STUDIES OF THE SIDEREAL DAILY VARIATION OF COSMIC RAY INTENSITY WITH PARTICULAR REFERENCE TO OBSERVATIONS UNDERGROUND

by

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PREFACE

This investigation has formed part of a joint programme of experimental studies of cosmic radiation carried out by the Australian National Antarctic Research Expeditions (ANARE) and the University of Tasmania, coordinated under the general supervision of Dr.A.G.Fenton of the Department of Physics.

I have been a member of the research staff of the Antarctic Division for a number of years. In 1951 I was employed as a cosmic ray physicist at Macquarie Island, being responsible in the main for the operation of a pair of crossed counter telescopes, the data from which were being used by K.B.Fenton in his investigation of the high-latitude East-West asymmetry at Hobart and Macquarie Island.

In 1956 I was employed as cosmic ray physicist at Mawson and initiated experiments concerned with time variations of the East-West asymmetry, using an inclined telescope that had been installed by N.R.Parsons in the previous year.

From 1957 onwards I assisted Dr.Parsons in the training of physicists, who were to operate cosmic ray equipment at the ANARE bases, and in the joint supervision with the University of the ANARE cosmic ray projects. I succeeded Dr.Parsons in that work after he left the Antarctic Division in 1964.

The ANARE projects were incorporated in a wider programme, whose object was to further our understanding of time-variations in the primary flux, chiefly through comparisons of observations (a) from different geomagnetic locations, and (b) from detectors which responded to different energy bands in the primary spectrum. While participating in these research activities of the Hobart Cosmic Ray Group, I became particularly interested in the underground experiments at Tunnel Hill, near Hobart. The underground experiments were initiated by Dr.A.G.Fenton and in July 1957 the two vertical telescopes were installed by R.B. Taylor. From the first six months' data, Taylor made the preliminary findings concerning the atmospheric effect and some of the time-variations. I have operated the equipment subsequently, have been responsible for the analysis of the barometric effect reported in paper 3 (Appendix III) and have supervised the processing of the data.

I noticed an apparent sidereal effect in the data for 1958, the first complete calendar year of operations underground. I have been essentially responsible for the conception, planning and installation of the directional experiments underground which followed and for the development of relevant techniques of analysis, to be described in the thesis.

With the assistance of I.B.McNaughton, two cubical telescopes were erected in 1960, one being inclined 45° south and the other 30° north of the zenith, for the purpose of studying the latitude dependence of response to the

evident solar and sidereal anisotropies. In 1964 the north-pointing cubical telescope was converted to a narrow-angle telescope, inclined 70° north of the zenith, to obtain observations of the phase of the sidereal effect from a detector situated in the southern hemisphere, but which scanned asymptotically in the northern hemisphere.

The material presented in this thesis is my own, except where due reference is made in the text.

I am grateful to Dr.P.G.Law who, as Director of ANARE when the investigation commenced, kindly allowed me to continue with it. I also wish to thank Professor G.R.A.Ellis, head of the Department of Physics at the University, for his keen interest and support.

I am greatly indebted to my supervisor, Dr.A.G.Fenton, for encouraging the investigation from its inception, for readily providing the experimental facilities and for many critical discussions.

It is a pleasure to acknowledge the cooperation of other past and present members of the Hobart Cosmic Ray Group, notably Drs.K.B.Fenton, N.R.Parsons, K.G.McCracken and P.J.Edwards. In many instances the instrumental and analytical techniques used here have developed out of their work or from discussions with one or other of them. I also wish to thank another member of the group, Mr.J.E.Humble,

for consultations and for his generous help, particularly with computer programmes, and, more recently Mr.A.Vrana for valuable assistance with electronic techniques.

More than nine complete years of data have now accumulated from the vertical telescopes underground and on only 3.8% of days have there been less than 20 hours of useful data. The low rate of loss of data is in part due to the uniformly high standard of performance of the Geiger counters produced by Mr.M.G. Mason and his assistant Mr.M.P.Cronly. Their work is greatly appreciated.

Efficient data-processing over many years has been due to the careful work of the members of the cosmic ray computing staff, led by Mrs.P.S.James and Mrs.F.R.Chappell and I wish to thank them for the services they have provided.

Finally I am grateful to Mrs.B.I.H.Scott for typing this thesis.

R.M.JACKLYN

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SUMMARY OF PROCEDURES AND RESULTS

CHAPTER 2

A theoretical treatment is given of the anti-sidereal technique, by which spurious annual sidereal components, generated by seasonal and secular modulation of solar components of the daily variation, may be detected. It is also shown that significant anti-sidereal effects may be generated by modulation of a genuine sidereal component.

CHAPTER 3

Experimental studies of the apparent sidereal daily variation at sea level are reviewed. There is great dispersion in the reported values of phase and amplitude of the deduced sidereal component and the causes of this are discussed. The uniform effect observed from year to year with the vertical G-M telescopes underground is compared with the variability of the effect observed with the neutron monitor at Mt.Wellington, the vertical G-M telescope at sea level, Hobart and the neutron monitor at Mawson.

CHAPTER 4

The sensitivity to the incoming radiation is worked out in detail for one of the underground vertical semi-cubical telescopes at Hobart, taking into account such factors as the geometric sensitivity, the zenith angle dependence of intensity and the distribution of material absorber within the cone of viewing.

The average material absorber is now estimated to be approximately 36 m.w.e. and mean cut-off energy for mesons at the production level is estimated to be approximately 10.6 GEV.

CHAPTER 5

The main body of evidence for the existence of a sidereal anisotropy in the primary radiation is presented. The evidence, relating essentially to observations at several latitudes of viewing underground, derives from

(i) the annual averaged daily variations in solar, sidereal and anti-sidereal time, with particular reference to the long-term trends observed with the vertical telescopes at Hobart;

(ii) phase anomalies;

(iii) the 12-hour difference in the time of the sidereal diurnal maximum that is observed when viewing asymptotically in different hemispheres;

(iv) the observed relation between the diurnal and semi-diurnal components of the sidereal daily variation.

It is concluded that spurious contributions, if any, constitute only a small part of the main effect, which is attributed to a two-way sidereal anisotropy. As observed at the earth, the intensity maxima in opposite directions would be unequal, the greater maximum being from the southward direction.

There is evidence for a gradual decrease of amplitude of the sidereal diurnal maximum over the period of observation with the vertical telescopes at Hobart, and for semi-annual modulation which became rapidly attenuated after 1958.

CHAPTER 6

1. The Sidereal Anisotropy

A simple empirical model for the anisotropy is described, consisting of

(a) a two-way component with equal intensity maxima in opposite directions along an axis, yielding characteristic diurnal and semi-diurnal variations of intensity in the rotating terrestrial frame of reference; and

(b) a superimposed "streaming" component, the direction of the intensity maximum being parallel with the axis of the two-way component.

The asymptotic constants of response for a sidereal anisotropy of this type are worked out in detail in relation to the various underground detectors, for two values of γ, the index of the variation spectrum, and for a range of values of R_C, the primary threshold rigidity for observation of the anisotropy.

2. The Solar Anisotropy

On the basis of the prevailing empirical model for the solar anisotropy, it is shown that the seasonal characteristics of the daily variation of response at the earth would be semi-annual and therefore would not generate a spurious sidereal effect.

Experimental estimates are made of the upper limiting primary rigidity R_u of the solar anisotropy, from comparisons of response from underground, vertical telescopes and high-latitude neutron monitors. It is concluded provisionally that the value of R_u in 1958 was approximately 95 GV and that between 1958 and 1962 its value decreased by approximately 30 GV.

CHAPTER 7

On the basis of the model proposed in Chapter 6, a method is described of determining the declination $\underline{d}$ of the axis of the sidereal anisotropy, as specified in interplanetary space beyond the earth's field region. The sidereal diurnal and semi-diurnal variations observed with independent detectors scanning two widely separated latitude bands are employed in the determination. To test the validity of the result, observations from a detector scanning a third latitude band are used.

Using observations from the underground vertical detectors at Hobart and Budapest, estimates were obtained of the declination and right ascension specifying the southward direction of the anisotropy. These estimates were found to conform closely with the response from the third detector, an underground cubical telescope directed 30° north of the zenith at Hobart, and to be only weakly dependent on values assigned to γ, in the range 0 to -2, and on values assigned to R_C, in the range from 50 GV to at least 200 GV.

The coordinates of the axis of the anisotropy are provisionally estimated to be (RA = 0640 ± 0115, Dec. = $-38^{o} \pm 7^{o}$) in the southward direction and (RA = 1840 ± 0115, Dec. = $+38^{o} \pm 7^{o}$) in the northward direction. The direction of the axis of the anisotropy beyond the earth's field would therefore appear to be approximately aligned with the direction of the local galactic magnetic field, as determined by optical and radio methods.

It is concluded that the character of the underground response associated with the proposed two-way component is consistent with an anisotropy due to the propagation of excess fluxes of primaries with steep helices along the galactic magnetic field, It is submitted in that case, that on the annual average the interplanetary medium does not introduce serious distortion of direction, above a cut-off rigidity for observation which it presumably imposes.

CHAPTER 8

The method of determining the amplitudes of the free-space intensity maxima, due separately to the two-way component and the "streaming" component, is described. While counting rates are not high enough at present to allow significant estimates of the separate maxima to be made, it is shown that, within the ranges of values assumed for γ and R_C, the intensity maxima due to the two-way component are unlikely to have exceeded 0.6%, while the maximum due to the "streaming" component should not have exceeded 0.2%.

CHAPTER 1

INTRODUCTION

1.1 Introductory Remarks

There is now considerable evidence that the charged primary cosmic radiation to which detectors at the earth respond is essentially isotropic in free space, at least in the seven decades of rigidity from 10^{10} v to 10^{17} v. In fact, many careful investigations over some thirty years have failed to produce convincing evidence for even a small excess of charged particle flux from any fixed galactic direction. More particularly, if the amplitude of the observed anisotropy is specified as

$$A = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} ,$$

there would appear to be zero anisotropy, with a margin of uncertainty of amplitude varying from less than 0.2% at the lowest primary rigidities to about 3% at the highest rigidities. Nevertheless, small anisotropies have been predicted, of special interest being those that should be associated with the propagation of cosmic rays through the galaxy in the presence of large-scale magnetic fields.

In the first part of this thesis, up to the end of Chapter 5, evidence is given that there is indeed a small sidereal anisotropy, such as would be expected to be associated specifically with a relatively uniform local galactic magnetic field (see section 2 below). The evidence derives from daily variations of intensity, obtained in the main from meson telescopes located at

depths of approximately 40 m.w.e. underground, where it is believed most of the response would be due to primary protons in the rigidity range 5×10^{10} v - 10^{12} v. The observations differ in a number of respects from those obtained at ground level. Of particular note is the fact that the amplitude of the solar daily variation is much smaller, and that the characteristics of the apparent sidereal effect reproduce themselves remarkably well from year to year. The initial problem is to determine to what extent the observed sidereal effect is due to a sidereal anisotropy, extracting, if necessary, spurious contributions that may have been generated by systematic modulation of solar components of the daily variation. If, as a result, there is significant evidence for a genuine sidereal component, it should be possible to ascertain the general character of the anisotropy, provided that observations can be obtained from several well-separated latitudes of viewing, preferably spanning both hemispheres.

The remaining chapters lead to estimates of the mean direction of the anisotropy in free space beyond the earth's field region, using assumed values for the index of the variation spectrum and for the expected cut-off rigidity due to the interplanetary magnetic field. While the time of maximum of the observed sidereal effect, after removal of any spurious sidereal components, gives some indication of the right ascension, the declination is not approximated by the latitude of maximum amplitude that might be obtained from a latitude survey, being in fact, independent of it. The determination of declination has to be made in three stages. Using observations from a number of detectors, scanning different strips of latitude, transformations are effected from the observed sidereal daily variations to the corresponding daily variations in free space, using the technique of the asymptotic cone of response.

Independently of these calculations, a model that is appropriate to the type of anisotropy is formulated, yielding a free-space sidereal daily variation and an average intensity term. The amplitude of the harmonics of the daily variation term depends in part on the latitude of observation and on the declination of the anisotropy. Using the expressions for the harmonics, a value of the declination can be so determined as to give a best fit agreement between the free-space amplitudes of the model and the amplitudes derived from the observations.

It will be shown that the estimated directions of the intensity maxima in free-space, when taken together with the type of two-way anisotropy that is indicated by the free-space harmonics, suggest that the primary cosmic radiation may be propagating along a rather uniform local galactic magnetic field.

1.2 Isotropy and Anisotropy

It appears that the overall high degree of isotropy of cosmic radiation must be due to the randomizing of the directions of motion of the charged primaries as they travel from their region or regions of origin to the solar system, rectilinear propagation being disallowed in the light of the evidence that large-scale magnetic fields of 10^{-5} to 10^{-6} gauss permeate the interstellar medium in the galactic disc. Optical and radio measurements indicate that the galactic magnetic field in the vicinity of the solar system lies approximately in the direction (to within 20°) of the local spiral arm, towards $\ell'' = 70°$ or 250° [optical polarization of starlight in dust clouds (e.g. Hiltner, 1951; Hall, 1958); Zeeman measurements relating to dense neutral hydrogen clouds (e.g.

Davies, Shuter, Slater and Wild, 1964); linear polarization of radio emission from nearby "synchrotron" regions (e.g. Mathewson and Milne, 1965; Mathewson, Broton and Cole, 1966); Faraday rotation in HII regions, of polarized emission from radio sources (e.g. Morris and Berge, 1964; Gardner and Davies, 1966)] and that it is a well-ordered field on the average, with typical local deviations of direction of about 0.1 radians. It is thought that the mean distance to the "synchrotron" region is $\sim$ 150 parsecs, while radio and optical data in general refer to regions within 3 kiloparsecs of the sun. That interstellar magnetic fields probably extend throughout the galactic disc had been suggested earlier from the requirements for storage of cosmic radiation in the galaxy (Fermi 1949, 1954) and from the need to explain the dynamical stability of the spiral arms, the kinetic pressure of the diffused matter being insufficient to counterbalance the gravitational pressure exerted on it (Chandrasekhar and Fermi, 1953).

Although the most recent measurements of linear polarization of synchrotron emission and of Faraday rotation give further evidence of a uniform magnetic field in the local spiral arm, it has been argued that the presence of random as well as ordered fields would be compatible with the type of uniformity of direction that has been observed as an average of contributions distributed along the arm (Ginzburg and Syrovatskii, 1964). It appears that the scale of a disordered segment of field would be so large in relation to the gyroradius of the average particle, however, that in the absence of small scale irregularities the motions of particles would be regarded as being tied to magnetic field lines in the highly conducting interstellar medium.

At the low end of the range of primary energies, the scattering and trapping of particles in the interplanetary medium would be expected to cause galactic anisotropies to be smeared out and it will be of importance to enquire here as to what the upper limiting energy for such an effect would be. At higher energies it may be inferred from the considerations given above that the charged primaries in the vicinity of the solar system spiral along a relatively uniform magnetic field yet exhibit a high degree of randomization of pitch angles, a situation which must prevail over distances of the order of a parsec if it applies over the range of particle rigidities up to 10^{17} v.

By what process isotropy is brought about at these higher energies is not clear, yet it is evidently a characteristic of cosmic radiation on the galactic scale. Conjectured mechanisms range from the propagation of small-scale irregularities along the spiral arm field, or reflections and diffusion along a uniform field containing randomly distributed constrictions, to the mixing of cosmic rays in the weak random fields of the galactic halo in association with a free interchange of particles between the halo and the galactic disc. The choice of a suitable mechanism is governed by a number of inter-dependent factors. Notably, if, as is widely believed, all but the most energetic cosmic rays originate in the galactic disc, they must be allowed to diffuse out of the plane of the galaxy in a time of the order of 10^6 years (Parker, 1965) and yet not exhibit pronounced anisotropy due to streaming. Consistent with this requirement, the cause of isotropy seems to depend on the distribution of sources, on the configuration of magnetic fields in the disc and the halo and on the dynamical properties of the medium (including cosmic

radiation). In this last respect, it has been pointed out (e.g. Parker, 1965) that on the galactic scale cosmic radiation is a hot collisionless gas whose pressure ($\sim 10^{-12}$ dynes/cm^2) is comparable to that of the magnetic field and considerably greater than that of interstellar hydrogen ($\sim 10^{-11}$ dynes/cm^2) and that is must therefore contribute very largely to the dynamical stability of the medium. By the same token, cosmic ray pressure, as well as motions of the interstellar gas, can produce local deformations in the field which may lead to dynamical instability, disordering of the field and the mixing of particle trajectories. It has also been suggested that plasma instabilities may occur in the cosmic ray gas, associated with excess pressure normal or parallel to the magnetic field, and that these could assist in the achievement of isotropy.

It is evident that the observed cosmic ray isotropy does not in itself greatly restrict conjecture as to the configuration of magnetic fields in the galaxy and the distributions of sources. On the other hand, it is known that rather definite information may be given by small anisotropies, notably those associated with particular configurations of magnetic fields. For example, in the model for propagation of cosmic rays put forward by Ginzburg and Syrovatskii, who favour disordered fields in the galactic arms, cosmic rays diffuse outwards from the central regions and a small excess of intensity should be observed in the direction of the galactic centre. On the other hand, if the magnetic fields are essentially uniform, an intensity maximum along the axis of the local field due to net streaming would be expected, if, for instance, diffusion out of the galaxy along the arms were important or if particles were diffusing away from a nearby source. Of particular interest would be the intensity maxima

connected with the reflection from regions of strong field and the leakage, through these regions, of particles whose motions were essentially adiabatic (although they may be subject to some degree of scattering by small-scale irregularities). In these situations the distribution of pitch angles would not be entirely random and a balance of factors would decide whether intensity maxima occurred along the axis of the field or at right angles to it. A possibility of great importance here is the two-way anisotropy that may occur if, as the result of the trapping between constrictions in the magnetic field, there are excess fluxes of particles with steep helices (large pitches) in both directions along the axis of the field. Leverett Davis (1954) has given a treatment of this possibility in connection with statistical acceleration processes whereby particles are reflected from moving inhomogeneities or in regions of time-varying field. The two-way anisotropy produces a terrestrial sidereal daily variation consisting essentially of a diurnal component and a semi-diurnal component whose characteristics exhibit a well-defined dependence on the latitude of observation.

It should be mentioned that an effect additional to simple streaming (see below) occurs if cosmic radiation which is isotropic in a region of strong field moves adiabatically into a region of weaker field. It becomes anisotropic in the moving frame of reference, there being an excess flux of particles with steep helices in the direction of motion. It has been noted that this is likely to give rise to first and second harmonics in the sidereal daily variation (L.Davis, private communication). Thus it would appear that a

two-way anisotropy may be connected with streaming of previously isotropic radiation from both directions into a region of weaker field in which the observer is located.

In simple streaming (Compton and Getting, 1935) which originates outside the solar system, the latter moves with uniform velocity relative to some frame of reference in which the cosmic radiation is isotropic - it may be the frame of reference of neighbouring stars, that of the extra-galactic medium or that which moves with the velocity of diffusion of cosmic radiation. Ignoring local modifying influences, the resulting terrestrial effect that is of interest here is a sidereal diurnal variation, corresponding to enhanced intensity in the upstream direction of viewing and reduced intensity in the opposite direction, the amplitude being proportional to the velocity of streaming. This is partly due to the apparent steepening of helices when viewing upstream and the apparent flattening of helices when viewing downstream. It seems likely that a net effect would be observed, noting particularly that in the event of streaming motions along the spiral arm there may be contributions from both directions. Thus the observed intensity anisotropy may be an unreliable indicator of individual streaming velocities.

It is evident that the two-way anisotropy is the one which specifically indicates that particles are being propagated along a relatively well-ordered magnetic field and that any indications of associated streaming along the field would relate to a balance of contributions from opposite directions.

Only a small fraction of the primary protons thought to be responsible for the mesons observed at a depth of 40 m.w.e. would have gyroradii in excess of hundreds of astronomical units (a.u.) in magnetic fields of 10^{-5} to 10^{-6} gauss. Therefore, if a type of anisotropy is found that is indicative of a relatively uniform galactic field, it may also enable the direction of the axis of the field in the immediate vicinity of the solar system to be estimated, provided that distortions of the effect that would occur within the solar system are not too serious or can be taken adequately into account.

An anisotropy of a different character should also be mentioned. It has been suggested that mesons produced by high-energy (BEV) γ-radiation may have been responsible for the large but variable intensity spike observed in the approximate direction of W-ORIONIS by Sekido et alia, using very narrow angle alt-azimuth detectors, horizontally directed (e.g. Murahama,1960; Sekido et alia, 1963).

With their high counting rates, the wide-angle meson detectors employed at moderate depths underground are designed specifically to detect very small anisotropies that give rise to a world-wide pattern of response. While there may be an extremely inefficient response to point sources of γ-radiation, the observations to be described here give no indication of it. The possibility would have to be considered more seriously, however, if a genuine sidereal component was observed that was not accompanied by the characteristic effects due to the deflections of primary charged particles in the terrestrial magnetic field.

Table 1.1 lists possible sources of anisotropy in the charged primary cosmic radiation that have been suggested in the literature, together with the estimated directions of maximum intensity.

1.3 The detection of a sidereal anisotropy

The methods that are employed to identify a sidereal anisotropy are conditinned to some extent by the type of anisotropy that is expected to be observed. This is reflected in the present investigation by a change in approach to the problem when it was realised that the observed sidereal effect was evidently conforming, not with a single direction of maximum intensity in free space as would be associated with simple streaming, but with intensity maxima in opposite directions such as might derive from trapping in regions of magnetic field that were of large scale by comparison with the average gyroradius.

At first, the investigations to be described here were concerned with observations at Hobart of the apparent sidereal effect in the vertical direction underground. The main difficulty of interpretation of the effect is caused by the presence of a solar daily variation of cosmic ray intensity. Some of the observed meson flux responds to solar modulation of the primaries and it has become clear in recent years that this accounts in part for the solar daily variation (dv), there being additional contributions of local (e.g. atmospheric) origin. At depths of about 40 m.w.e. underground the amplitudes of the observed solar and apparent sidereal dvs are roughly equal, about 0.1% or less, while at ground level the solar dv is usually considerably the greater of the two, with typical amplitudes of about 0.3%. Changes in the solar daily variation within a year, particularly seasonal modulation, may produce a residual spurious sidereal

TABLE 1.1

Predicted sources of sidereal anisotropy in the charged cosmic radiation at moderate primary energies. Galactic coordinates refer to the new system.

Source	Reference	First harmonic Direction of maximum				Second harmonic
		Celestial RA (hr)	Dec (deg)	Galactic Lat (deg)	Long (deg)	
Diffusion outwards from central regions of Galaxy	Ginzburg and Syrovatskii (1964)	1800	−30	0	0	Absent
Streaming due to motion of solar system relative to neighbouring stars	Hogg (1949) Parker (1964)	1800	+30	25	55	Absent
Streaming due to motion of solar system relative to extra-galactic medium	Compton and Getting (1935)	2100	+50	0	90	Absent
Propagation along Spiral Arm field	Davis (1954)					
(a) Simple streaming		0830	−35	0	250	Absent
		2030	+35	0	70	
(b) Excess of particles with steep helices		0830	−35	0	250	in phase
		2030	+35	0	70	
(c) Excess of particles with flat helices		0830	+35	30	190	6 hours out of phase
		2030	−35	−30	10	
(d) Inhomogeneities of cosmic ray energy density normal to direction of field		1230	+30	+90		Absent
		0030	−30	−90		

effect when the daily variation is arranged in sidereal time and averaged over a complete year. However, the smaller the average amplitude of the solar dv, the smaller this effect is likely to be. Thus it is important to note that unless the solar dv at moderate depths underground comprises rather large contributions that are out of phase, spurious sidereal effects should be a factor of about 3 smaller than they are at ground level for the same percentage modulation.

Close attention is given to the problem of seasonal modulation of the solar daily variation and the well-known anti-sidereal technique for calculating the spurious sidereal effects due to seasonal variation of amplitude of a solar diurnal variation (Farley and Storey, 1954) is extended to include seasonal and secular changes of various kinds. The application of the anti-sidereal technique to secular and seasonal modulation of a sidereal diurnal variation is developed and it is found that semi-annual changes in particular could give rise to large anti-sidereal effects. It is shown that the distinctive seasonal changes observed in 1958 could be explained rather better on the basis of the existence of a sidereal component exhibiting a semi-annual variation than by attributing seasonal modulation to a solar component.

Seasonal modulation is difficult to interpret when the daily variation contains solar and sidereal components of comparable magnitude and it will be shown that in seeking an explanation it is desirable to examine trends in the seasonal effect over a number of years.

Very significant evidence for the existence of a genuine sidereal component is obtained if, as might happen when solar activity decreases, the amplitude of the solar component becomes so small that the sidereal component dominates the daily variation and the resultant time of maximum becomes orientated in sidereal time. The phenomenon is referred to here as a phase anomaly. The appearance of such an effect in the daily variation underground during 1961 is examined closely. It is important that the sidereal time of maximum given by the phase anomaly should be compatible with the phase of the annual apparent sidereal effect observed over a number of years and it is also desirable to have evidence that the occurrence of the anomaly synchronises with secular trends of amplitude of the individual solar and sidereal components.

The indications from the underground telescopes at Hobart for a persistent sidereal component with an averaged time of maximum between 0500 and 0700 local sidereal time (LST) contrasted with observations from the northern hemisphere that had accumulated over many years from detectors at ground level. Although the observed times of maximum varied considerably, both from year to year and from place to place, there was a tendency, particularly from the ionization chambers, for the annual sidereal diurnal maximum in the northern hemisphere to occur somewhere between 1800 and 2100 local sidereal time (LST). Relatively little evidence was available from the southern hemisphere over the same period, certainly not enough to substantiate the indications from the ionization chamber at Christchurch (New Zealand) for a much earlier time of maximum at southern latitudes of viewing.

The evidence for a phase difference of approximately 12 hours between the apparent sidereal diurnal maxima in the two hemispheres was to be substantiated by the results from the high counting rate detectors underground at Budapest and London. Two phenomena are notably associated with a 12-hour phase difference of this type:

(1) A spurious sidereal diurnal variation, produced by seasonal modulation of a solar component of the daily variation and appearing as a residual when the data are arranged in sidereal time and averaged over a complete year. The reversal of phase between hemispheres follows from the fact that the time of maximum of the spurious effect is tied to the time of the year when maximum seasonal change occurs.

(2) A two-way sidereal anisotropy.

When it became clear that the essential problem of detection was to distinguish between possibilities (1) and (2) above, experimental studies were commenced that would give more specific evidence of the nature of the sidereal effect, as distinct from existing evidence for a genuine sidereal component in the vertical direction.

A simple two-way anisottopy lends itself quite well to identification. It produces a sidereal daily variation comprising two harmonics. If the anisotropy is due to an excess of primary particles with steep helices, the two harmonics are in phase, whereas in the case of an excess of particles with flat helices the harmonics are six hours out of phase (Table 1.1). In both cases

the harmonics exhibit a distinctive latitude dependence of amplitude. However, it should be noted that these characteristics relate to free-space conditions and as mentioned in section 1.1 it is necessary to employ the technique of the asymptotic cone of response to arrive at the corresponding characteristics that should be observed with underground detectors. The cone of acceptance technique has already been developed by McCracken and others (McCracken, Rao and Shea, 1962; Rao, McCracken and Venkatesan, 1963) for the interpretation of the solar daily variation of neutron intensity at ground level and the treatment is extended here in the somewhat different application to data from meson telescopes underground.

In calculating the response to a sidereal anisotropy, an empirical method is used in an attempt to take some account of the influence of the interplanetary magnetic field. The magnetic field should impose a primary threshold rigidity for observation of the anisotropy, which is assumed for the present to originate beyond the interplanetary field and independently of it. To a first approximation this threshold rigidity is equated with the upper limiting rigidity of response to the solar anisotropy. There is now considerable evidence to show that the extra-terrestrial part of the observed annual solar diurnal variation is due to charged primaries which are constrained by the spiral interplanetary field to stream past the earth with azimuthal velocity of rigid rotation with the sun (Rao, McCracken and Venkatesan, 1963; Parker, 1964). A method is described of determining the upper limiting rigidity of the solar anisotropy by comparison of observations from meson telescopes underground

and high latitude neutron monitors, and average values for some individual years are estimated.

To obtain free-space first and second harmonics of sidereal daily variation as functions of asymptotic latitude of viewing an underground latitude survey is required. This was undertaken during the years 1961 and 1962, with telescopes scanning three different strips of asymptotic latitude near the plane of the meridian at Hobart. The results were combined with the data from vertical telescopes located at approximately the same depth underground at Budapest. The latitude dependence of amplitude of the estimated free-space harmonics gives important evidence for the existence of a two-way anisotropy and provides a means of determining the declination (arbitrarily specified looking southward). The resulting estimated direction (RA and dec.) of the axis of the anisotropy should be appropriate to the observed phase relation between the two harmonics. That is to say, if the harmonics are in phase (steep helices) the direction might be expected to approximate that of the local galactic field, whereas if the harmonics are six hours out of phase (flat helices) the analysis should give a direction normal to the field (Table 1.1).

A further experiment was designed to distinguish specifically between the 12-hour phase difference in the first harmonic that would be connected with a two-way anisotropy and the 12-hour phase difference associated with a spurious sidereal effect produced by seasonal modulation of a solar component. Towards

the end of 1963 a narrow angle telescope was mounted at a sufficiently steep inclination to the north of zenith underground to enable it to scan asymptotically entirely within the northern hemisphere. If the apparent sidereal maximum observed in the vertical direction differed from the maximum observed in the north-pointing direction by 12 hours, a genuine two-way anisotropy would be indicated. On the other hand, if the maxima were in phase, this, combined with the existing evidence for a 12-hour difference in the two hemispheres, would conform with a spurious sidereal effect. A significant result for the first two complete years of observation is presented.

In summary, although at first the object of the investigation was to determine whether or not there was a genuine sidereal component in the daily variation observed underground, it will be shown that this has led to evidence for intensity maxima in both directions along an axis running parallel with the estimated direction of the local galactic magnetic field. While second-order features (e.g. year-to-year changes, seasonal variations, rigidity dependence of amplitude) give indications of a more complex structure, the evidence to be presented here is centred on the fulfilment of the following observational requirements for a sidereal component produced by a simple two-way anisotropy:

1. There should be evidence from the anti-sidereal analysis of annual averages that there is a sidereal component of the daily variation as distinct from spurious effects produced by seasonal modulation of a solar component. It is desirable that the identifying features - time of maximum, nature of harmonics (see 4. below) - should be observed to maintain approximate constancy from year to year.

2. A phase anomaly should be compatible with secular trends of amplitude of the solar and sidereal components and should give a sidereal time of maximum that conforms with the characteristic annual average.

3. The 12-hour phase difference between the sidereal diurnal maxima observed in the northern and southern hemispheres should be demonstrated to be consistent with a two-way anisotropy and not with a spurious sidereal effect.

4. The sidereal component should comprise first and second harmonics that should be approximately in phase or six hours out of phase, depending on the type of two-way anisotropy that is being observed. The corresponding free-space harmonics should exhibit the appropriate latitude dependence of amplitude.

5. The direction of the anisotropy estimated from the free-space harmonics should be appropriate to the type of anisotropy indicated by 4. above.

1.4 Definitions

To clarify some of the terms already used and to introduce the notion of sidereal time and its connection with solar time, the following definitions are appended.

A *sidereal day* is the time interval between successive transits of the observer's meridian by a fixed star. Because of the motion of the earth around the sun, the meridian transit of a star in solar time occurs approximately four minutes earlier on successive days, with the result that there are 366 transits in a year of 365 solar days. It follows that the unit of sidereal time is shorter than solar time by a factor of 365/366. Since it is necessary to

designate some moment in the year when solar and sidereal time are the same, solar time and *local sidereal time* (LST) are synchronised at the September equinox (September 22 or 23). In other words, sidereal time is reckoned from the noon transit (upper meridian passage) of the First Point of Aries, which occurs at the March equinox. Since *right ascension* (RA) is measured eastward of the First Point of Aries, the sidereal time of meridian transit of any fixed point in the sky is also its right ascension.

As far as the present study is concerned, some of these definitions need not be applied with astronomical strictness. For example, although solar and sidereal times should be synchronised at the moment of the equinox, it is sufficient to synchronise them at any time of the day of the equinox, conveniently at GMT zero, since this does not involve more than two minutes of error at any one place. Again, mean solar time is used in calculating the solar daily variation, whereas strictly speaking it is orientated in apparent local solar time - this introduces maximum seasonal errors of timing of approximately $\pm$ 18 minutes.

The maximum of a genuine sidereal daily variation of cosmic ray intensity must occur progressively earlier day by day in solar time, in such a manner that on being averaged in solar time over a complete year the daily variation cancels itself out - provided that, in sidereal time, it is of constant amplitude and phase. Similarly, the maximum of a genuine solar daily variation occurs progressively later in sidereal time on successive days and the daily variation

cancels itself out when averaged in sidereal time over a complete year - again, provided that it is a constant daily variation in solar time. The annual averaging of daily variations with respect to solar and sidereal time is therefore a completely effective method of separating out solar and sidereal components of all degrees of complexity provided that the components are constant.

A daily variation that is arranged in solar time and averaged over a period of less than a year is referred to here as *the daily variation in solar time*, *the total daily variation* or *the daily variation*, and usually relates to a single day or a monthly average. It may contain both solar and sidereal components.

A *sidereal component* of a daily variation is defined as a contribution which is orientated in sidereal time, the maximum having characteristically a fixed right ascension. It should perhaps be added that *orientated* implies that it exists independently of other components of the daily variation.

A *solar component* is defined as a contribution which is orientated in solar time, the identifying maximum having characteristically a fixed direction relative to the earth-sun line.

The annual average of a daily variation arranged in sidereal time is known as *the annual apparent sidereal daily variation*, since, as we shall see, it may not have been entirely produced by a genuine sidereal component.

The annual average of a daily variation arranged in solar time is known only as *the annual mean solar daily variation.* More strictly, it is the annual apparent solar dv, since there may be a spurious contribution due to modulation of a sidereal component. If, as is usually the case at ground level, the solar component of the daily variation is very much larger than any suspected sidereal component, the contamination of the annual average solar dv would be insignificant. The situation is rather different when the two components are of comparable amplitude and there is modulation of the sidereal component.

1.5 The Influence of the Interplanetary Magnetic Field

Proceeding from the evidence that a genuine sidereal component of the daily variation exists it is proposed to obtain a description of the sidereal anisotropy as it manifests itself immediately beyond the earth's field region, to the extent that the observation will allow. To do this, it will be necessary to assume that the observed effect is due to charged primaries, to assume a spectrum of variation and to estimate a cut-off for observation imposed by the interplanetary magnetic field. The resulting free-space characteristics, as they will be called for the present, should in themselves give some indication that the sidereal effect is indeed due to charged primaries. Naturally it is desirable to demonstrate this at an early stage, since it would constitute further evidence that the effect was not spurious and that it was not due to neutral radiation to which there might be a response at the earth. Anisotropies of the latter kind appear to be suggested in very different ways, by the experiments of Sekido et alia (1963) and of Cowan et alia (1965).

A quantitative extrapolation to the effective outer boundary of the interplanetary field will not be attempted in this thesis. It will be shown that this would concern in the main the calculation of asymptotic trajectories for primaries of rigidities greater than $\sim$ 100 GV which arrived at the magnetosphere from directions steeply inclined to the plane of the ecliptic. Of crucial importance here would be a knowledge of the manner in which the field strength changes as one proceeds outward from the plane of the ecliptic (where the value is $\sim 5 \times 10^{-5}$ gauss at the earth's orbit) to distances of the order of astronomical units. It would seem from the lack of observations in these regions that an adequate description of the field in three dimensions cannot be given yet. On the other hand, it is suggested that the underground observations in themselves may give new and significant information as to the properties of the field. We now briefly mention three relevant types of observation connected with the present investigation.

In the first place, a large decrease in amplitude of the solar daily variation underground has been observed to occur during the years following the recent solar maximum. It is believed that this constitutes evidence of a great reduction in the ability of the interplanetary field to trap primaries of high rigidity. Values of the maximum primary rigidity of response, in relation to the solar anisotropy, are estimated from comparisons of the solar daily variations observed above and below ground. These give some indication of the threshold rigidity for observation of a sidereal anisotropy.

The second type of observation that might prove to be of considerable relevance is seasonal modulation of a sidereal daily variation. The Right Ascension of the outward (or inward) direction of the interplanetary field as seen from the earth must rotate through 360° in the course of a year. It also appears, from the Imp - I measurements, that during that time there may be as many as 50 roughly equally spaced reversals of the observed direction of the field (Wilcox and Ness, 1965). Therefore it seems that, except for the imposition of a threshold rigidity for observation of a sidereal anisotropy and a considerable smoothing of amplitude, the influence of the field would be mainly seasonal. If, as will be proposed, strong seasonal modulation of the sidereal daily variation was observed underground at solar maximum, this may have been connected with a tendency for the primaries to mirror in the interplanetary field as they neared the sun. A discussion of this possibility is given in section 5.7.

Support for the view that the interplanetary field may not produce important directional changes other than those of a seasonal character will come from a consideration of annual average free-space characteristics. A corollary to the evidence for a sidereal anisotropy, derived from a presumed charged particle response integrated over all rigidities above a threshold of $\sim$ 100 GV, will be that there appears to have been no large distortion of the directions of maximum intensity between the source region and the magnetosphere.

It will be indicated, then, that distortions of the anisotropy produced by the interplanetary field may be considerably less, over annual averages, than the observed field strength at the earth's orbit might at first suggest.

CHAPTER 2

MODULATION OF COMPONENTS AND THE SPURIOUS SIDEREAL EFFECT

2.1 Modulation of a solar component

Almost any type of modulation of a solar component of the daily variation is liable to give rise to a spurious sidereal effect when the data are arranged in sidereal time and averaged over a complete year. Included are irregular changes, uniform linear changes, cyclic changes with a period of one year (referred to as annual seasonal changes), and cyclic changes with a period of more than one year or an odd sub-multiple of a year. Notable exceptions are the cyclic changes with periods of one half-year or sub-multiples of this. The guiding rule which covers most eventualities is that changes during the year that cause a solar component to be different, at any time, from its value six months previously are liable to give rise to a residual annual apparent sidereal effect. In most cases the residuals would be expected to be very small, but more significant effects may be generated by the annual seasonal changes and by sizeable linear displacements of phase.

Generally speaking, if all seasonal changes are regarded as ultimately deriving from the annual variation of the earth's ecliptic longitude, it is convenient to classify them (a) as being connected with the obliquity of the ecliptic (e.g. annual variation of the declination of the sun or semi-annual variation of the modulus of the declination) or (b) as

not belonging to (a) above (e.g. changes connected with the position of the earth relative to the solar equator, its heliocentric position, or with its distance from the sun, or with its position in the cavity swept out by the solar wind).

Seasonal modulation has been detected or predicted quantitatively in two solar components of the c/r daily variation and in both cases derives from the annual variation of the declination of the sun. In the first instance, there are the annual seasonal changes in the diurnal variation of atmospheric temperature that are of crucial importance in the study of the apparent sidereal daily variation of meson intensity at ground level. As Dorman (1957) has shown, the component of the daily variation due to atmospheric temperature is properly speaking the integrated effect of contributions from all levels of the atmosphere up to the production level for pi-mesons. It is difficult to predict theoretically the effect for a given locality, because it appears to vary markedly from place to place according to local conditions. In those places where it can be calculated, using radiosonde data, it seems that the true effect in the past has been seriously masked by radiation errors in the data (see Dorman, 1965). Again, the nature of the contribution to the daily variation depends greatly on the energy-response of the detector. Consequently it seems not quite clear yet what the characteristics are. The time of maximum (of the induced cosmic ray diurnal variation) once thought to be about midnight has recently been estimated to be near 0600 local

solar time. (Quenby and Thambyahpillai, 1960). The amplitude varies considerably from place to place [e.g. see (a) in the discussion by Kitamura and Kodama (1960) on the difference between Syoba Base and Mawson in relation to the negative temperature effect and (b) the conclusion reached by Bercovitch (1963) that the effect at Deep River was extremely small]. However, it is generally agreed that the diurnal variation is a maximum in summer. The daily variation of neutron intensity is, of course, not subject to temperature effects of this kind. It is hoped to show in later chapters that what contribution there is from atmospheric temperature in the daily variation of meson intensity underground must be relatively small at Hobart.

The other type of seasonal modulation of a solar component concerns the diurnal variation due to a solar anisotropy. The anisotropy is assumed to have a fixed orientation with respect to the E-S line and to be constant in time. The seasonal effect is a frame of reference phenomenon and is essentially a semi-annual variation of amplitude of the diurnal variation of the primaries. Thus, although the magnitude is small, it is inherent in the daily variation from any detector which responds to the anisotropy. The transformation from the frame of reference of the anisotropy to the rotating terrestrial coordinate system is described in paper 1 (Appendix I) and will be referred to again in Section 6B.1. The procedure is relevant in that it demonstrates that the seasonal effect is essentially of semi-annual character, not annual as might be at first supposed, and therefore cannot be responsible for spurious annual sidereal components.

There are two very important indicators of a spurious sidereal effect produced by seasonal modulation of a solar component -

(1) an anti-sidereal effect (not necessarily matching in amplitude). which is described below; and

(2) a difference of 12 hours between the time of maximum of a spurious sidereal effect observed in the northern hemisphere and the time of maximum observed in the southern hemisphere, because of the six months difference in the seasons. Now it so happens that there appears to be a genuine sidereal component of the daily variation that exhibits precisely these two characteristics. This constitutes the essence of the problem of interpretation of observations from the point of view of seasonal modulation. Consequently, as mentioned in the Introduction, it becomes necessary (a) to devise an experiment to distinguish specifically between the two possibilities and (b) to find supporting evidence for this particular type of sidereal component, independently of seasonal modulation.

2.2 Modulation of a sidereal component

Modulation of a sidereal component has not been considered by others, perhaps because there has been no compelling evidence for it and because it may have been thought that if there is a genuine sidereal daily variation it should remain constant over periods of time of the order of a solar cycle. However, to mention one situation that could arise, the interplanetary magnetic field may impose the effective primary cut-off rigidity, R_c, for observation of a sidereal anisotropy by the given detector, and R_c may be greater than R_ℓ the lower limiting rigidity of the anisotropy. It would be reasonable to

expect both seasonal and year-to-year changes in the cut-off to occur and it seems likely that the consequent seasonal changes in the observed sidereal effect would belong to category (a) described in Section 2.1, with a tendency to be semi-annual in character.

It will be necessary to consider modulation of a sidereal component very closely when examining the evidence for the existence of a sidereal anisotropy. It will be shown below that seasonal modulation of a sidereal component can give rise to a very significant anti-sidereal effect. If this possibility is not taken into account, the presence of a significant annual component in anti-sidereal time comes to be regarded as unambiguous evidence that at least part of the observed sidereal daily variation is spurious. It is stressed that this view of the role of the anti-sidereal effect is incorrect.

2.3 The anti-sidereal technique

A theoretical treatment of this topic is given in paper 2 (Jacklyn, 1962), which is included as Appendix II. Only the essential ideas are discussed here.

(a) Seasonal and secular modulation of a solar component

The anti-sidereal technique was originally devised by Farley and Storey (1954) as a means of determining the spurious annual sidereal component produced by an annual variation of amplitude of a solar diurnal variation. We use their case to illustrate a method that applies in much the same manner to other types of modulation.

The diurnal variation is regarded as a carrier wave whose frequency is 365 cycles per year. The annual variation of amplitude becomes amplitude modulation at a frequency of 1 cycle per year. It follows from the theory of modulation of radio waves that, upon averaging over complete cycles of modulation, matching sideband components will appear at frequencies of 366 cycles per year and 364 cycles per year. The former is the sidereal frequency, there being 366 sidereal diurnal cycles in one year. Thus, by arranging the modulated solar diurnal variation in sidereal time, by the day or group of days, and averaging over a complete year, the appropriate sideband components will appear as an apparent sidereal diurnal variation. However, it has none of the characteristics of a genuine sidereal component.

The matching sideband at 364 cycles per year is known as the anti-sidereal component. It is extracted by arranging the data, for individual days, in units of time according to day lengths that are approximately four minutes longer than a solar day and averaging over a complete year. Note that anti-sidereal time, like sidereal time, is synchronised with solar time at the September equinox.

In the case we are considering, the anti-sidereal component is very simply related to its counterpart, the spurious sidereal diurnal variation. Denoting the carrier wave by A sin ϕ, the modulated solar dv is given by

$$i_d = A \{1 + k \cos(\theta - \theta_o)\} \sin \phi \qquad 2.1$$

where k is the maximum fractional change of amplitude, θ is time of year in angular measure, and θ_o is the phase constant of the annual variation. It is given by

$$\theta_o = \frac{2\pi T_o}{12}$$

where T_o is time of the year in months measured from the September equinox. Following the usage in paper 2, where the distinction was found to be convenient in some sections, i_d is replaced by I_d to denote the annual average and ϕ is replaced by Φ, although the two symbols are synonymous.

The annual mean diurnal variations in solar, sidereal and anti-sidereal time are

Solar: $I_d = A \sin \Phi$ 2.2

Sidereal: $I_s = \frac{Ak}{2} \sin(\Phi' - \theta_o)$ 2.3

Anti-sidereal: $I_{as} = \frac{Ak}{2} \sin(\Phi'' + \theta_o)$ 2.4

where Φ, Φ' and Φ'' represent solar, sidereal and anti-sidereal time respectively.

We note that the sideband terms I_s and I_{as} have equal amplitudes and that, within the respective time frameworks, the times of maximum are displaced symmetrically about the time of maximum of the carrier wave. In practice the annual averaging of the data in solar and anti-sidereal time would give I_d and I_{as}, whereas in sidereal time we would have I_s plus, perhaps, a genuine sidereal component. The equations show that I_s can be deduced

from I_{as} and I_d very simply. This allows I_s to be subtracted from the annual mean apparent sidereal d.v., leaving the genuine part as the residual. This is usually done vectorially on a first harmonic dial. Knowledge of I_d and I_{as} also allows us to determine k and θ_o, the constants of seasonal modulation.

In general, annual seasonal changes and secular changes (e.g. uniform displacement of phase) in a solar diurnal variation give rise to sidereal and anti-sidereal sideband components. Semi-annual changes do not. Various cases of interest are examined in paper 2. The figures reproduced here show how the annual characteristics are related, in respect of -

(1) annual linear displacement of phase (Figures 2.1, 2.2 and 2.3); and

(2) annual sinusoidal variation of phase (Figure 2.4).

A treatment of combined annual variation of amplitude and phase is given in paper 4 (Appendix IV) and will be considered in Chapter 5.

(b) Seasonal and secular modulation of a sidereal component

In the context of modulation of a sidereal component the carrier wave is at the sidereal frequency, 366 cycles per year. We are particularly concerned here with modulations which produce sidebands at the anti-sidereal frequency, 364 cycles per year. Simple annual variations cannot do this,

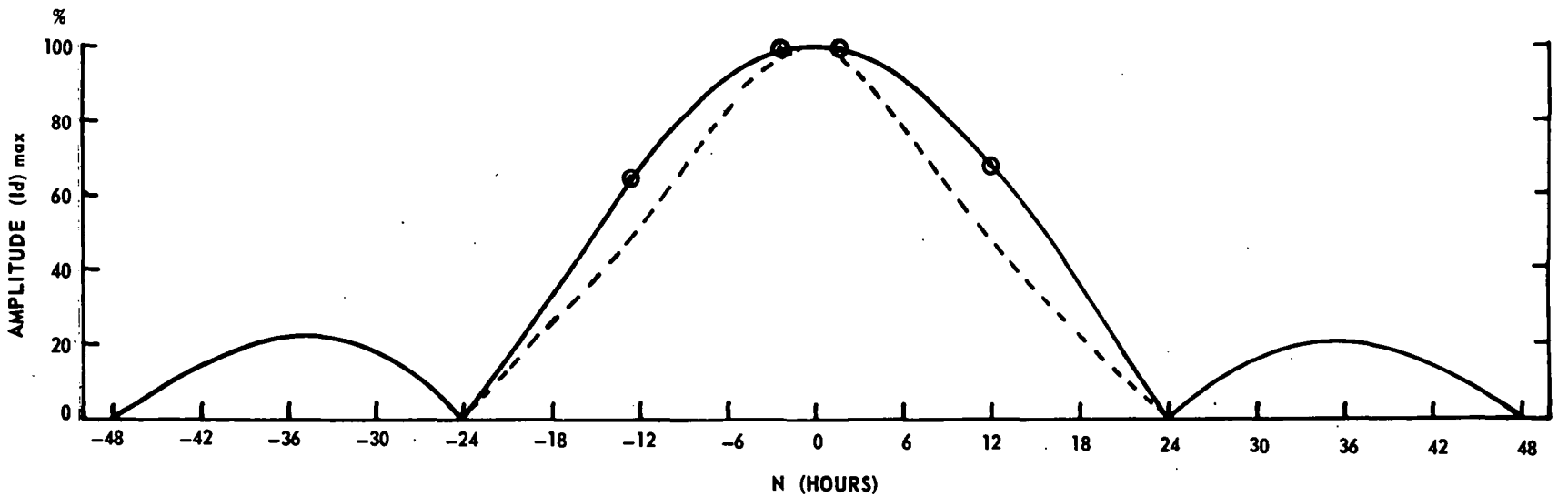


Figure 2.1 The annual mean amplitude of a solar daily variation versus total annual displacement (N) of phase.

Full lines : theoretical relationship
Dashed lines : relationship obtained from the "underground" model
Circled points: values from a sinusoidal model.

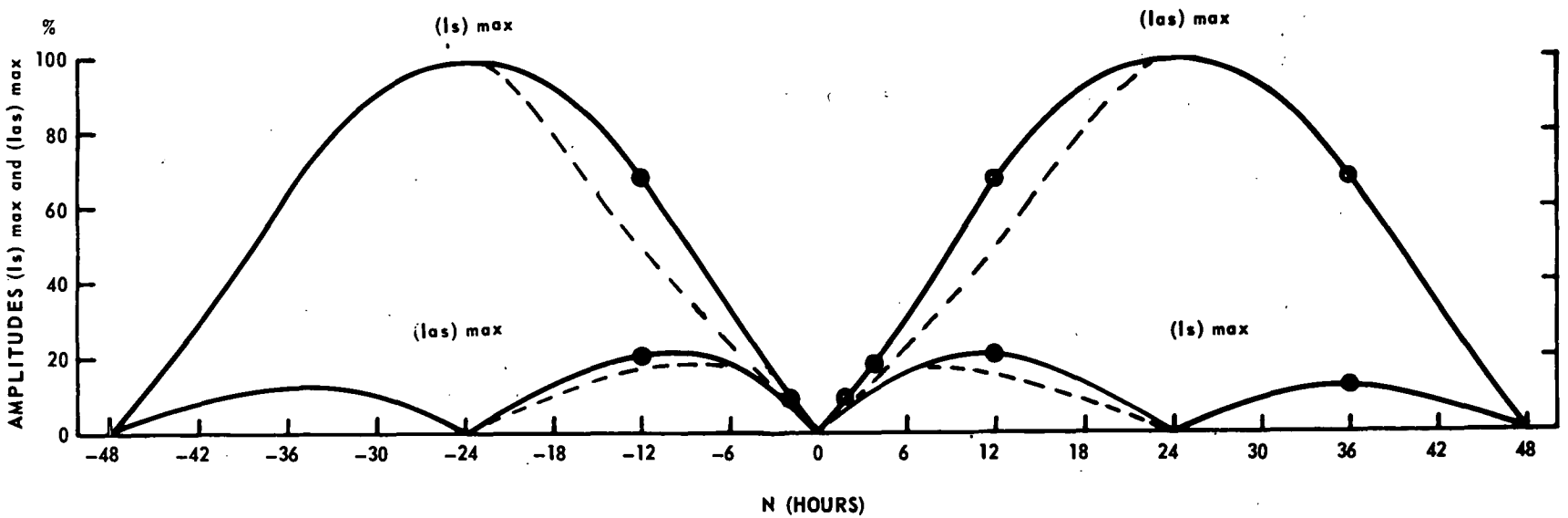


Figure 2.2 The amplitudes (Is) max and (Ias) max, versus total annual displacement N of phase of a solar diurnal variation. (Is) max is the amplitude of the annual mean d.v. in sidereal time and (Ias) max is the corresponding amplitude in anti-sidereal time.

Full lines : Theoretical relationship
Dashed lines : Relationship obtained from the "underground" model
Circled points : Values from a sinusoidal model.

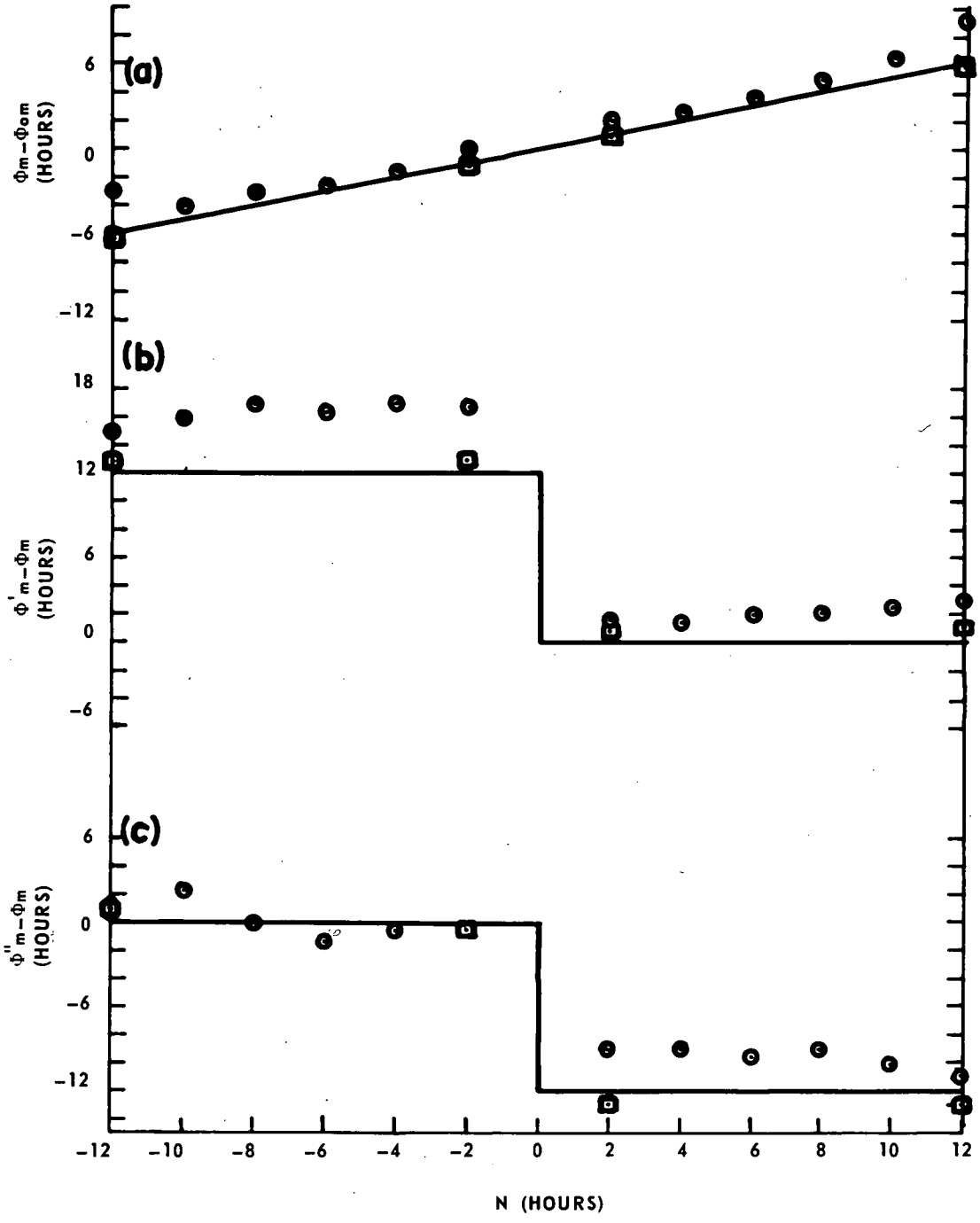


Figure 2.3 The relationships between the annual mean solar (Φm), sidereal ($\Phi' m$) and anti-sidereal ($\Phi'' m$) diurnal maxima for a solar daily variation with linear displacement of phase, versus N, the total annual phase displacement. Φom is the time of maximum of the unmodulated d.v.

Full lines : Theoretical relationships
Squared points : Values from a sinusoidal model
Circled points : Values from the "underground" model.

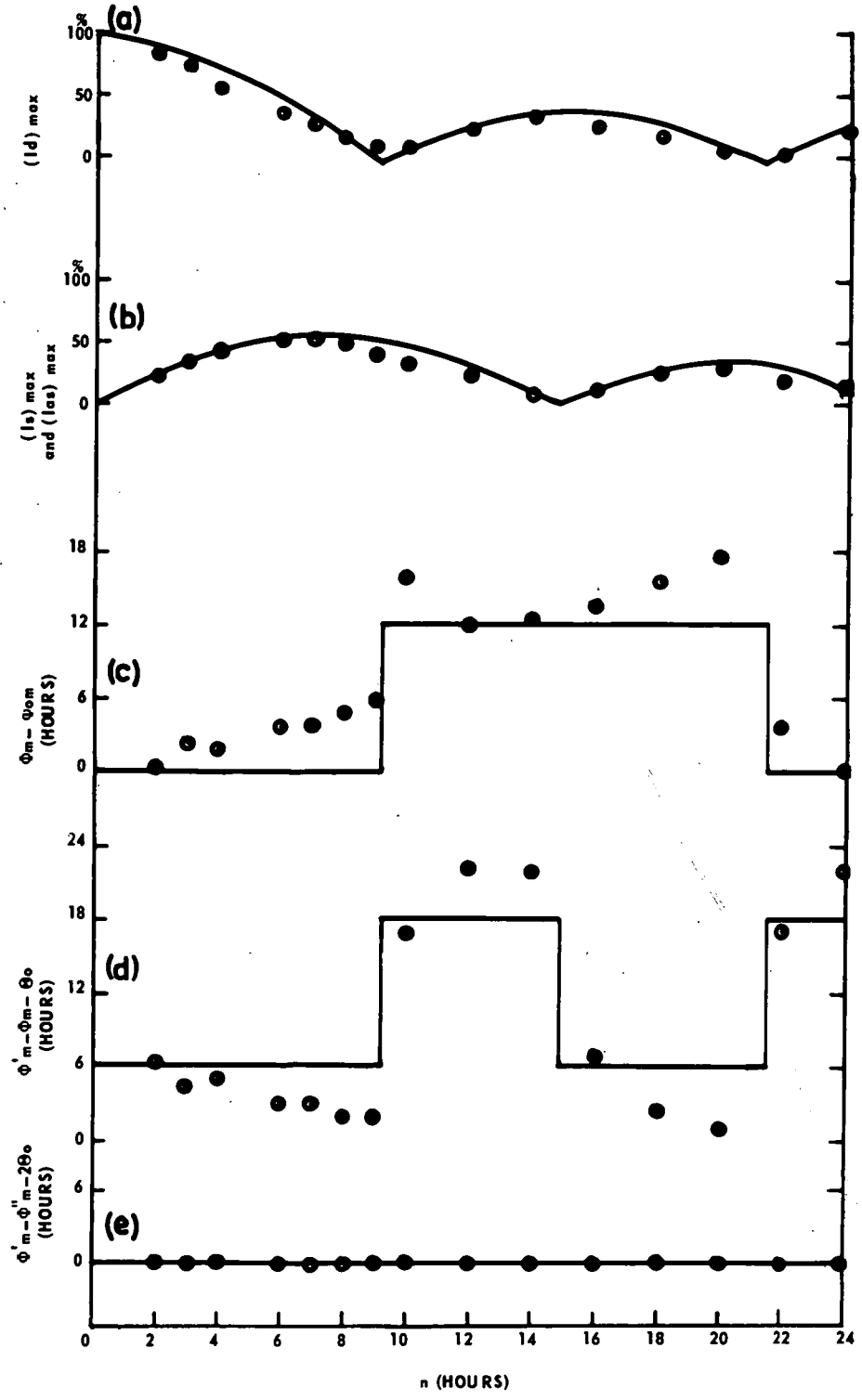


Figure 2.4 The amplitude and phase relationships for the annual mean solar (Id, Φm), sidereal (Is, $\Phi'm$) and anti-sidereal (Ias, $\Phi''m$) diurnal variations produced by a solar d.v. with an annual variation of phase, versus n, the maximum phase displacement. Φ_{om} is the time of maximum of the unmodulated d.v. and Θo is the phase constant determined by the time of year when the maximum displacement of phase occurs.

Full lines : theory
Circled points : values from the "underground" model.

but an obvious possibility is a semi-annual variation of amplitude, the modulating frequency being 2 cycles/year. This produces sidebands at 368 cycles/year and 364 cycles/year. It is perhaps the most important case and is dealt with on page 18 of paper 2. In brief, if the carrier is denoted by $A \sin \psi'$, the modulated sidereal d.v. is given by

$$i_s = A \{1 + k \cos(2\theta - \theta_o) \} \sin \psi' \qquad 2.2$$

where k, θ and θ_o have the appropriate meanings. The annual mean diurnal variations in sidereal and anti-sidereal time are

$$\text{Sidereal:} \qquad I_s = A \sin \psi' \qquad 2.6$$

$$\text{Anti-sidereal:} \qquad I_{as} = \frac{Ak}{2} \sin(\psi' + \theta_o) \qquad 2.7$$

A relatively large anti-sidereal effect may be generated in this manner. It is not at all easily distinguished from an effect produced by annual variation of amplitude of a solar d.v. if the total daily variation contains both solar and sidereal components. In either situation, if the solar d.v. is the greater of the two components, the end points of the monthly mean diurnal vectors trace out an ellipse in an anti-clockwise manner. The critical point of difference is that a spurious annual sidereal component is generated in the case of modulation of the solar component.

Other types of modulation of a sidereal component are considered in paper 2, as for example -

(a) linear displacement of phase of a sidereal diurnal variation (Figures 2.5 and 2.6),

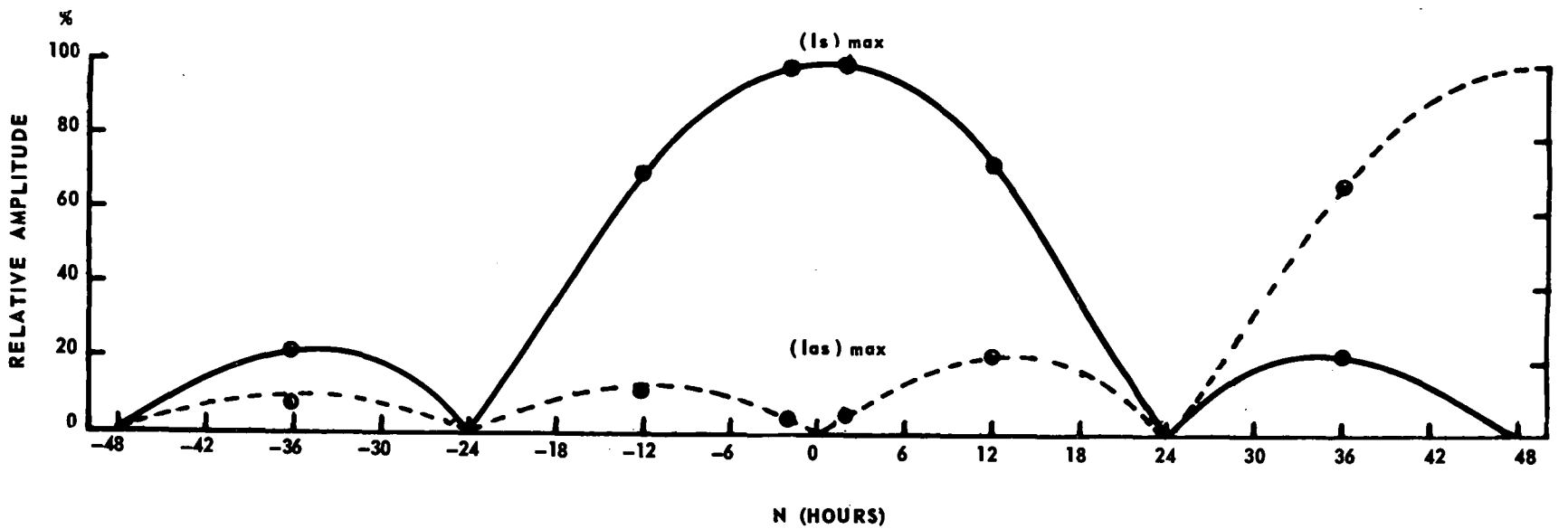


Figure 2.5 The amplitudes of the annual mean sidereal (Is) and anti-sidereal (Ias) diurnal variations associated with a linear displacement of phase of a sidereal diurnal variation, versus N, the total annual phase displacement. The amplitudes are measured relative to the amplitude of the unmodulated sidereal d.v.

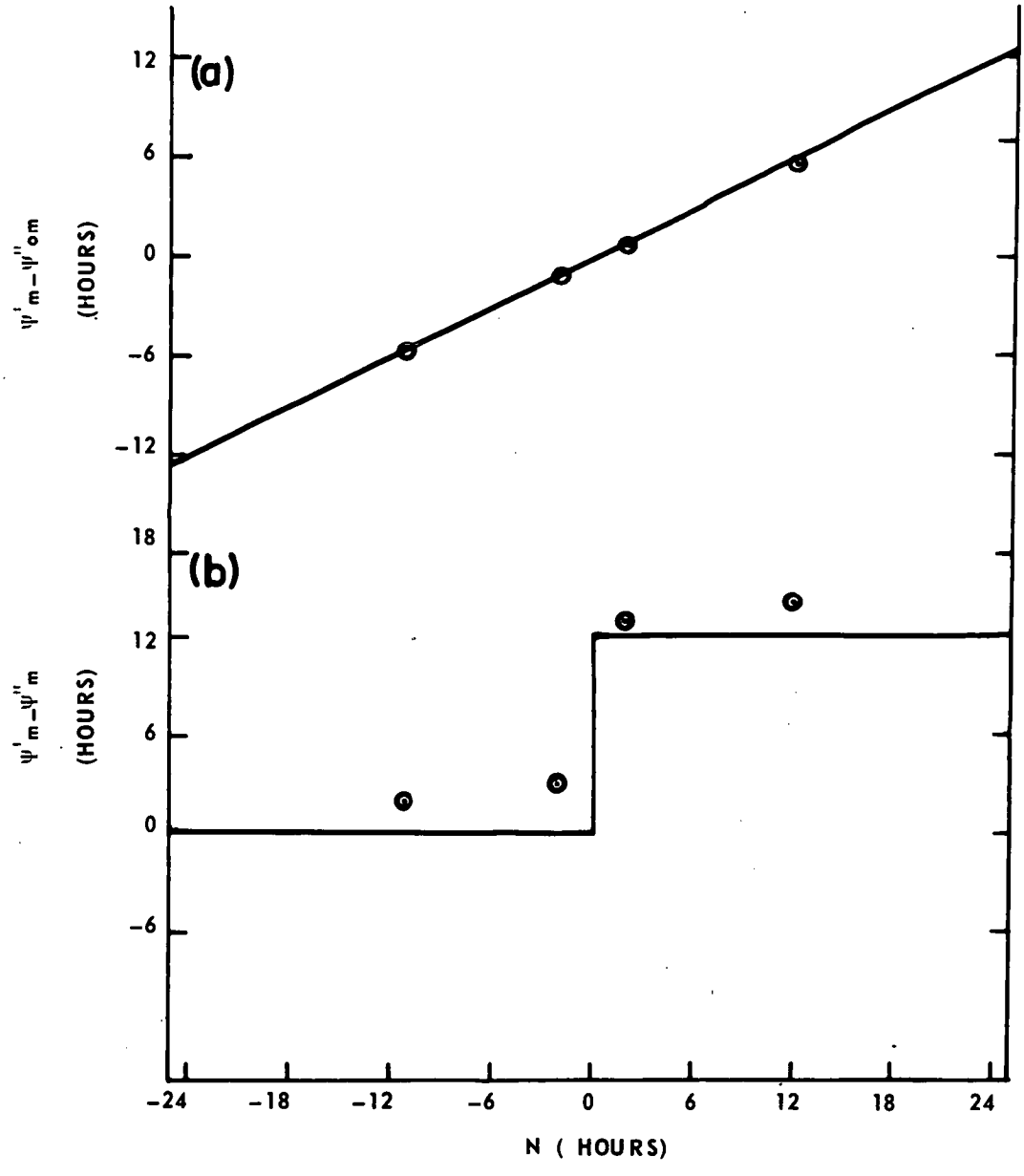


Figure 2.6 The relationships between the annual mean sidereal (Ψ'_m) and anti-sidereal (Ψ''_m) diurnal maxima for a sidereal diurnal variation with linear displacement of phase, versus N, the total annual phase displacement. Ψ'_{om} is the time of maximum of the unmodulated d.v.

Full lines : theoretical

Circled points: values from the "underground" model.

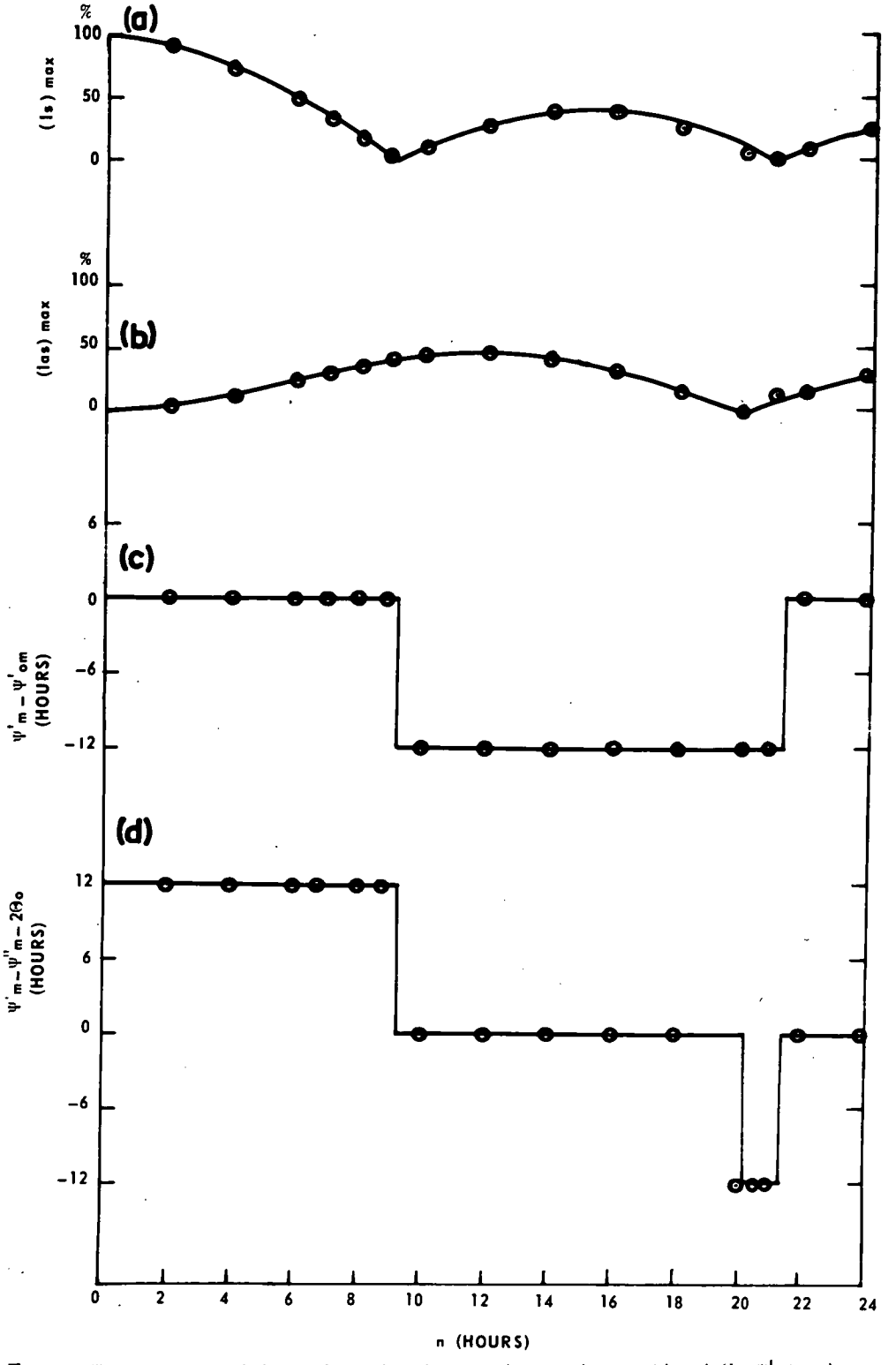


Figure 2.7 The amplitude and phase relationships between the annual mean sidereal (I_s, Ψ'_m) and anti-sidereal (I_{as}, Ψ''_m) diurnal variations for a sidereal d.v. with an annual variation of phase, versus n, the maximum phase displacement. Ψ'_{om} is the time of maximum of the unmodulated d.v. and the phase constant Θ_o is determined by the time of the year when maximum displacement of phase occurs.

Full lines : theory
Circled points : values from the "underground" model.

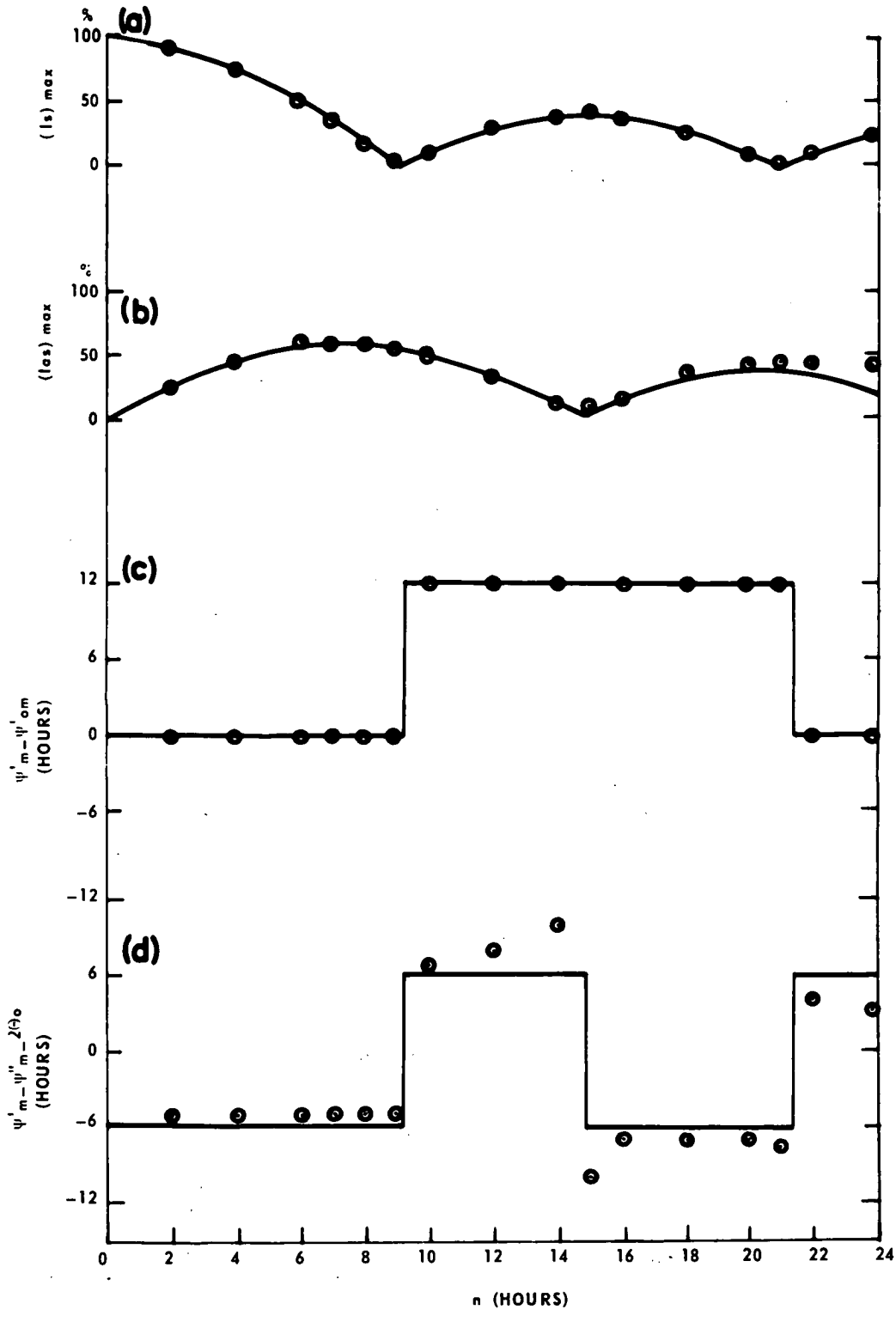


Figure 2.8 The amplitude and phase relationships between the annual mean sidereal (I_s, Ψ'_m) and anti-sidereal (Ias, Ψ''_m) diurnal variations for a sidereal d.v. with semi-annual variation of phase, versus n, the maximum displacement of phase. Ψ'_{om} is the time of maximum of the unmodulated d.v. and the phase constant Θ_o is determined by the time of the year when maximum displacement of phase occurs.

