $\nu_1 = p - 1$ and $\nu_2 = n - p$. Tables of z are given by Fisher (3, p.236) and Kendall (4, p. 443).

Experimental Results.

10. The data obtained at Macquarie Island between June 1950 and March 1952 in the course of investigations into the high latitude East-West asymmetry are available for analysis. The Island is in geomagnetic latitude 60.7°S and the station is about 4 metres above sea level.

The directional intensities were measured with two identical telescopes which we have labelled A and B. These were always operated at equal zenith angles, one pointing to the geomagnetic East while the other was at geomagnetic West. At the end of each hour the positions of the telescopes were interchanged by rotating the horizontal turntable on which they were mounted. Thus there are four sets of data for each zenith angle, viz. A_W, A_E, B_W, and B_E, where A_W refers to telescope A pointing West, etc.

The rotation of the turntable occupied approximately one minute, and this interval remained constant. Counts recorded during the rotation have not been included in the results. A correction for showers has been made by counting the rate of coincidences between the two telescopes.

The equipment will be described in greater detail in Part III, but attention will be drawn to other features of it where necessary in this Part.

During the period June 1950 to May 1951, the zenith angles were varied and results for $Z = 15^\circ$, 30°, 45°, 60° and 70° were accumulated. During the period June 1951 to March 1952 the telescopes were set at 45°. In all the results we shall be considering here 12 cm Pb absorber was used in each telescope. No prolonged measurements were made with the telescopes vertical. However, another set of equipment, which had telescopes fixed in the vertical direction, was

operated at Macquarie Island and we shall be able to compare the results of the directional measurements with these.

11. The meteorological data. A first class meteorological station has been established on the Island and all data required were readily available. The mean station level pressure for each hour was estimated to 0.1 mb from the barograph charts. In carrying out this work, the charts were frequently checked against readings of the mercury barometer made at 3-hourly intervals by the Meteorological Staff. No appreciable errors are likely.

The mean sea level temperature was estimated for each hour using the thermograph charts. These instruments were not as accurate as the barographs and it was nearly always necessary to refer to the 3-hourly observations to obtain suitable corrections. It is unlikely that errors exceeding 1°F are present. The hourly means were estimated to the nearest degree.

Upper air data were obtained from radiosonde flights made each day at about 0800 GMT. The information required included the heights of the 500, 300, 150, 100 and 80 mb pressure levels together with the mean temperatures in the intervals 500 to 550, 500 to 600, 300 to 350, 300 to 400, 150 to 200, 100 to 150, 100 to 200 and 80 to 100 mb. The heights were measured to the nearest ten feet and the mean temperature to 0.1°C.

12. Computing methods. The data obtained between June 1951 and March 1952 have been transferred to Hollerith punched cards. The use of the Hollerith sorting and tabulating machines has greatly simplified the work of computing the sums of squares and products required for the multiple correlation analyses. The data collected during this period have been analysed in monthly groups.

The results obtained between June 1950 and May 1951 have been dealt with by ordinary calculating machine methods. A model ESA-O Facit has been found admirable for this purpose because of the ease with which the sums of squares and products may be determined. This procedure is, of course, much slower than the punched card methods and for this reason the 1950-51 data have been correlated with pressure only.

A typical sheet of the 3-fold correlation analysis is appended here. The upper part of the 5th (unlabelled) column contains check totals.

Sheet No.

AUSTRALIAN NATIONAL ANTARCTIC RESEARCH EXPEDITION

CORRELATION ANALYSIS

Station No. 1 (Macquarie Is.)
Year 3 (1951)
Month Y (December)

$x_i = X_i - \overline{X}_i$

$b_{ij} = \Sigma x_i x_j / \Sigma x_j x_j$

$r_{ij} = b_{ij}\sigma_j/\sigma_i = \sqrt{b_{ij}b_{ji}}$

$X_1 =$ BE - 300, cts/hr
$X_2 =$ P - 9400, 1/10 mb
$X_3 =$ T, °F
$X_4 =$

No. of Observations n = 373

C.D.O.—547

i	1	2	3	4	
ΣX_i	39,348	213,068	16,207		268,623
$\overline{X}_i$	105.4906	571.2278	43.4504		720.1689
$\Sigma X_1 X_i$	4,323,550	22,244,489	1,704,533		28,272,572
$\frac{1}{n}\Sigma X_1 X_i$	11,591.286	59,636.69	4,569.793		75,797.78
$\overline{X}_1\overline{X}_i$	11,128.267	60,259.163	4,583.609		75,971.049
$\frac{1}{n}\Sigma x_1 x_i$	463.019	- 622.473	- 13.816		- 173.269
$\Sigma X_2 X_i$	22,244,489	124,913,908	9,285,277		156,443,674
$\frac{1}{n}\Sigma X_2 X_i$	59,636.69	334,889.8	24,893.504		419,419.99
$\overline{X}_2\overline{X}_i$	60,259.163	326,308.37	24,820.076		411,380.55
$\frac{1}{n}\Sigma x_2 x_i$	- 622.473	8,588.43	73.428		8,039.44
$\Sigma X_3 X_i$	1,704,533	9,285,277	707,823		116,976,33
$\frac{1}{n}\Sigma X_3 X_i$	4,569.793	24,893.504	1,897.649		31,360.946
$\overline{X}_3\overline{X}_i$	4,583.609	24,820.076	1,887.937		31,291.626
$\frac{1}{n}\Sigma x_3 x_i$	- 13.816	73.428	9.712		69.320
$\Sigma X_4 X_i$					
$\frac{1}{n}\Sigma X_4 X_i$					
$\overline{X}_4\overline{X}_i$					
$\frac{1}{n}\Sigma x_4 x_i$					
σ_i	21.5179	92.6738	3.1165		
b_{1i}		- 0.072478	- 1.422570		
b_{2i}	- 1.344379		7.560543		
b_{3i}	- 0.029839	0.0085496			
b_{4i}					
$r_1^2{}_i$		0.097438	0.042448		
$r_2^2{}_i$			0.064640		
$r_3^2{}_i$					
$r_4^2{}_i$					
r_{1i}		- 0.312151	- 0.206030		
r_{2i}			0.254244		
r_{3i}					
r_{4i}					
ω_{1i}		0.259769	0.126668	ω	0.828176
ω_{ii}	0.935360	0.957552	0.902562	ω/ω_{11}	0.885409
$\omega_{1i}\omega_{ii}$		0.895656	0.844220	$R^2_{1.23}$	0.114591
$\sqrt{\omega_{11}\omega_{ii}}$		0.946391	0.918814	$R_{1.23}$	0.338513
ω_{1i}/ω_{11}		0.277721	0.135422		
σ_1/σ_i		0.232190	6.904508		
$b_{1i.j}$		- 0.064484	- 0.935022		
$r_{1i.j}$		-0 .274484	- 0.137860		
$\sigma^2_{i.j}$		417.9034	443.3648	$\sigma^2_{1.23}$	409.9611

TABLE II.2 SUMMARY OF CORRELATION ANALYSES

X_1 = counting rate period period of 59 minutes at 45°, 12 cm Pb. X_2 = mean surface pressure, mb. X_3 = mean surface temperature, °F. The errors with the regression coefficients are the 95 percent fiducial limits, those with $\overline{X_2}$ and $\overline{X_3}$ are the standard deviations. For the meaning of the probability, P, given with each residual variance, see par. 14(c).

MONTH	TEL.	n	b_{12} %/mb	b_{13} %/°C	$b_{12.3}$ %/mb	$b_{13.2}$ %/°C	r_{23}	$R_{1.23}$	$\overline{X_1}$	σ_1^2	$\sigma_{1.23}^2$ P	$\overline{X_2}$	$\overline{X_3}$
JUN '51	A_W	297	-0.169 ±0.055	+0.13 ±0.37	-0.172 ±0.056	-0.09 ±0.36	-0.20	0.335	404.811	582.049	516.758 ~0	1007.99 ±11.76	37.23 ±3.30
	A_E	312	-0.176 ±0.051	+0.24 ±0.35	-0.175 ±0.052	+0.05 ±0.33	-0.17	0.358	398.494	530.889	463.018 0.037	1008.21 ±11.74	37.48 ±3.33
	B_W	312	-0.142 ±0.050	+0.37 ±0.33	-0.136 ±0.051	+0.22 ±0.32	-0.17	0.308	400.859	499.667	452.223 0.090	1008.21 ±11.74	37.48 ±3.33
	B_E	297	-0.186 ±0.050	+0.20 ±0.35	-0.188 ±0.051	-0.04 ±0.33	-0.20	0.393	396.650	489.553	413.904 0.504	1007.99 ±11.76	37.23 ±3.30

TABLE II.2, cont'd

JUL '51	A_W	323	-0.134 ±0.046	-0.27 ±0.29	-0.140 ±0.046	-0.35 ±0.28	-0.10	0.331	416.675	525.228	467.767 0.099	995.48 ±12.43	37.83 ±3.70
	A_E	313	-0.173 ±0.047	-0.04 ±0.28	-0.178 ±0.047	-0.20 ±00.26	-0.16	0.389	411.137	463.519	393.417 0.686	994.60 ±11.56	37.67 ±3.71
	B_W	313	-0.157 ±0.049	-0.10 ±0.29	-0.165 ±0.049	-0.25 ±0.28	-0.16	0.352	412.783	490.123	429.289 0.525	994.60 ±11.56	37.67 ±3.71
	B_E	323	-0.124 ±0.042	-0.02 ±0.27	-0.125 ±0.043	-0.10 ±0.26	-0.10	0.307	410.978	430.430	389.850 0.600	995.48 ±12.43	37.83 ±3.70
AUG '51	A_W	355	-0.217 ±0.035	-0.76 ±0.28	-0.203 ±0.036	-0.38 ±0.25	+0.26	0.557	415.569	624.839	430.685 0.566	998.15 ±15.03	37.31 ±3.85
	A_E	378	-0.251 ±0.035	-1.00 ±0.30	-0.231 ±0.037	-0.44 ±0.27	+0.33	0.595	407.310	705.865	455.780 0.086	1000.22 ±15.10	37.40 ±3.75
	B_W	378	-0.212 ±0.036	-0.88 ±0.29	-0.193 ±0.038	-0.41 ±0.27	+0.33	0.530	412.201	660.147	474.889 0.032	1000.22 ±15.10	37.40 ±3.75
	B_E	355	-0.228 ±0.035	-0.55 ±0.29	-0.223 ±0.036	-0.13 ±0.25	+0.26	0.565	407.324	615.547	419.041 0.608	998.15 ±15.03	37.31 ±3.85

TABLE II.2. cont'd

SEP '51	A_W	350	-0.207 ±0.044	-0.36 ±0.27	-0.206 ±0.046	-0.03 ±0.25	+0.28	0.445	411.843	580.163	465.122 0.073	998.32 ±12.56	38.79 ±4.09
	A_E	348	-0.213 ±0.044	-0.39 ±0.27	-0.211 ±0.046	-0.04 ±0.26	+0.29	0.457	408.940	574.103	454.468 0.118	998.43 ±12.53	38.76 ±4.04
	B_W	348	-0.192 ±0.043	-0.63 ±0.26	-0.174 ±0.045	-0.34 ±0.25	+0.29	0.444	413.471	547.712	439.920 0.335	998.43 ±12.53	38.76 ±4.04
	B_E	350	-0.194 ±0.042	-0.50 ±0.25	-0.183 ±0.04	-0.21 ±0.24	+0.28	0.444	406.351	516.114	414.339 0.695	998.32 ±12.56	38.79 ±4.09
OCT '51	A_W	361	-0.178 ±0.048	+0.09 ±0.24	-0.179 ±0.048	-0.03 ±0.23	-0.15	0.360	405.798	462.064	402.209 0.089	1001.87 ±10.70	37.60 ±4.06
	A_E	372	-0.139 ±0.046	-0.00 ±0.23	-0.141 ±0.046	-0.10 ±0.22	-0.14	0.298	405.602	418.809	381.591 0.490	1001.54 ±10.75	37.63 ±4.08
	B_W	372	-0.100 ±0.047	+0.15 ±0.23	-0.098 ±0.048	+0.08 ±0.23	-0.14	0.214	407.309	429.614	409.905 0.829	1001.54 ±10.75	37.63 ±4.08
	B_E	361	-0.156 ±0.048	+0.09 ±0.24	-0.156 ±0.049	+0.01 ±0.23	-0.15	0.317	402.742	448.759	403.539 0.875	1001.87 ±10.70	37.60 ±4.06

TABLE II.2. cont'd

NOV '51 A_W	283	-0.146 ±0.047	-0.19 ±0.33	-0.154 ±0.050	+0.15 ±0.33	+0.34	0.344	411.170	527.333	464.974 0.102	994.46 ±13.04	40.25 ±3.60
A_E	290	-0.178 ±0.043	-0.56 ±0.30	-0.169 ±0.046	-0.17 ±0.30	+0.36	0.434	407.107	483.315	392.339 0.768	994.20 ±13.01	40.13 ±3.67
B_W	290	-0.183 ±0.045	-0.74 ±0.30	-0.162 ±0.047	-0.37 ±0.30	+0.36	0.446	409.545	518.730	415.710 0.760	994.20 ±13.01	40.13 ±3.67
B_E	283	-0.177 ±0.045	-0.69 ±0.31	-0.160 ±0.048	-0.33 ±0.31	+0.34	0.431	404.565	504.607	410.752 0.758	994.46 ±13.04	40.25 ±3.60
DEC '51 A_W	373	-0.194 ±0.055	-0.47 ±0.31	-0.183 ±0.057	-0.22 ±0.30	+0.25	0.346	409.231	469.657	413.556 0.787	997.12 ±9.27	43.45 ±3.12
A_E	371	-0.166 ±0.055	-0.70 ±0.29	-0.142 ±0.056	-0.53 ±0.29	+0.23	0.342	402.871	449.809	397.117 0.947	997.04 ±9.28	43.44 ±3.26
B_W	371	-0.211 ±0.055	-0.31 ±0.30	-0.208 ±0.051	-0.07 ±0.29	+0.23	0.365	407.213	480.623	416.726 0.665	997.04 ±9.28	43.44 ±3.26
B_E	373	-0.179 ±0.055	-0.63 ±0.31	-0.159 ±0.057	-0.42 ±0.30	+0.25	0.339	405.491	463.019	409.961 0.781	997.12 ±9.27	43.45 ±3.12

TABLE II.2. cont'd

JAN '52	A_W	235	-0.192 ±0.057	-1.39 ±0.60	-0.165 ±0.059	-0.85 ±0.60	+0.33	0.431	412.434	612.460	498.842 0.020	986.03 ±12.45	43.43 ± 2.21
	A_E	235	-0.194 ±0.055	-1.12 ±0.59	-0.177 ±0.059	-0.43 ±0.60	+0.38	0.423	410.038	563.084	462.530 0.132	985.98 ±12.38	43.43 ± 2.20
	B_W	235	-0.191 ±0.054	-1.23 ±0.58	-0.169 ±0.058	-0.58 ±0.58	+0.38	0.431	417.132	561.429	456.980 0.240	985.98 ±12.38	43.43 ± 2.20
	B_E	235	-0.215 ±0.050	-1.09 ±0.56	-0.201 ±0.053	-0.43 ±0.54	+0.33	0.490	410.000	516.740	392.555 0.761	986.03 ±12.45	43.43 ±2.21
FEB '52	A_W	333	-0.083 ±0.058	-0.16 ±0.59	-0.091 ±0.063	+0.21 ±0.64	+0.39	0.158	411.712	498.231	485.877 0.018	998.51 ±10.04	44.14 ±1.76
	A_E	346	-0.151 ±0.058	-0.61 ±0.60	-0.149 ±0.063	-0.05 ±0.62	+0.37	0.266	407.697	525.940	488.651 0.009	998.43 ± 9.90	44.08 ±1.78
	B_W	346	-0.131 ±0.053	-0.71 ±0.54	-0.121 ±0.057	-0.25 ±0.57	+0.37	0.257	409.853	440.074	411.111 0.862	998.43 ± 9.90	44.08 ±1.78
	B_E	333	-0.113 ±0.056	-0.66 ±0.58	-0.104 ±0.061	-0.24 ±0.62	+0.39	0.218	406.228	465.041	443.042 0.201	998.51 ±10.04	44.14 ±1.76

TABLE II.2. cont'd

MAR '52	A_U	156	-0.243 ±0.073	-0.20 ±0.58	-0.243 ±0.073	-0.17 ±0.52	+0.02	0.469	405.891	531.999	414.940 0.693	996.24 ±10.92	41.77 ±2.76
	A_E	158	-0.167 ±0.070	+0.14 ±0.54	-0.167 ±0.071	-0.17 ±0.50	+0.03	0.352	409.127	453.309	397.047 0.947	996.26 ±10.89	42.08 ±2.74
	B_U	158	-0.155 ±0.066	-0.235 ±0.50	-0.154 ±0.066	-0.21 ±0.47	+0.03	0.351	415.038	412.043	361.376 0.322	996.26 ±10.89	42.08 ±2.74
	B_E	156	-0.206 ±0.067	-0.09 ±0.53	-0.206 ±0.068	-0.06 ±0.48	+0.02	0.436	411.692	451.612	365.647 0.414	996.24 ±10.92	41.77 ±2.76

13. Before discussing the results, we must consider a point relating to the significance tests. In paragraphs 8 and 9 methods have been described of assigning fiducial limits to the regression and correlation coefficients. These methods are dependent upon having data drawn from multivariate normal populations. Checks have not been made on all the data to see whether this condition holds. Checks made on most of the 1950-51 results show that there is no reason to suppose that the counting rates are not normally distributed. The distribution of mean hourly barometric pressures for December 1951 has been plotted. Again, there is no strong suggestion of a departure from normality. No similar tests have been made for the surface temperatures or the upper atmospheric data.

14 (a). The June 1951 - March 1952 results. We shall consider these first because during this period the telescopes remained at the same setting, viz. 45°, 12 cm Pb. We will therefore be able to see how variable the coefficients are, thus giving us a basis for comparing the results from the previous year when the telescopes were operated at several different settings. We will also be able to see how important the temperature effect is at Macquarie Island. This is necessary because the results for the previous year have been correlated with pressure only.

In analysing the data, only those results have been considered where both telescopes and also the shower recorder were operating together. The number of showers recorded in each hour has been subtracted from the corresponding telescope rates.

The results of the analyses are set out in Table II.2.

14 (b). The barometer effect and its variability. The values of $b_{12.3}$ and b_{12} are negative and significantly different from zero for all forty cases. Corresponding pairs of $b_{12.3}$ and b_{12} do not differ significantly from one another in any case. The greatest differences occur, of course, when $r_{13.2}$ is appreciable, as in January 1951. It is evident that at Macquarie Island it is fairly safe to disregard the effect of surface temperature in determining the barometer effect. This is to be expected because of the small range of tem peratures experienced.*

There are some significant differences between the values of $b_{12.3}$. The maximum value is -0.243 ± 0.073 percent/mb (in March) and the minimum is -0.091 ± 0.063 (in February). Duperier (5) has drawn attention to the fact that barometer coefficients determined from observations taken over short periods often show a degree of variability greater than would be expected from the statistical precision of the measurements. He considers that this may be due to ignoring the influence of other factors, such as the changes in the height of the region where most of the mesons are produced. The heights of the various pressure levels in the stratosphere are sometimes strongly correlated with the surface pressure and sometimes not. When there is a marked positive correlation, effects properly attributable to the height of production enhance the barometer effect. In the present work, correlation analyses of the upper air data have not been carried out, so far, in monthly groups. Hence we cannot tell whether such effects account for the observed variability.

* The lowest monthly averages during this period were in June and August (37.36°F) and the highest was in February (44.11°F). The extreme mean hourly temperatures were 21°F on September 19 and 51°F on December 15.

However, it should be mentioned that Parsons (6), using data from another set of equipment operated at Macquarie Island during the 1950-51 period, found that there is no apparent relationship between the month to month variations in the pressure coefficient with variations in the mean height of the 100 mb level, with the mean thickness of the 1000-100 mb layer nor with the mean surface pressure.

Since we have four sets of data per month, we can get some idea of how variable the values are under identical atmospheric conditions. We note that the four values usually do not vary much from one another in a month, but there are some instances (June, October, December and March) where there are variations between values of $b_{12 \cdot 3}$ within a month which are significant at the 5 percent level (between the coefficients for A_W and B_W). The reason for this is not clear.

Barometer coefficients have also been worked out with all the East data taken together and with all the West data taken together. The result for A_W and B_W combined for the 10 months is -0.1697 ± 0.0100 percent per mb, and for A_E and B_E combined is -0.1792 ± 0.0098. Similar analyses made with the results from June to November 1951 gave -0.1766 ± 0.0122 (West) and -0.1897 ± 0.0118 (East). Thus, over the whole period, there is no significant difference between the West and East coefficients, although the difference is significant over the first 6 months of the period. A reason why we might expect the East coefficient to exceed the West one will be mentioned in Part III in the section where the variations of the East-West asymmetry are discussed.

The average value of the barometer coefficient of the cosmic rays which can penetrate 12 cm Pb and which arrive at sea level at 45° is

$$-0.1745 \pm 0.0070 \text{ percent per millibar.}$$

14 (c). The temperature effect. On only ten occasions are the values of $b_{13.2}$ significantly different from zero and on all these occasions the coefficient is negative. There are six cases where the coefficient is positive but not significantly different from zero. In the remaining twenty four cases the values are negative but insignificantly different from zero.

Values of b_{13} are negative and significantly different from zero on twenty two occasions. Of these, all which correspond to insignificant values of $b_{13.2}$ occur in months when the pressure and temperature are positively correlated. On one occasion a b_{13} is positive and significant. This occurs in June when the temperature and pressure are negatively correlated. Such examples are to be expected because of the more marked barometer effect on cosmic rays. We therefore see that if we are interested in finding the temperature coefficient, the effect of pressure must be taken into account.

The temperature coefficient obtained from the combined A_W and B_W results for the whole period is -0.20 ± 0.05 percent/$^{\circ}$C, and from the combined A_E and B_E results is -0.19 ± 0.05.

14 (d). The residual variances. The probability quoted with each residual variance in Table II.2 gives the chance of a variance observed in random sampling from a normal population deviating, in absolute value, from the mean by an amount equal to or exceeding the observed difference between the residual variance and the mean, assuming that the mean and the variance of the population are equal. When this probability is less than 0.05, we may infer that we have not adequately accounted for the systematic variations in terms of pressure and temperature changes. There are only six cases where this occurs. However, for the two combined sets of results over the whole period, the residual variances differ markedly from the means. Hence, it is evident that the effect of other factors is important.

There is one example to which attention should be drawn, viz. B_W for March. The initial variance is 412 and the mean is 415. The chance of obtaining such a variance (or a larger one) is 0.94. Thus there is no evidence for systematic changes in the intensity. Yet the correlation analysis leads to a significant pressure coefficient. No adequate explanation has been found for this effect. The possibility that the anomaly is due to instrumental faults must be ruled out because the B_E results do not show the same effect, and the telescope operated in the East and West direction during alternate hours.

If the variability of the barometer coefficient can be attributed to factors which are sometimes correlated with pressure and sometimes not, it may be expected that on those occasions when the residual variance is close to the mean, the barometer effect would be larger than when the residual variance is large. There does not seem to be any evidence from these results that such is the case.

15 (a). The June 1950 - May 1951 results. During this period the equipment was operated at several different settings. We shall consider results obtained at zenith angles of 15°, 30°, 60° and 70°, with 12 cm Pb absorber. Measurements were also made at 45° during this period, but correlation analyses have not been carried out with these because of the larger number of results which became available the following year.