Full lines : theory
Circled points : values from the "underground" model.

(b) annual variation of phase (Figure 2.7), and

(c) semi-annual variation of phase (Figure 2.8).

Combined semi-annual variation of amplitude and phase of a sidereal diurnal variation is treated in paper 4 (Appendix IV) and will be referred to in Chapter 5.

(c) Application of the technique

In order to use the anti-sidereal technique in a given situation it is necessary to decide as to the component or components being modulated and the nature of the modulation. This seems to be the chief source of error in the use of the technique. The tendency is to assess modulation of the daily variation on the basis of physical mechanisms that are thought likely to apply, the result being that the anti-sidereal effect is generally attributed to seasonal changes in the diurnal variation of atmospheric temperature. In paper 4 (Appendix IV) an attempt is made to elucidate a pronounced seasonal effect in the daily variation underground by comparing the observations with the predictions of several widely differing hypotheses of seasonal modulation. The hypothesis of best fit is then provisionally chosen as the correct one.

Strictly speaking, the anti-sidereal technique in its present form applies only to first harmonic components of the daily variation. It is conceivable that serious departures from the prediction of the theory might occur if the component being modulated consisted of more than a first harmonic.

In paper 2 an attempt was made to investigate this problem. The mathematical predictions from the integration of modulated sine waves were compared with the results from two types of models (see page 5 of the paper). In one of the models the carrier was a sine wave and it was processed in the same way as the cosmic ray data. The purpose was to detect any significant bias or smoothing that might result from records that were analysed in the form of monthly groups of bi-hourly averages. Comparison with the mathematics revealed no significant discrepancies. The basis of the other model was a carrier constructed from the long term average solar daily variation observed underground. It is referred to as the "underground" model in the paper and contains a considerable second harmonic. In most cases the results from the model were in good agreement with the mathematics (Figures 2.7 and 2.8). However, in the important case of a solar component with annual variation of amplitude the disagreement was significant. The case is considered in paper 2 (see Chapter 5). This kind of difficulty may not be relevant to the atmospheric temperature effect, which appears to derive from a simple diurnal variation, but it would certainly apply if significant annual variations were found in the daily variation produced by the solar anistropy.

If seasonal m odulation happens to be non-sinusoidal it should suffice to break it down into annual and semi-annual terms and treat them separately.

2.4 Second order sidebands

Dorman (1965) has extended the sideband technique to include the second order sideband components produced by combined phase- and amplitude - modulation of the solar diurnal variation. These sideband components, at the frequencies 365 ± 2 cycles/year, appear in the expansion, by means of Bessel functions, of the expression for the modulated diurnal variation (equation 4.3 of paper 4). A simpler example, the expansion of a phase-modulated diurnal variation, is given in section 11 of paper 2. Dorman shows that the seasonal phase and amplitude constants can be derived from the harmonic coefficients of the first and second order sidebands, obtained by suitably arranging observations and averaging over complete years.

In the application of this interesting method, it is evident that essentially the same difficulties that have just been mentioned in connection with the anti-sidereal technique must be considered. In addition, it should be noted that the higher order sidebands tend to be smaller, so that unless the carrier is of large amplitude there are liable to be difficulties in obtaining significant harmonic coefficients. Thus the technique may be better suited to the analysis of observations from ground level than from underground.

The over-riding problem, of course, is that of identifying phase- and amplitude-modulation of a solar component in the first place. Sideband components at one or more of the four frequencies of interest (363, 364, 366

and 367 cycles/year) can be generated by other types of seasonal modulation of a solar diurnal variation (e.g. a semi-annual variation of phase, treated in paper 2, section 11) and by annual and semi-annual variations of a sidereal component (e.g. see paper 2, sections 10 and 12).

CHAPTER 3

REVIEW OF GROUND-LEVEL STUDIES AND COMPARISON OF RECENT GROUND-LEVEL AND UNDERGROUND OBSERVATIONS

3.1 Review of experiments at ground-level

The history of experimental studies of the sidereal effect at ground level goes back in the main to the 1930's, although as early as 1923 D. Kolhorster and von Salis (1926) observed an apparent effect with an ionisation chamber on the Jungfrau, the intensity maximum occurring at about 2000 local sidereal time (LST). Longer term studies became possible with the accumulation of data from ionisation chambers, notably from instruments on the Hafelekar, from 1932 to 1934 (W.Illing, 1936) and from 1936 to 1937 (Demmelmair, 1937); at Capetown, from 1933 to 1935 (Schonland, Delatitsky and Gaskell, 1937); at Canberra, from 1936 to 1940 (Hogg, 1949);and at Cheltenham (U.S.A.), Huancayo and Christchurch (N.Z.), from 1938 onwards (Lange and Forbush, 1948).

From the mid-1930's the theory of Compton and Getting(1935) appears to have greatly stimulated the studies at ground level. Following a suggestion of Lowry, the authors predicted a world-wide diurnal maximum of 0.1% within a factor of two, at about 2000 LST, in connection with the motion of the earth towards the constellation Cygnus, in the direction 47°N, 20 hours 40 minutes RA. The motion, with a velocity of 300 km/sec., was being imparted chiefly by the rotation of the galaxy and it was proposed that there should be a sidereal

effect due to cosmic rays coming from outside the galaxy and not sharing in the rotation.

Their evidence for the existence of the predicted effect came from ionisation chamber measurements on the Hafelekar (Lat.47°N, 2300 m), published by Hess and Steinmaurer (1933). Their Figure 2 showed that there was good agreement with the predictions.

In a later review of the evidence for the Compton-Getting effect, Wollan (Revs.Mod.Phys.11, 160(1939)) showed that when the data from ionisation chambers were averaged over several years (either that from the Hafelekar or from Capetown) the amplitudes of the apparent sidereal diurnal variations were not statistically significant. His Figure 8 gives a useful summary of results up to 1939. He also demonstrated the difficulty of detecting a sidereal component, superimposed on an amplitude-modulated solar component, by Thompson's method of fitting an ellipse to the end-points of monthly mean solar diurnal vectors on an harmonic dial (Thompson, 1939. His method is also discussed in paper 2, page 2).

The Hafelekar, Canberra and Capetown results when compared by Hogg attested to the year-to-year variability of the apparent sidereal effect at any one place. His Figure 17 shows that it was not possible to detect over three or four years either a reproducible time of maximum or, for that matter, a 12-hour difference between the two hemispheres. Thus the evidence gave little support to the

theory of cosmic ray streaming put forward by Compton and Getting, nor was the sidereal effect obviously fictitious, arising from seasonal modulation of a solar component.

In a well-known study of the sidereal effect at ground level, Elliot and Dolbear (1951) attempted to minimise effects due to seasonal changes of the solar component of the daily variation. They examined -

(a) the difference of intensity between crossed Geiger counter telescopes, inclined 45° south and 45° north of zenith at Manchester in 1949;

(b) the ion chamber data from Huancayo for the nine complete years available, assuming that seasonal effects would be small at the low latitude concerned (12°S); and

(c) the combined ion chamber data from Christchurch (Geog.Lat. 35°S) and Cheltenham (38.7°N) for the six complete years available. They assumed that spurious sidereal components at the two places would cancel out in the combining process because of the 12-hour difference in phase expected between the two hemispheres.

The time of maximum of the annual apparent sidereal wave found by methods (a), (b) and (c) were in reasonably good agreement, being 0800, 0400 and 0600 LST respectively. However, the result was at variance with the predictions of Compton and Getting and with most of the earlier ion chamber measurements.

The authors were able to demonstrate the anti-clockwise cyclic arrangement of the bi-monthly vectors in respect of the Huancayo and Christchurch/Cheltenham data, indicative of a sidereal vector superimposed on a relatively constant solar vector. In the case of the Manchester N-S difference the same test was not successful, the statistical errors apparently masking any positive indication of this type either for or against a genuine sidereal maximum at 0800 LST. As it now appears, the two results which derived partly or wholly from the southern hemisphere were understandable and it was the essentially northern hemisphere result from Manchester that was not.

The variability of the combined Cheltenham/Christchurch result becomes evident when one examines data averaged over a different period, namely the eight complete years from 1947 to 1954. The time of maximum of the apparent sidereal effect from these data is approximately 2200 LST, differing by eight hours from the result obtained by Elliott and Dolbear. It would appear (Figure 3.1) that the maximum of the combined daily variation tends to coincide with the maximum at either Christchurch or Cheltenham, depending on which has the greater average amplitude at the time of observation.

Very significant and somewhat complex developments accompanied the minimum of solar activity in 1954. By now, G-M counter telescopes had become the prevailing instruments for recording the intensity of μ-mesons and the first neutron monitors had been put into operation. Without enquiring into the many complexities of the observations, it seems that, broadly speaking, there were two major occurrences of interest:-

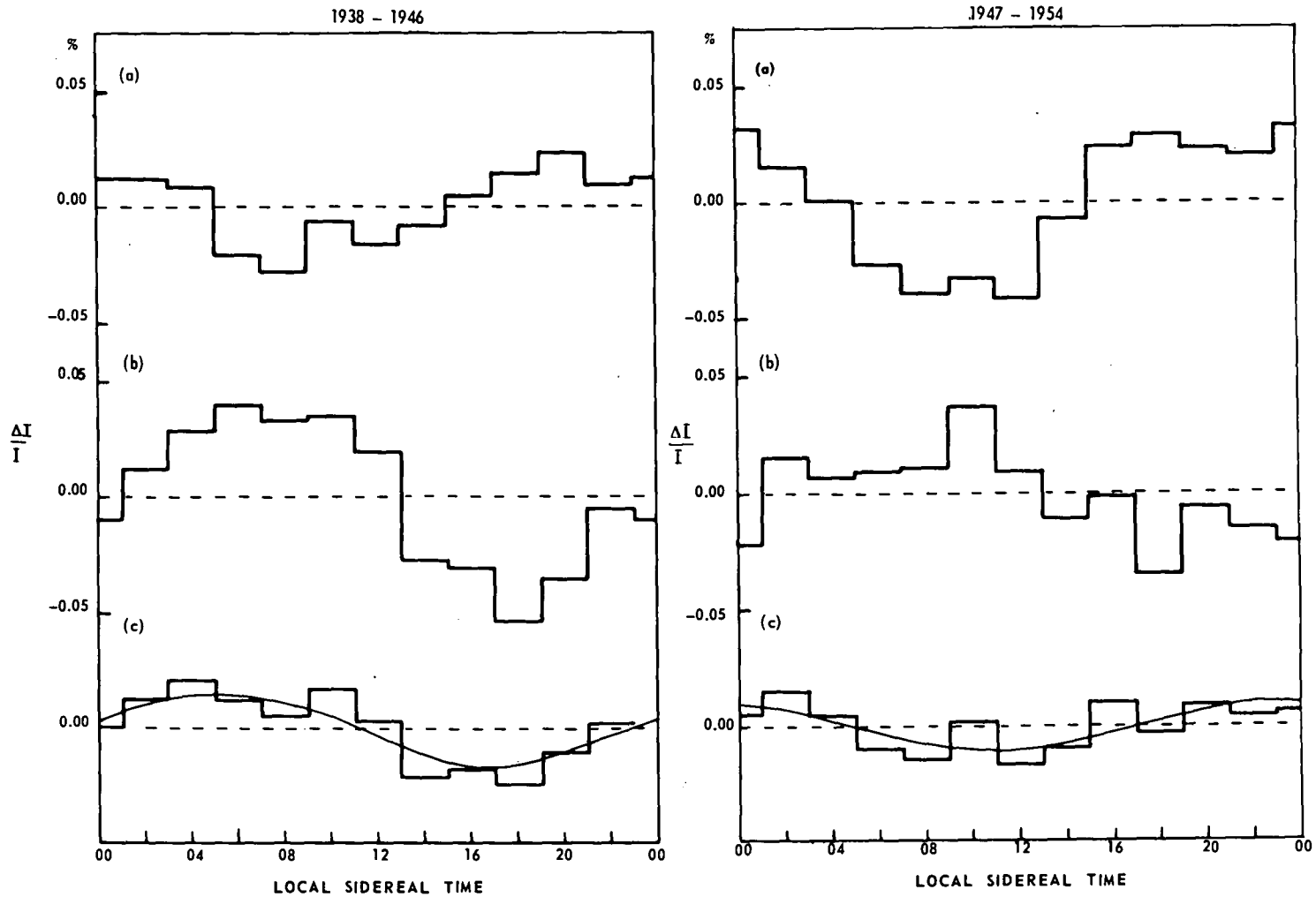


Figure 3.1 The apparent sidereal daily variation of pressure-corrected ionisation averaged over the 9 years 1938–1946 and over the 8 years 1947–1954.

(a) Cheltenham(38.7°N)

(b) Christchurch (35°S)

(c) Cheltenham and Christchurch combined, with the first harmonics of best fit.

(1) The time of maximum of the annual mean solar diurnal variation of meson intensity, having maintained a relatively constant value, approximately 1400 hours solar time, over the previous twelve years, began to advance after 1950 and reached its earliest value, about 0700, in 1954. Thereafter the trend reversed and over a period of three years the time of maximum returned to something like its former value. Clearly, these large secular, but non-linear, displacements of phase, varying in character from year to year, would most likely be responsible for annual sideband components in sidereal and anti-sidereal time.

(2) At the solar minimum a phase anomaly occurred in the data from neutron monitors, ionisation chambers and counter telescopes in the northern hemisphere. Simpson and Conforto (1957) noted that the phase of the solar diurnal variation in 1954 had advanced progressively at the rate of approximately two hours a month for upwards of six months in the manner of a genuine sidereal component superimposed on a smaller solar component, the sidereal maximum occurring at approximately 2000 LST. However, the figure they give is approximately 0800. It should be noted that the authors had synchronised solar and sidereal time at the March equinox instead of the September equinox, so that 12 hours must be added to sidereal times of maximum given in their paper.

Because the phase anomaly occurred in the diurnal variation of neutron intensity it could not have arisen from seasonal modulation of an atmospheric temperature component. Again, because of the magnitude and specific character of the phase anomaly it seemed unlikely to be merely an unusual seasonal

variation of phase of the solar diurnal variation of the primaries, although the possibility could not be ruled out. The obvious alternative was more plausible in at least one respect : if there were a constant sidereal component in the daily variation it would be more likely to dominate the daily variation during a particularly quiet solar minimum on the expectation that the solar diurnal variation of the primaries would have greatly diminished. However, if this was what had happened, it was necessary to reconcile it with the fact that there had been large year-to-year displacements of phase of the diurnal variation of meson intensity (see (1) above), while there was little evidence of prominent systematic changes of amplitude.

The difficulty has been studied by many workers. We refer to the more recent conclusion reached by Dorman (1963) in a review concerned essentially with the work of Glokova (1960), Kuzmin (1960) and Quenby and Thambyahpillai (1960), all of whom had made use of Dorman's method for determining the atmospheric temperature effect. It seemed that at many places an important and relatively constant component of the annual solar diurnal variation of meson intensity was of atmospheric origin. As the solar diurnal variation of the primaries weakened with the approach of solar minimum, so the atmospheric component began to dominate the total annual diurnal variation. This interplay between the two components was responsible for much of the secular change in phase mentioned in (1) above and to some extent masked the decrease in amplitude of the extra-terrestrial solar component. Much of the seasonal effect in the solar diurnal variation of meson intensity was also attributed to a seasonal change in the diurnal variation of atmospheric temperature. However,

it would contribute only in part to the phase anomaly in the hard component of intensity and not at all to that in the nucleonic component. At this point it was concluded that there probably was a genuine sidereal component in the daily variation, but that in the hard component there would also be a spurious annual sidereal effect of atmospheric origin and perhaps of extra-terrestrial origin as well. Dorman's assessment of the temperature effect as it applied to the events at the 1954 solar minimum will be referred to again in Chapter 5, in connection with phase anomaly observed in the daily variation of meson intensity underground. Here, the influence from atmospheric temperature variations appears to have been very small.

In a later chapter of the same publication, Dorman (1963) goes on to outline an attempt made in collaboration with Inozemtseva (Dorman and Inozemtseva, 1961) to discover extra-terrestrial sources of the daily variation using all available data from crossed telescopes. They hoped to be able to account for the many inconsistencies in the observations from inclined telescopes at different phases of the solar cycle over the period 1948 to 1960. They emphasised the importance of taking into account the diurnal variation of atmospheric origin and showed that the subtraction of this component had a very different effect at solar minimum than at solar maximum. Following a qualitative study of cones of viewing as functions of primary energy of response for inclined telescopes at ground level, they concluded (p.323) that two extra-terrestrial sources of the average diurnal variation were effective at solar minimum : a low-latitude source (I) to the left of the earth-sun line, influencing mainly low-energy primaries ($\lesssim$ 30-40 GEV), and a high-latitude

source II to the right of the earth-sun line (between 0 and 6 hours) influencing mainly particles of energy greater than 50-100 GEV. It might seem contradictory that source II should at the same time be regarded as sidereal. However, it appears that the authors' argument was as follows. Source II had a "fixed" direction with respect to the earth-sun line only in the sense that the inclined telescopes responded to it in the northern summer months, but not at other times of the year. On the other hand, the neutron monitor at Huancayo responded to it throughout the year and the observations from Huancayo were compatible with an annual anti-clockwise rotation of the source directions. Its RA would therefore appear to be about 18 hours.

It is unfortunate that confusion has arisen in the literature through the use of two definitions of sidereal time. This is particularly noticeable in Progress in Cosmic Ray Physics, Vol.VII. Table XV (p.308) of that volume summarises the apparent sidereal effects observed with meson telescopes at medium- and high-latitude stations from 1953 to 1957, according to the investigations of Berdichevskaya and Zhukovskaya (1960). The sidereal times of maximum range from 0300 to 1200 and include 0700 as the result from Huancayo in 1954. It is evident that the authors had synchronised solar and sidereal time at the March equinox as had Simpson and Conforto in their treatment of the phase anomaly. Consequently, their tabulated times of maximum differ from the conclusions of Baliga and Thambyahpillai (1959) reported on page 307, by twelve hours. Baliga and Thambyahpillai had examined the low- and mid-latitude data

for 1954, including the observations from Huancayo, and their times of maximum are quoted as ranging from 19 hours at the equator to 22 hours at the medium-latitude stations. These authors had synchronised solar and sidereal time correctly at the September equinox.

The ground level studies as a whole seem to suggest that the time of maximum of the apparent sidereal diurnal variation changes unpredictably, both from year-to-year and from place to place, although the amplitudes are often highly significant. Undoubtedly there may be considerable variability at any one place and this will be examined in the next section. However, in many cases the great and apparently unsystematic differences in reported times of maximum are equally traceable to methods of treating the data. Three factors in particular seem to have led to a confused picture of the effect :

(1) Generally, experimental studies have been concerned with the search for the sidereal diurnal variation associated with a single galactic intensity maximum. Consequently, allowing for the cones of viewing, it has been expected that the same time of maximum would be observed at all places. This has led to the tendency to group the observations from the northern hemisphere (which have predominated) with those from the southern hemisphere indiscriminately.

(2) The anti-sidereal effect has been attributed almost invariably to seasonal changes in a diurnal variation of atmospheric temperature. Spurious components have been calculated accordingly and subtracted from the apparent sidereal vector. It is of interest that Baliga and Thambyahpillai (1959), in their discussion of the 1954 phase anomaly, have compared anti-sidereal components

in the northern and southern hemispheres and have noted that they could not have entirely originated from seasonal modulation of a diurnal variation of atmospheric origin.

(3) When sidereal time has been synchronised with solar time at the March equinox, an error of 12 hours in the sidereal time of maximum has been incurred.

3.2 Comparison of recent observations at ground level and underground in the southern hemisphere

The following comparison anticipates consideration of the underground experiments at Hobart, but there seem to be advantages at this point in presenting a broad picture of the response in solar, sidereal and anti-sidereal time from detectors above and below ground.

The basic material comprises pressure-corrected data from the following detectors:

(a) The vertical semi-cubical G-M telescopes (1 sq. metre trays) at a depth of approximately 40 m.w.e. underground at Hobart (43°S, 147°E geographic).

(b) The standard vertical cubical G-M telescope (1 sq.metre trays) under 10 cm. lead absorber at sea level, Hobart.

(c) The standard 12-counter neutron pile at Mt.Wellington (725 m).

(d) The standard 12-counter neutron pile at sea level at Mawson (68°S, 63°E geographic).

Relatively few data are missing or have been rejected. Consequently, the observations may be considered to be simultaneous as between stations. Monthly averages of hourly data are processed in bi-hourly groups and expressed as percentage deviates from the monthly mean intensity. Considerable smoothing is effected by averaging the deviates in progressive groups of three, to form six-hour running averages. In this form the observations are expressed in solar, sidereal and anti-sidereal time and averaged over complete years centred on successive months, to give annual running averages. Sufficiently accurate curves of fit to the annual points are drawn by eye. In Chapter 5, annual running amplitudes and times of maximum obtained in this way from six-hour running average deviations are compared with values obtained from computed sums of first and second harmonics of best fit to the observed hourly deviates, and from the first harmonics of best fit. It should be explained that annual running averages are used for qualitative comparisons only. If they extend over a number of years they should serve the following purposes:

(a) They should distinguish between the type of daily variation in which a persistent component preponderates and the type which tends to be controlled by transient components, whether systemmatic or statistical.

(b) They should enable a genuine long-term trend in a parameter of the daily variation to be distinguished from an apparent trend that is influenced by transient effects of less than one year's duration. In this respect they possess an advantage over a set of independent annual averages.

The three figures for each type of daily variation relate to three independent biennial periods: January 1958 to December 1959, January 1960 to

December 1961 and January 1962 to December 1963. Thus they span a period from solar maximum to within approximately 18 months of the following minimum.

(1) The annual mean solar daily variations (Figures 3.2, 3.3 and 3.4)

Note that the solar daily variation is much smaller at 40 m.w.e. than above ground and that the ordinate scale has been adjusted to allow for this.

At all stations there is, as to be expected, a slow variation with a single maximum. The time of maximum is approximately the same at each of the three Hobart stations and is approximately three hours in advance of that at Mawson. The phase difference between the solar daily variations of neutron intensity at Mawson and Mt.Wellington has been explained as a longitudinal difference between the mean directions of the respective asymptotic cones of response to the solar anisotropy (Rao, McCracken and Venkatesan, 1963).

There have been no significant trends in the times of maximum over the period or any noticeable shorter term secular changes.

The figures do not allow amplitude changes to be easily detected, but on examination one important long-term trend should be apparent : the amplitude of the solar daily variation underground decreased by a factor of approximately two between the year ending December 1958 and the year ending December 1963, but without any noticeable displacement of phase. Thus there is no immediate indication of interplay between an atmospheric component and an extra-terrestrial solar component.

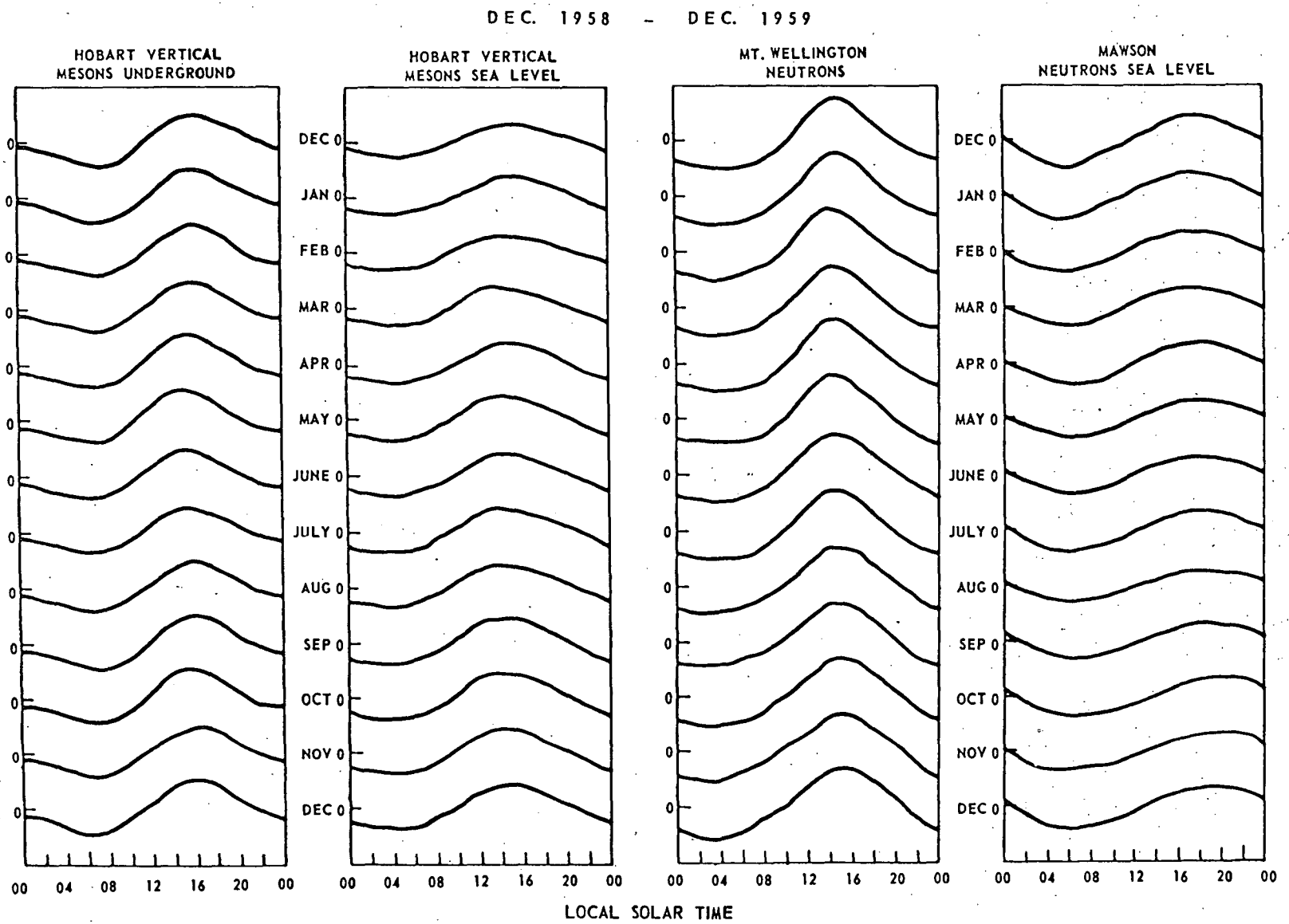


Figure 3.2 The solar daily variation. Annual running average curves for successive months, from the year ending December 1958 to the year ending December 1959 inclusively. The ordinate scale is 0.42 % per division, except for Hobart underground mesons where the scale is 0.21 % per division.

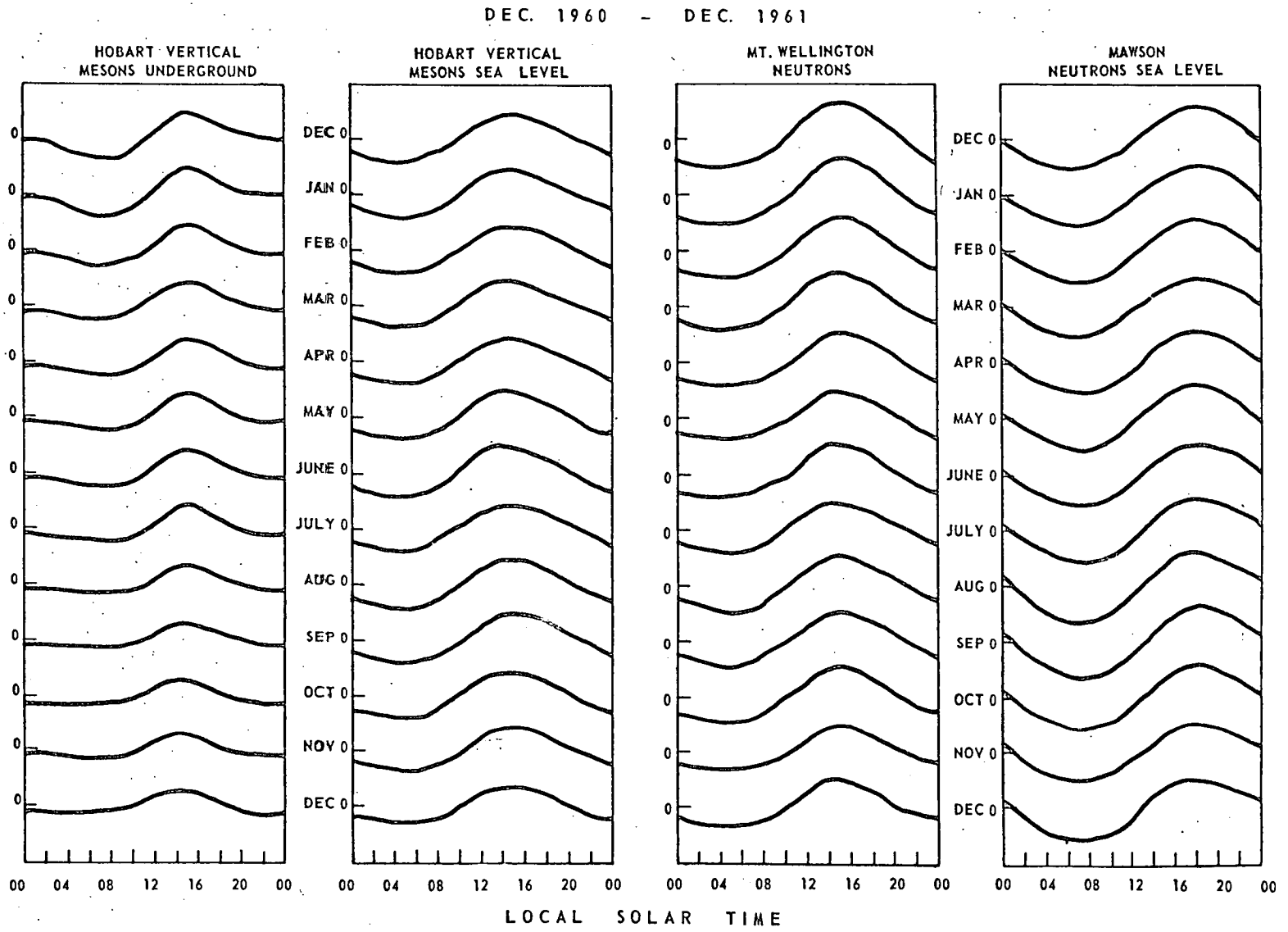


Figure 3.3 The solar daily variation. Annual running average curves for successive months, from the year ending December 1960 to the year ending December 1961 inclusively. The ordinate scale is 0.42 % per division, except for Hobart underground mesons where the scale is 0.21 % per division.

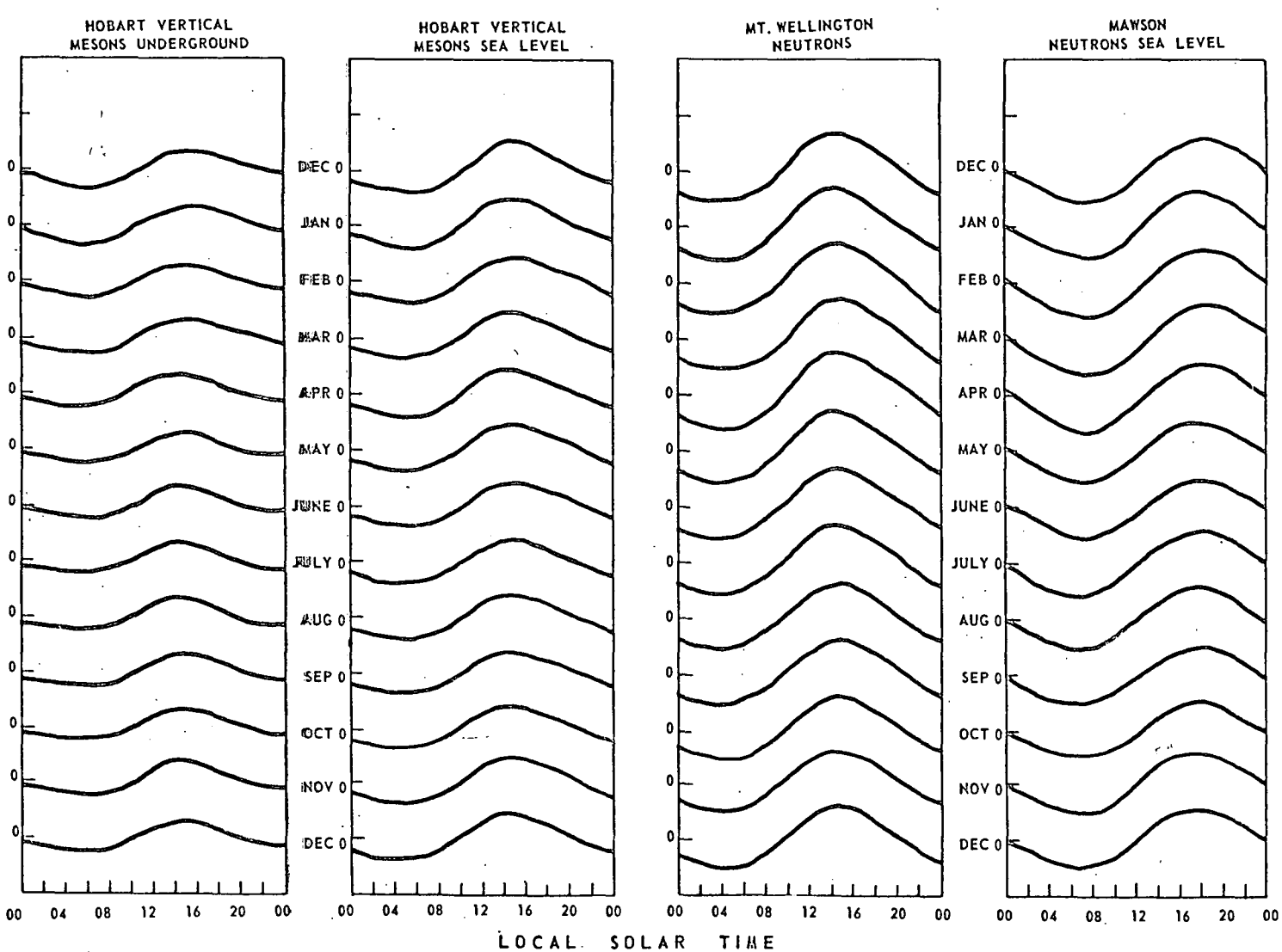


Figure 3.4 The solar daily variation. Annual running average curves for successive months, from the year ending December 1962 to the year ending December 1963 inclusively. The ordinate scale is 0.42 % per division, except for Hobart underground mesons where the scale is 0.21 % per division.

(2) The annual apparent sidereal daily variations (Figures 3.5, 3.6 and 3.7)

The apparent sidereal daily variations at all stations are shown to the same scale. It should be appreciated that, in contrast with the underground observations, the amplitudes above ground are generally very much smaller than the amplitudes of the corresponding solar daily variations, so that the influence of spurious sidereal effects produced by seasonal or irregular fluctuations of the solar components should be greater accordingly, as seems to be the case.

We draw attention first to the situation in 1958. It is remarkable that the apparent sidereal effect was relatively so large at all the stations at solar maximum. The times of maximum were approximately the same, although that at Mt. Wellington was somewhat earlier than the others, and there is no evidence of the characteristic phase difference between the Mawson and Hobart observations that was seen in the solar daily variation. It would be surprising if a common mechanism did not link the observations at the four stations and yet it is clear that there would be difficulties in attributing the result to seasonal modulation of a solar daily variation of atmospheric temperature.

The phase of the sidereal effect underground advanced somewhat during 1959, but thereafter remained fairly constant. There is also evidence

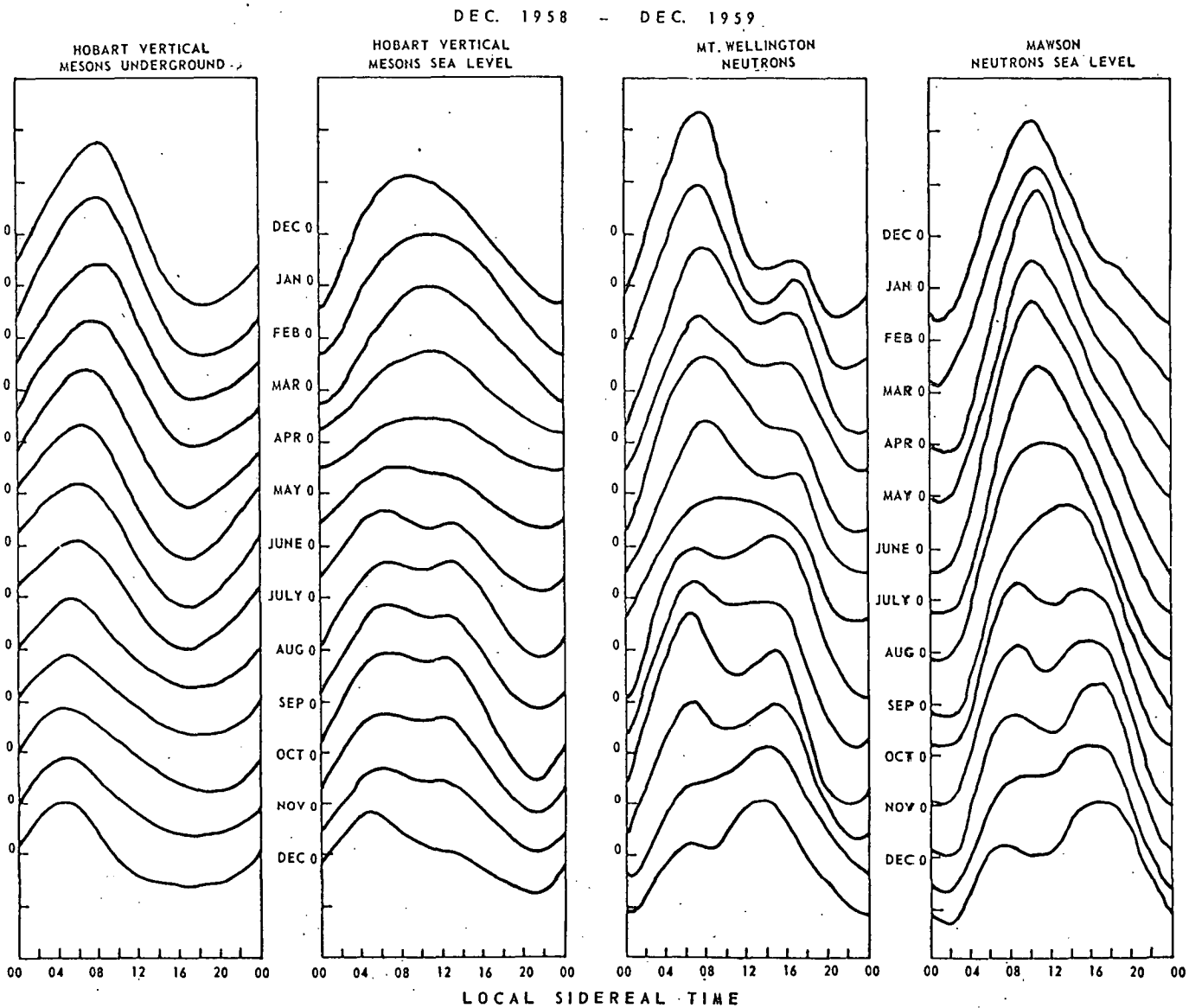


Figure 3.5 The daily variation in sidereal time. - Annual running average curves for successive months, from the year ending December 1958 to the year ending December 1959 inclusively. The ordinate scale is 0.042 % per division.

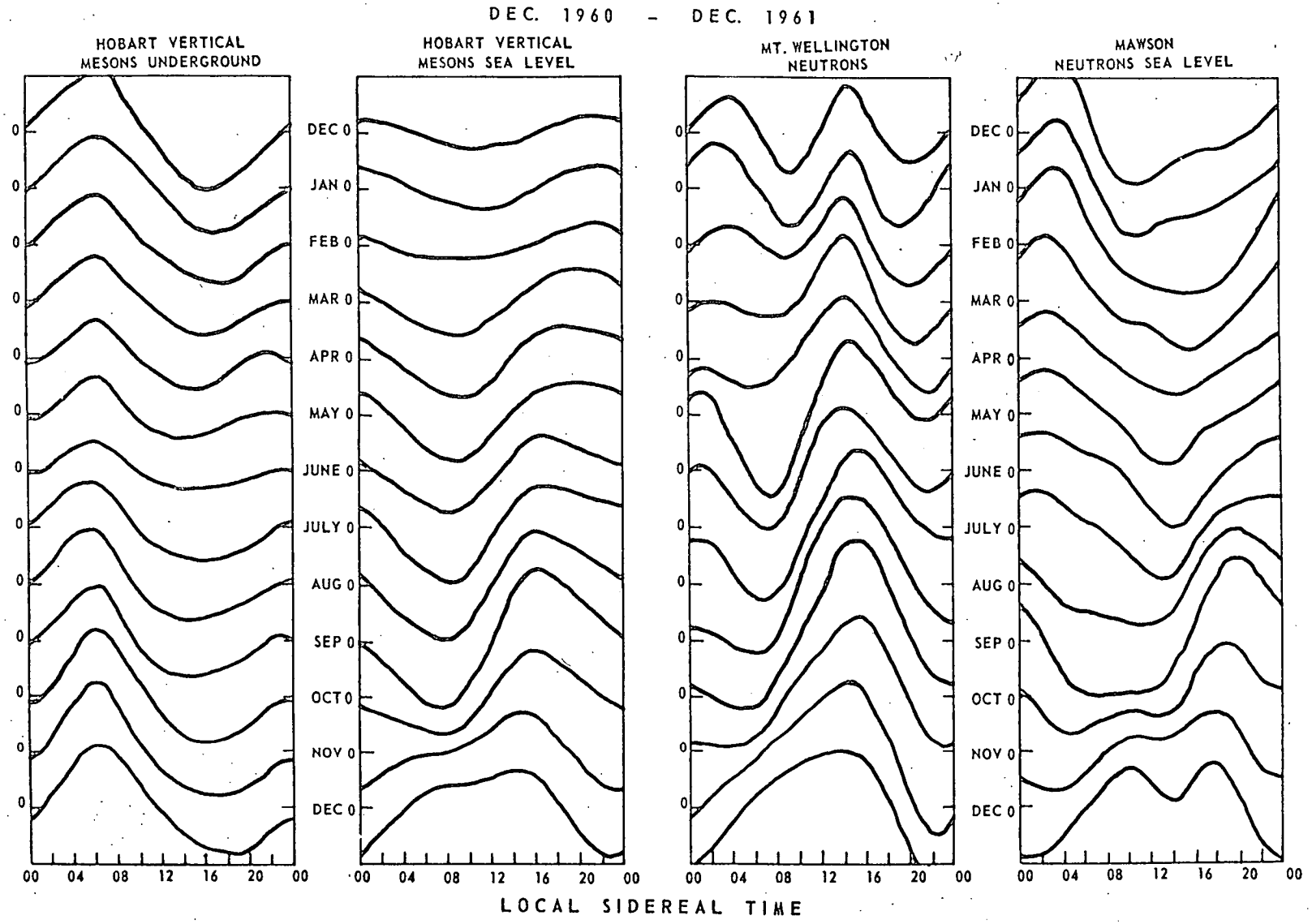


Figure 3.6 The daily variation in sidereal time. Annual running average curves for successive months, from the year ending December 1960 to the year ending December 1961 inclusively. The ordinate scale is 0.042 % per division.

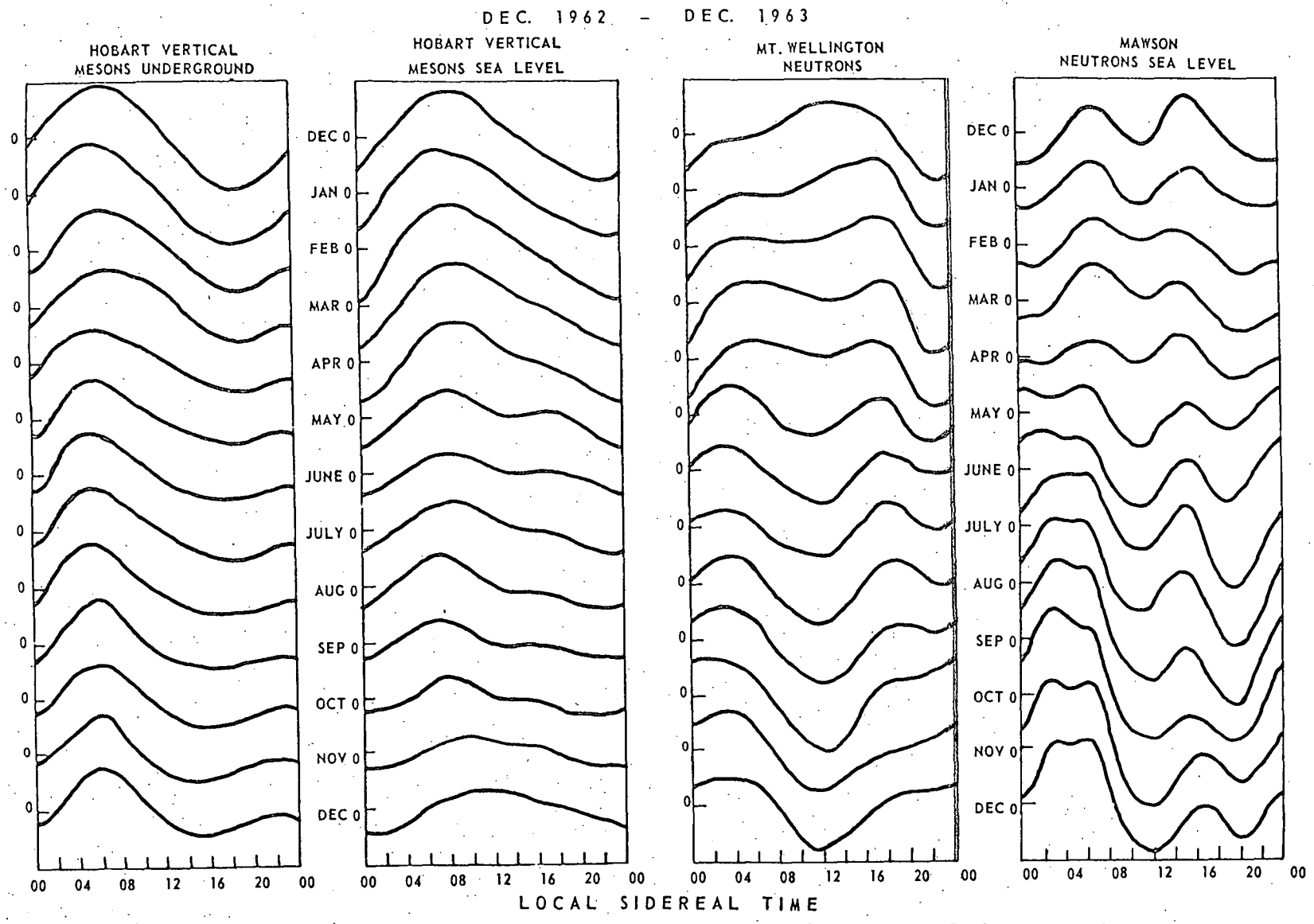


Figure 3.7 The daily variation in sidereal time. Annual running average curves for successive months, from the year ending December 1962 to the year ending December 1963 inclusively. The ordinate scale is 0.042 % per division.

of amplitude changes and some changes in the balance of harmonics, but overall it can be seen that there is a characteristic annual daily variation, reproducible from year to year and not subject to large transient fluctuations.

Above ground the effect in the meson intensity is variable and it can be seen that a complete reversal of phase may occur in a short time. Perhaps a persistent component might be detectable if the data were averaged over a number of years, but it is evident that the observations from a single year or even two years would not be sufficient.

The effect in the neutron intensity is extremely variable and after 1959 it is not obvious that there is any connection between the Mt. Wellington and Mawson results. It would seem that some of the variability must have been due to irregular fluctuations in the solar daily variation.

(3) The annual mean daily variations in anti-sidereal time (Figures 3.8, 3.9 and 3.10)

The effects in anti-sidereal and sidereal time are all shown to the same scale.

Little need be said about the anti-sidereal effect in the neutron intensity. It is much too variable and complex to be of use in a conventional anti-sidereal analysis.

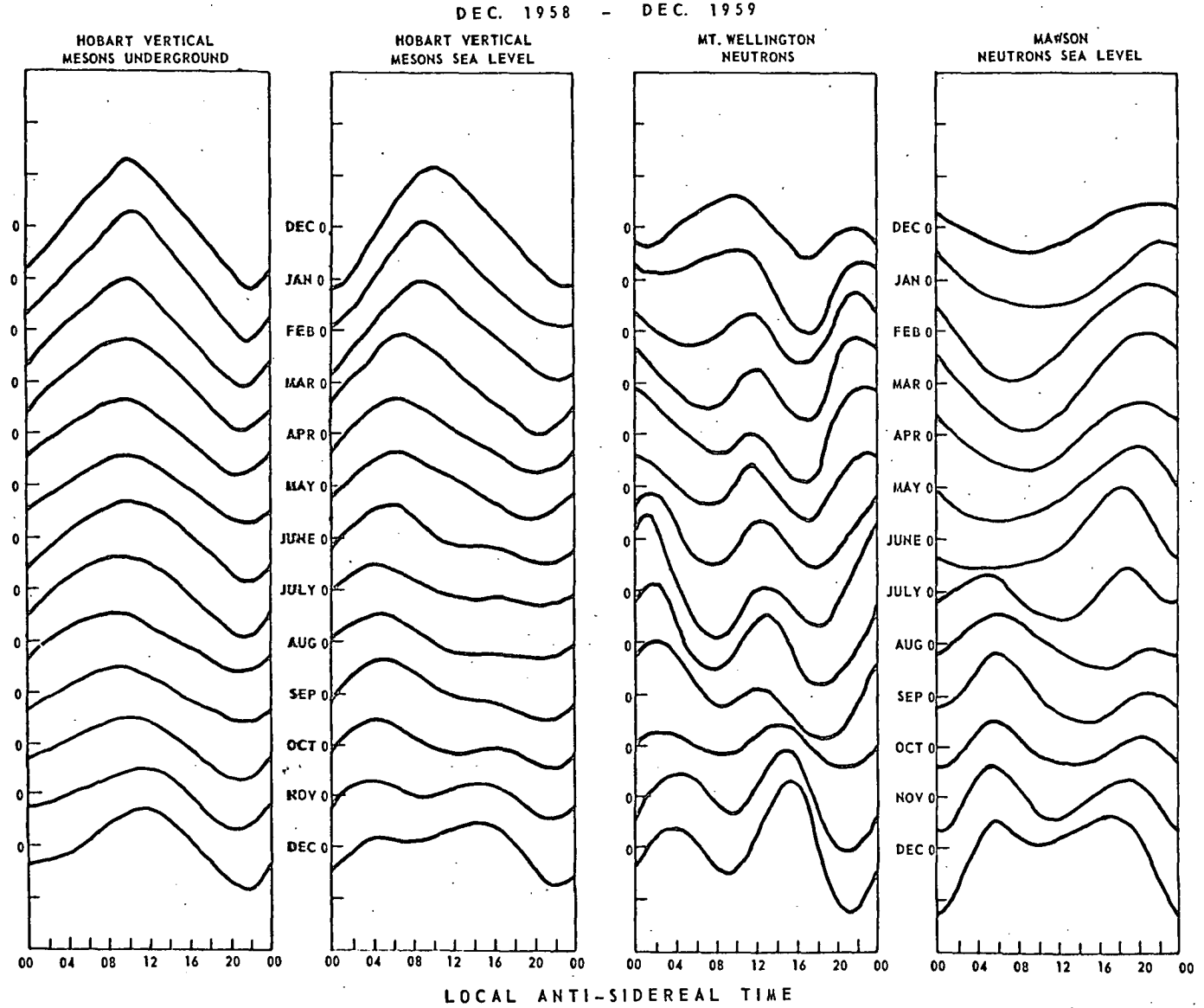


Figure 3.8 The daily variation in anti-sidereal time. Annual running average curves for successive months, from the year ending December 1958 to the year ending December 1959 inclusively. The ordinate scale is 0.042 % per division.

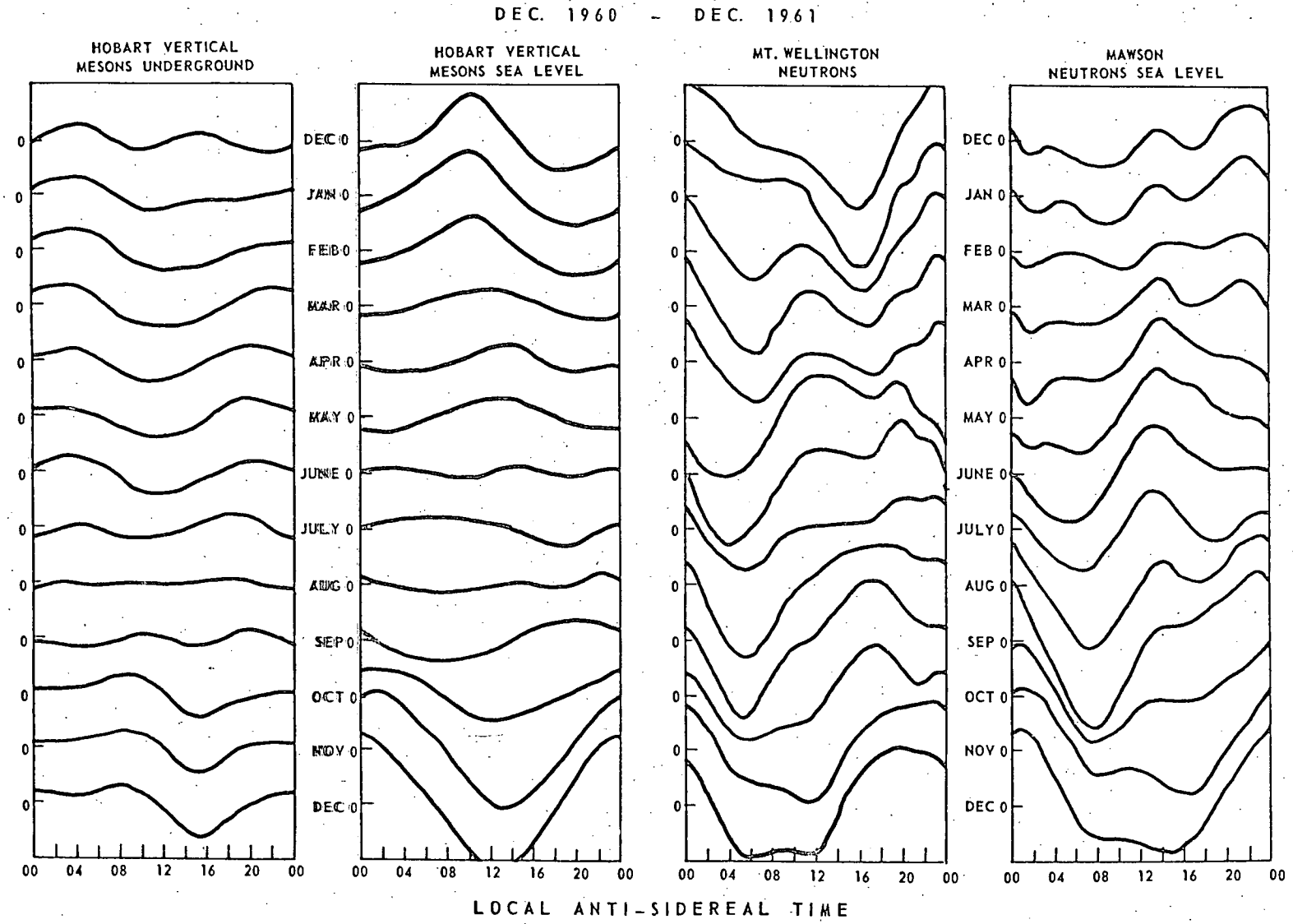


Figure 3.9 The daily variation in anti-sidereal time. Annual running average curves for successive months, from the year ending December 1960 to the year ending December 1961 inclusively. The ordinate scale is 0.042 % per division.

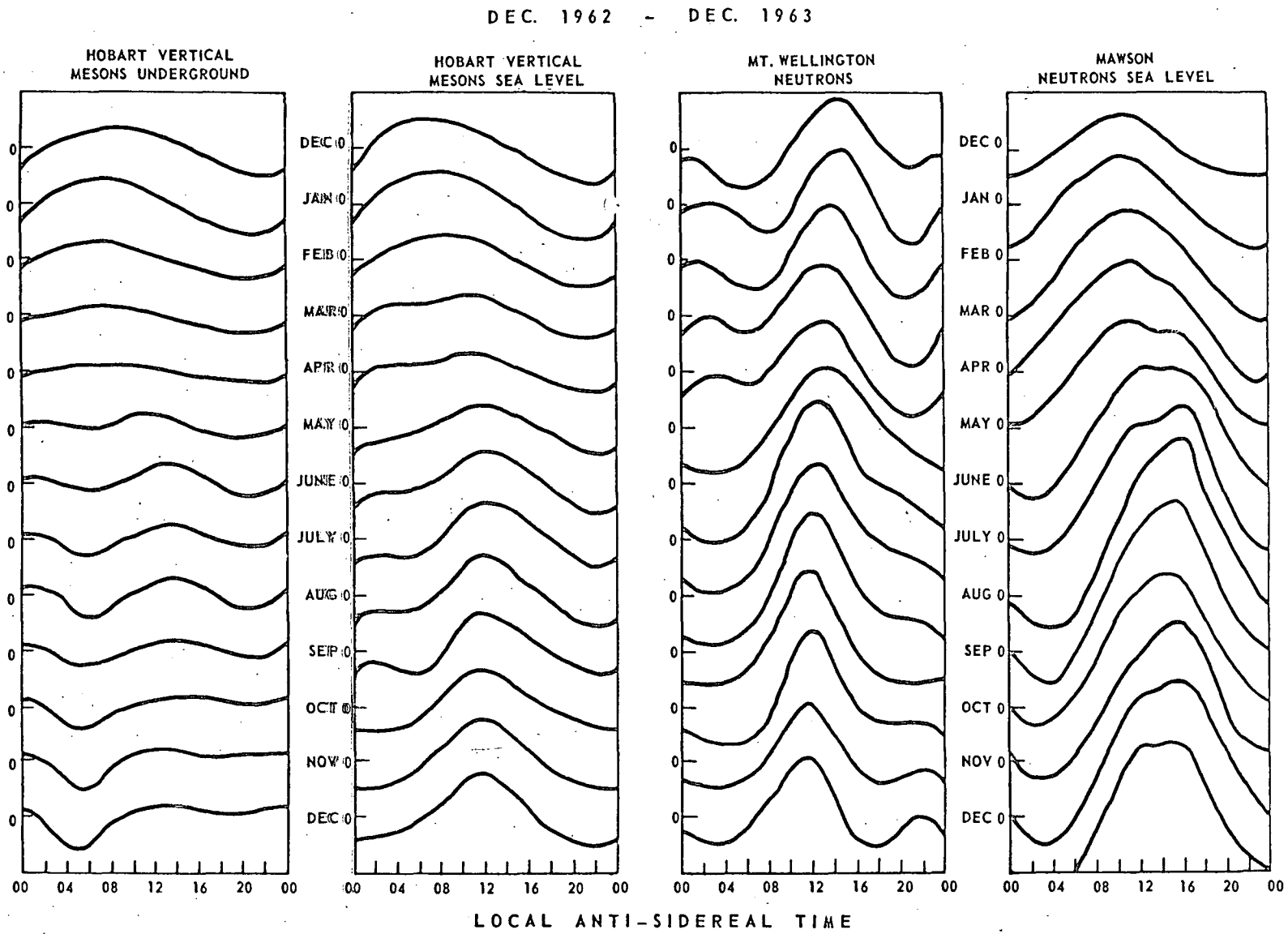


Figure 3.10 The daily variation in anti-sidereal time. Annual running average curves for successive months, from the year ending December 1962 to the year ending December 1963 inclusively. The ordinate scale is 0.042 % per division.

There is noticeable similarity between the relatively large anti-sidereal effects in the meson intensity above and below ground in 1958. However, the amplitudes decreased considerably during 1959 and the effect became small and variable. In later years the amplitude of the anti-sidereal effect at the underground station did not exceed 0.02% and was statistically insignificant. It would seem, then, that modulation of components of the daily variation underground became unimportant after 1960, although very significant annual variations in solar and sidereal time were observed.

CHAPTER 4

THE UNDERGROUND LABORATORY AT HOBART

4.1 Description of the underground site

(a) Location

The underground laboratory is situated six miles north east of Hobart in a disused railway tunnel, which passes approximately 60 feet beneath the Tunnel Hill saddle, 400 feet above sea level. The geographic position is 42° 51' S, 147° 25' E.

(b) The Tasman Highway and other roads

Figure 4.1 is reproduced from a contour map of the tunnel area supplied by the Department of Lands and Surveys. It shows a single lane highway (the Tasman Highway) passing over the centre of the tunnel. The road to Mt.Rumney (1240 feet, 1½ miles due east) branches off to the east at the top of the saddle. An access road to the farmhouse on Wiggins Hill branches off to the west and crosses above the tunnel immediately south of the cosmic ray building at O. The other road gives access from the highway through a farmyard to the northern entrance of the tunnel.

(c) The tunnel

The tunnel is straight, 530 feet long and, looking north, is orientated in the direction 352° 27' true. In cross-section it is 12 feet wide between the sandstone block walls at ground level, opening out to a maximum width of 14 feet at a height of 7 feet. Above this is a semi-circular roof

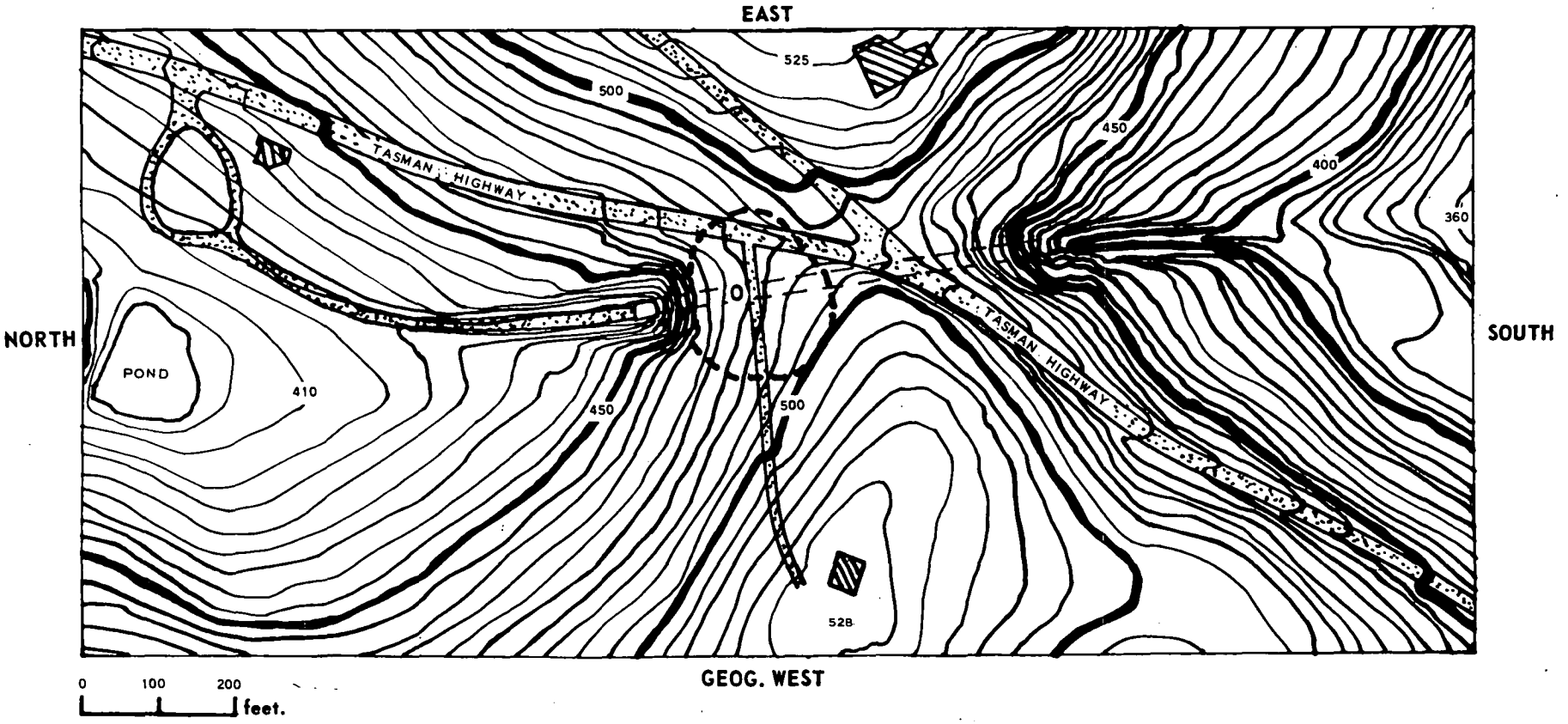


Figure 4.1 Diagram of the Tunnel Hill saddle near Hobart, showing contour levels at intervals of five feet. The numbers give heights in feet above sea level. The cosmic ray building is situated at O inside the tunnel, which is indicated by dashed lines. The dashed circle delineates the boundary of the cone of acceptance of one of the semi-cubical telescopes at intersection with the land surface.

of concrete, 18 inches thick. The height from floor to the centre of the ceiling is 14 feet 6 inches.

For the first 230 feet from the northern entrance the dirt floor is level and dry, an open concrete drain running flush against the western wall for approximately this distance.

The northern entrance is completely framed by a massive iron grille with a central 4' × 6'5" double-barred door. An identical structure secures the other end of the tunnel, 100 feet from the southern entrance.

(d) The laboratory

The equipment is housed in an oblong wooden framed hut of internal dimensions 12' × 28' × 7' high. The unlined walls are of hardboard and the ceiling is of cane-ite. The building is roofed with floorboards under a layer of malthoid. The northern wall is 95 feet from the concrete sill at the tunnel entrance. The hut is serviced electrically by the 240 volt A.C. mains supply.

4.2 Description of the equipment

(a) The G-M counter telescopes

There are four telescopes in operation at present, each comprising three trays of G-M counters in triple coincidence. A single tray contains 24 Maze-type external cathode glass counters, each of effective length 1 metre and approximately 4 cm in diameter, making up a total sensitive area of 1 sq. metre.

The two vertical semi-cubical telescopes with their electronic circuitry are essentially of the same design and construction as the standard cubical telescopes operating in the cosmic ray laboratories at Hobart and Mawson. Since the latter have been fully described in A.N.A.R.E. Interim Report No. 17, entitled *The Design and Operation of A.N.A.R.E. Cosmic Ray Recorder "C"* (N.R. Parsons, 1957), the details of construction and circuitry will not be reproduced here. The telescope trays are horizontal and are mounted above one another, care being taken to line up the ends of the counters so that the sensitive areas of the trays are correctly aligned vertically. The distance between top and bottom trays is 50 cm. The middle tray is positioned close to the top tray to allow room for a lead mount and space for 10 cm. of lead. After some initial tests the lead was removed and in all of the experiments reported here the telescopes have operated without any lead absorber.

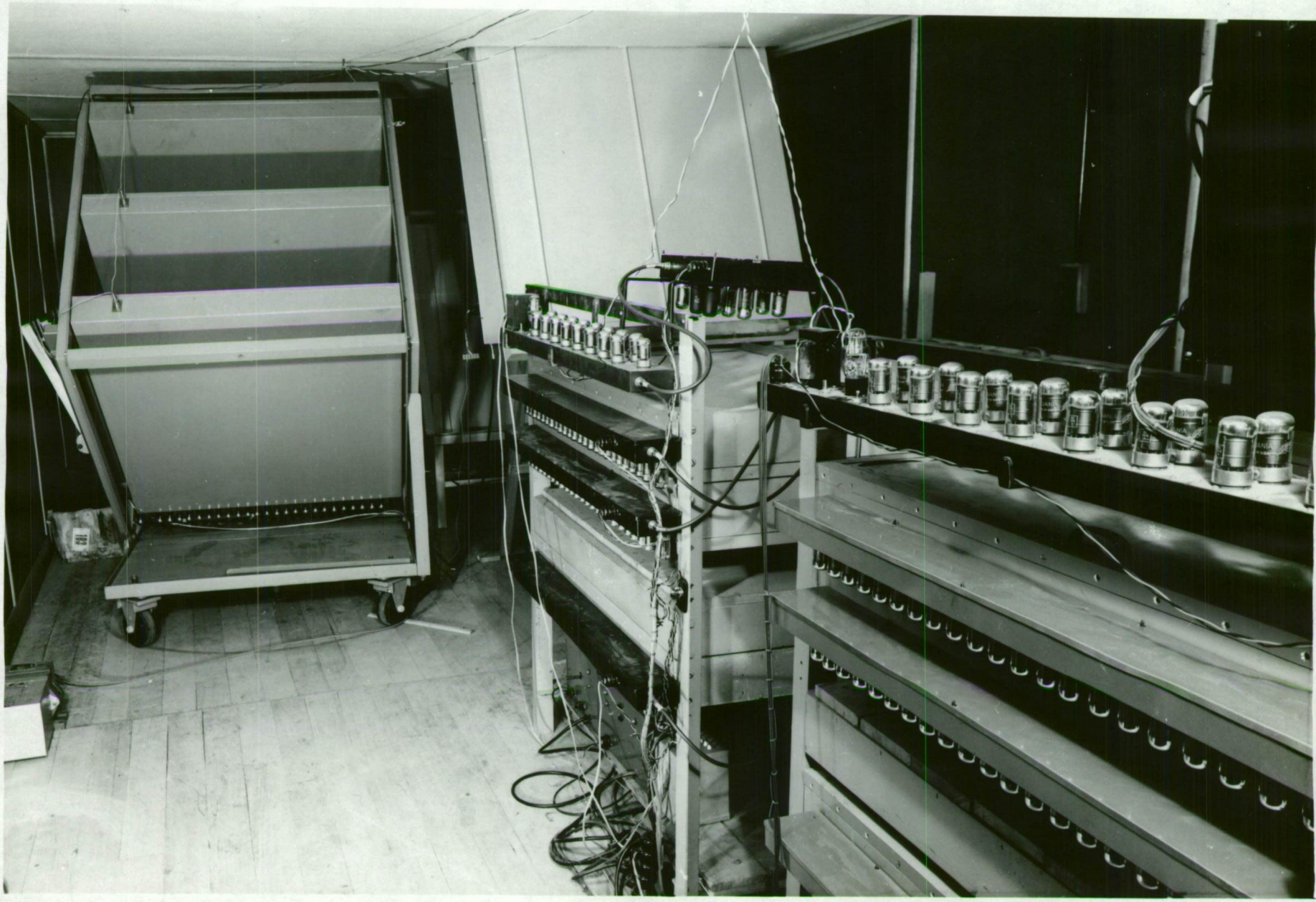
The constructional features of the two inclined telescopes will be described in the chapters relating to the inclined experiments. They have the same circuitry as the vertical semi-cubical units, consisting essentially of three tray strips, a three-fold coincidence circuit and a scaler, as described by Parsons.

A photograph (Figure 4.2) taken from the northern end of the laboratory gives a general view of the four telescopes.

(b) Voltage stabilisation, variations of room temperature

Until August 1958 an electronic type A.C. mains stabiliser

Figure 4.2



(manufactured by Stabilac Pty.Ltd., Sydney) protected the semi-cubical telescope circuits against voltage fluctuations of the mains supply. From August 1958 to July 1960 a stepwise-type voltage regulating B.A.T. transformer placed in series with the Stabilac brought about improved stability. However, because of the risk of losing data if the Stabilac circuit failed, a more rugged type of mains stabiliser was sought. In August 1960 the Stabilac was replaced by a saturated core voltage regulating transformer, which has proved to be efficient and trouble-free.

The mean temperature in the laboratory is approximately 55°F with maximum excursions of $\pm$ 10° over a year. The daily variation of temperature is not known, but would be expected to be small.

(c) The recording system

The main recording system is photographic, in conjunction with a chart trace. The scaled counts from each telescope are accumulated on three mechanical registers in series. Two of the registers are photographed and the records from them are compared from time to time to check against any losses of efficiency in the registers. The third register is a modified type, actuating a voltage pulse at the instant of every tenth scaled count. This pulse produces a deflection of one of the pens on the chart of a mechanically driven "Evershed" operations recorder. Six chart recorder channels are used, four of them recording scale-of-ten pips from the four telescopes. The two flanking channels record hourly time pips from a self-winding chronometer (manufactured by Thomas Mercer and Sons, St.Albans, England).

At the end of each GMT hour the chronometer contacts actuate the camera drive, shutter mechanism and the lights in the photographic recording panel. Thereby, hourly photographs are taken of the four pairs of registers and of an electric clock, which enables the approximate time-sequence of frames to be monitored.

The photographic unit and the electro-mechanical system for winding the chronometer operate from a 12-volt accumulator and the chart recorder solenoids are in parallel with a 50-volt A.C.-D.C. supply.

At weekly intervals the film and the chart have to be replaced and the Evershed recorder rewound. Consequently, general maintenance of the equipment is carried out on a weekly basis.

(d) Timing

Before the Mercer chronometer was installed in July 1960, timing was effected by a synchronous one hour Venner motor. Power breaks excepted, timing would have been accurate to within approximately 5 seconds. However, because of the possibility that average daily variations of timing might occur (amplitudes of the order of 1 second could not be tolerated) the system was inherently unsatisfactory. The battery-wound Mercer chronometer operates virtually independently of interruptions to the mains supply. It has performed with great reliability and for a long period has maintained a steady rate of gain of approximately 2.5 seconds per day. The time is checked periodically

against the National Bureau of Standards (U.S.A.) time signal WWV or the time signals transmitted by the Australian Broadcasting Commission.

A small, regular, 2-hour periodicity in timing has been found since the paragraph above was written. It is described in Section 4.11(b).

4.3 The material absorber

The results of a survey of the Tunnel Hill region made by the Geology Department of the University show that the rock structure of the tunnel area is a uniform Triassic Siltstone of dry density 2.41 ± 0.04 gm/cc. The nearest dolerite is approximately 1000 feet to the east. It is assumed, therefore, that in all directions above the tunnel the material is essentially siltstone of density 2.41 gm/cc, plus water that might be held by the rock and the layer (two to four feet) of soil above it.

A map showing contour levels at intervals of five feet became available in 1965. This has allowed the mean absorber within the viewing cone of any one of the telescopes to be calculated rather more accurately than had been possible earlier. It has also become possible to assess, with reasonable accuracy, other effects connected with the distribution of the absorber in azimuth and zenith angle, and the influence of traffic and the parking of vehicles within the viewing cones of the telescopes. Up to the present these more accurate calculations relate only to the vertical semi-cubical telescopes.

The profiles of the absorber in various azimuth planes are shown in Figures 4.3 and 4.4. The viewing cone of the vertical equipment is delineated

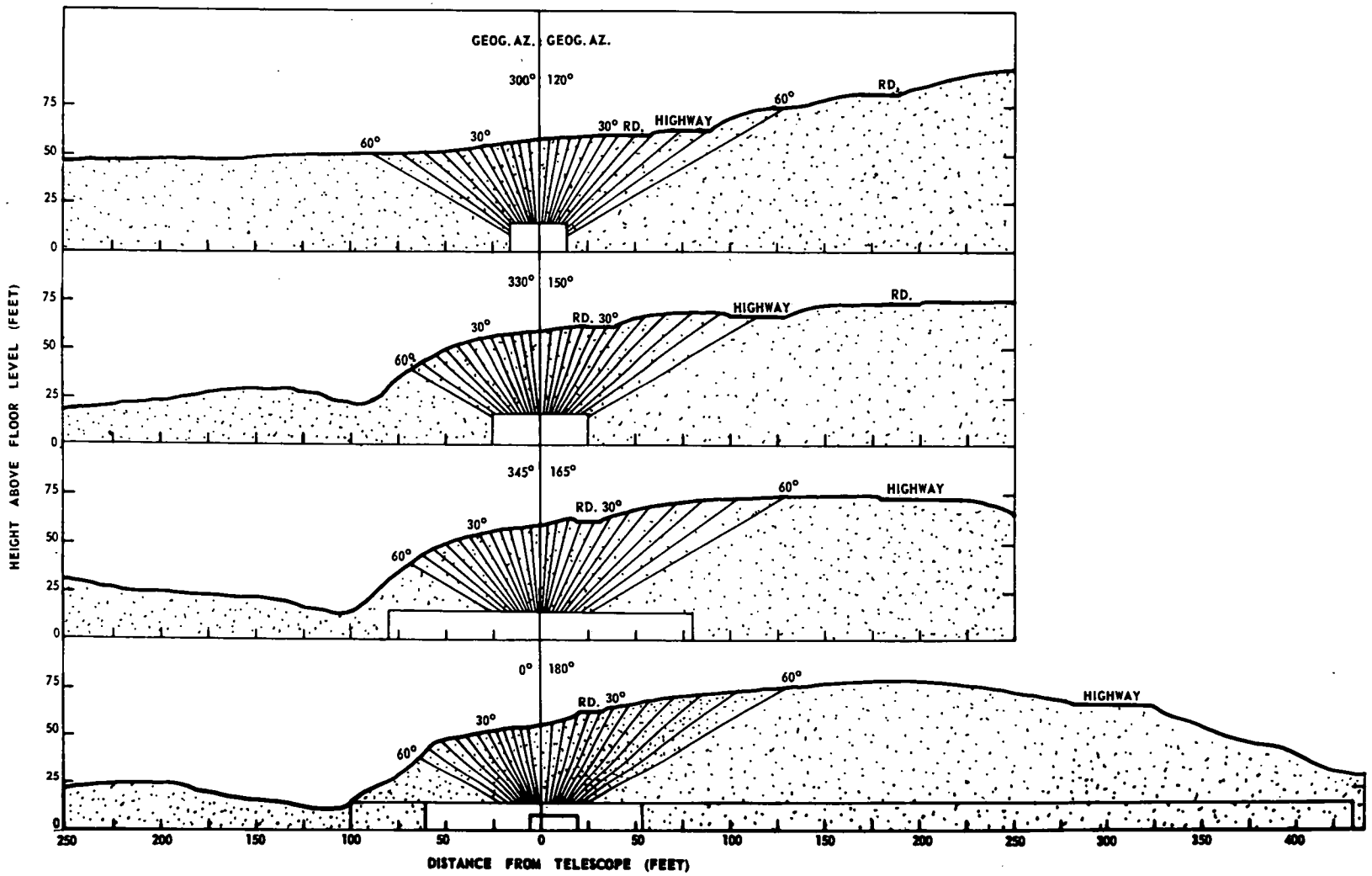


Figure 4.3 Hobart underground station. Profiles of the material absorber in the azimuthal planes 300° – 120°, 330° – 150°, 345° – 165° and 0° – 180°. The dimensions of the tunnel and of the cosmic ray laboratory in elevation, are shown to scale in the bottom diagram. Cross-sections of the tunnel in the various planes are indicated by undotted areas. The bearings in zenith, at 5° intervals, delineate the local viewing cone of one of the vertical semi-cubical telescopes in relation to the absorber, a private access road, the Tasman highway and the road to Mt. Rumney.

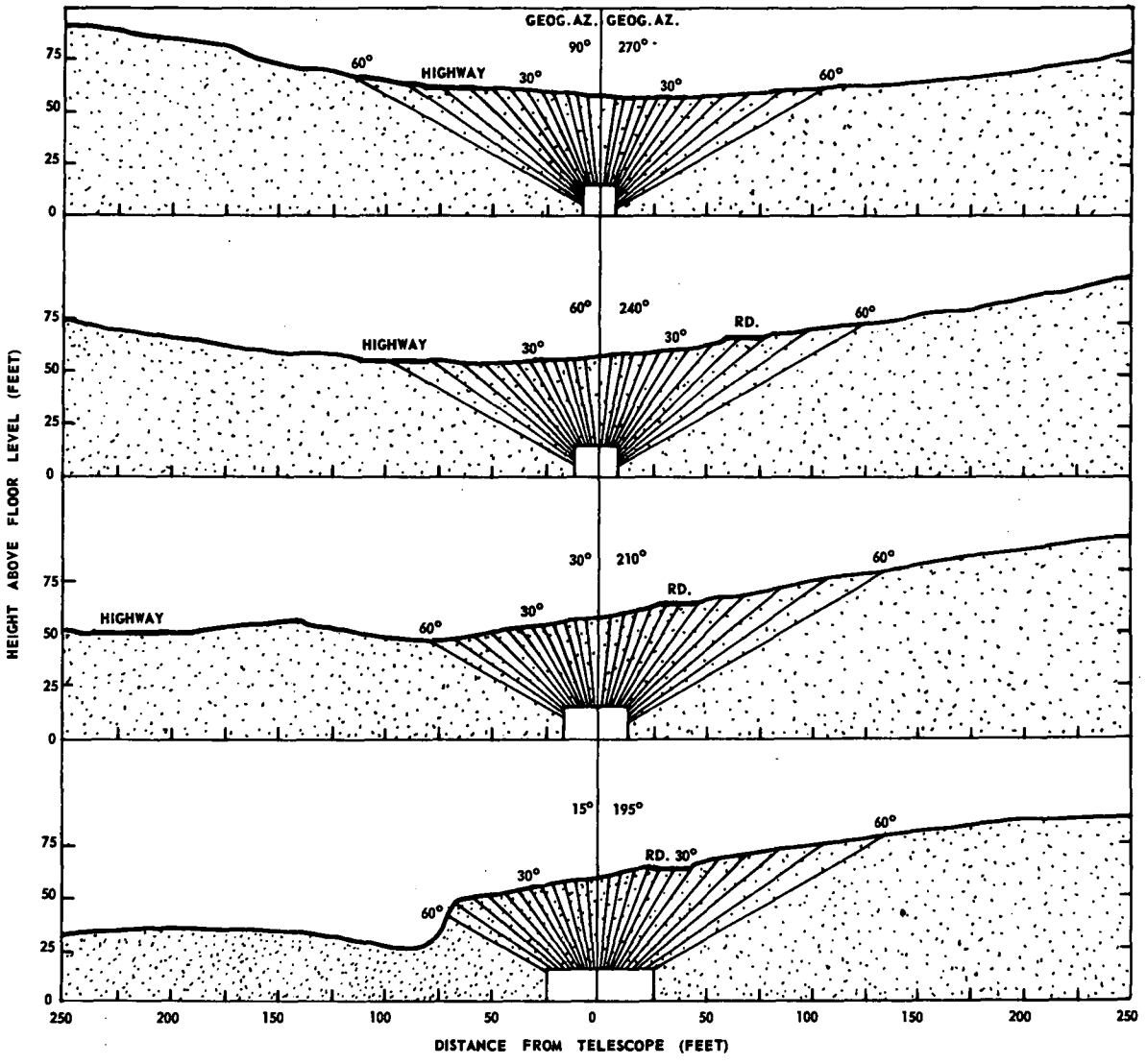


Figure 4.4 Hobart underground station. Profiles of the material absorber in the azimuthal planes $90^{\circ} - 270^{\circ}$, $60^{\circ} - 240^{\circ}$, $30^{\circ} - 210^{\circ}$ and $15^{\circ} - 195^{\circ}$

in cross-section by 5° bearings in zenith angle. Note where the boundary of the cone intersects the land surface at the zenith angle Z = 60°. Seen from above, the points of intersection form a rough circle, shown in Figure 4.1. This circle is the approximate boundary of the area of the surface penetrated by the particles which are detected by the vertical telescopes.

4.4 The local cone of acceptance of the vertical telescopes

The calculations of the sensitivity of a vertical semi-cubical telescope to the incoming secondary radiation, as a function of azimuth and zenith angle of arrival, derives from a theoretical treatment given by Parsons (1957) for telescopes at sea level. Parsons defines the sensitivity of a telescope to the radiation from a given direction as the product of the cross-sectional area presented by the telescope in that direction and the intensity of radiation arriving from that direction (particles per unit area, per unit solid angle, per unit time), and he calculates it in two stages. First, the influence of the cross-sectional area presented in a given direction is determined, on the assumption that the radiation above the top tray is uniform in all directions. This gives the *geometric sensitivity (G.S.)*. Then, the geometric sensitivity is multiplied by a factor specifying the observed zenity angle dependence of intensity, to give the *radiation sensitivity (R.S.)*.

(a) The geometric sensitivity (GS)

He considers an incident parallel beam of radiation inclined at an angle Z to the zenith (the direction of the telescope axis) from azimuth a.

Then the effective cross-sectional area presented by the semi-cubical telescope normal to the beam is given by

$$A = \cos z \left(1 - \frac{\tan z}{2} |\cos a|\right) \left(1 - \frac{\tan z}{2} |\sin a|\right) \qquad 4.1$$

He shows that the relative geometric sensitivity of the telescope to radiation from directions within a volume element (z to z + dz, a to a + da) is given by

$$GS_{z,a}\, dzda = A \sin z dzda \qquad 4.2$$

That is to say *the geometric sensitivity per unit solid angle of arrival*, $GS_{z,a}$, is given by

$$GS_{z,a} = A \sin z = \sin z \cos z \left(1 - \frac{\tan z}{2} |\cos a|\right)\left(1 - \frac{\tan z}{2} |\sin a|\right) \qquad 4.3$$

The value of $GS_{z,a}$ for a given value of Z varies somewhat with azimuth, the more so as Z increases. The total sensitivity for a given z is $GS_z = \int A \sin z\, da$, the integration being taken over all values of a for which the cross-sectional area A is positive. It may also be regarded as the average geometric sensitivity for the given z.

In later chapters the integrated responses to solar and sidereal anisotropies will be estimated. Whether $GS_{z,a}$ was used in the calculations or was replaced by the values GS_z, integrated over all azimuths and

appropriately normalised, the final result was found to be essentially the same. The large azimuthal variations at the high zenith angles are outweighed by the reduced overall sensitivities at these angles. Consequently, it is sufficient for our purposes to consider the semi-cubical telescope as having circular cross-section and to replace $GS_{z,a}$ by GS_z.

(b) The radiation sensitivity (RS)

It is well known that the μ-meson intensity at sea-level varies with zenith angle z as $\cos^{\lambda}z$, so that

$$Iz = I_o \cos^{\lambda}z$$

where I_o is the vertical intensity.

Parsons finds that the zenith angle dependence of intensity at sea-level is well approximated if $\lambda = 2.2$, at least up to $z = 70^o$.

Allowing for the zenith angle dependence of intensity, the total sensitivity of the telescope to the incoming radiation as a function of zenith angle is known as the *radiation sensitivity (RS)* and is given by

$$RS_z = GS_z \cos^{2.2}z \qquad 4.5$$

At moderate depths underground the zenith angle dependence of intensity appears to be of the same form as at sea-level, although λ seems to

be closer to 2.0 than 2.2 (Cousins, Nash and Pointon, 1957; Cousins and Nash, 1962). The asymptotic cones of response (Chapter 6) were calculated for both these values of λ and the difference in the final response to anistropies was found to be negligible.

It is concluded, that as far as present studies are concerned, the radiation sensitivity underground would be sufficiently well approximated by that calculated for sea-level telescopes, if we were to disregard directional inequalities in the material absorber.

(c) The modified radiation sensitivity (RS')

In fact, as Figures 4.3 and 4.4 show, the amount of material absorber underground varies considerably with direction of viewing within the cone of acceptance of the telescope. It is therefore necessary to calculate a modified radiation sensitivity, RS', of the form

$$RS'_{za} = \left(\frac{I}{I_o}\right)_{z,a} RS_z$$

where $\left(\frac{I}{I_o}\right)_{z,a}$ is the ratio of the intensity (I) in the direction (z,a) to the intensity (I_o) in the vertical direction, due only to the difference in amount of material absorber. The ratios for all the directions of interest have been estimated from the intensity-depth curve relating to the vertical direction, published by Thorndike (1952) and based on the measurements of Clay and others (Clay, 1939). For example $\left(\frac{I}{I_o}\right)_{50,180}$ is calculated as follows. The depth of material absorber directly above the vertical equipment is 42 feet (Figures 4.3

and 4.4). We assume a mean density of 2.41 gm/cc as for dry siltstone. The amount of absorber is therefore approximately 30.9 m.w.e. From the intensity-depth curve the relative intensity at a depth of 30.9 m.w.e. below ground is 0.165. This is I_o. At $z = 50^o$, $a = 180^o$, the depth of absorber is 88 feet (Figure 4.4) or 64.5 m.w.e. The relative intensity at this depth in the vertical direction is 0.054. This is I. Hence $(\frac{I}{I_o})_{50,180}$ is 0.327.

Since the rate of decrease of intensity with depth below ground is somewhat less at a given depth in an inclined direction than in the vertical direction (e.g. see Clay, 1939, Figure 2) the use of the vertical intensity-depth curve tends to overestimate effects due to the directional inequalities of the material absorber.

(d) Specification of the local cone of acceptance

In the various applications that will follow, the solid angle of acceptance of a vertical semi-cube is divided up into 576 elements (ω_r) of dimensions 7.5° in azimuth (centred on 0°, 7.5°, 15°, . . . 352.5°) and 5° in zenith angle (centred on 5°, 10°, 15°, . . . 55°), and the radiation sensitivity RS' is calculated for each of these elements. This provides an assembly of directional sensitivities that is used in the determination of the mean absorber and its directional effects and in the calculation of the asymptotic cones of acceptance (Chapter 6).

4.5 Characteristics of the absorber viewed by the vertical telescopes

(a) Directional inequalities

1. The effect in zenith angle

The dashed curve in Figure 4.5 shows the zenith angle dependence of RS, summed over all azimuths, for a semi-cubical telescope at sea-level, as calculated by Parsons. The material absorber above the underground telescopes reduces the sensitivity at the higher zenith angles. The full line shows the modified zenith angle dependence of RS', summed over all azimuths, so that it takes into account the average inequalities of the absorber in zenith directions.

2. The effect in azimuth

If the values of RS'_r at a given azimuth are averaged over all zenith angles within the cone of acceptance at that azimuth, the average response of the vertical equipment to inequalities of the absorber in azimuth may be plotted at intervals of 7.5°. The result is shown in Figure 4.6, where the relative intensity of response has been normalised to 100% at the azimuths where the absorber is a maximum. It can be seen that inequalities of the absorber, as they influence the counting rate, are rather symmetrically distributed about the N-S plane. In fact, it is found from the curve that 49.4% of the counting rate is due to particles arriving east of the meridian plane, 50.6% arriving from the west. Consequently, no significant bias of the local cone of acceptance either east or west of the meridian accrues from the distribution of the absorber.

Figures 4.5 and 4.6 indicate that the distribution of the absorber

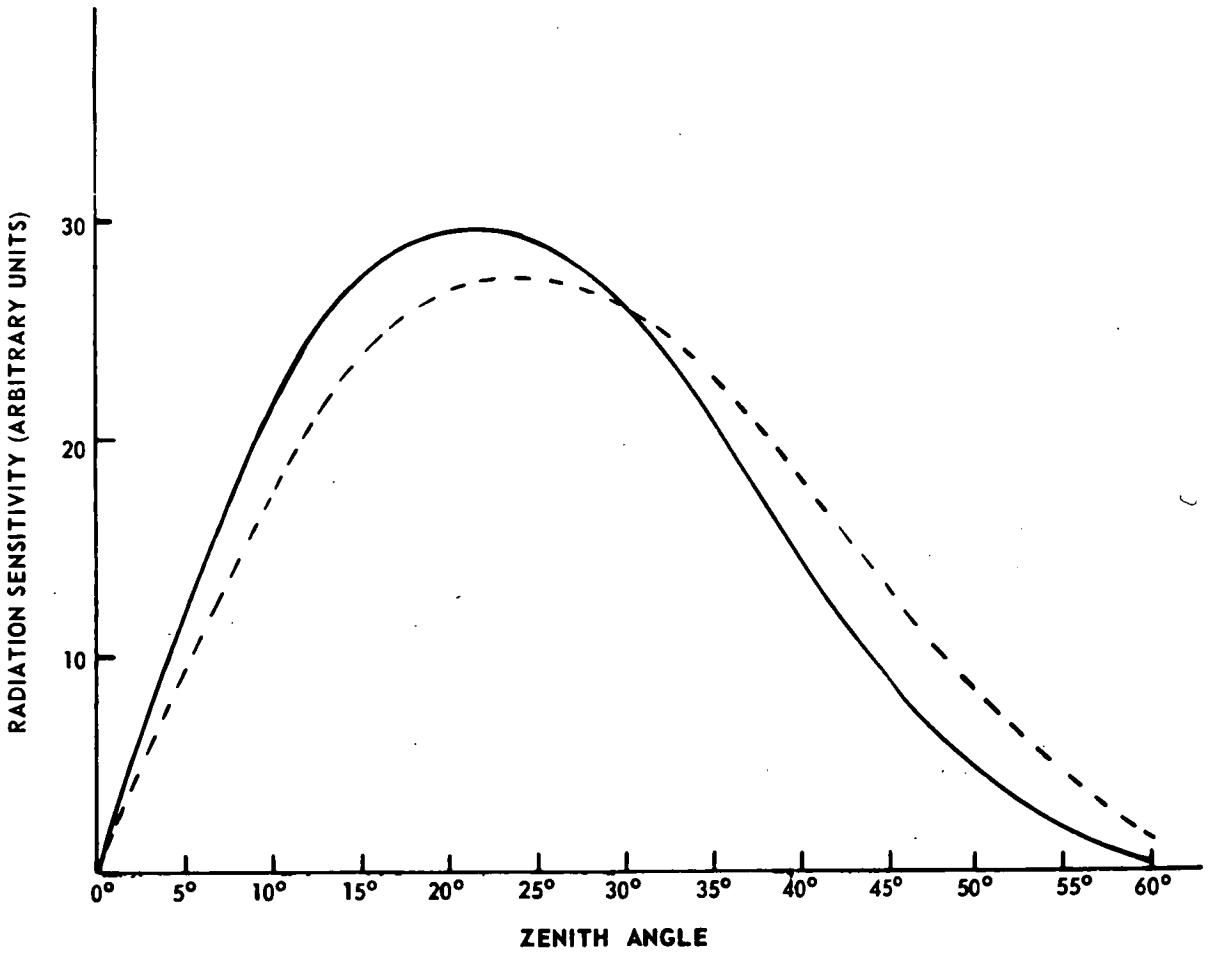


Figure 4.5 Mean radiation sensitivity as a function of zenith angle of arrival for a vertical semi-cubical telescope. The dashed line gives the calculated effect at sea level (from Parsons). The full line gives the calculated effect for a semi-cube at the underground station at Hobart, taking into account the distribution of material absorber above the telescope.

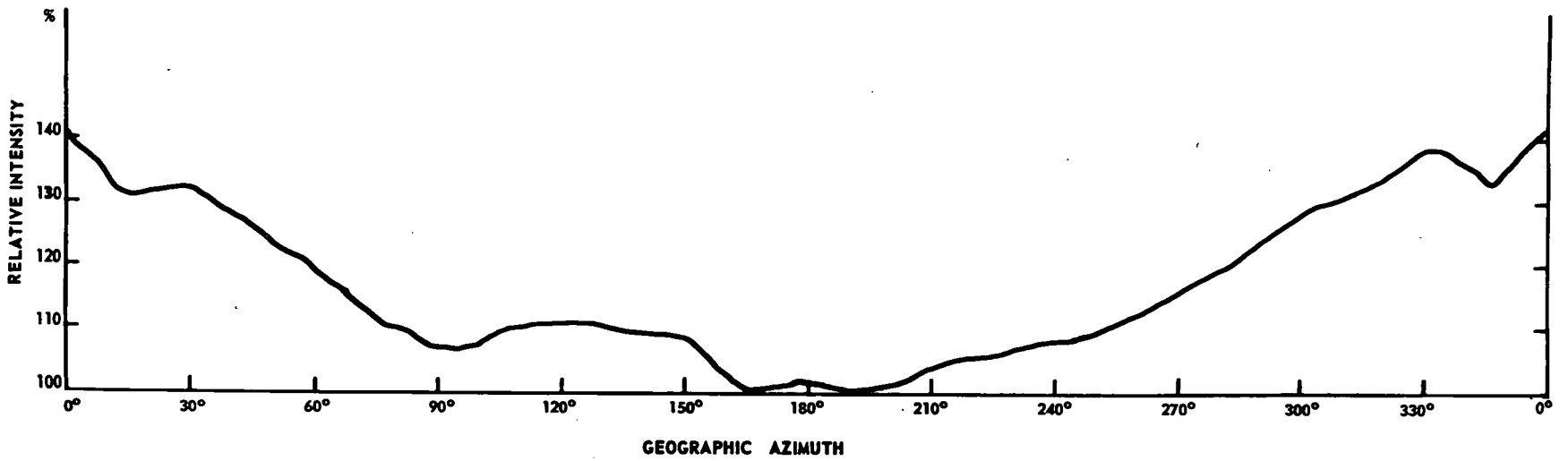


Figure 4.6 Radiation sensitivity of a vertical semi-cubical telescope at the Hobart underground station. Azimuthal variation due to the distribution of material absorber. The differential intensities have been estimated relative to the minimum value, at 165° of azimuth.

must be taken rather carefully into account in studies of the observed daily variation underground. If not, the tilting and other distortions of the local cone of acceptance that may be produced by the absorber could lead to serious errors in the calculation of the response to an anisotropy. On the other hand, if the influence of the absorber is included in the cone of sensitivity, any directional bias which results is eventually incorporated in the asymptotic cone of acceptance (Chapter 6).

A plaster model (Figure 4.7) demonstrates the characteristics of the local cone of sensitivity of the semi-cubical vertical telescope underground at Hobart. Ideally, the solid cone should meet the flat base at the point where the converging inside and outside surfaces join the axis of the telescope which is envisaged as emerging from the centre of the cone normal to the base. Zero sensitivity in the vertical (axial) direction (see also Figure 4.5) follows from the fact that the distribution of *sensitivity per unit solid angle* is being represented here, not the sensitivity to parallel radiation. Per degree of azimuth, per degree of zenith angle, the area of surface of a volume element penetrated by the radiation becomes vanishingly small as the vertical direction is approached. On the other hand, at high zenith angles the effective cross-sectional area A (equation 4.1) presented to the incoming radiation, becomes vanishingly small as the boundary of the telescope is approached. These competing influences cause maximum sensitivity to occur at a zenith angle of approximately 25^{o}. The reduced amount of absorber in the northerly azimuths (Figures 4.3 and 4.5) is reflected in the distortion of the model.

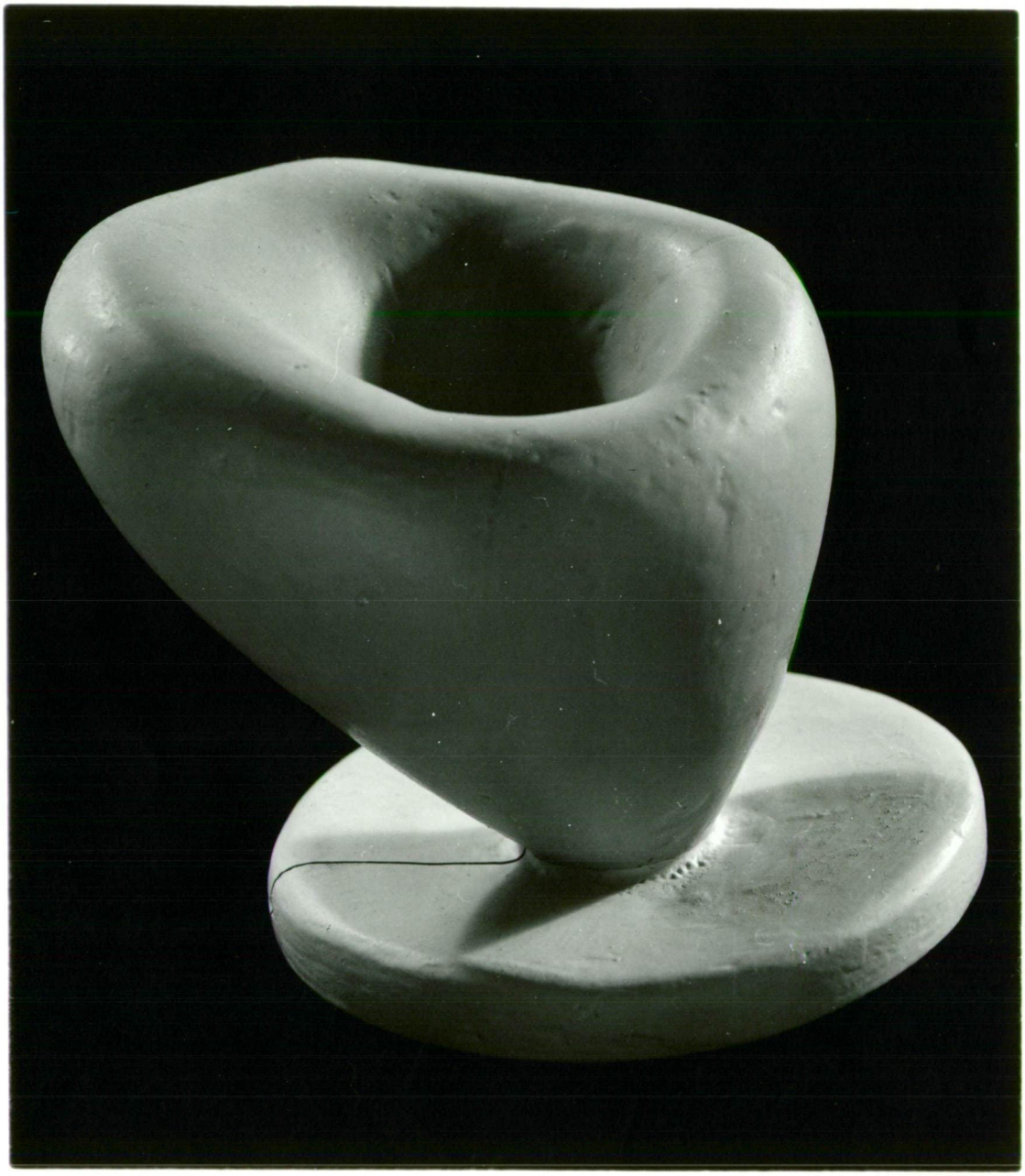


Figure 4.7

In the treatment of asymptotic cones of acceptance (Chapter 6) the determination of the asymptotic constants is based on the distribution of RS, rather than RS', since it is found that the constants would be very nearly the same in both cases. For instance, it will be seen in Table 6.1 that the asymptotic constants for the lowest rigidity (R = 50 GV), at which the greatest discrepancies should occur, are (A_R = 0.87, δ_{1R} = -34°, ϕ_{1R} = 30°). Based on the distribution of RS', the constants are (A_R = 0.90, δ_{1R} = -34°, ϕ_{1R} = 29°).

(b) The amount of dry material absorber above the vertical telescopes

If the radiation sensitivity appropriate to the volume element ω_r is RS'_r, and the amount of material absorber in that direction is m_r, then the average amount of absorber within the viewing cone of the telescope is

$$\bar{m} = \frac{\sum_r m_r \, RS'_r}{\sum_r RS_r} \qquad 4.6$$

where the summations are taken over the 576 elements of the local acceptance cone. By this method, the weighted average amount of dry absorber above the semi-cubes up to ground level was calculated to be 35.9 m.w.e., or 48 feet of material.

Material above the rock, consisting of two to three feet of clay and fragmented rock and one foot of topsoil, has not been distinguished from the rock itself in the calculation, so that it would be necessary to subtract perhaps 0.3 m.w.e. to allow for the difference in density. Therefore, the estimate of the amount of dry absorber above the telescopes is

$$\bar{m} = 35.6 \text{ m.w.e.}$$

(c) The average water content

It is very difficult to estimate precisely the average amount of water held in the rock and in the soil above it. Only a rough guess will be offered here.

Murdoch, Ogilvie and Rathgeber (1959) have estimated that, on the average, the water content of the sandstone above their underground laboratory in Sydney increased the density of the rock by 0.05 gm/cc, equivalent to 0.015 m.w.e./foot. It is known that the porosity of Triassic siltstone is low, very much less than that of sandstone. When we consider also that an annual average rainfall of 1.2 metres in Sydney compares with 0.6 metres at Hobart, it is concluded that the average depth of rock above the vertical telescopes, somewhat less than 49 feet, holds considerably less than 0.7 metres of water. It is suggested that half that figure would be an upper limit and would include moisture held in fissures in the rock.

Some indication of the average moisture content in the layer of crumbled rock, clay and topsoil above the siltstone may be gained from the following. As reported in paper 3, phenomenal rainfalls (approximately 40% of the annual total), which ducceeded a long period of dry weather, were experienced over a few days in April 1960 and appeared to be responsible for a decrease of approximately 0.8% in the counting rate underground. This was equivalent to an increase of approximately 0.25 metres of water in the material absorber. It is suggested that this roughly represented the transition from the dry to the saturated state of the material above the rock, so that on the average it would

not be expected to hold more than about half of that amount of water.

It is concluded that the average water content of the material absorber above the equipment is not likely to be greater than approximately 0.5 m.w.e.

(d) The total material absorber

Adding the water content to the amount of dry absorber, the mea absorber above the vertical telescopes to ground level is now taken to be approximately

$$\bar{m} = 36 \text{ m.w.e.}$$

Successive estimates of the mean absorber have followed a downward trend. In the earlier papers (appendices III and IV) the figure gi was 42 m.w.e. Overestimation of the depth of the material was the main sour of error. Later papers (appendices V, VI and VII) specify 40 m.w.e. as the better approximation. This was deduced from an inter-comparison of counting rates above and below ground, using the intensity-depth relationship. Normalisation for the different telescope geometries was perhaps the greatest factor of uncertainty here. Since most of the calculations described in thi thesis had been made before the contour map of the tunnel area became availab it will be found that the mean absorber is usually taken to be 40 m.w.e.

(e) Road traffic and parking of vehicles above the equipment

The Tasman Highway (Figures 4.1, 4.3 and 4.4) lies within the

viewing cone of one of the vertical telescopes for a distance of approximately 170 feet, from $a = 60^{o}$, $z = 60^{o}$ and distance 105 feet from the telescope, to $a = 150^{o}$, $z = 60^{o}$ and distance 115 feet. The zenith angle decreases to 50^{o} at $a = 90^{o}$. Accordingly, the sensitivity to traffic will not be underestimated if the segment of highway is considered to lie in a quadrant of azimuth at a zenith angle of 50^{o} from the telescope axis.

Influence of the daily variation of the volume of traffic. The Traffic Engineering Section of the Transport Commission has kindly made available the hourly records, from an automatic counter, of the volume of traffic that passed over Tunnel Hill from midnight, 6th November 1963, to midnight, 15th November. The average flow was 113 vehicles/hour. The average daily variation is shown in Figure 4.8. The amplitude of the first harmonic is 73% of the mean, or 82 vehicles/hour. Allowing for heavy vehicles, the average speed is assumed to be 30 ft/sec and the average weight 5700 lbs (2600 Kg). In the calculations that follow, a vehicle is thought of as a sheet of lead 14 feet long, 6 feet wide and 3 cm. high. At the peak of the first harmonic:

Time interval between excess vehicles = 44 sec.
Therefore average spacing between vehicles = 1320 feet, and
the numbef of excess vehicles within view of telescope = 0.13

Distributing the mass of 0.13 vehicles over 170 feet of highway, 6 feet wide, the equivalent lead thickness (T) = 0.0322 cm., or 0.354 cm. water equivalent.

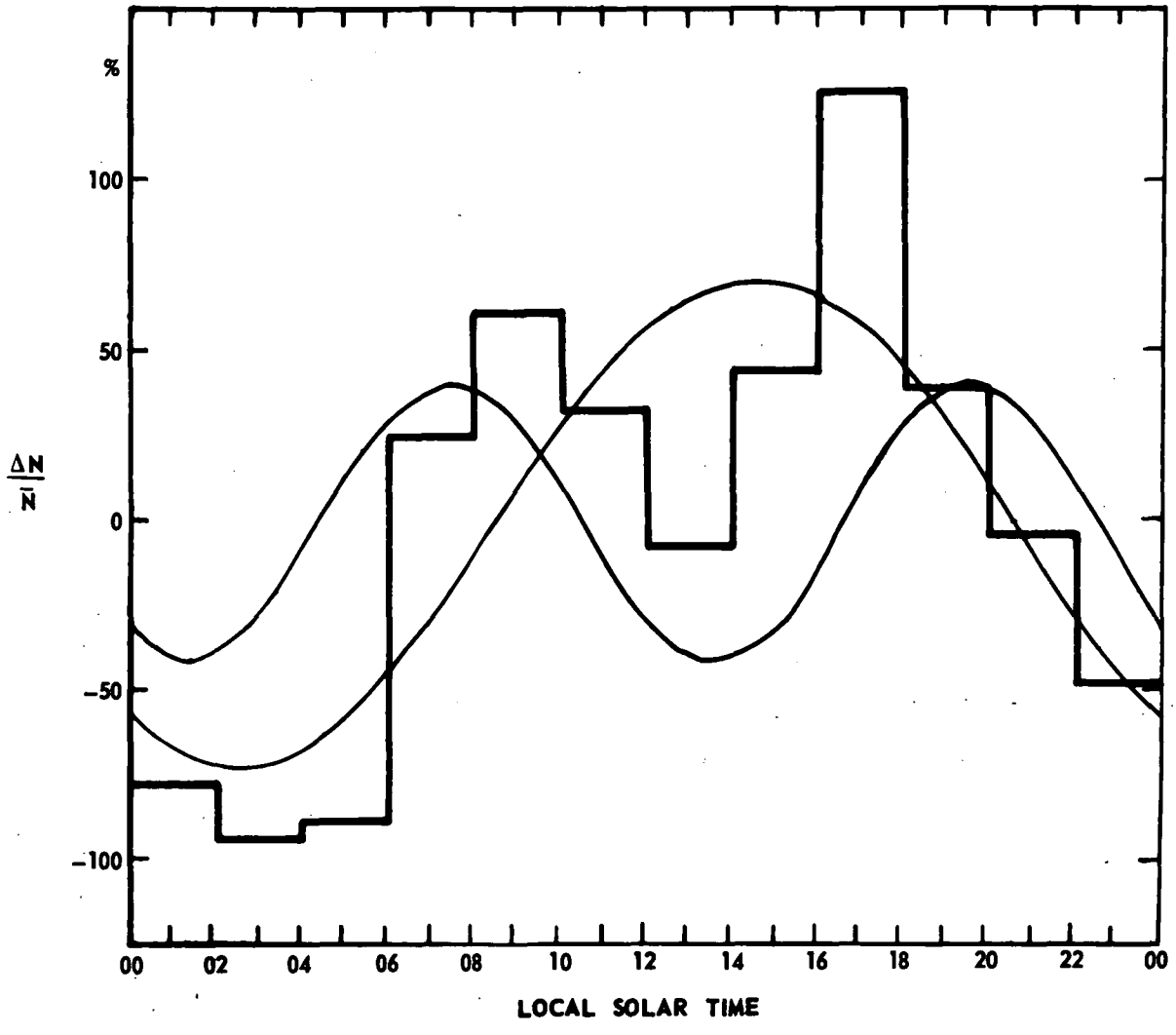


Figure 4.8 The daily variation of vehicular traffic over Tunnel Hill, near Hobart, during November 1963. The percentage deviations refer to a mean rate ($\bar{N}$) = 113 vehicles per hour. The curves are the first and second harmonics of best fit.

Per quadrant of azimuth the average radiation sensitivity (RS'_z) at $z = 50^\circ$ is 0.13%/deg.z. The ribbon of lead subtends an angle of approximately 3.5° in zenith angle at the telescope. Consequently, 0.45% of the total particle flux (I) would pass through the area covered by the lead. From the intensity-depth relationship (Thorndike, 1952),

$$\frac{dI}{dT} = -0.0303\%/\text{cm. water equivalent}$$

Therefore the amplitude of the first harmonic of the solar daily variation due to the daily variation of the volume of traffic is

$$\left(\frac{\Delta I}{I}\right)_{max} = 0.0303 \times 0.354 \times 0.0045\%$$
$$= 0.049 \times 10^{-3}\%$$

and the time of maximum intensity is 0236 solar time. A similar calculation gives 0.027×10^{-3} % as the amplitude of the second harmonic, the maxima occurring at 1324 and 0134 solar time.

Clearly, the calculated effect is too small by three orders of magnitude to be of any significance, since the amplitudes of the observed annual mean daily variations underground are of the order of 0.05%. It may be concluded that the influence of the daily variation of the volume of traffic is negligible, while conceding a wide margin of error in estimation of the various constants.

Parking of vehicles. The average motor vehicle parked on the side of the highway within view of the vertical telescopes subtends an angle of approximately 8^{o} in azimuth and 3.5^{o} in zenith angle at the equipment. Representing the vehicle as we have done, by a slab of lead, 3 cm. thick, the absorber is 33 cm. water equivalent. The radiation sensitivity per 8^{o} of azimuth, per 3.5^{o} of zenith angle at $z = 50^{o}$ is $RS' = 0.0416\%$. Consequently the decrease of intensity produced by the vehicle is

$$\frac{\Delta I}{I} = (0.0303 \times 33 \times 0.00042)\%$$
$$= 0.042 \times 10^{-2}\%$$

If a single vehicle is parked near the highway for eight consecutive hours of the day, the amplitudes of the first and second harmonics of the daily variation it produces at the equipment will be approximately

$$\frac{0.00042}{2} = 0.021 \times 10^{-2}\%$$

Parking in the zone of maximum sensitivity (e.g. on the access road to Wiggins' farm) could increase the response by as much as a factor of 25, but it is evident that even then the daily variation observed at the equipment would be imperceptible. It is concluded that the occasional parking of vehicles that does occur near the junction of the highway and Wiggins' access road would have negligible influence on the daily variation underground. Nevertheless, parking is potentially the more serious of the two effects we have considered.

4.6 The mean cut-off energy at production, $\overline{E}_c$

The cut-off energy at production is the initial energy of the μ-meson at the production level (assumed to be 100 mb) that is just sufficient to allow the meson to penetrate the atmosphere and material absorber and be reocrded by the vertical equipment. The value that matters is not the cut-off in the vertical direction, where the sensitivity per unit solid angle (RS') is zero, but the weighted mean value, $\overline{E}_c$, averaged over the cone of sensitivity:

$$\overline{E}_c = \frac{\sum_{z,a} (E_c)_{z,a} \, RS'_{z,a}}{\sum_{z,a} RS'_{z,a}} \qquad 4.7$$

A sufficiently accurate value for the present can be obtained by disregarding the azimuthal variations of RS' due to the absorber and using representative values for each zenith angle, as can be obtained from the full curve in Figure 4.5 . Then

$$\overline{E}_c \simeq \frac{\sum_z (E_c) \, RS'_z}{\sum RS'_z} \qquad 4.8$$

The summations are taken over 12 values of zenith angle, selected at 5° intervals.

The cut-off energy $(E_c)_z$ at the zenith angle z must be estimated in its two parts, that due to the material absorber and that due to the atmosphere. In column 2 of Table 4.1, the material absorber at each zenith angle is the value averaged over all azimuths. In column 3 the corresponding cut-off energy has been calculated on the assumption that the average μ-meson

energy loss for low energies of arrival underground is 2.1 Mev $gm^{-1}cm^2$. This has been estimated from the work of (a) Murdoch, Ogilvie and Rathgeber (1959), who calculate the momentum losses versus momentum for μ-mesons, arriving under 7000 $gm.cm^{-2}$ of sandstone, increasing from 1.9 Mev/c $gm^{-1}cm^2$ near zero momentum of arrival to e.g. 2.2 Mev/c gm^{-1} cm^2 at 10 Gev/c; and (b) Cousins and Nash (1962) who give calculated average energy losses in quartz, increasing from 1.6 Mev gm^{-1} cm^2 at the lowest energies of arrival at their depth of 36 m.w.e. (dry) to e.g. 2.3 Mev gm^{-1} cm^2 at an arrival energy of 20 Gev.

The energy loss in the atmosphere at various zenith angles and for the different cut-off energies at the surface (column 4) has been estimated from a set of tables of range in air versus initial momentum, compiled by K.B. Fenton (1951), based on the Bethe/Bloch expression for energy loss by ionization and excitation (Bethe, 1932; Bloch, 1935). A flat, exponential atmosphere has been employed in the calculation of the mass of air to 100 mb at the different zenith angles.

TABLE 4.1

Meson cut-off energies and radiation sensitivity (RS'_z) versus zenith angle within the cone of acceptance of a vertical semi-cubical telescope underground. The values are averaged over all azimuths.

Zenith angle degrees	Depth of material absorber (m.w.e.)	Cut-off energy at surface (Gev)	Energy loss in atmosphere (Gev)	Cut-off energy at production $(E_c)_z$ (Gev)	Radiation sensitivity RS'_z%
0	30.9	6.5	2.5	9.0	0.00
5	31.6	6.6	2.5	9.1	0.062
10	31.6	6.6	2.5	9.1	0.111
15	33.1	6.9	2.6	9.5	0.140
20	33.8	7.1	2.7	9.8	0.149
25	35.3	7.4	2.8	10.2	0.147
30	36.0	7.6	2.9	10.5	0.130
35	38.9	8.2	3.2	11.4	0.103
40	43.3	9.1	3.5	12.6	0.073
45	47.7	10.0	3.9	13.9	0.046
50	53.6	11.3	4.7	16.0	0.025
55	61.7	12.9	5.7	18.6	0.012
60	74.9	15.7	6.7	22.4	0.002

Columns 5 and 6 give the pairs of values of cut-off energy at production $(E_c)_z$ and radiation sensitivity RS'_z from which the weighted mean value of the cut-off energy, $\overline{E}_c$, is calculated. Using equation 4.8, the value obtained was

$$\overline{E}_c = 10.6 \text{ Gev}$$

This figure is accurate to within 1 Gev if we allow a margin of error of $\pm$ 0.5 m.w.e. in estimation of the depth of absorber and $\pm$ 0.2 Mev/c gm^{-1} cm^2 for

uncertainty in estimation of the average μ-meson momentum loss.

A knowledge of the cut-off energy at production is of importance in the determination of the effective primary spectrum responsible for the mesons that are able to be detected underground. The differential coupling coefficients specifying the fractions of the counting rate due to the primary protons per interval of primary energy, have been derived by Fenton from his calculated effective primary spectrum (Fenton, 1963) and enter critically into the computations of the asymptotic cones of acceptance, to be described in Chapter 6. In his determination of the spectrum, Fenton used 15 Gev as the best estimate available at the time for the cut-off energy E_c. An important refinement of the treatment of the asymptotic cones of acceptance given here may result from the re-evaluation of the coupling coefficients using the latest value of E_c given above.

4.7 The atmospheric effect

It has been assumed in the calculation of the atmospheric correction coefficients that the meteorological effect on the meson intensity underground is adequately described by Duperier's regression equation (Duperier, 1949)

$$\frac{dI}{I} = \beta_{1B\cdot HT}\, dB + \beta_{1H\cdot BT}\, dH + \beta_{1T\cdot BH}\, dT \qquad 4.10$$

where B is the barometric pressure at sea level, H is the height of the assumed mean pressure level for production (100 mb) and T is the temperature

in the vicinity of the production level (taken to be the mean temperature in the interval 100 - 200 mb).

An account of the evaluation of the coefficients and a discussion of their significance is given in paper 3 (appendix III). The observed coefficients, derived from the analysis of 432 sets of daily mean values of the variables, are, with the standard errors of estimate,

Total barometer coefficient, $\frac{dI}{dB} = \beta = -0.65 \pm 0.02\%/cm$

Partial barometer coefficient, $\beta_{1B \cdot HT} = -0.59 \pm 0.02\%/cm$

Negative temperature coefficient $\beta_{1H \cdot BT} = -0.46 \pm 0.13\%/km$

Positive temperature coefficient $\beta_{1T \cdot BH} = +0.02 \pm 0.005\%/^{o}C$

Positive 3-fold temperature coefficient $\beta_{1T \cdot B} = +0.03 \pm 0.005\%/^{o}C$

The only other known determinations at approximately the same depth relate to the vertical semi-cubical telescopes at the Budapest underground laboratory. Sandor, Somagyi and Telbisz(1962) report the following values for the partial regression coefficients:

$\beta_{1B \cdot HT} = -0.78 \pm 0.04\%/cm$

$\beta_{1H \cdot BT} = -1.42 \pm 0.08\%/km$

$\beta_{1T \cdot HB} = -0.19 \pm 0.06\%/^{o}C$

The errors given are total values with contributions from count rate statistics and from instability in the equipment.

Dutt and Thambyahpillai (1965), in a recent treatment of the barometric effect, have indicated that the Hobart values of the total and partial barometer coefficients are in accord with observations at London at 60 m.w.e. while the Budapest values are too high. It is not known whether the average amounts of absorber at Hobart and Budapest are strictly comparable or whether the absorber is distributed in approximately the same manner, but it would seem from the close agreement between the respective values of the observed solar daily variation (Table 6, Appendix VII) that the effective absorber is almost the same at both places.

The differences between the Hobart and Budapest coefficients may have come about through the short-term variability of the atmospheric effect. In an earlier paper (Sandor et al, 1959), the authors showed that over the period of 13 months of their analysis, month to month fluctuations in the coefficient at Budapest were far in excess of the variability due to counting rate statistics. For instance, their average total barometer coefficient, $\beta = -0.86 \pm 0.01\%/cm$, was considerably influenced by a very large value ($-1.72 \pm 0.04\%/cm$) in August 1958. In its absence, the mean value of β would have been $-0.76\%/cm$.

It is likely that the coefficient β at Hobart would also vary considerably over short periods of time, but the value given here, averaged over two years of data, would seem to be reliable, judging by the values obtained for the two individual years:

1958 $\beta = -0.69 \pm 0.05\%/cm$ (single telescope)

1959 $\beta = -0.65 \pm 0.02\%/cm$ (duplex telescope)

Since observations of the daily variation of atmospheric structure are not available at Hobart, no attempt has been made to correct the observed daily variation of intensity for atmospheric temperature changes. Even if it were possible to do this, it appears that the method of Dorman or Maeda would be more accurate than that of Duperier (e.g. see Bercovitch, 1963). We note that the observed negative temperature coefficient underground ($-0.46 \pm 0.13\%/km$) is very much less than that at sea level at Hobart ($-4.9 \pm 0.9\%/km$). As Dutt and Thambyahpillai have observed (1965), after comparison of the widely differing experimental values at London, Budapest and Hobart with the theory, the decay effect appears to be very weak at these depths underground.

If there was an out-of-phase contribution to the solar daily variation underground due to variations of atmospheric temperature, it should have revealed itself over the six years of observation, because of the large changes that have occurred in the annual mean amplitude. It will be shown in the following chapters that there is no indication of a significant effect of this kind.

On the other hand, a daily variation due to the positive temperature effect would be expected to be more nearly in phase with the solar daily variation of extra-terrestrial origin and would be difficult to detect from examination of long-term trends. However, a study of the daily variation

underground in relation to the solar anisotropy (Chapter 6) indicates that if there was any contribution of atmospheric origin it would have been small compared with the total solar daily variation.

4.8 Data processing

(a) Barometric correction

The data, in the form of daily values of intensity, are corrected for pressure variations only, using the total barometer coefficient $\beta = -0.65\%/\text{cm}$ ($-1.65\%/\text{inch}$). Corrections to the nearest scaled count introduce maximum errors of $\pm$ 0.1%. Hourly mean values of surface pressure are obtained from daily barographs at the Hobart office of the Bureau of Meteorology. To correct for instrumental errors, the barograph records are checked against spot readings taken with a mercury column barometer at 0600, 0900, 1200, 1500, 1800 and 2100, local time, each day.

(b) Corrections for changes in counting efficiency

A continuous record is kept of the daily mean values C_1/C_2, C_1/C_3, C_3/C_4 where C_1 and C_2 are the counting rates of the two vertical telescopes underground and C_3 and C_4 are the counting rates of the inclined telescopes. Sudden changes or short-term drifts in the ratios can normally be traced to a change in the counting efficiency of a particular telescope. In doubtful cases, where it appears that more than one of the counting efficiencies has changed or where continuity has been interrupted because of lack of data from one of the telescopes, graphs of the pressure-corrected daily mean intensities are examined and from these it is almost

always possible to detect a sustained change in counting level in either of the vertical telescopes.

Whenever sustained efficiency changes of magnitude approximately 0.1% or greater are detected in the vertical intensities, a normalising factor is computed for the combined intensity $C = C_1 + C_2$ and two other factors are computed which will give the equivalent of C if C_1 or C_2 is missing. When the hourly values of the counting rate C (or C_1 or C_2 if only one telescope is operating) have been corrected for pressure, they are multiplied by the normalising factor that prevails at that time and are thereby adjusted to a standard level of counting effieiency. The adjustments are made to the nearest scaled count and for that reason fluctuations of $\pm$ 0.1% in the final counting levels are liable to occur.

The overall stability of the normalised vertical intensity is difficult to assess. Small long-term drifts in the counting efficiencies cannot very well be taken care of by inter-comparison of counting rates with only two telescopes in the vertical direction. It would probably be fair to say that the counting level has been adjusted for short-term changes to within 0.2% of the standard level in most cases.

(c) Selection of data for daily variation studies

The data are arranged in groups of one month in the form of bi-hourly precentage deviates from the mean value. A single day's data comprises the twelve bi-hourly values plus the first bi-hourly value of the following day.

Thereby a linear adjustment for secular change can be effected when the data are averaged over a month.

Unusable data are rejected when the normalising factors are determined. Individual days are rejected from the tabulated data if more than two bi-hourly values are missing.

When a major Forbush-type intensity decrease occurs, the day of minimum intensity is rejected as is a number of days on either side of it, depending on the magnitude and duration of the decrease.

4.9 Harmonic analysis

In the derivation of the Fourier coefficient the least squares method of Whittaker and Robinson (1944) is followed. The particular treatment given here is based very largely on that given by Parsons (1959).

(a) The Fourier coefficient

We have $m + 1$ experimental values $U_0, U_1, \ldots U_{12}, \ldots U_m$ spaced at equal intervals of time (t) over the daily period T, the corresponding times of occurrence being $0, T, \ldots kt \ldots mt$. The values U_k are in the form of percentage deviations from the mean intensity, so that

$$U_k = \frac{(I_k - \bar{I})}{\bar{I}} \%$$

A progressive linear adjustment for secular (non-periodic) change causes U_o to become equal to U_m.

After the adjustment for secular change, the curve of best fit to the observed daily variation of the **m** values $U_o, U_1, \ldots U_k \ldots U_{m-1}$ is expressed as the sum of harmonics of period T/i where $i = 1,2,3$ etc. Our *estimate* of U_k is then specified by ΔI_k where

$$\Delta I_k = \sum_{i=1}^{\infty} \left(a_i \frac{\cos\ 2k\pi it}{T} + b_i \frac{\sin\ 2k\pi it}{T}\right)$$

$$= \sum_{i=1}^{\infty} \left(a_i \frac{\cos\ 2k\pi i}{m} + b_i \frac{\sin\ 2k\pi i}{m}\right) \qquad 4.11$$

$$= \sum_{i=1}^{\infty} R_i \cos\ i(\theta - f_i) \qquad 4.12$$

where $\theta = 2\pi k/m$

(b) Determination of the constants a_i, b_i, R_i, f_i

The average daily variation of intensity und er consideration here is adequately described by the sum of the first two harmonics of best fit. The first harmonic (diurnal component) is specified by the Fourier coefficients for $i = 1$ and we have

$$(\Delta I_k)_1 = a_1 \cos\ \theta + b_1 \sin\ \theta$$

$$= R_1 \cos\ (\theta - f_1)$$

Likewise, putting $i = 2$, the second harmonic (semi-duirnal component) is

$$\Delta(I_k)_2 = a_2 \cos 2\theta + b_2 \sin 2\theta$$

$$= R_2 \cos 2(\theta - f_2)$$

The curve of best fit is accordingly described by

$$\Delta I_k = a_1 \cos \theta + b_1 \sin \theta + a_2 \cos 2\theta + b_2 \sin 2\theta$$

The constants a_1, b_1, a_2 and b_2 are determined in such a way that the mean of the squares of the deviation of the observed values U_k from the estimates ΔI_k is a minimum, i.e.

$$\frac{\Sigma(\Delta I_k - U_k)^2}{m}$$

is minimised.

This requirement is fulfilled if

$$a_1 = \frac{2}{m} \sum_{k=o}^{m-1} U_k \cos \frac{2\pi k}{m} \qquad b_1 = \frac{2}{m} \sum_{k=o}^{m-1} U_k \sin \frac{2\pi k}{m}$$

$$a_2 = \frac{2}{m} \sum_{k=o}^{m-1} U_k \cos \frac{4\pi k}{m} \qquad b_2 = \frac{2}{m} \sum_{k=o}^{m-1} U_k \sin \frac{4\pi k}{m}$$

The amplitude of the first harmonic is

$$R_1 = \sqrt{a_1^2 + b_1^2}$$

and the phase f_1, including the sign, is determined from

$$\cos f_1 = a_1/R_1 \quad \text{and} \quad \sin f_1 = b_1/R_1$$

Similar considerations apply to the determinations of R_2 and f_2.

In the present investigation all observed daily variations of intensity are arranged in the form of bi-hourly deviates from the mean value. Consequently the harmonic coefficients are calculated on a 12-ordinate scheme ($m = 12$). For this purpose, computing sheets (Tables 4.2 and 4.3) have been drawn up to facilitate the calculations when desk machines are used.

(c) Adjustments for smoothing

As the number of ordinates in the ordinate scheme decreases, the amplitude of a given harmonic suffers increasingly from a smoothing error so that the true harmonic of best fit is underestimated. To adjust for smoothing of the i^{th} harmonic in an m-ordinate scheme, the amplitude R_i must be multiplied by $\omega/\sin\omega$ where $\omega = i\pi/m$. The multiplying factors for R_1 and R_2 in a 12-ordinate scheme are 1.012 and 1.047 respectively.

(d) Adjustments for change of time scale

The procedures for determining the harmonic coefficients assume that the first intensity value u_o refers to time 0000. However, where bi-hourly values are used, for instance, the first value usually refers to the mean of hours 01 and 02 and is taken as the value for the time 0100. The phase angle

TABLE 4.2 - HARMONIC COEFFICIENTS a_1, b_1, a_2, b_2 for 12-ORDINATE SCHEME

Multiplier M	U_k selection	Tunnel Vertical Solar 1959											
		ΣU_k	$M\Sigma U_k$	ΣU_k	$M\Sigma U_k$	ΣU_k	$M\Sigma U_k$	ΣU_k	$M\Sigma U_k$	ΣU_k	$M\Sigma U_k$		
0.16667	1-13	-104	-17.334										
0.14434	3-11-15+23	-215	-31.033										
0.08333	5-9-17+21	-152	-12.666										
	a_1		-61.033										
0.08333	3+11-15-23	-183	-15.249										
0.14434	5+9-17-21	-314	-45.323										
0.16667	7-19	-149	-24.834										
	b_1		-85.406										
0.16667	1-7+13-19	107	17.834										
0.08333	3-5-9+11+15 -17-21+23	123	10.249										
	a_2		28.083										
0.14434	3+5-9-11+15 +17-21-23	289	41.714										
	b_2		41.714										
	$a_1+b_1+a_2+b_2$		-76.642										
CHECK													
0.3334	1-19	-73	-24.334										
0.45534	3	-54	-24.588										
0.28868	5-21	-75	-21.651										
0.16667	-9-17	-43	-7.167										
0.12202	-11	9	1.098										
	$a_1+b_1+a_2+b_2$		-76.642										

N.B. EXAMPLE TUNNEL VERTICAL SOLAR 1959

HOUR(CENTRE OF INTERVAL)	0100	0300	0500	0700	0900	1100	1300	1500	1700	1900	2100	2300
$\Delta I/I$ (% × 10^3)	-23	-54	-92	-99	-98	-9	81	145	141	50	-17	-25
U_k number	1	3	5	7	9	11	13	15	17	19	21	23

ϕ = (0100-0000) in degrees = 15o

TABLE 4.3 - FIRST AND SECOND HARMONICS R_1, T_{1max}, R_2, T_{2max}

$\Delta I/I = R_1\cos(\theta-f_1) + R_2\cos 2(\theta-f_2) = a_1\cos\theta + b_1\sin\theta + a_2\cos 2\theta + b_2\sin 2\theta$

$(\Delta I/I)^* = R^*_1\cos(\theta-\theta_1 max) + R^*_2\cos 2(\theta-\theta_2 max)$

		Tunnel Vertical Solar 1959						
	a_1	-61.033						
	b_1	-85.406						
$R^2_1 = a^2_1 + b^2_1$		11019						
	R_1	104.97						
$R^*_1 = 1.012$	R_1	106.23						
	a_2	28.083						
	b_2	41.714						
$R^2_2 = a^2_2 + b^2_2$		2529						
	R_2	50.29						
$R^*_2 = 1.047$	R_2	52.65						
$\cos f_1 = a_1/R_1$		-0.5814						
	f_1	234°27'						
	ϕ	15°						
$\theta_1 max = f_1 + \phi$		249°27'						
$T_1 max$		1638						
$\cos 2f_2 = a_2/R_2$		0.5584						
	$2f_2$	56°03'						
$\theta_2 max = f_2 + \phi$		43°03'						
$T_2 max$		0252 1452						

f_i must be shifted forward accordingly, by the addition of a constant ϕ. In the case referred to, $\phi = 15^o$. In the accompanying form it can be seen at what stage ϕ must be added to f_2.

4.10 Statistical errors of amplitude and phase

Standard errors of estimate of amplitude and phase of the first and second harmonics of the observed daily variations are calculated on the assumption that, in the absence of systematic changes, the number of particles detected by the equipment in a given interval of time is a Poisson variable.

(a) The standard error (SE) of u_k

A given u_k value is derived from N particles and is expressed as a percent deviation from the mean $\overline{N}$. The SE of estimate, $\sigma(u_k)$, is

$$\sigma(u_k) = \frac{100\sqrt{\overline{N}}}{\overline{N}} \quad \%$$

The daily variation is very small so that $N \simeq \overline{N}$ and we can put

$$\sigma(u_k) = \frac{100\sqrt{\overline{N}}}{\overline{N}} = \frac{100}{\sqrt{\overline{N}}}$$

If the mean counting rate is $\overline{C}$, derived from n hourly values and containing a scale factor F,

$$\overline{N} = n\overline{C}F$$

and

$$\sigma(u_k) = \frac{100}{\sqrt{n\overline{C}F}}$$

(b) The standard error of a_1

$$a_1 = \frac{2}{m} \sum_{o}^{m-1} u_k \cos \frac{2\pi k}{m}$$

Since all the u_k's have the same variance $\sigma^2(u_k)$,

$$\sigma^2(a_1) = \left(\frac{2}{m}\right)^2 \sum_{o}^{m-1} \cos^2 \frac{2\pi k}{m} \sigma^2(u_k) = \left(\frac{2}{m}\right)\sigma^2(u_k)$$

therefore $$\sigma(a_1) = \sigma(b_1) = \sqrt{\frac{2}{m}}\ \sigma(u_k)$$

(c) The standard errors of R_1 and R_2

We use another expression for the variance of a function $F(a,b)$.

$$\sigma^2[F(a,b)] = \left(\frac{dF}{da}\right)^2 \sigma^2(a) + \left(\frac{dF}{db}\right)^2 \sigma^2(b)$$

since $$R = \sqrt{a^2 + b^2}$$

$$\sigma^2(R) = \left(\frac{a^2}{R^2} + \frac{b^2}{R^2}\right) \sigma^2(a) = \sigma^2(a)$$

therefore $$\sigma(R_1) = \sigma(a_1) = \sqrt{\frac{2}{m}}\ \sigma(u_k)$$

$$= 100 \sqrt{\frac{2}{mn\bar{C}F}}$$

$$= \sigma(R_2)$$

(d) The standard error of time of maximum

The error circle attached to the end-point of R_1 or R_2 on the harmonic dial subtends an angle at the origin equal to twice the SE of estimate of time of maximum. If small, $\frac{\sigma(R)}{R}$ approximates the SE of phase, in radians.

4.11(a) Observed versus calculated variability of the counting rate

After correction for pressure changes, the systematic time-averaged variations of intensity underground are relatively small. For instance, the amplitude of the daily variation is usually less than 0.1%, Forbush-type decreases rarely exceed 1%, and the change in the level of the daily average intensity from maximum to minimum of solar activity is only about 2.5%. Moreover, time-dependent processes that appear to produce rather small short-term irregular fluctuations that are of significance at ground level may not be nearly so important at the higher energies of response. Consequently it seemed that fluctuations of the underground intensity from one hour to the next might obey Poisson statistics, at least during periods of minimum solar activity, if the influence of the solar daily variation were removed. Since direct evidence of this was desirable, so that the applicability of the usual Standard Error tails could be assessed, it was decided to determine experimentally the variance of the count rate due to random fluctuations, using the pressure-corrected vertical intensity that related to the two quiet years 1964 and 1965. Table 4.4 is a typical monthly data sheet.

The observed solar daily variation averaged over the two years is shown in Figure 4.9(a), in the form of bi-hourly percentage deviates from the mean intensity. It is clear that the solar daily variation on the average produces scarcely any change in the counting rate from about 1900 hours to 0900 hours. Therefore, since it seemed that day-to-day fluctuations of counting rate during

COSMIC RAY DATA. PHYSICS DEPARTMENT, UNIVERSITY OF TASMANIA.

STATION: CAMBRIDGE TUNNEL (HOBART), TASMANIA. (42° 51'S, 147° 25'E). Altitude 110 metres. DECEMBER, 1964.
RECORDER: VERTICAL SEMI-CUBICAL MESON TELESCOPE. Absorber 40 m.w.e. Barometer Coefficient -0.65%/cm Hg.
TABULATED: Hourly count totals corrected to standard pressure of 28·0 inches of Hg.
REAL COUNTS = 128 x tabulated counts.

TABLE 4.4

Hour GMT.	Date. 01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
01	558s	557	559	560	558	558	558	561	561	562	560	558	557	559	563	561	560	565	558	559	558	557	556	559	554	555	557	560	560	558	558s
02	560s	559	559	560	560	563	562	560	563	561	557	560	564	554	561	559	558	563	559	561	555	558	560	555	559	558	554	559	556	554	554s
03	565s	561	560	561	563	558	560	564	564	562	554	558	562	557	560	558	562	561	554	560	556	554	557	560	555	556	558	560	562	556	559
04	558s	555	559	561	554	562	557	561	561	563	559	556	561	557	561	557	556	559	558	556	562	555	558	559	558	557	558	560	557	559	556
05	565s	561	562	564	562	559	557	558	563	562	558	559	557	558	560	559	555	563	560	553	555	555	556	556	556	557	556	558	561	558	558
06	RF	562	558	558	560	561	561	561	563	560	561	563	556	561	558	562	559	562	557	561	557	555	557	560	557	560	561	558	559	558	556
07	558s	559	559	558	561	559	557	558	561	561	562	561	558	556	557	560	560	565	560	555	558	556	556	561	555	557	558	556	559	559	556
09	561s	557	555	559	561	559	562	559	561	562	563	559	556	557	561	558	561	560	563	557	559	553	557	559	553	559	554	561	554	553	563
09	561s	558	562	561	558	557	558	562	561	559	558	560	558	561	558	562	562	559	555	560	558	557	558	556	555	557	553	564	559	556	555
10	555s	558	557	560	559	558	559	564	566	556	558	561	560	562	563	560	558	560	564	558	557	559	554	557	558	557	559	560	554	556	556
11	569s	554	559	559	558	559	558	565	561	560	557	560	558	554	558	561	558	562	560	558	556	557	559	562	556	554	556	560	560	557	558
12	562s	561	558	557	560	557	559	561	562	565	560	560	556	558	560	565	560	558	557	558	560	559	556	555	556	557	556	560	553	560	562
13	562s	557	564	559	561	561	563	562	557	559	559	559	559	558	558	556	561	564	560	554	555	555	562	561	560	561	559	559	559	557	559	558
14	558s	563	562	561	562	559	561	562	560	558	563	562	558	561	561	558	557	562	561	558	556	557	558	559	560	561	556	554	558	558	558
15	558s	563	561	562	560	558	557	557	557	564	559	558	560	559	562	562	566	561	559	558	557	554	556	557	556	562	557	556	559	556	556
16	556s	558	561	553	559	559	562	564	563	563	557	563	558	560	561	562	561	561	561	556	558	558	556	559	560	560	559	559	556	556	564
17	560s	561	558	559	561	562	559	559	562	563	559	556	558	558	560	562	562	561	561	560	553	556	559	560	559	558	557	555	556	560	559
18	560s	562	559	563	562	557	563	562	560	562	558	562	554	556	559	561	563	561	558	559	560	554	560	560	555	559	554	555	556	557	560
19	558s	556	560	562	558	556	560	560	557	562	559	560	559	557	561	563	561	559	558	556	559	559	554	557	558	559	559	560	559	555	560
20	552s	560	562	560	560	563	561	566	556	561	554	558	558	563	561	563	563	561	560	559	560	558	562	556	556	562	556	560	560	558	560
21	550s	561	557	563	562	557	561	563	560	561	565	556	559	556	561	559	558	556	557	554	559	553	556	560	552	553	557	555	557	558	556
22	562s	557	560	558	561	561	564	561	561	561	560	560	560	563	564	564	560	562	557	553	557	558	557	560	556	558	556	558	555	561	556
23	558s	562	559	558	558	560	562	563	558	561	563	560	558	559	560	556	559	559	564	558	562	561	556	559	555	555	560	556	559	RF	562
24	564s	562	564	559	562	558	559	561	558	556	560	558	560	560	559	563	561	560	559	555	560	558	557	557	558	555	557	559	557	557	559
Σ	(12870)	13434		13440		13440		13456		13423		13403		13445		13444		13414		13387		13376		13358		13367		13383		13399	
M		559.7		560.0		560.0		560.7		559.3		558.5		560.2		560.2		558.9		557.8		557.3		556.6		557.0		557.6		558.3	
	13424		13435		13421		13474		13464		13427		13404		13456		13460		13377		13363		13403		13385		13402		(12819)		
	559.3		559.8		559.2		561.4		561.0		559.5		558.5		560.7		560.8		557.4		556.8		558.5		557.7		558.4				

___s = Estimated from single telescope. RF = Recording failure.

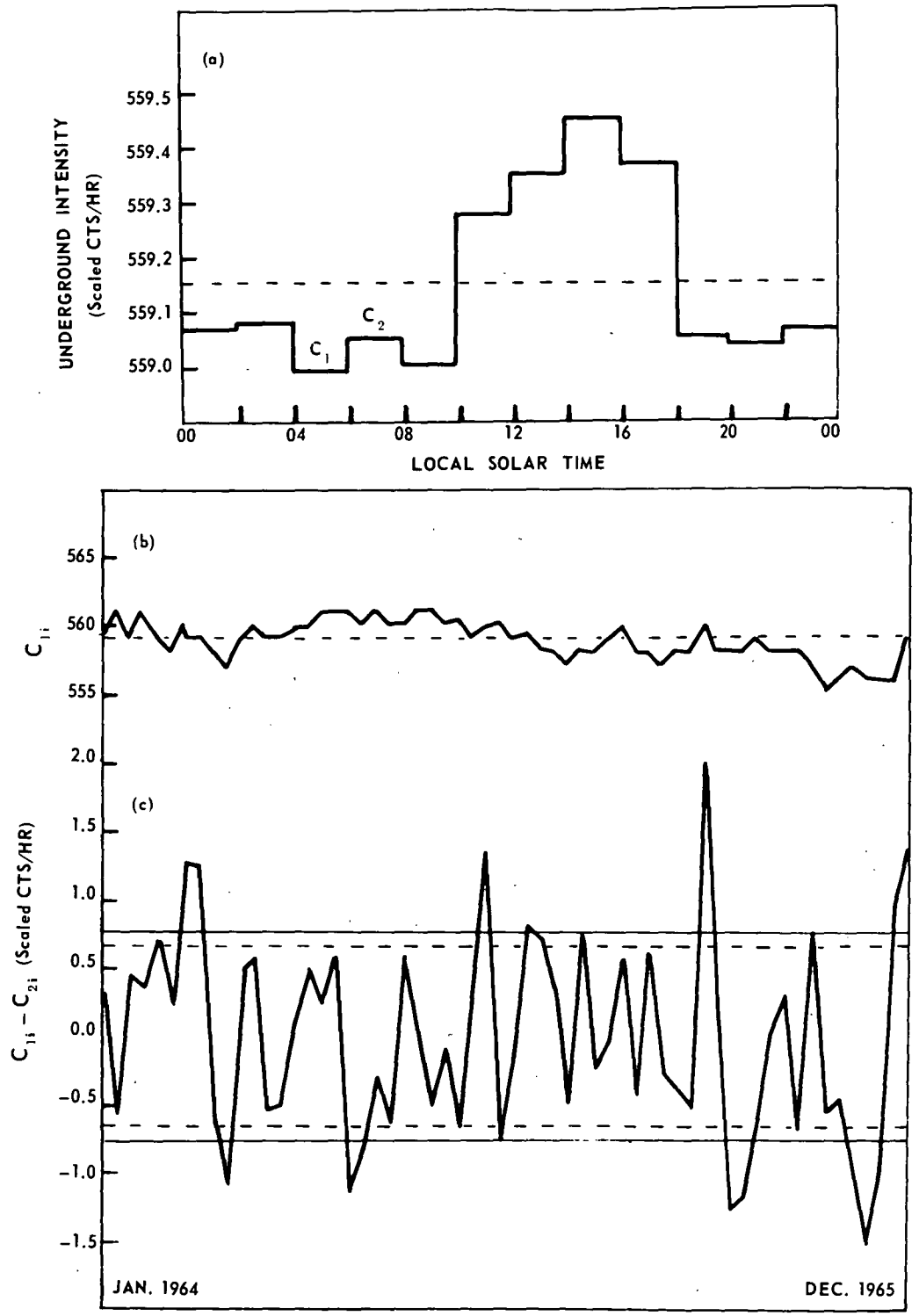


Figure 4.9 Variability of the pressure-corrected bi-hourly counting rate in the vertical direction underground.

(a) The solar daily variation averaged over 1964 and 1965. The average bi-hourly counting rates centred on 0500 and 0700 are shown as C_1 and C_2.

(b) Successive 20-day averaged values of C_1, from January 1964 to December 1965. The full line is the biennial mean value of C_1, as shown in (a).

(c) Successive 20-day averaged values of $C_1 - C_2$. The full lines represent the observed standard deviation and the dashed lines the S.E. of estimate, assuming the difference to follow a Poisson distribution.

these hours would be least influenced by day-to-day changes in the solar daily variation, two neighbouring bi-hourly periods from this relatively undisturbed part of the day were selected for the statistical test. It was decided to examine fluctuations of the difference of pressure-corrected intensity between the two bi-hourly periods (05, 06) and (07, 08). By taking differences the influence of day-to-day changes in the average level of intensity would be minimised.

The data were arranged in successive groups of 10-day averages, so that a sample value of intensity would be the average of 20 hours of counting. We represent the scaled counting rate for the bi-hourly period (05,06), averaged over the i^{th} group of 10 days, by C_{1i} and that for the period (07,08) by C_{2i}, while the difference is given by $d_i = C_{1i} - C_{2i}$. Regarding C_{1i} as a Poisson variable, the estimated variance is $V(C_{1i}) = C_{1i}/nF$, where n is the number of hours over which the average is taken and F is the scale factor. Averaged over the two years, $\overline{C}_1$ was found to be approximately 559 scaled counts while departures of the sample values from this did not exceed 4 counts (see Figure 4.9(b)). Therefore it is adequate to assign a constant estimated variance to each sample value, representing C_{1i} by its mean, $\overline{C}_1$, in the calculation of the variance.

Putting $C_{1i} = 559$, $n = 20$ and $F = 128$, we get $V(C_{1i}) = 0.218$. The correction of the count rate to a standard pressure of 28 inches of mercury

artificially raises the counting level by 2.75% on the average, so that the estimated variance must be multiplied by $(1.0275)^2$, giving us a revised $V(C_{1i}) = 0.230$. Then the estimated variance of the difference is $V(d_i) = 0.460$ and the standard estimate of error is $SE(d_i) = \pm 0.68$.

The 60 observed sample values of d_i of the difference, each relating to $\sim 2.9 \times 10^6$ detected particles, are shown in Figure 4.9(c) in their order of occurrence. The observed standard deviation is

$$SD(d_i) = \sqrt{\overline{(d_i^2)} - (\overline{d_i})^2}$$

It was found that $\overline{d_i}$ was zero and that

$$SD(d_i) = \pm 0.75$$

The ratio $\frac{\text{observed SD}}{\text{SE of estimate}}$ was therefore 1.10. In the figure the observed SD and the SE of estimate are shown as full lines and dashed lines respectively.

It would seem that the SE tails that are given with the individual harmonics of best fit to the observations adequately represent the errors of estimate arising from random fluctuations of the observed pressure-corrected intensity.

(b) A bi-hourly periodicity in timing

When the influence of the solar daily variation is minimised, the annual bi-hourly deviates in solar time appear to obey Poisson statistics. However, the same cannot be said of the hourly deviates. Very recently, an examination of the annual hourly deviates that had been computed by J.E.Humble showed that the deviates would tend to be consistently negative on odd hours G.M.T. and positive by about the same amount on even hours, if the daily variation were removed. That is to say, successive pairs of hourly deviates would exhibit approximately a constant difference and in the same sense.

It was concluded from a closer examination of this effect that over the period since July 1960, when the Mercer Chronometer was installed, until May 1965, when it was overhauled, odd hours had been very regularly shorter than even hours by approximately 2.5 seconds, and that after the overhaul even hours were shorter than odd hours by that same amount. This latter difference was confirmed by taking successive hourly photographs, initiated by the chronometer, of the position of its own second hand. It became evident, by taking photographs with other timing devices, that the fault originated in the chronometer and not in the timing control circuit.

When bi-hourly deviates of intensity are used, as has been almost exclusively the case in the analyses of the underground data described in this thesis, a 2-hour periodicity in timing is averaged out. Care has been taken to

point out in the text (Chapter 5) the two occasions on which hourly data have been used, but it should be noted that even if hourly data had been used throughout,

(1) a 12^{th} harmonic in the daily variation, such as that introduced by the periodicity in timing, would not have compromised determinations of first and second harmonics;

(2) if diurnal modulation of the 12^{th} harmonic had occurred it would have given rise not to an apparent *diurnal* variation of intensity, but to 11^{th} and 13^{th} harmonics;

(3) if seasonal modulation of the 12^{th} harmonic had occurred it would not have given rise to sideband components at the sidereal and anti-sidereal frequencies, since the frequency of the carrier is much too high (4380 cycles/year).

To confirm that the 2-hour periodicity would not affect calculations of first and second harmonics based on hourly deviates, the harmonic components of the annual daily variations were compared when the same underground data had been arranged in both hourly and bi-hourly form. The year 1961 and the biennial periods 1962-1963 and 1964-1965 were considered. For each of these periods the results obtained were the same, whether one used hourly or bi-hourly deviates.

A modification of the method of timing, which it is hoped will eliminate the timing periodicity, is at present under test.

CHAPTER 5

THE UNDERGROUND EVIDENCE FOR A TWO-WAY SIDEREAL ANISOTROPY

5.1 Introduction

In this chapter a summary is given of the qualitative evidence from underground, acquired up to the end of 1965. Some of the evidence has been published, and the following relevant papers which will be referred to in the text are attached as appendices:

Paper 4 (Appendix IV). "The apparent sidereal daily variation of cosmic ray intensity at 42 m.w.e. underground at Hobart, Tasmania. I. Results of observations in 1958".

Paper 5 (Appendix V). "The apparent sidereal daily variation of cosmic ray intensity at $\sim$ 40 m.w.e. at Hobart, Tasmania. II. Results of observations 1958-1962".

Paper 8 (Appendix VIII). "Evidence for a two-way sidereal anisotropy in the charged primary cosmic radiation".

It is proposed to arrange the summary under headings which tend to relate to particular papers and which follow the order of the list of observational requirements given in Chapter 1.3. These are considered to be requirements that must be satisfied by the present observations if a sidereal component produced by a two-way anisotropy is to be identified - as against a spurious effect arising from seasonal modulation of a solar component of the daily variation. The headings are as follows:

(a) the evidence from annual averages, relating to seasonal modulation as detected by the anti-sidereal technique;

(b) the evidence from phase anomalies;

(c) the evidence from the 12-hour phase difference between the hemispheres;

(d) the evidence from the observed harmonics. Under (d) only the general character of the harmonics is noted in this chapter. Employing the asymptotic cones of response (Chapter 6), we go on in Chapter 7 to examine the latitude dependence of the deduced free-space harmonics. This leads to an estimation of the free-space direction of the anisotropy and provides a useful test of the coherence of the evidence, obtained as it was from several detectors with differing response characteristics.

Since the evidence is not being presented in strict chronological order it might be helpful to begin with a brief description of the experimental arrangements, elaborating where necessary in later sections.

5.2 The Underground Experiments at Hobart

Observations have been obtained from four directions in the plane of the geographic meridian. Figure 5.1 shows the axial inclinations to the zenith, namely 70°N, 30°N, 0° and 45°S. Four counter telescopes have been used, each essentially comprising three metre-square trays in triple coincidence. However only two telescopes have been available for the measurements in the three inclined directions, the other two being premanently tied up in the vertical experiment.