There are some points which must be mentioned concerning the equipment during this period. At the beginning the telescopes were 2-fold and it was not till February 1951 that they were converted to 3-fold ones. This means that the correction for accidental coincidences is very important in the period before February 1951. These corrections have been made in the following way. The correlation analyses proceded in the usual way with the

determination of the sums of the squares of the observed counting rates (uncorrected for accidentals) and the sums of their products with pressure. The values of b_{12} so obtained (in counts per millibar) were divided by the average counting rates with the estimated accidental rate subtracted to give b_{12} in percent per millibar. To obtain the fiducial limits, it is necessary to know the correlation coefficients which, in turn, depend upon the variances of the genuine counting rates. If these variances are taken as those of the observed counting rates, a contribution due to the accidentals is included, so that we assign a greater fiducial range than we should. If we regard the observed counting rate as the composition of two independent variables, viz., the genuine and the accidental rates, we can take it that the observed variance will be the sum of the variances of these two variables. Thus, the variance of the genuine counting rate will be the difference between the observed variance and that of the accidental rate. Assuming that there are no systematic variations of the accidental rate, we take its variance to be equal to its mean.

There are two separate ways of estimating the accidental rate. Firstly, we can estimate it from the observed counting rates of the trays (about 3,300 per minute) and the measured resolving time of the coincidence circuit (4 μsec). This gives a value of about 85 counts per interval of 59 minutes (the interval spent by the telescopes in each direction before rotation). Secondly, we can take the difference between the rates measured at the same angle before and after conversion of the telescopes from 2-fold to 3-fold. This method is most sensitive at the large zenith angles where the counting rate is small, the 2-fold accidental rate being independent of zenith angle. For the 70° 12 cm Pb measurements, this method gives a difference of about 75 counts per 59 minutes. We have there-

TABLE II.3

Summary of results of correlation analyses using Macquarie Island data obtained during the period June 1950 - May 1951 for various zenith angles and with 12 cm Pb. The Table indicates whether the telescopes during a run were 2-fold or 3-fold.

Description of Run	TEL.	n	b_{12} (%/mb.)	r_{12}	$\bar{X}_1$	σ_1^2	$\sigma_{1.2}^2$	$\bar{X}_2$
15° 1st Run, 2-Fold	A_W	240	-0.179 ±0.039	-0.50	706.158	1067.2	802.2	990.18 ±12.85
26/11/50-19/12/50	A_E	243	-0.189 ±0.037	-0.54	701.811	1008.5	717.4	990.36 ±12.87
	B_W	241	-0.207 ±0.039	-0.56	720.668	1182.5	812.8	990.34 ±12.92
	B_E	239	-0.229 ±0.038	-0.61	711.100	1189.6	749.4	990.17 ±12.87
15° 2nd Run, 3-Fold	A_W	252	-0.085 ±0.044	-0.23	756.052	1440.3	1363.1	999.13 ±13.73
9/2/51-6/3/51	A_E	254	-0.134 ±0.043	-0.36	755.689	1405.3	1221.4	999.51 ±13.36
	B_W	255	-0.206 ±0.035	-0.56	784.090	1354.1	889.5	999.49 ±13.34
	B_E	252	-0.197 ±0.035	-0.57	778.489	1342.9	901.9	999.13 ±13.73

TABLE II.3. cont'd

30° 1st Run, 2-Fold	A_W	190	-0.167 ±0.091	-0.25	656.294	1355.8	1268.9	1005.39 ±8.50
3/10/50-20/10/50	A_E	196	-0.251 ±0.088	-0.37	651.918	1364.4	1174.4	1005.38 ±8.42
	B_W	178	-0.266 ±0.079	-0.45	648.978	1165.5	934.7	1005.50 ±8.80
	B_E	174	-0.247 ±0.081	-0.41	640.046	1138.7	942.9	1005.37 ±8.84
30° 2nd Run, 3-Fold	A_W	61	-0.137 ±0.162	-0.21	631.934	735.5	702.3	1000.22 ±6.75
28/3/51-2/4/51	A_E	59	-0.111 ±0.154	-0.18	622.356	643.0	621.2	1000.17 ±6.75
6/3/51-2/4/51	B_W	303	-0.208 ±0.048	-0.43	626.066	887.2	722.2	999.93 ±10.15
	B_E	309	-0.139 ±0.046	-0.32	619.194	772.7	694.4	999.88 ±10.30
60° 2-Fold	B_W	301	-0.154 ±0.055	-0.30	209.483	421.012	382.4	996.75 ±14.76
17/8/50 – 14/9/50	B_E	294	-0.151 ±0.092	-0.27	202.998	456.3	421.8	996.96 ±14.73

TABLE II.3. cont'd.

70° 1st Run, 2-Fold	A_W	232	-0.377 ±0.132	-0.35	96.013	118.0	104.0	997.69 ±10.37
7/11/50-26/11/50	A_E	206	-0.221 ±0.039	-0.21	92.233	89.85	85.75	998.19 ±9.93
	B_W	206	-0.218 ±0.145	-0.20	102.272	119.4	114.4	998.19 ±9.93
	B_E	232	-0.361 ±0.143	-0.31	99.539	144.6	130.8	997.69 ±10.37
70° 2nd Run, 2-Fold	B_W	218	-0.133 ±0.154	-0.11	90.849	77.3	76.3	1005.65 ±8.35
7/1/51-25/1/51	B_E	219	-0.070 ±0.193	-0.05	87.936	117.9	117.6	1005.56 ±8.51
70° 3rd Run, 3-Fold	A_W	326	-0.223 ±0.135	-0.18	106.034	96.201	93.183	999.66 ±7.35
2/4/51-8/5/51	A_E	320	-0.221 ±0.149	-0.16	102.397	103.234	100.544	999.61 ±7.24
	B_W	320	-0.012 ±0.148	-0.09	104.047	103.015	102.208	999.61 ±7.24
	B_E	326	-0.158 ±0.149	-0.12	100.850	103.605	102.231	999.66 ±7.35

fore taken the accidental rate as the average of these two estimates, viz., 80 counts per 59 minutes. interval. Hence we have subtracted 80 from the observed counting rate and from the observed variance in each of the experiments before February 1951.

Another feature of the equipment during the early part of this period must be mentioned. The clock which initially controlled the timing of the photographs of the registers turned out to be very poor. Its rate varied so much with spring tension that corrections were necessary. The rate of the clock was determined by comparison with time signals from radio station WWVH in Honolulu, or with a chronometer on another piece of equipment on the Island. The counting rate was then found in counts per minute by dividing the rate per '59' minutes by the corrected clock rate, taking this to be constant over the period between checks against WWVH or the chronometer, which were made once a day. Fortunately, these corrections only apply to one set of results considered here (viz. those for 60°), because it was possible to have a chronometer sent down on a ship which visited the Island in September, 1950. This was quickly fitted with electric contacts and put into operation in place of the original clock.

15 (b). Since the analyses for 1951-52 show that the temperature effect is unimportant in determining the barometer effect at Macquarie Island, correlation has been carried out with pressure only with the 1950-51 data. The results are set out in Table II.3.

It will be seen that all values of b_{12} are negative, though there are a few cases where they are not significantly different from zero. There is a greater degree of variability

than was found with the 1951-52 results. This is most noticeable for the 70° runs, where the smallest value is -0.01 and the largest -0.38 percent per millibar. There is no reason to doubt the performance of the equipment, since the results obtained with the same telescope pointing in the opposite direction during the same run gave results much closer to the average.

15 (c). Some 0° results. Another cosmic ray recorder was operated at Macquarie Island by Mr. N.R. Parsons during the period June 1950 to May 1951. This recorder had several telescopes fixed in the vertical direction. One of the 3-fold ones had 10 cm Pb absorber in it. The counting area was the same as in the equipment with which the observations discussed above were made (i.e. 400 cm^2), but the separation of the extreme trays was 30 cm instead of 75 cm. The vertical counting rate was therefore about four times greater, so that the statistics were somewhat better. The values of $b_{12.3}$ for the twelve months are set out in Table II.4.

TABLE II.4

Values of $b_{12.3}$ in percent per millibar obtained at Macquarie Island by Parsons (6) using 10 cm Pb in a vertical telescope. The 95 percent fiducial ranges are given.

June 1950	-0.182 ± 0.021	December 1950	-0.171 ± 0.013
July	-0.155 ± 0.022	January 1951	-0.214 ± 0.017
August	-0.152 ± 0.018	February	-0.174 ± 0.012
September	-0.171 ± 0.009	March	-0.146 ± 0.020
October	-0.121 ± 0.012	April	-0.153 ± 0.020
November	-0.157 ± 0.013	May	-0.158 ± 0.009

It would be of interest to know whether the variations in the values of $b_{12.3}$ for vertical incidence occur in the same direction as the variations of the inclined values determined

from the measurements during the same year. A proper comparison is not possible because the analyses of the vertical data are in monthly groups and not for the periods when the inclined measurements were made. However, some of the inclined measurements cover approximately a calendar month. Thus the first 70° run during November gave values of b_{12} greater than average, while the vertical value is just below average. The second 70° run during January gave small values of b_{12} whereas the largest vertical value obtained was for January. The third 70° run in April gave an average value only slightly different from the vertical one. The 30° run during October gave large values while the least vertical value was found then. Thus, there do not seem to be any definite relationships between the variations.

16. Summary of the barometer effects. Because of the variability of the coefficients, which shows that some values are drawn from different populations, it is not strictly permissable to take an average or weighted mean for a particular zenith angle and call it the barometer coefficient for that angle. However, we desire to know whether the results give any indication of a systematic change of barometer coefficient with zenith angle. Hence estimates have been made of the 'best' values at each angle. These are given in Table II.5. The values for 15°, 30°, 60° and 70° are the weighted means of the values set out in Table II.3. The method of weighting is the same as that given in Part I, par. 2. For 45° the value is given at the end of par. 14(b), which is the average of the combined West measurements and the combined East measurements during 1951-52. The 0° value is the average of Parsons' determinations. No estimate ofthe fiducial limits of this value has been made.

TABLE II.5.

The barometer coefficients at Macquarie Island for various zenith angles. The values are in percent per millibar. The 95 percent fiducial limits are given.

0°	-0.165	$b_{12.3}$
15°	-0.180 ± 0.023	b_{12}
30°	-0.192 ± 0.040	b_{12}
45°	-0.175 ± 0.007	$b_{12.3}$
60°	-0.153 ± 0.042	b_{12}
70°	-0.209 ± 0.082	b_{12}

If we regard these values as the best estimates we can make from the data available, it is clear that there is no definite indication of a trend towards larger values at large angles.

17. Comparison with the results of other workers. Although a great number of investigations of the barometer effect have been made, very few have been concerned with the inclined directions.

Trumpy and Orlin (7), at Bergen (approximately sea level, geomagnetic latitude 60°N), obtained the values set out in Table II.6 They used a 3-fold telescope in which

TABLE II.6

Barometer coefficients measured by Trumpy and Orlin (7) with 12 cm Pb.

Z	b_{12}, percent/cm Hg	percent/mb
0°	-3.0	-0.23
30°	-3.3	-0.25
45°	-3.9	-0.29
60°	-5.0	-0.38

the separation of the extreme counters was 19 cm and in which 12 cm Pb was used. The counting rate was low, being about 16 counts per hour at 60°. The contribution due to showers would be very considerable with such a counting rate. Parsons (6) found the rate of showers (measured by registering coincidences between two telescopes separated by one metre in the horizontal direction) to be about 6 per hour and the barometer coefficient for these was -0.82 percent/mb, or -10.9 percent/cm Hg. Trumpy and Orlin make no mention of corrections for showers. If corrections were not made, it is clear that the barometer effect determined from the observed counting rates would increase as the genuine directional rate decreased, i.e. with increasing zenith angle, due to the increasing contribution of showers which have a high barometer coefficient.

Barnothy and Forro (8), at Budapest (altitude 124 m, geomagnetic latitude 46.7°N) obtained values set out in Table II.7.

TABLE II.7

Barometer coefficients measured by Barnothy and Forro (8) with 36 cm Pb. (The errors are presumably the standard deviations).

Z	b_{12}, percent/cm Hg	b_{12}, percent/mb
0°	-3.12 ± 0.39	-0.235 ± 0.029
50°	-3.44 ± 0.77	-0.258 ± 0.058
64°	-3.82 ± 0.63	-0.287 ± 0.047

A 2-fold telescope was used with 36 cm Pb placed in it. The half angle was $5^{\circ} \times 20^{\circ}$ and the counting area 96 cm^2. The counting rates are not given, but they are likely to be low. It is probable that the rate would not exceed 20 per hour at 64°. Table II.7 shows us that, although no value differs significantly from any other, there is a trend towards higher values at larger angles. If the counting rate is low, the contribution due to showers would be important, as in the

experiment of Trumpy and Orlin. The values of b_{12} are considerably higher than some obtained by Duperier (to be mentioned shortly) with 40 cm Pb, and most other measurements with large thicknesses of Pb show that the coefficient decreases with increasing thickness. For this reason also it appears likely that in Barnothy and Forro's experiment the contribution of showers was important.

Although these are the only results which seem to have been published where the barometer coefficient has been measured at several zenith angles with the one telescope, we can compare some measurements made by Duperier for vertical incidence with some by Dolbear and Elliot at 45°. Duperier (9) at London (approximately sea level, geomagnetic latitude 54°N), used a 3-fold telescope with 40 cm Pb, some of which was placed above the telescope. The half angle was $18^{\circ} \times 35.5^{\circ}$ and the counting rate about 12,400 per hour. Duperier does not give the values of b_{12} obtained, but does give values of r_{12}, σ_1 and σ_2 for six periods of observation. From these we have calculated the corresponding values of b_{12}. The average value is -1.63 percent/cm Hg or -0.122 percent/mb. Dolbear and Elliot (10), at Manchester (approximately sea level, geomagnetic latitude 57°N), used 3-fold telescopes of half angle $38.7^{\circ} \times 38.7^{\circ}$ with 35 cm Pb. One of these was inclined at 45° to the North and the other at 45° to the South and measurements were made for 12 months. The counting rate was about 7,000 per hour. The average value of b_{12} was -1.88 ± 0.03 percent/cm Hg or -0.141 ± 0.002 percent/mb (the error is the standard deviation). This value is higher than that of Duperier for vertical incidence. However, if proper account could be taken of the effect of the difference thicknesses of Pb used, it is likely the two values would be more nearly equal.

It is of interest to know how the omnidirectional measurements made with ionization chambers compare with those

made with counter telescopes. One set of results will be considered, viz. that of Hogg (11) working at Canberra (altitude 800 m, geomagnetic latitude 45°S). A cylindrical ionization chamber shielded with 10 cm Pb was operated continuously for 5 years. Correlations with surface pressure and temperature were carried out. The values of $b_{12.3}$ are listed in Table II.8.

TABLE II.8

Values of $b_{12.3}$ obtained by Hogg (11) with an ionization chamber shielded with 10 cm Pb. The errors are the standard deviation.

Period	$b_{12.3}$, percent/mb
Sept. - Dec. 1935	-0.223 ± 0.031
1936	-0.218 ± 0.023
1937	-0.201 ± 0.011
1938	-0.212 ± 0.014
1939	-0.179 ± 0.016
Jan. - Aug. 1940	-0.219 ± 0.018
Average	-0.206 ± 0.018

Most of these values are higher than those listed in Table II.5 for Macquarie Island. However, a table has been drawn up by Hogg giving the results of a large number of observations by other workers at a number of different altitudes. There is a marked tendency for the barometer coefficient to increase with altitude. Hence it is likely that part of the difference between Hogg's coefficients and the Macquarie Island ones could be accounted for in terms of the difference in altitude.

The conclusion we draw from this survey and from the experimental work at Macquarie Island is that, although there are indications that the barometer effect increases with zenith angle, the evidence is insufficient to make this definite.

Theory of the Barometer Effect.

18. Preliminary remarks. An increase in atmospheric pressure may be expected to have two chief effects on μ-mesons. Firstly, because of the increased air mass which they have to penetrate, some of the low energy ones are removed. Secondly, with an increase in pressure, the height of the region of production tends to increase, so that the chance of mesons decaying in flight increases. Since the charged decay products (electrons) are not part of the penetrating component, the intensity of the penetrating component is reduced. (We shall not consider here why the soft component itself shows a negative correlation with pressure.)

19. Janossy (12, p.194) has calculated the expected barometer effect by treating these two effects separately. Although we do not propose to discuss his method in detail, it should be pointed out that it contains an error. In determining the magnitude of the second effect (loss by decay), Janossy assumes that a change in pressure alters the height of the homogeneous atmosphere, whereas, in fact, this height is independent of pressure. Because of this, the magnitude of the barometer coefficient determined (-3.5 percent per cm Hg) is considerably overestimated. A calculation along the general lines of Janossy's method, but with this error not present, gives a value of -2.4 percent per cm Hg, or -0.18 percent/mb for vertically incident mesons.

20. A more satisfactory method of calculating the barometer coefficients has been given by D.C. Rose (13). In this method the coefficient is determined for mesons arriving at the station level in a narrow momentum band by considering the effects of decay and energy loss by ionization together. By integrating over the momentum spectrum the integral barometer coefficient

is found. The treatment of this method about to be given differs somewhat in detail from Rose's.

We desire to determine the barometer coefficient as $\delta N/N\,\delta P$, where δN is the change in the intensity when the pressure is changed by an amount δP. This quantity is to be determined for μ-mesons arriving at the station level with momentum in a certain narrow momentum band.

We assume that the mesons are produced after the primaries have traversed 100 g cm^{-2} of atmosphere. Let the distance along the path of the particles from the station to the point beyond which 100 g cm^{-2} of air remain in the direction of the path be s_1 in the normal atmosphere and s_2 in the pressure altered atmosphere. Let the μ-mesons which reach station level in the desired momentum band have initial momenta in the range p_1, $p_1 + dp$ in the normal atmosphere, and in the range p_2, $p_2 + dp$ in the pressure altered atmosphere. We assume that the production spectrum is of the form

$$N(p)dp = kp^{-\gamma}dp.$$

Since the fraction of mesons which survive after travelling a distance s is $\exp\left[-\frac{1}{\tau c}\int_0^s \frac{ds}{p/\mu c}\right]$, the barometer coefficient is

$$\frac{\delta N}{N\delta P} = \frac{k\,p_2^{-\gamma}\,dp\;e^{-\frac{1}{\tau c}\int_0^{s_2}\frac{ds}{p/\mu c}} - k\,p_1^{-\gamma}\,dp\;e^{-\frac{1}{\tau c}\int_0^{s_1}\frac{ds}{p/\mu c}}}{\delta P\,kp_1^{-\gamma}\,dp\;e^{-\frac{1}{\tau c}\int_0^{s_1}\frac{ds}{p/\mu c}}}$$

This expression may be simplified by assuming that

$$\int_0^{s_2}\frac{ds}{p/\mu c} = \int_0^{s_1}\frac{ds}{p/\mu c} + \frac{\delta s}{\langle p/\mu c\rangle}$$

where $\delta s = s_2 - s_1$, and where $\langle p/\mu c\rangle$ is the average value of $p/\mu c$ over δs. Because δs is small compared with s_1 and s_2

and because the difference between p_1 and p_2 is small for the changes in pressure we shall be considering, this assumption introduces much smaller errors than would be present if the integrals themselves were evaluated by numerical methods. With this assumption the barometer coefficient becomes

$$\frac{\delta N}{N\delta P} \qquad \frac{\left[\frac{p_1}{p_2}\right]^{\gamma} e^{-\frac{1}{\tau c}\frac{\delta s}{\langle p/\mu c\rangle}} - 1}{\delta P} \quad . \qquad -(iv)$$

As Rose has pointed out, the barometer coefficient is critically dependent upon the exponent, γ, of the initial momentum spectrum. In Part I, par. 10, we found that the exponent was very close to 3, this estimate being based on Rossi's sea level spectrum. This value has been adopted here.

To make the process used for the calculations clear, we consider a specific case, viz., the determination of the barometer coefficient of μ-mesons arriving at sea level with $p/\mu c = 2.25$ (momentum 245 Mev/c) at 45° at Macquarie Island.

As discussed in the Appendix to Part I, the mean Macquarie Island atmosphere may be represented by

$$P = 1005\, e^{-h/7.11}$$

where P is the pressure in millibars at the height h in kilometres. Hence the mass of air, x, beyond a point at distance s from sea level in the direction inclined at an angle Z to the vertical is

$$x = \frac{13.6}{13.33} \cdot \frac{1005}{\cos Z}\, e^{-(s \cos Z)/7.11} \text{ g cm}^{-2}$$

(since 13.33 mb = 1 cm Hg, and since 1 cm Hg = 13.6 g cm^{-2}).

The increment in pressure in the present calculations is +10 mb. Hence we require to find s_1 and s_2 from the equations

$$x = 100 = 1449\, e^{-s_1/10.05}$$

$$\text{and} \quad x = 100 = 1464\, e^{-s_2/10.05}$$

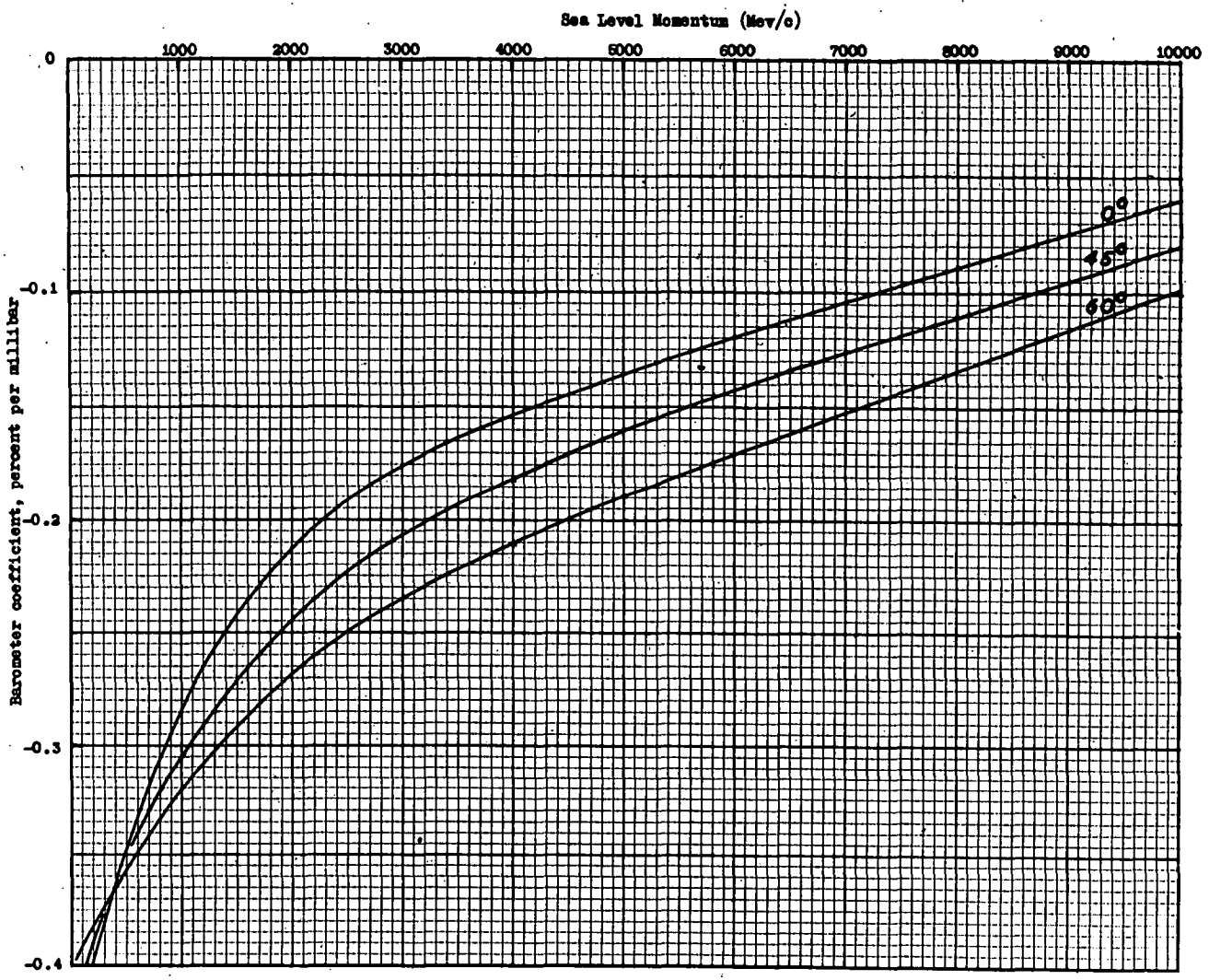

FIG. II.1 Differential barometer coefficient at sea level as a function of zenith angle.

which give s_1 = 26.8687 km, and s_2 = 26.9722 km. Hence δs = 0.1035 km for the 10 mb change.