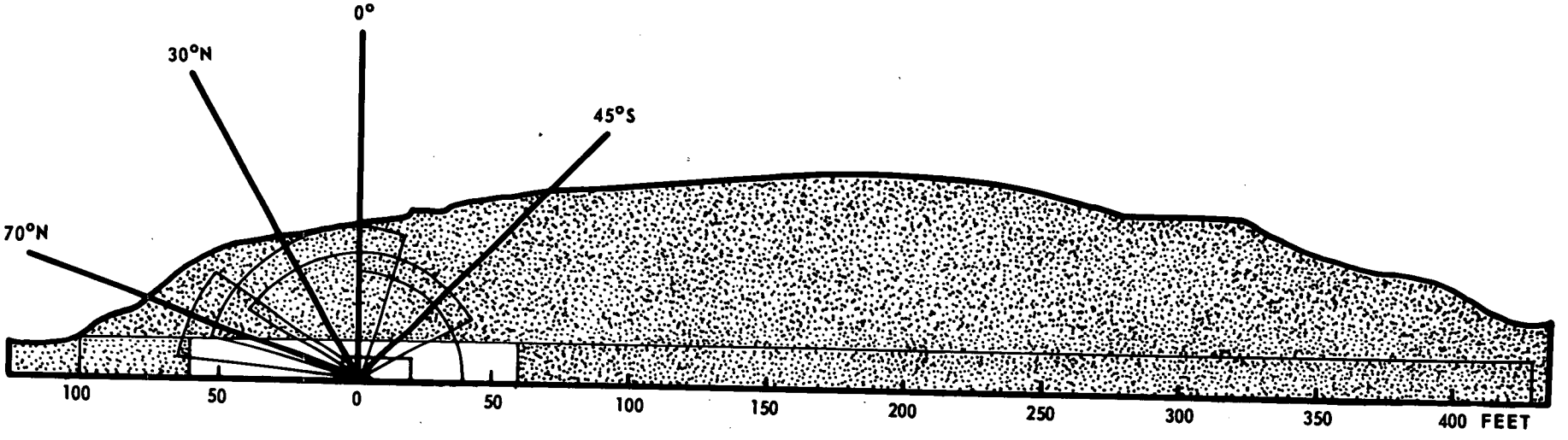


Figure 5.1 Cross-section of tunnel in the meridian plane, showing axial headings and geometric apertures of the detectors in relation to the material absorber.

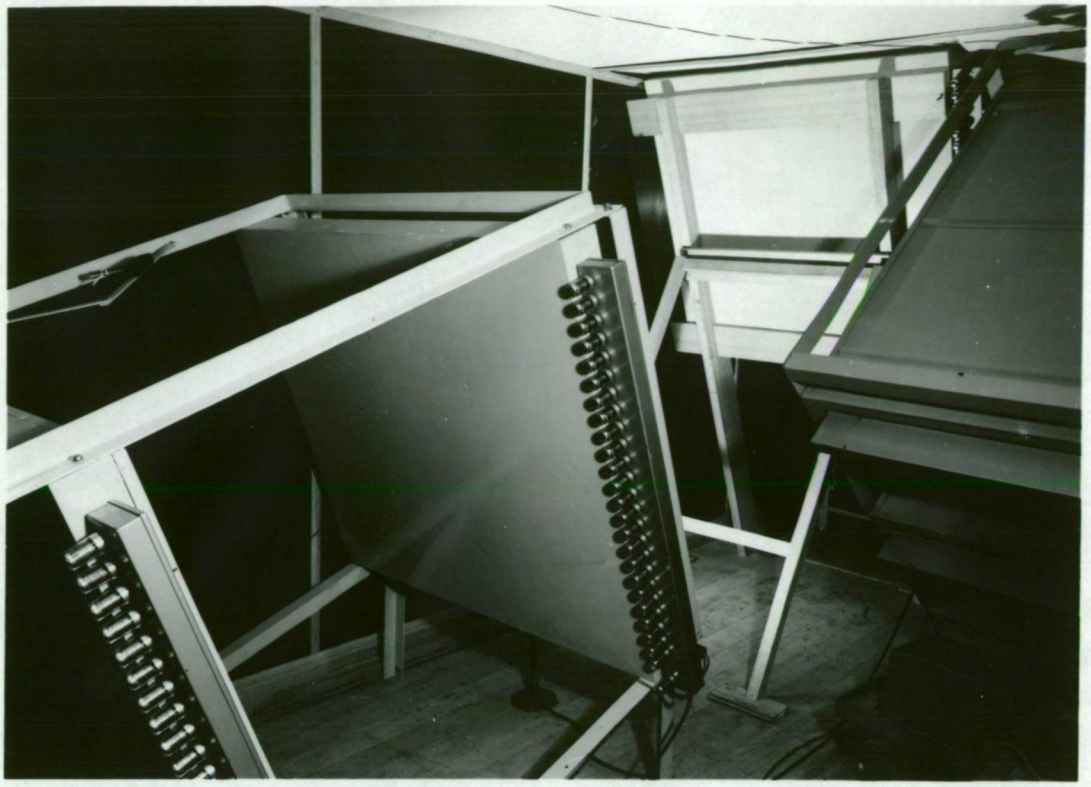
(a) The vertical semi-cubical experiment

The two vertical semi-cubical telescopes were installed in July 1957 by R.Taylor, and continuous observations date from October of that year.

(b) The cubical telescopes inclined 30°N and 45°S of zenith

Towards the end of 1960 two additional telescopes were put into operation. The tray assemblies and electronic circuits were copies of the vertical semi-cubical construction in all but minor detail. The cubical framework comprised two square side-frames of welded angle-iron, bolted to each other at the four corners by cross bars. This provided a rigid structure to which the trays could be fastened. In Figure 5.2 one of these cubical telescopes can be seen on the left, mounted on a rotatable platform and tilted at an angle of 45°S of zenith in the geographic meridian plane. One of the two pivoting points at which the telescope was attached to the platform is seen more clearly in Figure 5.2(b). Two vertical supports, cut to the appropriate length, bolted the telescope securely to the platform fore and aft, at the given inclination to the zenith.

A spirit level attached to the frame of the telescope allowed variations of the zenith setting, that might result from equipment tests and from movements in the floor level, to be read off to an accuracy of 10 seconds of arc. Occasional slight adjustments were effected by raising the platform with a car jack (Figure 5.2(b)) and placing pieces of shim under the wheels.



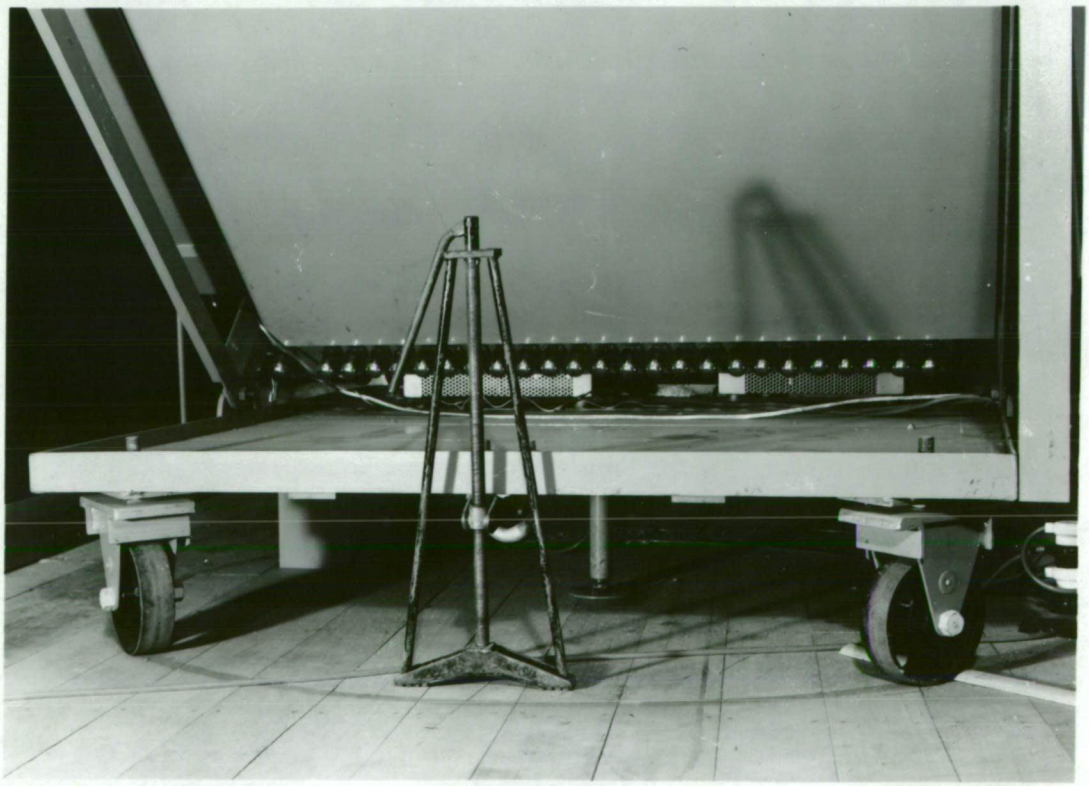


Figure 5.2

A cylindrical metal sleeve projecting downwards under the centre of the platform was fitted over a vertical peg screwed into the floor, thereby fixing the axis of rotation of the telescope. The rotatable platform was intended for investigations of azimuthal effects and was of no particular relevance for the experimental studies of the sidereal effect.

The two directional telescopes were equipped with independent HT and EHT supplies and were connected to the mains through a common voltage regulating transformer. They shared with the vertical telescopes the same photographic recording panel, timing system and chart recorder (Chapter 4).

Over the two complete calendar years 1961 and 1962 one telescope was set at an inclination 30° N of zenith and the other at 45° S of zenith so that simultaneous results could be obtained from low-, mid- and high-latitude scans. The calculation of the asymptotic cones of viewing will be described in Chapter 6. It suffices to say here that the mean asymptotic latitudes of viewing in the three directions are equally spaced, being approximately 17° S, 39° S and 60° S geographic.

Of the two inclined telescopes, the south-pointing had much the lower counting rate due both to its greater inclination to the zenith and to the greater amount of material absorber within its cone of viewing.

(c) The narrow angle telescope inclined 70°N of zenith

The experiment at the zenith angle setting 30°N was discontinued in

1963. The cubical telescope was converted to a narrow angle detector of dimensions 3m × $1m^2$, at the zenith angle setting 70°N. To effect this, the telescope was removed from the turntable and set up vertically on the floor. It was then tilted over on one edge to 70°N of zenith and supported in that position by two angle-iron uprights at the corners. The middle tray was removed and placed on a wooden easel, in line with the other two trays, and at a distance of 3 metres from the bottom tray. The arrangement is shown in Figure 5.2(a) and can also be seen in Figure 4.2, to the right of the south-pointing telescope and beyond the two semi-cubes. Further details of this experiment are given in section 5.5.

Some constants of interest relating to the four experiments are listed in Table 5.1. The absorber within the viewing cone of the narrow angle telescope has been approximated by the absorber in the direction of the axis, since a detailed cone of acceptance has not yet been calculated for this detector. In the case of the inclined cubical telescopes, the technique of the local cone of acceptance (see Chapter 4.4) has enabled the effective absorber to be calculated as a weighted average of contributions from a large number of elements of solid angle, according to the radiation sensitivity of each element.

TABLE 5.1

Telescope Geometry	Single cube (1×1×1m)	Duplex semi-cube (1×1×0.5m)	Single cube (1×1×1m)	Narrow angle (1×1×3m)
Inclination of axis	30°N	0°	45°S	70°N
Approximate mean asymptotic latitude of viewing	17°S	39°S	60°S	∿ 20°N
Average material absorber (m.w.e.)	31	36	44	∿ 33
Average absorber, including atmosphere to 100 mb (m.w.e.)	42	46	56	∿ 60
Particles/hour	16,700	70,000	8,700	1,200

5.3 The Evidence for a Sidereal Component from the Annual Average Effect in the Vertical Direction

The evidence presented under this heading relates to the interpretation of seasonal modulation of the daily variation underground. As explained in Chapter 2, the treatment differs from that given by others in that the possibility of seasonal modulation of a sidereal component is examined. The reason for considering this possibility in the first place was that the observed behaviour of the anti-sidereal effect appeared to be incompatible with seasonal modulation of a solar component. Naturally, it is not suggested that the sidereal anisotropy itself would exhibit seasonal changes - what is of concern here is the response at the earth, where a sidereal anisotropy is expected to be viewed through the interplanetary medium, the spatial and temporal properties of which are not at all well known.

It will not be presumed to present answers to the complex problem of seasonal changes here. Rather, although it is proposed to outline the distinctive nature of seasonal modulation that may be inferred from the underground observations and to suggest a possible explanation that conforms with the present observations, the main concern here is to decide whether or not the observed annual apparent sidereal effect contains a significant spurious component.

Two aspects of seasonal modulation are examined. There are the seasonal changes that are effective over a particular year and there are the long-term trends in seasonal modulation that may become evident over a number of years of observation. These trends are observable to a marked degree in the underground data from Hobart. Consequently an hypothesis should not only account for seasonal modulation that may apply over a given year, but should also accommodate the year to year trends.

(a) Hobart, 1958

The pressure-corrected daily variations averaged over the first complete year of operation of the vertical telescopes underground were notable for the large amplitudes in solar, sidereal and anti-sidereal time - 0.126%, 0.090% and 0.052% respectively. This was the only year when the anti-sidereal effect was so pronounced that a quantitative treatment of seasonal modulation seemed to be worth attempting. In an analysis of this kind it is necessary to determine rather precisely the constants specifying the seasonal variations of amplitude and phase of the total daily variation and to be able to apply

realistic tests of theory against observation month by month. The analysis of observations presented in paper 4 is now briefly reviewed.

It was found that the seasonal characteristics were more easily recognised if the total daily variation in solar time (V_r) for each month r was subtracted from the daily variation for month $r + 6$, giving an average difference variation

$$D_r = \frac{V_r - V_{r+6}}{2}$$

as a monthly unit of information applicable to the first six months of the year (paper 4, section 2)

The annual average daily variations in solar, sidereal and anti-sidereal time, the monthly values of amplitude and phase of the total daily variation in solar time and the monthly values of amplitude and phase of the D_r variation constituted a set of seven quantities characterising the daily variation phenomena in 1958 (Figure 5.3). As mentioned in paper 4, considerable smoothing was achieved with 6-hour running average deviations. The use of running averages arose from the need to obtain a large number of curves of fit to observed daily variations at a time when the facilities of a high-speed computer were not available. The monthly parameters have since been obtained from the harmonics of best fit to the daily variations of observed bi-hourly deviates. In Figure 5.4 the parameters obtained from the first harmonics and from the running averages are compared [(d) and (e)]. Sums of harmonics differ in giving rather better agreement with the phase changes in solar time and rather worse agreement with the phase changes in sidereal time. In (a), (b) and (c) of

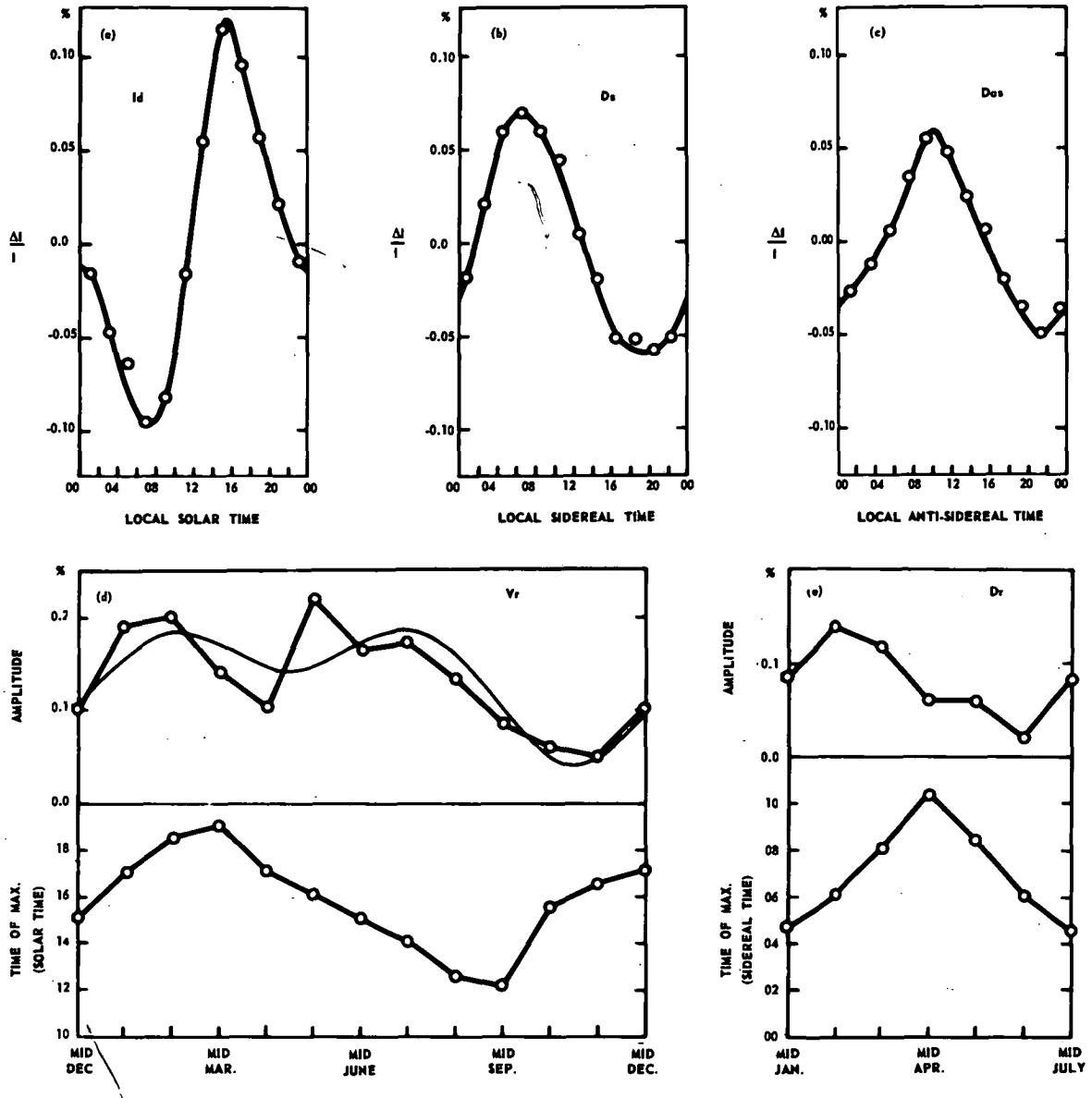


Figure 5.3 Characteristics of the daily variation at 40 m. w. e. underground, Hobart, in 1958. The circled points are obtained after conversion of the observed bi-hourly mean deviations of intensity to 6-hour running means.

(a) The annual mean daily variation in solar time.
(b) The annual mean difference, D_r, variation in sidereal time.
(c) The annual mean D_r variation in anti-sidereal time.
(d) Month by month mean values of amplitude and phase of the solar daily variation. A curve of best fit to the amplitude values is also shown.
(e) Month by month mean values of amplitude and phase of the D_r daily variation in sidereal time.

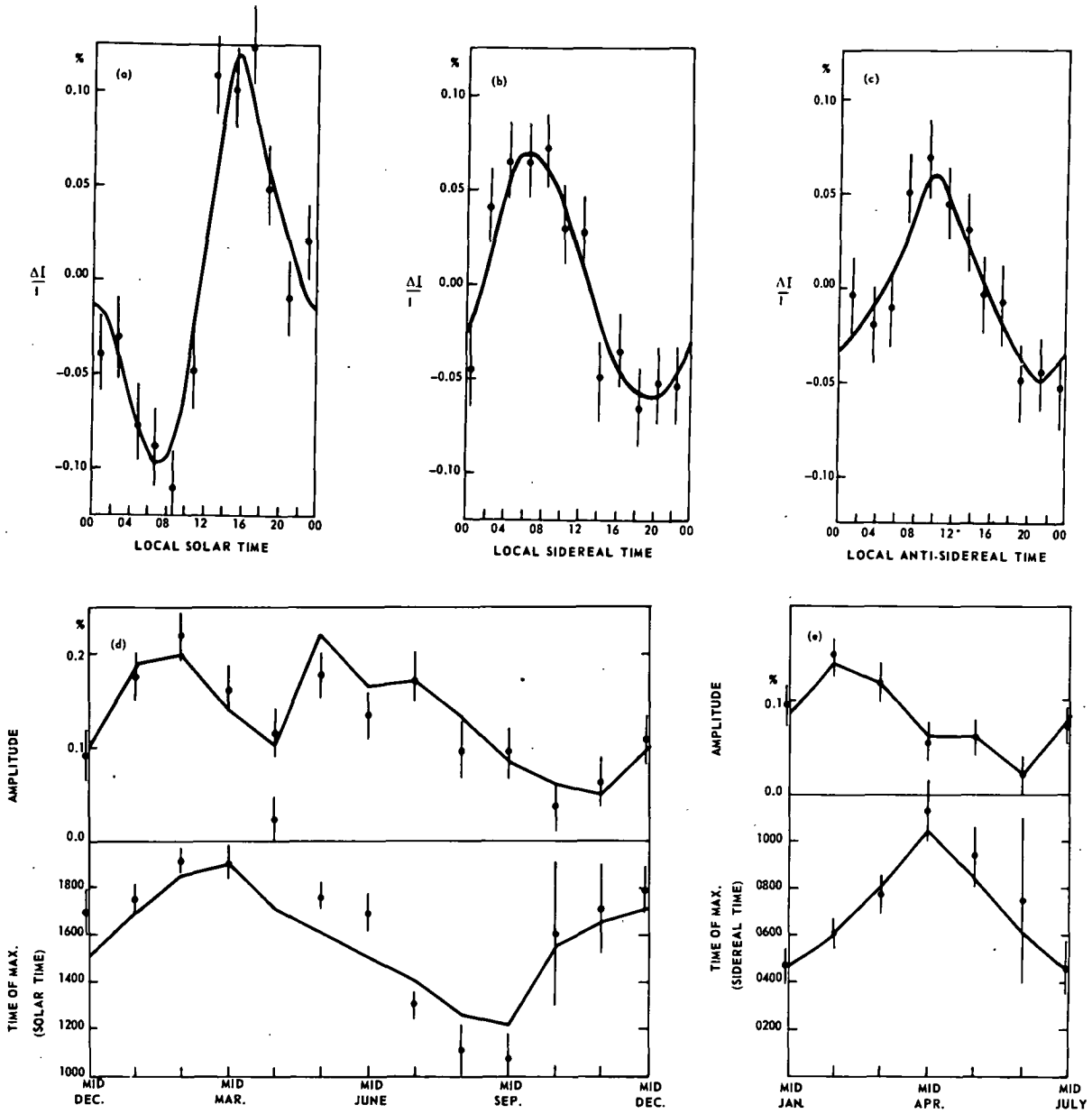


Figure 5.4 Hobart, 1958. Characteristics of the daily variation of vertical intensity underground. Full lines are drawn through values obtained after the conversion of bi-hourly deviations of intensity to 6-hour running averages. For comparison, values obtained from the unsmoothed deviates are shown, together with the S.E.s of estimate.

(a) The annual mean daily variation in solar time.

(b) The annual mean difference, D_r, variation in sidereal time.

(c) The annual mean D_r variation in anti-sidereal time.

(d) Month by month mean values of amplitude and phase of the solar daily variation. A curve of best fit to the amplitude values is also shown.

(e) Month by month mean values of amplitude and phase of the D_r daily variations in sidereal time.

of Figure 5.4 the annual daily variation curves drawn through the running averages are shown against the observed bi-hourly deviates. It appears that the method of running averages specifies the chosen characteristics of the daily variation in 1958 as accurately for our purposes as would other methods. It was proposed to attempt to reconstruct the situation in 1958, month by month, using facsimile daily variations to express various hypotheses of seasonal modulation and to find if there was any one simple hypothesis which would predict, markedly better than any other, the seven characteristics shown in Figure 5.3. The hypotheses were of three types according as the observed sidereal effect was considered to be entirely spurious (I), partly spurious and partly genuine (II), or entirely genuine (III).

The particular hypotheses, describing the components of the total daily variation, were :

I Spurious

I(a). A solar daily variation with combined annual sinusoidal variations of amplitude and phase.

I(b). A solar diurnal variation with combined annual sinusoidal variations of amplitude and phase, plus a constant solar daily variation.

II Partly spurious

II(a). A solar daily variation with an annual sinusoidal variation of amplitude, plus a constant sidereal daily variation.

II(b). A solar diurnal variation with a sinusoidal annual variation of amplitude, plus a constant solar daily v ariation, plus a constant sidereal diurnal variation.

III Genuine

III(a). A sidereal diurnal variation with semi-annual variation of amplitude add phase, plus a constant solar daily variation.

III(b). A sidereal diurnal variation with combined annual and semi-annual variation of phase, plus a constant solar daily variation.

Although only selected data were used (Chapter 4.8), the value of the analysis was limited by the observational uncertainties, due not only to statistical fluctuations, but to possible irregularities and secular changes that may have been produced by processes of various kinds. Again, the possibility of a second harmonic in the hypothetical carrier had to be considered, leading to the use of models (paper 2.3) based on the observed solar daily variation, which contains an important second harmonic. Because of the presence of the second harmonic, the sidebands may differ considerably from those generated by modulation of a simple diurnal variation (e.g. see Figure 8 of paper 4, relating to hypothesis II(a)).

Considering the above limitations, the fact that *hypothesis* III(a), relating to modulation of a sidereal component, accounted for the observations rather better than any other hypothesis was perhaps not very significant. It is the fact that it conformed with the observed characteristics, both seasonal and annual, as well as it did that is important. The agreement may be seen in Figure 5.5, where the observational quantities shown in Figure 5.3 are compared with the predictions. Not shown is the annual mean solar daily variation,

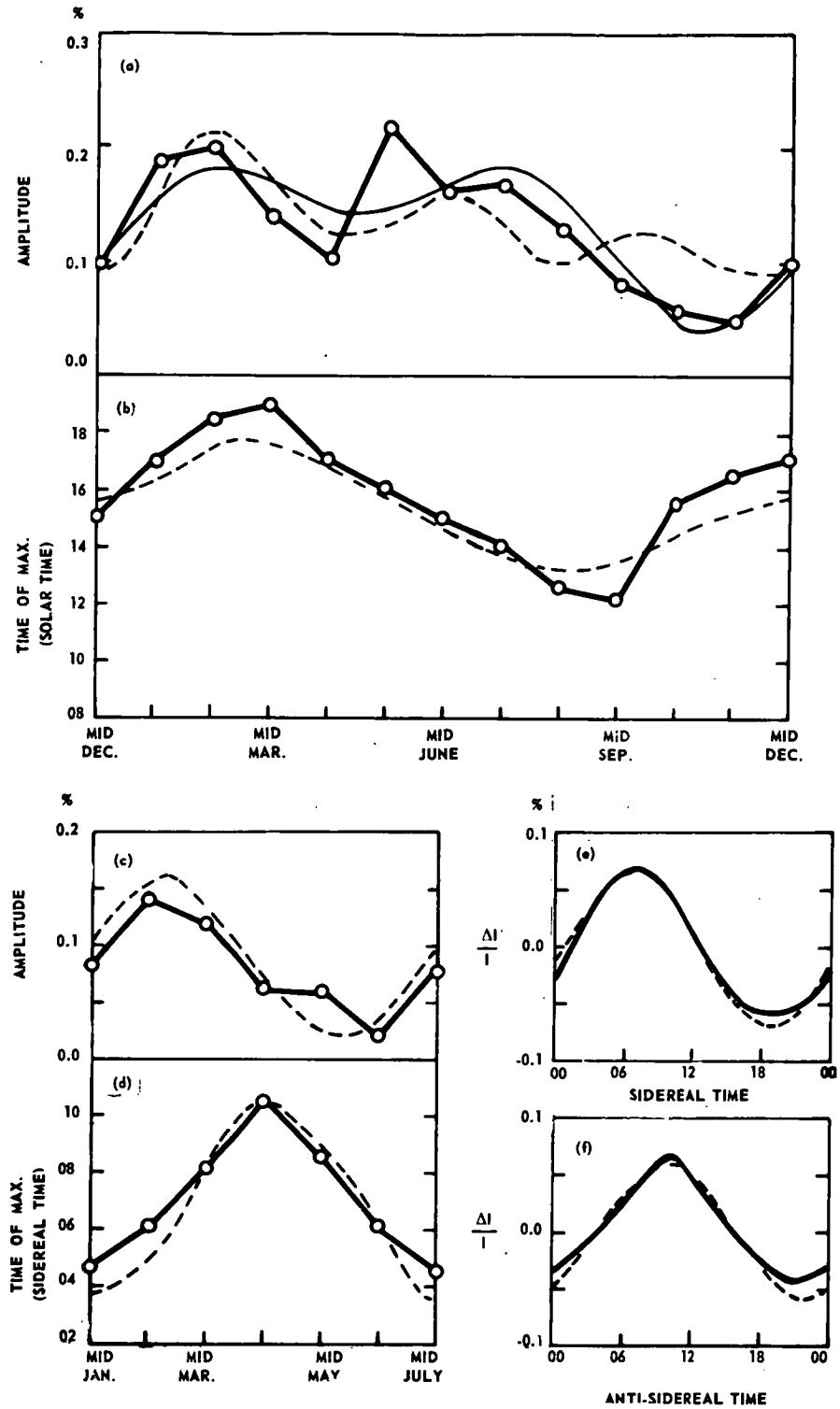


Figure 5.5 Hypothesis III. The results of model analyses (dashed lines) compared with the observed characteristics of the daily variation reproduced from Fig. 3. The values of the sidereal constants k and n used in the model are estimated from observations and the model is designed to give the correct values for the amplitudes and times of maximum of D_s and D_{as} and to reproduce I_d indentically. In (a) the curve of best fit to the amplitude variations is also shown.

since it is not affected by this type of modulation. The solved characteristics of the phase- and amplitude-modulated sidereal diurnal component were :

unmodulated amplitude: 0.094%

unmodulated time of maximum: 0700 LST

degree of semi-annual modulation of amplitude: 80%

maximum semi-annual displacement of phase: 3 hours

months of minimum amplitude: December and June

months of maximum advancement of phase: October and April

Also of interest is *hypothesis* II(b), which would be invoked if the anti-sidereal effect were attributed to a diurnal variation of atmospheric temperature exhibiting an annual variation of amplitude. Best fit to the observations is obtained (Figure 5.6) if the amplitude-modulated solar component has the following characteristics:

amplitude: 0.120/k%, where k is the constant of seasonal modulation

time of maximum: 2000 local solar time

date of maximum amplitude: mid February (Tasmanian summer)

As mentioned in Chapter 2, it seems to be generally agreed that maximum amplitude due to atmospheric temperature effects occurs in summer (e.g. Dorman, 1965) and that the maximum of the cosmic ray diurnal variation due to the integrated temperature effect occurs at about 0600 local solar time (e.g. Quenby and Thambyahpillai, 1960). As far as the positive temperature effect (connected with π-μ decay) is concerned, Thambyahpillai et alia (1965) report that the

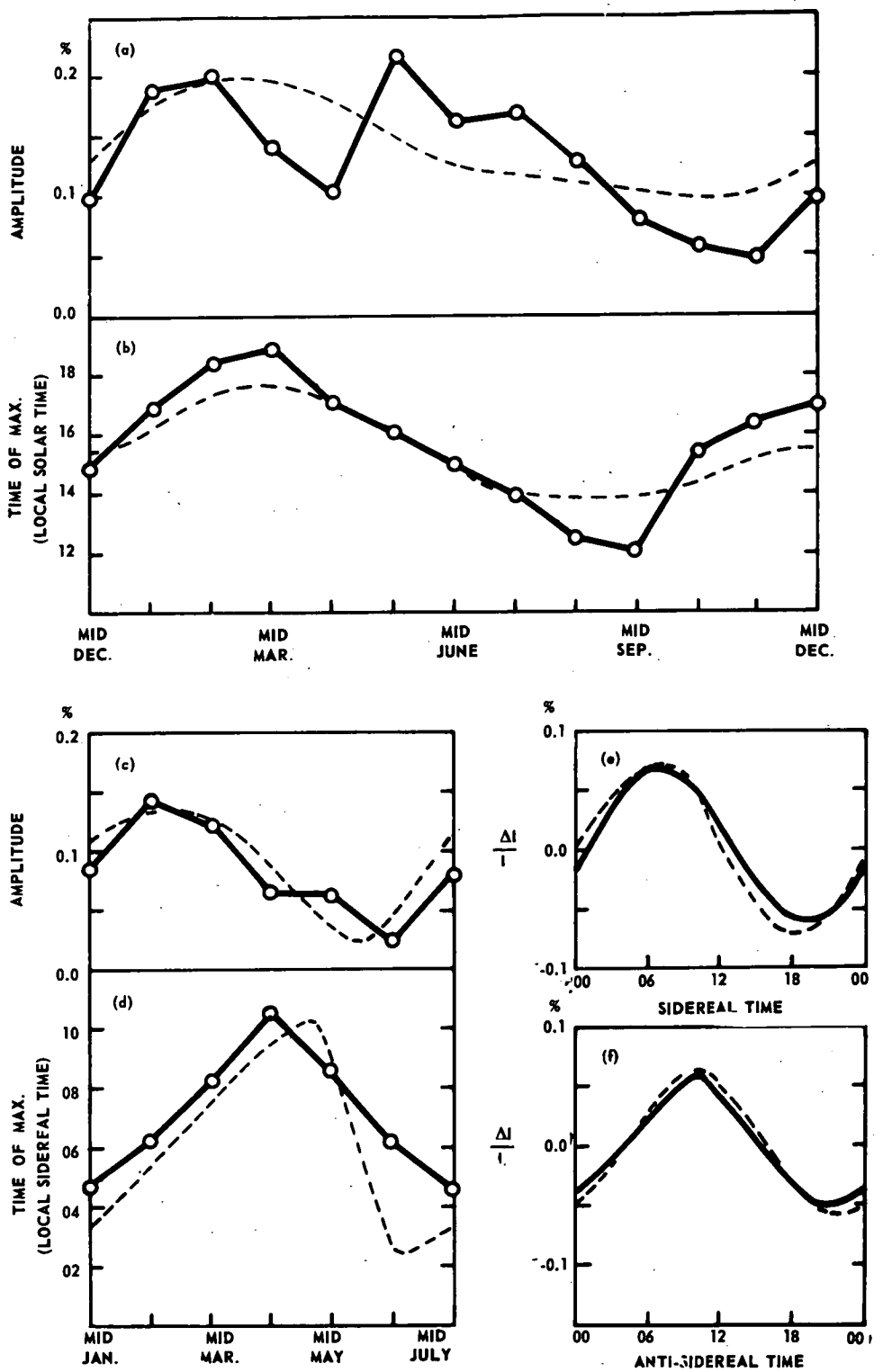


Figure 5.6 Hypothesis II (b). The results of model analyses (dashed lines) compared with the observed characteristics of the daily variation reproduced from Fig. 3. The model is designed to give the correct amplitudes and times of maximum of D_s, and D_{as} and to reproduce I_d indentically.

induced cosmic ray diurnal maximum would be expected to occur at ∿ 1300 local solar time in Britain, while amplitudes might be expected to vary considerably from place to place, the times of maximum of diurnal temperature effects should be relatively constant. Thus there is a difficulty with the phase of the carrier if the anti-sidereal effect observed at Hobart is to be attributed to seasonal changes in a diurnal variation of atmospheric temperature. Perhaps the discrepancy should not be regarded too seriously, since as yet we have no direct knowledge of daily variations of upper atmospheric structure in Tasmania.

A more general difficulty connected with an interpretation based on hypothesis II(b) concerns the magnitude of the amplitude-modulated component. It is clear that on the annual average its amplitude ($0.12/k$ %) would be greater than that of the total solar daily variation (0.13%) if seasonal modulation of amplitude were less than 100% (i.e. $k < 1$). Thus there would be a tendency for this component to have a controlling influence over the daily variation as a whole. This will be shown to be at variance with the very significant trends of the daily variation over the eight years 1958 - 1965 (section 4(b) below), and with the apparent influence of the solar anisotropy at this depth underground (Chapter 6).

(b) Hobart, 1958-1965

A description of the observations in the vertical direction over the years 1958 - 1962 has been given in paper 5. Results from the subsequent three

years have confirmed that very substantial changes in the component daily variations have occurred, apparently connected with changes in solar activity.

Figure 5.7 gives an impression in cross-section of some of the most important changes. The observations, in three bi-ennial groups, represent in succession the situation near solar maximum (1958-59), during the declining phase (1960-61) and at the minimum of activity (1964-65). Curves of fit to the histograms of bi-hourly deviates are shown. Sums of harmonics, rather than first harmonics, are compared because of the evidence, to be discussed in later sections, that both solar and sidereal anisotropies produce significant second harmonics. We consider first the solar daily variation.

In paper 7, Appendix VII (see also Chapter 6), the decrease in amplitude of the solar daily variation is largely attributed to the diminishing effectiveness of the solar anisotropy at high rigidities of response as solar activity declines. In a quantitative treatment it will be shown that the observed first harmonics in the vertical and inclined directions represent a realistic underground response to the free-space solar diurnal variation that was formulated by Rao et alia (1963) from observations with neutron monitors. It has also become apparent from the analyses of neutron intensity (McCracken et alia, 1965; J.E.Humble, private communication) that the average direction of the anisotropy has not changed appreciably over the eight years 1958-65. Consequently, there should be no secular displacement of phase of the diurnal response underground except that which is due to the contraction of the range of

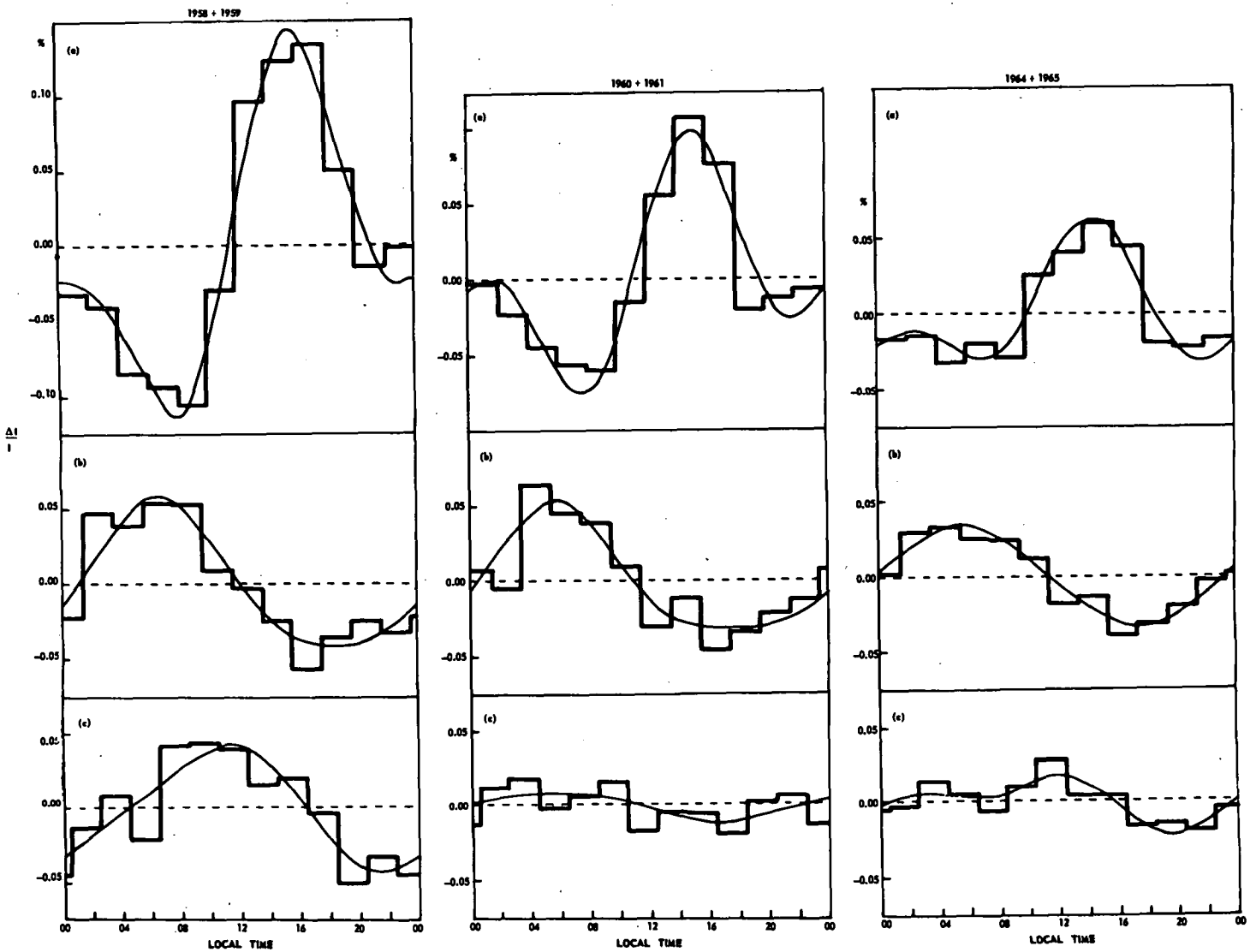


Figure 5.7 Observed biennial daily variations of vertical intensity underground, Hobart, relating to three periods of interest between 1958 and 1965, in

(a) solar time

(b) sidereal time

(c) anti-sidereal time.

The curves of best fit to the histograms and the sums of first and second harmonics.

primary rigidities over which the response is integrated. This produces a large change in amplitude but only a small displacement in the time of maximum, as Figure 2 of paper 6 shows. With the proviso that the role of the second harmonic is not yet fully understood, the long-term behaviour of the solar daily variation indicated in Figure 5.7 can be explained without difficulty in terms of changes to be expected in the solar anisotropy between the maximum and minimum of activity. In the light of this, the observations are incompatible with the presence of a large diurnal temperature component whose seasonal changes might have explained the anti-sidereal effect in 1958. The temperature component vector would have to be balanced off by a residual diurnal vector whose amplitude progressivly increases and phase angle decreases as solar activity declines, the time of maximum of the total daily variation remaining constant. Figure 5.8 shows the first harmonic vector triangles that would have to be satisfied for a minimal temperature effect.

The discussion of the temperature effect will not be carried any further here, since we now wish to consider other evidence from Figure 5.7 that tends to refute the possibility of a significant spurious sidereal effect, whether this comes from a temperature-induced diurnal variation or from any other kind of modulated solar component.

The sidereal and anti-sidereal effects observed in 1958-59 and in 1960-61 are compared. We note first that a very significant anti-sidereal component was observed in 1958-59 (the SE of amplitude of individual harmonics

Application of Silver Diethyldithiocarbamate Method to Determination of Arsenic

—D. G. Ballinger, R. J. Lishka, and M. E. Gales Jr.—

A contribution to the Journal by D. G. Ballinger, Supervisory Chemist; R. J. Lishka, Chemist; and M. E. Gales Jr., Chemist, all of the R. A. Taft San. Eng. Center, USPHS, Cincinnati, Ohio.

THE determination of arsenic has been of importance to public health agencies for many years because of the toxicity of arsenic compounds. Adequate evaluation of toxicity hazard requires arsenic measurement in the microgram range.

Gutzeit Method

Traditionally, the classic Gutzeit method has been used in public health work. The method is based on the formation of arsine in a hydrogen generator, passing of the gas stream through lead acetate to remove sulfides, and subsequent formation of brown stains on paper strips impregnated with mercuric bromide. Studies of the recovery of known amounts of arsenic have shown that there is complete conversion to arsine gas in the Gutzeit generator.[1] The difficulty in the method lies in the inability to accurately reproduce and measure the staining of the paper strips.[2] The staining is apparently affected by temperature, humidity, size and positioning of the paper, and by stability of the mercuric reagent. While arsenic test papers are commercially available, best results are obtained when test strips are prepared immediately before use.[3]

Because of the lack of reproducibility of the Gutzeit procedure, the method is considered to be only semi-quantitative.[3] The minimum detectable concentration is 1 μg in the sample aliquot. For this reason water samples must be concentrated before analysis. When the sample is concentrated 10:1, the minimum detectable limit becomes 0.01 mg/l As and the accuracy is ±0.05 mg/l. The concentration step is a significant source of error in the method, but granting no loss during digestion, the precision and accuracy of the method are not adequate for the determination of arsenic levels in drinking water.

TABLE 1

Recovery From Demineralized Water

Arsenic Added mg/l	Arsenic Found mg/l	Range mg/l	Standard Deviation mg/l
0.050	0.049	0.034–0.091	±0.0[illegible]

Heteropoly Blue Method

In an effort to increase the reliability of the arsenic determination, a heteropoly blue colorimetric method was adopted.[3,4] Because of the inefficiency of the Gutzeit generator, this step is retained and the arsine gas is oxidized to the pentavalent state in a solution of sodium hypobromite in the scrubber. The solution is removed from the absorption tube, treated with ammonium molybdate, in the presence of hydrazine sulfate, to form heteropoly blue complex.

After a color development period of hr, the optical density of the complex is measured at 820 mμ in a spectrophotometer. Since many instruments do not reach the 820 mμ range, color is generally measured at 650 with considerable reduction in sitivity. Similar complexes are med by phosphate and silicon. s, these substrates represent significant positive interferences. It has found that traces of residual phosphate on the glassware are a source error in the method.

The procedure is sensitive to approximately 1 μg in the sample aliquot therefore, concentration of the ple by acid digestion is required. centrating 250 ml of water sample ds a minimum detectable limit of mg/l As and a precision of ±0.08 l when the arsenic level in the inal sample is between 0.4 and mg/l.

s in the older Gutzeit method, the ssity for concentrating the sample desirable. In addition, the pro-re normally requires approxiely 2 hr to complete.

d for Improved Method

e recent publication of the re-USPHS Drinking Water Standemphasizes the long recognized for a more sensitive arsenic od.[5] The standards recommend arsenic concentrations in excess of mg/l should not be present, and that 0.05-mg/l arsenic shall conse grounds for rejection of the supply. It is evident that neither utzeit method nor the heteropoly blue procedure have sufficient sensitivity or precision to permit adequate evaluation of arsenic levels in the range of interest in the water quality field. In routine use these methods could not detect arsenic which might be present in concentrations at or near the permissible level and cannot satisfactorily measure compliance with the rejection standard when trace amounts of the element are present.

Silver Diethyldithiocarbamate Method

In 1952 Vasak and Sedivek proposed a new arsenic method based on a color reaction with silver diethyldithiocarbamate (SDDC).[6] They used the standard Gutzeit generator and lead acetate scrubber but adsorbed the arsine in an SDDC solution. In the absorber, arsenic replaces an equivalent amount of silver, yielding a highly colored red complex. The complex is measured photometrically at 560 mμ. The procedure was applied to the determination of arsenic in biological materials and the sensitivity was found to be 0.5 μg.

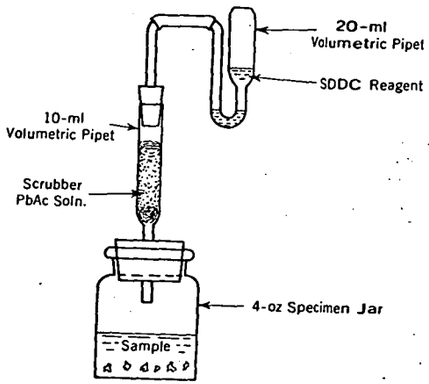


Fig. 1. Arsenic Generator and Absorber Assembly

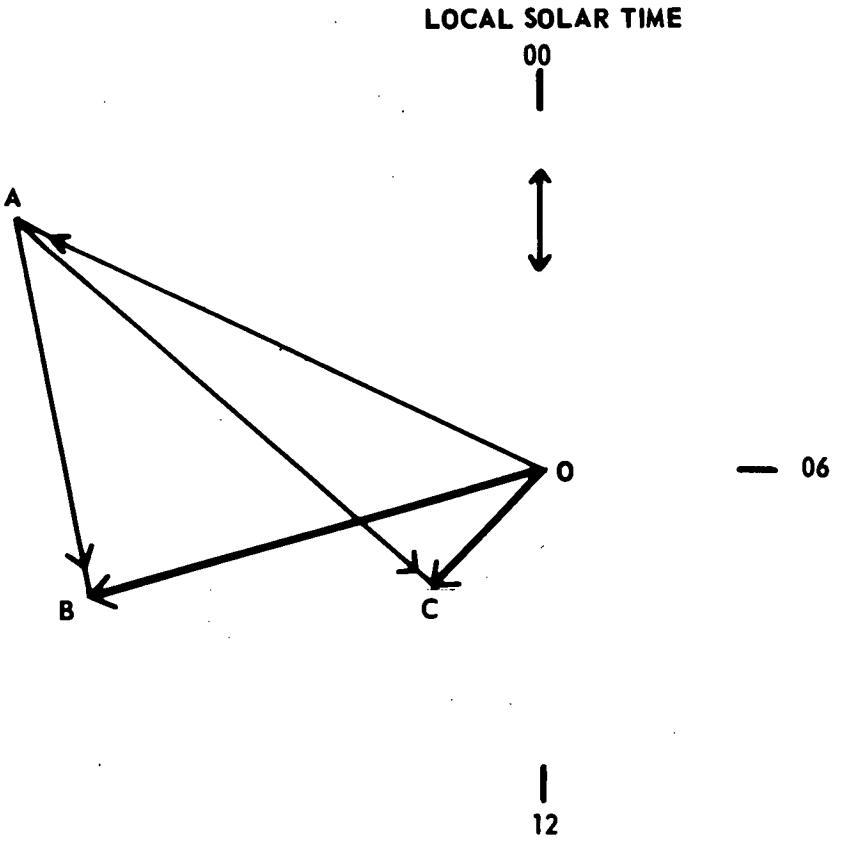


Figure 5.8 Vector triangles showing the residual first harmonic vectors $\overline{AB}$ and $\overline{AC}$ that are obtained by subtracting an assumed temperature vector $\overline{OA}$ from the solar diurnal components of vertical intensity underground averaged over the respective biennial periods 1958–59 ($\overline{OB}$) and 1964–65 ($\overline{OC}$).

was ± 0.005%), whereas in the average of the years 1960-61 the effect was almost completely absent. The change was by no means erratic. The amplitude decreased quite rapidly after 1958 and by 1960 had reached the low level shown for the two years 1960-61, and, as the result for 1964-65 indicates, has remained at approximately that level ever since. We next note that the amplitudes of the apparent sidereal and anti-sidereal components were comparable over the two years 1958-59, the ratio anti-sidereal/sidereal being 0.75, whereas in 1960-61 the ratio was approximately 0.2. The point we now wish to make is this : if the apparent sidereal daily variation in 1958-59 contained a significant spurious component, comparable to the anti-sidereal effect, it should have been evident from the parameters of the sidereal daily variation in 1960-61 that the spurious component had been removed. There is no indication whatever of this. Except for a slight difference of amplitude, it is clear that the sidereal effect observed in each of the two periods was essentially the same. There is no difference either in time of maximum or in the relationship between the harmonics (the individual harmonics are discussed in section 5.6 below and are shown in Figure 5.16). In fact it is evident that if the spurious sidereal effect in 1958-59 is obtained as the difference between the two observed sidereal effects, it must be negligible.

In seeking an explanation of the anti-sidereal effect in 1958-59 that would be compatible with later observations, the only possibility that seems worth considering seriously at present is that of semi-annual modulation of a

sidereal component. Unfortunately there is a lack of useful data from other latitudes of viewing at the peak of the effect. One possible mechanism for semi-annual modulation of amplitude will be suggested at the end of this chapter (5.7), but little can be said with confidence until a more complete description of the effect is obtained. It is considered that this must be an important objective in a continuing experimental study of the sidereal daily variation.

A general impression of the changes that have occurred in the component daily variations is given in the month to month sequence of annual running averages of amplitude (Figure 5.9) and phase (Figure 5.10). An individual pair of values of amplitude and phase is obtained from the curve drawn through the set of twelve 6-hour running average deviates that represents an annual daily variation. Some of these curves are shown in Figures 3.2 to 3.10.

It may be of interest to compare annual running averages obtained from smoothed annual daily variations with those derived from harmonic analyses of unsmoothed hourly deviates. The comparison relating to the solar daily variation, in which the long-term changes were most significant, is shown in Figures 5.9(a) and 5.10(a)*. As to be expected, the amplitudes obtaine d from 6-hour running

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* All harmonics derived from hourly deviates have been made available by J.E. Humble, using the facilities of an Elliott 503 computer.

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average deviates are smaller than the amplitudes of sums of harmonics of fit to the unsmoothed deviates, but the same trends are followed in each case. The sequence

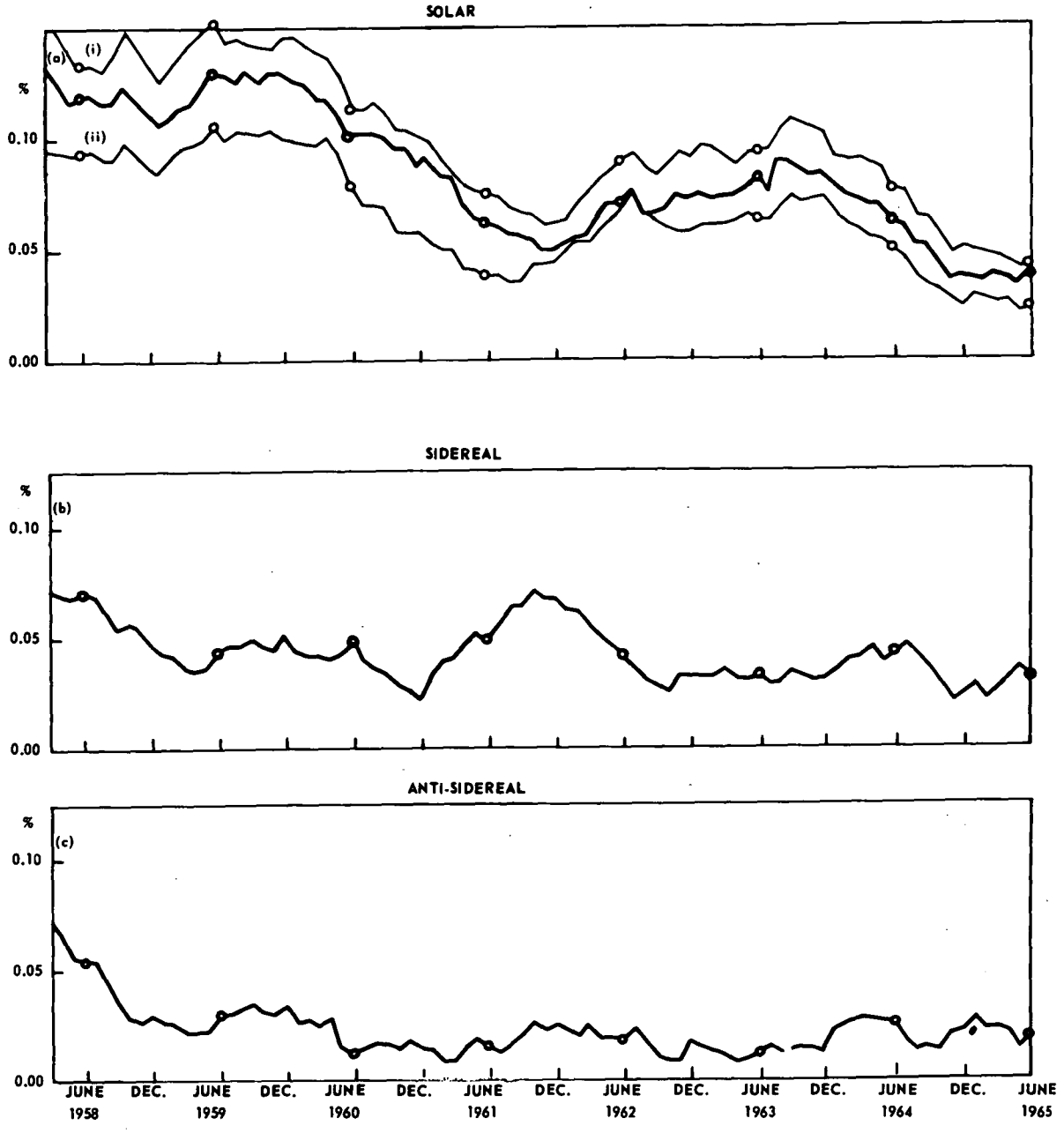


Figure 5.9 Twelve-month running averages of amplitudes of the daily variations in solar, sidereal and anti-sidereal time observed in the vertical direction underground. The dates refer to the centre-months and the circles indicate the conventional annual mean values.

Thick lines – amplitudes derived from smoothed bi-hourly deviates (see text).

Thin lines – amplitudes in solar time (i) from the sums of first and second harmonics of best fit to unsmoothed hourly deviates and (ii) from the first harmonics.

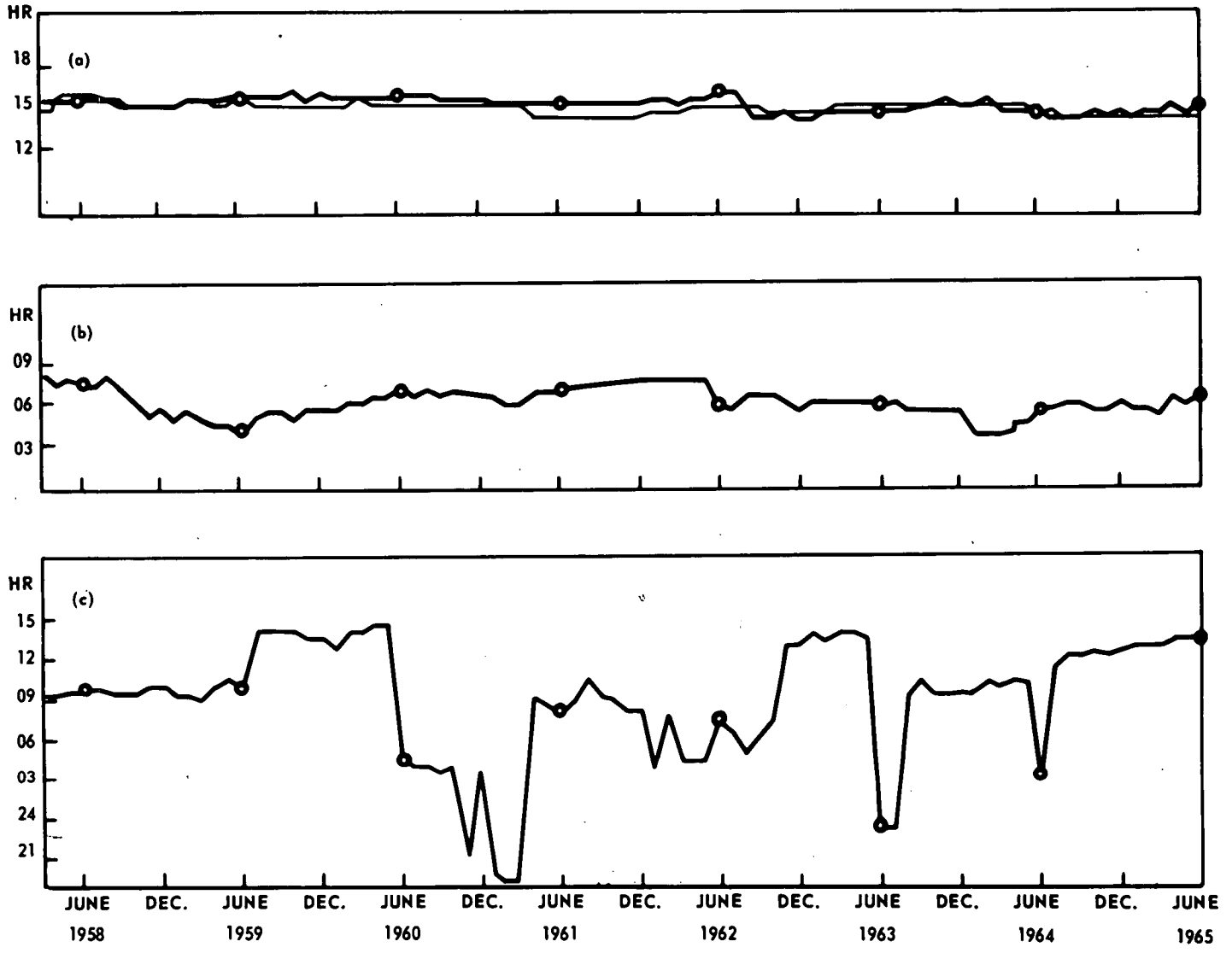


Figure 5.10 Twelve-month running averages of times of maximum of the daily variations in solar sidereal and anti-sidereal time observed in the vertical direction underground, corresponding to the amplitudes shown in Figure Thin line – times of maximum of the solar daily variation from the sums of first and second harmonics of best fit to unsmoothed hourly deviates.

of diurnal amplitudes is also shown and it is clear from the SE tails that the values of the parameters, in solar and sidereal time at least, could not have been greatly affected by statistical fluctuations.

Reasons for using annual running averages were briefly mentioned earlier (Chapter 3.2). In the short run they are liable to give a false impression, irregular changes tending to appear as quasi-persistent features. However, taken over a number of years, running means portray long-term changes more effectively than do the independent annual averages. They answer the following question: would trends that might be apparent in the set of independent averages (eight in our case) have been different if the averages had been centred on another month of the year? In the case of the amplitudes of the solar daily variation it is evident that the trend indicated by the annual averages centred on June (circles) would not have differed greatly if the average had been centred on any other month. On the other hand, independent annual averages would have represented trends in amplitude of the sidereal effect rather differently if they had been centred on October as against June. In fact, it appears from the running averages that there was a transient maximum of the sidereal effect in 1962, superimposed on the tendency for a gradual decline of amplitude.

From the point of view of seasonal modulation, the most important of the long-term changes that must be taken into account have already been discussed. We note the features of relevance in Figures 5.9 and 5.10 :

(i) The great decrease in amplitude of the solar daily variation and the constancy of the time of maximum.

(ii) The less clearly defined but, by 1965, significant decrease of amplitude of the sidereal daily variation, associated with an essentially constant time of maximum, within the limits of a year to year variability.

(iii) The attenuation of the anti-sidereal effect to a level of amplitude that was generally $< 0.02\%$ after 1960. The annual histograms (Figure 5.7) show perhaps more clearly that this effect becomes small and irregular while the sidereal daily variation, like the solar daily variation, remains well-defined and of uniform character.

5.4 Phase Anomalies

In Figure 5.11 the graphs of annual running averages of amplitude of the solar daily variation (Figure 5.9(a)) and of the apparent sidereal daily variation (Figure 5.9(b)) are referred to the same origin. It will be noted there were two periods when the averaged sidereal amplitude either exceeded the solar amplitude or was only slightly less than it, namely a period of about eight months following June 1961 and a period of about one year following June 1964. The first period in particular would be expected to give important evidence as to the nature of the apparent sidereal effect. For if the latter were not spurious then some time late in 1961 the sidereal component should have commenced to dominate the observed monthly average daily variation. It would then depend on the phase relationship between the solar and sidereal components at that time of the year as to whether an apparently enhanced solar daily

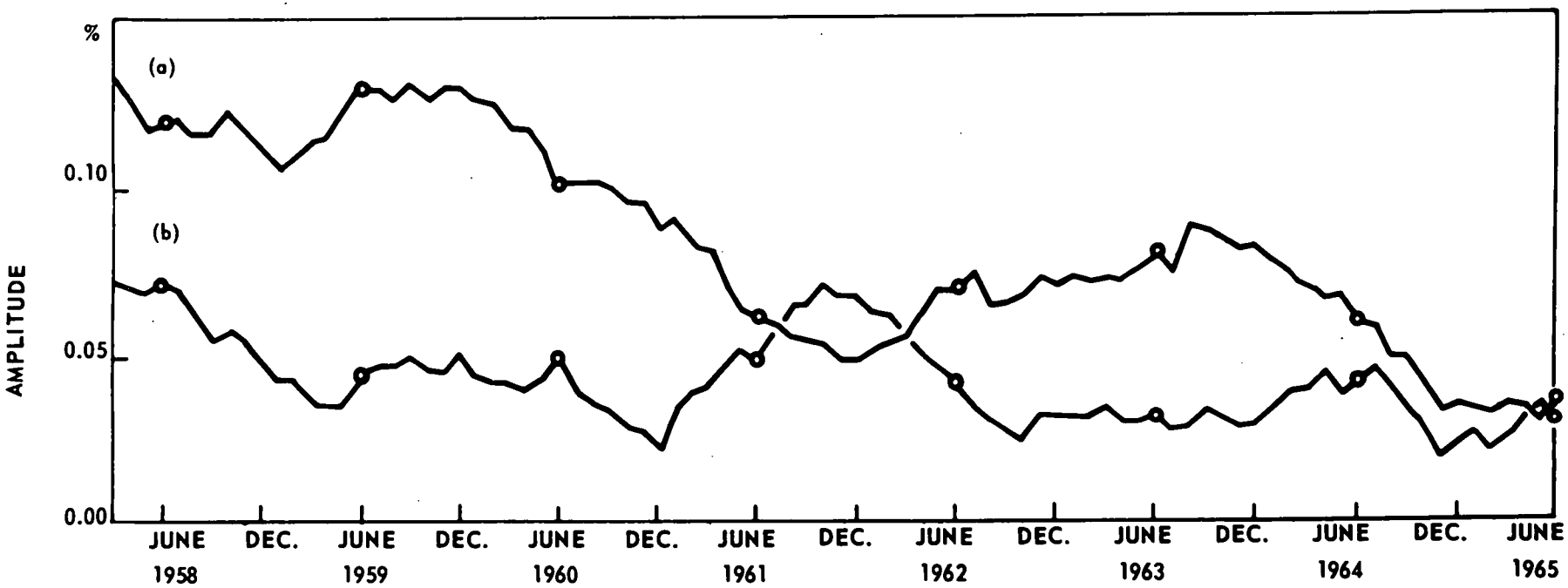


Figure 5.11 Twelve-month running averages of amplitude of (a) the solar daily variations and (b) the apparent sidereal daily variation, reproduced from Figure and referred to a common ordinate.

variation would be observed at first (both components in phase) or whether there would be a sudden large displacement of phase as the net daily variation became orientated in sidereal time (components out of phase). The period following June 1964 might also present opportunities of observing the dominant effect of the sidereal component provided that there was assistance from relatively modest seasonal or irregular changes. If, on the other hand, in spite of the evidence already given to the contrary, the apparent sidereal effect was essentially spurious, we should expect to observe during both periods only enhanced seasonal changes in a small solar daily variation.

The sequences of amplitude and phase of the observed monthly average daily variation (arranged in solar time) are shown in Figure 5.12. The well-defined phase anomaly late in 1961, the absence of this effect at the same time of the year in 1962 and 1963 and the incomplete effects in 1964 and 1965 are clearly in accord with the trends of amplitude shown in Figure 5.11 if the annual amplitudes in sidereal time are attributed to a genuine sidereal daily variation. Confirmation is given by the equally good accord with the annual times of maximum depicted in Figures 5.10(a) and (b). A genuine sidereal daily variation with a maximum at approximately 0700 local sidereal time and a solar daily variation whose maximum occurs at approximately 1500 local solar time become 12 hours out of phase with each other in November. It is noted that on the occasions of the anomalies the large displacement of phase took place either in October or November. In fact, as the dashed line in Figure 5.12 indicates, the time of maximum of the daily variation during the phase anomaly in 1961

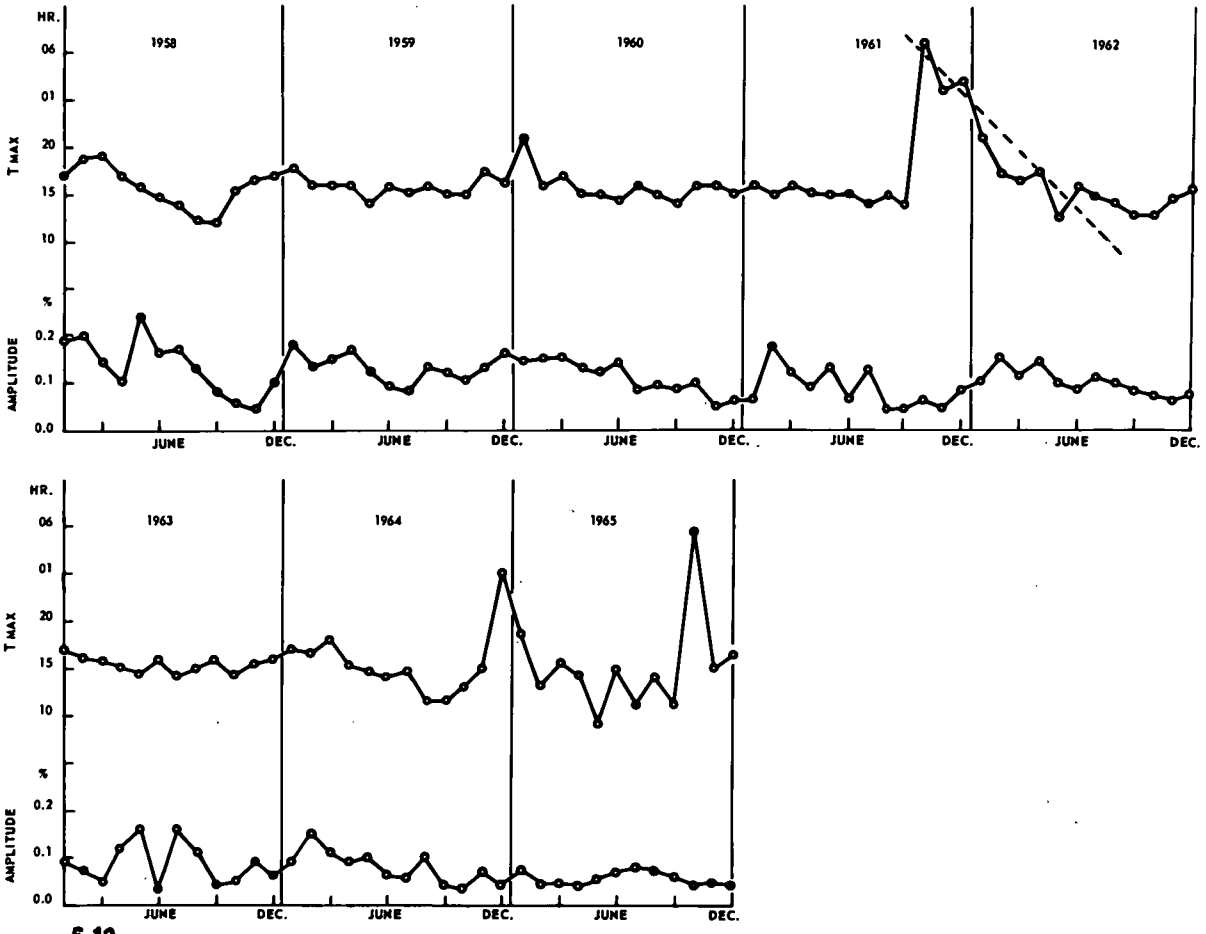


Figure 5.12 Monthly mean values of the time of maximum (in local solar time) and amplitude of the daily variation of vertical intensity underground, Hobart. The dashed line associated with the anomalous times of maximum in 1961–62 represents 0700 local sidereal time.

followed 0700 local sidereal time quite closely. Supposing the sidereal diurnal maximum to have occurred at 1900 sidereal time, on the other hand, the phase anomaly would have commenced in April.

It is submitted that these phenomena not only constitute strong evidence for a genuine sidereal component in the daily variation, but indicate also that any spurious contribution to the annual apparent sidereal effect must have been very small.

An analysis of the phase anomaly in 1961 is given in paper 5 where it is shown that when suitable models for the solar and sidereal components are used, the significant features of the daily variation in 1961 are reproduced.

The only other known instance of a phenomenon that was apparently of this kind relates to observations at ground level in the northern hemisphere during 1954. The effect, reported by Conforto and Simpson (1957) has been discussed in Section 3.1. Monthly average daily variations of both meson and neutron intensities exhibited progressive displacements of phase towards earlier values in the manner of a sidereal component with a diurnal maximum at approximately 2000 local sidereal time. In Figure 5.13 the effect on the daily variation of meson intensity at Rome is compared with the phase anomaly observed at Hobart in the daily variation of meson intensity underground during 1961-62.

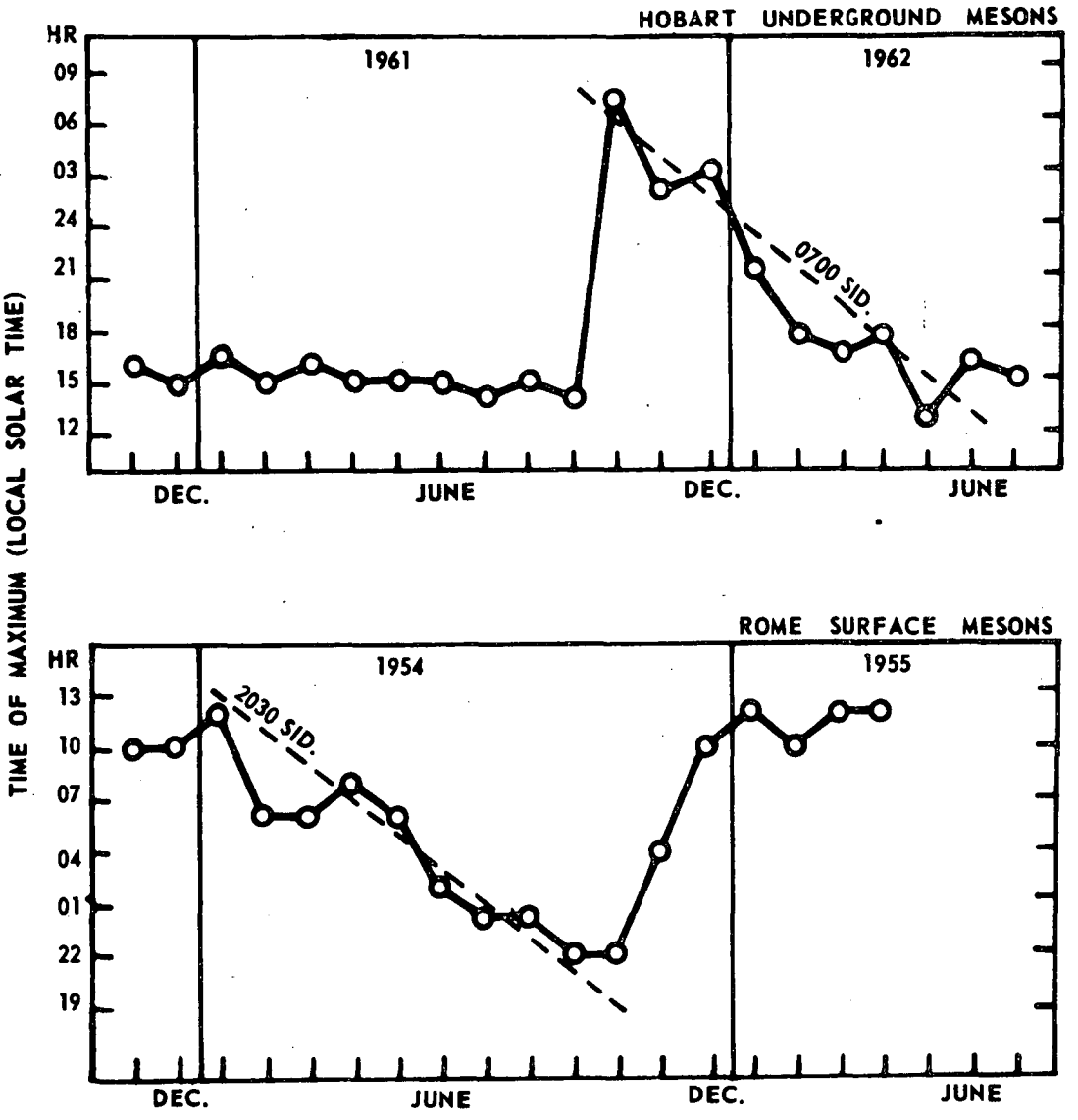


Figure 5.13 Monthly averages of the time of maximum, in solar time, of the pressure-corrected daily variation of vertical meson intensity at the Hobart underground site (1961–1962) and at Rome (1954). The dashed lines represent the solar time equivalents of 0700 local sidereal time and 2030 local sidereal time as shown.

As mentioned in section 3.1, the time of maximum of the annual average solar daily variation advanced considerably after 1950 and reached a turning point in 1954. Thereafter it began to return to later values. There seems little doubt that important changes in the solar daily variation of extra-terrestrial origin had occurred, because of the effect on the neutron component at Huancayo and Climax where the end-points of the annual running average diurnal vectors exhibited an anti-clockwise rotation between the years 1953 and 1955 (Glokova, 1960). Glokova has shown that the mean-year solar diurnal variation of meson intensity did not behave in quite the same way and evidently consisted of a varying component due to the solar anisotropy and a constant component of atmospheric origin. The latter would appear to have been consistent with a diurnal variation due to atmospheric temperature as deduced by Quenby and Thambyahpillai (1960) from an analysis of the nucleonic and hard components at Huancayo. The estimated temperature component at Huancayo had a maximum of 0.11% at 0540 local time.

It would seem from the above considerations that greatly weakened solar modulation of the primaries was responsible for the small and complex *mean-year diurnal variation* of neutron intensity, while the corresponding solar diurnal variation of meson intensity tended to come under the control of a constant component (due to atmospheric temperature) as the extra-terrestrial contribution weakened. On the other hand, although the *monthly average diurnal variations* were probably influenced by changing solar modulation and (in the case of the

hard component) by seasonal changes in the temperature effect, they appear to have been more substantially affected by a sidereal component.

There was considerable variation of individual response in respect of the phase anomaly of 1954. This would probably depend in the main on the nature of the temperature vector at the place of observation (for mesons), on the primary cut-off rigidity of the detector, and on the spatial characteristics of the two anisotropic components, particularly their respective latitude dependences of amplitude. As we have seen, the anomalous variations of phase in the hard component at Rome followed constant sidereal time quite well, perhaps in consequence of the rather high cut-off rigidity at that place and a sufficiently small diurnal variation due to atmospheric temperature. In contrast to this, the anomaly was not seen in the hard component at ground level at Hobart. On the latter occasion this was evidently connected with the fact that the average amplitude of the sidereal effect observed underground in the southern hemisphere was greater than that in the northern hemisphere by a factor of about two. The point will be discussed in section 5.5 where annual averaged observations from the two hemispheres are compared. In later chapters the unsymmetrical nature of the latitude dependence of amplitude will be considered more closely, being one of the major features of the sidereal effect observed underground.

One cannot discuss the latitude dependence of amplitude in 1954 with confidence because of the difficulty of assessing the nature of the apparent

sidereal effect in the hard component at sea level. However, it may be of interest to refer once again to Figure 3.1 which shows that over the period 1947-54 the amplitude of the apparent sidereal effect in the ion chamber data at Cheltenham was greater than it was at Christchurch, while the reverse was the case during the period 1938-1946. If a genuine sidereal component had been responsible for these events the absence of the phase anomaly at Hobart in 1954 would appear to have a simple explanation. It is considered that there is sufficient in the evidence to suggest that the type of asymmetry in the latitude dependence of amplitude observed in the recent underground data should not necessarily be regarded as a constant feature.

It is noted that the time of maximum of the sidereal component deduced from the phase anomaly in 1954 agrees well with the time of maximum (approximately 2000 sidereal time) of the annual apparent sidereal effect averaged over a large number of years (see Figure 3.1 relating to the ionization measurements at Cheltenham). Thus, while the phase anomalies give evidence of a genuine sidereal component, acceptance of the evidence requires that there be a 12-hour phase displac ement depending on whether the observations are made in the northern or the southern hemisphere. We now proceed to discuss other evidence, sone of which indicates unequivocally that the 12-hour phase difference does indeed relate to a genuine sidereal daily variation.

5.5 The 12-hour Difference of Phase Observed in the Sidereal Maxima from Opposite Hemispheres

Among the many observations that have accumulated from experiments in the northern hemisphere at ground level there is an overall tendency for the maximum of the apparent sidereal effect to occur at apptoximately 2000 local sidereal time. The variability of these observations, and of observations at ground level generally, has been noted in Chapter 3. It seems likely that the spurious effects, due severally to secular, seasonal and irregular changes in the large solar component, are frequently important and would be very difficult to extract. This might raise considerable doubt as to the genuineness of even the more persistent effect at ground level. Thus it would be reasonable to suspect that the difference of roughly 12 hours between the apparent sidereal diurnal maxima at Christchurch (43.5°S) and Cheltenham (38.7°N) from ion chamber measurements (see Figure 3.1) was a sideband effect, due to seasonal modulation of the solar daily variation. Nevertheless, the average time of maximum from Christchurch was not very different from that observed underground at Hobart, while, as we noted in the previous section, the time of maximum from Cheltenham agreed quite well with that deduced from the phase anomaly in 1954. Thus there was clearly a suggestion that a genuine sidereal daily variation may have been responsible for the 12-hour phase difference.

In an examination of this aspect of the sidereal effect it is obviously desirable to have simultaneous observations from similar detectors located at approximately the same depths underground in the separate hemispheres. For the purpose of comparison with the underground observations at Hobart it was most fortunate that data were available from two (and later, three) semi-cubical

vertical telescopes situated at a depth of approximately 40 m.w.e. underground at Budapest (47.5°N, 18.9°E geographic). Each telescope consisted of four trays, giving fourfold coincidences and was of dimensions 1.264 × 1.264 × 0.632 m. From February 1958 to August 1961 the duplex counting rate was approximately 100,000 pcles/hr. Thereafter three telescopes were used, with a combined counting rate of approximately 160,000 pcles/hr.

The only complete calendar years for which continuous records were available from Budapest at World Data Centre WDC2 were the years 1959 and 1961. The results of harmonic analyses of the daily variations in sidereal time observed at Hobart and Budapest over the two years are listed in Table 5.2. In Figure 5.14 the individual harmonics are shown against the histograms of the bi-hourly deviates. The 12-hour phase difference between the first harmonics is clearly indicated, the actual difference being 11 hours $\pm$ 50 minutes (SE). The sums of harmonics also exhibit this difference, the first and second harmonics being in phase.

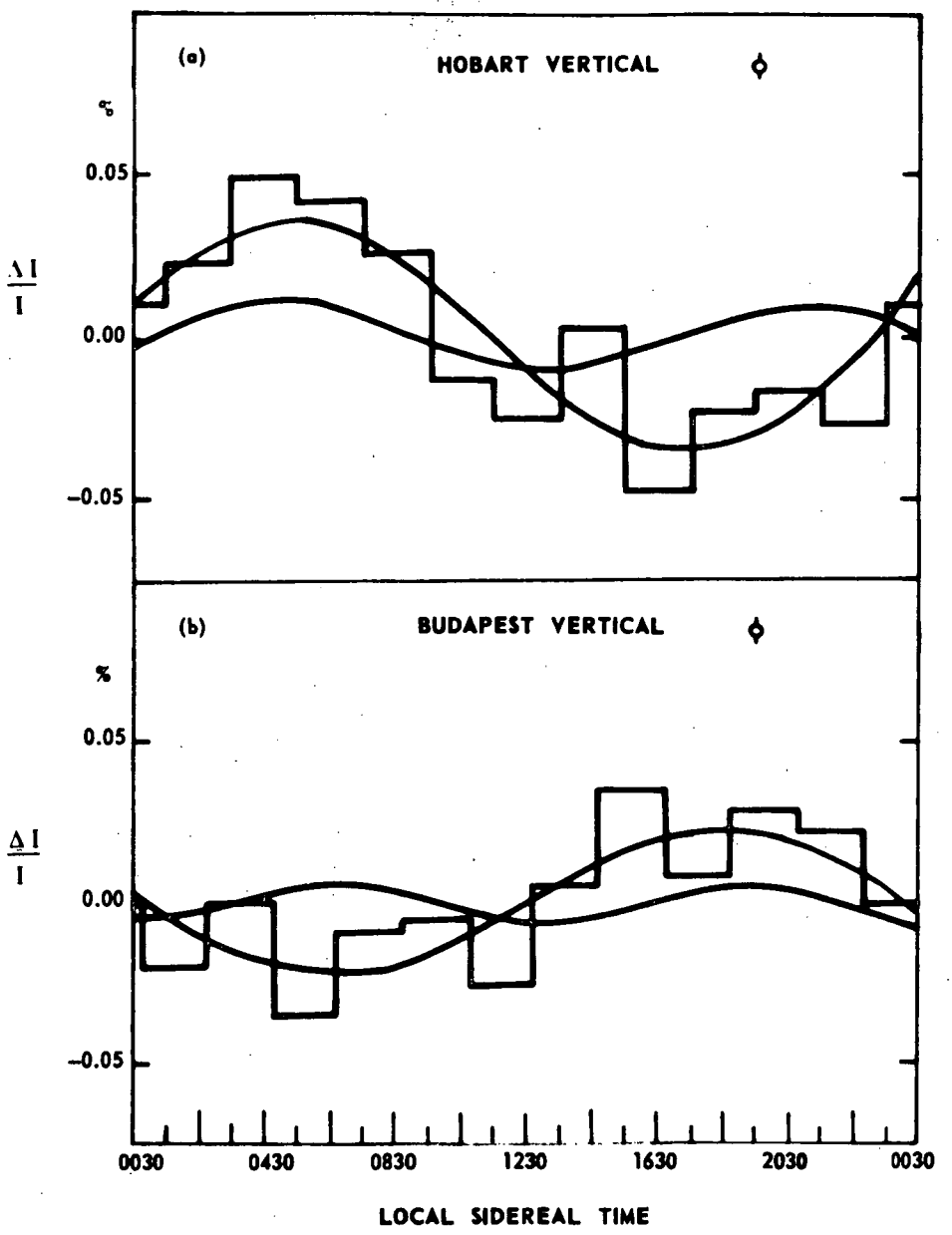


FIGURE 5.14 The observed daily variation in sidereal time, averaged over the years 1959 and 1961, of pressure-corrected meson intensity from vertical semi-cubical telescopes at a depth of ≈ 40 m.w.e

(a) at Hobart, scanning asymptotic latitudes in the vicinity of 39°S geographic
(b) at Budapest, scanning asymptotic latitudes in the vicinity of 44°N geographic.

The error tails shown are the S E's of amplitude associated with the first and second harmonics of best fit.

TABLE 5.2

Hobart and Budapest. Harmonics of best fit to the apparent sidereal daily variations of pressure-corrected bi-hourly deviates, averaged over the two years 1959 and 1961.

	1st harmonic		2nd harmonic		Sum of harmonics	
	Amplitude %	Tmax hr	Amplitude %	Tmax hr	Amplitude %	Tmax hr
Hobart	0.036	0520	0.012	0500	0.048	0520
SE	± 0.004	± 0020	±0.004	±0120		
Budapest	0.023	1830	0.006	1900	0.028	1830
SE	±0.004	±0040	±0.004	±0240		

Observations from the two underground cubical telescopes inclined 30°N and 45°S of the zenith at Hobart will be discussed in later sections. It is noted here that significant daily variations in sidereal time were observed in the two directions and that the times of maximum, whether of first harmonics or sums of harmonics, conformed with the result from the vertical direction. Averaged over the two years of operation, 1961 and 1962, the sidereal diurnal maximum in the north-pointing direction occurred at 0700 ± 0040. In the south-pointing direction the diurnal maximum, averaged over the four years 1961 - 1964, occurred at 0600 ± 0110.

A result which corroborates the observations from Budapest has recently been reported from London (Thambyahpillai et alia, 1965) where two (and later, six) semi-cubical vertical scintillator telescopes, each of detecting area 1.44 m^2, had been installed at a depth of approximately 60 m.w.e. underground. The time of maximum of the observed sidereal diurnal variation, averaged over the three years from 1961 to 1964, was 1800 $\pm$ 55 minutes, differing by 12 hours from the time of maximum at Hobart averaged over the eight years 1958 - 1965.

Table 5.3 summarises the times of occurrence of the observed sidereal diurnal maxima that have been discussed here. From the point of view of reproducibility and statistical accuracy this is believed to be the best information available.

TABLE 5.3

Times of maximum of the first harmonics of daily variations in sidereal time, from pressure-corrected data, averaged over the years indicated.

Northern hemisphere	Time of maximum
Cheltenham (38.70°N geog) sea-level ionisation 1938-1946 (9 years)	2120
Cheltenham sea-level ionisation 1947-1954 (8 years)	2110
Budapest (47.5°N geog) vertical semi-cubical telescopes at ∿ 40 m.w.e. underground, 1959, 1961 (2 years)	1830
London (50°N) vertical semi-cubical telescopes at ∿ 60 m.w.e. underground, 1961-1964 (3 years)	1800
Southern hemisphere	
Christchurch (43.5°S geog) sea-level ionisation 1938-1946(6 years)	0630
Christchurch sea-level ionisation,1947-1954 (8 years)	0750
Hobart (43°S geog) ∿ 40 m.w.e. underground vertical semi-cubical telescopes 1958-1965 (8 years)	0610
cubical telescope inclined 30°N 1961-1962 (2 years)	0700
cubical telescope inclined 45°S 1961-1964 (4 years)	0600

The 12-hour phase difference in the underground observations is unmistakable. From the Cheltenham-Christchurch comparison a similar effect at ground level is suggested, although there appears to be a tendency for the maxima to occur later and for the phase difference to be somewhat less than 12 hours.

Considered aside from any other evidence, the 12-hour phase difference underground demonstrates that the apparent sidereal daily variation is controlled either by seasonal modulation of a solar component or by a two-way sidereal anisotropy or by a combination of these two phenomena. (Note that a uni-directional sidereal anisotropy could not have been responsible for the effect.) If we now add the evidence indicating that the observed sidereal effect in the vertical direction at Hobart was essentially genuine it would appear that a two-way anisotropy was exerting the controlling influence and that the sidereal effects at Budapest and London were essentially genuine also.

The above conclusion would have been strengthened if the absence of significant spurious components in the underground observations from the northern hemisphere could have been demonstrated independently of the 12-hour phase difference and the observations from Hobart. However, some of the significant indications of a genuine sidereal effect at Hobart were not to be found in the available observationsffrom the northern hemisphere, and it appears that this was because the sidereal amplitudes were smaller. (The amplitude at Budapest was about 60% of the amplitude at Hobart, while at London the amplitude was less by a

factor of at least two, but was not directly comparable, having been obtained at a considerably greater depth.) Notably, a phase anomaly was not observed at Budapest (or at London) in 1961, although it was evident from the available data that the solar daily variation at Budapest was the same as at Hobart and exhibited the same large changes (e.g. see Table 6, paper 7). Thus the non-appearance of the phase anomaly seems to have been directly attributable to the much smaller sidereal effect. Again, an analysis of the anti-sidereal effect would be handicapped, in the first place, because of the small observed amplitudes ($\sim$ 0.015% both at Budapest and London). In the second place, there were not, as far as is known, substantial year to year changes of the kind which would either indicate the presence of the appropriate solar carrier and annual sidereal sideband to confirm suggested modulation of a solar daily variation, or indicate the absence of these if the anti-sidereal effect had been largely caused by modulation of a sidereal daily variation.

It was desirable then, to obtain supporting evidence that the 12-hour phase difference was of sidereal origin by some quite different method. It was therefore decided to set up a telescope inclined in the plane of the meridian so as to view asymptotically entirely within the northern hemisphere from the underground laboratory at Hobart. If the phase difference that had been observed between the two hemispheres was a genuine sidereal effect there should be a displacement of approximately 12 hours between the annual apparent sidereal maxima observed in the vertical and inclined directions. As distinct from this, the maxima from these two directions should be approximately in phase (1) if the results from Hobart and Budapest in the vertical direction had both been spurious,

as the result of seasonal modulation of a solar component, or (2) if the result from Hobart in the vertical direction had been genuine, but not due to a two-way anisotropy, while the result at Budapest had been spurious, or vice versa. Thus, in principle, the experiment would be able to distinguish unambiguously between a 12-hour phase difference of sidereal origin and one that was either partly or completely produced by spurious sidereal components resulting from any kind of local seasonal modulation of a solar daily variation. A narrow angle telescope was set up underground in the direction 70°N of zenith (see 5.2 above) and was put into operation towards the end of 1963. The location of the asymptotic cone of acceptance is indicated sufficiently well from the following considerations. The total geometric aperture being 36°, both in zenith angle and azimuth, the estimated meson cut-off energy at production increases from approximately 14 GEV to 70 GEV between the lower and upper limits of zenith angle acceptance. (The values may be calculated in the manner described in section 4.6, except that the energy loss must be determined for a curved atmosphere at high zenith angles. For near-horizontal incidence the atmospheric energy loss given by Wilson (1958) has been used.) From the coupling coefficients applicable to the production of mesons of energies > 15 GEV in the vertical direction (see Chapter 6) it follows that the mean energy of response to the average primary spectrum in the inclined direction must be considerably in excess of 100 GEV and we can be confident that this would also be true of the response to the variation spectrum of a sidereal anisotropy. Charged primaries of these energies arriving from the north at the latitude of Hobart experience only small deflections in the terrestrial magnetic field. It can be seen from the diagrams of Brunberg

and Dattner (1953) that deflections would not be greater than about 3° for all directions of arrival within a cone of half angle 18° centred on the zenith angle 70°N. Consequently, the asymptotic cone is centred only a few degrees southward, in latitude, and eastward, in longitude, of the geometric cone of acceptance. Latitudes of viewing which range from 9°N to 45°N within the geometric cone are therefore estimated to range from about 7°N to 42°N in the asymptotic cone. While the boundary cannot be precisely defined because of scattering in the material absorber, particularly of those mesons which are near the end of their range on arrival at the equipment, it is clear that the asymptotic cone must be confined essentially to the northern hemisphere, being centred at about 25°N geographic latitude and not more than 3°E of the meridian through Hobart. On the other hand, when the integrated cone of acceptance for a semi-cubical vertical telescope is calculated for a sidereal anisotropy, it is found to be centred approximately 10°E of the meridian at the latitude 39°S (Chapter 6). Therefore, the resulting distortion due to this cause in the observation of a 12-hour phase difference of sidereal origin, is estimated to be less than half an hour.

Although only a qualitative result was to be expected from the experiment, it was hoped that it would clearly distinguish between the two possibilities. In the event of a spurious sidereal effect, due to a modulated solar daily variation, the first harmonics from the two directions should be essentially in phase. Alternatively, in the event of an effect due to a simple two-way sidereal anisotropy that was relatively undistorted beyond the earth's

field region, not only the first harmonics, but the sum of first and second harmonics should be in phase. Moreover, a relatively large second harmonic would be expected from the north-pointing detector, which scans near the equator. (In advance of the quantitative treatment it is noted that the semi-diurnal component must increase to a maximum at the equator, while the diurnal component falls to zero, either at the equator or very close to it.) Thus the best indication of the 12-hour phase difference, if it existed, would come from a comparison of sums of harmonics.

The first complete year of observation, 1964, yielded a significant daily variation in sidereal time. The time of maximum of the first harmonic, 1600 $\pm$ 0140 (SE), when compared with the time of maximum in the vertical direction, 0500 $\pm$ 0030, unmistakably favoured the 12-hour phase difference. The corresponding times of maximum of sums of harmonics were 0400 and 1340 respectively, giving a phase difference of 9 hours 40 minutes.

The sidereal maxima in the two directions were again significantly out of phase in 1965, the difference being more pronounced in the sum of harmonics ($\sim$ 10 hours) than in the first harmonic ($\sim$ 8 hours). These results were accompanied by improved evidence for a significant second harmonic in the north-pointing direction.

From the average of the two complete years 1964 and 1965, the observed phase difference between sums of harmonics was 12 hours 14 minutes and

between first harmonics was 9 hours 14 minutes. The daily variations of bi-hourly deviates are shown, with the sums of harmonics of best fit, in Figure 5.15. Table 5.4 summarises the results of harmonic analysis of the observations from the narrow angle inclined telescope*.

*Following the installation at the University of the Elliott 503 computer, it has become the general practice to analyse hourly, rather than bi-hourly, count totals. Thus some of the more recent groups of harmonics presented in this thesis have been computed from hourly deviates. Care is taken to ensure that all harmonics employed in a given comparison are derived in the same manner.

TABLE 5.4

Harmonic characteristics of the apparent sidereal daily variations observed with the narrow angle telescope inclined 70°N of zenith. The harmonics are derived from hourly deviates of pressure-corrected intensity. The errors shown are the SE's of estimate.

	1964	1965	1964 + 1965
First harmonic			
Amplitude (%)	0.104 ± .044	0.095 ± .044	0.092 ± .031
Tmax (Hr)	1600 ± 0130	1310 ± 0140	1430 ± 0120
Second harmonic			
Amplitude (%)	0.029 ± .044	0.087 ± .044	0.055 ± .031
Tmax (Hr)	1700	1830 ± 0140	1810 ± 0200
Sum of harmonics			
Amplitude (%)	0.091	0.112	0.110
Tmax (hr)	1340	1740	1730

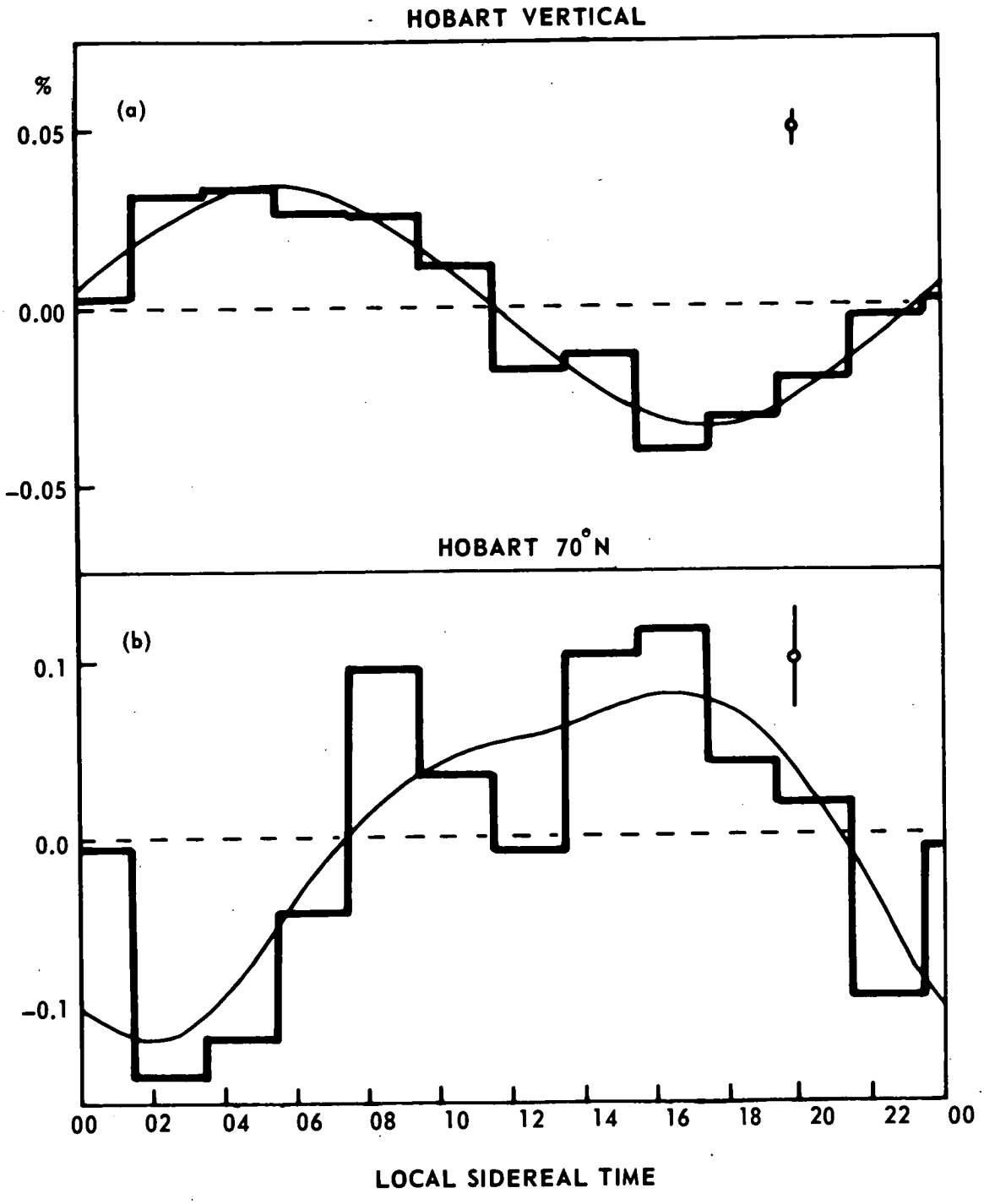


Figure 5.15 The observed sidereal daily variations of underground meson intensity at Hobart, averaged over the years 1964 and 1965
(a) from the vertical semi-cubical telescopes, scanning asymptotic latitudes in the vicinity of 39° S geographic and
(b) from a narrow angle telescope inclined 70° to the north of zenith, scanning asymptotic latitudes in the vicinity of 20° N geographic.

The error tails shown are the S.E.'s of amplitude of the individual harmonics.

An indication of the consistency of the phase difference observed between the sums of harmonics in the two directions at Hobart is given by the annual running averages listed in Table 5.5. It appears that the phase differenc associated with the year ending in December 1964 and December 1965 were representative of a presistent effect. The only important divergence from a large phase displacement occurred in the year ending in February 1965. It came about through a somewhat early maximum (∿ 1400) of the first harmonic in the inclined direction, favouring the first maximum of the second harmonic .

TABLE 5.5 (see over)

TABLE 5.5

Comparison of the apparent sidereal daily variations observed with the vertical semi-cubical telescope and the narrow angle telescope inclined 70°N of zenith. Times of maximum of sums of first and second harmonics of best fit to the hourly deviates are tabulated in the form of annual running averages.

Year Ending		Vertical Tmax HR	Inclined 70°N Tmax HR	Difference (inclined-vertical) HR
1964	December	0400	1340	0940
1965	January	0450	1810	1320
	February	0540	0900	0320
	March	0520	1840	1320
	April	0610	1830	1220
	May	0800	1820	1020
	June	0820	1820	1000
	July	0710	1740	1030
	August	0720	1820	1100
	September	0720	1830	1110
	October	0740	1840	1100
	November	0650	1830	1140
	December	0710	1730	1020

It is clear that greatly improved counting statistics are needed in the inclined direction if useful information concerning the relative amplitudes of the harmonics is to be obtained. It will be shown later that a two-way anisotropy which conforms with observations from other latitudes of viewing would require the amplitude of the second harmonic appropriate to the cone of viewing of the inclined telescope, to be somewhat greater than the amplitude of the first harmonic. The only suggestion of this in the present data is that on eight of the thirteen occasions to which Table 5.5 refers, the amplitude of the second harmonic exceeded that of the first.

5.6 The Observed First and Second Harmonics

In his considerations of acceleration mechanisms associated with the galactic magnetic field, Davis (1954) has described the two-way anisotropy that results when excess fluxes of particles with steep helices are being propagated in both directions along the field. He shows that the sidereal daily variation observed terrestrially should consist of first and second harmonics that are in phase (see Table 1.1). In Chapter 6 of this thesis the harmonics that Davis had obtained in his more generalised treatment of anisotropies will be derived independently, employing an empirical expression for the two-way anisotropy. On the assumption that the intensity maxima are equal in the opposite directions and that the anisotropy is being observed directly (i.e. ignoring the influence of local magnetic fields) the amplitude of the observed sidereal diurnal variation would reach its maximum value at the latitudes of viewing 45°S and 45°N and would be zero at the equator and the poles. On the other hand the amplitude of the second harmonic would reach its maximum value at the equator and would be zero at the poles. The maximum amplitudes of the two harmonics would be equal.

In reality, because local magnetic fields intervene, the above characteristics more properly relate to the free-space harmonics as functions of asymptotic latitude of viewing. Moreover, the evidence will suggest that the intensity maxima in the opposite directions along the axis of anisotropy are not equal. It will be shown that the sidereal diurnal variation may in fact consist of two parts, one part being identifiable with the two-way anisotropy and hence associated with a semi-diurnal variation to which it bears a fixed relation at the

given latitude of viewing. It will be proposed that there is an additional contribution to the diurnal variation from a single intensity maximum along the axis of the anisotropy, due perhaps to a balance of streaming. Thus if the two components of the diurnal variation changed with respect to each other, the ratio of amplitudes of the observed first and second harmonics would vary accordingly, It follows that a survey of the latitude dependence of the observed harmonics should relate to observations that are as nearly as possible simultaneous with each other.

The factors that have just been mentioned will be given full consideration in the later chapters. Our main concern in this section is to demonstrate that the observed sidereal daily variation does comprise first and second harmonics, which, when significant, are in phase, both from observations in the vertical direction at Hobart relating to individual years and from simultaneous observations at different latitudes of viewing.

(a) Observations in the vertical direction at Hobart 1958-1965

Figure 5.16 shows the results of harmonic analysis of the observations averaged over each of the four biennial periods and their sum. It can be seen that when the small second harmonic was detectable it was in phase with the first harmonic and that in three of the four biennial periods and in the eight-year average the amplitude was just significant at the 5% level of probability. From annual running averages derived from hourly deviates (Figure 5.17) there is some indication that the amplitude decreased over the eight years, with a

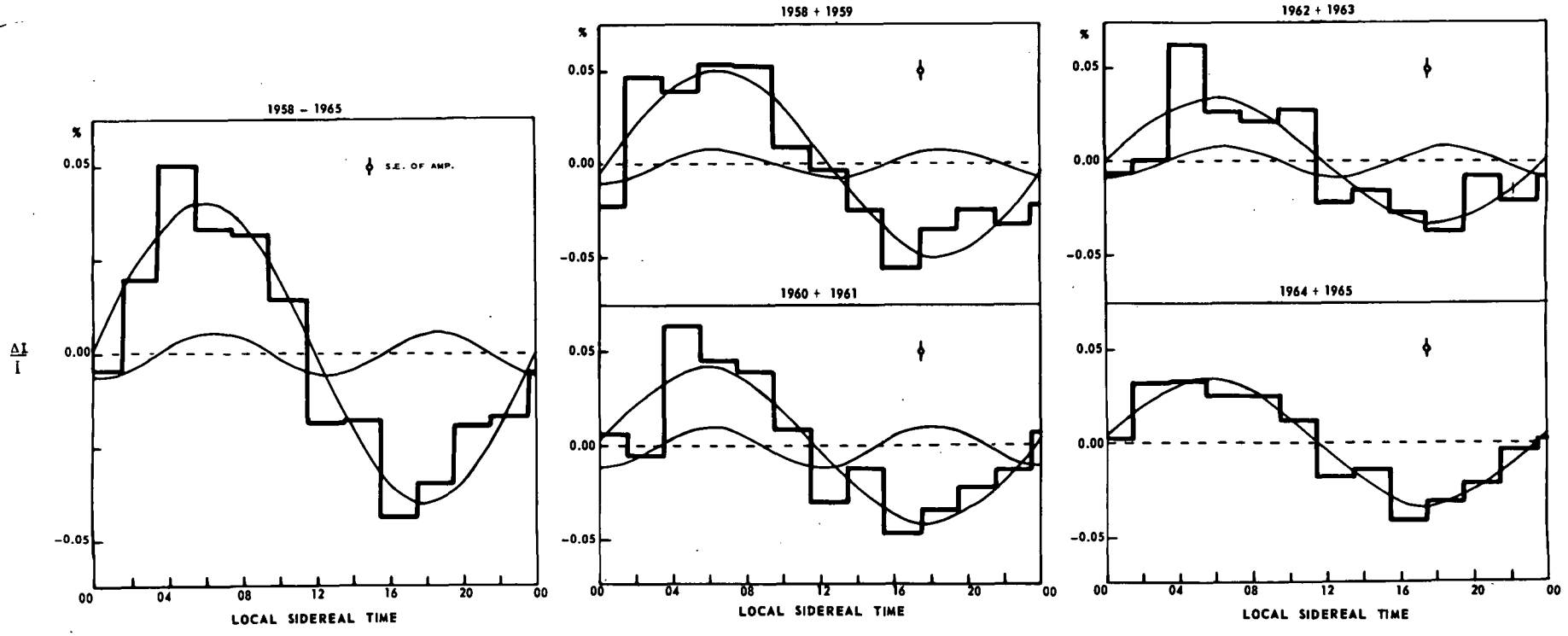


Figure 5.16 The apparent sidereal daily variation of vertical intensity underground, Hobart. The histograms, relating to the eight-year average and the four biennial periods, are shown with first and second harmonics of best fit.

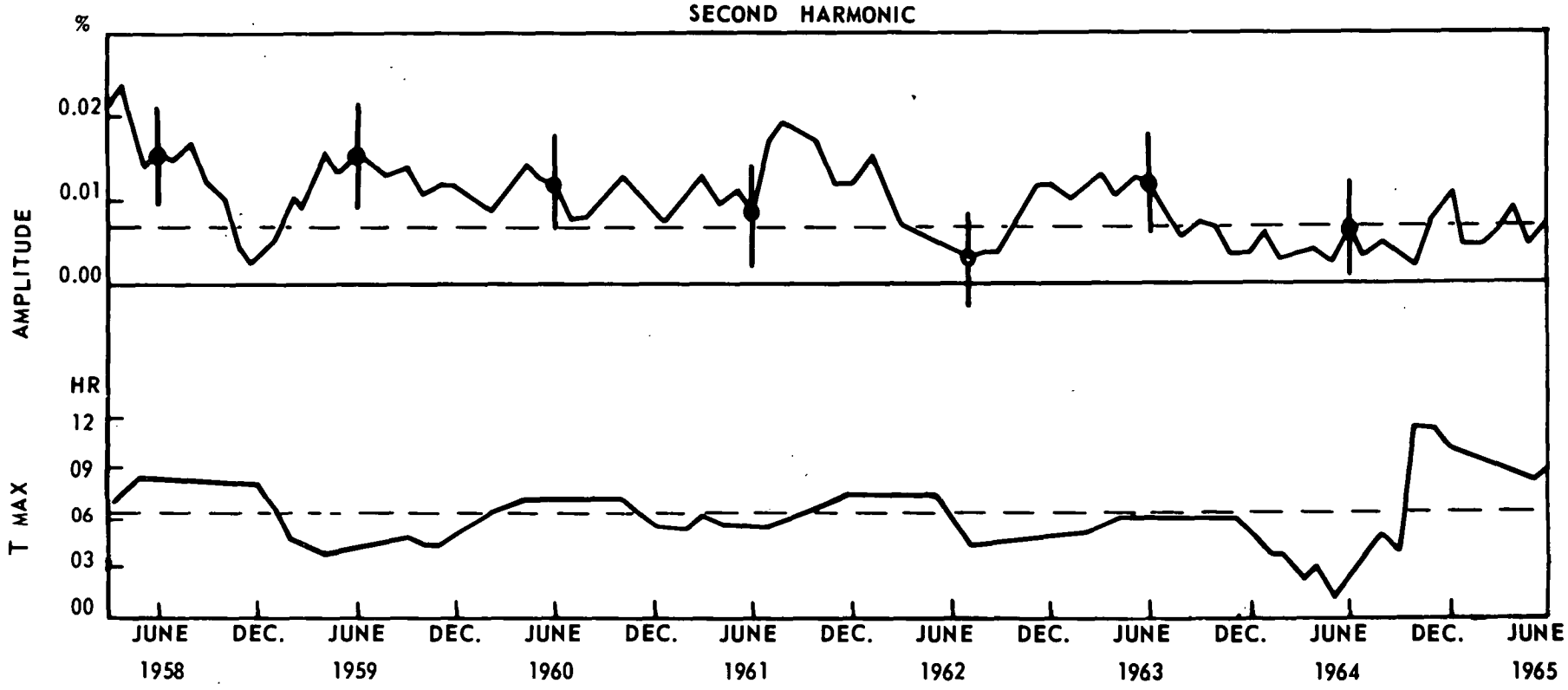


Figure 5.17 The apparent sidereal daily variations of vertical intensity underground. Twelve-month running averages of amplitude and time of maximum of the second harmonic, derived from hourly deviates. The dashed lines represent the long-term averages. S E tails are attached to the calendar-year values of amplitude.

tendency for large excursions in the time of maximum to occur when the amplitude was persistently less than about 0.01%. This is about what would be expected for an annual S.E. of amplitude of ± 0.006%.

(b) Observations at other latitudes of viewing

Under this heading the most useful information to date has come from the vertical semi-cubical telescope at Budapest (asymptotic latitude ∿ 43°N) and from the cubical telescope inclined 30°N of zenith (asymptotic latitude ∿ 17°S) at Hobart. In Figure 5.18 the harmonics relating to the two complete years of operation of the north-pointing cubical telescope are compared with simultaneous observations in the vertical direction at Hobart and with the available observations from Budapest. Not only are the observed harmonics in phase at each of the three different latitudes of viewing but, as will be shown, the corresponding free-space harmonics exhibit a latitude dependence of amplitude that conforms completely with the predictions of the model for the anisotropy. This set of observations is the one that will be employed in the estimation of the direction of the anisotropy.

Thambyahpillai et alia (1965) have found that at the depth of 60 m.w.e. at London there was no significant second harmonic in the observed sidereal daily variation averaged over the years 1961-1964. Thus if it was present it appears that the amplitude of the second harmonic must have been less than about 0.008%.

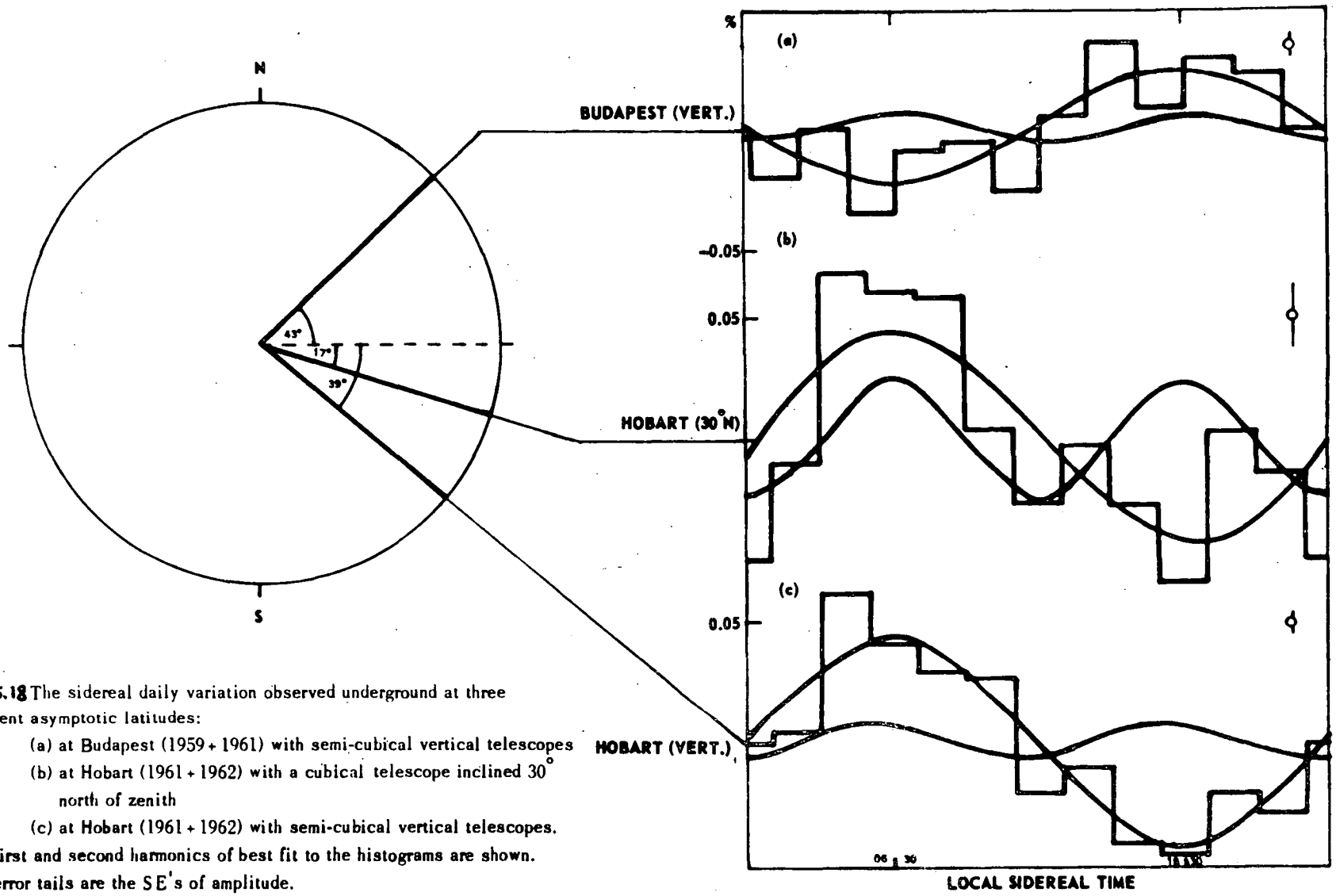


FIG. 5.18 The sidereal daily variation observed underground at three different asymptotic latitudes:

(a) at Budapest (1959 + 1961) with semi-cubical vertical telescopes

(b) at Hobart (1961 + 1962) with a cubical telescope inclined 30° north of zenith

(c) at Hobart (1961 + 1962) with semi-cubical vertical telescopes.

The first and second harmonics of best fit to the histograms are shown. The error tails are the SE's of amplitude.

The other result that should be mentioned was obtained from the cubical telescope inclined in the direction 45°S of zenith (mean asymptotic latitude ∿ 60°S) at Hobart. As shown in Table 5.1, the counting rate was less than that in the direction 30°N of zenith by a factor of about two, because of the higher zenith angle and the greater amount of material absorber. The counting rate was insufficient, in fact, for the effective observation, on an annual basis, of the small daily variations (solar and sidereal) that would be expected at the high latitude of viewing. Over the two years of operation of the experiment in the north-pointing direction the harmonics of the sidereal effect observed with the south-pointing telescope were not statistically significant. The south-pointing run was continued for a further two years in the hope of obtaining a more definite result. The final values of the harmonics and of their sum, averaged in sidereal time over the four complete years 1961 - 1964 are as follows:

	Amplitude (%)	Tmax (Hr)
First Harmonic	0.025 ± .008	0610 ± 0110
Second Harmonic	0.020 ± .008	0550 ± 0050
Sum of Harmonics	0.045	0600

While the observations from the south-pointing telescope give useful supporting evidence for the 0600 maximum of the sidereal daily variation in the southern hemisphere (albeit an earlier value would be expected - see below) and for harmonics that are in phase, the amplitude of the observed second harmonic is clearly anomalous. At the high asymptotic latitude of viewing the amplitude of any second harmonic of extra-terrestrial origin should be very small indeed,

independently of the nature of the anisotropy. With improved counting statistics a more realistic value should be observed.

In principle, a south-pointing experiment at Hobart is an important one. High energy primaries which arrive from the south approach the earth transverse to the terrestrial magnetic field and the longitude coordinates of the velocity vectors are appreciably displaced eastwards. Accordingly there should be a phase-shift in the time of maximum of the sidereal daily variation observed in the south-pointing direction relative to observations in the vertical direction. This would constitute valuable evidence that the sidereal anisotropy was due to charged primaries and would give information as to the variation spectrum. From a high counting rate experiment that is being planned with this object in view, it is hoped that relatively accurate estimates of the amplitudes of the diurnal and semi-diurnal sidereal components will be obtained. In the following chapters the asymptotic response in the south-pointing direction will be closely examined.

5.7 (a) Summary of Evidence

Table 5.6 summarises the annual averaged observations from underground that have been discussed in this chapter and that, for one reason or another, are of relevance. For the sake of completness, all known harmonics are shown although not all of the experiments have given, or were intended to give, useful information concerning individual harmonics, particularly the second.

TABLE 5.6

Harmonics of the apparent sidereal daily variation observed underground in the two hemispheres, estimated from bi-hourly deviates of pressure-corrected intensity. S.E. tails are shown.

Location and detector	Geog. latitude of viewing*	Absorber† (m.w.e.)	Period of obs.	First Harmonic Amplitude %	First Harmonic Tmax Hr	Second Harmonic Amplitude %	Second Harmonic Tmax Hr	Sum of Harmonics Amplitude %	Sum of Harmonics Tmax Hr
			NORTHERN ASYMPTOTIC LATITUDES						
Budapest vertical	43°N	50	1959, 1961 (2yrs)	0.023 ±0.004	1830 ±0040	0.006 ±0.004	1900 ±0240	0.028	1830
London vertical	46°N	70	1961-1964 (3yrs)	0.015 ±0.004	1800 ±0055			0.015	1800
Hobart 70°N of zenith	20°N	60	1964-1965 (2yrs)	0.092 ±0.031	1430 ±0120	0.025 ±0.031	1930	0.080	1710
			SOUTHERN ASYMPTOTIC LATITUDES						
Hobart vertical	39°S	46	1958-1965 (8yrs)	0.040 ±0.002	0554 ±0012	0.006 ±0.002	0640 ±0120	0.046	0600
Hobart 30°N of zenith	17°S	42	1961-1962 (2yrs)	0.043 ±0.008	0650 ±0040	0.024 ±0.008	0630 ±0050	0.067	0650
Hobart 45°S of zenith	60°S	56	1961-1964 (4yrs)	0.025 ±0.008	0610 ±0110	0.020 ±0.008	0550 ±0050	0.045	0600

* Estimated mean asymptotic latitude of viewing

† Approximate mass to the top of the atmosphere

From the analyses of the underground data there is evidence in the first place for an annual apparent sidereal effect that is both statistically significant and reproducible, i.e. neither statistical nor quasi-statistical fluctuations obscure the main effect. Thus a phenomenon is being observed which, like the solar daily variation, is continuously present and requires an explanation in terms of some physical process.

The observations indicate that there is in fact a genuine sidereal daily variation of intensity, associated with a two-way anisotropy, and that if there are any spurious contributions to the annual average they are relatively unimportant. In particular, if there is a sideband of atmospheric origin it must be small in comparison with the genuine sidereal component.

Three of the most important lines of evidence - from the annual averages relating to the vertical intensity at Hobart, from the phase anomalies and from the phase difference of 12 hours in the sidereal diurnal variations as viewed in the different hemispheres - were of widely differing character. Each would seem to have required the presence of a sidereal component in the daily variation. Of particular significance, however, were two other features of this evidence:

(i) It was found that the phase anomalies at Hobart were synchronised with the trends of the annual average amplitudes in solar and sidereal time. This was mandatory if the annual sidereal effect was to be essentially genuine. On the other hand, if there had been a substantial spurious sidereal component

in the annual average the degree of coordination of events could scarcely have occurred.

(ii) The evidence from Hobart relating to asymptotic scans in the southern hemisphere (in the axial directions 30°N, 0° and 45°S underground) when compared with the observations from Budapest indicated the existence of a two-way sidereal anisotropy. The vertical - 70°N experiment underground constituted an acid test of this hypothesis. Both from individual years and from the biennial average the result from this experiment was significant and unequivocally in favour of the hypothesis. Moreover, of all the evidence this most directly indicated that the apparent sidereal effect could not have been due to local seasonal modulation of a solar component.

The evidence for a two-way anisotropy led to a consideration of the two harmonics and the latitude dependence of their amplitudes. For a proper study of the relationship between the harmonics a simultaneous latitude survey with much higher counting rates than we have at present is needed. Nevertheless, the present experiments have given significant evidence from the observed first and second harmonics for the existence of the two-way anisotropy. It has been shown in this chapter that there is a presistent tendency for the two harmonics to be in phase. In the final chapters it will be shown how the amplitudes of the free-space harmonics deduced from the observations in the vertical direction at Hobart and at Budapest and in the axial direction 30°N at Hobart conform with the requirements of the anisotropy. The mean asymptotic latitudes of viewing

of the three telescopes are particularly well located in the latitude range from approximately 40°N to approximately 40°S, over which the greatest variation in the ratio of amplitudes of harmonics is to be expected, the amplitudes of one or other of the harmonics should be relatively large and maximum resolution of amplitude is achieved from the response characteristics of the telescopes. A value for the declination will be calculated from the harmonics, leading finally to an estimate of the direction of the anisotropy in free space.

There seems to be no doubt from equivalent observations at the same depths underground that at present the amplitude of the sidereal diurnal variation is much greater in the southern than in the northern hemisphere. In the long term it will be most desirable to monitor the ratio of amplitudes in the two hemispheres to determine the constancy or otherwise of the amplitude asymmetry. In the quantitative treatment it will be assumed that there are in effect two parts to the anisotropy - one part being the two-way anisotropy with equal amplitudes in opposite directions along an axis and the other part being a superimposed intensity maximum (e.g. due to net streaming) in one of these two directions.

There has been evidence that the sidereal daily variation exhibits two important time-variations. In the first place there was an overall tendency for the amplitude in the vertical direction at Hobart to decrease by a factor of perhaps two between 1958 and 1965. This must be of great significance for the interpretation of the anisotropy, but for a true appreciation of this effect it

would be essential to have simultaneous observations of the long-term trends of amplitude from both hemispheres. Atbrief discussion of the phenomenon will be given later.

Again, it has been necessary to rely on the observations from Hobart for the evidence of seasonal modulation of the sidereal component. Although the explanation must be regarded as tentative, it is suggested that pronounced semi-annual modulation of amplitude and, to a lesser extent, of phase, occurred at sunspot maximum, that there was a rather rapid attenuation of modulation thereafter and that the phenomenon possibly survived in a greatly weakened form over the years of minimum activity. This hyopthesis appears to be the only one which is compatible with all of the changes that have occurred in the components of the daily variation of vertical intensity underground and is not compromised by the evidence for a genuine sidereal effect from other measurements (e.g. the narrow angle north-pointing experiment) that the sidereal effect at Hobart is essentially genuine.

The interplanetary field at the sun as a possible source of semi-annual variation of the sidereal effect will be discussed below, in an attempt to visualise what the influence of the field might be. However, it seems that a correct assessment will have to wait until substantial evidence for the type of seasonal modulation observed at Hobart is to be had from both hemispheres simultaneously, perhaps at the next sunspot maximum, and until we have a more complete description of the interplanetary field.

(b) Semi-Annual Modulation of the Sidereal Effect in Relation to the Interplanetary Magnetic Field

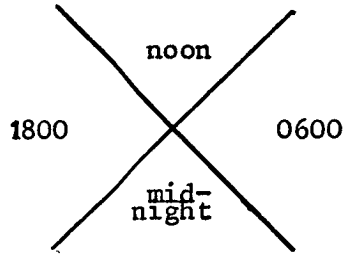
It will be recalled that the large anti-sidereal effect observed in 1958 was compatible with semi-annual modulation of the sidereal component (hypothesis IIIa). On this interpretation the amplitude of the sidereal component decreased almost to zero in June and December, as the difference (D_r) variations show (Figure 5.6). The sidereal maximum at these times would have occurred at approximately 0600 local sidereal time (Figure 5.5). It is noted that 0600 is also approximately the sidereal time of midday in June and of midnight in December. In other words, when the direction of maximum intensity approximately coincided with the direction of the earth-sun line, towards the sun in June and away from the sun in December, the intensity maximum was not observable. In this connection mirroring in the interplanetary field may be important and some of the factors involved should be mentioned.

If a sidereal anisotropy is to be observed at the earth's orbit, it is to be inferred that the charged primaries responsible for it could not have approached with guiding-centre motions along lines of the interplanetary magnetic field. Moreover, the threshold rigidity for observation should be not less than the upper limiting rigidity for observation of the solar anisotropy. A method of estimating annual values, to be described in Chapter 6, gives approximately 100 GV as the upper limiting rigidity in 1958. For the present, we assume that this also approximated the threshold rigidity, R_c, for observation of sidereal anisotropy during that year. It will be shown later that approximately 75% of the underground response relates to primaries in the rigidity range 100 - 500

GV, for $R_c \simeq 100$ GV and a flat spectrum of variation.

It appears, then, that at the earth's orbit where the interplanetary field strength is $\sim 5 \times 10^{-5}$ gauss the gyroradii of the majority of primaries of interest would be between 0.4 A.V. and 2.0 A.U. provided that there was no substantial spatial variation of the field, normal to its direction in the plane of the ecliptic, over these distances. Although it seems that great variation would be likely, the nature of the field over distances of astronomical units above and below the plane of the ecliptic is not known. On the other hand, the Imp-I data gives evidence that in the ecliptic plane there are four sectors in heliographic longitude, the field in neighbouring sectors being oppositely directed (Ness and Wilcox, 1965). At the earth's orbit the width of a sector must on the average be approximately 1.5 AU, although one sector was observed to be much narrower than this and the others somewhat wider. Therefore the high energy primaries with relatively large pitch angles would usually not complete one tuen of a helix near the earth's orbit without crossing from one field sector to another, and this would prevent mirroring along the direction of the spiral field from the earth ($\sim 45^{o}$W of the earth-sun line). However there would be a greater tendency for trapping and mirroring in a field sector as the particles approached the sun. That is to say, for a field strength B_r proportional to r^{-2}, where r is distance from the sun, the gyroradius ρ is proportional to r^2 and thus $\rho/r \propto r$. This means that as r decreases the gyroradius at mirroring becomes progressively smaller in relation to the width of a field sector, thus materially increasing the probability of reflection in addition to scattering. It appears,

then, that high energy primaries which approach the earth from the anti-sun direction in the plane of the ecliptic will have a tendency to be subsequently reflected as well as scattered from the general direction of the sun. It follows that when the asymptotic cone of the detector scans in the direction of the sun a proportion of the flux to which it responds will have originated from the anti-sun direction. We now divide the solar day into four zones of time, arbitrarily shown to be equal in the sketch. When the asymptotic cone of the detector is in the noon zone and in the midnight zone the primary response tends to be the same, there being a tendency to view in the anti-sun direction in the noon zone.



Let us now suppose that there is a sidereal anisotropy at the earth's orbit and that the detector scanning asymptotically in the plane of the ecliptic records a sidereal diurnal variation $V_1 \simeq A \cos \phi$ at the times of the year when there should be least interference due to mirroring, i.e. when the sidereal maximum occurs at 0600 and at 1800 solar time respectively. Now, when the sidereal maximum occurs in the midnight zone it can easily be seen that mirroring tends to cause rectification of the diurnal wave, giving rise to a semi-diurnal component $V_2 \simeq \frac{A}{2} \cos 2\phi$ instead. Six months later the intensity minimum occurs in the midnight zone and, because of rectification of the negative half of the diurnal wave, it tends to be replaced by a semi-diurnal component, $V_2 \simeq \frac{A}{2} \cos 2(\phi - \frac{\pi}{2})$. Over these two particular periods of the year six months apart, then, the sidereal diurnal variation would tend to vanish and be replaced

by a smaller semi-diurnal component, exhibiting a phase displacement of six hours as between the two periods. It will be noted that in the difference (D_r) type of observation, which gives the sidereal effect averaged over pairs of months six months apart, the semi-diurnal sidereal components produced by mirroring must vanish. Therefore, depending on the efficiency of the process, the sidereal daily variation observed in this form should tend to vanish twice a year, when the sidereal maximum occurs at about noon and at about midnight respectively.

The times of occurrence of minimum amplitude predicted above agree very well with the underground observations in 1958 if, as it seems, the difference (D_r) variations were essentially due to a genuine sidereal component. As described, however, the reflection process relates to near-equatorial observations of a uni-directional anisotropy. In reality we are concerned with mid-latitude observations of an apparent two-way anisotropy whose axis is steeply inclined to the plane of the ecliptic, as will be shown later. The main difference here is that if the process of reflection and scattering near the sun is to be of relevance in this situation, it seems that the smoothing out of intensity differences between the noon and midnight zones must be effective for directions of arrival inclined as much as approximately 60^{o} to the plane of the ecliptic.

Although the sidereal diurnal maximum occurs approximately 12 hours later in the northern hemisphere, on the above hypothesis June and December should have been the months of minimum amplitude in that hemisphere also.

Because of the small annual amplitudes in the northern hemisphere and the lack of a complete year's data from Budapest in 1958, the use of difference daily variations to obtain evidence of a semi-annual modulation of amplitude was not practicable. However, another indication is given by the anti-sidereal effect, which should exhibit 12 hours difference in time of maximum between the two hemispheres, although it should be remembered that there could be other reasons for this. We can simplify the argument, without serious error, by neglecting the relatively minor semi-annual variations of phase that appears to have been associated with amplitude modulation in 1958. The time of maximum τ_m'' of the anti-sidereal sideband due to semi-annual modulation of amplitude of a sidereal diurnal variation (time of maximum τ_m') is given by

$$\tau_m'' = \tau_m' - 2T_o$$

where T_o is the time of the year when maximum amplitude occurs. The derivation is given in Paper 2, section 10. Clearly, if T_o is the same in both hemispheres of viewing, but τ_m' is 12 hours different, the phase of the anti-sidereal effect must also be 12 hours different. For what it is worth, it is noted that during years of pronounced seasonal modulation the maximum of the anti-sidereal effect observed underground at Hobart occurred at approximately 1000, while (a) at London, averaged over the three years 1961-1964, the time of maximum was approximately 2200 and (b) at Budapest, averaged over the two available years 1959 and 1961, the maximum occurred at approximately 2100. While the result is consistent with the type of semi-annual modulation we are considering, it is recognised that there may have been other contributions to the anti-sidereal effects at London and Budapest. To assess the true nature of these

effects it would be most desirable to have evidence of significant long-term changes of seasonal modulation at the two places.

On the evidence, the mechanism for seasonal modulation of the sidereal component must have operated with great efficiency in 1958, but with rapidly decreasing effect thereafter (Figure 5.9(c)). From the overall constancy of the solar diurnal variation of neutron intensity over the years 1958-65 it is clear that solar modulation of lower energy primaries in the plane of the ecliptic had maintained its level. Therefore it is suggested that the lessening of seasonal modulation of the sidereal component may have been connected with considerable changes in the field in regions away from the plane of the ecliptic, perhaps a lateral contraction of the volume of the field. There is an indication of this, independently, from the great decrease in amplitude of the solar daily variation observed underground, as will be discussed later.

CHAPTER 6

TRANSFORMATION OF COORDINATES AND THE ASYMPTOTIC CONE OF ACCEPTANCE

It is the purpose of this chapter to describe the daily variation that is expected to be observed underground, in response to an intensity anisotropy of the primaries as specified in the frame of reference in which it is fixed. There are three parts to the treatment :

(1) An empirical description of the intensity anisotropy outside the earth's field in the fixed frame of reference.

(2) Transformation to the rotating celestial coordinate system, to give the free-space harmonics and an average intensity term.

(3) Estimation of the harmonics that would be observed underground in response to the free-space harmonics.

We consider first a model for the sidereal anisotropy (subdivision A of the chapter) and then go on to consider the relevant features of the solar anisotropy (subdivision B).

A. The Sidereal Anisotropy

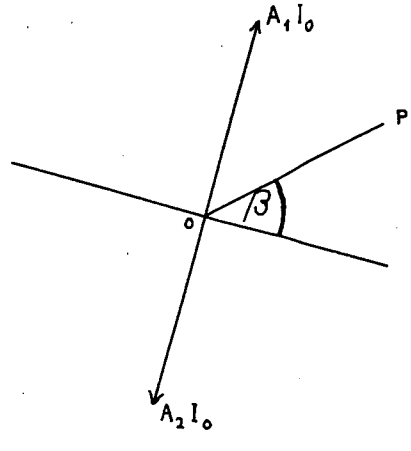
6A.1 Description of the Anisotropy

It will be assumed from the underground evidence that there is (a) a

two-way sidereal anisotropy such as might be associated with a galactic magnetic field when there are equal excess fluxes of particles with steep helices in both directions along the field; and (b) an additional intensity maximum in one of the two directions, such as might be associated with net streaming of the cosmic ray gas.

For simplicity of the present treatment it will be assumed that both contributions to the total anisotropy exhibit the same rigidity dependence of amplitude. It is also assumed for the present that the anisotropy results from modulation of the isotropic flux and that the omnidirectional intensity is unaffected.

Suppose that observations of the directional intensity of high energy primaries is made from some position O_1 on the earth's orbit, remote from the earth and its magnetic field. The point O is the origin of a frame of reference determined by the axis of the anisotropy and the plane normal to it. The frame of reference moves like the earth on an orbital path round the sun, but does not rotate. We proceed to an empirical model which describes the intensity anisotropy that is expected to be observed on the annual average at O. In this model the differential intensity for primaries of rigidity R in the direction of viewing OP, which makes an angle β with the plane normal to the axis of the anisotropy (see sketch), is given by



$$I_{R,\beta} = I_{oR}[1 + R^{\gamma}\{m \sin \beta + n(\sin^2\beta - \tfrac{1}{3})\}] \qquad 6.1$$

where I_{oR} is the isotropic flux, γ is the index of the variation spectrum of the anisotropy, and m and n are amplitude constants. The term $-\frac{1}{3} I_{oR}R^{\gamma}n$ allows for the fact that I_{oR} is also the omnidirectional flux, obtained by averaging $I_{R,\beta}$ over all directions of viewing, OP, within a sphere centred on O.

The unequal and oppositely directed differential intensity maxima will therefore be

$$A_{1R}\, I_{oR} = \{1 + \frac{R^{\gamma}}{3}(2n + 3m)\}\, I_{oR} \qquad \text{when } \beta = +\frac{\pi}{2}$$

$$\text{and} \quad A_{2R}\, I_{oR} = \{1 + \frac{R^{\gamma}}{3}(2n - 3m)\}\, I_{oR} \qquad \text{when } \beta = -\frac{\pi}{2}$$

6A.2 Transformation to the Rotating Celestial Frame of Reference

The well-known celestial system of spherical coordinates is coincident with the rotating terrestrial system, but is fixed with respect to neighbouring stars. A point P on the celestial sphere is specified by declination (dec.), synonymous with geographic latitude, and Right Ascension (RA), measured eastward of the First Point of Aries, γ.

We now wish to describe the anisotropic intensity in terms of celestial coordinates, in the direction OP when it partakes of terrestrial rotation. We then have a description of the anisotropy as observed at any time in the

vertical direction at some point P on the earth, when all environmental factors, such as the terrestrial magnetic field, the atmosphere and the material absorber underground, are absent. That is to say, we have a free-space description in celestial coordinates, as a function of local sidereal time.

In Figure 6.1 the frame of reference of the anisotropy and the celestial frame of reference are referred to the common origin O. In the celestial system the coordinates of the direction of the intensity maximum $A_1 I_0$ at $\beta = \pi/2$ are specified by Right Ascension α_s and declination d. It can be seen that the equatorial plane of the anisotropy intersects the celestial equator along the line whose RA is $\alpha_s + 90$ in the direction coming out of the page from O, and that the transformation from one frame of reference to the other is effected by simple rotation of axes about the $\alpha_s = 90$ direction through the angle $\varepsilon = 90 - d$. Using the appropriate transformation formula we now relate the coordinate β of the direction OP, in the anisotropic frame of reference, to the coordinates of OP in the celestial system. Since the anisotropy is described entirely in terms of β we then immediately have the means of describing it in terms of celestial coordinates.

In the celestial system the direction OP is specified by declination δ and Right Ascension ϕ', ϕ' being the local sidereal time at P. However, to transform from the one coordinate system to the other, it is necessary to refer bearings in RA to the common axis at $\alpha_s + 90$. Referred to this direction the RA of the direction OP is $(\phi' - \alpha_s - 90)$.

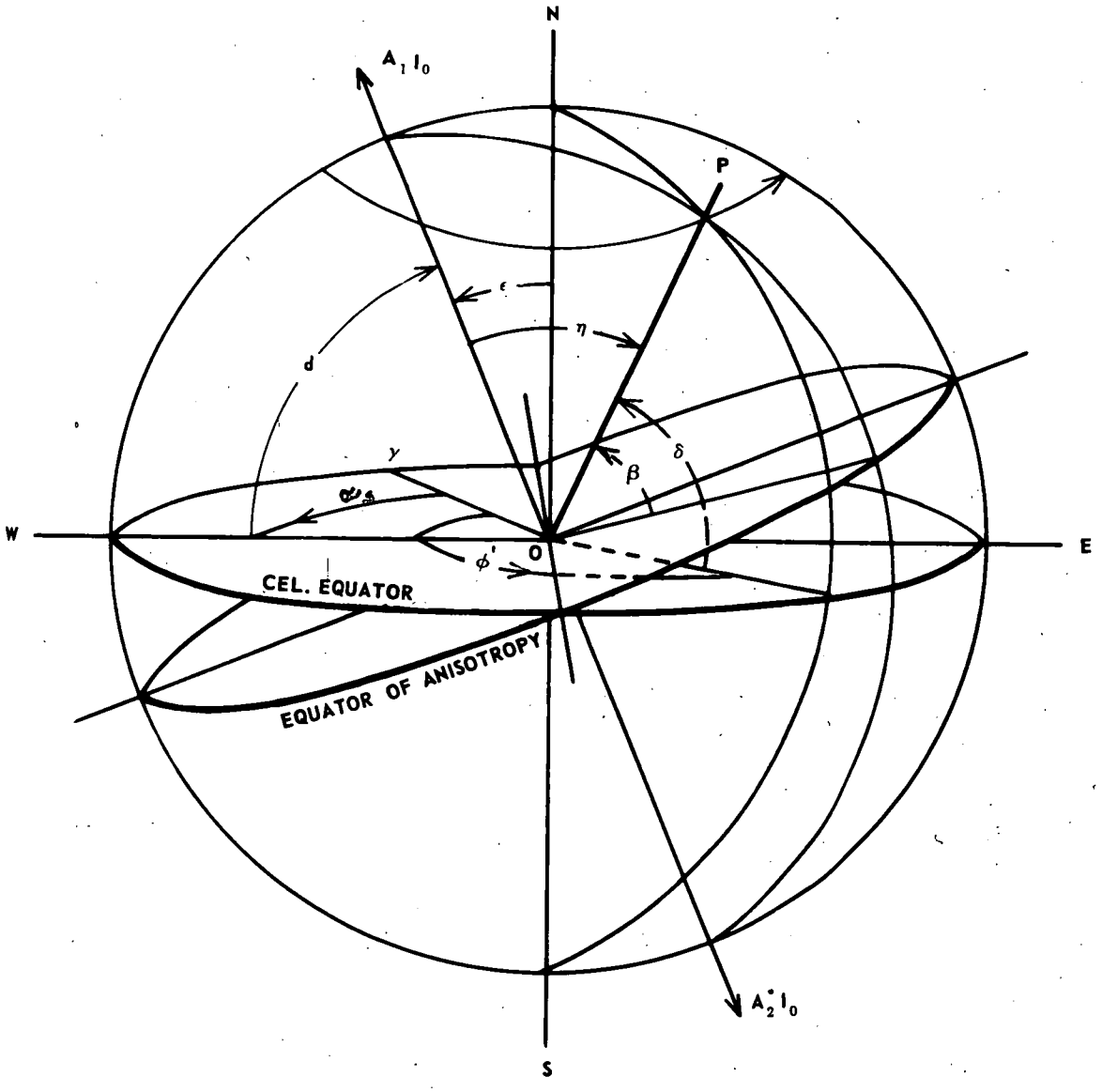


Figure 6.1 The frame of reference of the sidereal intensity anisotropy, in relation to the frame of reference of the rotating direction of observation O P.

The appropriate coordinate conversion formula for expressing β in terms of celestial coordinates (e.g. see Explanatory Supplement to the Ephemeris, 1961) is then

$$\sin \beta = \cos d \cos \delta \cos(\phi'-\alpha_s) + \cos^2 d \cos^2\delta \cos 2(\phi'-\alpha_s) \quad 6.2$$

It will be noted that once we have the corresponding expression for $\sin^2\beta$ the anisotropic intensity $I_{R,\beta}$ as observed in the rotating terrestrial system can be expressed in terms of celestial coordinates and local sidereal time, using equation 6.1.

Squaring 6.2:

$$\sin^2\beta = \frac{\sin 2d \sin 2\delta}{2} \cos(\phi'-\alpha_s) + \frac{\cos^2 d \cos^2\delta}{2} \cos 2(\phi'-\alpha_s)$$
$$+\{ \frac{\cos^2\delta}{2} (3 \cos^2 d-2) - \cos^2 d + 1\} \quad 6.3$$

Therefore the intensity anisotropy specified in the terrestrial framework is, from 6.1, 6.2 and 6.3:

$$I_R = I_{oR}\{1 + R^\gamma(m+2n \sin d \sin\delta)\cos d \cos\delta \cos(\phi'-\alpha_s)$$
$$+R^\gamma \frac{n}{2} \cos^2 d \cos^2\delta \cos 2(\phi'-\alpha_s) + m \sin d \sin\delta$$
$$+ n (\frac{\cos^2\delta}{2} (3 \cos^2 d-2) - \cos^2 d + 1 - \frac{R^\gamma}{3})\} \quad 6.4$$

$$= I_{oR}(1 + T(\phi') + T(\delta))$$

where $T(\phi')$ is the daily variation term and $T(\delta)$ is the latitude-dependent daily

average term. In the present context $T(\delta)$ must be small compared with I_{oR} and may be disregarded in the expression for

$$T(\phi') = \frac{\Delta I_R(\phi')}{I_{oR}}$$

where $\Delta I_R(\phi') = I_R - I_{oR}(1 + T(\delta))$.

The daily variation term consists of first and second harmonic components V_{1R} and V_{2R}:

$$V_{1R} = \frac{\Delta I_{1R}}{I_{oR}} = R^{\gamma}(m + 2n \sin d \sin \delta) \cos d \cos \delta \cos(\phi'-\alpha_s) \qquad 6.5$$

$$V_{2R} = \frac{\Delta I_{2R}}{I_{oR}} = R^{\gamma} \frac{n}{2} \cos^2 d \cos^2 \delta \cos 2(\phi'-\alpha_s) \qquad 6.6$$

These are the *differential free-space harmonics* that we will be concerned with. They are identical with the composite of terms due to diffusion and acceleration derived by L.Davis (1954) in his generalised treatment of high energy anisotropies associated with the galactic magnetic field,

We may represent V_{1R} by

$$V_{1R} = R^{\gamma} M \cos(\phi' - \alpha_s)$$

where M is the rigidity-independent constant $(m + 2n \sin d \sin \delta) \cos d \cos \delta$. Then the *total free-space first harmonic* will be

$$V_1 = \frac{\int_{R_c}^{R_m} \Delta I_{1R} \, d R}{\int_{R_c}^{R_m} I_{oR} \, d R} = \frac{\int_{R_c}^{R_m} I_{oR} R^{\gamma} dR}{\int_{R_c}^{R_m} I_{oR} \, d R} \cdot M \cos(\phi' - \alpha_s)$$

$$= \frac{\int_{R_c}^{R_m} R^{\varepsilon} R^{\gamma} \, d R}{\int_{R_c}^{R_m} R^{\varepsilon} \, d R} M \cos(\phi' - \alpha_s)$$

$$= k_c \, M \cos(\phi' - \alpha_s)$$

where k_c is an amplitude constant depending on R_c and R_m, these being the upper and lower cut-off rigidities for observation of the anisotropy at the earth's orbit, and ε is the index of the average primary differential spectrum. Since only negative values of γ will be considered here, it suffices to put $R_m = \infty$, and integration gives

$$k_c = \frac{\varepsilon + 1}{\varepsilon + \gamma + 1} \quad R_c^{\gamma}$$

Over the primary rigidity range of significance (approximately 10^{11} to 10^{13} ev) ε is thought to be approximately -2.5 (e.g. see Parker, 1965, Figure 1) and thus

$$k_c \simeq \frac{1.5}{1.5 - \gamma} \quad R_c^{\gamma} \qquad 6.6a$$

For the case $\gamma = 0$ it is noted that $k_c = 1$ and therefore

$$V_1 = V_{1R} = M \cos(\phi' - \alpha_s)$$

The *total free-space second harmonic* is similarly

$$V_2 = k_c \, N \cos 2(\phi' - \alpha_s)$$

where N is the amplitude constant $\frac{n}{2} \cos^2 d \cos^2 \delta$.

We now have to calculate the corresponding daily variation to be expected at the telescope. The following factors, some of which depend considerably on primary rigidity, must be taken into account:- the deflections of the primaries in the terrestrial magnetic field, the coupling between the primaries and the observed meson component, the influence of the atmosphere and the underground absorber on the counting rate as a function of direction of arrival of the mesons, and the physical geometry of the telescope. These are all taken care of, to the best of our knowledge, by the asymptotic constants of response.

The first and second harmonics of the sidereal daily variation expected to be observed will be

$$v_1 = K_1 M \cos(\phi' - \alpha_s + \phi_1)$$

and

$$v_2 = K_2 N \cos 2(\phi' - \alpha_s + \phi_2)$$

It can be seen that the asymptotic constants K_1, K_2, ϕ_1 and ϕ_2 and the free-space amplitude constant k_c connect the observed daily variation with the daily variation of primaries in free space. Thus, denoting the observed diurnal amplitude by v_{1max}, the corresponding free-space amplitude is $V_{1max} = \frac{k_c}{K_1} v_{1max}$ and the phase difference is ϕ_1. Similarly, for the second harmonic, $V_{2max} = \frac{k_c}{K_2} v_{2max}$ and the phase difference is ϕ_2.

It may be of interest to note that when $\gamma = 0$ the constants K_1 and ϕ_1 are analogous to B_1 and ρ_1 that have been calculated for the first harmonic of the solar daily variation as functions of the upper limiting rigidity R_u, for $\beta = 0$ (where β is the index of the variation spectrum of the solar anisotropy). Values of B_1 and ρ_1 versus R_u are shown in Figures 2 and 3 of paper 7.

6A.3 The Anisotropic Constants of Response

The given detector responds to a free-space first harmonic

$$V_1 = k_\epsilon (m \cos d \cos \bar{\delta} + \frac{n}{2} \sin 2d \sin 2\bar{\delta}) \cos(\phi' - \alpha_s) \qquad 6.7$$

at the mean asymptotic latitude of response, $\bar{\delta}$. The observed harmonic will be

$$v_1 = K_1 (m \cos d \cos \bar{\delta} + \frac{n}{2} \sin 2d \sin 2\bar{\delta}) \cos(\phi' - \alpha_s + \phi_1) \qquad 6.8$$

where K_1, ϕ_1 and $\bar{\delta}$ are to be determined.

Similarly, the observed second harmonic will be

$$v_2 = K_2 \frac{n}{2} \cos^2 d \cos 2\bar{\delta} \cos 2(\phi' - \alpha_s + \phi_2) \qquad 6.9$$

A simplification of the present treatment is that the mean asymptotic latitude of viewing for the second harmonic is defined to be the same as that for the first harmonic. The constants of response are therefore k_ϵ, K_1, K_2, ϕ_1, ϕ_2 and $\bar{\delta}$.

Alternatively, the observed harmonics of response may be written as

$$v_1 = (C_1 m \cos d + C_2 \frac{n}{2} \sin 2d) \cos(\phi' - \alpha_s + \phi_1) \qquad 6.10$$

and

$$v_2 = C_3 \frac{n}{2} \cos^2 d \cos 2(\phi' - \alpha_s + \phi_2) \qquad 6.11$$

where $C_1 = K_1 \cos \bar{\delta}$ 6.12

$C_2 = K_1 \sin 2\bar{\delta}$ 6.13

$C_3 = K_2 \cos^2 \bar{\delta}$ 6.14

Thus $\sin \bar{\delta} = \frac{C_2}{2C_1}$ 6.15

The method used here is to estimate approximate values of the amplitude constants C_1, C_2 and C_3, from which $\bar{\delta}$, K_1 and K_2 may be determined.

6A.4 Calculation of the Constants of Response. Outline of Method.

The calculation of the underground response to the solar anisotropy has been described in paper 7. Since the technique was designed to have a more general application, the following is essentially a recapitulation of the treatment given in the paper, suitably modified so as to apply to the sidereal anisotropy.

The fraction of the observed first harmonic underground that is due to anisotropic primaries of rigidity R will differ considerably from W_{1R} (equation 6.5), partly because at any instant the primaries involved do not approach the earth's magnetic field from a single direction (δ, $\phi'-\alpha_s$) but from a cone of directions that will allow them to arrive at the meson production level within the solid angle of viewing of the recorder, after penetrating the field. The cone is the asymptotic cone of acceptance of which the concept and general method of application to an anisotropy have been fully outlined by Rao, McCracken and Venkatasan (1963). The treatment given here differs somewhat in detail

from theirs and is perhaps only appropriate when dealing with high energy primaries. The cone of acceptance is divided up into N segments specified by intervals of asymptotic latitude. These intervals can be conveniently defined so that each segment of the cone contributes about the same amount to the fraction of the counting rate that is due to primaries of the given rigidity. The contribution v_{1R} to the first harmonic of the sidereal daily variation from primaries of rigidity R is then approximated by the summation.

$$v_{1R} = \frac{Y_R}{N} R \sum_{n=1}^{N} \{m \cos d\, A_{Rn} \cos \delta_{Rn} + \frac{n}{2} \sin 2d\, A_{Rn} \sin 2\delta_{Rn}\} \cos(\phi' - \alpha_s + \phi_{1Rn}) \qquad 6.16$$

In this expression Y_R is the differential coupling coefficient and A_{Rn} is an amplitude reduction factor determined by the manner in which the contributions to Y_R from the n^{th} segment of the cone are distributed in asymptotic longitude. An individual contribution will be defined more precisely in section 6A.5 as the "differential radiation sensitivity", $I(R,\omega)$. The distribution also determines an effective longitude of viewing ϕ_{1Rn} (east of the observer's meridian) with respect to a first harmonic of the anisotropy. The mean asymptotic latitude δ_{Rn} is obtained from the distribution in asymptotic latitude of the contributions to Y_R from the n^{th} segment of the cone.