For mesons to reach sea level from the 100 g cm^{-2} point they have to penetrate 1349 g cm^{-2} in the normal atmosphere and 1364 g cm^{-2} in the pressure altered atmosphere. By using the range table given in the Appendix to Part I, it may readily be seen that for μ-mesons to penetrate these masses of air and emerge with $p/\mu c$ = 2.25, their initial values of $p/\mu c$ must have been 28.26 and 28.58 in the normal and pressure altered atmosphere respectively (corresponding to momenta of 3080 and 3115 Mev/c respectively). By substituting these values into Eq.(iv), and taking τ = 2.15 μsec, we find the barometer coefficient for such mesons is -0.386 percent/mb.

Calculations of this kind have been made for about a dozen final values of $p/\mu c$ for each of the angles 0°, 45° and 60°. These are plotted in Fig.II.1. There are two points which must be mentioned concerning these curves and the calculations. Firstly, since the barometer coefficient is dependent upon $(p_1 p_2)^3$, slight errors in these momenta can have a noticeable effect. As mentioned when discussing the range table in the Appendix, there are some slight irregularities in that table, due, it is thought, to errors in the fourth digits. Because of these, some of the barometer coefficients did not lie on the curves in Fig.II.1 which have been drawn to fit as closely as possible the calculated points. No serious errors are likely however. Secondly, since the range table only extends to 10,000 Mev/c, it was possible to determine coefficients for mesons whose initial momenta do not exceed 10,000 Mev/c. For the 60° coefficients, the calculations could not be carried beyond 5800 Mev/c. For the 0° calculations the maximum momentum was about 7500 Mev/c. The curve for 0° was extrapolated to 10000 Mev/c by simply continuing it on in

the direction in which it was going at 7500 Mev/c. The other curves have been extrapolated by drawing them approximately parallel to the 0° one. This procedure is not highly satisfactory.

The differential barometer coefficients so determined are integrated over the sea level spectra given in Fig.I.5 to give the integral coefficients. That is

$$b_{12.3} = \frac{\int_{p_1}^{10000} b(p)N(p)dp + \int_{10000}^{\infty} b(p)N(p)dp}{\int_{p_1}^{10000} N(p)dp + \int_{10000}^{\infty} N(p)dp}$$

where b(p) is the barometer coefficient for mesons of momentum p. The lower limit p_1 has been taken as 250 Mev/c at which momentum μ-mesons can just penetrate about 12 cm Pb. The determination of the integrals up to 10,000 Mev/c by numerical methods is straightforward. The calculations beyond this call for comment.

We can fairly safely assume that the loss of μ-mesons by decay is unimportant in the high momentum region. This assumption implies that the sea level spectrum is of the same form as the production spectrum. We assume as usual that this varies as $p^{-\gamma}$. The barometer effect, in percent/mb, will then be

$$b(p) = \frac{100}{N(p)dp} \frac{d[N(p)dp]}{dP}$$

$$= \frac{100}{N(p)dp} \frac{d[N(p)dp]}{dp} \frac{dp}{dP}$$

$$= -\frac{100\,\gamma}{p} \frac{dp}{dP}$$

The rate of loss of momentum per mb, dp/dP, will vary with momentum. However, to simplify the calculations, we have taken this as constant and equal to the rate of loss for the highest

momentum mesons considered in the earlier vertical calculations. Thus, for mesons arriving vertically at sea level ($P = 1005$ mb) from the 100 g cm^{-2} level ($P = 98$ mb) with final momentum 7467 Mev/c, the initial momentum is 9888 Mev/c. Hence

$$\frac{dp}{dP} = \frac{9888 - 7467}{1005 - 98} = 2.76 \text{ Mev/c per mb.}$$

For the inclined directions we have taken dp/dP as 2.67 sec Z Mev/c per mb.

The next problem is to decide what value of γ should be taken. It is clear that if $\gamma = 3$ for the production spectrum, this value should be taken. However, in Part I, when integrating over the spectra to find the directional intensities, it was found desirable to assume $\gamma=2$ in the high momentum region. For vertically incident mesons of final momentum 10000 Mev/c, b(p) is -0.053 percent/mb if $\gamma = 2$, or 0.080 percent/mb if $\gamma = 3$. We note that the extrapolated 0° curve in Fig.II.1 passes through -0.06 percent/mb at 10000 Mev/c, suggesting that $\gamma = 2$ is more in keeping with the low momentum calculations than $\gamma = 3$ in the region near 10000 Mev/c.

George (14) quotes some unpublished results of MacAnuff who determined the barometer effect at the Holborn Station, London, at a depth of 60 m (water equivalent). The minimum momentum at this depth would be about 20000 Mev/c. The integral coefficient would be

$$\frac{\int_{20000}^{\infty} b(p)N(p)dp}{\int_{20000}^{\infty} N(p)dp} = -\frac{100(\gamma - 1)}{20000} \frac{dp}{dP} .$$

If we take $dp/dP = 2.7$ Mev/c per mb, we find the coefficient is -0.0135 percent/mb if $\gamma = 2$, or -0.027 percent/mb if $\gamma = 3$. These correspond to -0.18 and -0.36 percent/cm Hg respectively.

The value measured by MacAnuff was -0.47 percent/cm Hg. George states that the theoretical value based on a p^{-3} differential spectrum is -0.45 percent/cm Hg. Presumably a higher value of dp/dP was taken in his calculation (as it should be), or else the estimate of the minimum momentum is different from that taken by George. However the results suggests that a value of γ greater than 2 should be used in the high momentum region.

The values of the integral barometer coefficients have been worked out using the numerical results up to 10000 Mev/c and using both $\gamma = 2$ and $\gamma = 3$ for the integrals above 10000 Mev/c. The values of the coefficients so obtained are set out in Table II.9.

TABLE II.9

Calculated sea level values of the integral barometer coefficients $b_{12.3}$ for a cut-off momentum of 250 Mev/c. The values of γ refer to the sea level spectrum beyond 10,000 Mev/c.

	$b_{12.3}$, percent/mb	
Z	$\gamma = 2$	$\gamma = 3$
0°	-0.202	-0.216
45°	-0.188	-0.209
60°	-0.171	-0.203

We note that the values listed in this Table are of the same order as the experimental ones given in Table II.5. It may seem surprising that, although the coefficients for a given momentum increase with zenith angle, the integral coefficients show no such increase. This is because the average momentum increases with zenith angle, as may be seen from the calculated sea level spectra in Fig.I.5. The fact that the calculated coefficients do not show an increase with angle is in line with the conclusion drawn from the Macquarie Island investigations, and from the review of other results, that there is no experimental evidence for such an increase.

This agreement suggests that the penetrating component does become harder with increasing zenith angle, in accordance with the calculations in Part I. This seems to be the only evidence for such a hardening, since measurements of the spectra at inclined angles have not been made. However, as pointed out below (par. 23) this evidence cannot be regarded as conclusive.

We also note from the Table above that the effect of the high momentum particles is by no means negligible, suggesting that it would be desirable to perform the calculations in this region with the degree of accuracy used below 10,000 Mev/c. If this is done, it may be necessary to take into account effects due to π-mesons. We have been able to neglect π-mesons altogether in the low momentum region only because their mean path length is short compared with the distance from the assumed production level and most of the π-mesons can be assumed to decay to μ-mesons with only a small proportion being lost in nuclear encounters. However, when the energy of the π's is great, their mean lifetime (relative to a terrestial observer) is increased and the chance of such encounters increases, resulting in a reduction of the hard component (unless the cross-section for such interactions decreases with momentum). There could, therefore, be an appreciably greater barometer effect in the high momentum region than would be expected from processes involving μ-mesons only.

Correlation with Upper Air Data.

21. The 1951-52 Macquarie Island results have been correlated with the radiosonde data. The four sets of 45° data A_W, A_E, B_W and B_E have been kept separate in this work. At the present time, the calculations for only one set are complete, viz. the A_W data.

Because of the rapid changes in atmospheric conditions which take place at Macquarie Island, the cosmic ray results selected for this work were those obtained during a four hour period at the time of the sonde flight. This four hour period began one hour before the time the balloon was released and ended three hours after release. The flight was usually made at about 0800 hours GMT and lasted approximately two hours. Because the telescopes rotated from West to East or vice versa, at the end of each hour, only two hour's results were available. In cases when less than two hours' results were available due to equipment failure, the results have been disregarded. There were no occasions when the equipment failed to rotate, leaving telescope A pointing West for more than two hours, although there were some occasions when this telescope remained in the East position for more than two hours. The average counting rate for these two hours was taken for the correlations. The only sonde flights considered were those which reached the 80 mb level. There were 179 suitable flights when telescope A pointed West for two hours during the specified four hour period.

The following correlations have been performed

X_1	X_2	X_3	X_4
A_W	P	H_{500}	$T_{500-600}$
A_W	P	H_{300}	$T_{300-400}$
A_W	P	H_{150}	$T_{150-200}$
A_W	P	H_{100}	$T_{100-150}$
A_W	P	H_{80}	T_{80-100}

where A_W is the average counting rate of telescope A pointing West

P is the average surface pressure during the specified four hour period at the time of the flight

H_{500} is the height of the 500 mb level, etc.

$T_{500-600}$ is the average temperature in the layer of air between 500 and 600 mb, etc.

X_1, X_2, X_3 and X_4 are the symbols used in the 4-fold correlations to represent the quantities listed below them. Thus, the partial correlation coefficient $r_{13.24}$, for instance, for a particular correlation (say that for the 500 mb level) refers to the correlation between the cosmic ray intensity and the height of the 500 mb level, with the surface pressure and the average temperature in the 500-600 mb layer held constant. The results of these correlations are set out in Table II.10.

TABLE II.10

Results of the correlations of the A_W 45° rates for June 1951 - March 1952 with the upper air data. The errors given with the R's are the 95 percent fiducial limits. The 95 percent fiducial limits of all the z's are ±0.149.

Level	$r_{12.34}$	$z_{12.34}$	$r_{13.24}$	$z_{13.24}$	$r_{14.23}$	$z_{14.23}$	$R_{1.234}$
500 mb	-0.044	-0.044	-0.124	-0.125	+0.102	+0.102	0.496±0.110
300	-0.251	-0.257	-0.110	-0.110	+0.038	+0.038	0.499±0.110
150	-0.408	-0.433	-0.079	-0.079	+0.081	+0.081	0.515±0.111
100	-0.426	-0.455	-0.030	-0.030	-0.056	-0.056	0.498±0.110
80	-0.440	-0.472	-0.029	-0.029	-0.046	-0.046	0.499±0.110

To test the significance of the partial correlation coefficients, the z-transformation method mentioned in par. 9 was used. The corresponding values of z are set out in Table II.10. Since the same number of sets of data (179) were used for each correlation, the standard deviation of the a-values is the same in each case, viz. $1/\sqrt{179 - 2 - 3}$. The 95 percent fiducial limits of the z-values are therefore $1.96/\sqrt{174} = 0.149$. Hence, the only partial correlation coefficients which differ significantly from zero are the $r_{12.34}$ at and above 300 mb.

To find the 95 percent fiducial limits of the $R_{1.234}$, the method in par. 9 is used. Accordingly, the 95 percent limits are $1.96 \times 2R(1 - R^2)/\sqrt{n}$. Values determined in this way are given in Table II.10. It will be seen that all values are highly significant. The maximum value is reached at the 150 mb level, although the value for this level is not significantly greater than any of the others, if they may be compared by considering the fiducial ranges given. However, one feels that when portion of the data is common to all correlations (viz. A_V and P), it is perhaps not correct to compare values of R by assigning fiducial ranges in the above manner. This statistical problem has not been considered further.

If, as Duperier's work suggests (5, 9) the 100 mb level is an important one for the vertically incident radiation at sea level, then the 70 mb level may be expected to be important for radiation arriving at 45°. Hence measurements at 45° may be expected to show the maximum correlation with the height of, and the temperature near, the 70 or 80 mb level. However, the counting rate in the present experiment was too low to test this hypothesis. All that can be inferred from these results is that, if any level is more important than any other, it is the 150 mb level at which the multiple regression coefficient $R_{1.234}$ is slightly greater than elsewhere.

General Discussion on Atmospheric Effects.

Our object now is to consider the value of studies of variations of the cosmic ray intensity associated with atmospheric conditions, and to suggest some investigations which might be profitable in the future.

22. First of all, there is the obvious application of barometer and other coefficients in correcting results to standard atmospheric conditions. However, in view of the variability of these coefficients, it is much more desirable to determine these for the particular experiment where they are needed than to use someone else's values.

23. Although theoretical values of the barometer and other effects can be worked out fairly accurately for certain hypotheses, it would seem that no definite conclusions can be based on comparisons between these and the measured values until the problem of the variability of these coefficients is more thoroughly understood. While Duperier's explanation of the variability of the barometer effect in terms of the correlation of the height of the production region with pressure is appealing, one feels that this must not be regarded as definitely established. If it were, we would expect the absorption coefficients $b_{12.34}$ and the coefficients $b_{13.24}$ etc. to show a high degree of constancy, whereas this does not appear to be the case in either Duperier's results (9) or those of Dolbear and Elliot (10). One possible factor, which is likely to have an effect and which does not seem to have been considered is the water content of the atmosphere. The rate of loss of energy of a charged particle by ionization depends upon Z/A, the ration of atomic number to atomic weight. For most light elements Z/A is close to 0.5, but for hydrogen it is 1. Thus the average value of Z/A for water is 0.56, or 12 percent greater than for dry air. This means that a given mass of moist air has a greater

stopping power than the same mass of dry air. It is therefore likely to be worthwhile taking the water content of the atmosphere as a variable in correlations of intensity with radiosonde data (the radiosonde has a humidity element.)

24. It would seem that the methods used by Duperier, and others who follow his example, in analysing the atmospheric effects on cosmic rays do not lead to condusive results because no account is taken of intercorrelations between the heights of the various pressure levels etc. For instance, it is known that the temperature in the stratosphere is often negatively correlated with the temperature in the troposphere. This means that great caution must be exercised in inferring that a positive correlation between cosmic ray intensity and stratosphere temperature (Duperier's positive temperature effect) has a greater physical significance than a negative correlation with the troposphere temperature. The most desirable procedure in investigating the effects of atmospheric conditions on cosmic rays would be to perform a many-fold multiple correlation in which the heights of several pressure levels and the temperatures in their vicinity as well as surface pressure and water content were taken as variables. The computing difficulties would be enormous in such an undertaking. However, it is felt that this should be done when very high counting rates become available. The various sums of squares and products could still be determined by punched card methods. The labour involved in computing the partial regression and correlation coefficients with ordinary desk machines, however, is likely to be prohibitive. But it is almost certain that a scheme could be programmed for use with the modern electronic computing machines. Hartree (15) has specifically cited the problem of multiple correlation as one which should be soluble with these machines. Such a procedure should show fairly definitely what atmospheric factors are

important in producing variations in the cosmic ray intensity.

25. The Macquarie Island results suggest that if further experiments are to be performed to determine whether or not the barometer effect varies with zenith angle, an apparatus should be used which simultaneously measures the intensity from several different angles in order to be sure that the measurements are made under the same atmospheric conditions.

26. In our calculations of the barometer coefficients, we assumed that the μ-mesons were generated after their primaries traversed 100 g cm^{-2} of air. This idea of a definite production level is, of course, fictitious. It would be expected that the lower the final momentum of the μ-mesons the more unsatisfactory would the assumption become because so few can reach sea level from the 100 g cm^{-2} level before decaying. An important part of the low momentum mesons would be expected to arise in the lower reaches of the atmosphere. An indication of the extent to which production in the lower atmosphere is important may possibly be gained by measuring the barometer coefficient for mesons in a narrow momentum band. For instance, if we assume that mesons arriving vertically at sea level with momentum of 245 Mev/c arise at the 500 g cm^{-2} level, and assume that the production spectrum is of the same form as in our earlier calculations, i.e. a p^{-3} spectrum, the barometer coefficient is -0.612 percent/mb compared with -0.387 percent/mb if they arise from the 100 g cm^{-2} level. It may therefore be possible, by measuring differential barometer coefficients, to test an hypothesis of Rathgeber (16) that some mesons of low momentum are produced in pairs by photons at about 6 km. It is also likely that if differential counting rates are correlated with

atmospheric data, both surface and upper air, a good deal more information about the modes of production could be gained than is possible from integral measurements. The problem of obtaining a high differential counting rate is by no means easy, but it should be possible with modern techniques. A counting rate of about 7000 per hour could be obtained for particles penetrating 10 but not 20 cm Pb in a telescope of half angle 30° built from counter trays 1 metre square.

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PART III.

THE HIGH LATITUDE EAST-WEST ASYMMETRY.

1. Introduction. The fact that the intensity of cosmic rays at sea level remains almost constant from high geomagnetic latitudes to latitudes of about $\pm 40^{\circ}$ (the 'knee' of the latitude curve) is usually thought to be due to the absence of primary particles which would be field sensitive in these latitudes and which would be energetic enough to produce effects at sea level. If this is so, the Lemaitre-Vallarta theory enables us to conclude that in latitudes above the knee there should be no East-West asymmetry or other azimuthal variations due to the primary radiation, except perhaps at large zenith angles. However, an E-W asymmetry has been observed in high latitudes by several investigators. Some of these experiments will be considered later in this Part.

Johnson (1) has suggested that this asymmetry is due to the deflection of secondary particles in the earth's field. A somewhat more elaborate extension of this theory appears to describe the effect more adequately.

This Part deals mainly with the theory of the high latitude E-W effect and with experiments performed at Macquarie Island and Hobart.

2. Johnson's theory of the high latitude E-W asymmetry.

Very briefly, this theory is that positive particles are deflected by the earth's magnetic field in such a way that more enter a telescope when it is set at a certain zenith angle to the West than when it is set at the same zenith angle to the East. Negative particles are deflected in the opposite way with more arriving from the East than

the West. Since there are more positive particles than negative ones, the net effect is a preponderance of particles arriving from the West.

We now consider Johnson's theory in detail.

It is assumed that the rays reaching sea level are symmetrically distributed upon their arrival at the top of the atmosphere or at the point where they are produced as secondaries from such isotropic radiation. There is ample evidence that this assumption is justified, at least to a first approximation, from the observed fact that the diurnal variation at sea level is small (e.g. Elliot (2, p.455)), and from the direct experiments near the top of the atmosphere by Winckler et al. (3) and Swann et al. (4).

As the particles are slowed down due to ionization in the atmosphere, their paths become more and more curved until, when they reach the observer, they have been slightly deflected from the path they would have followed in the absence of this energy loss. Johnson assumes that there would be no asymmetry if the particles did not lose energy, even though they are influenced by the earth's magnetic field. This is taken to be a consequence of Liouville's theorem. Thus it is assumed that it is the additional deflection due to the slowing down which produces the asymmetry. This assumption will receive further consideration later.

The E-W asymmetry for a zenith angle Z, $\alpha(Z)$, is defined as

$$\alpha(Z) = \frac{N_w(Z) - N_e(Z)}{\frac{1}{2}\left[N_w(Z) + N_e(Z)\right]} , \qquad -(i)$$

where $N_w(Z)$ and $N_e(Z)$ are the counting rates observed at the same zenith angle to the West and East respectively. To calculate this quantity, we consider, firstly, only the positive particles travelling in the (geomagnetic) E-W vertical plane.

The force on a charged particle moving with velocity $\underline{v}$ in a magnetic field of strength $\underline{H}$ is proportional to the vector product $\underline{v} \times \underline{H}$. Therefore, for a down-coming positive particle travelling in the E-W vertical plane, the force is directed to the East, since $\underline{H}$ is directed from South to North. This means that the paths of the positive particles are concave to the East.

Since, as we shall see, the deflections are small, we can assume, to a first approximation, that absorption and decay processes are unchanged along the slightly changed paths. In the absence of the deflections, radiation which is isotropic at the top of the atmosphere would produce a distribution symmetrical about the vertical at any atmospheric depth. Therefore, in the absence of the effect, the counting rate $N_w(Z)$ would have been observed at the smaller zenith angle $(Z - \delta)$, where δ is the additional deflection suffered due to slowing down, and the counting rate $N_e(Z)$ would have been observed at the larger angle $(Z + \delta)$.

If we represent the counting rate at the angle Z in the absence of the deflections by $N(Z)$, without suffix, we therefore have

$$N_w(Z) = N(Z - \delta)$$
$$\text{and} \quad N_e(Z) = N(Z + \delta).$$

Expanding the right hand sides in a Taylor series and neglecting terms of higher order than the first,

$$N_w(Z) = N(Z) - \delta\, N'(Z)$$
$$\text{and} \quad N_e(Z) = N(Z) + \delta\, N'(Z)$$

Hence, for positive particles, the asymmetry $\alpha_+(Z)$ is given by

$$\alpha_+(Z) = -2\,\delta\, N'(Z)/N(Z).$$

Similarly, for negative particles we find

$$\alpha_-(Z) = 2\,\delta\, N'(Z)/N(Z).$$

Thus, if there were equal numbers of positive and negative particles, there would be no asymmetry. If a fraction F is positive (i.e. the fraction 1 - F is negative), the asymmetry is

$$\alpha(Z) = \frac{F\alpha_+(Z) + (1 - F)\,\alpha_-(Z)}{F + (1 - F)}$$

and since $\alpha_+(Z) = -\alpha_-(Z)$

$$\alpha(Z) = (2F - 1)\alpha_+(Z).$$

In terms of the more usual representation of the positive excess, viz. the ratio $r = N_+/N_-$ of the number of positive to negative particles.

$$F = N_+/(N_+ + N_-).$$

Therefore $$\alpha(Z) = -2\delta\,\frac{r - 1}{r + 1}\,\frac{N'(Z)}{N(Z)} \qquad \text{-(ii)}$$

If we are interested in the integral spectrum of the penetrating component, we may calculate the asymmetry from Eq.(ii) by taking

$$N(Z) = N(0)\cos^{\lambda}Z.$$

If $\bar{\delta}$ is the average additional deflection of the particles arriving at the angle Z, averaged over the spectrum, Eq.(ii) becomes

$$\alpha(Z) = 2\bar{\delta}\lambda\,\frac{r - 1}{r + 1}\tan Z. \qquad \text{-(iii)}$$

In this case, r is the average positive excess over the spectrum. It will be noted that by making use of the observed zenith angle distribution law $N(Z) = N(0)\cos^{\lambda}Z$, no account has to be taken of decay or absorption processes, since these determine this law.

3. Johnson's method of determining δ. The problem now reduces to that of calculating the additional deflection suffered by the particles in being slowed down in their flight through the atmosphere.