The total first harmonic v_1 is estimated as

$$v_1 = \sum_{R=R_c}^{R_m} v_{1R}$$

In the calculations of response the upper cut-off R_m is put equal to infinity, since the telescope response becomes negligible at very high rigidities.

A useful form of v_{1R} is the vector

$$\hat{v}_{1R} = |v_{1R}|\hat{\alpha}_{1R}$$
$$= m \cos d \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} A_{Rn} \cos \delta_{Rn} \hat{\alpha}_{Rn}$$
$$+ \frac{n}{2} \sin 2d \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} A_{Rn} \sin 2\delta_{Rn} \hat{\alpha}_{Rn} \qquad 6.17$$

where $\alpha_{1Rn} = \alpha_s - \phi_{1Rn}$

α_s is the phase angle on the harmonic dial. In turn, the total first harmonic vector is

$$\hat{v}_1 = |v_1|\hat{\alpha}_1 = \sum_{R=R_c}^{\infty} \hat{v}_{1R}$$

$$= C'_1 \, m \cos d \, \hat{\alpha}_{1a} + C'_2 \frac{n}{2} \sin 2d \, \hat{\alpha}_{1b} \qquad 6.18$$

where the constants C'_1 and C'_2 are determined as

$$C'_1 \hat{\alpha}_{1a} = \sum_{R=R_c}^{\infty} \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} A_{Rn} \cos \delta_{Rn} \hat{\alpha}_{1Rn} \qquad 6.19$$

and

$$C'_2 \hat{\alpha}_{1b} = \sum_{R=R_c}^{\infty} \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} A_{Rn} \sin 2\delta_{Rn} \hat{\alpha}_{1Rn} \qquad 6.20$$

It will be noted that $C'_1 m \cos d \, \hat{\alpha}_{1a}$ and $C'_2 \frac{n}{2} \sin 2d \, \hat{\alpha}_{1b}$ form a vector triangle with $|v_1|\hat{\alpha}_1$ and that $C'_1 = C_1$ and $C'_2 = C_2$ only if the unit vectors $\hat{\alpha}_{1a}$ and $\hat{\alpha}_{1b}$ are the same. In the various determinations of the

underground response it has been found that ϕ_{1a} and ϕ_{1b}, the deflection angles corresponding to α_{1a} and α_{1b}, rarely differ by more than two degrees and that therefore it suffices to approximate C_1 by C'_1, C_2 by C'_2 and to put

$$\hat{\alpha}_{1a} = \hat{\alpha}_{1b} \simeq \hat{\alpha}_1$$

The response to the free-space second harmonic is calculated in a like manner, the differential contribution v_{2R} being given by

$$v_{2R} = \frac{Y_R}{N} R^{\gamma} \frac{n}{2} \cos^2 d \sum_{n=1}^{N} B_{Rn} \cos^2 \delta_{Rn} \cos\, 2(\phi' - \alpha_s + \phi_{2Rn}) \qquad 6.21$$

where B_{Rn} is analogous to A_{Rn}, being the amplitude reduction factor for a second harmonic in relation to the n^{th} segment of the asymptotic cone.

The vector form of v_{2R} is

$$\hat{v}_{2R} = |v_{2R}| \hat{\alpha}_{2R}$$

and we have finally the total second harmonic vector

$$\hat{v}_2 = |v_2| \hat{\alpha}_2 = \sum_{R=R_c}^{\infty} \hat{v}_{2R}$$

$$= C_3 \frac{n}{2} \cos^2 d\, \hat{\alpha}_2 \qquad 6.22$$

where $$C_3 \hat{\alpha}_2 = \sum_{R=R_c}^{\infty} \frac{Y_R}{N} R^{\gamma} \sum_{n=1}^{N} B_{Rn} \cos^2 \delta_{Rn} \hat{\alpha}_{2Rn} \qquad 6.23$$

In summary, the integrated asymptotic amplitude and phase constants of the anisotropy are derived in sequence as follows:

$$\left.\begin{aligned} C_1 &= C_1' \text{ (equ.6.19)} \\ C_2 &= C_2' \text{ (equ.6.20)} \end{aligned}\right\} \rightarrow \bar{\delta},\ K_1$$

$$C_3 = C_3' \text{ (equ.6.23)} \rightarrow K_2$$

$$\hat{\alpha}_{1a} = \hat{\alpha}_{1b} = \hat{\alpha}_1 \rightarrow \phi_1$$

$$\hat{\alpha}_2 \rightarrow \phi_2$$

The constants $\bar{\bar{\delta}}$, K_1, K_2, ϕ_1 and ϕ_2 appropriate to the given detector may now be estimated for selected values of γ and R_c. Together with k_c (see equation 6.6a) these constants relate the observed harmonics v_1 and v_2 to the corresponding integrated free-space harmonics V_1 and V_2.

It will be shown in chapter 7 that if sets of estimated free-space harmonics are available from two, or preferably three, well-separated latitudes of viewing, these suffice for the determination of the *declination* $\underline{d}$ of the anisotropy. That is to say, $\underline{d}$ can be estimated without having to estimate $\underline{m}$ and $\underline{n}$, the amplitude constants of the anisotropy.

6A.5 The Calculations in Detail. Evaluation of δ_{Rn}, A_{Rn}, ϕ_{Rn}, B_{Rn}, ϕ_{2Rn}

(a) Method of determination

In calculating the response underground to a first harmonic of the anisotropy we first determine sets of values of A_{Rn}, δ_{Rn}, and ϕ_{Rn} for selected rigidities so that by curve-fitting they may be read off as functions of R. It should be noted that A_{Rn}, δ_{Rn} and ϕ_{Rn} are *constants of the detector* at the given location and may be used to determine the response to any free-space first harmonic, either solar or sidereal. Corresponding quantities A_{2Rn}, δ_{2Rn} and ϕ_{2Rn} relating to a second harmonic may be obtained by the same procedures.

Consider the μ-mesons, due to isotropic primaries of rigidity R, which arrive at the dtector within a small solid angle ω_r specified by zenith angle Z_r and azimuth a_r. They constitute a fraction of the total counting rate that is known as the differential radiation sensitivity $I(R, \omega_r)$ given by

$$I(R,\omega_r) = Y_R F(\omega_r) \cos^n Z_r \qquad 6.24$$

The factor $F(\omega_r)$, the geometric sensitivity (viz. Parsons, 1957), is the fraction of the total counting rate apportioned to ω_r by virtue of the geometry of the telescope, while $\cos^n Z_r$ expresses the well-known zenith angle dependence of intensity. The pprimaries of rigidity R that are responsible for $I(R,\omega_r)$ will have approached the earth's magnetic field from directions within some asymptotic volume element $(\Omega_r)R$ specified by latitude $(\delta_r)R$ and longitude $(\phi_r)R$. Such asymptotic coordinates are usually found either by

computing the trajectories of the primaries outward from the direction (Z_r, a_r) at the geomagnetic location of the recorder (viz. McCracken et alia, 1962) or else from observations of the deflections of charged particles in a physical simulation of the geomagnetic field (viz. the terrella experiments of Brunberg and Dattner, 1953). When $I(R, \omega_r)$ and the associated asymptotic coordinates $(\delta_r)R$ and $(\phi_r)R$ have been calculated for each of the volume elements ω_r which make up the solid angle of the recorder, the distribution of the fractional μ-meson intensity due to primaries of rigidity R can be determined with respect both to asymptotic latitude and asymptotic longitude of the primaries. The distribution with latitude can be simply divided up amongst N latitude intervals giving equal contributions to the differential counting rate, thereby defining the N segments of the total asymptotic cone. The mean latitudes δ_{Rn} are obtained from the latitude distribution within the individual segments. The distribution with respect to longitude within each segment, when applied to a first harmonic $\cos \phi$, provides the amplitude reduction factor A_{Rn} and the longitude displacement angle ϕ_{Rn}. However, it should be noted that the distributions themselves are derived on the assumption of isotropic primaries and an average cosmic ray spectrum.

When the constants A_{Rn}, δ_{Rn} and ϕ_{Rn} are applied to a particular model for the anisotropy, the important simplification in the present treatment is that the mean value δ_{Rn} replaces a latitude distribution. This suggests that the latitude intervals specifying the segments of the asymptotic cones should be small. Several factors are involved in the choice of intervals,

one of them being the manner of variation of the longitude distribution with respect to asymptotic latitude. As far as the underground vertical semi-cube is concerned, the final result does not seem to depend at all markedly on the number of latitude intervals used.

(b) Application to detectors

(i) The vertical semi-cube underground

The solid angle of the semi-cube was divided up into 576 elements (ω_r) of dimensions 5° in azimuth and 7.5° in zenith angles. In calculating the values of $I(R,\omega_r)$, Fenton's coupling coefficients and the geometric sensitivity characteristics worked out by Parsons were used. Both $n = 2.0$ and $n = 2.2$ were tried for the $\cos^n z$ zenith angle dependence of intensity but for all-practical purposes they each led to the same final asymptotic distribution.

Since about 90% of the counting rate underground appears to be due to primaries of energy > 50 GEV, it was considered that a centred dipole was a sufficient approximation to the real geomagnetic field for calculations of the asymptotic coordinates $(\delta_r)_R$ and $(\phi_r)_R$. [Calculations using a more complex representation of the field tended to confirm this view and are described in paper 7, section IV(a)* .] Consequently, the particle deflections were estimated from the diagrams of Brunberg and Dattner for the geomagnetic southern latitude of 50°. Considerable interpolation was necessary and for this reason it was found more convenient to express the asymptotic latitude and longitude data in the form

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* The comparison of different representations of the field was performed by J.E.Humble

of deflections in zenith angle and azimuth. A technique for converting coordinates by means of a terrestrial globe is described in Appendix IX, where a general method of obtaining radiation sensitivity $I(R,\omega_r)$ as a function of asymptotic direction of viewing is described.

At the higher rigidities, the differential cones of acceptance change rather slowly with rigidity and therefore it was decided to work them out only for the three most important rigidities, namely 50 GV (a practical lower limiting rigidity of response), 150 GV(near the mean rigidity of response) and infinite rigidity (providing an asymptote for the response constants). Figures IX.2 to IX.9 (Appendix IX) give the deflection data for particles of rigidity 50 GV and 150 GV at the geomagnetic latitude of observation $\lambda = -50^{o}$.

Each cone of acceptance was initially divided up into three segments defined by intervals of asymptotic geographic latitude. For example, it was found that of the 50 GV primaries contributing to the counting rate, one third came from the latitude range 90°S to 46°S, one third from the range 45°S to 28°S and the rest from the range 27°S to 20°N. These high, middle and low latitude intervals were found to be almost exactly the same at the higher rigidities.

The mean latitude δ_{Rn} was worked out for each segment from the distribution $I(R,\omega_r)$ versus $(\delta_r)_R$, after the data had been grouped at 3° intervals of latitude. The distribution $I(R,\omega_r)$ versus $(\phi_r)_R$ was based on longitude intervals of 20° and provided the amplitude constant A_{Rn} and phase

constant ϕ_{Rn} when a first harmonic cos ϕ was impressed on it. In addition, constants A_R, δ_R and ϕ_R were obtained for the total (undivided) cone. Values of the asymptotic constants for individual segments and for the total cone are given in Table 6.1 for each of the three rigidities. It can be seen that (a) above 150 GV the cone of acceptance does not change with increasing rigidity except for a gradual displacement towards the observer's meridian, and that (b) the distribution $I(R,\omega_r)$ versus $(\phi_r)_R$ changes only slightly with latitude, as would be expected.

TABLE 6.1

Asymptotic constants for the vertical semi-cube underground at 50° geomagnetic latitude. The constants specify the differential response to a free-space first harmonic at the primary rigidities 50 GV, 150 GV and infinity.

	Primary Rigidity (GV)								
	50			150			∞		
	A_{Rn}	δ_{Rn}	ϕ_{Rn}	A_{Rn}	δ_{Rn}	ϕ_{Rn}	A_{Rn}	δ_{Rn}	ϕ_{Rn}
High Latitude Range (H)	0.84	-56°	25°	0.75	-60°	12°	0.74	-62°	0°
Mid Latitude Range (M)	0.86	-36°	35°	0.86	-38°	10°	0.86	-41°	0°
Low Latitude Range (L)	0.91	-13°	33°	0.90	-15°	12°	0.89	-18°	0°
Total Cone	0.87	-34°	30°	0.83	-39°	11°	0.83	-39°	0°

Curves of fit were drawn through values of A_R, δ_R and ϕ_R given at the bottom of the Table and are shown in Figure 6.2. These curves were used in all subsequent calculations requiring values of the asymptotic constants as functions of R. In a similar manner, curves of B_R and ϕ_{2R} may be drawn as functions

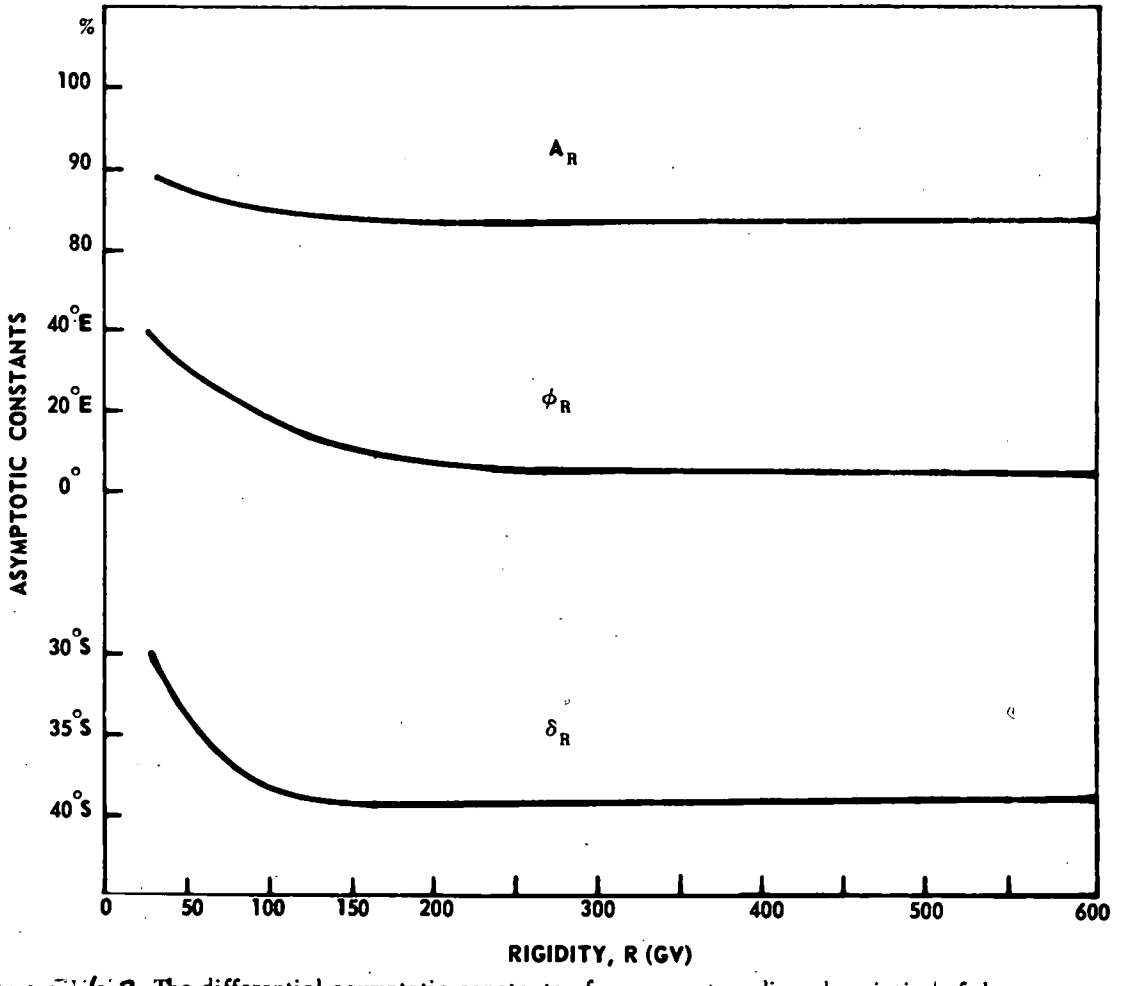

Figure 6.2 The differential asymptotic constants of response to a diurnal variation of the primaries in free space, for a vertical semi-cubical telescope at a depth of 40 m.w.e. at $\lambda = 50°S$.

of R, characterising the differential response to a generalised second harmonic.

(ii) The cube inclined 30°N underground

In the axial direction, the telescope happens to view upwards along the magnetic field at Hobart and at the same time this represents a low latitude of viewing. It is for this reason that the asymptotic acceptance cones are relatively compact, centred only slightly east of the observer's meridian and are not strongly dependent on rigidity, while the distribution of $I(R,\omega_r)$ with asymptotic longitude is virtually independent of asymptotic latitude. Consequently the constants A_R, δ_R and ϕ_R can be computed from the undivided cones of acceptance with rather less loss of accuracy than in the case of the vertical semi-cube.

(iii) The south-pointing cube underground

The axial direction of the telescope is 45° South of the zenith and this corresponds to a geographic latitude of 88°S. The situation is the reverse of that in the N-pointing direction. Directions of arrival tend to be transverse to the magnetic field and at higher zenith angles are accessible from all 360° of asymptotic longitude. The cones of acceptance, divided up into high, middle and low latitude segments (Figure 6.3), are centred far to the east of the observer's meridian at rigidities near the cut-off and change markedly with increasing rigidity. Figure 6.4 shows how the spread in asymptotic latitude varies with rigidity.

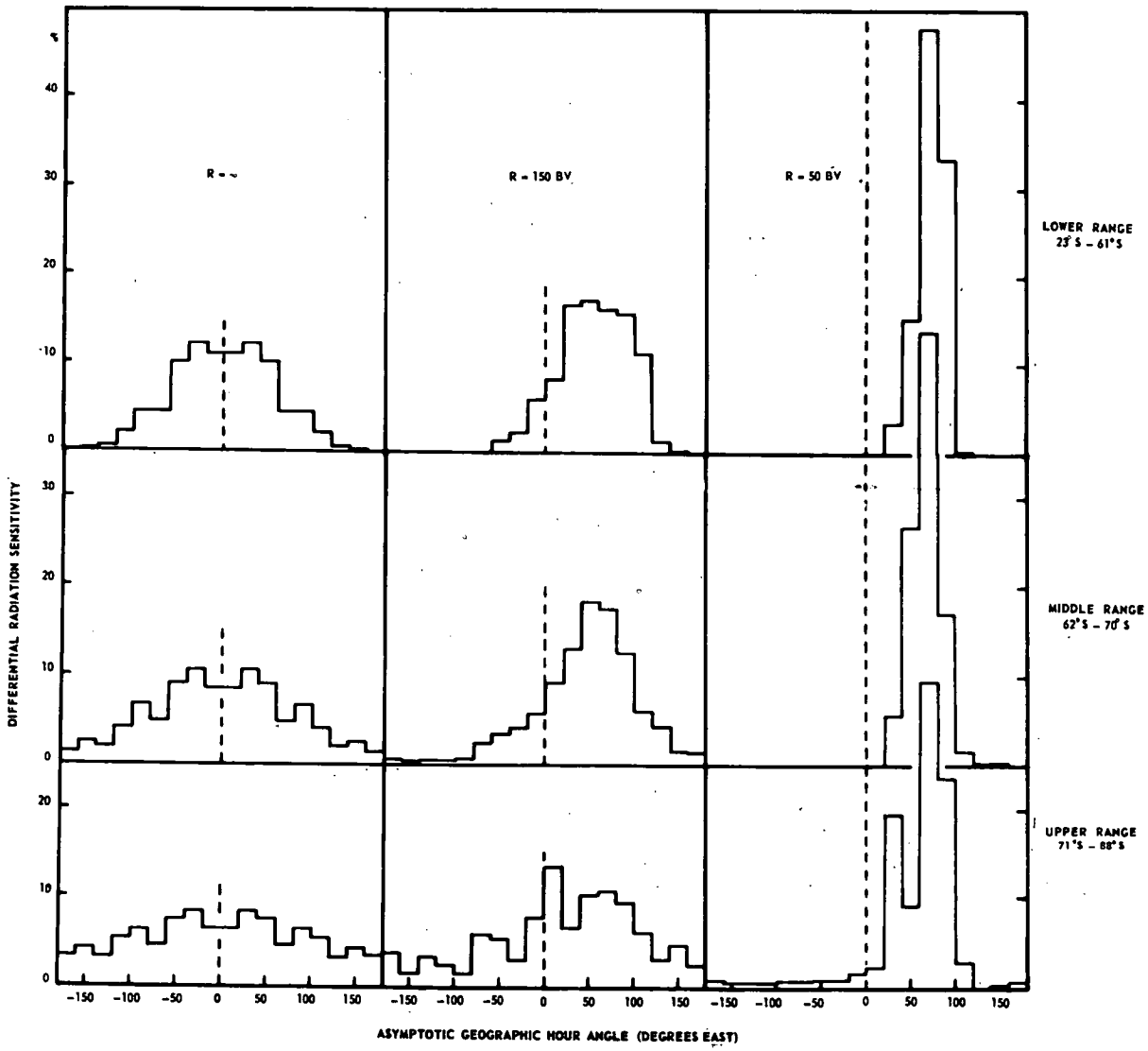

Figure 6.3 Differential asymptotic cones of acceptance for sectional latitude ranges of viewing, relating to a cubical telescope inclined 45° south of zenith at a depth of about 40 m.w.e. at Hobart. The asymptotic latitude ranges have been chosen to give approximately equal contributions to the differential counting rate that derives from primaries of the given rigidity.

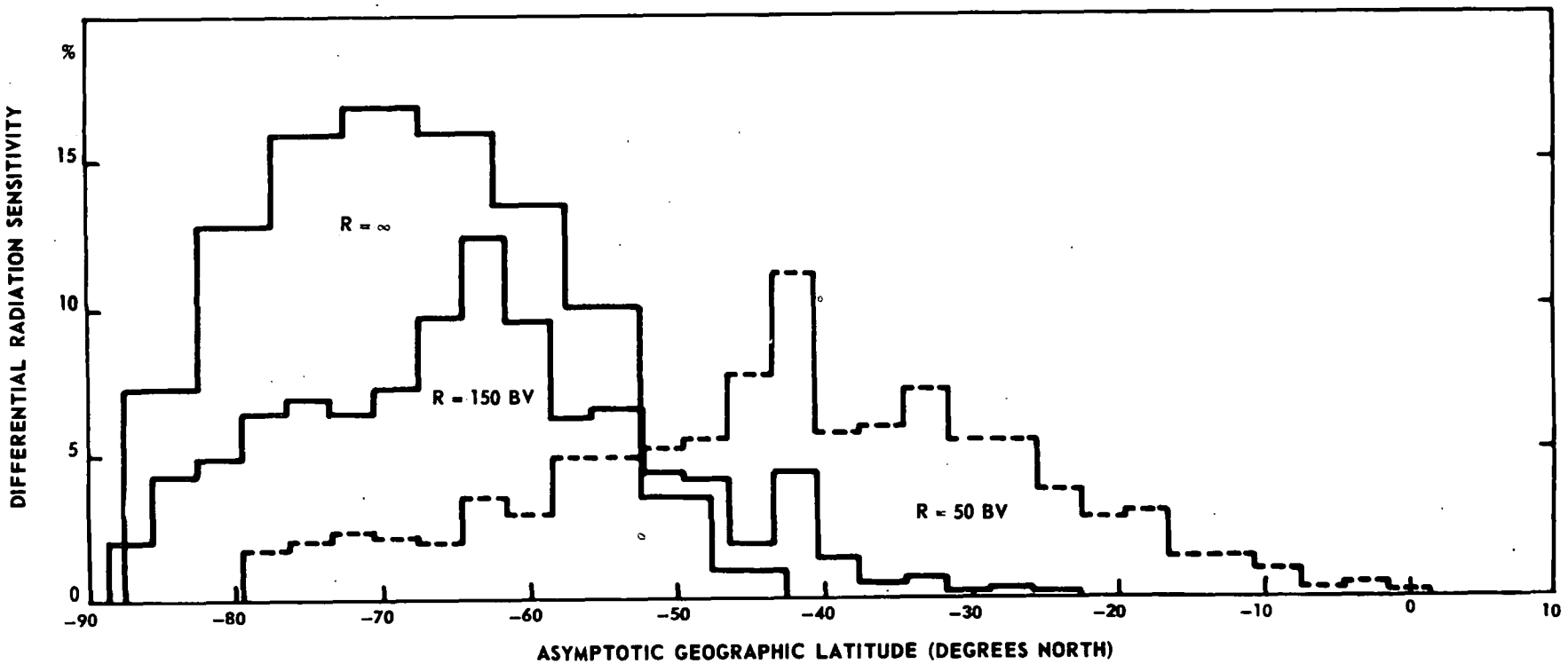

Figure 6.4 Response characteristics of a cubical telescope inclined 45° south of zenith at a depth of about 40 m.w.e. at Hobart. The estimated contributions to the differential counting rate are shown as functions of asymptotic latitude of viewing, for primaries of low, average and infinite rigidity respectively.

It is clearly neeessary to use divided cones of acceptance for the calculation of response in the south-pointing direction. It would also be desirable in an improved treatment to determine the response at other values of rigidity between 50 GV and 150 GV.

In Figure 6.5 are shown curves of the estimated differential response constants as functions of primary rigidity.

6A.6 The Calculations in Detail. Determinations of $\overline{\delta}$, K_1, K_2, ϕ_1 and ϕ_2

In order to proceed from the differential response constants of the detector to the integrated constants that relate to the anisotropy, it is necessary to obtain the differential coupling coefficients, Y_R, and to select values for γ, the index of the variation spectrum, and for R_c, the primary threshold rigidity for observation of the anisotropy.

(a) The coupling coefficients

Fenton's coupling coefficients relate the vertical intensity of μ-mesons at a depth of 40 m.w.e. to the primary proton spectrum. In Figure 6.6 the relationship is presented in the form of an integrated response curve. The corresponding curves for mesons at sea level (Quenby and Webber, 1959) and neutrons at sea level (Dorman, 1957) are shown for comparison. It would appear that only about 8% of the counting rate underground is due to protons of energy < 50 GEV as against 50% in respect of the meson intensity at sea level and 74% in respect of the neutron intensity.

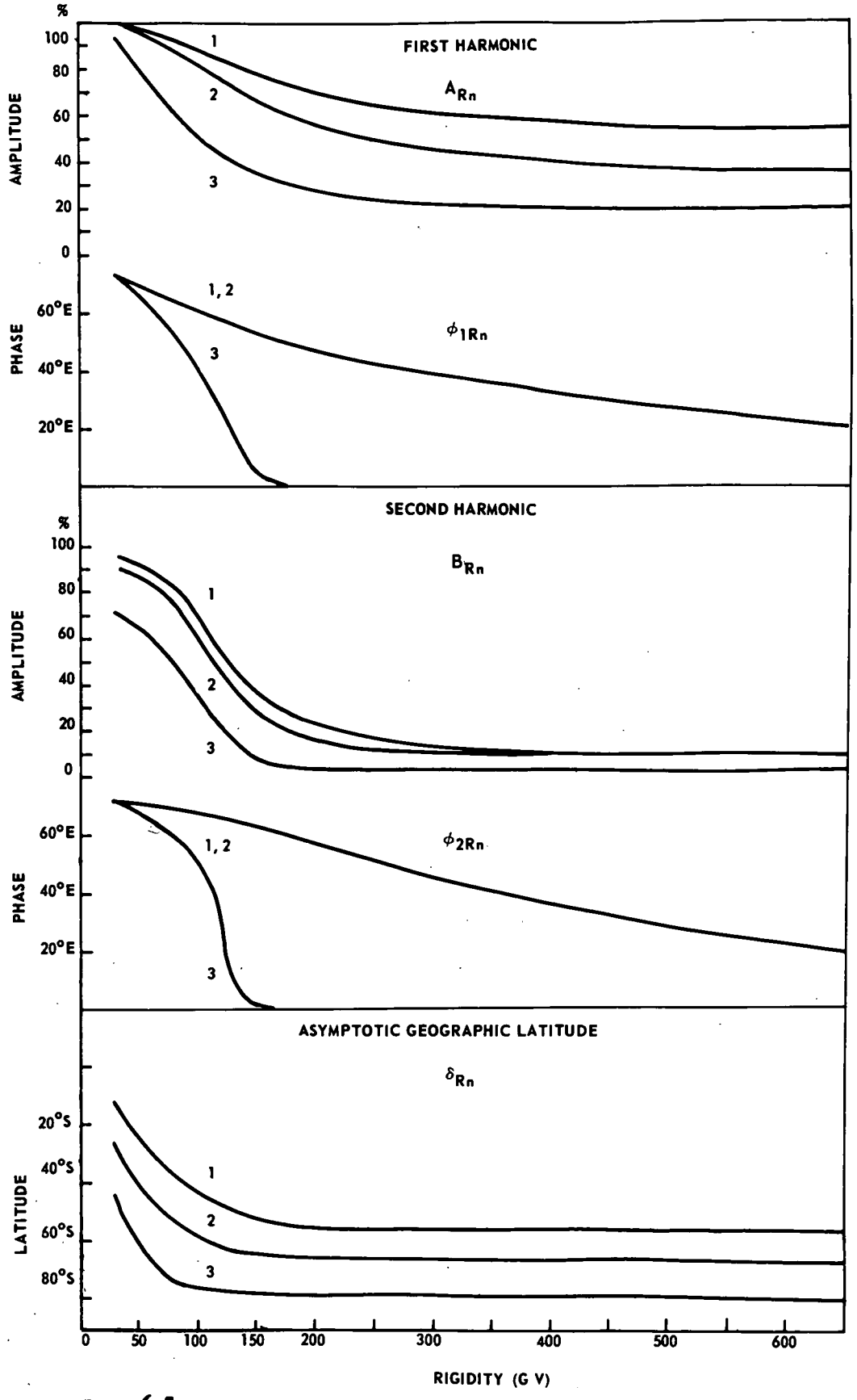

Figure 6.5 The differential response constants of the south-pointing cube underground, relating to cones of acceptance divided up into low, middle and high latitude segments according as n takes the values 1, 2 and 3 respectively.

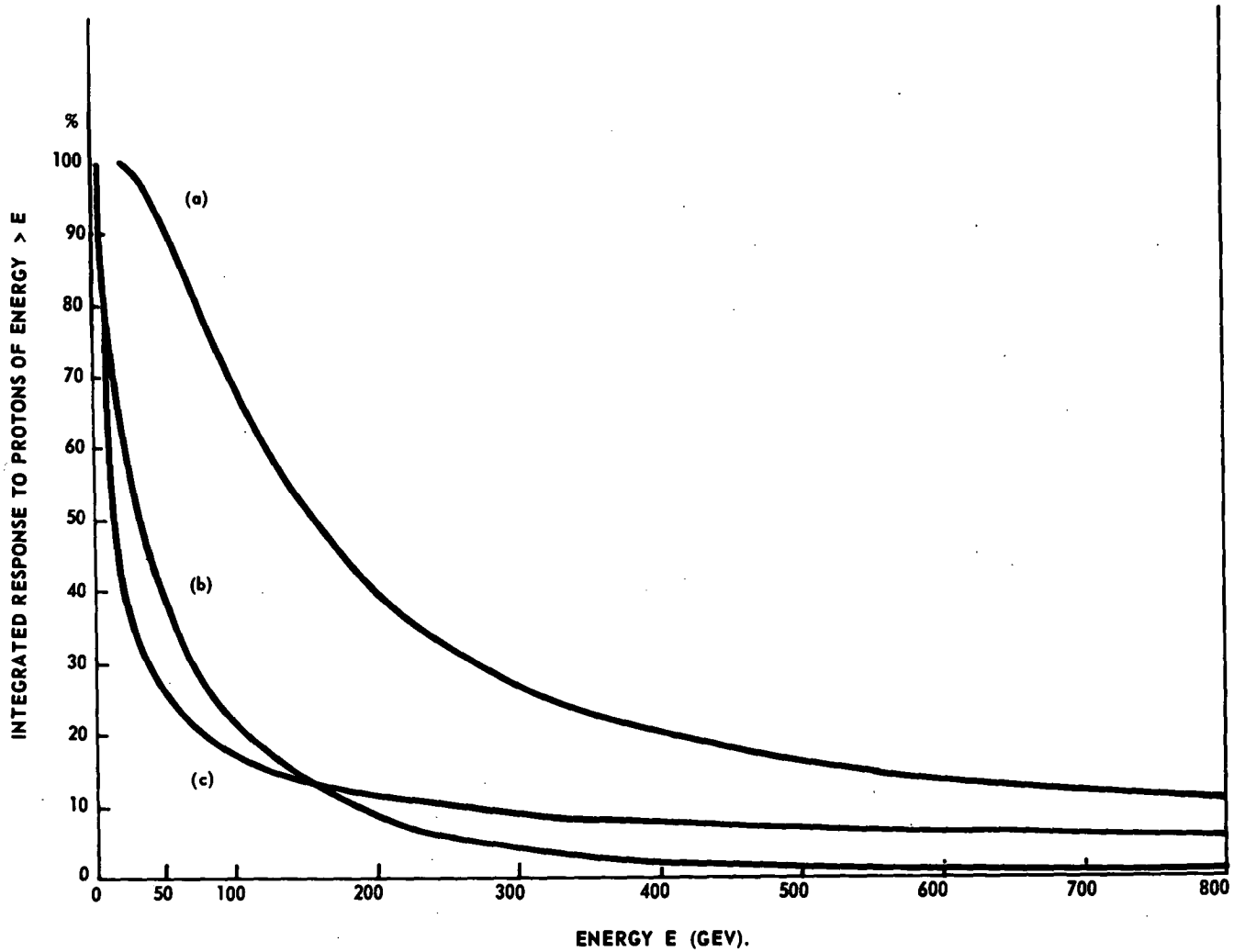

Figure 6.6 Estimated proportion of the counting rate due to primary protons of energy > E, in respect of:

(a) meson intensity at 40 m.w.e. underground (from Fenton, 1963)

(b) meson intensity at sea level (from Webber and Quenby, 1959).

(c) neutron intensity at sea level (from Dorman, 1957).

Fenton's coefficients derive from considerations of the theoretical yield of pions of energy $\geq$ 15 GEV(it being assumed that 15 GEV is the minimum energy required by a μ-meson at production to penetrate the atmosphere plus 40 m.w.e. of material absorber). A similar result was obtained by Mathews (1963) from an empirical response curve extrapolated from latitude-intensity data.

The provisional nature of the coefficients is an important factor inhibiting accurate estimations of response to anisotropies, in particular the solar anisotropy. In respect of the sidereal anisotropy the existing coefficients should suffice for the preliminary calculations of response. However, revised estimates of the underground absorber already suggest that the coupling coefficients for the Hobart detectors should be appropriately modified. In section 4.7 it was shown that the mean cut-off at production for the vertical semi-cube must be approximately 11 GEV. From this point of view the existing coefficients tend to underestimate the contributions to the counting rate from the lower energy primaries. As against this, Fenton (1963) has pointed out that his calculations of the yield of pions may have led to an overestimation of the response to the low energy primaries.

(b) The index, γ, of the variation spectrum

The little evidence that is available, from the comparison of

(i) observed sidereal diurnal amplitudes at sea level and underground (amplitudes comparable) and

(ii) the amplitude observed at approximately 60 m.w.e., London, and that observed at approximately 40 m.w.e., Budapest (amplitudes equal, within statistical errors of $\sim$ 25%), suggests that the amplitudes of the free-space harmonics of the sidereal anisotropy are not steeply rigidity- dependent.

Again, if the anisotropy is connected with a Fermi type of acceleration process - although as noted in Chapter 1, there are other possibilities to consider as well - the amplitude would be expected to decrease with increasing rigidity.

However, it is only of concern in this thesis to find out whether estimates of the direction of the anisotropy depend weakly or strongly on the values assigned to γ. Consequently, it has been decided to compare estimations based on $\gamma = 0$ and $\gamma = -2$, on the supposition that the true value of γ might lie somewhere between these two.

(c) The primary threshold rigidity, R_c, for observation of the anisotropy

We are guided here by the estimations of the upper limiting rigidity, R_u, for observation of the solar anisotropy. Provisionally R_c is equated with R_u, although it is recognised (see discussion in paper 7, section IX) that they are not necessarily the same. For this reason, and from the indications that R_u itself changes considerably during the course of the cycle of solar activity, the asymptotic constants are calculated over a range of values of R_c.

(d) Determination of the constants

Approximate values of the constants, referring to the vertical semi-cubical telescope and the north-pointing and sout-pointing cubical telescopes, were obtained by vector summations. Tables 6.1 and 6.2 list the results of the determinations for $\gamma = 0$ and $\gamma = -2$ respectively, and for the following values of R_c: 20, 50, 70, 100, 150 and 200 GV. In Figure 6.7, the relative (observed/free-space) diurnal and semi-diurnal amplitudes of response that apply to the vertical telescopes are shown versus R_c for the two values of γ. They are in fact the tabulated values of K_1/k_c (= K_1 when $\gamma = 0$).

Since the constants have been determined for the geomagnetic latitude $\lambda = -50^o$, it has been considered sufficient in this treatment to use the same asymptotic constants for the Hobart ($\lambda = -52^o$) and Budapest ($\lambda = +45^o$) vertical semi-cubical telescopes. It will be noted that the appropriate values of the mean asymptotic latitude of viewing at Budapest, $\overline{\delta}_{Bud}$, are listed under the Hobart values.

B. The solar Anisotropy

Brief reviews are given here of two features of the solar anisotropy: the seasonal characteristics of the *free-space* diurnal variation and the upper limiting rigidity for observation of the anisotropy, as described in papers 1 and 7 respectively.

TABLE 6.1

Integrated constants of response relating to underground observations of the sidereal anisotropy, for $\gamma = 0$.

		Cut-off Rigidity R_c(GV)					
		20	50	70	100	150	200
Semi-cube vertical	K_1	0.828	0.764	0.676	0.566	0.424	0.327
(Hobart and Budapest)	ϕ_1	15°	12°	10°	8°	7°	6°
	$\overline{\delta}_{HOB}$	-37°	-38°	-39°	-39°	-39°	-39°
	$\overline{\delta}_{BUD}$	+42°	+43°	+43°	+43°	+43°	+43°
	K_2	0.509	0.460	0.405	0.343	0.257	0.199
	ϕ_2	16°	12°	10°	8°	7°	6°
Cube 30°N of zenith	K_1	0.942	0.864	0.756	0.646	0.485	0.380
(Hobart)	ϕ_1	4°	3°	2°	1°	1°	1°
	$\overline{\delta}$	-18°	-18°	-18°	-18°	-17°	-17°
	K_2	0.812	0.754	0.656	0.567	0.415	0.326
	ϕ_2	4°	3°	2°	1°	1°	1°
Cube 45°S of zenith	K_1	0.465	0.415	0.341	0.269	0.189	0.146
(Hobart)	ϕ_1	53°	49°	45°	39°	31°	25°
	$\overline{\delta}$	-45°	-52°	-55°	-59°	-61°	-61°
	K_2	0.320	0.265	0.173	0.128	0.063	0.029
	ϕ_2	67°	65°	62°	56°	41°	34°

TABLE 6.2

Integrated constants of response relating to underground observations of the sidereal anisotropy, for $\gamma = -2$.

		Cut-off Rigidity R_c (GV)					
		20	50	70	100	150	200
	$k_c \times 10^3$	1.07	0.172	0.087	0.043	0.019	0.011
Semi-cube vertical	K_1/k_c	0.125	0.296	0.339	0.334	0.277	0.210
(Hobart and Budapest)	ϕ_1	33°	20°	17°	11°	7°	6°
	$\overline{\delta}_{HOB}$	-32°	-37°	-40°	-40°	-40°	-40°
	$\overline{\delta}_{BUD}$	+37°	+42°	+44°	+44°	+44°	+44°
	K_2/k_c	0.084	0.180	0.215	0.214	0.179	0.139
	ϕ_2	35°	21°	17°	11°	7°	6°
Cube 30°N of zenith	K_1/k_c	0.136	0.325	0.376	0.386	0.332	0.255
(Hobart)	ϕ_1	10°	5°	4°	1°	1°	1°
	$\overline{\delta}$	-19°	-17°	-17°	-17°	-17°	-17°
	K_2/k_c	0.111	0.273	0.319	0.330	0.281	0.217
	ϕ_2	10°	5°	4°	1°	1°	1°
Cube 45°S of zenith	K_1/k_c	0.121	0.249	0.217	0.186	0.134	0.098
(Hobart)	ϕ_1	69°	63°	56°	48°	47°	40°
	$\overline{\delta}$	-29°	-43°	-48°	-47°	-60°	-63°
	K_2/k_c	0.102	0.197	0.137	0.065	0.036	0.014
	ϕ_2	71°	69°	67°	65°	49°	42°

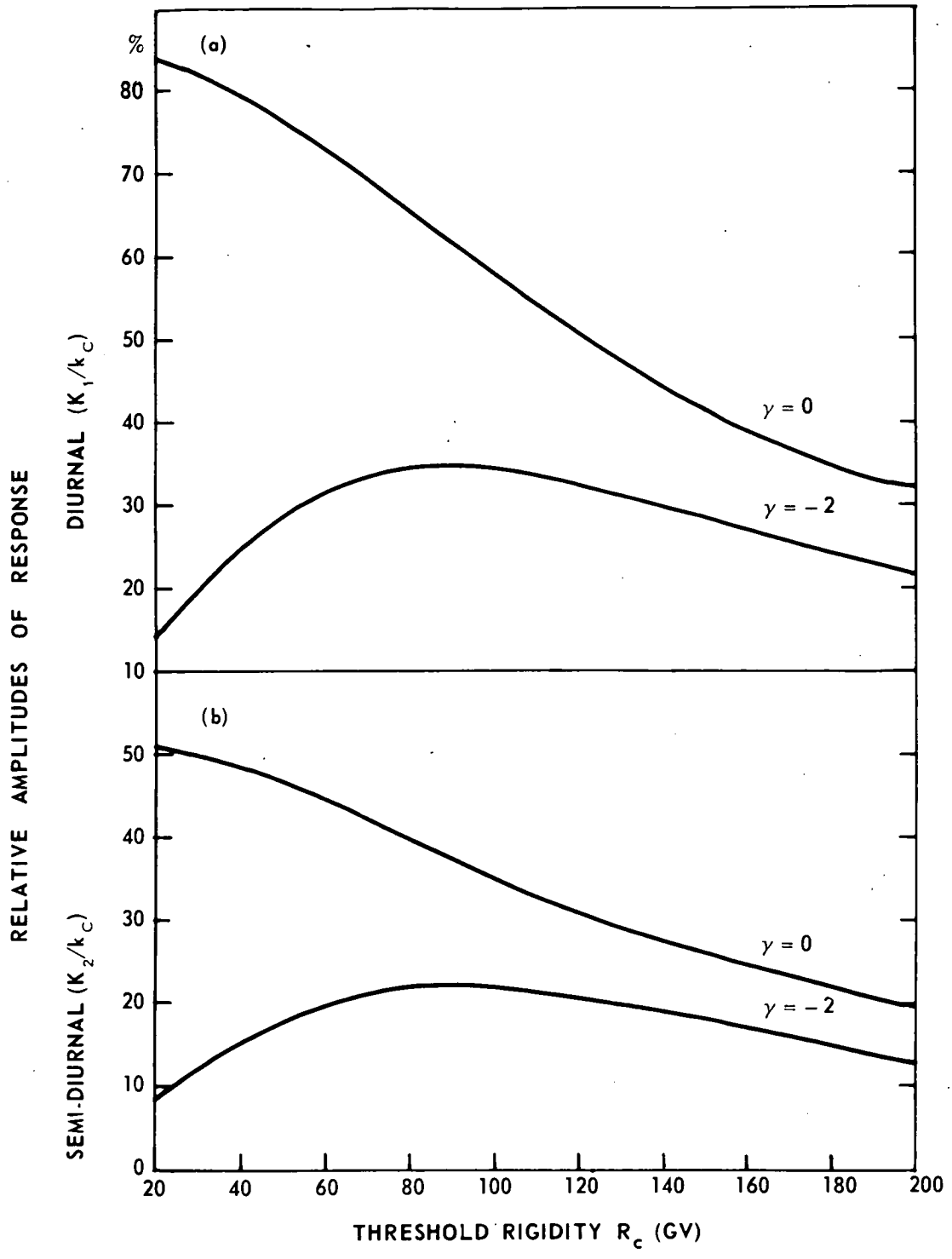

Figure 6.7 Calculated amplitudes of response to the two-way anisotropy from a vertical semi-cubical telescope at a depth of ~40 m.w.e. , Hobart, relative to the corresponding free-space amplitudes.

6B.1 The Seasonal Characteristics of the Free-Space Diurnal Variation

The solar anisotropy as originally described by Rao et alia (1963) was specified in the system of spherical coordinates whose reference planes were the ecliptic and the plane normal to it through the earth-sun line, this being the frame of reference in which the anisotropy was assumed to be fixed. The anisotropic component of intensity for primaries of rigidity R was specified as

$$\Delta I(R,\Lambda,\psi) = A\, I_o(R) \cos\Lambda \cos(\psi - \psi_o) \qquad 6.25$$

where A = the amplitude constant

$I_o(R)$ = the average differential rigidity spectrum

Λ = the ecliptic latitude of the detector

ψ = the direction of viewing in the plane of the ecliptic, east of the earth-sun line

ψ_o = the direction of maximum intensity

After transforming to the conventional ecliptic system, specified by the plane of the ecliptic and the equinox, it is necessary to transform to the frame of reference of the observer, the terrestrial equatorial system. This is achieved by rotation of axes through the angle ε, the obliquity of the ecliptic, as shown in Figure 6.8. In terrestrial coordinates the expression for the anisotropy outside the earth's field region then becomes (see equation (4) of paper 1) :

$$\begin{aligned}\Delta I(R,\alpha_s,\delta,\phi) = AI_o(R)\{&[1-0.0826\sin^2(\alpha_s+\psi_o)]\cos\delta\cos(\phi-\psi_o)\\ &- 0.0413\cos\delta\sin 2(\alpha_s+\psi_o)\sin(\phi-\psi_o)\\ &+ 0.3979\sin\delta\sin(\alpha_s+\psi_o)\}\end{aligned} \qquad 6.26$$

$$(\text{for } \varepsilon = 23^o27')$$

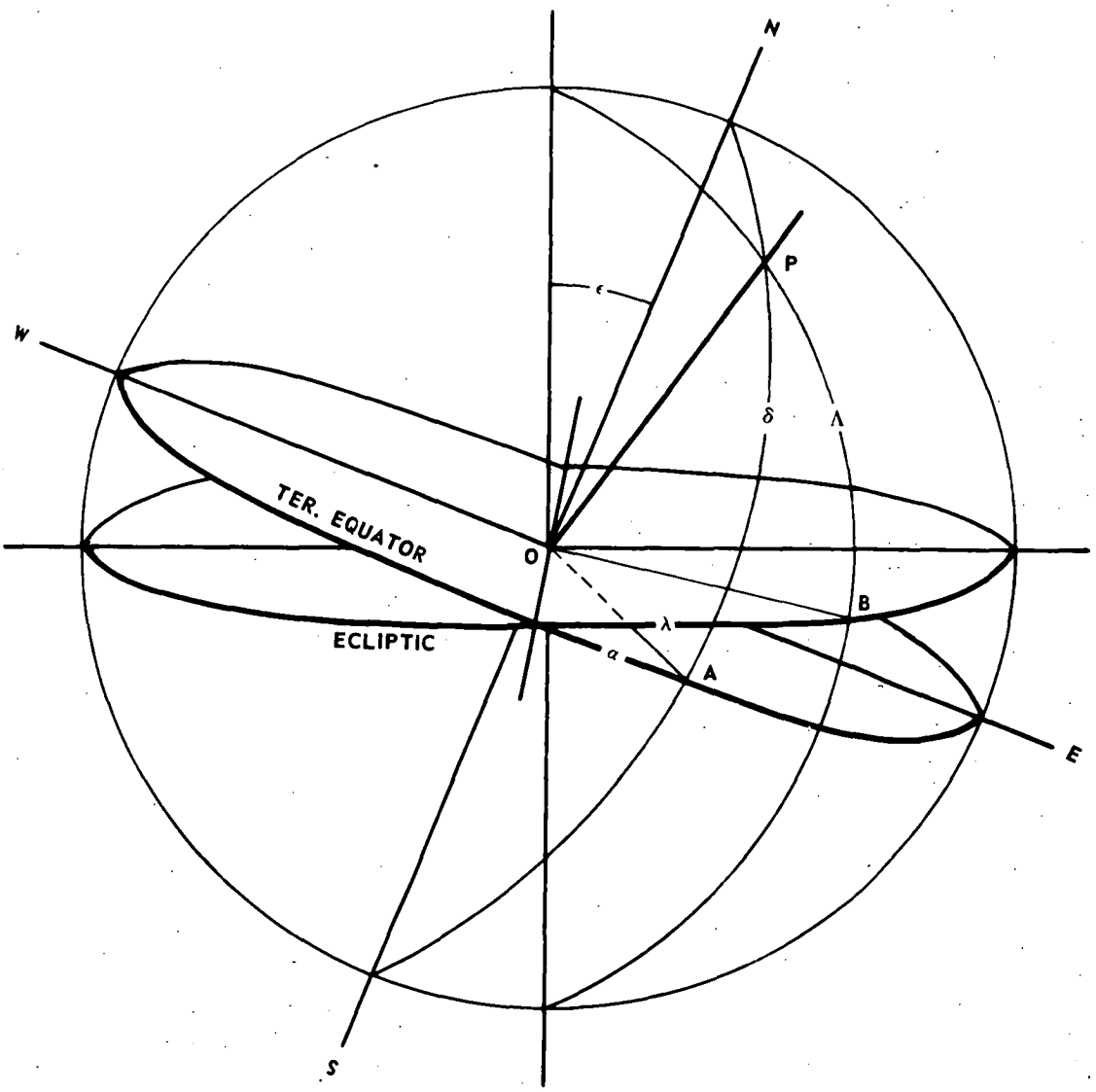

Figure 6.8 The spherical coordinates of the direction of observation, OP, in the terrestrial and ecliptic frames of reference.

THE DISPOSITION TOWARD ORIGINALITY[1, 2]

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THERE has been a marked tendency in psychological research on originality to focus attention upon the single original act in itself, rather than upon the total personality of the originator. This is understandable, for the birth and development of the original idea is usually more immediately interesting and dramatically vivid than the birth and history of the man who had the idea. Newton's apple and Archimedes' tub and the well of Eratosthenes are thus naturally the circumstances with which we associate the remarkable insights of these original geniuses; we do not often ask ourselves whether these men were for the most part disposed to express or to suppress erotic impulses, or whether their emotions were fluent or turgid, or how subject to intense anxiety they were, or how much given to violent action. We tend to disembody the creative act and the creative process by limiting our inquiry to the creator's mental content at the moment of insight, forgetting that it is a highly organized system of responding that lies behind the particular original response which, because of its validity, becomes an historical event.

There is good reason for believing, however, that originality is almost habitual with persons who produce a really singular insight. The biography of the inventive genius commonly records a lifetime of original thinking, though only a few ideas survive and are remembered to fame. Voluminous productivity is the rule and not the exception among individuals who have made some noteworthy contribu... Original responses, it would seem, recur ... larly in some persons, while there are ... individuals who do not ever depart from ... stereotyped and the conventional in ... thinking.

If, then, some persons are regularly ... while others are regularly unoriginal, it ... be the case that certain patterns of ... enduring traits either facilitate or impede ... production of original acts. Rather than ... ing on the immediate conditions which ... triggered the original response, the present study was concerned with the underlying ... position toward originality which it ... presumed exists in those persons who are ... larly original. The research was directed ... all toward identifying individuals who ... formed consistently in a relatively ... relatively less original way; when this had been done, the more original were compared ... the less original in terms of personality ... zation. Independent evidence concerning ... personalities of the *S*s was obtained ... through the use of standardized paper-and-pencil tests and through employment of ... living-in assessment method, with its emphasis upon observation of the *S*s through ... days of informal social interaction, situation tests, group discussions, psychodrama, and the like. The observers were of course kept ... norance of the scores earned by the *S*s ... of originality.

THE RELATIVITY OF ORIGINALITY

It is a basic assumption of this study ... acts are original only in relation to some ... fied commonality. The original must be ... relative to the usual, and the degree of ... nality must be specified statistically in ... incidence of occurrence. Thus the first ... terion of an original response is that it ... have a certain stated uncommonness ... particular group being studied. A ... example of this in psychological practice ... definition of an original response to the ... schach inkblots, the requirement there ... that the response should, in the examiner's ...

[1] Acknowledgment is made to the Rockefeller Foundation for its generous financial support of the program of research being carried on by the Institute of Personality Assessment and Research, of which this study is a part.

[2] This research is supported in part by the United States Air Force under Contract No. AF 18 (600) -8, monitored by Technical Director, Detachment #7, (Officer Education Research Laboratory), Air Force Personnel and Training Research Center, Maxwell Air Force Base, Alabama. Permission is granted for reproduction, translation, publications, use, and disposal in whole and in part by or for the United States Government. Personal views or opinions expressed or implied in this publication are not to be construed as necessarily carrying the official sanction of the Department of the Air Force or of the Air Research and Development Command.

...rate, occur no more often than once in 100 ...ations.

...the present study, we propose to deal ... relatively low order of originality, its ... being set by the nature of the sampling ... The *S*s are 100 captains in the United States ...rce, and originality as discerned here is ...ality in relation to the usual responses of ...00 persons. Furthermore, these 100 per... are not themselves especially selected for ...ality in relation to the population in ...ral. Nevertheless, as we shall show later, ... of the 100 captains are regularly original ...parison with the remainder, while others ...gularly unoriginal in relation to the entire ... Apart from their military status, the ... may be described as a group of normal, ...y young men, of average intelligence, ...conomically of the lower middle class in ... pre-army background, and similar to ... men in general in terms of the usualness ... the unusualness of their responses to the ... of originality employed in this experi...

...second criterion that must be met if a re... is to be called original is that it must be ... extent adaptive to reality. The intent ... requirement is to exclude uncommon ...ses which are merely random, or which ...ed from ignorance or delusion. An ex... of the application of this second criterion ...e taken from the scoring of one of the ...res of originality used in this experiment: ...easure is a count of the number of un... *and correct* anagram solutions to the ...word "generation." Many *S*s did not ... to offer solutions that were incorrect, ...at were usually unique. In such in..., the application of the second criterion ...ality was straightforward and decisive. ...l of the tests called for such purely cogni... responses with unambiguous denotative ..., however: in the case of inkblot tests, ... come closer to the problems involved ...ating fantasy or works of art, and veri... cannot be had by recourse to a dic...y. Instead, when *E* himself cannot "see" ...m pointed to by *S*, he must have re... to other psychologists who have given ...Rorschachs and who can be considered ...en to suggestions as to what the blots ...easonably look like. Consensual verifi... is thus sought for such imaginings. Poor forms, or uncommon responses that did not sufficiently respect the inkblot reality, were not credited as original in this study.

THE MEASUREMENT OF ORIGINALITY

Eight test measures were accepted here as indicative of originality. They are described below. The first three of these measures are taken from the creativity battery developed by Guilford and his associates (5, 6) in the Project on Aptitudes of High-Level Personnel at the University of Southern California. These three tests had significant loadings on the Originality factor in the Guilford researches.[3] Of the remaining five measures, two are derived from commonly used projective techniques, the Rorschach Psychodiagnostic (10) and the Thematic Apperception Test (9); another is a commonly used anagram test, and the remaining two tests were devised by the writer.

1. *Unusual Uses.* This test calls upon the subject to list six uses to which each of several common objects can be put. It is scored for infrequency, in the sample under study, of the uses proposed. Odd-even reliability in this sample is .77.
2. *Consequences B.* In this test, *S* is asked to write down what would happen if certain changes were suddenly to take place. The task for him is to list as many consequences or results of these changes as he can. The responses are scored according to how obvious the imagined consequences are, the less obvious responses receiving the higher scores. Interrater agreement is .71.
3. *Plot Titles B.* Two story plots are presented, and *S* is asked to write as many titles as he can think of for each plot. The titles are rated on a scale of cleverness from 0 to 5. The number of titles rated 2, 3, 4, or 5 constitutes the cleverness score. Interrater agreement in this study was .43.
4. *Rorschach O+.* This is a count of the number of original responses given by *S* to the 10 Rorschach blots and adjudged by two scorers, working separately, to be good rather than poor forms. Standard Rorschach administrative procedure was followed. Interrater agreement was .72, and only those responses scored by both scorers as 0+ were credited.
5. *Thematic Apperception Test: Originality rating.* Two raters, working independently of one another, rated the TAT protocols of the 100 *S*s on a 9-point scale, using approximate normal curve frequencies for each point along the scale. Interrater agreement was .70. The *S*'s score was the average of the two ratings.
6. *Anagrams.* The test word "generation" was used, and the anagram solutions were scored for infrequency of occurrence in the sample under study. If *S* offered a solution that was correct and that was offered by no more than two other *S*s, he received one point for originality. Total score is therefore the number of such uncommon but correct solutions.
7. *Word Rearrangement Test: Originality rating.* In this test, *S* is given 50 words which were selected at random from a list of common nouns, adjectives, and

[3] The present writer is indebted to Dr. Guilford and the personnel of the Project not only for permission to use the tests, but also for the actual scoring of the protocols.

where α_g = RA of the sun at transit at Greenwich

ϕ = time of day relative to local noon

δ = geographic latitude of observation

Bearings of interest in the ecliptic and terrestrial equatorial planes are shown in Figure 6.9.

The presence of the annually varying factor α_s leads to an annual variation of the secular component $0.3979 \sin\delta \sin(\alpha_s + \psi_o)$ and to a semi-annual variation of the diurnal component. If the diurnal component is the same on any given day as it was six months previously, it then must vanish when arranged in sidereal time and averaged over a complete year, since all days can be grouped in pairs six months apart. Therefore semi-annual modulation of this kind cannot possibly be responsible for a spurious sidereal effect.

By similar methods, some preliminary calculations have been made for a solar anisotropy whose latitude dependence of amplitude differs from $\cos\Lambda$. The possibility of latitude-independence of amplitude has been considered, and of latitude dependence specified by $\cos^2\Lambda$, $\cos^3\Lambda$ and $\cos^5\Lambda$ respectively. In all of these cases it appears that the seasonal variation of the free-space first harmonic must be very small and that spurious sidereal effects are negligible.

Much the same considerations apply if the solar anisotropy is assumed to be located in the solar equatorial plane, and as McCracken and Rao (1965) have indicated in their review of the diurnal anisotropy, the free-space diurnal

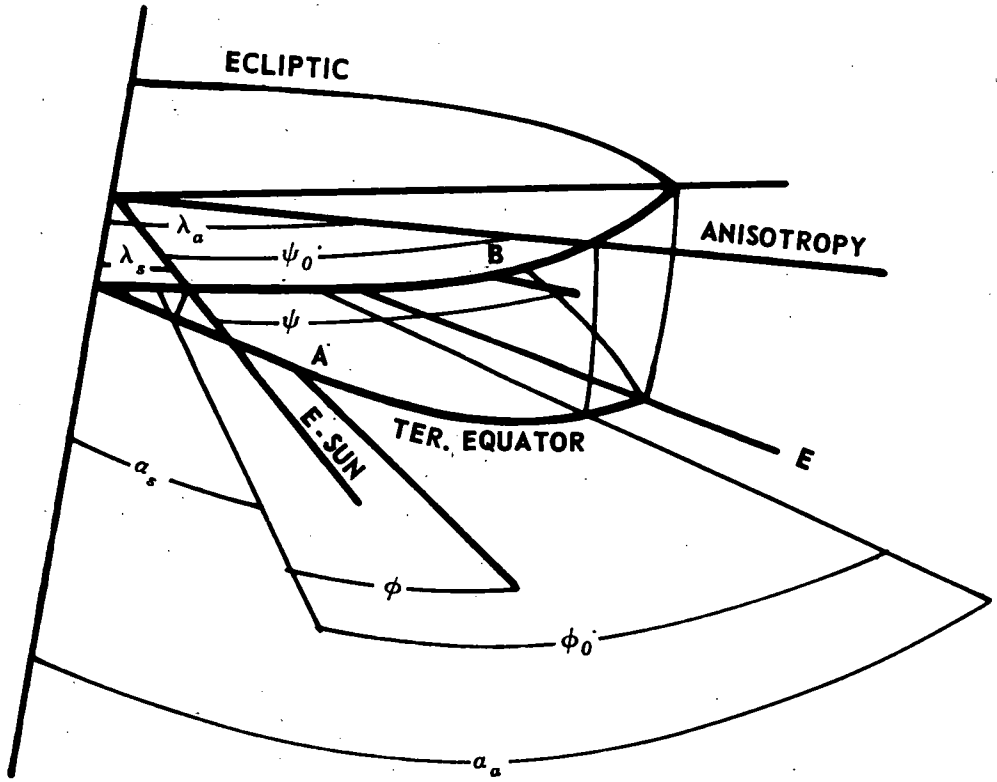

Figure 6.9 The ecliptic and terrestrial longitude bearings of the observational direction OP, in relation to the Earth-Sun line and to the direction of an anisotropy located in the plane of the ecliptic. The points A and B are the same as those shown in Figure

variation must exhibit a semi-annual variation, which they have specified as a semi-annual variation of amplitude. As we have seen, this cannot give rise to an annual sidereal sideband component.

It is concluded that on our present understanding of the solar anisotropy the transformation from the frame of reference in which it is orientated to the rotating terrestrial system of coordinates does not in itself bring into being any significant spurious sidereal effect.

6B.2 The Upper Limiting Rigidity of the Solar Anisotropy

A method of determining R_u is fully described in paper 7. It depends on the fact that changes in R_u would produce a much greater response in the solar diurnal variation as observed underground than in the diurnal variation as observed with neutron monitors at high latitude sea-level stations. Only a brief summary of the steps in the determination is given here.

The amplitude constant of response B_1, analogous to K_1, is calculated for the given detector as a function of R_u (Figure 6.10). Now, B_1 is $|v|/\alpha_1$, where $|v|$ is the amplitude of that part of the observed first harmonic that is caused by the primary anisotropy, and α_1 is the amplitude constant of the free-space first harmonic. Corresponding to a given value of $|v|$ (e.g. an observed annual amplitude, after the removal of estimated extraneous contributions) a curve can be obtained giving estimates of α_1 over a range of values of R_u.

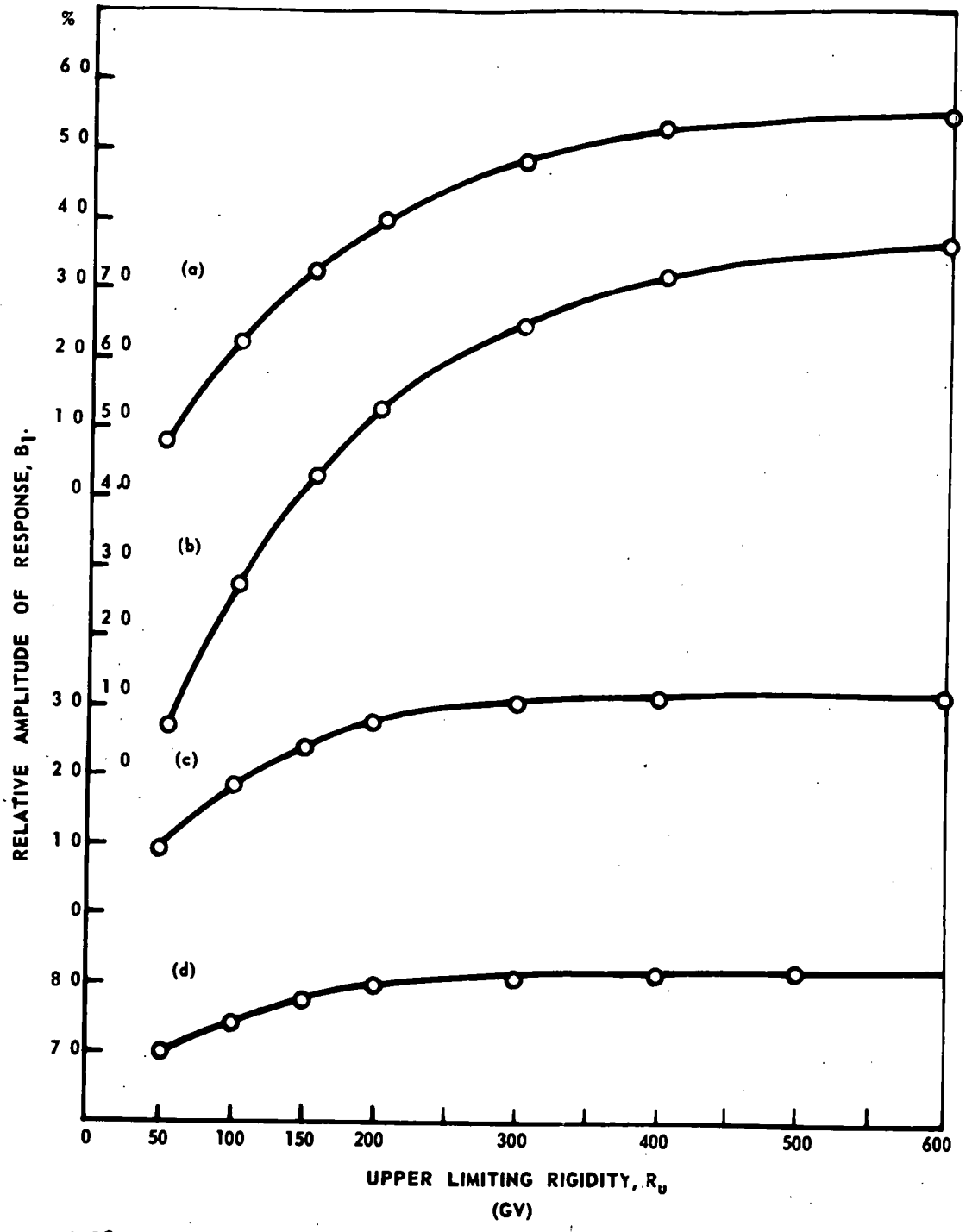

Figure 6.10 The estimated amplitude of the solar diurnal variation, relative to the amplitude of the average free-space first harmonic, versus upper limiting rigidity, relating to

(a) a vertical semi-cube at a depth of ≈40 m.w.e. at Hobart.

(b) a cube inclined 30° north of the zenith at a depth of ≈40 m.w.e. at Hobart.

(c) a cube inclined 45° south of the zenith at a depth of ≈40 m.w.e. at Hobart.

(d) a high-latitude neutron monitor at sea level.

Circled points indicate calculated values. The model for the free-space first harmonic is that proposed by Rao, McCracken and Venkatesen.

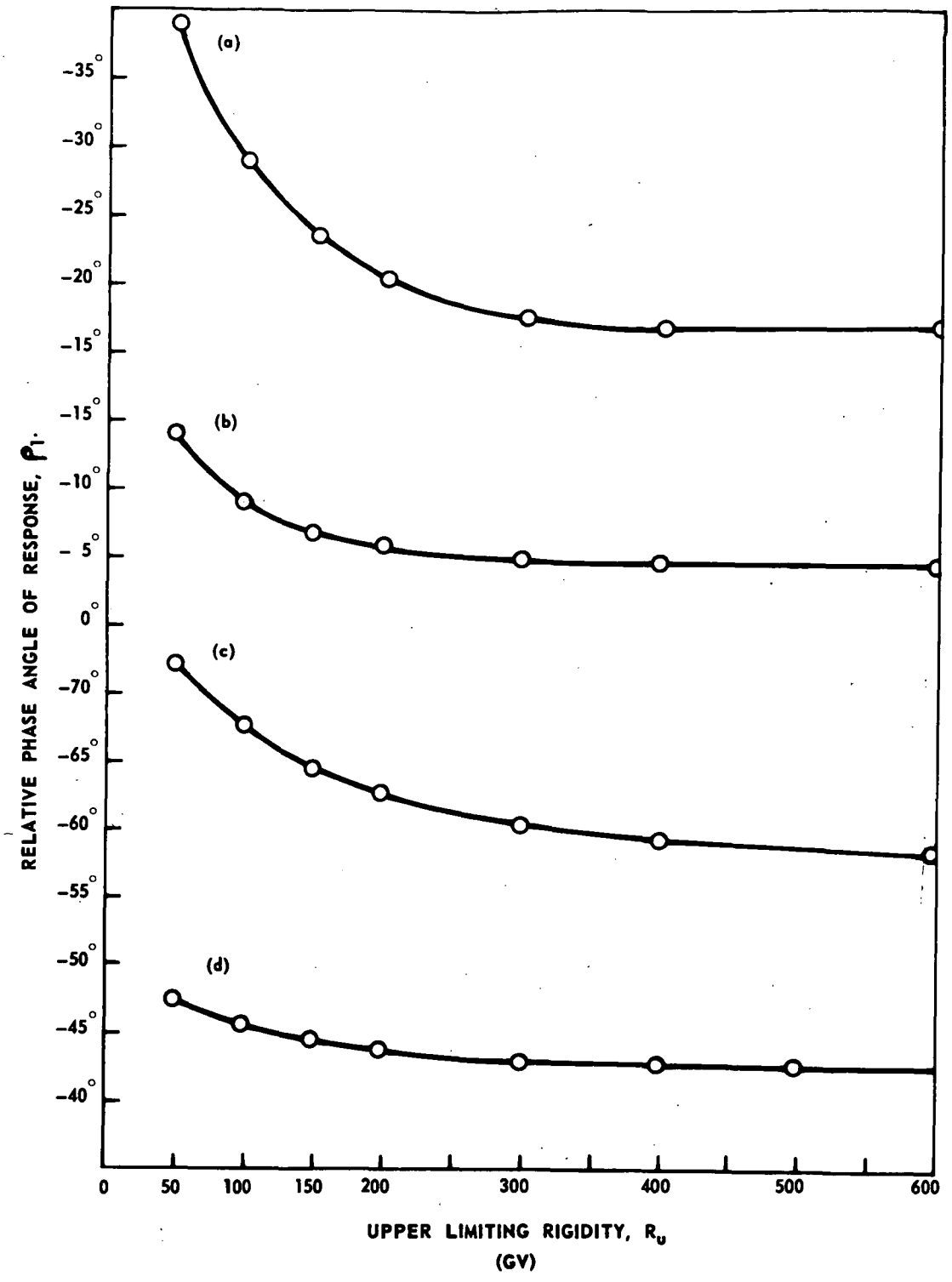

Figure 6.11 The estimated direction of the maximum of the solar diurnal variation, measured east of the direction of maximum intensity of the free-space first harmonic, versus upper limiting rigidity, relating to

(a) a vertical semi-cube at a depth of ≈40 m.w.e. at Hobart.
(b) a cube inclined 30° north of the zenith at a depth of ≈ 40 m.w.e. at Hobart.
(c) a cube inclined 45° south of the zenith at a depth of ≈ 40 m.w.e. at Hobart.
(d) a high-latitude neutron monitor at sea level.

Circled points indicate calculated values. The model for the anistropy is that proposed by Rao, McCracken and Venkatesen.

Provided R_u takes values in excess of, say, 50 GV, the curve relating to a high latitude neutron monitor should be relatively flat, since the observed amplitude will vary only slowly with R_u at these high rigidities. On the other hand, the curve relating to an underground telescope, for which the primary spectrum of response virtually commences at about 50GV, should be very steep. Consequently the intersection of the two curves should be clearly defined and should give the actual values of α_1 and R_u appropriate to the period of observation.

Figure 6.12 shows the result for the year 1958. Curve B, relating to high latitude neutron monitors, was estimated on the basis of the analysis by Rao et alia of the solar diurnal variation of neutron intensity observed that year at the network of sea level stations. The intersections of curves A and B gave R_u = 95 GV with an S.E. of estimate of $\pm$ 5 GV due to count rate statistics. Naturally, other uncertainties connected with the calculations of response would be expected to predominate and it was thought that overall the true value of R_u should lie within about 20 GV of 95 GV.

The other period examined was that of the underground latitude survey, 1961-1962. At the time there were no means of adequately estimating the new curve B, relating to the neutron monitors, but the displacement of the new curve A relative to that of 1958 indicated a significant decrease in the value of R_u (Figure 6.13). It was noted that the curves deduced from the observations in the inclined directions (particularly from the more accurate observations in the

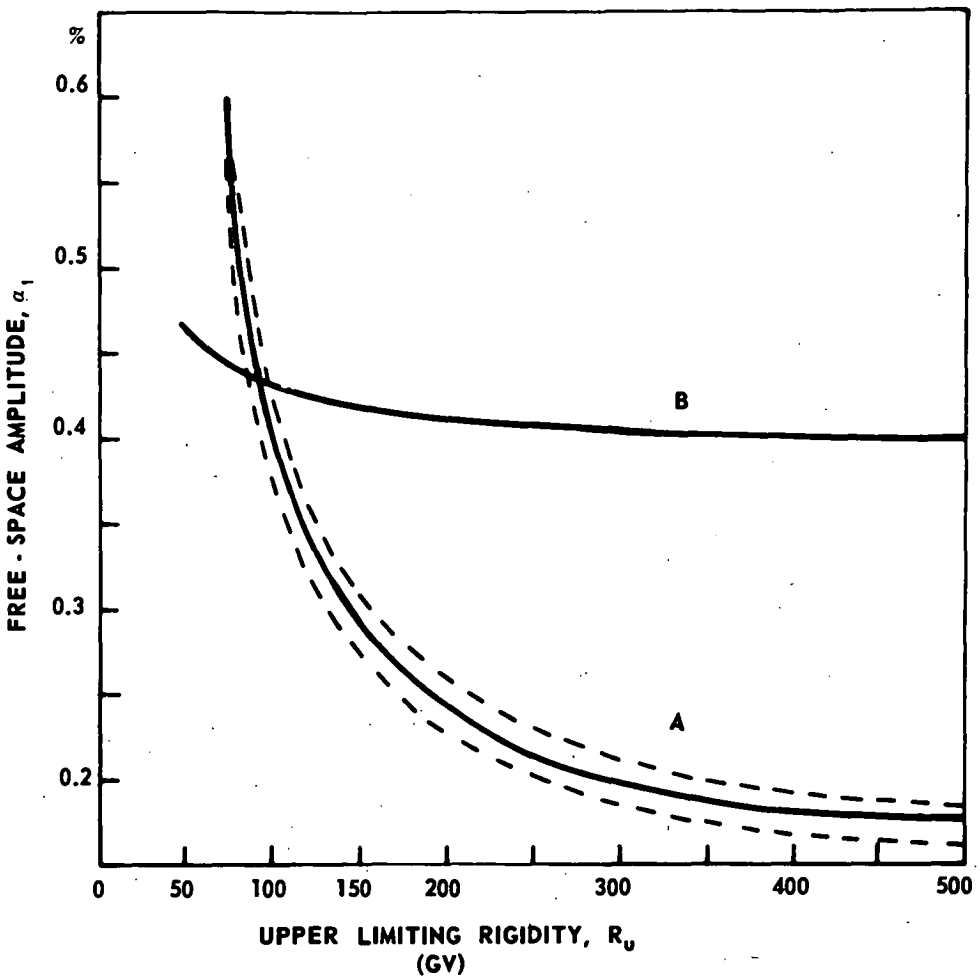

Figure 6.12 Estimates of the amplitude (a_1) of the average free-space solar diurnal variation of the primaries in 1958 for different values of the upper limiting rigidity (R_u) based on

(a) observations with a vertical semi-cubical telescope at a depth of 40 m.w.e. at Hobart. Curve A.

(b) the estimated solar diurnal variation of neutron intensity at a high-latitude sea level station, deduced from world-wide observations of neutron intensity. Curve B.

The average values of R_u and a_1 for 1958 are given by the intersections of the two curves.

All the estimates derive from the model for the free-space first harmonic proposed by Rao et alia. The dashed lines represent the S.E.s of error of individual points on Curve A.

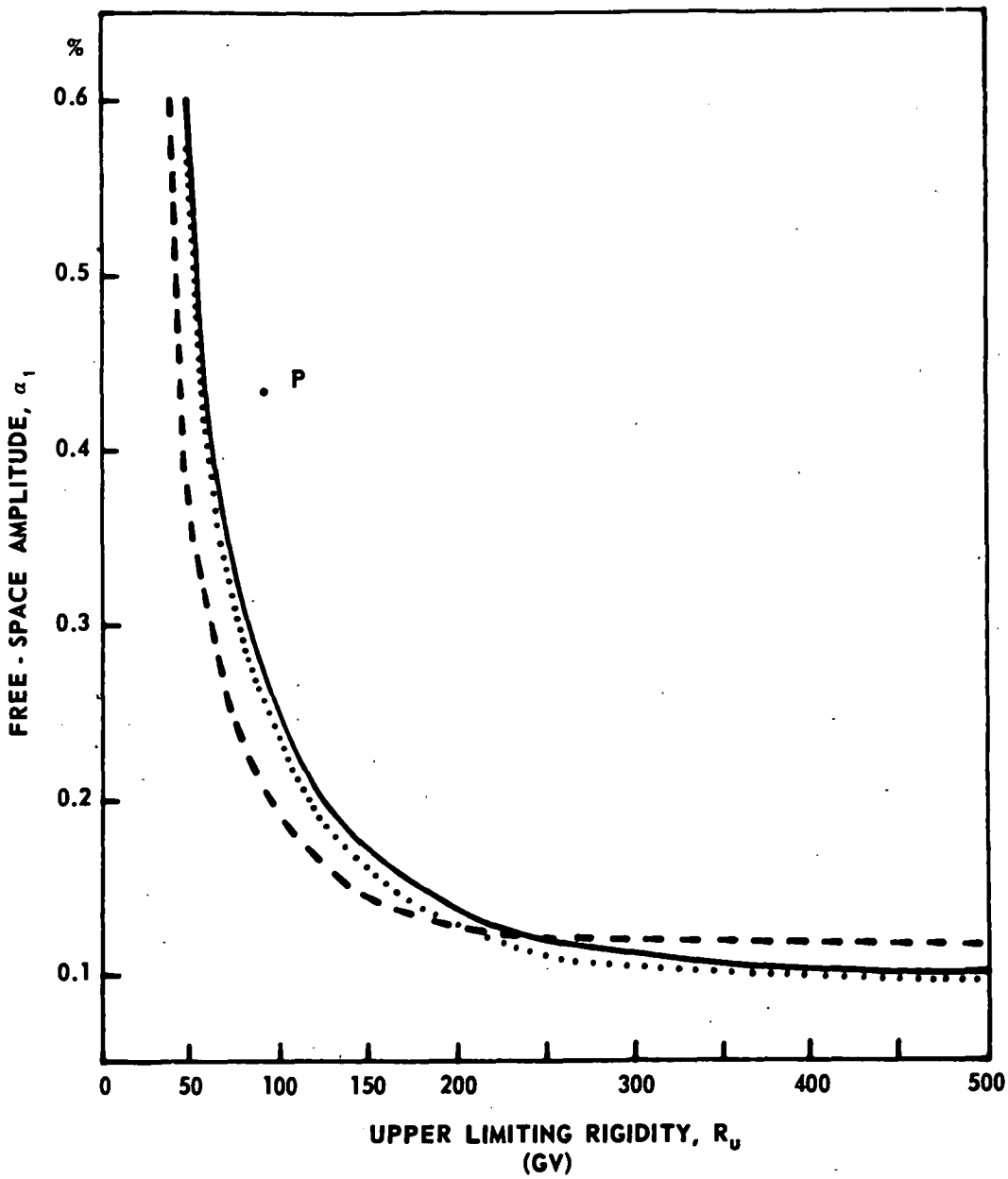

Figure 6.13 Estimates of the amplitude (a_1) of the free-space solar diurnal variation of the primaries, averaged over 1961 and 1962, for different values of R_u and for $\beta = 0$, based on observations underground at Hobart with

(a) the vertical semi-cubes. Full line.
(b) the north-pointing cube. Dotted line.
(c) the south-pointing cube. Dashed line.

The point P represents the intersection of curves A and B for 1958 (Figure 4).

axial direction 30°N of zenith) agreed quite closely with the curve obtained from the vertical observations. This tended to confirm the validity of the method of calculation of response and also demonstrated that the observed amplitudes in the three directions must have been essentially due to the solar anisotropy. Figure 6.14 depicts the compatibility in respect of amplitude and phase, the observed first harmonic vectors in the three directions being compared with curves of estimates (dashed lines) calculated as functions of R_u. The estimates were derived from amplitude and phase constants of response appropriate to each detector (Figures 6.10 and 6.11), on the assumption that the free-space constants β, α, and ψ_o had not changed appreciably between 1958 and 1961-1962.

On the above assumptions it was concluded that R_u had decreased by approximately 30 GV over the period from 1958 to 1962. There was supporting evidence for this from the behaviour of the solar diurnal variation observed underground at Budapest (see Table 6, paper 7) and from the differences in behaviour of the solar daily variations of neutron intensity at Huancayo and Churchill, as reported by Sarabhai and Subramanian (1963).

It now seems (McCracken and Rao, 1965; J.E.Humble, private communication) that the amplitude constant α, and the direction ψ_o of the diurnal part of the solar anisotropy, did not change significantly at any time, on the annual average, between 1958 and 1965 and that as far as could be judged the differential free-space amplitude was independent of rigidity ($\beta = 0$). It is submitted that these findings confirm the view that the large decreases of amplitude observed at high mean rigidities of response over the same period were due essentially to a decrease in the upper limiting rigidity R_u.

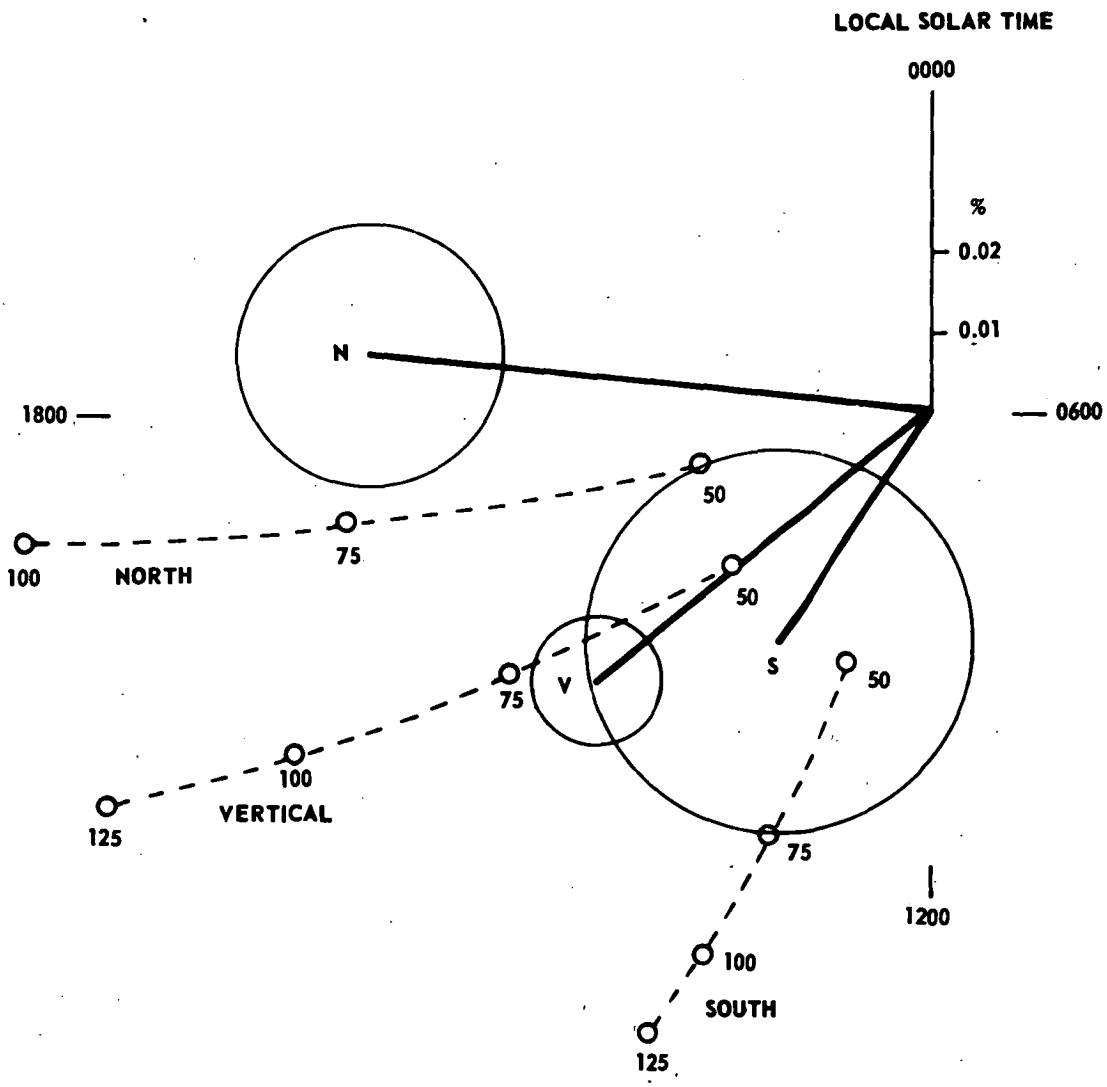

Figure 6.14 The first harmonics of the pressure-corrected daily variations observed in the north (N), vertical (V) and south-pointing (S) directions underground at Hobart, averaged over the years 1961 and 1962. The radius of each error circle is 2σ.

The dashed lines give estimates of the first harmonics to be expected in each of the three directions, for different values of upper limiting rigidity (R_u). Some values of interest that are shown are in GV. The estimates are based on the model for the free-space solar diurnal variation of the primaries proposed by Rao et alia.

It seems that even existing counting rates underground would warrant more accurate calculations of R_u. It is suggested that the following points in particular should be taken into account in an improved treatment:

(1) It would be necessary to revise the calculations of the coupling coefficients, basing them on a more sophisticated model for the yield of mesons and using more accurate estimates of the cut-off energy for mesons at production as these relate to given detectors. The coupling coefficients used in the present investigation have probably led to overestimation of R_u.

(2) As Thambyahpillai et alia (1965) have pointed out, the streaming of cosmic radiation due to the orbital motion of the earth around the sun with a speed of 30 km/sec should generate a solar diurnal variation, with maximum intensity at 0600 local solar time in free-space. They have estimated the amplitude to be 0.02%. Compensation for this effect increases the underground amplitude of response to the solar anisotropy by a little less than 0.02% and has the effect of increasing the estimated value of R_u by approximately 20 GV.

(3) No calculations have yet been made of the effect of the scattering of μ-mesons in the material absorber. It has been assumed that the integrated amplitude constants of response for wide-angle detectors would require little modification because of scattering. However, by not taking into account the smoothing of amplitudes produced by scattering, existing values of R_u have been underestimated.

CHAPTER 7

THE ESTIMATED DIRECTION OF THE ANISOTROPY

The direction of the sidereal anisotropy outside the earth's magnetic field will now be estimated as a function of the cut-off for observation (R_c) and the index (γ) of the variation spectrum. In the process, the latitude dependences of the free-space diurnal and semi-diurnal amplitudes are determined. These latitude effects give important indications as to the validity of the model for the anisotropy and as to the presence or otherwise of spurious components in the sidereal daily variation.

The estimated direction is compared with recent optical and radio determinations of the direction of the local spiral arm magnetic field.

7.1 The Estimated Declination

(a) Outline of procedure

Let us suppose that simultaneous annual averaged observations of the two-way anisotropy are available from two detectors A and B, which are located at approximately the same depth underground, but which scan widely differing asymptotic strips of latitude. Now, provided that there are (a) no important spurious contributions to the observations and (b) no additional genuine contributions from other directions (e.g. due to diffusion outward from the centre of the galaxy), the observations from A and B suffice for the determination of the *declination* d, as a function of threshold rigidity R_c and

index of the variation spectrum γ. While it would appear from the evidence given in Chapter 5 that if there are spurious effects in the underground observations they are not significant, it is nevertheless desirable to test against possibilities (a) and (b) with observations from a third detector C. This detector should scan a latitude strip which differs considerably from the strips scanned by A and B. In effect, having estimated $\underline{d}$ with the data from detectors A and B one notes whether such a value of d is compatible with the observations from detector C. If there is a significant discrepancy, various possible reasons for this, including possibilities (a) and (b) above, must be examined. If there is no significant discrepancy it is concluded that, within the limits of error of observation, the anisotropy being observed is the one described by the model.

Evidently, then, the procedure not only gives an estimate of the declination, but provides a test of the genuineness of the observed sidereal effect.

(b) The expression for the declination

It is proposed to evaluate the declination of the *southward* direction of the axis of the two-way anisotropy. Thus, following the usual convention, the value of $\underline{d}$ will be negative. If we now go back to equation 6.7 and put $\phi' = \alpha_s$ this gives a diurnal maximum for detector A scanning asymptotically in the southern hemisphere (δ negative), depending on the influence of the additional component $m \cos d \cos \delta \cos(\phi' - \alpha_s)$. It should be explained (see equation 6.1)

that the amplitude constant $\underline{n}$ is positive, while $\underline{m}$ may take a positive or a negative value depending on whether the intensity maximum from this streaming type of component is from the southward or northward direction of the axis of the two-way anisotropy.

The respective observed diurnal and semi-diurnal amplitudes of response from detectors A and B are denoted by (v_{1Amax}, v_{2Amax}) and (v_{1Bmax}, v_{2Bmax}) and relate to the mean asymptotic latitudes of viewing δ_A and δ_B. The corresponding free-space amplitudes are (V_{1Amax}, V_{2Amax}) and (V_{1Bmax}, V_{2Bmax}).

Putting $\phi' = \alpha_s$ in equation 6.7 we may write in respect of detector A,

$$V_{1Amax} = k_c(m \cos d \cos \delta_A + \frac{n}{2} \sin 2d \sin 2\delta_A) \qquad 7.1$$

Similarly $$V_{2Amax} = k_c \frac{n}{2} \cos^2 d \cos^2 \delta_A \qquad 7.2$$

Substituting for n from equation 7.2 in equation 7.1, we get

$$V_{1Amax} = k_c(m \cos d \cos \delta_A + \frac{V_{2Amax} \sin 2d \sin 2\delta_A}{k_c \cos^2 d \cos^2 \delta_A}) \qquad 7.3$$

Likewise, if detector B scans asymptotically in the same hemisphere as A and the observed diurnal components from the two detectors are *in phase*, putting $\phi' = \alpha_s$ gives

$$V_{1Bmax} = k_c(\frac{m \cos d \cos \delta_B + V_{2Bmax} \sin 2d \sin 2\delta_B}{k_c \cos^2 d \cos^2 \delta_B}) \qquad 7.4$$

However, it is important to note that if B scans in the opposite hemisphere

from A and the diurnal maximum is 12 hours out of phase with that from A, then $\phi' = \alpha_s$ is the condition for $V_{1Bmin} = -V_{1Bmax}$ and it is necessary to use minus the observed amplitude in the subsequent calculations. Thus, in what follows, negative values of amplitude denote intensity minima that may occur in the northern hemisphere when $\phi' = \alpha_s$.

We now substitute for m in equation 7.3 from equation 7.4. Thereby the declination **d** may be expressed solely in terms of the free-space amplitudes and associated asymptotic latitudes of viewing that are deducible from the harmonics obtained from detectors A and B. After substitution for m and rearrangement of terms we finally get, in terms of *free-space harmonics*,

$$\tan d = \frac{V_{1Bmax} \frac{\cos^2 \delta_A}{\cos \delta_B} - V_{1Bmax} \cos \delta_A}{4 V_{2Amax} (\sin \delta_B - \sin \delta_A)} \tag{7.5}$$

Now $$V_{1Bmax} = \frac{k_c}{k_{1B}} v_{1Bmax} \tag{7.6}$$

$$V_{1Amax} = \frac{k_c}{K_{1A}} v_{1Amax} \tag{7.7}$$

and $$V_{2Amax} = \frac{k_c}{K_{2A}} v_{2Amax} \tag{7.8}$$

Therefore, in terms of the *observed harmonics*,

$$\tan d = \frac{\frac{K_{2A}}{K_{1B}} \frac{\cos^2 \delta_A}{\cos \delta_B} v_{1Bmax} - \frac{K_{2A}}{K_{1A}} \cos \delta_A v_{1Amax}}{4 v_{2Amax} (\sin \delta_B - \sin \delta_A)} \tag{7.9}$$

(c) S E of estimate of tan d

We may write tan d as

$$\tan d = ax + by$$

where a and b are asymptotic constants, and

$$x = v_{1Bmax}/v_{2Amax}$$

and $$y = v_{1Amax}/v_{2Amax}$$

In respect of random fluctuations of intensity, v_{1Bmax} and v_{1Amax} may be regarded as independent Poisson variables and the variance of x may be calculated accordingly. It is not quite clear that y can be treated in the same way, being the quotient of the diurnal and semi-diurnal amplitudes from detector A. That is to say, it might be asked if there is any association between the amplitudes of the first and second harmonics of best fit to random values of the bi-hourly deviates. [A similar question arises earlier, in the analysis of variance of an individual harmonic $R_m = a_m \cos m\theta + b_m \sin m\theta$, where the harmonic coefficients a_m and b_m are regarded as being statistically independent (e.g. see section 4.10(c))]. Some indication should be given by Fourier analysing sets of random numbers. Ninety-nine sets of such numbers were formed by adding a constant large base value to random numbers (0 to 9) listed in Table 8 of "Cambridge Elementary Statistical Tables" (Lindley and Miller, 1953) and arranging them in groups of thirteen. Harmonic analyses were then carried out in the same way as for normal groups of bi-hourly intensity values. A scatter diagram of pairs of first and second harmonic amplitudes is shown in Figure 7.1. The result suggests that if there is an association between R_1

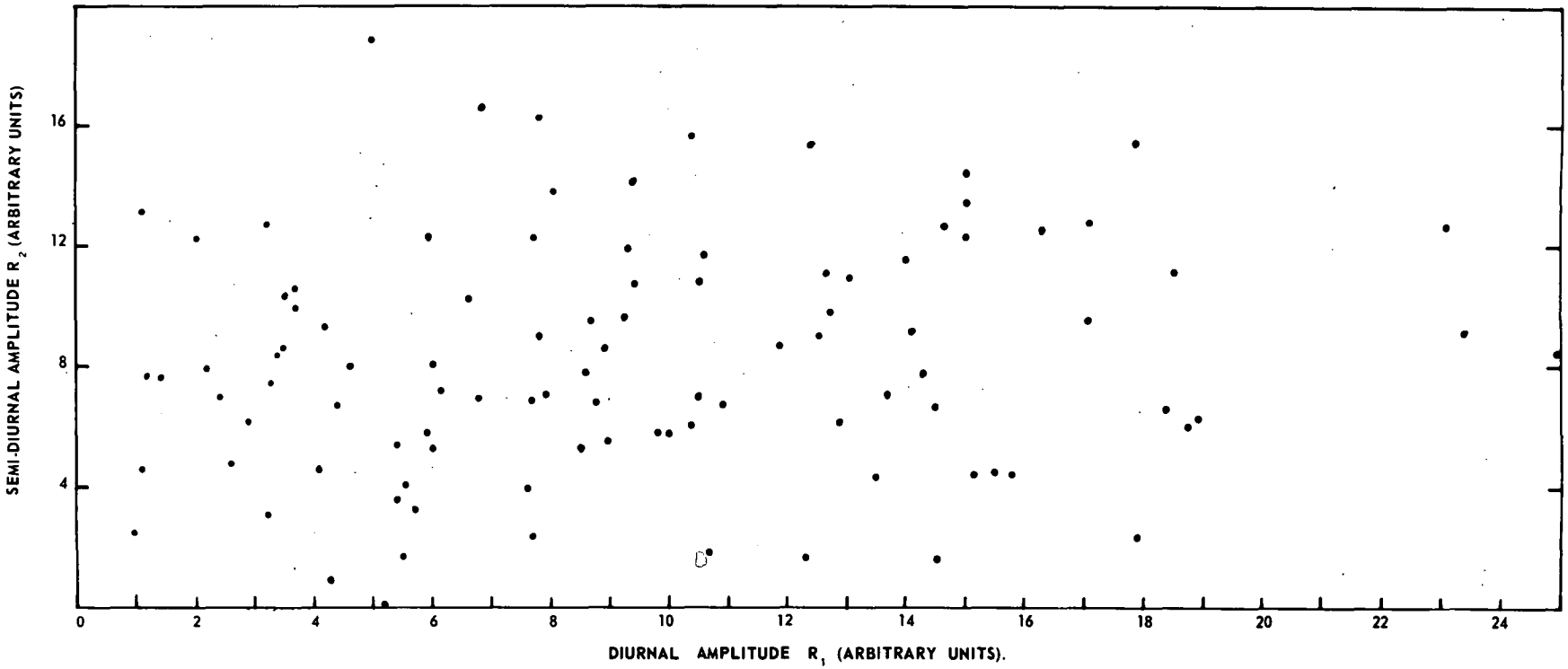

Figure **7.1** Scatter diagram of pairs of values of R_1 and R_2 from Fourier analysis of 99 independent groups of 12 randomnumbers, representing bi-hourly deviates.

and R_2 it must be very weak. Therefore it is assumed for the present that if v_{1Amax} and v_{2Amax} are treated as independent chance variables, in respect of random fluctuations of intensity, this will not lead to serious error in calculating the variance of their quotient, y.

Accordingly

$$\sigma(\tan d) = \pm \sqrt{a^2\sigma_x^2 + b^2\sigma_y^2} \qquad 7.10$$

where $$\sigma_x^2 = \frac{1}{(v_{2Amax})^4}\left[(v_{2Amax})^2\sigma^2(v_{1Bmax}) + (v_{1Bmax})^2\sigma^2(v_{2Amax})\right]$$

and $$\sigma_y^2 = \frac{1}{(v_{2Amax})^4}\left[(v_{2Amax})^2\sigma^2(v_{1Amax}) + (v_{1Amax})^2\sigma^2(v_{2Amax})\right]$$

The estimated variances of the observed amplitudes v_{1Amax}, v_{1Bmax} and v_{2Amax} are found by the method described in 4.10.

(b) Evaluation of d, using data from detectors A and B

The vertical semi-cubical telescopes at Hobart ($\delta \simeq -39^\circ$) and the vertical semi-cubical telescopes at Budapest ($\delta \simeq +43^\circ$) were chosen as detectors A and B respectively and the cubical telescope directed 30°N of the zenith at Hobart ($\delta \simeq -18^\circ$) was chosen as detector C.

Data from detector A were available from 1958 onwards, from detector B they were available for the years 1959 and 1961, and from detector C they were available for the years 1961 and 1962. Since there did not appear to be a marked trend in the amplitude observed with the vertical telescopes at Hobart between 1958 and 1962, it was decided that the greatly improved statistical accuracy gained by using all data that were available from 1958 to 1962

outweighed the advantage of using only the data from 1961, the one year when simultaneous observations could be obtained from all three sets of detectors.

Allowance was made for an apparent decrease in the cut-off rigidity R_c over the period, as deduced from observations of the solar diurnal variation (e.g. see section 6 B.2). However, as will be shown below, the value estimated for $\underline{d}$ depends only weakly on the value assigned to R_c.

The observational data and asymptotic constants that were required are listed in Table 7.1. The constants have been extracted from Table 6.1 and 6.2, the values for R_c = 85 GV having been obtained by graphing and interpolating.

TABLE 7.1

The observed amplitudes and asymptotic constants used in the determination of d, in respect of the detectors A (Hobart semi-cube, vertical), B(Budapest semi-cube, vertical) and C(Hobart cube, 30°N of zenith).

Detector		A	B	C
Period		1958-1962	1959;1961	1961-1962
Assumed R_c(GV)		85	85	70
$v_1$max%		0.0450	0.0233	0.0430
$v_2$max%		0.0080	0.0055	0.0240
SE%		±0.0025	±0.0027	±0.0080
γ=0	K_1	0.620	0,620	0.756
	K_2	0.370	0.370	0.656
	δ	-39°	+43°	-18°
γ=2	$K_1 \times 10^6$	20	13	28
	$K_2 \times 10^6$	13	13	28
	δ	-40°	+44°	-17°

The estimate for $\gamma = 0$

Equation 7.9 yields the following value for tan d, based on the constants derived from Table 6.1 and the observed amplitudes v_{1Amax}, v_{1Bmax} and v_{2Amax}:

$$\tan d = -0.7710$$

From equation 7.10, the S.E. of estimate is

$$6(\tan d) = \pm 0.1822$$

Therefore the estimate of $\underline{d}$ from the vertical observations at Hobart and Budapest, for $\gamma = 0$, is

$$d = -37.6 \pm 7^{o}$$

The estimate for $\gamma = -2$

Using the observed amplitudes and the appropriate constants derived from Table 6.2, it is found that

$$d = -38.7 \pm 7^{o}$$

Table 7.2 shows that determinations of $\underline{d}$ are not seriously affected by the choice of γ and R_c, for values of γ in the range 0 to -2, except for the combination of negative γ and a value of R_c close to the absolute primary cut-off for detection underground. On present evidence of the influence of the interplanetary magnetic field such a low value for R_c must be regarded as being unlikely.

TABLE 7.2

Estimates of the declination of the anisotropy, in the southward direction, versus threshold rigidity R_c, for $\gamma = 0$ and $\gamma = -2$. The determinations are based on the amplitudes of the harmonics of the sidereal daily variations observed in the vertical direction at Hobart (1958-1962) and at Budapest (1959, 1961), and on the constants of response given in Tables 6.1 and 6.2.

	THRESHOLD RIGIDITY, R_c(GV)					
	20	50	70	85	100	200
Estimated d ($\gamma = 0$)	-39°	-38°	-38°	-38°	-38°	-38°
Estimated d ($\gamma = -2$)	-48°	-40°	-38°	-39°	-39°	-39°

Aside from errors of measurement, the estimated value of d is clearly defined by the relationship between the amplitudes observed with detectors A and B. This may be shown by calculating v_{1Bmax} as a function of d for an observed pair of values of v_{1Amax} and v_{2Amax}. By re-arranging equation 7.9, v_{1Bmax} (specified for $\phi' = \alpha_s$) is expressed as

$$v_{1Bmax} = \frac{\cos \delta_B}{\cos \delta_A} \left\{ \frac{K_{1B}}{K_{1A}} v_{1Amax} + \frac{4K_{1Bmax}}{K_{2Amax}} \frac{\sin \delta_B - \sin \delta_A}{\cos \delta_A} \tan d \; v_{2Amax} \right\} \quad 7.11$$

In terms of the vertical observations at Hobart and for ($\gamma = 0$, $R_c = 85$ GV) equation 7.11 gives

$$v_{1Bmax}(\%) = 0.042 + 0.085 \tan d \quad 7.12$$

The curve v_{1Bmax} versus d shown in Figure 7.2 is specified by equation 7.12. Its intersection with the dashed line representing the observed value of v_{1Bmax} in the vertical direction at Budapest gives the estimated

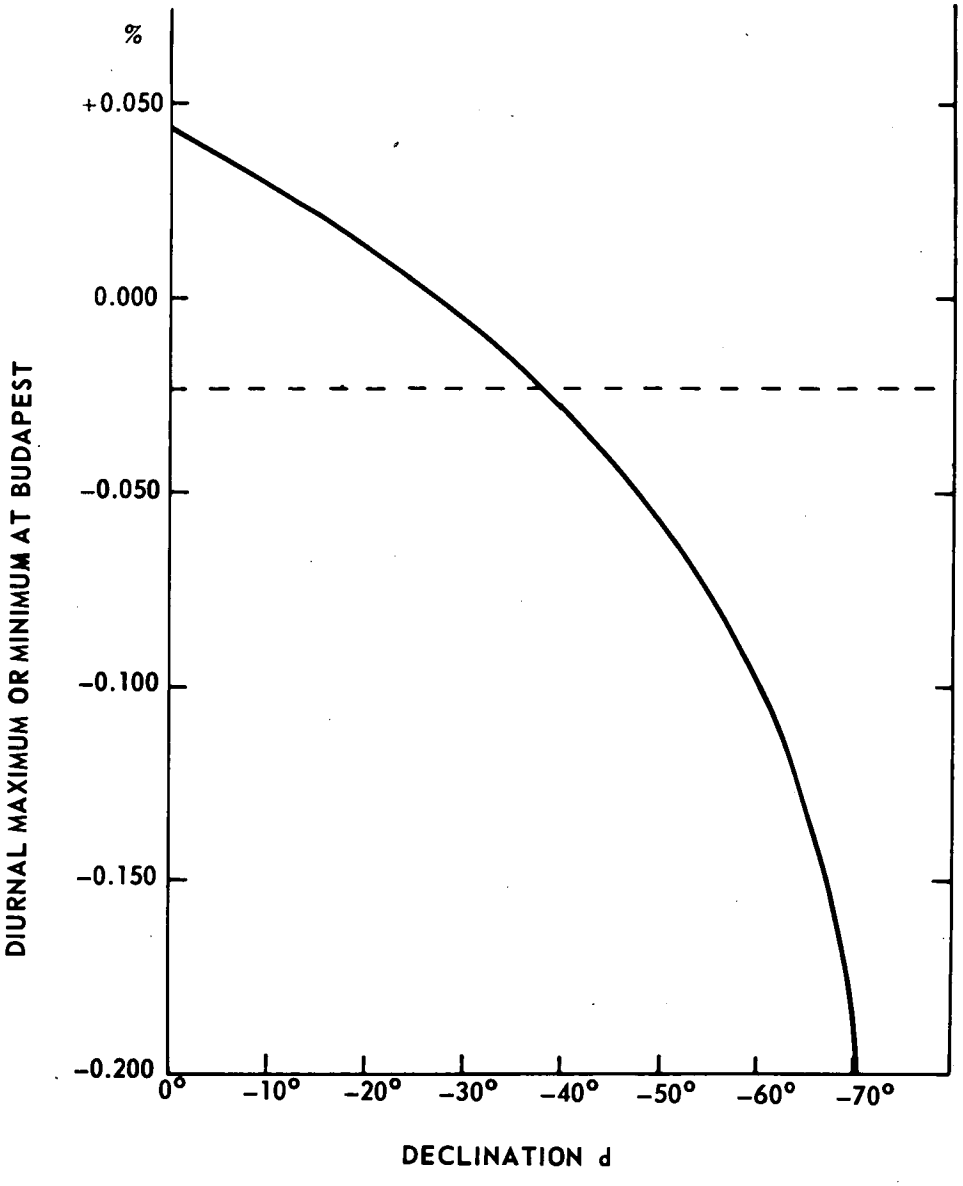

Figure **7.2** The sidereal diurnal maximum or minimum that would be observed in the vertical direction at Budapest simultaneously with the sidereal diurnal maximum in the vertical direction at Hobart, calculated from the observed amplitude at Hobart, 1958–1962, as a function of the (southward) declination of the two-way anisotropy, for $\gamma = 0$. The dashed line represents the diurnal minimum which occurred at Budapest at the same sidereal time as the maximum observed at Hobart.

declination $d = -38^{\circ}$. Because of the presence of the term $K_{1B}\, m \cos d \cos \delta_A \cos(\phi - \alpha_s)$ representing streaming, it can be seen from the figure that if d had been smaller than approximately -27° the observed diurnal variation at Budapest would have been in phase with the diurnal variation at Hobart.

(e) Test of the validity of the estimate of d, using detector C

Having obtained an estimate of d from detectors A and B one may calculate the diurnal and semi-diurnal amplitudes that should be observed with detector C, scanning a different asymptotic strip of latitude. Agreement with the observations from C would indicate that, within limits of error of measurement, the model for the anisotropy was basically the correct one and would constitute supporting evidence that there was not an important spurious contribution to the observed sidereal effect.

The underground cubical telescope inclined 30°N of zenith at Hobart was well suited for use as detector C. As Table 7.1 shows, the telescope exhibits a very good response to first and second harmonics of the anisotropy, while the free-space diurnal and semi-diurnal amplitudes would be expected to be relatively large at the asymptotic latitude of viewing $\delta \simeq -18^{\circ}$.

The amplitudes v_{1Cmax} and v_{2Cmax}, relating to detector C, have to be expressed in terms of the estimated declination d, the amplitudes observed with one of the detectors A and B (we choose A) and the appropriate asymptotic constants. Equation 7.11 is therefore used for the diurnal amplitude and we have

$$v_{1Cmax} = \frac{\cos \delta_C}{\cos \delta_A} \left\{ \frac{K_{1C}}{K_{1A}} v_{1Amax} + \frac{4K_{1C}}{K_{2A}} \frac{\sin \delta_C - \sin \delta_A}{\cos \delta_A} \tan d \; v_{2Amax} \right\} \quad 7.13$$

The expression for the semi-diurnal amplitude v_{2Cmax} is obtained by substituting $\frac{k_C}{K_2} v_{2max}$ for the free-space amplitude V_{2max} in equation 7.2, so that

$$v_{2Amax} = K_{2A} \frac{n}{2} \cos^2 d \cos^2 \delta_A$$

and $$v_{2Cmax} = K_{2C} \frac{n}{2} \cos^2 d \cos^2 \delta_C$$

therefore $$v_{2Cmax} = \frac{K_{2C}}{K_{2A}} \frac{\cos^2 \delta_C}{\cos^2 \delta_A} v_{2Amax} \quad 7.14$$

It is noted that the relationship between the semi-diurnal amplitudes observed at two different latitudes is independent of d.

The expressions for v_{1Cmax} and v_{2Cmax} given by equations 7.13 and 7.14 were evaluated for $\gamma = 0$ and $\gamma = -2$, using the appropriate constants from Table 7.1. Table 7.3 shows that the observed and calculated amplitudes were in close agreement, particularly for the case $\gamma = 0$. The discrepancy when $\gamma = -2$ is not significant and from considerations of counting rate statistics alone it could not be said that one index was to be preferred to the other.

TABLE 7.3

Observed and calculated values of sidereal, diurnal and semi-diurnal amplitudes relating to the north-pointing cubical telescope (detector C). The calculated values are derived from the observations with detectors A and B, for the cases $\gamma = 0$ and $\gamma = -2$.

	Observed	Calculated ($\gamma = 0$)	Calculated ($\gamma = -2$)
$v_{1Cmax}(\%)$	0.043 ± 0.008	0.042	0.050
$v_{2Cmax}(\%)$	0.024 ± 0.008	0.021	0.027

(f) The latitude dependence of the free-space diurnal amplitude

A more comprehensive variant of method (e) of testing the validity of the estimate of d comes from the determination of the free-space amplitude $V_{1\ell max}$ as a function of asymptotic latitude of viewing δ_ℓ, based on the estimate of d which satisfies observations from detectors A and B, for given values of γ and R_c.

Substituting the variable subscript ℓ for B in equation 7.5 and rearranging, we obtain

$$V_{1\ell max} = \frac{\cos \delta_\ell}{\cos \delta_A} \{V_{1Amax} + 4 \tan d \frac{\sin \delta_\ell - \sin \delta_A}{\cos \delta_A} V_{2Amax}\} \qquad 7.15$$

Consider the solution $d = -38.5^\circ$, for $\gamma = 0$ and $R_c = 85$ GV, presented in 7(d) above. Having derived

$V_{1Amax} (= \frac{v_{1amax}}{K_{1A}})$ and $V_{2Amax} (= \frac{v_{2a\ max}}{K_{2A}})$ from Table 7.1,

we use equation 7.15 to obtain the curve $V_{1\ell max}$ versus δ_ℓ. Naturally V_{1Amax} (Hobart vertical) and V_{1Bmax} (Budapest vertical) must give two points on the curve, at latitudes δ_A and δ_B respectively. Now, to test the validity of the model for the anisotropy, and of the estimate of $\underline{d}$, we determine V_{1Cmax} from observations with the north-pointing cubical telescope at Hobart and note whether the result agrees with the prediction of the latitude-curve for the appropriate asymptotic latitude of viewing.

In Figure 7.3(a) it can be seen that the point representing V_{1Cmax} does in fact fall on the latitude curve that derives from the observations with the semi-cubical telescopes at Hobart and Budapest. It is noted that the free-space amplitude deduced from observations with the south-pointing telescope also falls on the curve at $\delta_S = -55^o$, although the statistical error is much too large for this result to be of significance.

The dashed line in Figure 7.3(a) is the latitude-curve which conforms with V_{1Amax} (Hobart vertical) for the declination $d = -66^o30'$. Since the coordinates of the normal to the plane of the ecliptic, looking south, are RA = 0600 and $d = -66^o30'$, this direction was thought to be worth examining because of its Right Ascension. However, it is clear from the figure that as an axis for the anisotropy the normal to the ecliptic would be quite inconsistent with the observations from Budapest and from the north-pointing cubical telescope at Hobart. In fact, the two curves serve to illustrate

(a) that the declination of fit can be rather precisely determined,

for given values of γ and R_c, if sufficiently accurate observations are available from several well-spaced latitudes of viewing; and (b) that compatibility between the free-space amplitudes would be unlikely if the amplitude deduced from any one of the detectors was based on a false observation.

Figures 7.3 and 7.4 show that the estimate of $\underline{d}$ and the compatibility between amplitudes V_{1Amax}, V_{1Bmax} and V_{1Cmax} does not depend at all strongly on the values assigned to γ and R_c, at least in the ranges 0 to -2 for γ and 85 - 200 GV for R_c.

(g) The latitude dependence of the free-space semi-diurnal amplitude

On the basis of the model for the anisotropy it is evident from equation 7.14 that the ratio $\frac{V_{2\ell max}}{\cos^2\delta_\ell}$ is constant. Consequently, if V_{2Amax} is estimated from observations with detector A, for a pair of values of γ and R_c, the free-space amplitude at any other latitude of viewing may be determined without requiring knowledge of the declination $\underline{d}$. Using the semi-diurnal amplitude observed at Hobart with the vertical semi-cubical telescopes (detector A) as the basic observation, we have

$$V_{2\ell max} = \frac{\cos^2\delta_\ell}{\cos^2\delta_A} \quad V_{2Amax} \qquad 7.16$$

The latitude curves in Figure 7.5 have been calculated so as to conform with V_{2Amax} for the pairs of values of γ and R_c that are indicated. It is clear that the curves also conform with V_{2Bmax} and V_{2Cmax}. Thus, within the errors of the measurements, it is evident that the three most accurate semi-

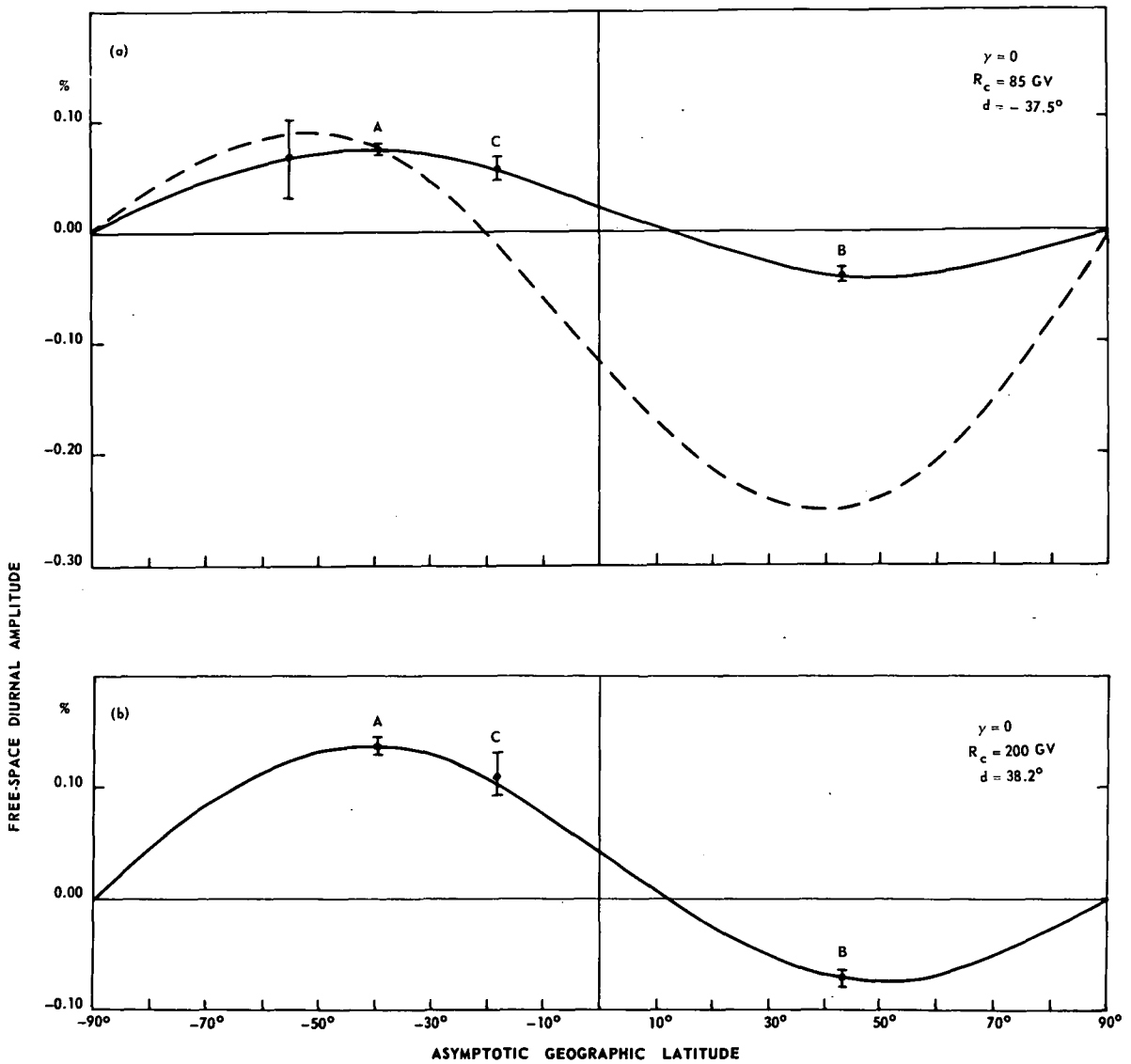

Figure **7.3** Sidereal anisotropy. Dependence of free-space diurnal amplitude on asymptotic latitude of viewing, calculated for the value of the declination d which conforms with the estimated amplitudes A and B, when

(a) $\gamma = 0$, $R_c = 85$ GV

(b) $\gamma = 0$, $R_c = 200$ GV

The dashed line in (a) is calculated for $d = -66°30$, and so as to conform with the estimated amplitude A.

A. – Hobart vertical, 1958 – 1962

B. – Budapest vertical, 1959 + 1961

C. – Hobart 30°N, 1961 + 1962

Unlettered – Hobart 45°S, 1961 + 1962.

Standard error tails are shown.

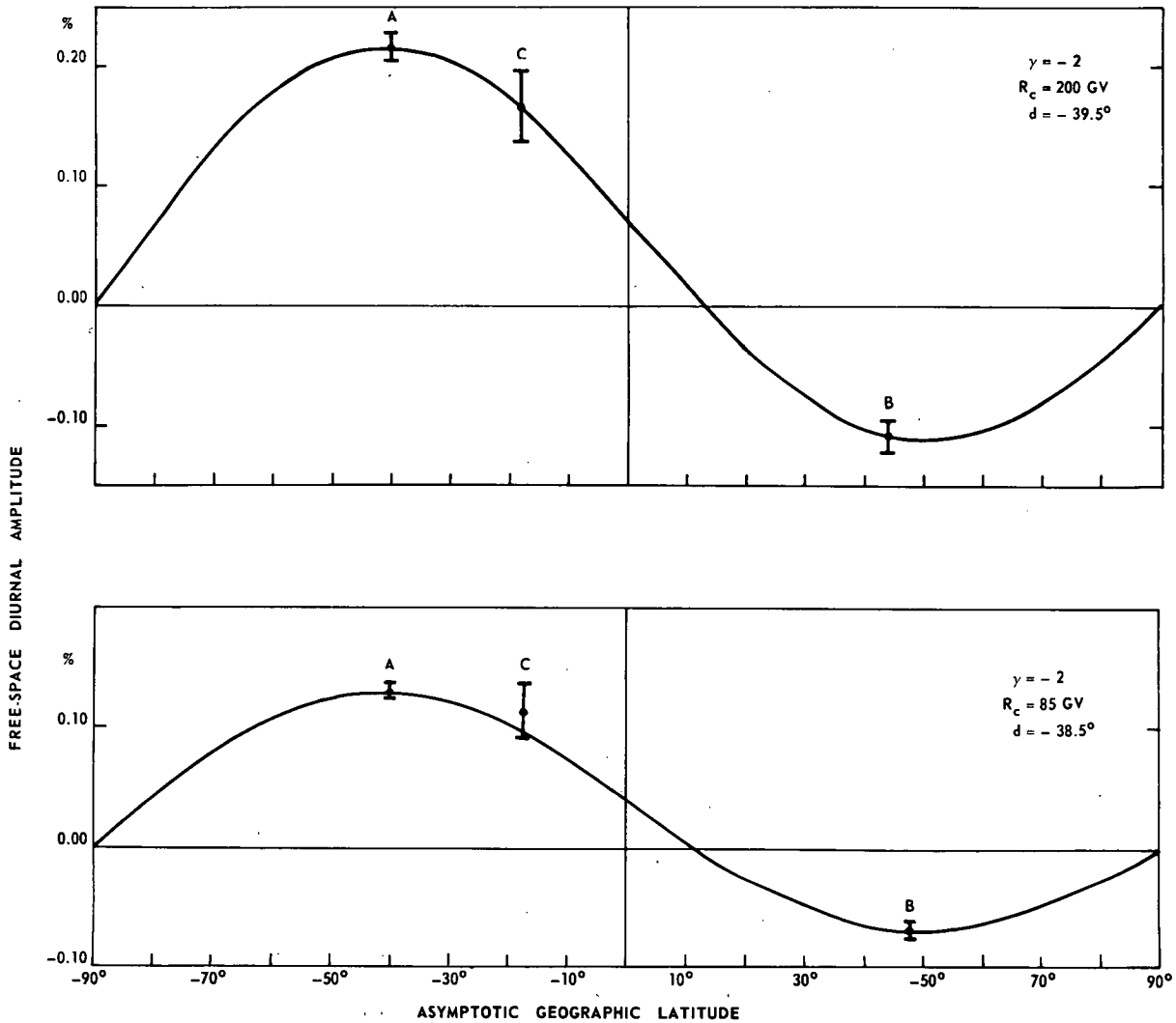

Figure **7.4** Sidereal anisotropy. Dependence of free-space diurnal amplitude on asymptotic latitude of viewing, calculated for the value of the declination d which conforms with the estimated amplitudes A and B, when (a) $\gamma = -2$, $R_c = 200$ GV
(b) $\gamma = -2$, $R_c = 85$ GV

Standard error tails are shown.

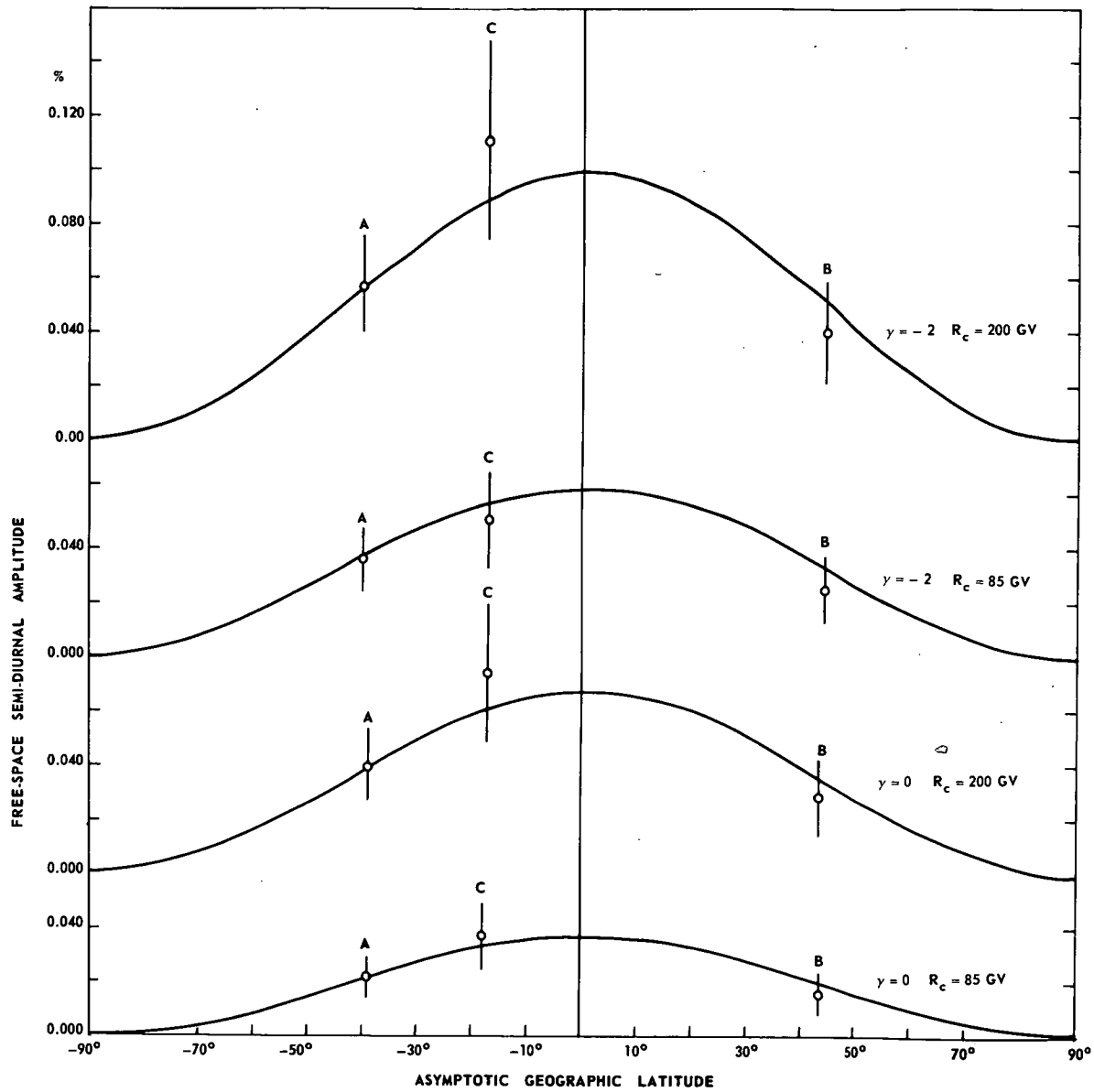

Figure **7.5** Sidereal anisotropy. Dependence of free-space semi-diurnal amplitude on asymptotic latitude of viewing, calculated for the values of γ and R_c shown, so as to conform with the estimated amplitude A. Standard error tails are shown.

diurnal amplitudes derived from the observations underground are compatible with the model for the anisotropy. The free-space amplitude obtained from observations with the south-pointing telescope is not shown. Although anomalously large, it is not significantly different from zero.

From analysis of amplitudes of the second harmonic, it cannot be said that any one value of γ or R_c is to be preferred, within the ranges considered.

7.2 The estimated Right Ascension (R.A.)

The RA of the anisotropy in the southward direction beyond the earth's field region is estimated from the time of maximum of the sidereal daily variation observed in the vertical direction underground at Hobart, averaged over the eight years 1958-1965 (Table 5.6) and from the phase displacements ϕ_1 (diurnal component) and ϕ_2 (semi-diurnal.

Calculated values of ϕ_1 versus R_c and γ that are listed in Tables 6.1 and 6.2 for the vertical telescopes are seen to be practically the same as the corresponding values of ϕ_2. Further, the observed first and second harmonics in the vertical direction are in phase, within the errors of observations, and therefore on the basis of the model are in accord with the calculations of ϕ_1 and ϕ_2. Consequently the time of maximum, 0600, of the sum of harmonics is taken as the best estimate of the phase at Hobart and from this the RA of the anisotropy versus R_c and γ is determined by application of

the phase displacement $\phi_1 (\sim \phi_2)$. The result is shown in Figure 7.6. It is clear that the dependence of the estimate on the nature of the variation spectrum only becomes important for threshold rigidities below about 100 GV. In lieu of a better criterion, the choice of a value for R_c shall for the present be centred on the upper limiting rigidity of the solar anisotropy, R_u. The RA would then be 0640 with an uncertainty of perhaps a quarter of an hour depending on the range of values within which R_c and γ might reasonably be thought to lie.

Much of the error of estimate would seem to come from a year to year variability in the time of maximum of the observed sidereal daily variation. The phase of the smoothed annual average daily variation shown in Figure 5.10 exhibits a fluctuation of approximately ± 2 hours about the average, somewhat in excess of the variation to be expected from count rate statistics. Therefore it would seem fair to assign to the eight year average an uncertainty of approximately ± 1 hour, accruing from statistical and quasi-statistical fluctuations. Thus only a rough figure can be given for the error of estimate of RA, and it is suggested that this would be ± 1.25 hours, noting that some of this might arise from genuine fluctuations in the direction of the anisotropy as specified outside the region of the earth's field.

7.3 The Direction of the Anisotropy and of the Galactic Magnetic Field

The coordinates of the axis of the anisotropy in the southward direction

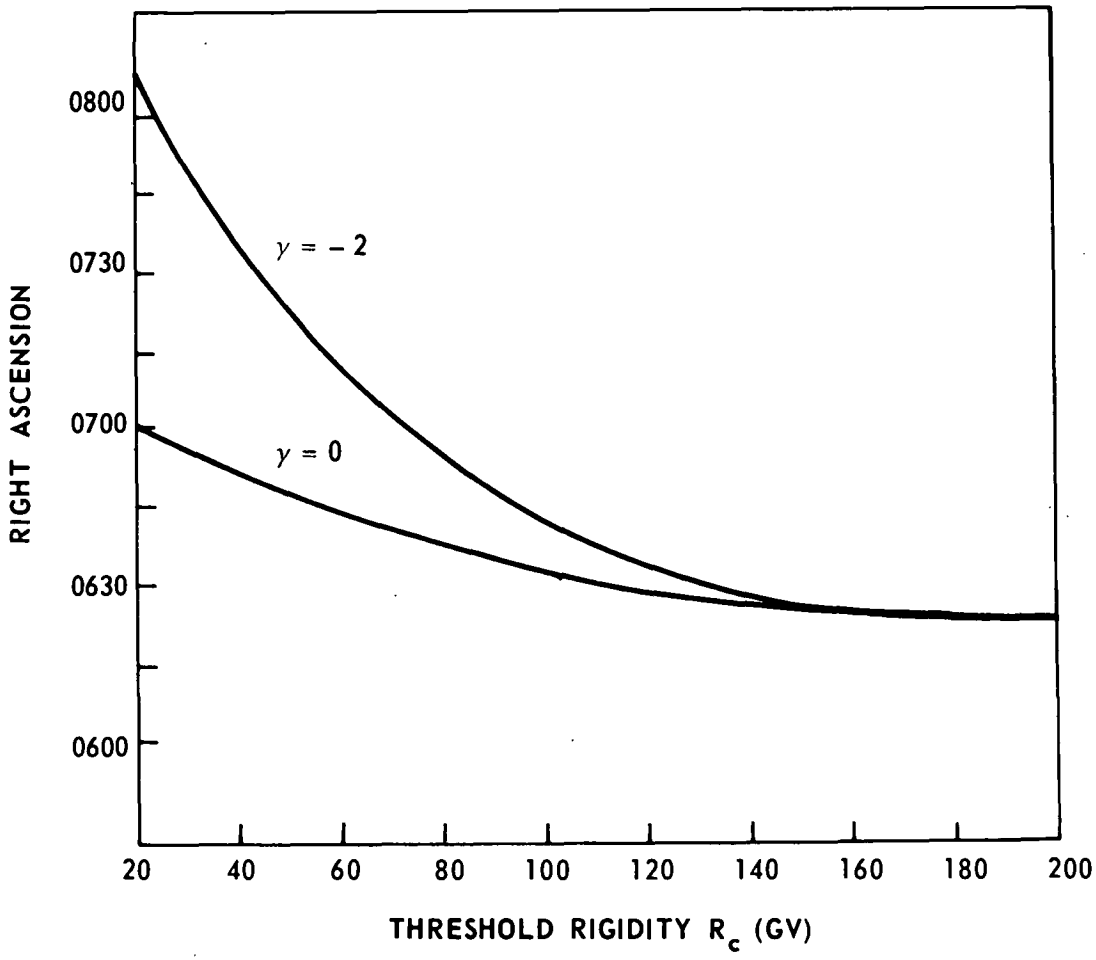

Figure **7.6** The estimated right ascension of the two-way anisotropy, versus threshold rigidity, for two values of the index of the variation spectrum.

are provisionally estimated to be (RA = 0640 $\pm$ 0115, Dec. = -38 $\pm$ 7°). The corresponding galactic longitude and latitude coordinates in the new system (Blaauw et alia, 1959) would be approximately (ℓ^{II} = 245°, b^{II} = -15°). In the northward direction of the axis the coordinates would be (RA = 1840, Dec. = +38°; and ℓ^{II} = 65°, b^{II} = +15°).

In Chapter I reference was made to the optical and radio measurements, which suggest that there is a relatively uniform galactic magnetic field in the vicinity of the solar system, lying approximately in the direction of the local spiral arm, towards ℓ^{II} = 70°, b^{II} = 0 in the southward, or outward, direction. Table 7.4 lists some of the most recent determinations of the direction of the local galactic magnetic field by radio and optical methods. The optical polarization measurements (except those of Behr) give the direction of the field within about 3 kiloparsecs (kpc) of the sun, while the radio determinations appear to refer to much closerregions. Notably, the regions of polarized synchrotron emission are thought to be located at distances of perhaps only 100 to 200 parsecs from the sun (Mathewson and Milnes, 1965).

TABLE 7.4

Recent optical and radio determinations of the galactic longitude (new system) of the local spiral arm field. Galactic latitude $b^{II} \simeq 0^{o}$ in all cases.

		Longitude (ℓ^{II})
Optical polarization of starlight		
Northern hemisphere		
Stars > 600 parsecs from sun	Serkowski (1962)	230° - 50°
Stars < 250 parsecs from sun	Behr (1964)	242° - 62°
Southern hemisphere	Smith (1956)	240° - 60°
Faraday rotation of source polarization		
	Gardner and Davies (1966)	275° - 95°
Linear polarization of synchrotron emission		
	Mathewson and Milne (1965)	250° - 70°

Arising largely out of these determinations, opinion now seems to favour a local galactic field which preserves a uniform direction out to large distances from the sun. The direction appears to be, within about 20 degrees, approximately parallel with the local spiral structure as outlined by the concentrations of H II regions, OB stars and galactic clusters (e.g. Becker, 1964) although it is not clear that this is coincident with the directions given by the distribution of neutral hydrogen (e.g. Beer, 1961; Kerr, 1966).

In Figure 7.7 the apparent difference between the direction of the galactic magnetic field and of the cosmic ray anisotropy is shown on a celestial-galactic coordinate grid. The estimated northward and southward directions of

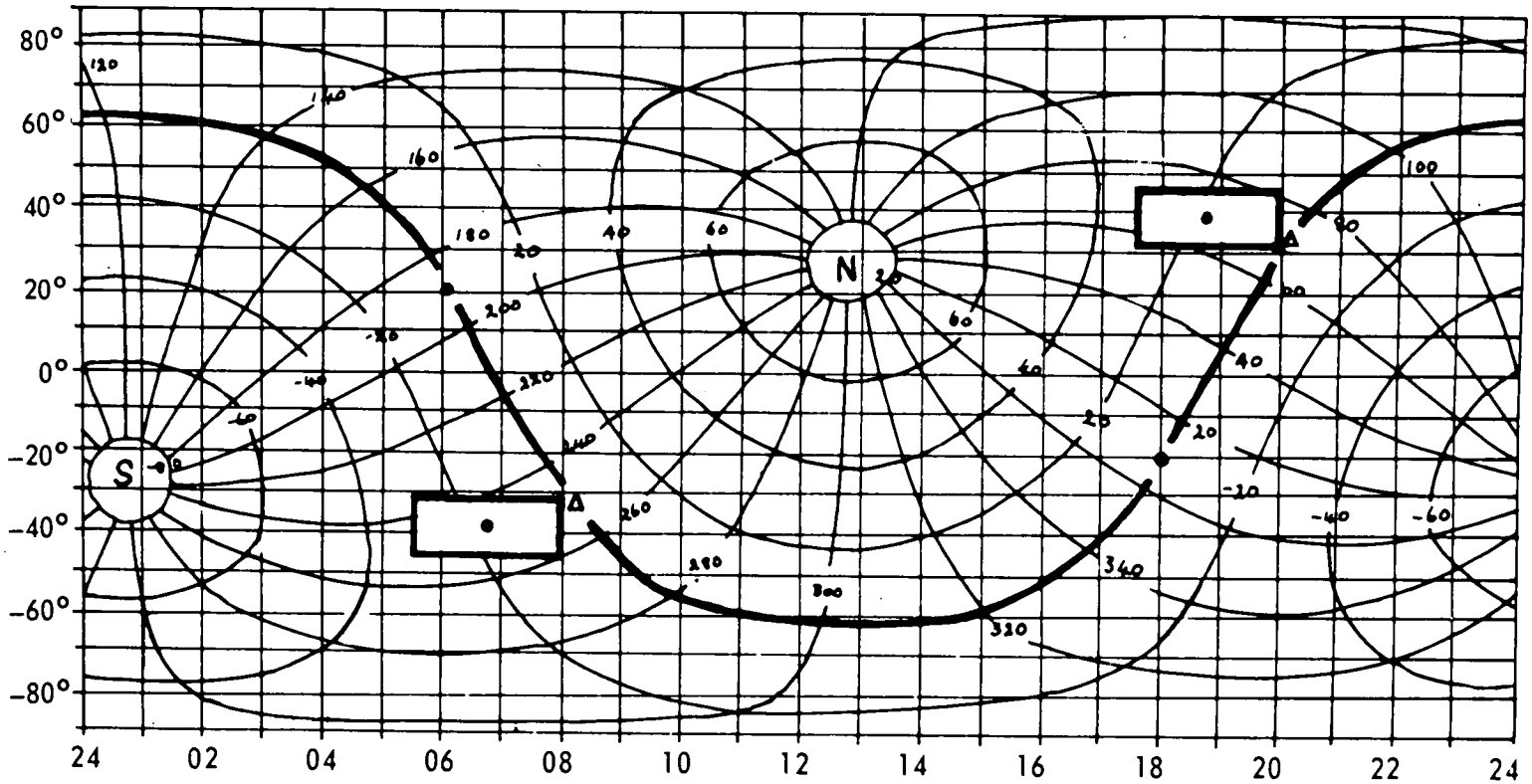

Figure **7.7** The estimated direction [•] of the proposed cosmic ray anisotropy compared with a representative estimate, from optical and radio measurements, of the direction Δ of the local galactic magnetic field. The directions are referred to the new system of galactic coordinates (latitude b^{II}, longitude l^{II}) superimposed on the celestial grid (declination d, right ascension a).

the anisotropy, positioned within approximate zones of uncertainty, may be compared with a representative direction ($\ell^{II} = 70^{o}$, $b^{II} = 0^{o}$) of the field. Because of the uncertainties of measurement it cannot be said at present that there is a significant difference.

The estimated direction of the anisotropy is consistent with the propagation of excess fluxes of primaries with steep helices along the field, since the maxima of the diurnal and semi-diurnal components coincide (Table 1.1). If the anisotropy does originate in the galactic field, as its type and direction suggest, and if a real difference is established between the cosmic ray and the other estimates of direction, two possibilities suggest themselves:

(1) The direction of the galactic field immediately beyond the solar system may not be the same as that derived from measurements extending over relatively vast distances remote from the sun.

(2) The interplanetary magnetic field may produce a distortion of direction of the anisotropy, of a type that would be evident in annual averaged observations.

CHAPTER 8

THE AMPLITUDE OF THE ANISOTROPY, THE AMPLITUDE OF RESPONSE AT GROUND LEVEL AND CONCLUDING REMARKS

8.1 The Estimated Amplitudes of the Components of the Anisotropy

(a) Derivation

The proposed description of the anisotropy in the frame of reference in which it is fixed (section 6.A1) gives the differential intensity maxima in opposite directions as

$$A_{IR}\, I_{OR} = \{1 + \frac{R^{\gamma}}{3}(2n + 3m)\}\ I_{OR}$$

and $$A_{2R}\, I_{OR} = \{1 + \frac{R^{\gamma}}{3}(2n - 3m)\}\ I_{OR}$$

The amplitude of the integrated intensity maximum in the A_1 direction, expressed as a fraction of the total intensity, is therefore (for the non-positive values of γ considered here)

$$\frac{\Delta I_1}{I_o} = \frac{2n + 3m}{3} \quad \frac{\int_{R_c}^{\infty} R^{\gamma} I_{OR}\, dR}{\int_{R_c}^{\infty} I_{OR}\, dR}$$

$$= k_c \frac{2n + 3m}{3} \qquad 8.1$$

Similarly, in the opposite direction, where streaming effects a reduction of intensity,

$$\frac{\Delta I_2}{I_o} = k_c \frac{2n - 3m}{3} \qquad 8.2$$

It can be seen then that the contribution from streaming is $S = mk_c$ and from the symmetrical two-way anisotropy is $T = \frac{2n}{3} k_c$.

Now, the free-space diurnal amplitude derived from observations with detector A is (equation 6.7)

$$V_{1A_{max}} = k_c\, m \cos d \cos \delta_A + k_c \frac{n}{2} \sin 2d \sin 2\delta_A \qquad 8.3$$

Equation 7.19 may be conveniently expressed as

$$V_{1A_{max}} = X \cos \delta_A + Y \sin 2\delta_A \qquad 8.4$$

where X is the (equatorial) maximum of the latitude-dependent diurnal amplitude term due to streaming, and Y is the corresponding (mid-latitude) maximum amplitude due to the two-way diurnal component. Given X and Y, the anisotropic intensity maxima are

$$\text{streaming:} \quad S = \frac{X}{\cos d} \qquad 8.5$$

$$\text{two-way:} \quad T = \frac{4Y}{3 \sin 2d} \qquad 8.6$$

To determine the two component maxima of the anisotropy as functions of statistically independent variables, it is necessary to estimate the declination $\underline{d}$ from two of the detectors (A and B) and employ that value of $\underline{d}$ in the estimations of S and T from observations with a third detector (C).

The amplitude terms X and Y appear quite simply by suitably rearranging equation 7.15. In fact, it is the most convenient way of determining the total diurnal amplitude. From the equation we obtain

$$X = \frac{1}{\cos \delta_C} V_{1Cmax} - \frac{4 \tan \delta_C}{\cos \delta_C} \tan d.V_{2Cmax} \qquad 8.7$$

and $$Y = \frac{2}{\cos^2 \delta_C} \tan d.V_{2Cmax} \qquad 8.8$$

The two components of the latitude curve of the free-space *diurnal* amplitude are shown, with their maxima X and Y, in Figure 8.1 for the case $\gamma = 0$ and $R_C = 85$ GV. The sum curve is the same as that shown in Figure 7.3(a).

It follows from the equations above that the streaming and two-way components of the intensity maxima in the frame of reference of the anisotropy are

$$S = \frac{X}{\cos d} = \frac{1}{\cos \delta_C} \frac{V_{1Cmax}}{\cos d} - \frac{4 \tan \delta_C}{\cos \delta_C} \frac{\tan d}{\cos d} V_{2Cmax} \qquad 8.9$$

and $$T = \frac{4Y}{3 \sin 2d} = \frac{8}{6 \cos^2 \delta_C} \frac{1}{\cos^2 d} V_{2Cmax} \qquad 8.10$$

Treating S as a function of the independent chance variables V_{1Cmax}, V_{2Cmax} and d, we have

$$\sigma^2(S) = \left(\frac{\partial S}{\partial (V_{1Cmax})}\right)^2 \sigma^2(V_{1Cmax}) + \left[\frac{\partial S}{\partial (V_{2Cmax})}\right]^2 \sigma^2(V_{2Cmax}) + \left(\frac{\partial S}{\partial d}\right)^2 \sigma^2 d \qquad 8.11$$

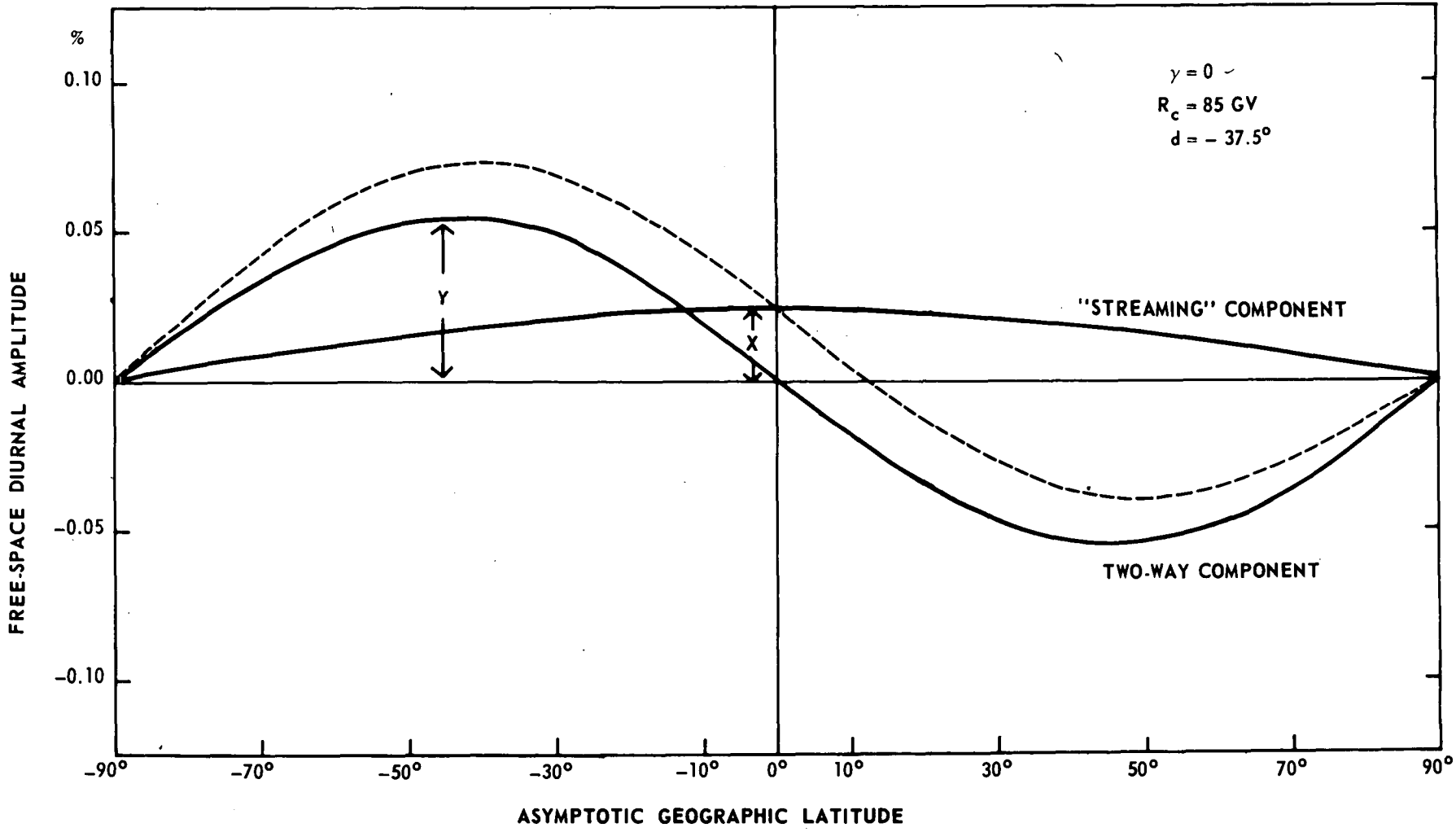

Figure **8.1** Sidereal anisotropy. Dependence of free-space diurnal amplitude (dashed line) on asymptotic latitude of viewing, reproduced from Figure **7.3 (a)** and showing the two diurnal components predicted by the model, the dashed line representing their sum.

Thus $$\sigma^2(S) = \left(\frac{1}{\cos\delta_C \cos d}\right)^2 \sigma^2(V_{1Cmax}) + \left(\frac{4\tan\delta_C \tan d}{\cos\delta_C \cos d}\right)^2 \sigma^2(V_{2Cmax})$$

$$+ \left[\frac{1}{\cos\delta_C \cos^2 d} V_{1Cmax} + \frac{4(2-\cos^2 d)\tan\delta_C}{\cos^3 d \cos\delta_C} V_{2Cmax}\right]^2 \sigma^2 d \qquad 8.12$$

Similarly,

$$\sigma^2(T) = \left(\frac{8}{6\cos^2\delta_C \cos^2 d}\right)^2 \sigma^2(V_{2Cmax}) + \left(\frac{4}{\cos^2\delta_C \cos^3 d} V_{2Cmax}\right)^2 \sigma^2 d \qquad 8.13$$

(b) Estimation

The declination was estimated from the sidereal diurnal and semi-diurnal amplitudes observed in the vertical direction at Hobart and Budapest (detectors A and B). The component intensity maxima S and T were then estimated from observations with the north-pointing cubical telescope (detector C) using the value of $\underline{d}$ obtained from detectors A and B.

The results of the computations are listed in Table 8.1 for different values of γ and R_C.

TABLE 8.1

Estimates of the component sidereal intensity maxima S and T in the frame of reference of the anisotropy outside the earth's field region, for different values of γ and R_C. The estimates derive from observations in the vertical direction at Hobart (1958-1962) and Budapest (1959,1961) and in the direction 30°N of zenith at Hobart (1961-1962). The errors shown are the S.Es of estimate.

		$\gamma = 0$		$\gamma = -2$	
	R_C(GV)	85	200	85	200
Streaming maximum % ($S = mk_c$)		0.027 ±0.022	0.056 ±0.043	0.053 ±0.043	0.076 ±0.068
Two-way maximum% ($T = 2/3\ nk_c$)		0.086 ±0.047	0.175 ±0.096	0.179 ±0.096	0.272 ±0.150
Total maximum, northward direction (T+S)%		0.059	0.119	0.126	0.196
Total maximum, southward direction (T-S)%		0.113	0.231	0.232	0.348

None of the estimates of anisotropic intensity maxima listed in the Table differ from zero by as much as two standard errors. It is clear that a very large increase in counting rate is needed if a reliable estimate of the contribution due to streaming is to be found. However, it is interesting to note that about 95% of $\sigma^2(S)$ is due to $\sigma^2(V_{1Cmax})$ and $\sigma^2(V_{2Cmax})$. On the other hand, about 65% of $\sigma^2(T)$ is due to $\sigma^2(d)$. Taking these factors into account, it would seem from Table 8.1 that an increase in counting rate with cubical telescopes in the north-pointing direction by a factor of ten, to $\sim$ 170,000 pcles/hour, would reduce $\sigma(S)$ by a factor of about 2.6 and give a significant result for S over a biennial period. At the same time, $\sigma(T)$

would be reduced by a factor of only 1.2 and this should give a barely significant result for the two-way component. If, together with the increase in count rate in the north-pointing direction at Hobart, the counting rates of each of the vertical semi-cubical arrays A and B were increased by a factor of two, the reduction factor for $\sigma(T)$ would increase to about 1.7, although $\sigma(S)$ would not be appreciably altered. In terms of a biennial period, the actual counting rates required from the semi-cubical vertical arrays at each of the two latitudes of viewing would be $\sim$ 350,000 pcles/hour.

On the basis of the model and of the observations that have been used here, it seems unlikely that the intensity maximum due to streaming would have exceeded approximately 0.2% outside the earth's field region or that the two-way maximum would have exceeded 0.6%, if the actual values of γ and R_C lay somewhere in the ranges considered. The corresponding upper limits beyond the interplanetary field would be expected to be higher, because of the smoothing of amplitudes effected by the field.

8.2 The Relative Response at Ground Level

The procedures for determining the asymptotic cones of acceptance for underground meson telescopes were applied to the determination of the cones of acceptance of a vertical cubical telescope at ground level, Hobart, using the coupling coefficients of Webber and Quenby (Figure 6.6). Integrated constants of response analogous to those listed in Tables 6.1 and 6.2 were computed. From these, the relative amplitudes of response $\frac{\text{sea-level}}{\text{underground}}$ could be calculated

in respect of a sidereal anisotropy, for different values of γ and R_C. The calculations were made for three values of threshold rigidity (50, 70 and 100 GV) and for γ = +1, 0, -2 and -4. The resulting curves of fit (Figure 8.2) give an approximate indication of the relative diurnal and semi-diurnal amplitudes of response.

It appears from the calculations that unless γ is negative and the anisotropy is observable at a very low threshold rigidity, the amplitudes of the harmonics at sea level should be $\lesssim$ 50% of the amplitudes underground. If γ = +1, amplitudes at sea level must be less than 20% of the amplitudes underground, i.e. actual values would be typically less than 0.01%.