Johnson defines the additional deflection as

$$\delta = \underset{s_1 \to \infty}{L_t} \left[\int_0^{s_1} \frac{ds}{\rho} - \int_0^{s_1} \frac{ds}{\rho_0} \right] \qquad -(iv)$$

where s is the orbital distance measured from the observer backwards along the path, ρ is the radius of curvature over the element ds of the path, and ρ_0 is the initial radius of curvature of the path of the particle.

To evaluate this integral we need to know ρ as a function of s. Two simplifying assumptions are made by Johnson, firstly, that the rate of loss of energy by ionization is independent of the energy, and, secondly, that the atmosphere may be described by an exponential pressure-height relation.

Particles travelling in the geomagnetic E-W vertical plane are considered. The horizontal component, H, will therefore determine the deflection. The radius of curvature for a particle of rest mass μ, electronic charge e and energy $q\mu c^2$ is

$$\rho = \frac{\mu c^2}{He} \sqrt{q^2 + 2q}$$

Let $\mu c^2/He = R$. Then

$$d\rho/dq = R\sqrt{1 + R^2/\rho^2}$$

Taking the rate of loss of energy as $\alpha\mu c^2$ per cm air at atmospheric pressure, dq/ds at pressure p is

$$dq/ds = \alpha p. \text{ where p is in atmospheres.}$$

Therefore

$$\frac{d\rho}{ds} = \frac{d\rho}{dq}\frac{dq}{ds} = \alpha R p \sqrt{1 + R^2/\rho^2}\ . \qquad -(v)$$

Let the atmosphere be represented by the exponential relation

$$p = e^{-h/h_o},$$

where h is the height at which the pressure is p, and h_o the height of the homogeneous atmosphere. Because the deflection of a particle which can just reach the observer is only a few degrees, and because the deflection of all other (more energetic) particles must be smaller, it is permissable to take

$$h = x_o + s \cos Z,$$

where x_o is the height above sea level of the observer and Z is the zenith angle of the path of the particle when it reaches the observer. This assumption is equivalent to regarding the path as straight as far as determining the pressure at any distance s along the path is concerned. Hence Eq.(v) may be written

$$d\rho/ds = \alpha R \sqrt{1 + R^2/\rho^2} \; e^{-(x_o + s \cos Z)/h_o}.$$

This differential equation may be readily solved to obtain ρ in terms of s. The solution is

$$\rho = \left[(a - be^{-\gamma s})^2 - R^2\right]^{\frac{1}{2}}, \qquad -(vi)$$

where $a = \sqrt{\rho_o^2 + R^2}$,

ρ_o = the radius of curvature of the ray upon entry into the atmosphere at a distance great compared with h_o but small compared with the radius of the earth,

$$b = \frac{\alpha R h_o \, e^{-x_o/h_o}}{\cos Z},$$

and $\gamma = (\cos Z)/h_o$.

We may now determine the first integral in Eq.(iv). If we put $z = be^{-\gamma s}/a$ and $k = R/a$ in Eq.(vi), this integral becomes

$$\frac{ds}{\rho} = -\frac{1}{\gamma a}\int_{z=b/a}^{(b/a)e^{-\gamma s_1}} \frac{1}{z}\left[(1-z)^2 - k^2\right]^{-\frac{1}{2}} dz$$

where s_1 is some distance large compared with $h_o \sec Z$ but small compared with the radius of the earth. Upon evaluating this integral* and inserting the limits, we find

$$\int_0^{s_1} \frac{ds}{\rho} = \frac{1}{\gamma a\sqrt{1-k^2}}\left\{\log\left[\sqrt{\left[1-(b/a)e^{-\gamma s_1}\right]^2 - k^2} + \sqrt{1-k^2} - \frac{be^{-\gamma s_1}}{a\sqrt{1-k^2}}\right]\right.$$

$$\left. - \log\left[\sqrt{(1-b/a)^2 - k^2} + \sqrt{1-k^2} - \frac{b}{a\sqrt{1-k^2}}\right]\right\}$$

$$+ \frac{s_1}{a\sqrt{1-k^2}} \qquad -(vii)$$

The second integral of Eq. (iv) is

$$\int_0^{s_1} \frac{ds}{\rho_o} = \frac{s_1}{\rho_o} = \frac{s_1}{a\sqrt{1-k^2}}$$

---

* The integral $I = \int \frac{dz}{z\left[(1-z)^2 - k^2\right]^{\frac{1}{2}}}$ may be evaluated by putting $(1 - z) = k \cosh \nu$ which makes the integrand $-1/(1 - k \cosh \nu)$. By now letting $t = \tanh \frac{1}{2}\nu$, the integral reduces to

$$I = -\frac{2}{1+k}\int \frac{dt}{(1-k)/(1+k) - t^2}$$

$$= -\frac{2}{1-k} \operatorname{artanh} t(1+k)/(1-k).$$

This is also the last term of Eq. (vii). Hence we shall find δ by omitting this term from Eq. (vii) and letting $s_1 \to \infty$. Therefore

$$\delta = \frac{1}{\gamma a\sqrt{1-k^2}} \log \frac{2}{1 - b/a(1-k^2) + \sqrt{[(1-b/a)^2 - k^2]/(1-k^2)}} \qquad \text{-(viii)}$$

To compute the values of δ for particles of any final energy, we must get the terms in Eq. (viii) in a form involving the final energy. It may be shown quite simply that if the energy of a particle as it reaches the observer is E, its initial energy at the top of the atmosphere (calculated on Johnson's assumption of constant energy loss) is

$$E_o = E + b\mu c^2/R$$

We can now list all the terms required for the calculation of δ from Eq.(viii).

$R = \mu c^2/eH$

μ = rest mass of particles
c = velocity of light
e = charge of the electron
H = horizontal component of the earth's field

$b = (\alpha R h_o \sec Z)e^{-x_o/h_o}$

$\alpha\mu c^2$ = energy loss per cm in air at atmospheric pressure (Johnson takes $\alpha = 2.5 \times 10^{-5}$ for the μ meson
h_o = height of homogeneous atmosphere (taken by Johnson as 8×10^5 cm)
x_o = altitude of observer
Z = zenith angle at which particle arrives.

$\gamma = (\cos Z)/h_o$

$\rho_o^2 = (ER/\mu c^2 + b)\ (E/\mu c^2 + 2)R + b$ E = final energy of particle

$a^2 = \rho_o^2 + R^2$

$k^2 = R^2/a^2$

We shall not quote results computed from Eq.(viii) until we are ready to compare them with values determined by a modified method.

4. Discussion of Johnson's theory. The measurements of the high latitude E-W asymmetry at Hobart and Macquarie Island have been carried to a fairly high degree of statistical accuracy. It was thought desirable to have theoretical values with which to compare these results calculated as precisely as possible. Johnson's theory has therefore been re-examined to see whether any changes are needed.

The first question we shall consider is whether the asymmetry should be regarded as due to the additional deflection suffered because the particles are slowed down by the atmosphere. As previously stated, Johnson assumes that there would be no asymmetry if the particles did not lose energy, even though they are deflected by the earth's magnetic field. This is a consequence of Liouville's theorem, which, applied to this problem, states that if the cosmic rays are isotropic at the top of the atmosphere, this isotropy will not be disturbed by any process (such as deflection in a static magnetic field) in which the energy of the particles is conserved. The applicability of this theorem is subject to the further proviso that the number of particles shall remain constant. This is not the case for mesons, which undergo spontaneous decay. Therefore the assumption that the asymmetry is due to the difference between the deflections with energy losses considered and with losses not considered does not rigorously hold.

If we consider the hypothetical case in which the atmosphere is absent but the earth is surrounded, at some distance above its surface, by a thin shell of material in which mesons may be generated from the primaries, we can see that an asymmetry would still be expected. For, the earth's field deflects the mesons in such a way that positives coming in at a certain zenith angle from the West have travelled a shorter distance from the shell than those arriving at the

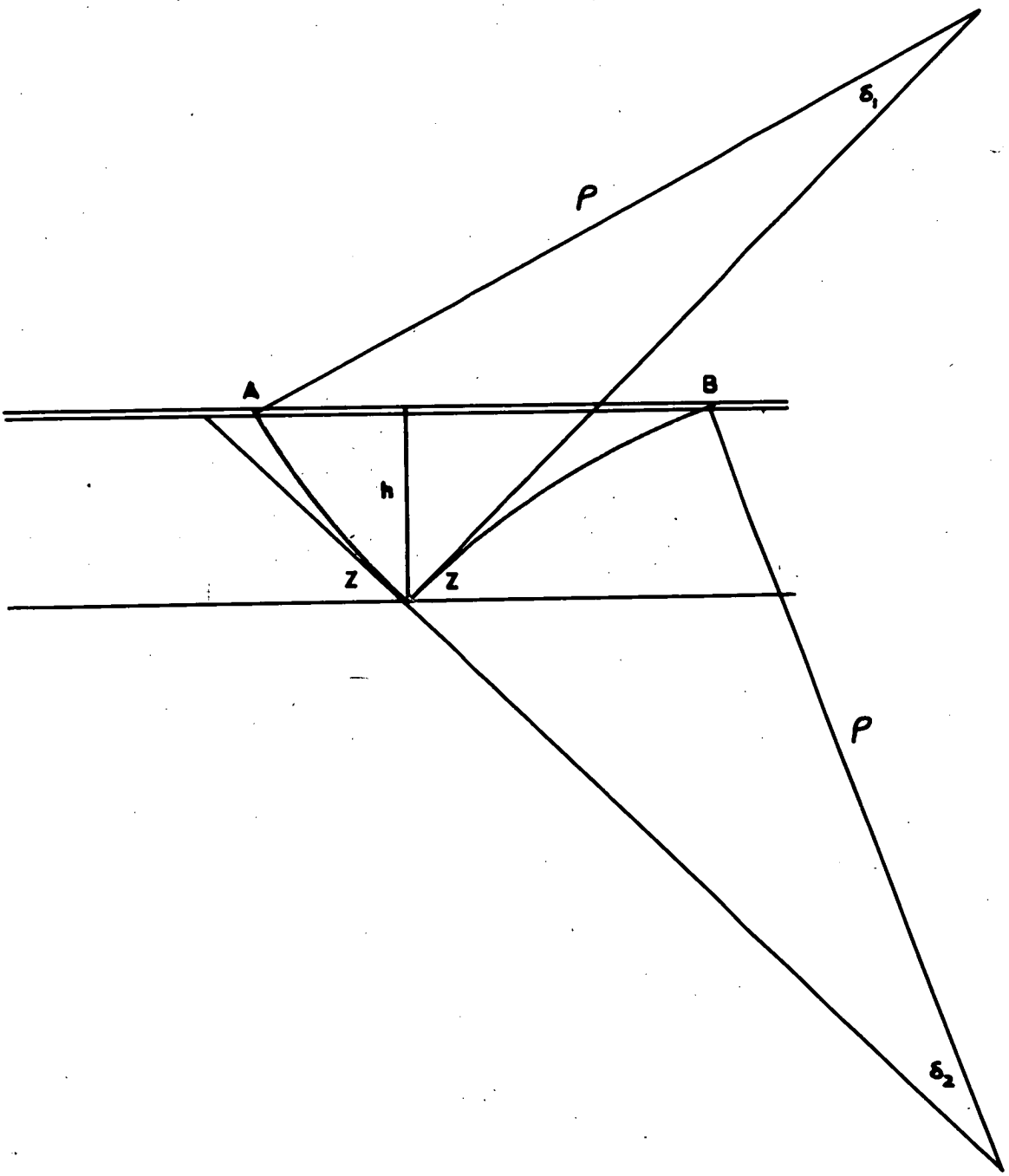


FIG. III.1

same zenith angle from the East. Therefore, due to decay, the intensity from the West will be greater than that from the East although both beams were initially of the same intensity.

The magnitude of this effect may be calculated easily. It is necessary, first of all, to know the path lengths for mesons arriving from the West and East at an angle Z. These may be found from Fig. III.1 in which positive mesons of radii of curvature ρ are considered. Such mesons arriving at O from the West must have come from the region around A in the shell while ones arriving from the East must have come from B. If h is the height of this shell and if δ_1 is the deflection of the mesons from A and δ_2 that of the mesons from B, it may be seen that

$$\sin(Z - \delta_1) = \sin Z - h/\rho$$

$$\text{and} \quad \sin(Z + \delta_2 = \sin Z + h/\rho.$$

Thus δ_1 and δ_2, and hence the path lengths, may be found.

We now consider a numerical example, calculated for Macquarie Island where H = 0.133 oersted. We take $Z = 45^\circ$ and h = 19 km (this figure is 27 cos 45°, 27 km being the distance at Macquarie Island to the point beyond which 100 g cm^{-2} of air remains in the 45° direction). We consider μ-mesons whose value of $p/\mu c$ is 28.31 (this figure is the initial value for a μ-meson which, when energy losses are taken into account, arrives at sea-level at 45° from 27 km with $p/\mu c = 2.25$, or energy = 270 Mev). Using these values, we find $\delta_1 = 0.0339$ radian and $\delta_2 = 0.0350$ radian. The path lengths are 26.46 km from the West and 27.37 km from the East. The fraction of mesons surviving after travelling a distance s with momentum p is $\exp(-\frac{1}{\tau c}\frac{s}{p/\mu c})$ which, in this case, means that 23.4781 percent survive from the West and 22.3420 percent survive from the East (the value

taken for the mean lifetime, τ, was 2.15 μsec.). This leads to an asymmetry, as defined in Eq. (i), of 0.05, which may be compared with the value of 0.131 calculated by Johnson's method for mesons which arrive at sea level with an energy of 270 Mev (and therefore have the same energy at 27 km along the path as those considered above). Considering μ-mesons which reach sea level at 45° with momentum 4×10^{9} Mev/c, we find in the same way that if there were no loss of energy there would be an asymmetry of 0.0085 compared with 0.012 calculated from Johnson's theory.

Hence, by attributing the asymmetry to the additional deflection only, its magnitude is underestimated quite appreciably.

A few further comments on Johnson's theory are required. His method takes no account of the currently accepted theory that the μ-mesons which reach sea level in the vertical direction are produced, on the average, near the 100 mb level. In fact, to make certain integrals calculable, the path length has to be large compared with $h_0 = 8$ km. The method also ignores the fact that the rates at which particles lose energy by ionization is dependent upon their energy.

It seemd that it would be worthwhile investigating what differences would be made by not making these simplifying assumptions.

5. Modifications of Johnson's theory. The preceding discussion enables us to see that a more correct theory of the high latitude E-W asymmetry would take into account the fact that Liouville's theorem is not strictly applicable.

The theory we envisage would therefore require the determination of the actual shape of the trajectories, and the corresponding integrals $\int_0^s \frac{ds}{p/\mu c}$, of mesons arriving at the same zenith angle from the East and West. Since the positives from the East travel further, on the average, from the place of production than positives from the West, those arriving with a certain momentum from the East must have a higher initial momentum than those arriving with the same momentum from the West (and vice versa for negatives). Therefore, the fact must be taken into account that the momentum spectrum (at production) of mesons which can reach sea level decreases with increasing momentum. The asymmetry of mesons of a given final momentum will then be determined by the production spectrum, by the decay loss along the unequal paths (i.e. by the integrals $\int_0^s \frac{ds}{p/\mu c}$), and by the positive excess.

Although this suggested theory is simple in principle, it is not simple to determine the asymmetry numerically. To determine the integrals for the actual trajectories, it would be necessary to work back in short steps along the paths from the observer to the average production level, calculating the deflection and energy loss suffered in each short step. It can be seen that this would involve some very laborious computations. Therefore, no attempt has been made to introduce these refinements. However, indications from the study of variations of the E-W asymmetry and of differences in pressure coefficients for the West and East intensities, to be dealt with later in this Part, suggest that such refinements may be desirable when results from cosmic ray recorders of much larger counting area become available.

For present purposes, we have contented ourselves with calculations of the deflections as $\delta = \frac{He}{\mu c^2}\int_0^s \frac{ds}{p/\mu c}$, with the integrals determined as in Part I, over the straight paths. The average value of δ calculated from these was then used in Johnson's formula, derived earlier, Eq.(iii), to give the asymmetry as

$$\alpha(Z) = 2\lambda\bar{\delta}\ \frac{r-1}{r+1}\ \tan Z.$$

The theory along these lines has been published by the Hobart group (Burbury and K.B. Fenton (5)).

It should be mentioned that this method estimates the actual deflection and not the additional deflection due to energy loss only. It is therefore to be expected that the asymmetries will be overestimated. If we attempted to correct this by subtracting the deflection which the particles would suffer in the absence of energy loss, we would be committing the error of assuming that Liouville's theorem is strictly applicable.

6. Calculation of the deflections. Comparison with Johnson's values.

Some values of the integrals $\int_0^s \frac{ds}{p/\mu c}$ have been listed in Table I.9, and the solid curves in Fig.I.4 have been plotted from these. To obtain the total deflection suffered by a meson of certain final momentum, we multiply the corresponding integral by $He/\mu c^2$. For Macquarie Island, where H = 0.133 oersted,

$$\frac{He}{\mu c^2}\ \text{(Macquarie Island)} = 3.63 \times 10^{-7}\ \text{cm}^{-1}.$$

For Hobart, where H = 0.19 oersted,

$$\frac{He}{\mu c^2}\ \text{(Hobart)} = 5.17 \times 10^{-7}\ \text{cm}^{-1}.$$

Table III.1 lists some values of the deflection calculated for Macquarie Island for a zenith angle of 45°. The column headed 'Total' gives values obtained by multiplying the integrals read off from the 45° curve in Fig. I.4 by the above factor.

TABLE III.1

Values of the deflection, δ, in radians, as a function of final energy calculated by the numerical method and by Johnson's method for Macquarie Island for a zenith angle of 45°.

Final Energy Mev.	Numerical Method. Total	No loss	Additional	Johnson's Method
250	0.0600	0.0335	0.0265	0.0278
500	0.0493	0.0303	0.0190	0.0202
1000	0.0370	0.0258	0.0112	0.0128
2000	0.0261	0.0199	0.0062	0.0068
4000	0.0170	0.0141	0.0029	0.0030
8000	0.0099	0.0089	0.0010	0.0011

The column headed 'No loss' gives the deflection which the mesons would have suffered if there were no energy loss during their flight. In this case the deflection is $\frac{He}{\mu c^2} \frac{s}{p/\mu c}$, where p is the momentum of the mesons at production. Values of $\frac{s}{p/\mu c}$ for these calculations were read off from the broken curve in Fig. I.4 in which values of this quantity for a zenith angle of 45° are plotted against the final momentum which the mesons actually have when energy loss occurs.

The column headed 'Additional' gives the difference between the total and the no loss deflections. The final column of Table III.1 gives values calculated from Johnson's theory (our Eq. viii) using the same constants (height of the homogeneous atmosphere, mass of the μ-mesons , etc.) as were used in the numerical calculations.

It will be noted that the values determined by the numerical methods are a good deal larger than those obtained by Johnson's method. However, the additional deflections determined by the numerical method differ very little from Johnson's. Therefore, if the asymmetry were determined by the additional deflection only, as assumed by Johnson, there would be no need to use the numerical methods.

7. The average deflections. These have been calculated from

$$\bar{\delta} = \frac{\int_{250}^{10000} \delta(p)N(p)dp}{\int_{250}^{\infty} N(p)dp} \qquad -(ix)$$

where $\delta(p)$ is the total deflection suffered by a meson of final momentum p. There should be the additional term $\int_{10000}^{\infty} \delta(p)N(p)dp$ in the numerator of Eq.(ix). However, we have chosen to ignore this contribution. A rough estimate of this additional term may be made in the following way. Assume that the average momentum of a meson which reaches sea level with momentum p is $p+\alpha$ during its flight. That is, α will be half the momentum loss in traversing the atmosphere. If we assume a spectrum of the form

$$N(p)dp = kp^{-2}dp,$$

we have

$$\int_{p}^{\infty} \delta(p)N(p)dp = \frac{Hes}{c} \int_{p}^{\infty} \frac{kp^{-2}}{p+\alpha} dp$$

$$= \frac{Hesk}{c} \left[\frac{1}{\alpha p} - \frac{1}{\alpha^2} \log_e (1 + \alpha/p)\right]$$

If we take the momentum loss to be 3700 Mev/c for mesons arriving at 45°, we find that the average deflection determined by incorporating this contribution into the numerator of Eq.(ix) is 0.0219 radian compared with 0.0209 radian if it is ignored. The actual contribution from the high momentum mesons will be less than this because, in this region, where loss by decay becomes unimportant, it is the additional deflection which determines the asymmetry. Our rough estimate is of the total deflection which will be a good deal greater than the additional deflections. It therefore seems to be quite legitimate to omit from the numerator of Eq.(ix) the contribution from mesons of momentum greater than 10000 Mev/c.

The values of the mean deflections are listed in Table III.2. These were calculated using the sea level spectra for the inclined directions given in Fig.I.5. The contribution to the denominator of Eq.(ix) for mesons of momentum exceeding 10000 Mev/c has been estimated using a $p^{-\gamma}$ spectrum with $\gamma = 2$ and $\gamma = 3$.

TABLE III.2

Average deflection as a function of zenith angle of μ-mesons reaching sea level at Macquarie Island in the vertical East-West geomagnetic plane.

Z	$\bar{\delta}$, radian	
	$\gamma = 2$	$\gamma = 3$
30°	0.0202	0.0220
45°	0.0209	0.0233
60°	0.0198	0.0236

It will be seen that the mean deflection does not vary much with zenith angle, although the deflection for a meson of particular momentum increases with zenith angle. As with our calculations of the integral barometer effect, this is a consequence of the hardening of the radiation with increasing zenith angle which is evident in our derived sea level spectra for inclined directions.

We note here that Johnson obtained his values of the average deflection by using an energy spectrum proportional to E^{-3}. In one case, the lower limit of integration was 200 Mev. Rossi's spectrum (given in our Fig. I.5) reaches a maximum at 500 Mev/c (400 Mev.) and it does not follow a power law till about 2000 Mev/c. Hence, by integrating over an E^{-3} spectrum, the contribution from low energy mesons, which are those which suffer the greatest deflection, is considerably overestimated. This has had the effect of giving average values which are comparable with those given in Table III.2, although Johnson's

deflection for a given momentum is considerably less.

We should also mention that in the paper by Burbury and K.B. Fenton (5), the integrations for each zenith angle were performed with the same spectrum, viz. that for vertical incidence. The average deflections therefore showed an increase with zenith angle in contrast with the results of the more recent computations given here.

8. The positive excess. The value of the ratio of positives to negatives which we have used is that given by Owen and Wilson (6), viz.,

$$r = 1.268 \pm 0.023.$$

This is the average over the range 1000 - 10000 Mev/c for vertically incident particles. The experiment on which this estimate was made gave no indication of a variation of positive excess with momentum. Subsequent measurements by these authors (7) indicated that the ratio varies from about 1.17 at 1000 Mev/c to about 1.26 at 4000 Mev/c. At higher momenta, the results suggest a decrease. We have decided to use their earlier work to avoid having to perform an integration over the 'positive excess spectrum'. When the shape of this spectrum becomes more definitely established and is extended to higher momenta, it may be worthwhile performing such an additional integration.

9. The asymmetries. We assume that the E-W asymmetry is given by our Eq. (iii), viz.,

$$\alpha(Z) = 2\bar{\delta}\lambda \frac{r-1}{r+1} \tan Z,$$

where Z is the zenith angle,

$\bar{\delta}$ is the average total deflection (we shall take the values for $\gamma = 2$ given in Table III.2).

λ is the exponent in the zenith angle distribution law (we shall take $\lambda = 2.135$, the value given in Part I, par. 8, which is based on the measurements of Greisen and of Rogozinski and Voisin).

r is the ratio of positives to negatives (taken as 1.268).

Values of the asymmetry calculated in this way are given in Table III.3. The values for Hobart were obtained by multiplying those for Macquarie Island by the ratio of the values of the horizontal component of the earth's field at these two places, viz. 0.19/0.133 = 1.43.

TABLE III.3.

Theoretical values of the E-W asymmetry at Macquarie Island and Hobart, based on the assumption that the asymmetry is due to the total deflection suffered by μ-mesons in their flight through the atmosphere.

Z	α, Macquarie Island	α, Hobart
30°	0.0059	0.0084
45°	0.0105	0.0150
60°	0.0173	0.0247

It may be mentioned (as Burbury (8) has pointed out) that Johnson (1) has made an error in calculating his asymmetries, the mistake being the factor involving the positive excess. Johnson uses a value of r given by Hughes (9), viz., r = 1.21. The factor $(r - 1)/(r + 1)$ should therefore be 0.095 and not 0.20, which is the value Johnson takes.

EXPERIMENTAL WORK.

10. Reasons for the experiments at Macquarie Island.

Measurements of the E-W asymmetry at Hobart (sea level, geomagnetic latitude 51.7°S) had been in progress about two years when the opportunity was afforded by the Australian National Antarctic Research Expedition of extending the experiments to Macquarie Island (geomagnetic latitude 60.7°S). There were several reasons why investigations at Macquarie seemed worthwhile.

Firstly, it was desirable because no other measurements had been carried out at a geomagnetic latitude as high as 60°. We felt that until such measurements were made at latitudes well beyond the knee of the latitude curve, we could not be sure that the effect was not a residual of the low latitude E-W asymmetry. Secondly, if the Hobart results proved to be entirely due to deflections of the secondary radiation, comparison with the Macquarie Island results would enable us to determine how the effect varies with latitude. Thirdly, since Macquarie Island is in or near the auroral zone where magnetic changes are frequent and considerable, it was thought that some interesting correlation work might be possible. It was also thought that variations of the asymmetry due to meteorological changes might be detected. We shall now describe the equipment used in these experiments.

11. General features of the equipment.

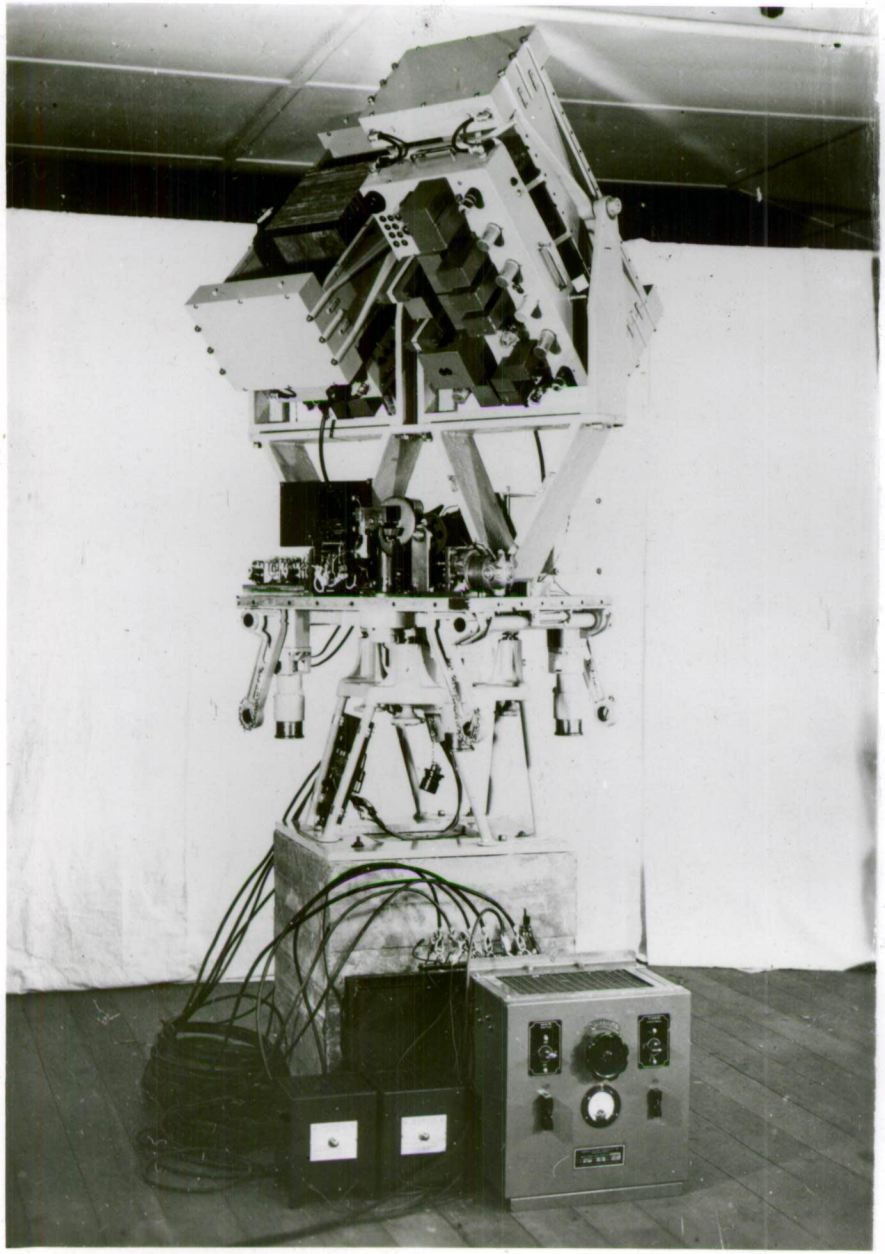
A double telescope apparatus was used, one telescope pointing to the East and the other at an equal zenith angle to the West. The telescopes were mounted on a turntable which was automatically rotated through 180° at the end of each hour, so that the

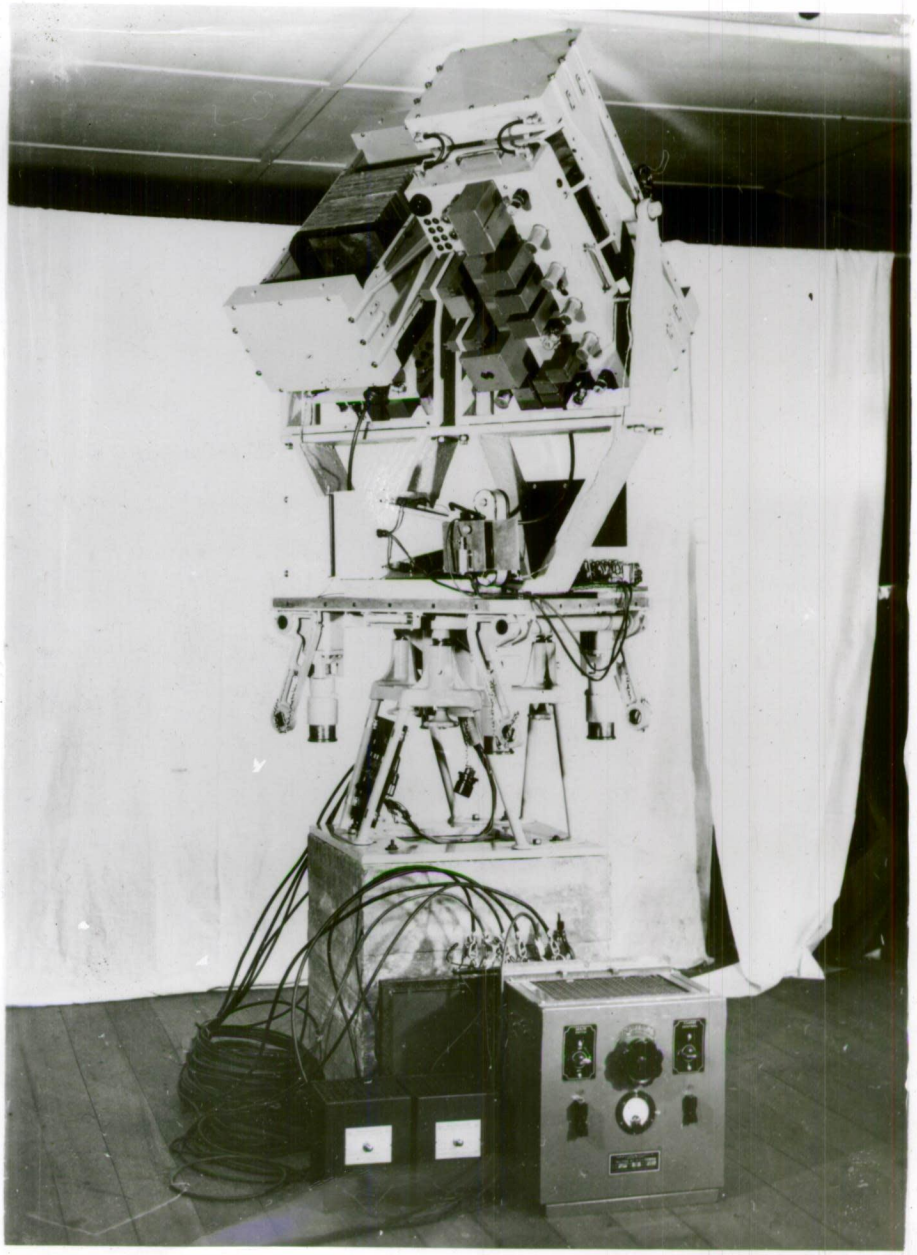
telescope which pointed West before rotation pointed East afterwards, and vice versa. Photographs of the mechanical registers were taken immediately before and immediately after each rotation. The whole equipment was run from accumulators which were maintained by battery chargers from the 240 V AC mains or from a small petrol generator when a mains failure occurred. Independent electronic circuitry, including power supplies, was used for each telescope.

The turntable used was from an Army Predictor Unit which was admirable for the purpose. It was sturdy enough to support quite easily the half ton or so which the telescopes and their mounts, the circuits and the Pb absorber weighed. Another important feature of the predictor unit was the system of slip-ring contacts which enabled power to be brought in from the accumulators. In time, ordinary leads would have become frayed due to the repeated rotation back and forth through 180°.

Photographs of the complete unit are shown in Plate 1.

PLATE V. Two photographs of the equipment taken at Hobart before shipment to Macquarie Island, showing views before and after rotation. In these pictures, the telescopes are both set at zenith angles of 46° and both contain 24 cm Pb which can be seen in the left hand telescope of each photo. The chassis on which the recording circuits and power supplies are mounted are supported on one side of each telescope. The left hand view shows the relay control system on the turntable, while the right hand view shows the camera and lamp housing. Power from batteries (not shown) is fed to the apparatus via slip-rings in the turntable base. The battery chargers are shown in front of the concrete block. A 12V system operates the recording circuits and a 24V system operates the camera and rotating mechanism.





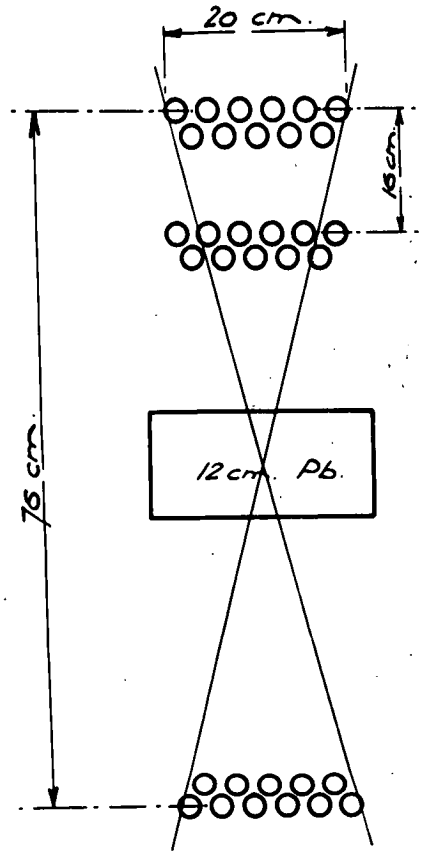


FIG. III 2. COUNTER TELESCOPE

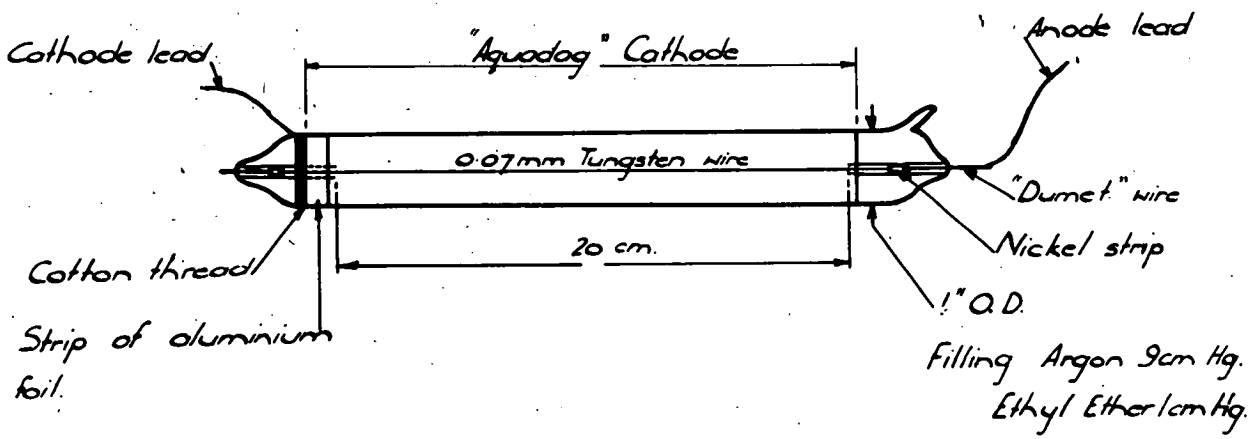


FIG. III 3. GEIGER COUNTER.

12. The telescopes. The telescopes, which were identical, as far as possible, consisted of square trays of counters 20 cm x 20 cm the outer trays being separated by 76 cm. This gave a half angle of 14.75° in each direction.

It was thought to be wise to use square trays and to keep the opening angle small in each direction. Very often, experiments on the directional effects are conducted with telescopes having a narrow angle in the zenith direction but a wide one in the azimuth direction. This procedure was not adopted because it seemed likely that the motion of skew rays in the earth's field would render more difficult the interpretation of the results. If narrow angles are used, it is fairly safe to regard all the particles as travelling in the E-W vertical plane.

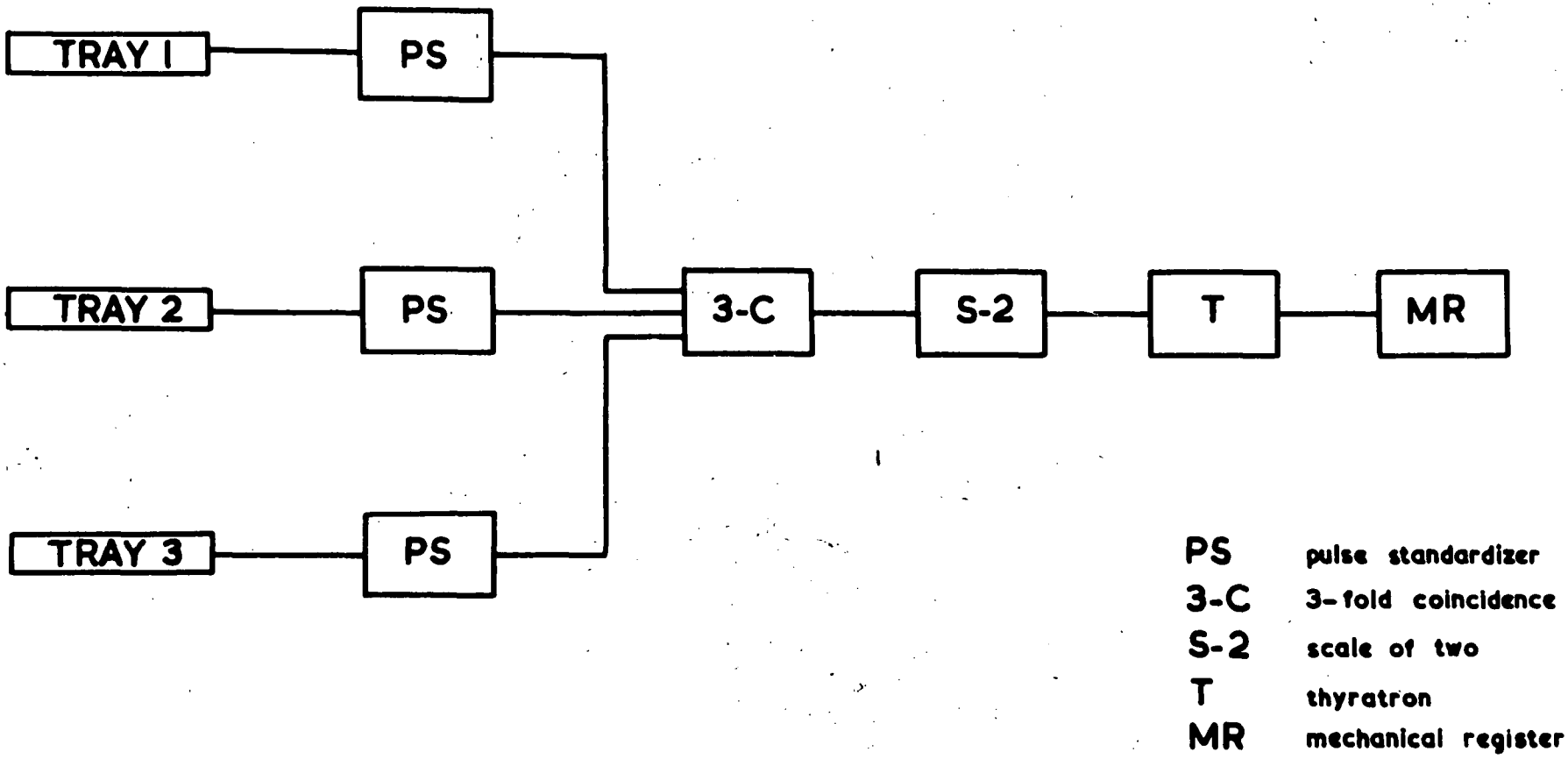
To obtain a counting rate as high as possible with the geometry chosen, the efficiency of the trays had to be as great as possible. One of the chief sources of inefficiency in conventional arrays is the loss of particles passing through the gaps between the counters in the trays. These gaps were therefore filled with counters. This procedure further increases the efficiency of the trays because rays, which, in the absence of the overlapping counters, would just graze the edge of a counter with high probability of not producing an ion pair in it, would have a substantial path length in the overlapping counter.

The telescope geometry is illustrated in Fig.III.2.

13. The Geiger -Muller Counters. External cathode counters (Maze, 10) were used because of their long life, good plateau and simplicity of construction. Fig. III.3 gives most of the constructional details. The cathode was of colloidal graphite (Acheson 'Aquadag') painted on the outside of the soda glass envelope. The cathode lead consisted of a short length of copper braid bound onto the 'Aquadag' with a piece of aluminium foil, cellulose tape and cotton thread. The whole cathode assembly was painted with a protective coat of clear 'Dulux' varnish. The effective length was determined by the separation of two glass capillaries over the anode. This separation was within 1 mm of 20 cm in all the counters.

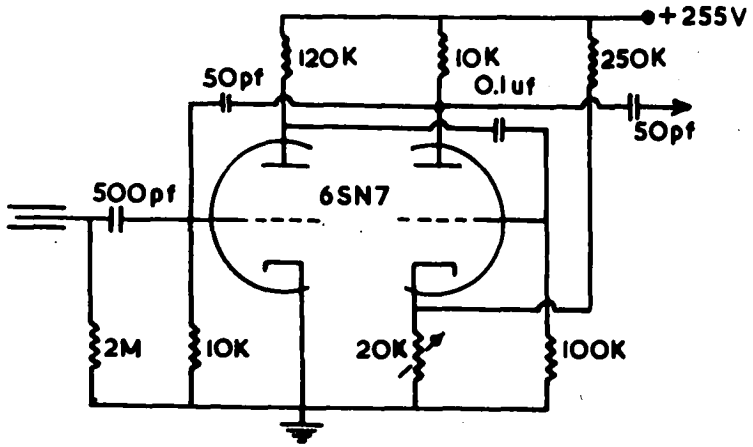
The filling was of argon (9 cm Hg) and ethyl ether (1 cm Hg). The counters were filled in such a way that all had very nearly the same starting voltage (1000 V). Plateaus were not plotted for every counter used (although a visual check was made by observing the pulses on a CRO screen), but those for which a plot was made had plateaus about 400 V long with a slope of 0.02 percent per volt. The counters were operated at about 1100 V. With ether as the quenching agent, the temperature effect was slight. The temperature in the cosmic ray hut at Macquarie Island remained fairly constant. Although an accurate measurement of the efficienty of the counters has not been made, experiments that were carried out in this laboratory indicated that it was about 99 percent. Since the dead time of these counters was of the same order as that of conventional counters, there is no reason to suppose that they were less efficient from this cause.

The background rate was higher than that of pyrex counters of the same area, being about 300 counts per minute. This was almost certainly due to the presence of K^{40} in Australian soda glass which contains about 1.8 percent of potassium. Australian pyrex contains about 0.1 percent.

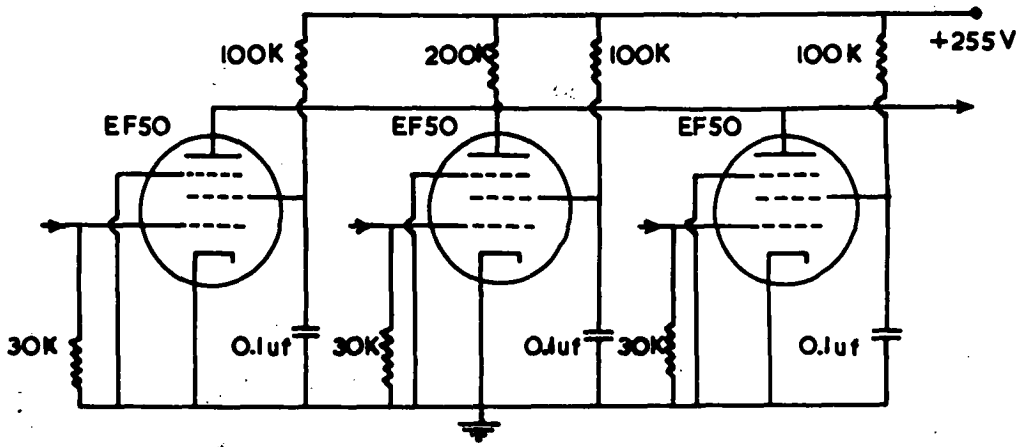


BLOCK DIAGRAM OF RECORDING SYSTEM

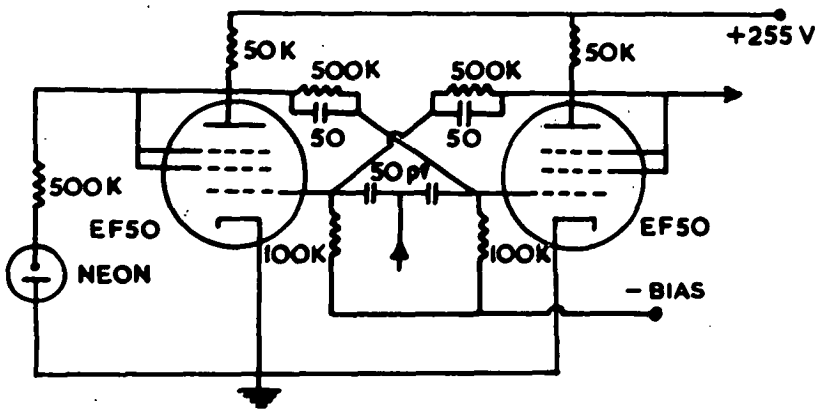
FIG. III.4



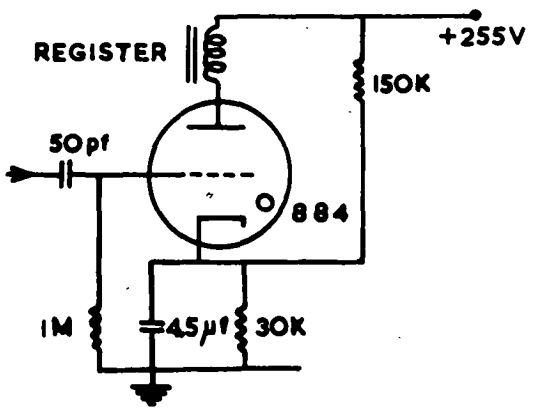
UNIVIBRATOR



3 - FOLD COINCIDENCE



SCALE PAIR



THYRATRON REGISTER CIRCUIT

FIG. III.5

14. The recording system. Fig. III.4 gives the block diagram of the circuitry. The circuit diagrams are shown in Fig. III.5. The recording system for one telescope was completely separate from that for the other.

The pulse standardizers were univibrators, adjusted to trigger on -0.5 V pulses from the counter trays. The duration of the output pulses was 4 μsec and their height -20 V. The dead time of the circuit was about 10 μsec.

The resolving time of the coincidence circuit was measured in the usual way by observing the counting rate due to chance coincidences between pulses from two trays so placed that the genuine coincidence rate was small. Such measurements showed that the resolving time was about 4 μsec.

The scale-of-two was so designed that it triggered only on the triple coincidence pulses. A discriminator tube between the coincidence circuit and the scaler to eliminate pulses due to double and single coincidences was therefore unnecessary. The scale-of-two was needed because the mechanical recorder used (a telephone message register driven by a thyratron circuit) could count only about 8 evenly spaced pulses per second. The maximum counting rate (with no Pb and with the telescope vertical) was about 1000 per hour, or 0.28 per sec. Alaoglu and Smith (11), who have considered the statistics of counting randomly occurring pulses with a scale-of-n circuit, have shown that the efficiency, E, of the counting system, when the only loss of counts is due to the inability of the register to respond to each of two pulses which arrive in an interval τ is

$$E = e^{-\mu\tau}\left[1 + \mu\tau + \frac{(\mu\tau)^2}{2!} + \dots + \frac{(\mu\tau)^{n-1}}{(n-1)!}\right],$$

where μ is the average rate of arrival of pulses at the input to the scale-of-n. In our case, this means that the efficiency

of a scale-of-1 (i.e. just the register) would be 96.56 percent. However, the addition of a scale-of-2 increases the efficiency to 99.94 percent. It is evident that it is quite unnecessary to have more than a scale-of-2.

15. Correction for showers. After the equipment had been in operation for some months on the Island, it was decided that some idea should be gained of the contribution due to side showers. It had been noted that the registers for the two telescopes quite often operated at nearly the same time, and it was thought that the effect of showers might not be unimportant. Such an error would be of increasing importance as the zenith angle of the telescopes was increased with consequent decrease of the genuine directional counting rate.

It is not easy with telescopes of this size to get an accurate idea of the contribution due to showers. If the middle tray were moved its own width to the side, the geometry would be quite different. The simplest thing to do appeared to be to register coincidences between the telescopes. Although it was fully realized that this would underestimate the contribution due to showers, this method was adopted. For a zenith angle of 70°, the shower rate was about 2 percent of the directional rate.

The circuit used consisted of a two-fold coincidence unit of the same type as in the main circuits. Its input pulses were the output pulses from the two coincidence circuits of the telescopes.

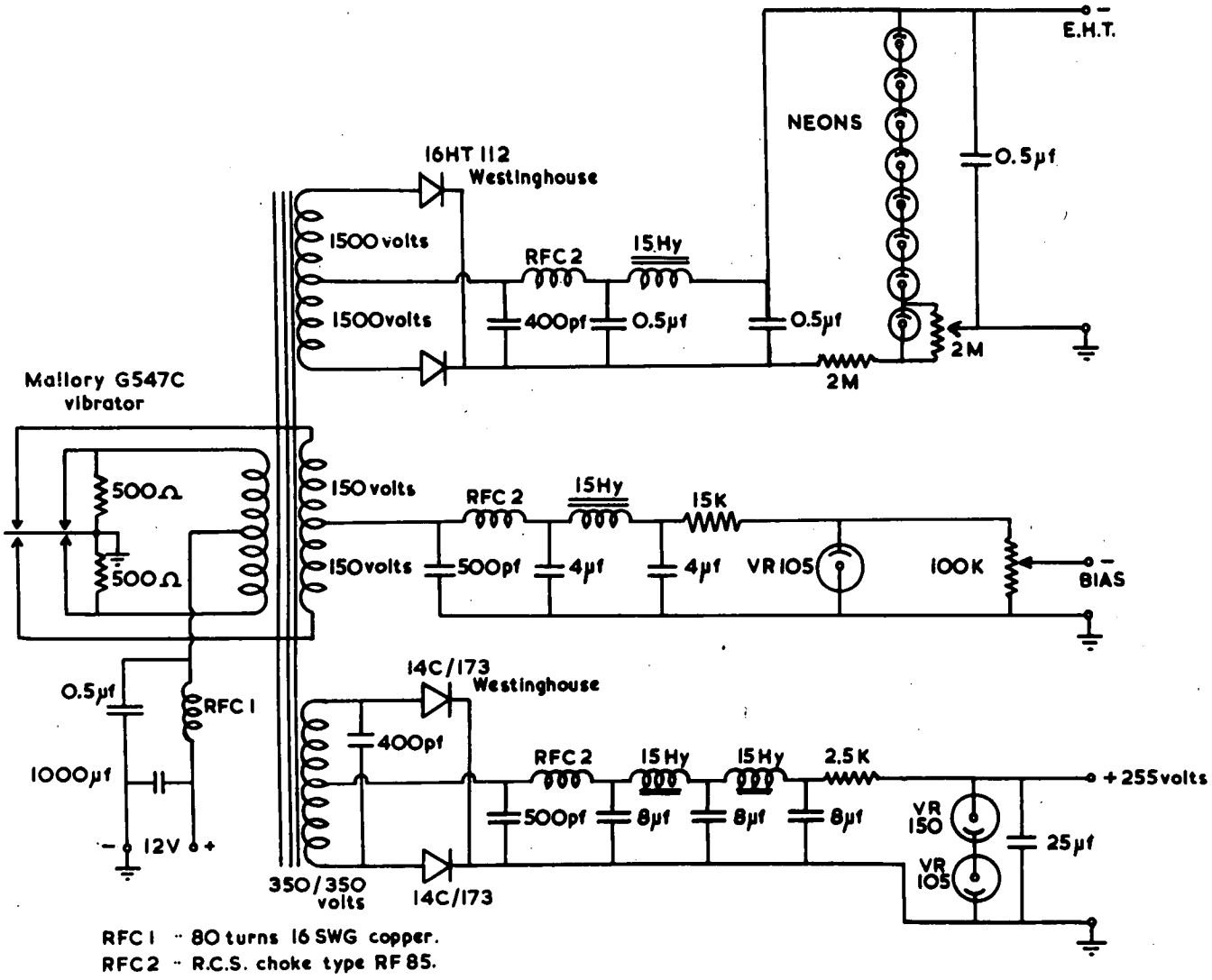


FIG. III.6 POWER SUPPLY

16. The power supplies. The EHT (-1100 V), the HT (255 V) and the bias (-105 V) were derived from a vibrator power supply run from 12 V accumulators. The circuit diagram is shown in Fig. III.6. A separate supply was used for each telescope.

The synchronous vibrator was used as the recitfier for the bias supply. Metal rectifiers were used for the HT and EHT supplies. Stabilization was by VR tubes for the bias and HT and by small neon lamps for the EHT.

One of the main troubles with vibrator supplies is 'hash' due to sparking at the vibrator contacts. This was reduced as far as possible by the correct choice of the timing condenser, which was placed across the HT winding. Further filtering by the RF chokes reduced the hash to a level where it did not seriously interfere with radio equipment on the Island (ionospheric recorder, radiosonde receiver and the communication receivers).

17. The control system. The operation of the system of relays which controlled the camera and the rotating mechanism was governed by two platinum contacts on a ship's chronometer. These contacts were separated by one minute and each was about ten seconds long. When the minute hand came into contact with the first, the recording camera operated and the equipment began to rotate. The motor driving the turntable continued to operate until rotation through 180° was complete when a relay cut it off. The rotation took less than a minute. When the second chronometer contact was made the camera operated again. The instrument panel which the camera photographed housed the registers for the two telescopes and for the shower recorder, a clock, an aneroid barometer and a dial thermometer which measured the room temperature.

18. Modifications to the original equipment. The description given in the preceding paragraphs is of the equipment in its form during the period February 1951 to March 1952. It commenced operation on the Island in June 1950. In the first six or seven months several features became apparent which required modification. Wherever possible, with the facilities available, these were put into effect immediately the need for them was realized.

The most important modification was the change from two-fold to three-fold telescopes since this greatly reduced the accidental rate. The counting rate, N, of a tray was about 3300 per minute. With a resolving time τ (= 4 μsec), the accidental rate is approximately

$$A = 2N^2\tau$$
$$= 0.024 \text{ per sec.}$$

In a 59 minute period (the interval during which measurements were made in one direction before rotation of the equipment to the opposite direction), the number of accidentals recorded would therefore be about 85. The genuine counting rate at 45° with 12 cm Pb was about 400 per 'hour', and at 70° the genuine rate was about 100 per 'hour'. Although it is possible to correct for accidentals in the way we have in Part II, par. 15(a), it is much better to reduce the accidental rate to negligible proportions. The accidental rate with a 3-fold telescope is approximately

$$A = 2NN_{12}\tau + 2NN_{23}\tau,$$

where N is the counting rate of a tray,

N_{12} is the genuine 2-fold rate between the top and middle trays,

and N_{23} is the genuine 2-fold rate between the middle and bottom trays.

In the modified telescope, the rate N_{12} for vertical incidence was about 12,000 per hour and N_{23} about 1000 per hour. Hence the accidental rate was about 6 per hour, or about 0.6 percent. Since the rates N_{12} and N_{23} vary with zenith angle in the same way as the genuine 3-fold coincidence rate, the accidental rate remained approximately the same percentage of the genuine rate for any zenith angle. On the other hand, with 2-fold coincidences the accidental rate is independent of zenith angle.

19. The site and the installation of the equipment. Macquarie Island is in latitude 54.5°S and longitude 159°E. Its geomagnetic latitude is 60.7°S.

The cosmic ray hut is situated about 12 feet above sea level on a narrow isthmus which runs approximately magnetic North-South. The site was chosen so that hills subtended the smallest possible angles. Hills to the North and South subtend about 8°. A massive rock on the isthmus to the East of the hut subtends 5°. Hence the maximum zenith angle at which E-W measurements could be made was 70°, since the half angle of the telescopes was 15°.

The hut, which was prefabricated, is 36 ft. x 12 ft. and has a gable roof. The wall and roof panels, which are about 6 ft. x 6 ft., have an outer layer of 5-ply separated from an inner layer of 3-ply by about 2" of an insulating material known as 'Onazote'. The frame onto which these panels are bolted is of oregon pine. Except for the brass bolts no metal is used in its construction. When the equipment was mounted on a cement block in the middle of the building, absorbing material was symmetrically placed with respect to it.

The site for the hut had been properly surveyed. Its long side lies on a line 12°E of N. The magnetic declination at

Macquarie Island is about 24°E of N. Hence, to set the axes (of rotation) of the telescopes in the geomagnetic N-S line, the equipment was oriented a further 12°E of N with respect to the walls of the hut. It is not likely that the setting was more than a degree or two in error and it was probably aligned with the same degree of accuracy as that to which the declination itself is known.

The equipment was carefully levelled so that the axis of rotation of the turntable was vertical. A very serious error could be caused if this axis were not vertical. It may be shown simply that, if the axis differs from the vertical by an angle ϕ, an apparent asymmetry will result of magnitude $4\phi \tan Z$ (assuming a $\cos^2 Z$ distribution), Z being the zenith angle. Thus, for measurements at 45°, if $\phi = 0.0025$ radian (i.e. an error of approximately 10' in the verticality of the axis), an apparent asymmetry of 0.01 would result. This is of the order expected for the true asymmetry. The spirit levels used in adjusting the turntable were sufficiently sensitive to allow an error of about 1' to be detected. As we shall see later, the results themselves indicate that any error present in the levelling must have been quite negligible.

20. The results for June 1951 - March 1952. We consider these first because during this period, results were collected at the one setting, viz. zenith angle 45°, 12 cm Pb absorber. We can thus get an indication of the variability of the asymmetry.

The results are set out in Table III.4 which is a condensation of the relevant part of Table II.2. The errors given with the counting rates are the standard deviations of the means, i.e. $\sigma/\sqrt{n}$, where σ is the standard deviation of a single result, and n is the number of results (hours). The asymmetries have been worked out from

$$\alpha = \frac{2(W - E)}{W + E}$$

where W represents the counting rate of one telescope (either A or B) pointing West and E represents the counting rate of the same telescope pointing East.

The standard deviation of each estimate of α has been calculated from the relation

$$\sigma_\alpha = \sqrt{\left[\left(\frac{\partial \alpha}{\partial W}\sigma_W\right)^2 + \left(\frac{\partial \alpha}{\partial E}\sigma_e\right)^2\right]^{\frac{1}{2}}}$$

$$= \frac{4}{(W + E)^2}\sqrt{\left[(W\sigma_e)^2 + (E\sigma_W)^2\right]^{\frac{1}{2}}}$$

As we shall see in par. 24, the sampling distribution of the quantity $\alpha' = W/(W + E)$ is very nearly normal. Since $\alpha = 4(\alpha' - \frac{1}{2})$, the sampling distribution of α is also very nearly normal. Hence, in testing whether any value differs singificantly from another value or from zero, the 95 percent fiducial limits will be $\pm$ 1.96 σ_α. The errors given with the values of α in Table III.4 are these 95 percent limits.

It will be observed that 13 values of α are positive and significantly different from zero. Of the seven which are not different from zero, only one (α_A, March) is negative.

TABLE III. 4

Values of the E-W asymmetry at 45° with 12 cm Pb at Macquarie Island. The errors with the counting rates are the standard deviations of the means and those with the values of α are the 95 percent fiducial limits.

		Counting Rate	α	Pressure mb
June 1951	A_W	404.811 ± 1.400	α_A = 0.0157 ± 0.0093	1007.99
	A_E	398.494 ± 1.304		1008.21
	B_W	400.859 ± 1.266	α_B = 0.0106 ± 0.0089	1008.21
	B_E	396.650 ± 1.284		1007.99
July 1951	A_W	416.675 ± 1.275	α_A = 0.0134 ± 0.0084	995.48
	A_E	411.137 ± 1.217		994.60
	B_W	412.783 ± 1.251	α_B = 0.0044 ± 0.0081	994.60
	B_E	410.978 ± 1.154		995.48
Aug. 1951	A_W	415.569 ± 1.327	α_A = 0.0201 ± 0.0091	998.15
	A_E	407.310 ± 1.367		1000.22
	B_W	412.201 ± 1.322	α_B = 0.0119 ± 0.0089	1000.22
	B_E	407.324 ± 1.317		998.15
Sept. 1951	A_W	411.843 ± 1.287	α_A = 0.0071 ± 0.0087	998.32
	A_E	408.940 ± 1.284		998.43
	B_W	413.471 ± 1.255	α_B = 0.0174 ± 0.0084	998.43
	B_E	406.351 ± 1.214		998.32
Oct. 1951	A_W	405.798 ± 1.131	α_A = 0.0005 ± 0.0075	1001.87
	A_E	405.602 ± 1.061		1001.54
	B_W	407.309 ± 1.075	α_B = 0.0113 ± 0.0075	1001.54
	B_W	402.742 ± 1.115		1001.87

TABLE III.4 (contd).

			Counting Rate	α	Pressure mb
Nov.	1951	A_W	411.170 ± 1.365	α_A = 0.0099 ± 0.0090	994.46
		A_E	407.107 ± 1.291		994.20
		B_W	409.545 ± 1.337	α_B = 0.0122 ± 0.0091	994.20
		B_E	404.565 ± 1.335		994.46
Dec.	1951	A_W	409.231 ± 1.122	α_A = 0.0157 ± 0.0076	997.12
		A_E	402.871 ± 1.101		997.04
		B_W	407.213 ± 1.138	α_B = 0.0042 ± 0.0077	997.04
		B_E	405.491 ± 1.114		997.12
January	1952	A_W	412.434 ± 1.614	α_A = 0.0058 ± 0.0107	986.03
		A_E	410.038 ± 1.548		985.98
		B_W	417.132 ± 1.546	α_B = 0.0173 ± 0.0102	985.98
		B_E	410.000 ± 1.483		986.03
Feb.	1952	A_W	411.712 ± 1.223	α_A = 0.0098 ± 0.0083	998.51
		A_E	407.696 ± 1.233		998.43
		B_W	409.853 ± 1.128	α_B = 0.6689 ± 0.0079	998.43
		B_E	406.228 ± 1.182		998.51
March	1952	A_W	405.891 ± 1.347	α_A = -0.0079 ± 0.0121	996.24
		A_E	409.127 ± 1.694		996.26
		B_W	415.038 ± 1.615	α_B = 0.0081 ± 0.0111	996.26
		B_E	411.692 ± 1.701		996.24

TABLE III.4 (contd).

		Counting Rate	α
June-March 1951-52 (inclusive)	A_W	410.7387 ± 0.4248	$\alpha_A = 0.0102 \pm 0.0027$
	A_E	406.5847 ± 0.4132	
	B_W	410.1002 ± 0.4105	$\alpha_B = 0.0104 \pm 0.0027$
	B_E	405.8552 ± 0.4068	

There are several cases where significant differences between values occur. Such differences occur not only between months but also within months (i.e. there are half a dozen months for which α_A and α_B are significantly different.) However, the values of α_A and α_B obtained by taking the results for the ten months together are identical.

A careful check of the performance of the equipment has shown that there is no reason to suppose that the differences between the values of the asymmetry are due to instrumental faults. Of the cases where there are significant differences within a month, α_A exceeds α_B on two occasions and α_B exceeds α_A on four occasions. This suggests that the differences are not attributable to one telescope.

We may consider the possibility that the axis of rotation of the turntable was not vertical. If we suppose that the turntable rotated about an axis which inclined to the East, then, when Telescope A pointed East, its zenith angle would be greater than when it pointed to the West. When Telescope B pointed West, its zenith angle would be less than when it pointed to the East. Therefore, due to this supposed error, both telescopes would register a higher counting rate in the West than in the East. This means that both α_A and α_B would be increased. In other words, an off-vertical axis would not produce a difference between α_A and α_B.

A difference between the average pressures while one telescope pointed West and while it pointed East could cause an apparent asymmetry. Furthermore, since the only results used in these calculations were those obtained when both telescopes operated satisfactorily simultaneously, such a pressure difference would affect the two telescopes in the opposite way.

However, in the present results, the only case where there was an appreciable pressure difference was in August, when the difference was about 2 mb. This occurred because of the failure of the rotating mechanism for a few consecutive hours during the month. If we correct for this difference by using the pressure coefficient calculated from the August results, we find that α_A is reduced from 0.0201 to 0.0153, while α_B is increased from 0.0119 to 0.0167. This clearly illustrates the importance of operating in such a way that pressure changes affect the measurements in the two directions in the same way. However, the August values of α_A and α_B were not significantly different even before correction for the pressure difference. The next largest pressure difference was in July, viz. 0.88 mb. α_A and α_B do differ significantly this month, but if the correction for pressure is made, the difference is increased not decreased.

It would therefore appear that systematic variations of the asymmetry occur, which can result in significant differences between values obtained during short periods of observation, but which average out over long periods. The type of process which could produce this effect could be the occurence of the short-lived and infrequent increases or decreased in the radiation intensity. These would have to be of short duration (approximately one hour), or the rotation of the equipment would cancel out the effect. They would have to be infrequent for fluctuations in their occurrence to affect A_W more than A_E, or vice versa.

We have seen that the asymmetry is very sensitive to changes in intensity. Consider, for instance, the December results where there is a large and significant difference between α_A and α_B. One finds that a decrease in A_W and B_E (or an increase in A_E and B_W) of 0.2 percent would bring α_A and α_B into statistical agreement. If we suppose that during December there were 12 changes of intensity of magnitude 10 percent with 8 occurring when one telescope

pointed West and 4 when the same telescope pointed East, we find that the intensities from the West and East averaged over the month would differ by 0.1 percent from this cause.

Further discussion along these lines is unlikely to be profitable until results from a double telescope equipment of much higher counting rate are available, to establish with a greater degree of certainty whether these apparently significant differences in the asymmetry do occur. However, it is perhaps worth considering whether the proposed changes in intensity may have escaped detection in other investigations. For a single value in a series of hourly measurements to arouse suspicion, its deviation from the average rate would have to be about three times the standard deviation. Thus, with a counter array whose rate is 10000 per hour (of which several have been used in various parts of the world), deviations greater than 3 percent would be regarded as significant. It is therefore unlikely that hourly deviations as great as 10 percent would have gone unnoticed. It is possible that lesser effects may have been dismissed as statistical fluctuations, particularly if there did not appear to be any other terrestrial or solar phenomena (such as radio fadeouts) with which they may be correlated. We may remark that Dolbear, Elliot and Dawton (12) have found that an increase in cosmic ray intensity does accompany radio fadeouts, but the increase is only about 0.3 percent.

We note from Table III.4 that A_W and B_W, averaged over the year, are almost identical, as also are A_E and B_E. We may argue from this that any error in the verticality of the axis must have been negligible. Suppose the zenith angle of Telescope A when pointing West was Z_1, and of Telescope B when pointing East was Z_2. Suppose the axis of rotation was inclined at an angle ϕ to the East.

Then it may be readily shown that

$$\frac{A_W/B_W}{A_E/B_E} = \frac{\cos^2 Z_1}{\cos^2(Z_2 - 2\phi)} \cdot \frac{\cos^2 Z_2}{\cos^2(Z_1 + 2\phi)}$$

If we suppose that an error as great as 1' were made in levelling the turntable, and if we suppose that $Z_1 = Z_2 = 45^\circ$, we find that the right hand side of the above relation is 0.978. The ratio on the left hand side, however, is 0.99976 $\pm$ 0.00020. We conclude that any error in the levelling must have been slight. Since the level was adjusted once only during the two years of work on the Island, and this change was in the N-S direction and not E-W, we can safely assume that the results for the first year, about to be given, did not suffer from any error due to levelling.

We shall take as the best estimate of the 45° E-W asymmetry at Macquarie Island with 12 cm Pb, the average of α_A and α_B for the 10 months, viz.,

$$\alpha\,(45^\circ,\ 12\ \text{cm Pb}) = 0.0103 \pm 0.0020,$$

where the error gives the 95 percent fiducial range.

21. <u>The June 1950 - May 1951 Results</u>. These are set out in Table III.5 in which the relevant data have been condensed from Table II.3.

It will be noted that all values are positive, although some of those for the small zenith angles are not significantly different from zero. There is only one case where values are significantly different from one another for a given zenith angle, viz., for 15°.

TABLE III.5

Values of the E-W asymmetry at various zenith angles, with 12 cm Pb at Macquarie Island. The errors with the counting rates are the standard deviations of the means and those with the values of α are the 95 percent fiducial limits.

			Counting Rate	α
15°	1st run	A_W	703.138 ± 2.109	α_A = 0.0062 ± 0.0082
		A_E	701.811 ± 2.037	
		B_W	720.668 ± 2.215	α_B = 0.0134 ± 0.0086
		B_E	711.100 ± 2.231	
15°	2nd run	A_W	756.052 ± 2.391	α_A = 0.0005 ± 0.0087
		A_E	755.689 ± 2.352	
		B_W	784.090 ± 2.304	α_B = 0.0072 ± 0.0082
		B_E	778.489 ± 2.308	
30°	1st run	A_W	656.294 ± 2.671	α_A = 0.0067 ± 0.0112
		A_E	651.918 ± 2.638	
		B_W	648.978 ± 2.559	α_B = 0.0139 ± 0.0110
		B_E	640.046 ± 2.558	
30°	2nd run	A_W	631.934 ± 3.472	α_A = 0.0153 ± 0.0150
		A_E	622.356 ± 3.301	
		B_W	626.066 ± 1.711	α_B = 0.0110 ± 0.0073
		B_E	619.194 ± 1.581	
60°		B_W	209.483 ± 1.183	α_B = 0.0314 ± 0.0163
		B_E	202.998 ± 1.246	

TABLE III. 5 (contd).

			Counting Rate	α
70°	1st run	A_W	96.013 ± 0.7133	α_A = 0.0402 ± 0.0202
		A_E	92.233 ± 0.6604	
		B_W	102.272 ± 0.7612	α_B = 0.0402 ± 0.0213
		B_E	99.539 ± 0.7895	
70°	2nd run	B_W	90.849 ± 0.5955	α_B = 0.0326 ± 0.0208
		B_E	87.936 ± 0.7338	
70°	3rd run	A_W	106.034 ± 0.7371	α_A = 0.0349 ± 0.0174
		A_E	102.397 ± 0.5680	
		B_W	104.047 ± 0.5674	α_B = 0.0312 ± 0.0153
		B_E	100.850 ± 0.5637	

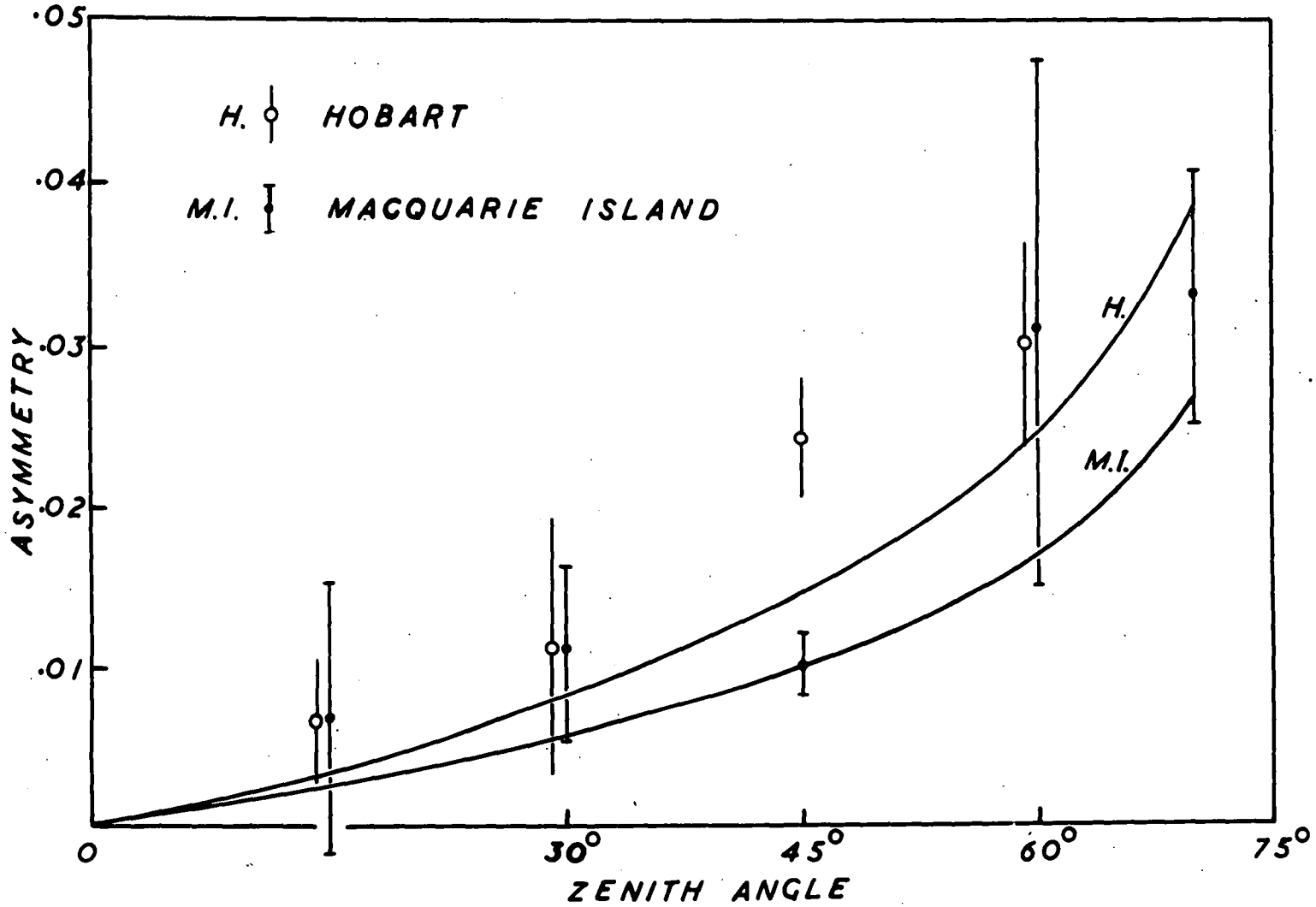

·05
·04
·03
·02
·01
ASYMMETRY
H. HOBART
M.I. MACQUARIE ISLAND
H.
M.I.
0
30°
45°
60°
75°
ZENITH ANGLE

FIG. III.7

22. Some Hobart Results. Table III.6 sets out some results obtained at Hobart by Burbury (5, 8) using a double telescope system of the same type as that used at Macquarie Island, but of lower counting rate. The times spent by each telescope in each direction were approximately as follows : 200 hours at 15°, 70 hours at 30°, 600 hours at 45° and 700 hours at 60°.

Table III.6 also includes our best estimates of the Macquarie Island results. These are the weighted means of the values given in Table III. (except for the 45° results which is the average of α_A and α_B for the year). We must remark that although this procedure has been adopted, it is not strictly correct to take weighted means when there are significant differences between the individual values. The weighted means and the errors given with them have been worked out according to the method given in Part I, par. 2.

TABLE III.6

East-West asymmetry with 12 cm Pb at Macquarie Island and Hobart. (geomagnetic latitudes 60.7°S and 51.7°S respectively). The errors are the 95 percent fiducial limits.

Z	α Macquarie Island	α Hobart
15°	0.0068 ± 0.0087	0.0066 ± 0.0041
30°	0.0112 ± 0.0054	0.0113 ± 0.0082
45°	0.0103 ± 0.0020	0.0245 ± 0.0037
60°	0.0314 ± 0.0163	0.0303 ± 0.0063
70°	0.0332 ± 0.0078	

The values have been plotted in Fig. III.7 in which the curves are the theoretical values obtained from Table III.3. Although theoretical calculations have not been made for a zenith angle of 70°, we have assumed that the mean deflection would be about the same as for other zenith angles, as suggested by Table III.2.

23. Discussion. There is surprisingly good agreement between the theoretical and measured 45° values for Macquarie Island. Since this value is based on 10 months' measurements with two telescopes, we consider it to be a good deal more accurate than any of the others. As may be seen by referring back to Table III.1, where we compared the 45° deflections predicted by Johnson's theory and by the numerical method, Johnson's theory would have predicted a much smaller asymmetry than the measured value. It therefore seems reasonable to suppose that the criticism we have made of his theory has the backing of these experiments. But it must be emphasized that if the theory by which we have replaced Johnson's gives a better representation of the asymmetry, it is to some extent accidental. For, the numerical calculations attribute the asymmetry to the total deflection which the mesons suffer, whereas we would expect it to be compounded of the additional deflection suffered as a result of energy loss, and of loss of mesons by decay over unequal paths. It is very difficult to estimate what the magnitude of these combined effects would be, but one would hardly expect it to be identical with that calculation from the total deflection. It is more likely that the asymmetry based on the total deflection is an overestimate.

We note from both the Hobart and Macquarie Island results that the trend towards higher values at the large zenith angles is unmistakeable. If we accept the above mentioned belief that the numerical method would overestimate the asymmetry, it seems likely that at large angles the asymmetry is not adequately accounted for in terms of deflections in the atmosphere of an isotropically produced secondary meson component.

Rose, Heikkila and Ford (13) have recently drawn attention to the fact that the Lemaitre-Vallarta theory of the

deflection of the primary cosmic rays leads us to expect an E-W asymmetry at large zenith angles in high geomagnetic latitudes. Referring to some curves published by Alpher (14) (which unfortunately are unavailable in Hobart at the time of writing), Rose et al. state that at Ottawa (goemagnetic latitude 56.8°N) " at zenith angles above about 50°, the minimum allowed energy of particles coming from easterly directions rises rapidly, and at 60° it is of the order of 10 Bev, while remaining quite low from the West". Some curves given by Vallarta (15) for a zenith angle of 45° show that at this zenith angle a primary asymmetry would not be expected at Macquarie Island, although at Hobart it might just be detectable.

We note from Table III.6 and Fig. III.7, that although the values of the asymmetry at 45° at Hobart and Macquarie Island do differ, there is almost no evidence for a variation with latitude. We would have expected larger values at Hobart due to both the primary and secondary effects. It may be possible to attribute this to variations of the asymmetry of the type discussed in par. 20.

We shall defer further discussion till we have considered some experiments of Groetzinger and McClure (16). Before doing this we wish to consider an investigation of the hourly variations of the asymmetry.

Variations of the Asymmetry.

24. It is of interest to search for systematic fluctuations of the asymmetry accompanying pressure changes or variations of the magnetic field strength. Changes associated with the latter would obviously be expected. Fluctuations due to pressure variations could also occur because an increase in pressure increases the mean path length of the particles and hence the distance over which they are influenced by the earth's field.

Observations over a long period are required for this purpose unless equipment with a very high counting rate is used. The 1951-52 Macquarie Island results for a zenith angle of 45° are suitable for this investigation. It is very unfortunate that the magnetometer on the Island was out of commission for almost the whole of this period. However, it is possible to search for variations correlated with pressure and with the heights of the various pressure levels.

For this purpose, the quantity α' was computed for each hour, α' being defined as

$$\alpha' = \frac{W}{W + E},$$

where W represents the number of counts recorded in an hour by one telescope (either A or B) pointing West, and E represents the number of counts recorded in the same hour by the other telescope (either B or A) pointing East. This quantity was chosen because it is always positive. The asymmetry as ordinarily defined is very often negative for hourly values and the Hollerith machines, which were used for the correlation analyses, cannot handle a mixture of positive and negative numbers as easily as they can numbers of one sign.

α' is simply related to the asymmetry, α, the relation being

$$\alpha = 4(\alpha' - \tfrac{1}{2}) .$$

We note that this investigation will throw no light on the significant variations between α_A and α_B discussed in par. 20. With the values of α_A and α_B we were dependent upon the rotation of the equipment averaging out any changes in pressure, magnetic field strength and cosmic ray intensity. Here, we take hourly values of α', combining the results of Telescopes A and B. Hence changes of this type will affect both telescopes similarly.

The correlation and regression coefficients may be determined by the methods set out in Part II. However, when we come to assign fiducial limits to the coefficients by the methods set out in Part II, we have to assume that the sampling distribution of α' is normal. It did not appear to be wise to make this assumption without first testing whether it was legitimate. The investigation of the sampling distribution proved to be an interesting study and will therefore be discussed fairly fully.

The actual distribution was found for the period 1 June - 30 November, 1951 by sorting 3982 values of α' into the 29 small equal ranges 0.435-0.439, 0.440-0.444, ..., 0.575-0.579. The frequencies are given in Table III.7, and the histogram obtained by plotting them is shown in Fig.III.8.

To see how closely these could be represented by a normal distribution, the expected frequencies were calculated using the mean value of α' and its standard deviation for this period. These values were

$$\overline{\alpha'} = 0.502729$$

$$\sigma = 0.017839$$

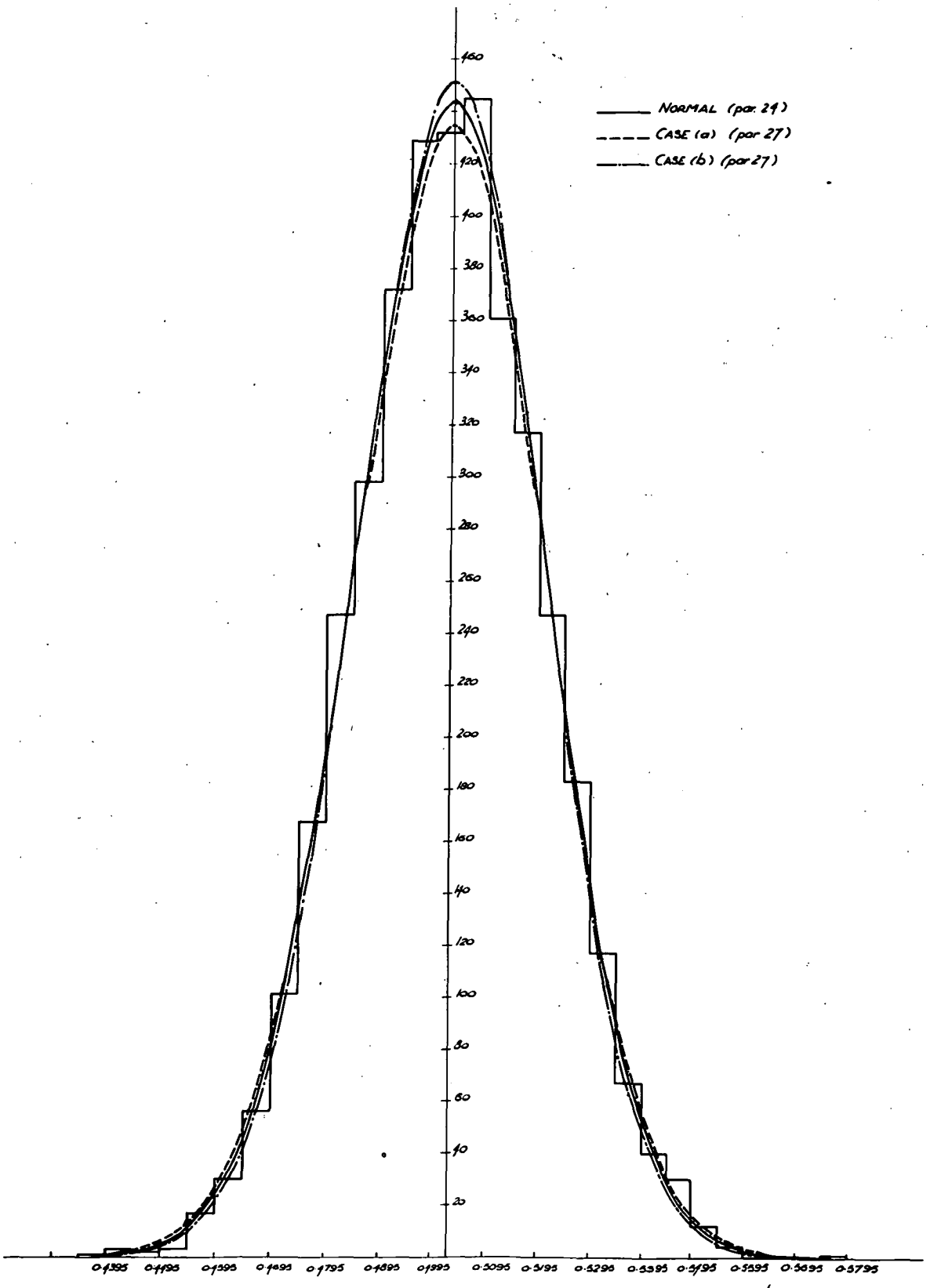


FIG. III 8. FREQUENCY DISTRIBUTIONS OF α'.

TABLE III.7

Frequency distributions of α' from 1 June - 30 November, 1951. The column headed "Normal" gives the normal distribution fitted to the observed data. Case (a) and Case (b) are frequencies calculated from the theoretical sampling distribution. Case (a) is when the observed means and variances are used in this distribution. Case (b) is when the variances are taken equal to the observed means.

Range of α'	No. Observed	Normal	Case (a)	Case (b)
0.435-0.439	1	0.5	0.8	0.5
0.440-0.444	3	1.4	1.9	0.7
0.445-0.449	2	3.5	4.4	3.5
0.450-0.454	3	7.4	9.6	7.1
0.455-0.459	17	17.5	19.4	15.2
0.460-0.464	30	33.3	36.4	30.8
0.465-0.469	56	60.4	64.3	56.9
0.470-0.474	101	101.9	105.4	97.6
0.475-0.479	167	157.8	160.2	153.3
0.480-0.484	247	226.7	227.1	244.2
0.485-0.489	298	301.7	298.6	301.8
0.490-0.494	372	370.8	364.5	374.3
0.495-0.499	429	421.7	413.0	428.5
0.500-0.504	432	443.5	434.3	451.6
0.505-0.509	445	431.4	423.2	439.2
0.510-0.514	361	388.0	382.7	394.2
0.515-0.519	317	323.0	321.1	325.3
0.520-0.524	247	248.3	249.6	248.9
0.525-0.529	183	177.4	180.7	174.4
0.530-0.534	117	116.4	120.5	117.1
0.535-0.539	67	70.9	75.6	62.1
0.540-0.544	40	40.1	43.6	39.4
0.545-0.549	30	20.8	24.8	19.9
0.550-0.554	12	10.0	10.6	8.5
0.555-0.559	4	4.5	5.6	4.2
0.560-0.564	0	1.9	2.4	1.3
0.565-0.569	0	0.7	1.0	0.8
0.570-0.574	0	0.3	0.4	0.2
0.575-0.579	1	0.1	0.1	0.1

The method of calculating the expected frequencies requires a word of explanation. If the distribution of α' is normal, the probability that a value lies in the range z, z+dz is

$$P(z)dz = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\gamma^2} d\gamma$$

where $\gamma = (z - \overline{\alpha'})/\sigma$. Let us now consider a specific case, that of finding the frequency of values lying in the range 0.500–0.504. The values of α' for each hour had been worked out correct to three decimal places. Therefore, in the process of rounding off the values for each hour, any α' lying in the range $0.4995 \leq \alpha' < 0.5045$ will be given a value in the range 0.500–0.504. Hence, to compute the probability that a value lies in this range, we require the integral

$$\int_{0.4995}^{0.5045} P(z)dz = \int_{-\infty}^{0.5045} P(z)dz - \int_{-\infty}^{0.4995} P(z)dz$$

Values of integrals of this form for the 29 ranges have been worked out using tables of the probability function. When these are multiplied by the number of observations in the sample, i.e. 3892, the expected frequencies are obtained. These are also listed in Table III.7 and are plotted as curve I in Fig.III.8. It will be seen that the normal distribution fits the histogram very well.

However, it appeared that it would be of interest to determine the sampling distribution of α' on the assumption that the West and East counting rates follow normal distributions. One could then substitute into this theoretical distribution function the observed values of the means and variances of the West and East rates and see how closely the expected frequencies so calculated fit the histogram. One could also calculate another set of frequencies by putting the variances equal to the means, which would be the case if there were no systematic

variations of the counting rates and, presumably, none of the asymmetry. This, it seemed, would allow one to see whether those factors which influenced the counting rates also affected the asymmetry and would provide an alternative method of detecting the presence of systematic variations.

In determining the sampling distribution of α', it was at first assumed that the West and East rates, W and E, were independent normal variables. The possibility of correlations between these was ignored because it simplified the somewhat involved algebra in the derivation. However, it was found that the fit was not at all good when the observed values of the means and variances were inserted into the distribution function so obtained. A much better fit was obtained with values of the variances equal to the corresponding means. This indicated that an error was present in the theoretical distribution function and it seemed likely that this was due to neglecting the correlation between W and E. Hence the sampling distribution was re-determined, taking the correlation into account, by the method about to be outlined.

25. Determination of the sampling distribution of $\alpha' = W/(W + E)$.

The problem is to determine the probability that α' lies in the range $z, z+dz$.

We assume that W and E are normal variables with means $\overline{W}$ and $\overline{E}$ and standard deviations σ_w and σ_e respectively, and that W and E are linearly correlated, the correlation coefficient being ρ.

The probability that W lies in the range $x, x+dx$ and that E lies in the range $y, y+dy$ is given by the bivariate normal distribution (see for example Kendall (17, p. 334)) and is

$$\frac{dx\,dy}{2\pi\sigma_W\sigma_e\sqrt{1-\rho^2}} \exp - \frac{1}{2(1-\rho^2)}\left[\frac{(x-\overline{W})^2}{\sigma_W^2} - \frac{2\rho(x-\overline{W})(y-\overline{E})}{\sigma_W\sigma_e} - \frac{(y-\overline{E})^2}{\sigma_e^2}\right]$$

To find the probability $P(z)dz$ that α' $(= W/(W+E))$ lies in the range $z, z+dz$, we must impose the restriction that when $W = x$, the value, y, which E assumes, is given by $z = x/(x + y)$. That is

$$y = x(1 - z)/z$$
$$\text{and } dy = -x\,dz/z^2.$$

By substituting these values into the expression (x) and integrating over all values of x, we find that

$$P(z)dz = -\frac{dz}{z^2}\;\frac{1}{2\pi\sigma_W\sigma_e\sqrt{1-\rho^2}}\int_{-\infty}^{\infty} x\,dx\;e^{-A}$$

$$\text{Where } A = \frac{1}{2(1-\rho^2)}\left[\frac{(x-\overline{W})^2}{\sigma_W^2} - \frac{2\rho(x-\overline{W})(\gamma x-\overline{E})}{\sigma_W\sigma_e} + \frac{(\gamma x-\overline{E})^2}{\sigma_e^2}\right]$$

Where $\gamma = (1 - z)/z$.

From this point the derivation of $P(z)dz$ is straightforward but the algebra is complicated and will not be included here. It turns out that the function ξ defined by

$$\xi = \frac{\overline{W} - (\overline{W} + \overline{E})z}{\sqrt{\sigma_W^2 - 2z(\sigma_W^2 + \rho\sigma_W\sigma_e) + z^2(\sigma_W^2 + 2\rho\sigma_W\sigma_e + \sigma_e^2)}} \qquad -(xi)$$

is a standard normal variable, i.e. it has zero mean and unit standard deviation. Thus

$$P(z)dz = \frac{1}{\sqrt{2\pi}}\;e^{-\frac{1}{2}\xi^2}\;d\xi$$

and the probability that α' lies in a certain range may be found by the same method that was used for determining the probabilities when it was assumed that the distribution was normal, if the values of ξ are worked out for the desired values of z. To do this the values of the parameters involved in Eq.(xi) are required. The means and standard deviations $\overline{W}$, $\overline{E}$, σ_w, σ_e are known from earlier calculations.

<u>26. Determination of ρ.</u> The determination of the correlation coefficient between W and E presents a new problem. In principle, it is, of course quite simple to obtain but, since its calculation involves finding the sum of the four thousand products WE, it is a calculation for the Hollerith machines which at the present time are too fully occupied on other work. However, a fairly good estimate can be made from the regression equations, the constants in which have been worked out previously. The method used is as follows.

We assume that, since the particles recorded by one telescope must be different from those recorded by the other, the rates would be independent were it not that meteorological changes simultaneously affect each rate. Thus we assume that the correlation between the rates is due to these changes. If we let $w = W - \overline{W}$ and $e = E - \overline{E}$, the regression equations are

$$w = b_{12.3}x_2 + b_{13.2}x_3$$

$$\text{and} \quad e = b'_{12.3}x_2 + b'_{13.2}x_3$$

where the rest of the symbols are as defined as in Part II. The primed regression coefficients are not necessarily equal to the unprimed ones.

The correlation coefficient ρ which we desire to find is

$$\rho = \frac{1}{n} \frac{\Sigma\, we}{\sigma_w \sigma_e}$$

From the regression equations we have

$$\Sigma\, we = \Sigma(b_{12.3}x_2 + b_{13.2}x_3)(b'_{12.3}x_2 + b'_{13.2}x_3).$$

Using the fact that $b_{12.3} = -\frac{\sigma_w \omega_{12}}{\sigma_2 \omega_{11}}$ etc., and bearing in mind that $\omega_{11} = \omega'_{11}$ we find

$$\rho = \frac{1}{\omega_{11}^2}\left[\omega_{12}\,\omega'_{12} + r_{23}(\omega_{12}\omega'_{13} + \omega'_{12}\omega_{13}) + \omega_{13}\omega'_{13}\right] \quad -(xii)$$

The combined West rates for telescopes A and B for the Macquarie Island results during the months June - November, 1951 lead to $\overline{W} = 410.2454$ counts per hour. The residual variance $\sigma^2_{w.23} = 459.2386$. The corresponding combined East rates give $\overline{E} = 405.6984$ and $\sigma^2_{e.23} = 424.5787$. Since the residual variances exceed the means by significant amounts, it is clear that the regression equations do not adequately describe the variations. Hence it is not expected that the correlation coefficient ρ calculated from Eq.(xii) accurately measured the interdependence of the West and East rates. However, in the absence of the computed value, it is the best estimate we can make. The value so obtained is

$$\rho = 0.190978$$

27. Expected frequencies based on the theoretical sampling distribution.

Two sets of expected frequencies have been calculated. The values of the parameters used (for the period 1 June - 30 November, 1951) were -

Case (a).

$$\bar{W} = 410.2454$$

$$\bar{E} = 405.6984$$

$$\sigma_w^2 = 557.8822$$

$$\sigma_e^2 = 534.9650$$

$$\rho = 0.190978$$

Case (b).

$$\bar{W} = 410.2454 = \sigma_w^2$$

$$\bar{E} = 405.6984 = \sigma_e^2$$

$$\rho = 0.$$

The expected frequencies are listed in Table III.7 and are plotted in Fig.III.8. It will be noted that case (b) slightly overestimates the frequency of values close to the mean and underestimates frequencies of the larger deviations. The fit with case (a) is somewhat better. This indicates that factors which influence the counting rate also affect the asymmetry, although the dependence appears to be slight.

28. Dependence of the asymmetry on pressure and temperature.

We return now to the correlation of α' with meteorological factors.

Analyses have been carried out for the whole period June 1951 to March 1952 and for the shorter period June - November 1951. The correlation and regression coefficients so obtained are set out in Table III.8. The symbolism is the same as used in Part II, except that the countingrate is replaced by α'. Thus the variable X_1 is α', X_2 ismean surface pressure, and X_3 is mean surface temperature.

TABLE III.8

Results of correlation of hourly values of α' with surface pressure and temperature. The errors are the 95 percent fiducial limits.

Period	n	$\overline{\alpha'}$	σ	$b_{12.3}$ percent per mb	$b_{13.2}$ percent per °C	$R_{1.23}$
1 June-30 Nov. 1951	3982	0.503	0.018	+0.0063 ±0.0081	+0.126 ±0.507	0.025
1 June 1951-15 March 1952	6189	0.503	0.018	+0.0047 ±0.0069	+0.102 ±0.369	0.017

The fiducial ranges assigned to the regression coefficients are the 95 percent ranges, calculated on the assumption that the sampling distribution is normal, an assumption amply justified by what has preceded.

It will be seen that, even for the whole period, the regression coefficients do not differ significantly from zero. Although the probability that the pressure coefficient is greater than zero is 0.91 (for the whole period), we would not be justified in inferring that the asymmetry does undergo variations associated with pressure. The variation with temperature is quite insignificant.

The pressure coefficient given in the Table is for the quantity α'. For the asymmetry itself, the coefficient is four times greater , and is therefore, for the whole period

$$+0.0188 \pm 0.0276 \text{ percent/mb.}$$

29. Correlation with radiosonde data. Average values of α' over the four hour period at the time of the radiosonde flight (Part II, par. 21) have been correlated with mean surface pressure and the heights of the 500, 300, 150, 100 and 80 mb levels. As may be expected with the low counting rates, no significant correlation coefficients were found. The multiple correlation coefficients obtained are listed in Table III.9.

TABLE III.9

Multiple correlation coefficients $R_{1.23}$ obtained in the correlation of α' with surface pressure and heights of the pressure levels.

Pressure level	$R_{1.23}$
500 mb	0.039
300 mb	0.012
150 mb	0.067
100 mb	0.046
80 mb	0.008

Although we cannot base any discussion on these values, it is interesting to note that the largest value occurs at the 150 mb level, where we also found the largest value in the correlation of the counting rate with sonde data.

30. Variations due to other causes. It is necessary to know whether the asymmetry is subject to any systematic variations. We have already seen from the examination of the theoretical frequency distributions that there is a suggestion of this. A further indication may be gained from the comparison of the observed variance of α' with the value which would be expected if there were not such influences.

The expected variance σ^2 has been calculated from the relation

$$\sigma^2 = \sigma_W^2(\partial \alpha'/\partial W)^2 + \sigma_e^2(\partial \alpha'/\partial E)^2$$

If there are no systematic variations of the counting rates, $\sigma_W^2 = \overline{W}$ and $\sigma_e^2 = \overline{E}$. Hence the expected variance of α' is

$$\sigma^2 = \overline{W}\,\overline{E}/(\overline{W} + \overline{E})^3.$$

The mean values of the West and East counting rates for the two telescopes over the period June 1951 - March 1952 were

$$\overline{W} = 410.4165 \text{ counts per hour}$$

$$\overline{E} = 406.2233 \text{ counts per hour.}$$

Thus the expected variance is

$$\sigma^2 = 0.000306124$$

compared with the observed value

$$\sigma_o^2 = 0.000320724 .$$

To see whether σ^2 is significantly greater than σ_o^2 we use a χ^2 test in the same way as in Part II. That is, we define χ_s^2 as

$$\chi_s^2 = n\sigma_o^2/\sigma^2 .$$

This test enables us to conclude that the probability of getting a sample variance as great as 0.000320724 or greater in random

sampling from a normal population of variance 0.000306124 is 0.004.

Hence we can safely conclude that some factors are operative which have caused the asymmetry to vary systematically.

We have seen that there is no significant correlation with surface pressure and temperature. The residual variance, after removing effects 'correlated' with these factors, is 0.000320629, which is practically identical with the initial variance. Some other factors must be present.

The variable with which the asymmetry is most likely to be correlated is the earth's magnetic field strength. As mentioned previously, H varies considerably at Macquarie Island which is near the auroral zone. Unfortunately the magnetometer on the Island was working for only a few weeks at the end of the period during which these experiments were made, so that no investigation along these lines is possible.

31. Expected effect of pressure on the asymmetry. It is possible to calculate the expected barometer coefficient of the asymmetry fairly simply using the same kind of method that was used to determine the coefficients for the intensity in Part II.

Since, in the form of the theory of the E-W effect adopted here, the asymmetry is proportional to the deflection of the mesons, which in turn, is proportional to the integral $\int_0^s \frac{ds}{p/\mu c}$, we may define the pressure coefficient of the asymmetry for a particular final momentum as

$$\frac{\delta a}{a\ \delta P} = \frac{\int_0^{s_2} \frac{ds}{p/\mu c} - \int_0^{s_1} \frac{ds}{p/\mu c}}{\delta P \int_0^{s_1} \frac{ds}{p/\mu c}}$$

where δP is the increment in pressure

s_1 is the distance along the path of the meson to the production region in the normal atmosphere.

and s_2 is this distance in the pressure altered atmosphere.

As in Part II, par. 20, we let $s_2 - s_1 = \delta s$ and take $\langle p/\mu c \rangle$ as the average value of $p/\mu c$ over δs. The coefficient then becomes

$$\frac{\delta \alpha}{\alpha\ \delta P} = \frac{\dfrac{\delta s}{\langle p/\mu c \rangle}}{\delta P \displaystyle\int_0^{s_1} \frac{ds}{p/\mu c}}$$

Using values of these quantities determined for earlier calculations in Parts I and II for a zenith angle of 45°, we obtain the coefficients set out in Table III.10.

TABLE III.10

The barometer coefficient of the asymmetry as a function of sea level momentum.

Final momentum Mev/c	Barometer coefficient of α percent/mb
245	+0.0194
849	+0.0243
1335	+0.0262
2691	+0.0292
3859	+0.0305
5814	+0.0320
7467	+0.0323

The mean value of the barometer coefficient of α over the range 250-10000 Mev/c, determined by a rough numerical integration is

+0.025 percent/mb.

Although this is not significantly different from the observed value ($+0.0188 \pm 0.0276$), it could be an overestimate, because the theory on which this calculation is based attributes the E-W asymmetry to the total deflection of the mesons. On the other

hand it assumes that the path legnth is changed by the same amount for particles coming from the East as from the West. As we have discussed in pars. 2 and 4, positive particles coming from the East travel over a longer path than those coming from the West, and the trajectories are concave to the East. An increase in pressure will increase the path length, and hence the deflection, for particles coming from the East by an amount greater than the increase for the West. In addition, the initial momenta will be increased more for mesons from the East than the West, and the spectrum decreases with increasing momentum. Thus, the intensity of positives at a given zenith angle in the East will be decreased by a greater amount than that at the same angle in the West. The reverse will apply for the negatives, but due to the positive excess the net effect will be an enhanced asymmetry. No attempt has so far been made to compute the magnitude of this effect, because it requires a knowledge of the actual paths followed by the mesons and the determination of these appears to be a very laborious undertaking.

It has been noted in Part II, par.14(b), that the barometer coefficients of the West and East rates differ. For the period June - November, 1951. the Macquarie Island results give

$$b_{12.3}\ (\text{West}) = -0.1766 \pm 0.0122 \text{ percent/mb}$$

$$b_{12.3}\ (\text{East}) = -0.1897 \pm 0.0118 \text{ percent/mb},$$

where the limits quoted are the 95 percent fiducial ranges. Thus an increase in the pressure decreases the East rate by a greater amount than it decreases the West rate. This is probably a result of the process suggested above.

Review of Other Experiments.

32. The first evidence that there is an East-West asymmetry in latitudes beyond the knee of the latitude curve was obtained by Johnson (Johnson (18), Johnson and Street (19)) in 1932 in some experiments on Mt. Washington (altitude 1917 m, geomagnetic latitude 57°N) in which the total radiation was studied. Further experiments, mainly on the total radiation, have confirmed the existence of the effect, e.g. Johnson and Stevenson (20) at Swarthmore (approximately sea level, geomagnetic latitude 51°N), Stearns and Froman (21) at Mr. Evans (altitude 4300 m, geomagnetic latitude 49°N).

One of the most recent experiments reported is that of Labonte (22) who examined the total radiation at Montreal (altitude 60 m, geomagnetic latitude 57°N). Measurements were made at zenith angles of 30° and 45°. From the counting rates given, the asymmetries are found to be $+0.0015 \pm 0.0058$ for 30° and -0.0000 ± 0.0088 for 45°.

33. The first experiments with Pb absorber to remove the soft component appear to be those of Seidl (23) at New York (approx. sea level, geomagnetic latitude 54°). His value with 14.5 cm Pb for a zenith angle of 20° was 0.0073 ± 0.0027 (probable error), which is close to the value predicted by our theory. Seidl also measured the asymmetry of the total radiation at the same zenith angle obtaining a value of 0.0010 ± 0.0022. Thus the value with the Pb absorber is greater than without it. If a symmetrical soft component is removed by the Pb, an increased asymmetry would be expected, because the difference between the West and East rates would remain the same but the average rate with absorber would be less, thus decreasing the denominator of the expression

2(W-E)/(W+E). But in Seidl's experiments the asymmetry with Pb is greater than would be expected by removing a symmetrical soft component. It is not known whether this is a real effect. The first experiments conducted at Hobart (A.G. Fenton and Burbury (24)) also showed the same effect, although subsequent ones here have not. Further experiments are at present being conducted at Hobart with the equipment used at Macquarie Island in an attempt to discover whether it is real. However, this work is incomplete. The problem is of interest because, if the effect is real, it may be due to a negative asymmetry of the electronic component, to a negative asymmetry of the slow meson component (due perhaps to an excess of negatively charged mesons instead of a positive excess), or to some process occurring in the Pb. An event of this last mentioned type could occur when a positive meson stops in the Pb and emits its decay electron in a time less than the resolving time of the coincidence circuit, and in a direction to produce a coincidence. Negative mesons which stop in the Pb would be captured before decay. Since positives are expected to predominate from the West, an enhanced asymmetry would occur.

34. An experiment which commands close attention is that of Groetzinger and McClure (16). Magnetised iron plates were used to select positives only or negatives only. Furthermore, mesons in a narrow energy band (at about 800 Mev) were investigated. This is an approach to the ideal type of experiment on the E-W asymmetry in which the need to know the positive excess and the shape of the spectrum is eliminated. Comparison with the theory should therefore be simpler.

The system consisted of two telescopes containing

magnetised iron plates and was such that simultaneous measurements were made at zenith angles of 24° and 58° in either the West or the East azimuth. The azimuth was changed approximately every six days. The polarity of the magnetised plates was reversed every 24 hours, so that presumably, during one day the intensity of positives would be measured with one telescope and negatives with the other, and vice versa during the next day. The results obtained at Chicago (altitude 260 m, geomagnetic latitude 51°N) and at Mr. Evans (altitude 4300 m, geomagnetic latitude 49°N) are set out in Table III.11. Groetzinger and McClure have quoted the probable errors. Since, in a normal distribution, the probable error is 0.67σ, we obtain the 95 percent fiducial limits by multiplying the probable errors by $1.96/0.67 = 2.93$. The errors quoted in our Table III.11 are the 95 percent fiducial limits.

TABLE III.11

E-W asymmetry of mesons of average energy 750 Mev with positives and negatives observed separately. Groetzinger and McClure (16).

	Z	a_+	a_-	$a_\pm$(theor.)	λ
Mt. Evans	24°	0.07 ± 0.073	-0.055 ± 0.059	± 0.057	3
	58°	0.37 ± 0.16	-0.375 ± 0.176	± 0.329	3
Chicago	24°	0.070 ± 0.070	-0.039 ± 0.070	± 0.037	2
	58°	0.128 ± 0.152	-0.192 ± 0.167	± 0.197	2

We therefore see that none of the 24° values is significantly different from zero, but that three of the 58° values are.

The theoretical values are those worked out by the authors on the basis of Johnson's theory. They have assumed a $\cos^{\lambda} Z$ zenith angle distribution law at both altitudes, with λ taken as 2 at Chicago and as 3 at Mt. Evans. Although their telescopes select μ-mesons of approximately 800 Mev., they allow a small North-South component. For their calculations

Groetzinger and McClure have therefore taken a lower average energy (750 Mev), corresponding to the projection of the particles on a vertical E-W plane.

The numerical theory presented here (pars. 6-9) gives a total deflection of 0.064 radian for μ-mesons of energy 750 Mev arriving at a zenith angle of 60° at Chicago (we have assumed Chicago is at sea level, and we have taken H = 0.172 oersted (Vestine et al. (25))). Assuming a $\cos^2 Z$ distribution law, this gives an asymmetry of 0.41% (The value at 58° would not be expected to be very different). This is about twice the value predicted by Johnson's theory, which appears to give a very close agreement with the measured values. The measured values are significantly less than 0.41. It may therefore seem that Johnson's theory gives a better description of the E-W asymmetry than the numerical theory. However, as pointed out earlier (pars. 5 and 23), the numerical theory is likely to overestimate the effect, although probably not by so wide a margin as the results suggest. We must bear in mind also, that if the mesons in the energy band studied are produced, on the average, nearer the observer than our numerical calculations assume (i.e. 43 km at 60°), the deflection would be less.

In estimating the average energy which their spectrometer records, Groetzinger and McClure do not appear to have taken into account the shape of the energy spectrum. We have used their curve giving the relative sensitivity of their telescope as a function of energy (curve W in their Fig.3) in combination with Rossi's vertical sea level spectrum (our Fig.I.5) and have found that the average energy recorded is about 900 Mev instead of 800 Mev. If we calculate the asymmetry expected from Johnson's theory for 900 Mev mesons, we find a value of 0.159, and the value expected from the numerical theory is 0.39. We therefore see that taking account of the shape of

the energy spectrum appreciably reduces the asymmetry calculated according to Johnson's theory. We must remark that in these calculations we have taken H = 0.172 oersted for Chicago (taken from the tables of Vestine et al. (25)) because Groetzinger and McClure do not state what value they used. Using this value of H, we find that the asymmetry at 58° at Chicago for 750 Mev μ-mesons is 0.177 instead of 0.197. The difference is probably due to Groetzinger and McClure taking some other value of H.

Following Groetzinger and McClure, all the calculations we have made for Chicago are based on a $\cos^2 Z$ distribution law. There is very little evidence that such a law is appropriate. As we saw in Part I, section B, Voisin (ref. I, (27)) found evidence for a law of the form $1 - a \sin^b Z$ in a momentum range somewhat lower than that investigated by Groetzinger and McClure. If this law holds, we find from Eq. (ii) that the asymmetry is

$$\alpha = \frac{2\delta ab \sin^{b-1} Z \cos Z}{1 - a \sin^b Z}$$

Taking $\delta = 0.0308$ for $Z = 58^{\circ}$ at Chicago (which we obtain by working back from the theoretical value given by Groetzinger and McClure), and taking $a = 1$ and $b = 1.6$ (after Voisin), we find that the asymmetry is 0.204, instead of 0.197 if a $\cos^2 Z$ law is followed. On the other hand Zar (ref. I, (28)) found values of λ close to 3 in a momentum range close to that used in the epxeriments of Groetzinger and McClure. By using $\lambda = 3$ in the calculations of the asymmetry for Chicago, the values are increased by a factor of 1.5.

If there is a primary asymmetry (par. 23) for large zenith angles, it should be detectable at 58° in geomagnetic latitude 51°N (although at a zenith angle of 24° it would not).

We now consider two possible sources of error in the measurements themselves, viz. pressure differences and levelling of the equipment.

No mention is made by Groetzinger and McClure about precautions taken to ensure that the axis of rotation of the telescopes was vertical. However, if there had been a serious error, it would have been expected to have the effect of increasing the asymmetry of the positives and decreasing the asymmetry of the negatives, or vice versa.

The equipment was rotated from West to East, or vice versa, every six days. The authors do not state how much time was spent on the measurements, but, judging by the counting rates and their errors, approximately 2 weeks would have been needed for each azimuth and for each polarity in the Chicago measurements. It is therefore highly probable that the pressure would not have averaged out with rotations every 6 days. We saw in par. 20 that, when the equipment used at Macquarie Island failed to rotate for a few consecutive hours in August 1951, a 2 mb pressure difference arose and correction for this difference appreciably altered the asymmetries. Groetzinger and McClure do not state whether pressure corrections were made or whether they were necessary. According to our Fig.II.1, the pressure coefficient for 800 Mev mesons arriving at a zenith angle of 60° would be about -0.35 percent/mb. If the pressures during the periods over which the West and East measurements were made at 58° at Chicago differed by 10 mb, the value of α_{-} could be changed from -0.192 to -0.227.

It is evident from this review that too many uncertainties are involved and that the statistical accuracy of the results is too low to allow us to decide whether Johnson's theory gives a better means of determining the E-W asymmetry than the numerical method.

CONCLUDING REMARKS.

The following main points have emerged from this investigation of the East-West asymmetry in high latitudes:

(a). An E-W asymmetry of the penetrating component of cosmic rays does exist in latitudes beyond the knee of the latitude curve.

(b). The asymmetry increases with zenith angle.

(c). Comparison of the results obtained at Hobart and Macquarie Island (goemagnetic latitudes 51.7°S and 60.7°S respectively) fails to show a latitude effect.

(d). Observed asymmetries measured during short periods (e.g. a month) frequently differ from one another by a greater amount than would be expected from the statistical accuracy of the experiments. Over a long period these differences average out.

(e). Variations of the asymmetry from hour to hour occur. There is no conclusive evidence that there is a pressure effect, although what evidence there is suggest a positive barometer coefficient.

(f). The review of Johnson's theory of the effect has shown up certain defects and it seems certain that his theory underestimates the magnitude of the effect. The asymmetries measured with coincidence telescopes are larger than those predicted by Johnson's theory, thus supporting this contention. The measurements by Groetzinger and McClure on a narrow energy band do not appear to lead to conclusive results.

(g). An attempt has been made to evaluate the expected asymmetries by numerical methods, assuming that it is due to the total deflection suffered by isotropically produced mesons in their flight through the atmosphere. We believe this theory would somewhat overestimate the effect, but it does lead to values of the order measured at intermediate zenith angles.

(h). At large zenith angles the measured values appear to exceed those expected from this theory, suggesting that at these large angles there may be an asymmetry of the primaries. However, the expected asymmetries are sensitive to the shape of the sea level momentum spectrum. If the same spectrum is assumed for each zenith angle, the asymmetry increases more rapidly with angle than the values calculated here.

We therefore see that the study of the high latitude E-W asymmetry must not be regarded as closed. On the theoretical side, attention should be given to working out a more precise method of calculating the asymmetries. We have outlined in par. 5 the lines which such an attempt might take. Such an undertaking would be worthwhile only when experimental results of much higher accuracy than most of ours become available. On the experimental side, therefore, one of the requirements is to construct equipment whose countingrate is very much higher, but whose angular resolution remains high. This can be done by developing telescopes of larger counting area. With such equipment it should be possible to establish whether the variations mentioned under (d) are real. This may be of considerable importance, because, as suggested in par. 20, such variations may be produced by short term changes in intensity hitherto unobserved. If such changes occur ~~and~~

and are of primary origin, they would be of fundamental importance.

Results of high statistical accuracy for large zenith angles when taken in conjunction with a more precise theory should establish whether a primary asymmetry exists in high latitudes.

On the experimental side also it is clearly desirable to perform differential measurements, preferably using mesons of one sign only. This could be done either by using a system similar to that of Groetzinger and McClure, or by using the delayed coincidence technique discussed in Part I, section B. If decay electrons were observed from mesons which come to rest in a Pb absorber, only positives would be recorded.

It may also be of interest to apply the nuclear emulsion technique to this problem. As mentioned in Part I, par. 17, Creamer has investigated the zenith angle distribution of slow mesons by this method. It would be of interest to know whether there is a negative asymmetry of slow mesons (par. 33). A momentum range not easily accessible by other techniques could be studied. Photographic plates have been used in an investigation of the low latitude E-W effect by Moucharrafieh and Rebaud (26).

It would also be of interest to know whether there is an asymmetry of the electronic component (par. 33). An experiment could be performed using a soft component telescope of the type which we are using in Hobart to investigate the zenith angle distribution of the soft component (Part I, par.24).

It is necessary to establish whether the high latitude E-W effect varies with latitude. We have found that the asymmetry varies from hour to hour (e) and we have supposed (par. 30) that these variations are likely to be associated with variations of H, although the absence of magnetic measurements gave us no evidence to support this notion. If variations with H do occur at one station, this would seem to be incompatible with the absence of a latitude effect. Measurements are proceeding at Hobart with the equipment returned from Macquarie Island and these should allow us to decide whether there is a latitude effect.

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