Averaged over the five years 1958-1962, the observed harmonic components from the telescope at ground level were:

diurnal amplitude 0.035 ± 0.002%, time of maximum 0900±0014

semi-diurnal amplitude 0.008 ± 0.002%, time of maximum 0540±0030

amplitude of sum of harmonics 0.037%, time of maximum 0730

The corresponding results from observations of vertical intensity underground over the same period are listed in Table I, paper 5, and lead to the following values for the amplitude ratios $\frac{\text{sea-level}}{\text{underground}}$:

diurnal: $\sim$ (80 ± 7) %

semi-diurnal: $\sim$ (100 ± 45) %

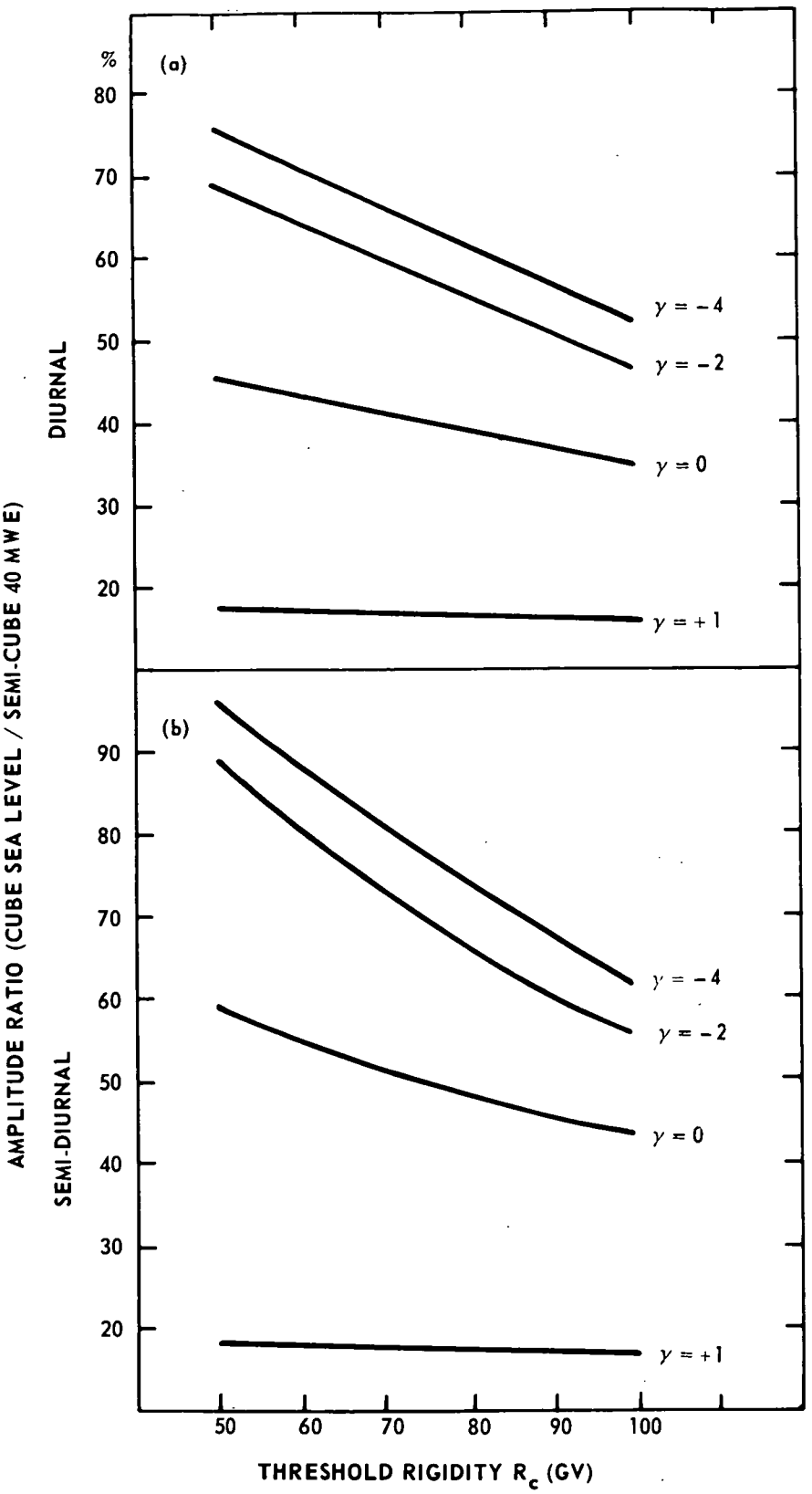

Figure **8.2** Sidereal anistropy. (a) Calculated ratios of amplitudes of first harmonics at Hobart for a vertical cubical telescope at sea level and a vertical semi-cubical telescope at a depth of 40 m.w.e. (b) Calculated ratios of amplitudes of the second harmonics.

The intercomparison implies a very low value for R_C and a negative value for γ. Alternatively, it could be interpreted as a further indication that a persistent and significant part of the observed effect at ground level is spurious, while being approximately in phase with the proposed anisotropic component. The great year-to-year and shorter term variability of the surface observations in sidereal time has been summarised in Figures 3.5 to 3.7 and emphasises the problem of identifying a sidereal component in the data either from meson telescopes or neutron monitors. It is likely that much of the variability arises from the fact that it does not require great percentage modulation (e.g. $\sim$ 20% of amplitude) to produce annual sideband amplitudes of $\sim$0.05% in sidereal time from significant components of the large solar daily variation at ground level. Short-term irregular changes may also be important.

Perhaps observations with crossed telescopes would remove much of the variable and presumably spurious content in the annual apparent sidereal effect above ground. It seems that it would be desirable to obtain long-term records from several crossed-pair arrays, each array being identified with its own well-defined asymptotic latitude strip. However, two points should be noted. At the important mid-latitude locations the differential asymptotic latitude of viewing δ_R varies considerably with rigidity over the range of rigidities where most of the response occurs. Typically, the range of variation of δ_R is about 40^o, as Baker et alia (1966) indicate in Figure 1 of their paper on the crossed telescope experiment at Denver. Consequently, an initial assumption has to be made as to the likely value of R_C, since this enters rather critically into the

design of experiments and the interpretation of the data. In the case of experiments underground, on the other hand, δ_R does not change greatly with R unless telescopes are directed towards very high latitudes. If we consider the vertical semi-cubical telescopes at Hobart and Budapest for example, Figure 6.2 shows that the total range of variation of δ_R with R is not more than 10°. Because of factors of this kind, a rudimentary description of the anisotropy can be obtained from underground data without having to specify R_C and γ. As to the second point, if R_C is in fact not less than about 100GV, very high counting rates would seem to be necessary at ground level, because of the small expected amplitudes (Figure 8.2). Putting this in another way, the anisotropy would affect less than 20% of the counting rate above ground as against 75% at 40 m.w.e., or 5% as against 25% if R_C were greater than about 300 GV (see Figure 6.6). Nevertheless, because of the information it would give concerning γ and R_C, unambiguous and significant evidence for the anisotropy from detectors with relatively low mean rigidities of response would be of great value.

8.3 Concluding Remarks

(a) General comments on the evidence

The investigation began with observations of a persistent annual apparent sidereal effect in the underground data from Hobart. The purpose of subsequent experiments and analyses of data has been essentially to determine the origin of the underground effect.

In the first place, there can be no doubt as to the existence of a characteristic annual sidereal daily variation at Hobart. The basic features can be identified in each of the eight years of data. In the long-term average, the diurnal amplitude is found to differ from zero by approximately twenty standard errors and the semi-diurnal amplitude by four.

Evidence has accumulated that the underground effect is due to a two-way sidereal anisotropy and, on the basis of the empirical model used, an additional streaming component. Four major lines of evidence have been presented:

1. The evidence from the annual daily variations of vertical intensity in solar, sidereal and anti-sidereal time at Hobart.
2. The evidence from the phase anomalies.
3. The evidence for a 12-hour phase difference of sidereal origin between the two hemispheres, as verified by observations in the vertical and inclined (70^{o}N) directions at Hobart.
4. The evidence from the diurnal and semi-diurnal components. They have been shown to conform with the empirical model for the anisotropy in free space, notably in respect of their latitude dependences of amplitude.

The possibility of a significant spurious component in the observation has been a primary consideration. The evidence concerning this was of two kinds:

(1) The sideband analysis of the vertical intensity at Hobart showed that if the persistent and uniform annual daily variation in sidereal time was spurious, it could not be reconciled with the behaviour of the daily variations in solar and anti-sidereal time. A major difficulty was the lack of evidence for a solar component large enough to generate the required sidereal sideband component. If there had been such a solar component, it would have been difficult to explain the absence, in most years, of a significant sideband component in anti-sidereal time. It was proposed that in 1958 and 1959, when the anti-sidereal effect was prominent, modulation of the sidereal component itself may have occurred.

(2) The elements of evidence for the two-way anisotropy showed significant compatibility. Thus the occurrence of the phase anomalies were consistent with the secular trends of amplitude of the observed solar and sidereal components; the sidereal times of maximum obtained from annual averages and from the phase anomalies agreed; the 12-hour difference in the sidereal diurnal maxima between the hemispheres agreed with the phase difference observed in the vertical and inclined (70^{o}N) directions at Hobart and was consistent with the presence of appropriate second harmonics; and the compatibility of the estimated free-space amplitudes relating to three widely spaced latitudes of viewing were demonstrated for both diurnal and semi-diurnal components. It is submitted that if the observed sidereal effect had been spurious, or had contained a large spurious component, there should instead have been an obvious lack of agreement amongst these varied contributions to the evidence. In fact, no significant incompatibilities were found.

The coherent nature of the evidence from the few detecting units was a main source of information that the data selected for analysis were reliable. A useful indication also came from the response to the solar anisotropy : subject to the estimated statistical errors, which were serious only in the data from the south-pointing telescope, there was good agreement amongst all detectors as to the response underground, particularly so between the Hobart and Budapest vertical telescopes. Again, analysis of variance indicated that if systematic effects were removed, the bi-hourly values of vertical intensity at Hobart would follow essentially the expected Poisson distribution.

The data from ground level that were examined showed much variability, both year-to-year and shorter term and it was concluded that the usual methods of analysis would be unsuitable. Nevertheless, there were some indications that the neutron and charged particle detectors were responding to the sidereal anisotropy.

Finally it will have been noted that, if we disregard the latitude dependence of free-space harmonics, the evidence that a sidereal anisotropy exists did not depend on assumptions as to the nature of the interplanetary magnetic field. In fact, it did not require that the primaries be charged, although it seemed unlikely from other considerations that the anisotropy would be due to neutral primaries. Even when the evidence relating to the free-space harmonics is included, it is apparent that a wide range of values of R_C would be tolerated.

(b) The anisotropy

(i) Spatial characteristics

The underground observations are consistent with a two-way anisotropy (equal amplitudes in opposite directions) and a superimposed streaming component. The southward direction of the two-way anisotropy would also be the direction of maximum intensity due to streaming.

The estimation of this direction from the data takes account of the influence of the earth's magnetic field only and thus gives an annual average direction in interplanetary space, the coordinates being (RA = 0640 ± 0118, Dec. = -38 ± 7°).

The calculations, based on a simple empirical model, require assumptions to be made as to the values of γ, the index of the variation spectrum, and R_C, the primary threshold rigidity for observations. From the few estimations made it is clear that γ may take values from 0 to -2 at the least, and the choice for R_C may range from about 50 GV to not less than 200 GV, without affecting the estimate of the direction of the anisotropy.

Two determinations of R_u, the upper limiting rigidity of the solar anisotropy, have been made and indicate that R_u may have decreased from approximately 100 GV in 1958 to perhaps 60 GV in 1962. It seems unlikely that R_C would be less than R_u and this has guided the choice of a range of values for R_C in the calculations of response.

There has been some evidence of a response to the anisotropy from the data at ground level, notably from the phase anomaly in 1954. If there has been a response, typical amplitudes can hardly have been less than approximately 0.02%. Guided by the response characteristics of the sea-level telescope at Hobart, this suggests that unless the amplitude of the anisotropy was much greater then than it is now, γ could not have been positive.

The counting rates from the underground experiments were too low to allow significant values of the individual intensity maxima in the frame of reference of the anisotropy to be determined. However, preliminary estimates gave approximately 0.05%, not exceeding 0.2% for the streaming component, and approximately 0.2% not exceeding 0.6% for the two-way component.

The conclusion that a streaming component explains the larger diurnal amplitudes in the southern hemisphere is a very tentative one. The observations may also be compatible with a somewhat different model for the anisotropy and a different interpretation of the so-called "streaming" term. For instance, the possibility should be considered that the opacity of the interplanetary medium differs between the two hemispheres defined, say, by the plane of the ecliptic, that this at present causes R_C to be smaller and the smoothing of free-space amplitudes to be less in southern directions of viewing and is sufficient to produce the observed asymmetry in the amplitudes. However, with existing observations it would clearly not be possible to distinguish between the present model and a better one.

As yet no differences, such as would be caused by the earth's magnetic field, have been detected in observed sidereal times of maximum. Naturally these phase displacements are expected to be small in most directions of viewing (see values of ϕ_1 and ϕ_2 in Tables 6.1 and 6.2), but they give direct evidence that the anisotropy is due to charged primaries. Since it is important to verify this, experiments are being planned with the detection of this type of phase displacement as a primary objective.

(ii) Time-variations

There are indications from the eight years' observations of vertical intensity underground that the sidereal daily variation at Hobart is subject to both seasonal and year-to-year changes. It is believed that pronounced semi-annual variations of amplitude and, less markedly, of phase, may have occurred during 1958 and 1959 and it was tentatively suggested in section 5.7(b) that these seasonal changes may have been connected with the scattering and reflection of primaries in the interplanetary field as they approached the sun. As to the year-to-year changes it is evident that the annual amplitude has decreased since 1958, although the trend is not a very obvious one (Figure 5.9) and that an amplitude peak occurred between 1961 and 1962. The latter, being associated with a minimum of amplitude of the solar daily variation and with the occurrence of a well-defined phase anomaly, has been interpreted as evidence of a temporary decline in the ability of the interplanetary field to trap the higher energy primaries and thus to restrict observation of the sidereal anisotropy. On the other hand, it is not yet clear that the longer term decrease of sidereal amplitude is associated with solar activity

For a proper assessment of the seasonal and long-term changes it will evidently be necessary to compare equivalent data from several latitudes of viewing in both hemispheres.

It is considered that a latitude survey of high statistical accuracy could over a number of years give results of great interest. A description of the free-space parameters of the sidereal anisotropy rests on a precise determination of the amplitudes of the harmonics versus latitude of viewing and from this point of view the survey would be essential. It is important to discover the origin of the apparent seasonal effect in the sidereal component and here it would be useful to have simultaneous observations from the equator and from mid-latitudes in both hemispheres. Studies of the solar anisotropy would benefit, notably as they concern the latitude dependence of amplitude and the rôle of the second harmonic. Finally, it should perhaps be mentioned that various cosmic ray phenomena connected with interplanetary modulation of the primary spectrum (e.g. Forbush decreases and the eleven-year variation of average intensity) are observed at moderate depths underground and, when comparisons with observations at sea level are made, significant differences in the characteristics can be seen. It would be desirable to obtain a more complete description of these differences from simultaneous observations in both hemispheres.

While it is doubtful if a latitude survey would give useful information concerning R_C and γ, a comparison of sidereal diurnal variation from underground data in the vertical and south-pointing directions at Hobart (or

from equivalent directions in the northern hemisphere) might enable some distinctions to be made. There could be an appreciable phase difference due to the earth's magnetic field, because of the greater deflections of primaries approaching from the south. However, on present calculations, it may only be possible to distinguish $R_C << 150$ GV from $R_C >> 150$ GV, subject to γ being negative. Nevertheless there should be a difference of more than one hour between the diurnal maxima in the two directions, indicating that the anisotropy was due to charged primaries, unless the spectrum is flat and $R_C >> 200$ GV. Very high counting rates would be needed in the south-pointing direction because of the small amplitude expected.

An experiment being planned for ground level at Mawson ($\sim 68^{\circ}$S geog.) employs telescopes of small aperture in zenith, directed in the plane of the meridian 75° south and 75° north of zenith respectively. The atmospheric absorber should be roughly equivalent to 40 metres of water. From comparisons of the apparent sidereal harmonics in both directions, it is hoped to be able to distinguish a genuine sidereal component from any appreciable spurious effect and also to obtain direct evidence that the sidereal component is due to charged primaries, as a consequence of the relatively greater deflections of primaries approaching from the south. The diurnal amplitude due to the sidereal anisotropy should be considerably larger from the south in this experiment ($\overline{\delta} \simeq -35^{\circ}$) than the amplitude expected from a south-pointing underground experiment at Hobart ($\overline{\delta} \simeq -60^{\circ}$).

In p rinciple, intercomparison of amplitudes at different depths underground would seem to offer the best prospects of determining R_c and γ experimentally. It would be necessary to have adequate knowledge of the asymptotic constants of response of the detectors in relation to depth underground, including accurate estimates of the material absorber and of the coupling coefficients appropriate to the different cut-off energies for production of mesons. While it may be convenient to choose approximately 40 m.w.e. as one of the observing depths, a comparative measurement at a greater depth would call for a very large detecting area (a) to take account of the decrease of intensity with depth, and (b) to allow for the possibility that the amplitude decreases with increasing depth. On the other hand, if measurements are to be made closer to ground level than 40 m.w.e., there should be no difficulty in achieving high counting rates, but the increased risk of contamination by a spurious sidereal component would have to be considered.

APPENDIX I

(Paper 1, attached)

"A solar anisotropy in cosmic radiation, viewed from a terrestrial frame of reference"

R.M.Jacklyn, Nature 200, 1306 (1963)

APPENDIX II

(Paper 2, attached)

"The apparent sidereal and anti-sidereal daily variations of cosmic ray intensity"

R.M.Jacklyn, Il Nuovo Cimento 24, 1034 (1962)

APPENDIX III

(Paper 3, attached)

"Cosmic ray observations at 42 m.w.e. underground at Hobart, Tasmania"

A.G.Fenton, R.M.Jacklyn and R.B.Taylor, Il Nuovo Cimento 22, 285 (1961)

APPENDIX IV

(Paper 4, attached)

"The apparent sidereal daily variation of cosmic ray intensity at 42 m.w.e. underground at Hobart, Tasmania. I. Results of observations in 1958"

R.M.Jacklyn, Il Nuovo Cimento 30, 40 (1963)

APPENDIX V

(Paper 5, attached)

"The apparent sidereal daily variation of cosmic ray intensity at ∿ 40 m.w.e. underground at Hobart, Tasmania. II. Results of observations, 1958 - 1962"

R.M.Jacklyn, Il Nuovo Cimento 36, 1135 (1965)

APPENDIX VI

(Paper 6, attached)

"Evidence for a genuine sidereal daily variation of cosmic ray intensity at ∿ 40 m.w.e. underground at Hobart in 1961"

R.M.Jacklyn, Proc.Jaipur Conf.Cosmic Rays 2, 345 (1963)

APPENDIX VII

(Paper 7, attached)

"The upper limiting primary rigidity of the cosmic ray solar anisotropy"

R.M.Jacklyn and J.E.Humble, Aust.J.Phys. 18, 451 (1965)

APPENDIX VIII

(Paper 8, attached)

"Evidence for a two-way sidereal anisotropy in the charged primary cosmic radiation"

R.M.Jacklyn, Nature 211, 690 (1966)

APPENDIX IX

RADIATION SENSITIVITY AS A FUNCTION OF ASYMPTOTIC DIRECTION OF VIEWING

IX.1 Radiation Sensitivity Specified in the Alt-azimuth Frame of Reference

In section 6A.5 (see also Parsons, 1957) the differential radiation sensitivity associated with the elementary solid angle of arrival ω_r is given by

$$I(R, \omega_r) = Y_R F(\omega_r) \cos^n Z_r$$

where Y_R, the differential coupling coefficient, gives the fraction of the counting rate due to primaries of rigidity R, $F(\omega_r)$ is the geometric sensitivity and $\cos^n Z_r$ expresses the zenith angle dependence of intensity. $F(\omega_r)$ is initially specified in the frame of reference of the telescope, referred to the axis and the plane of a tray. (In this respect the treatment of $F(\omega_r)$ for inclined telescopes differs somewhat from that given by Parsons.) To obtain $I(R,\omega_r)$ it is necessary to transform to the frame of reference of altitude and geographic azimuth referred to the zenith and the plane of the horizon. Naturally,if the telescope axis is vertical only a formal exchange of symbols is required (depending on the reference direction for azimuth in the plane of the tray), since the two frames of reference are then coincident.

IX.2 The deflection data Specified in the Alt-azimuth Frame of Reference

Brunberg and Dattner (1953) give asymptotic bearings in geomagnetic latitude and longitude for particles which arrive in directions specified by altitude and geographic azimuth. When the asymptotic bearings are expressed in

this form, interpolation from the diagrams becomes impracticable for some directions if the cone of viewing of the telescope, such as the south-pointing cube at Hobart, takes in the polar axis. It was therefore found more convenient to express the asymptotic latitude and longitude data in the form of deflections in zenith angle and geographic azimuth, thus transforming from the geomagnetic equatorial to the alt-azimuth system of coordinates.

IX.3 Conversion of Coordinates Using a Terrestrial Globe

Since the facilities of a high-speed computer were not available when the calculations were to be made, a terrestrial globe (Figure IX.1) was employed to convert directly from, say, bearings in geomagnetic latitude and longitude to bearings in altitude and geomagnetic azimuth, at the given geomagnetic latitude of observation. By drilling a suitable pair of conjugate holes for the axis of rotation, transformation from one spherical coordinate system to another, involving simple rotations of axes, was effected by tilting the vertical axis of one frame of reference (represented by the celluloid strips) with respect to that of the other (represented by the coordinate grid). With the exception of directions very close to the pole, coordinates of direction could be read off to an accuracy of a degree without difficulty.

IX.4 Radiation Sensitivity as a Function of Asymptotic Direction of Viewing

The steps are now listed by which sets of values of $I(R,\omega)$ and the associated asymptotic latitude and longitude coordinates, $(\delta_r)_R$ and $(\phi_r)_R$, may be obtained.

Figure IX.1

1. Initially, $F(\omega_r)$ is expressed as a function of inclination (Z') to the telescope axis and azimuth (A') in the plane of a tray.

2. Coordinates of directions are transformed from the telescope system to the alt-azimuth system (axis of rotation of globe set at Z = inclination of telescope axis to zenith). $I(R,\omega_r)$ is obtained as a function of altitude (a) and geographic azimuth (A).

3. Asymptotic bearings in geomagnetic latitude and longitude, from the diagrams of Brunberg and Dattner are converted to bearings in altitude and geomagnetic azimuth (axis of rotation of globe set at λ = geomagnetic latitude of place of observation). Deflections in altitude and geographic azimuth are obtained by adding to the bearings in geomagnetic azimuth the constant angle α = difference between true north and geomagnetic north at the latitude of the place of observation. Figures IX.2 to IX.9 give the deflection data referred to these coordinates, for particles of rigidity 50 GV and 150 GV which arrive at the geomagnetic latitude $\lambda = -50^{o}$, approximating the latitude of Hobart.

4. From 2. and 3. above, $I(R,\omega_r)$ is obtained as a function of asymptotic direction, referred to the alt-azimuth coordinate system.

5. $I(R,\omega_r)$ is obtained as a function of asymptotic geographic latitude and longitude of viewing by converting from the alt-azimuth to the terrestrial equatorial coordinate system (axis of rotation of globe set at δ = geographic latitude of place of observation).

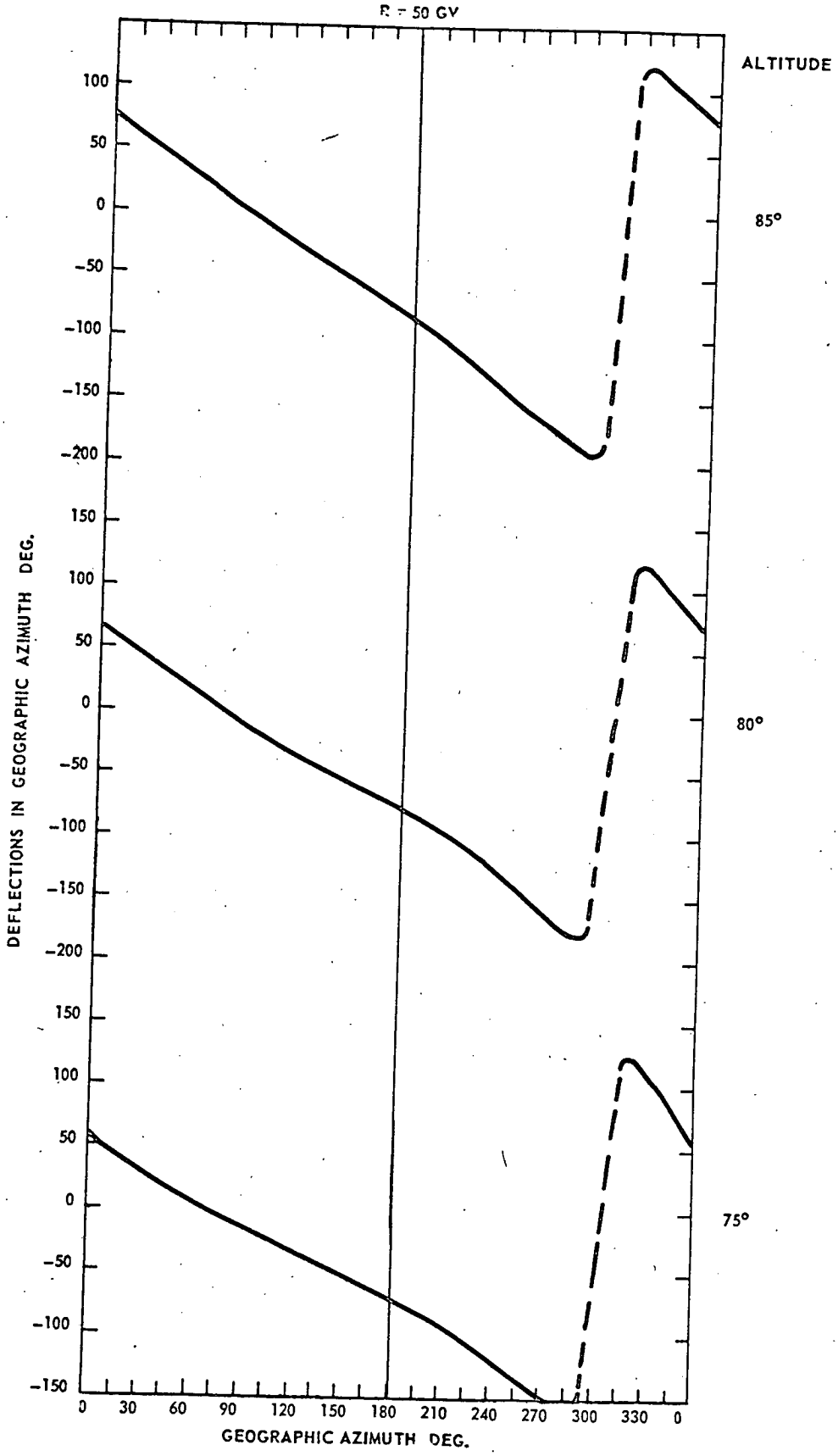

Figure IX.2 Deflections in azimuth of protons of rigidity 50 GV in centred dipole field for given directions of arrival at geomagnetic latitude -50°.

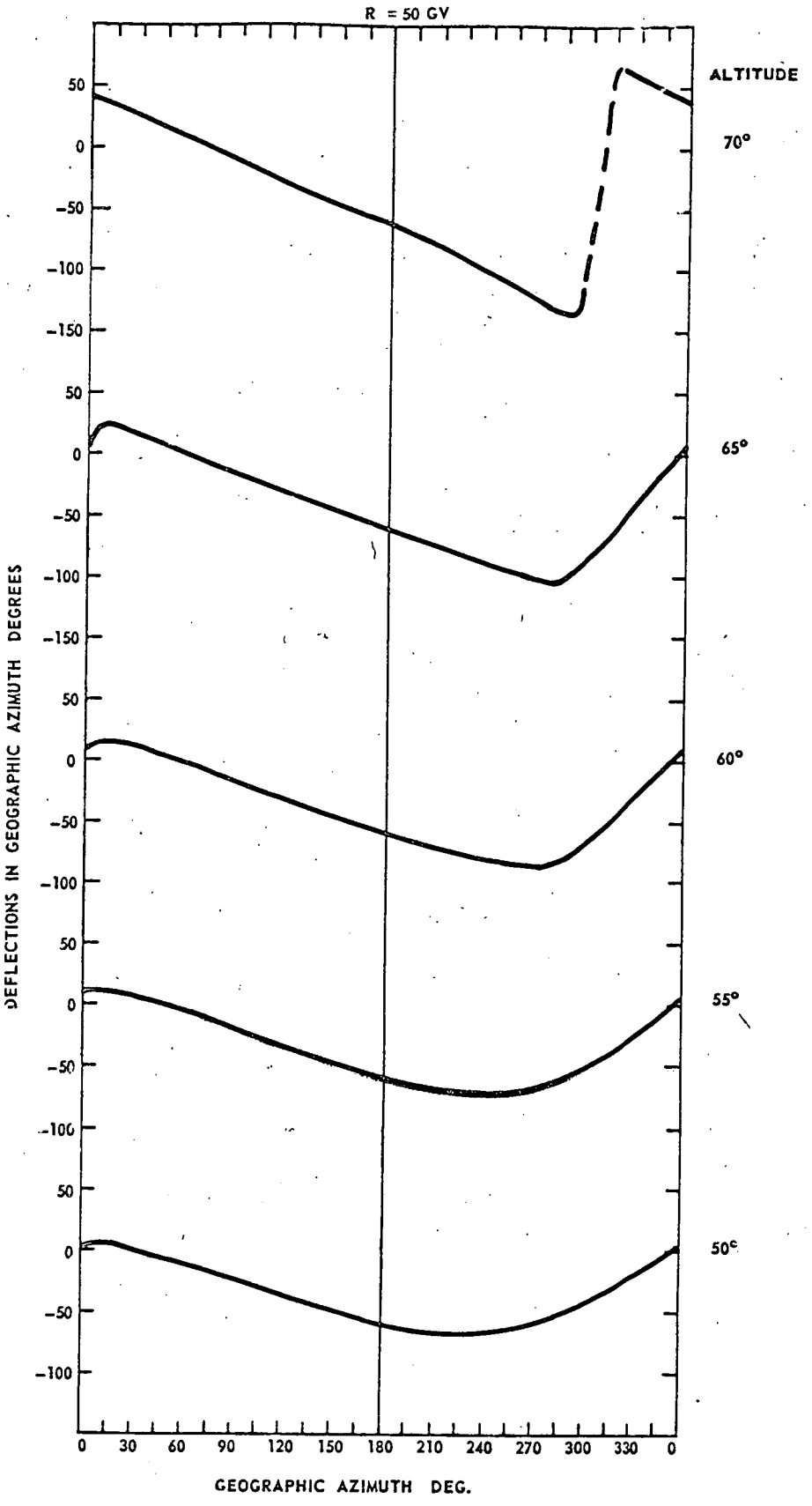

Figure IX.3 Deflections in azimuth of protons of rigidity 50 GV in centred dipole field for given directions of arrival at geomagnetic latitude -50°.

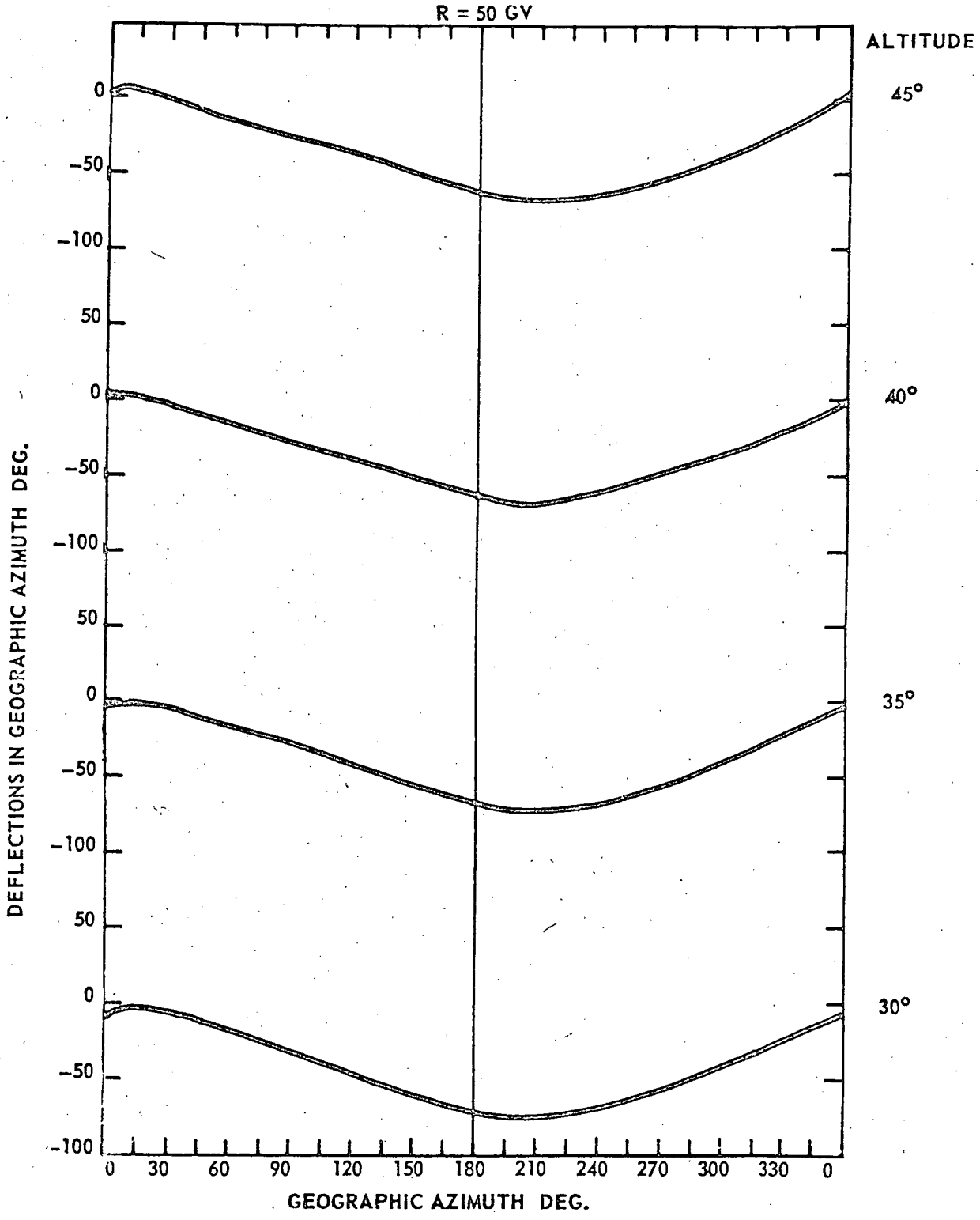

Figure IX.4 Deflections in azimuth of protons of rigidity 50 GV in centred dipole field for given directions of arrival at geomagnetic latitude −50°.

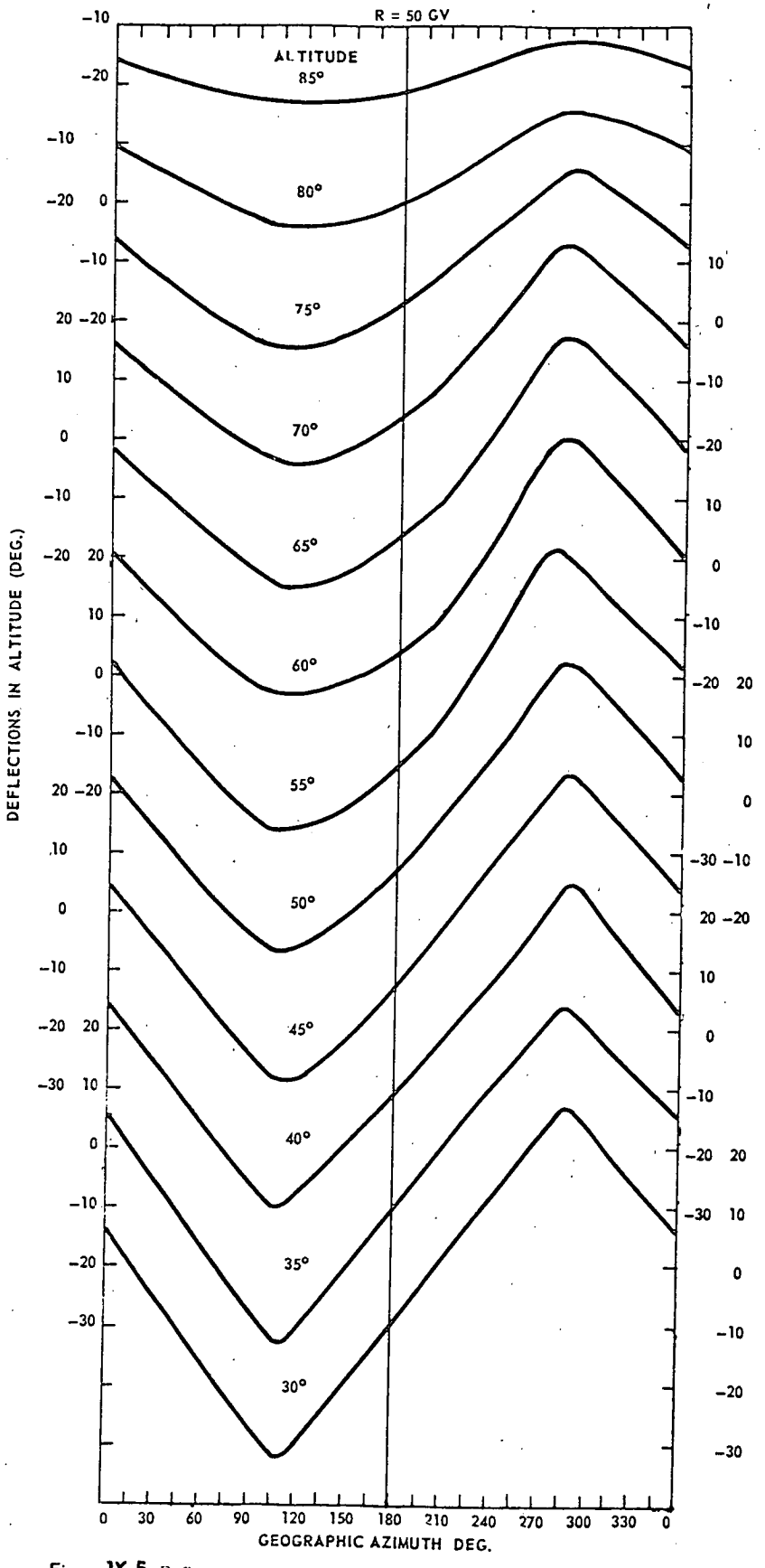

Figure IX.5 Deflections in altitude of protons of rigidity 50 GV in centred dipole field for given directions of arrival at geomagnetic latitude −50°.

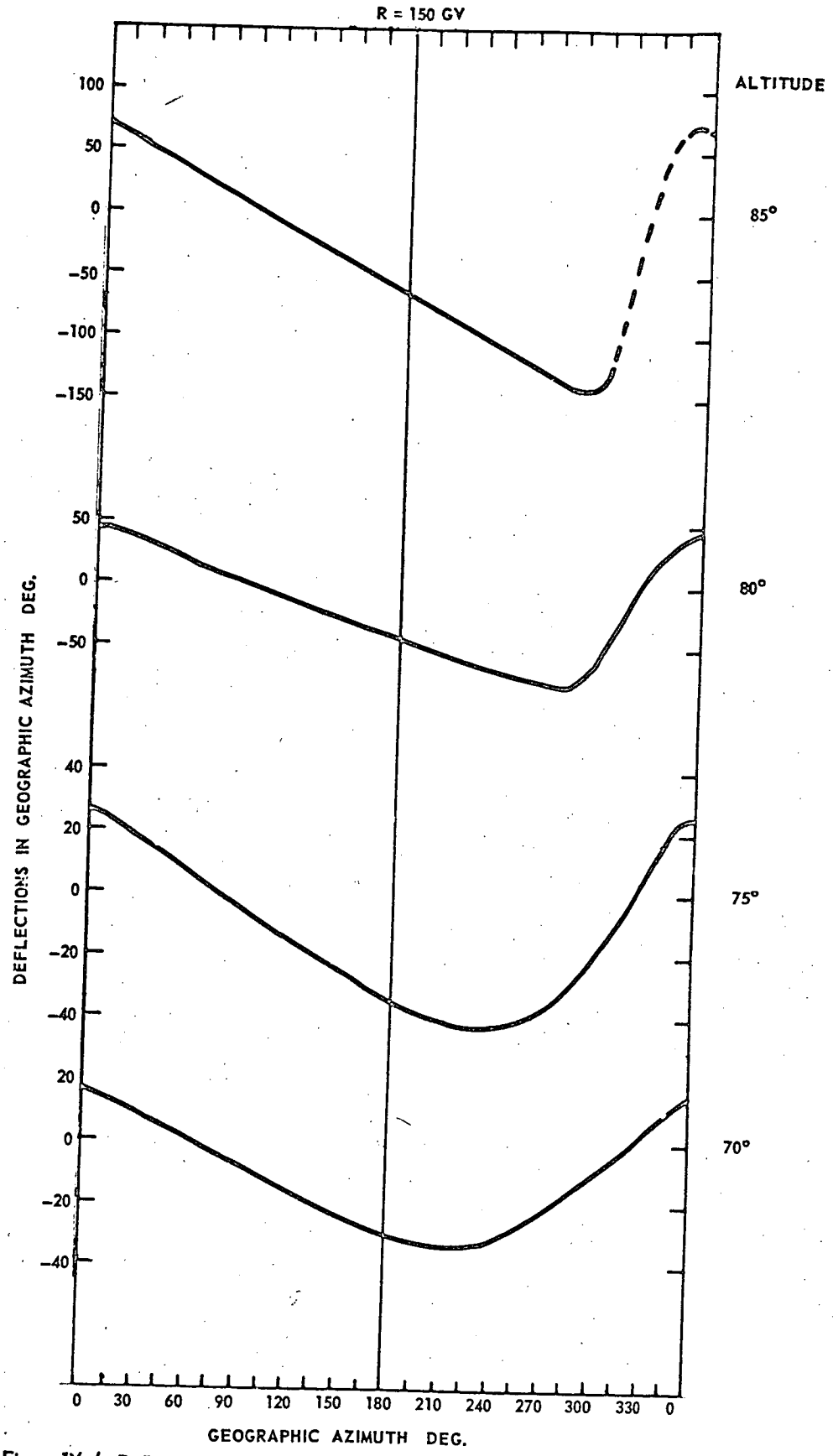

Figure IX.6 Deflections in azimuth of protons of rigidity 150 GV in centred dipole field for given directions of arrival at geomagnetic latitude −50°.

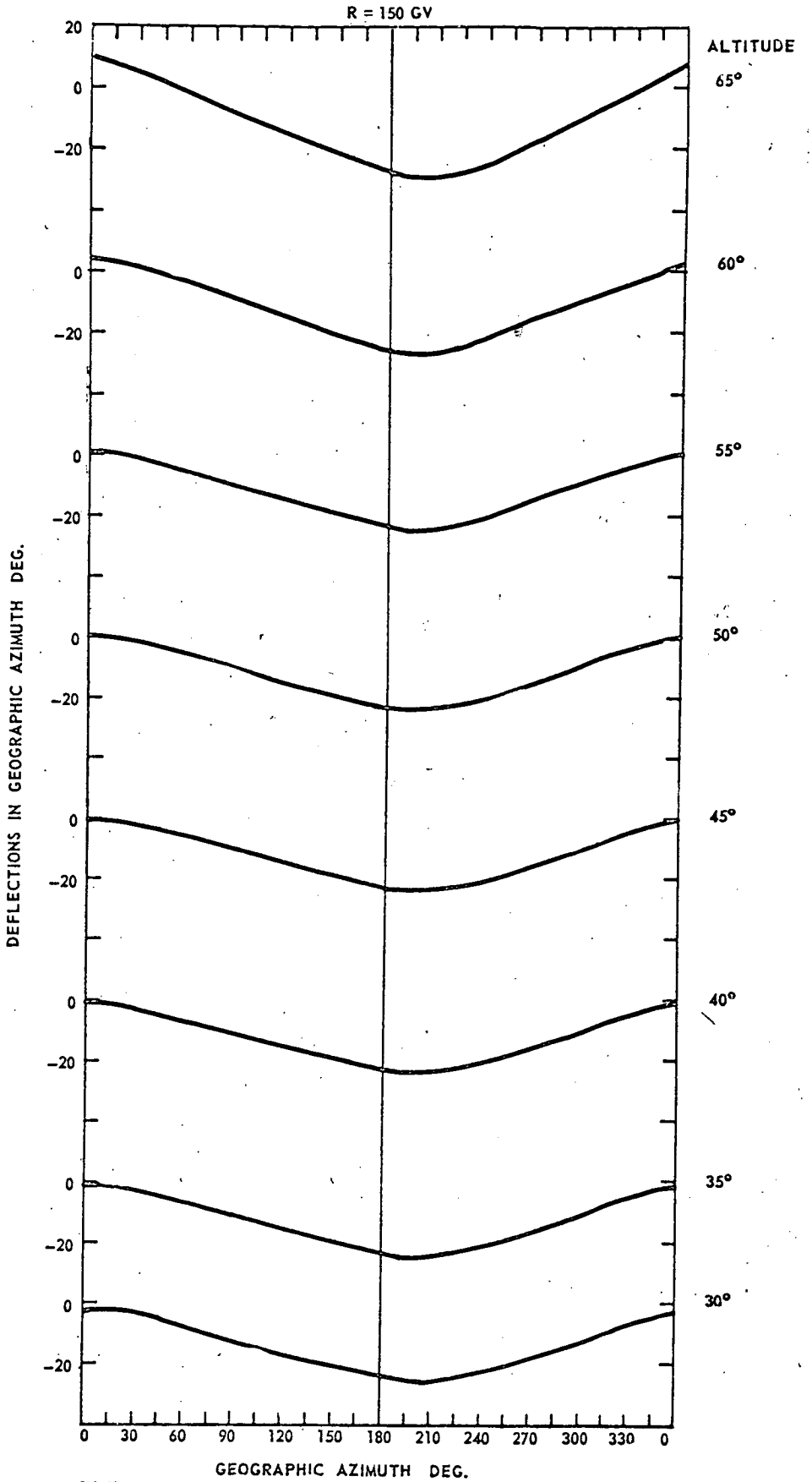

Figure IX.7 Deflections in azimuth of protons of rigidity 150 GV in centred dipole field for given directions of arrival at geomagnetic latitude −50°.

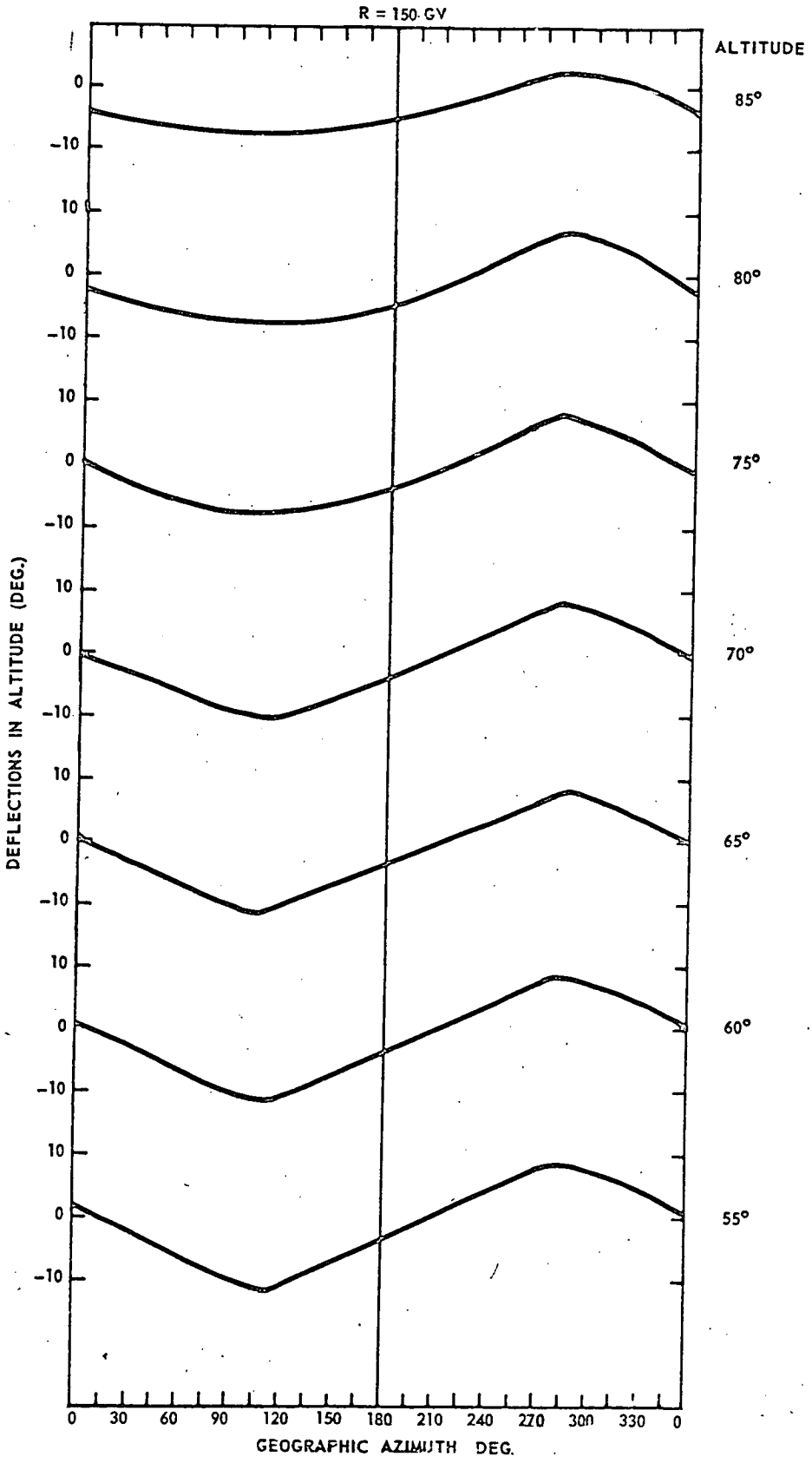

Figure IX. 8 Deflections in altitude of protons of rigidity 150 GV in centred dipole field for given directions of arrival at geomagnetic latitude –50°.

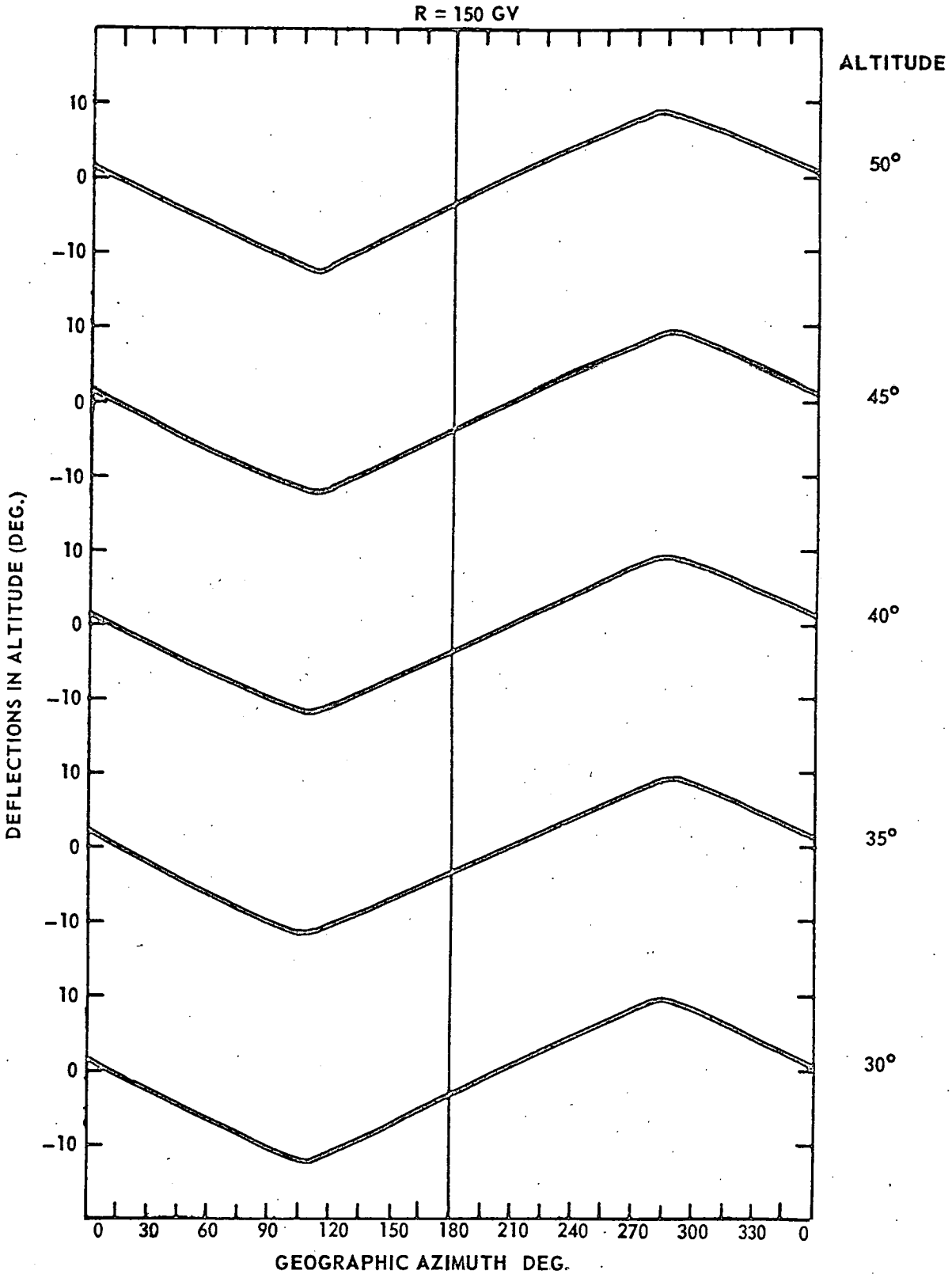

Figure IX.9 Deflections in altitude of protons of rigidity 150 GV in centred dipole field for given directions of arrival at geomagnetic latitude -50°.

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PAPER 1

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A Solar Anisotropy in Cosmic Radiation, viewed from a Terrestrial Frame of Reference

It is generally recognized that a solar anisotropy in the primary cosmic radiation must be responsible for at least part of the observed terrestrial daily variation of intensity, although it seems that the exact nature of this contribution is not yet clear. However, Rao, McCracken and Venkatesan[1] have recently presented a method of determining the characteristics of a daily variation of this type. Their method takes into consideration the influence of the Earth's magnetic field and in particular they emphasize the part played by the asymptotic cones of acceptance of cosmic ray detectors. Consequently, using experimental values of the daily variation of neutron intensity averaged over the year 1958, they have been able to postulate a type of anisotropy which appears to account for the complex manner in which the amplitude and phase of the first harmonic of the daily variation depends on the geographical location of the detector. However, it would perhaps be truer to say that they predict that if the Earth's atmosphere and its magnetic field were removed the annual mean diurnal variation of intensity observed at the Earth's surface would have the following characteristics: (1) the diurnal variation would be independent of rigidity in the range 1–200 GV.; (2) the amplitude would vary as the cosine of geographic latitude and the time of maximum would be in the direction 85° to the east of the Earth–Sun line; (3) the maximum amplitude, observed with narrow-angle detectors at the equator, would be 0·4 per cent of the average primary flux. It is proposed to describe here the co-ordinate transformations which relate these important characteristics to those of a solar anisotropy specified in the frame of reference in which its orientation is fixed.

Following Rao *et al.*, let us suppose that the intensity anisotropy is fixed in the co-ordinate system the reference planes of which are the ecliptic and the plane normal to it through the Earth–Sun line. In this framework let us specify the anisotropic component of the total intensity as $AI_0(R) \cos \Lambda \cos (\psi - \psi_0)$, where A is the amplitude constant, $I_0(R)$, the average cosmic ray differential rigidity spectrum, Λ, the ecliptic latitude of the detector, and ψ, its direction of viewing in the plane of the ecliptic, measured eastward of the Earth–Sun line, with ψ_0 the direction of maximum intensity. In what follows it will be more convenient to refer the direction of viewing in the plane of the ecliptic to the direction of maximum intensity and express it as $\psi - \psi_0$. We now have to transform to the

terrestrial co-ordinate system the reference planes of which are the equator and the plane normal to it through the Earth–Sun line.

Transforming first to the conventional ecliptic system, specified by the plane of the ecliptic and the equinox, the spherical co-ordinates of the direction of viewing become Λ and $\lambda_a+\psi-\psi_0$, where λ_a is the ecliptic longitude in the direction of maximum intensity. The corresponding co-ordinates in the terrestrial equatorial system, specified by the equator and the equinox, are the geographic declination δ and the right ascension $\alpha_a+\varphi-\varphi_0$, where α_a is the right ascension of the direction of maximum intensity and φ and φ_0 are the directions of viewing and of maximum intensity in the plane of the equator, both measured eastward of the Earth–Sun line. We can now transform simply from one set of co-ordinates to the other by rotation of the axes of one of the co-ordinate systems through the angle ε without change of origin, ε being the obliquity of the ecliptic. Projections on the appropriate axes yield:

$$\cos\Lambda\cos(\lambda_a+\psi-\psi_0)=\cos\delta\cos(\alpha_a+\varphi-\varphi_0) \qquad (1)$$

and

$$\cos\Lambda\sin(\lambda_a+\psi-\psi_0)=\cos\delta\sin(\alpha_a+\varphi-\varphi_0)\cos\varepsilon+\sin\delta\sin\varepsilon \qquad (2)$$

Since the maximum difference between ecliptic longitude λ_a and right ascension α_a never exceeds about 2·5 degrees (cf. values for the Sun in the *Nautical Almanac*) α_a can be substituted for λ_a in equations 1 and 2. Expanding and combining equations 1 and 2 to eliminate $\sin\psi$, we obtain:

$$\cos\Lambda\cos(\psi-\psi_0)=[1-(1-\cos\varepsilon)\sin^2\alpha_a]\cos\delta\cos(\varphi-\varphi_0)-\frac{1-\cos\varepsilon}{2}\cos\delta\sin 2\alpha_a\sin(\varphi-\varphi_0)+\sin\varepsilon\sin\delta\sin\alpha_a \qquad (3)$$

The right ascension of the anisotropy, α_a, is $\alpha_a=\alpha_s+\varphi_0$, where α_s is the Sun's right ascension. We can replace φ_0 by ψ_0 since, like λ_a and α_a, the two constants never differ by more than about 2·5 degrees. Therefore making these substitutions in equation 3 and multiplying through by $AI_0(R)$ we finally have:

$$AI_0(R)\cos\Lambda\cos(\psi-\psi_0)=AI_0(R)\left\{[1-(1-\cos\varepsilon)\sin^2(\alpha_s+\psi_0)]\cos\delta\cos(\varphi-\psi_0)-\left(\frac{1-\cos\varepsilon}{2}\right)\cos\delta\sin 2(\alpha_s+\psi_0)\sin(\varphi-\psi_0)+\sin\varepsilon\sin\delta\sin(\alpha_s+\psi_0)\right\} \qquad (4)$$

$$=AI_0(R)\left\{[1-0{\cdot}0826\sin^2(\alpha_s+\psi_0)]\cos\delta\cos(\varphi-\psi_0)-0{\cdot}0413\cos\delta\sin 2(\alpha_s+\psi_0)\sin(\varphi-\psi_0)+0{\cdot}3979\sin\delta\sin(\alpha_s+\psi_0)\right\}\text{ for }\varepsilon=23^\circ\,27'$$

On the right-hand side of equation 4 the anisotropic component of primary intensity is expressed as a function of geographic latitude, time of day relative to local noon, and day of the year as specified by α_s, which, for practical

purposes, can be regarded as the right ascension of the Sun at transit at Greenwich. The co-ordinates δ and φ refer, of course, to the asymptotic direction of viewing of a cosmic ray detector with respect to primaries of rigidity R.

It may be of interest to mention an application of equation 4 to observations of the intensity of sunlight falling on unit area of the Earth's surface. In this case, $\psi_0 = 0$ and the conditions for sunrise and sunset are that $\psi = \pm \pi/2$. The solutions for φ then give the approximate sundial times of sunrise and sunset at the latitude δ at the time of the year specified by α_s. The co-ordinates δ and φ may be likened to asymptotic directions of viewing since the atmosphere, and therefore refraction, has been disregarded.

We note that $AI_0(R) \cos\Lambda \cos(\psi - \psi_0)$ transforms to a solar diurnal variation and a total intensity term $AI_0(R) \sin\varepsilon \sin\delta \sin(\alpha_s + \psi_0)$ to which it reduces at the geographic poles. That is to say, at the poles the anisotropy manifests itself as a small seasonal variation of the total intensity. The expression for the diurnal variation at other latitudes allows the seasonal variations of amplitude and phase to be worked out. For example, at the latitude of Hobart (43° S.) these amount to a small semi-annual variation of amplitude, the maximum departure from the mean being less than 10 per cent, and an almost negligible (less than 5 degrees) semi-annual variation of phase. Spurious sidereal and anti-sidereal effects are absent.

The annual mean expression for the anisotropy is obtained by integrating equation 4 with respect to α_s, giving:

$$AI_0(R)\cos\Lambda\cos(\psi - \psi_0) = \underset{I(R)}{AI_0(R)} \left(\frac{1+\cos\varepsilon}{2}\right) \cos\delta\cos(\varphi - \psi_0)$$

$$= 0{\cdot}9587\, A_\Lambda \cos\delta \cos(\varphi - \psi_0) \text{ for } \varepsilon = 23°\ 27' \qquad (5)$$

Therefore it happens that on the yearly average the anisotropy postulated by Rao *et al.* would exhibit the same characteristics, except for a slight difference in amplitude, whether viewed from the terrestrial frame of reference or from that in which its orientation was fixed. As might be expected, a similar result would be obtained if the orientation were fixed in the plane of the Sun's equator instead of the plane of the ecliptic. However, seasonal variations of amplitude and phase would be more pronounced and small apparent sidereal and anti-sidereal effects would be generated.

If the latitude dependence differs from cos Λ, the characteristics of a solar anisotropy may become greatly modified on transformation to the terrestrial frame of reference. The cases $A \cos\psi$, $A \cos^2\Lambda \cos\psi$, $A \cos^3\Lambda \cos\psi$ and $A \cos^5\Lambda \cos\psi$ will be discussed elsewhere. Although transformation procedures for a simple type of sidereal anisotropy will also be mentioned a more detailed treatment may be found in a paper by L. Davis[2].

In conclusion, it is hoped that the method outlined here may have indicated the need for correct transformations: (*a*) when the characteristics of the anisotropy in its rest framework are to be deduced from the annual mean daily variation observed in the terrestrial frame of reference, after accounting for the influence of the atmosphere and the Earth's magnetic field; (*b*) when the day-by-day contributions which the specified anisotropy makes to the daily variation and to the total intensity of cosmic radiation in the terrestrial frame of reference are to be determined and the seasonal effects calculated.

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[1] Rao, U. R., McCracken, K. G., and Venkatesan, D., *J. Geophys. Res.*, **68**, 345 (1963).
[2] Davis, L., *Phys. Rev.*, **96**, 743 (1954).

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PAPER 2

R. M. JACKLYN
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The Apparent Sidereal and Anti-Sidereal Daily Variations of Cosmic Ray Intensity.

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(ricevuto il 2 Gennaio 1962)

Summary. — A theoretical treatment of the annual apparent sidereal daily variation and the anti-sidereal daily variation associated with the following phenomena is outlined: *a*) An annual linear displacement of phase of as olar daily variation. *b*) An annual linear displacement of phase of a sidereal daily variation. *c*) An annual linear change of amplitude of a solar daily variation. *d*) Combined linear changes of phase and amplitude of a solar daily variation. *e*) An annual linear change of amplitude of a sidereal daily variation. *f*) A seasonal variation of amplitude of a solar daily variation. *g*) A seasonal variation of amplitude of a sidereal daily variation. *h*) A seasonal variation of phase of a solar daily variation. *i*) A seasonal variation of phase of a sidereal daily variation. *j*) Curvature associated with the 11-year cycle of intensity variation. The theory is supplemented by analyses of facsimile daily variations, suitably modulated.

1. – Introduction.

In the search for the existence of a sidereal anisotropy in the cosmic ray flux, the investigator has been faced with two serious problems. The first has been the difficulty of obtaining statistically significant observations of an apparent sidereal effect. DELVAILLE *et al.* (1) have drawn attention to this problem in a recent survey of known determinations of asymmetries in the distribution of high energy primaries. The second difficulty has been that a

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(1) J. DELVAILLE, F. KENDZIORSKI and K. GREISON: *Proceedings of the 1960 Moscow Cosmic Ray Conference*, vol. 3, p. 143.

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sidereal anisotropy is not the only phenomenon that may give rise to an apparent sidereal daily variation. However, only one of the various ways of generating a spurious sidereal effect seems to have been seriously considered so far. It has been known for many years—*e.g.* MESSERSCHMIDT (2)—that if the amplitude of the solar daily variation changes uniformly with the seasons in a sinusoidal manner, this alone will produce a fictitious sidereal effect when the daily variations are arranged in sidereal time and summed over a complete year. J. L. THOMPSON (3) devised the first of the two methods generally used nowadays to distinguish between this particular type of spurious sidereal effect and a genuine sidereal variation. An ellipse of best fit is drawn about the end points of, say, the monthly mean diurnal vectors as displayed on a harmonic dial. If the end points of the twelve vectors for the year progress anti-clockwise in a full circle, then it is assumed that the solar diurnal variation of intensity is being modulated by a genuine sidereal variation with amplitude equal to the radius of the circle. If a seasonal change of amplitude of the solar diurnal variation is also present the circle will be elongated into an ellipse and if the seasonal vector is much greater than the pure sidereal vector the end-points of the monthly diurnal vectors may progress clockwise. The genuine sidereal component is separated out from the amplitude component trigonometrically.

The other method of dealing with the same problem is a development by FARLEY and STOREY (4) of Thompson's method. If the apparent sidereal daily variation happens to contain a component due to a seasonal change of amplitude of the solar diurnal wave, the fictitious part can be subtracted by rearranging the data in anti-sidereal time. Whereas the sidereal day is four minutes shorter than the solar day, the anti-sidereal day is four minutes longer. If a genuine sidereal daily variation is arranged in anti-sidereal time and summed over a complete year, it cancels itself out. If a seasonal change of amplitude of the solar diurnal variation is present, however, a year's data will show a diurnal variation in sidereal time and a matching variation, with the same amplitude, in anti-sidereal time. The time of maximum of the former can be readily deduced from that of the latter. Therefore, if a set of cosmic ray data yields a significant daily variation in anti-sidereal time, it is the practice to assume that this is entirely due to a seasonal change of amplitude of the solar diurnal wave. The corresponding fictitious sidereal variation is calculated and then subtracted from the total apparent sidereal variation. Any significant residual is accordingly taken to be a genuine sidereal effect. The method is attractive, in the first place because it enables one to achieve a significant result from, for example, extensive air shower data. The latter

(2) W. MESSERCHMIDT: *Zeits. f. Phys.*, **78**, 682 (1932).
(3) J. L. THOMPSON: *Phys. Rev.*, **55**, 11 (1939).
(4) F. J. M. FARLEY and J. R. STOREY: *Proc. Phys. Soc.*, **67**, 996 (1954).

could not be handled effectively by the first method because of the poor statistics of the monthly mean daily variations. In the second place, if a set of data shows a significant variation in sidereal time but not in anti-sidereal time, then it can be safely assumed that a genuine sidereal effect is present.

The weakness of both methods is that they only give a correct answer when the solar diurnal wave is modulated by a seasonal change of amplitude. In actual fact, an apparent annual mean sidereal daily variation and/or an anti-sidereal effect may originate in a variety of ways and yet be attributed to a seasonal modulation of amplitude of the solar daily variation. This would lead to an improper correction for the fictitious sidereal component. As a result, the residual « true » sidereal daily variation may be greatly in error since it is frequently found to be of the order of magnitude of the apparent spurious effect.

In this paper it will be shown how the correct phase and amplitude of the fictitious sidereal daily variation may be obtained when a daily variation in solar time has been modulated in one of a number of ways outlined. It will also be shown for a number of cases how a modulation of a genuine sidereal daily variation will give rise to an annual variation in anti-sidereal time. In such cases of course, the anti-sidereal variation does not indicate the presence of a spurious sidereal effect. Finally, consideration is given to the problem of curvature as a possible source of a spurious sidereal daily variation.

It should be mentioned that the following treatment of the sidereal and anti-sidereal daily variation is by no means comprehensive. Annual mean daily variations in sidereal and anti-sidereal time may be traceable to statistical fluctuations, for example, if the observational counting rates are rather low. Spurious effects may also arise from various sorts of short-term irregularities in phase and amplitude of the solar daily variation. However, there is a fair degree of consistency in reported observations of the apparent sidereal effect, particularly in the intermediate range of cosmic ray energies, yet there are strong suggestions that some kind of seasonal component is present (*e.g.* see HOGG (5)). Therefore, it has been decided to confine attention in this paper to the contaminating effects of systematic modulations of intensity rather than to effects due to irregularities in the intensity variation.

2. – Time relationships. Symbols and expressions for the daily variation.

Local sidereal time is, by convention, synchronised with local solar time at noon on the 22nd of September. It is therefore convenient to synchronise anti-sidereal time with sidereal and solar time at noon on the 22nd of Sep-

(5) A. R. HOGG: *Journ. Atmos. Terr. Phys.*, **1**, 56 (1951).

tember as well. A function in solar time is expressed in the form $f(t)$, where t is in units of a solar day. A function in sidereal time is $f(t')$, where t' is in units of a sidereal day, and likewise a function in anti-sidereal time is $f(t'')$, t'' being in units of an anti-sidereal day.

Expressing $f(t)$ in sidereal and anti-sidereal time we have

$$f(t) = f\left(t' - \frac{T}{12}\right) = f\left(t'' + \frac{T}{12}\right)$$

where T is in months measured from noon on the 22nd of September.

A diurnal variation in solar time on any day is denoted by i_d, in sidereal time by i_s, and in anti-sidereal time by i_{as}. The corresponding annual mean diurnal variations are denoted by I_d, I_s and I_{as} respectively.

For the theoretical treatment the unmodulated solar daily variation is considered to be adequately described by the function $i_d = A \sin 2\pi(t - t_0)$, where t_0 is the phase constant and A, the amplitude term, is a positive number.

More conveniently, i_d may be expressed by

$$i_d = A \sin \varphi ,$$

where $\varphi = 2\pi(t - t_0)$.

In sidereal time i_d becomes

$$i_s = A \sin 2\pi\left(t' - t_0 - \frac{T}{12}\right) = A \sin(\varphi' - \theta) ,$$

where $\varphi' = 2\pi(t' - t_0)$ and $\theta = (2\pi T/12)$

In anti-sidereal time i_d becomes

$$i_{as} = A \sin 2\pi\left(t'' - t_0 + \frac{T}{12}\right) = A \sin(\varphi'' + \theta)$$

where $\varphi'' = 2\pi(t'' - t_0)$.

An unmodulated sidereal daily variation is considered to be a simple sinusoidal function of sidereal time:

$$i_s = A \sin 2\pi(t' - t_0') ,$$

where t_0' is the phase constant,

$$= A \sin \psi' ,$$

where $\psi' = 2\pi(t' - t_0')$.

Expressed in anti-sidereal time i_s becomes

$$i_{as} = A \sin 2\pi \left(t'' - t_0' + \frac{2T}{12} \right) = A \sin (\psi'' + 2\theta)$$

where $\psi'' = 2\pi(t'' - t_0')$.

In order to distinguish clearly between the diurnal variation i_d for any day and the annual mean diurnal variation, I_d, it has been found convenient to use the symbol Φ instead of φ in the expression for I_d. Similarly Φ', Φ'', Ψ' and Ψ'' replace the symbols φ', φ'', ψ' and ψ'' respectively. This enables one to distinguish between, for instance, φ_m, the time of maximum of the solar diurnal variation on a single day, and Φ_m, the time of maximum of the annual mean diurnal variation. The time of maximum of the annual mean unmodulated diurnal variation is denoted by Φ_{0m} while that for the annual mean unmodulated sidereal daily variation is denoted by Ψ'_{0m} The corresponding amplitudes are $(I_{0d})_{max}$ and $(I_{0s})_{max}$. Since there is no ambiguity they are both replaced for convenience in the theory by the amplitude symbol A.

The abbreviation « d.v. » is frequently used in place of the phrase « diurnal variation ».

3. – Facsimile daily variations.

In the following sections the annual mean daily variations I_d, I_s and I_{as} are obtained theoretically by integration over a whole year of the appropriately modulated sinusoidal functions i_d, i_s and i_{as}. Apart from considerations of counting rate statistics there are two important reasons why the results from the theory may differ considerably from experimental observations of the same phenomena. Let us now look briefly at these in turn and at the methods used to determine how serious the discrepancies involved may be.

In the first place, the processing of records to obtain annual mean daily variations usually requires a sequence of averaging of data arranged in groups and sub-groups. For example, an observed annual mean d.v. is frequently the average of twelve sets of monthly mean d.v.s, each consisting of twelve by-hourly values. Such a result is only approximately equivalent to that obtained theoretically by integration of continuous functions. For comparison, therefore, the theory is supplemented, in all sections where it seems necessary, by analyses of facsimile d.v.s derived from a simple sinusoidal diurnal variation suitably modulated. The data from the sinusoidal model consist of twelve sets of bi-hourly values, each set representing a monthly mean daily variation. The data are arranged in solar time to give the amplitude, $(I_d)_{max}$, and the

time of maximum, Φ_m, of the annual mean d.v., and in sidereal time and anti-sidereal time to give $(I_s)_{max}$, Φ'_m, $(I_{as})_{max}$ and Φ''_m.

Secondly, an observed daily variation is not necessarily very well described by a simple diurnal wave. For example, in studies of the solar daily variation of cosmic ray intensity, it is often necessary to use the first and second harmonics to represent the daily variation adequately. Therefore, as a check on the efficacy of using only a first harmonic in that part of the theory which is concerned with modulations of a solar d.v., a second model has been developed. The basis of the model is the mean pressure-corrected daily variation of vertical intensity obtained at 40 m w.e. underground near Hobart, in the form of bi-hourly departures from the daily mean, averaged over the three years 1958-1960. As seen from the curve drawn through the bi-hourly points in Fig. 1 we have, in the shape of the curve, a fairly typical example of a long-term daily variation of c/r intensity. The intensity increases rapidly to its maximum of 0.110 % at 1500 hours local time and thereafter falls off gradually to a minimum of -0.092 % at 0700 hours. In order to use the curve as the basis of a model it is greatly expanded and values read off at intervals of 20 minutes. Appropriate modulations are impressed on these values and the resulting sets of monthly mean daily variations, each comprising twelve bi-hourly values, are then analysed in the usual way.

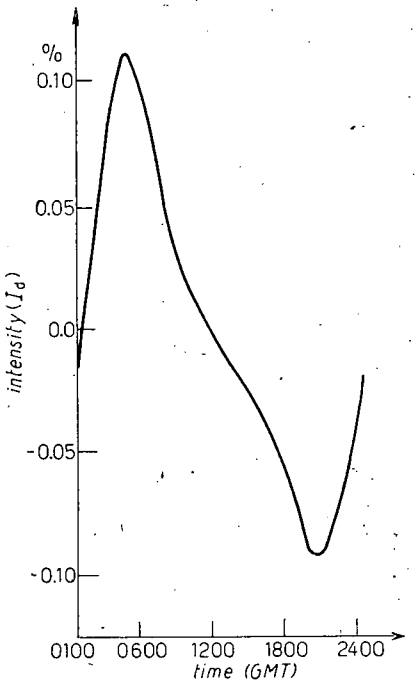

Fig. 1. – The pressure-corrected solar daily variation of vertical intensity at 40 m w.e. underground near Hobart, averaged for the years 1958, 1959 and 1960.

The use of facsimile d.v.s then, should enable us to judge the adequacy of the theory in relation to some of the actual conditions of experiment.

4. – An annual linear displacement of phase of a solar daily variation.

Year by year secular changes of phase may come about in various ways. For instance, as we shall see in a later section, a secular change of phase would accompany a seasonal change of phase having a year by year variability. But

in this section our attention will be confined to the case of a simple continuous linear displacement of phase. Consequently the solar daily variation becomes

$$i_d = A \sin 2\pi\left(t - t_0 - \frac{NT}{288}\right),$$

where N is the total annual displacement of phase in hours,

$$= A \sin\left(\varphi - \frac{N\theta}{24}\right). \tag{4.1}$$

Correspondingly, when the daily variation is expressed in sidereal time and in anti-sidereal time we have respectively,

$$i_s = A \sin\left(\varphi' - \frac{N+24}{24}\theta\right), \tag{4.2}$$

and

$$i_{as} = A \sin\left(\varphi'' - \frac{N-24}{24}\theta\right), \tag{4.3}$$

The resulting annual mean d.v. in solar time is

$$I^d = \frac{1}{2\pi} A \int_0^{2\pi} \sin\left(\varphi - \frac{N\theta}{24}\right) d\theta,$$

and we finally have, in solar time, sidereal time and anti-sidereal time respectively,

$$I_d = \frac{24A}{N\pi} \sin\frac{N\pi}{24} \sin\left(\Phi - \frac{N\pi}{24}\right), \tag{4.4}$$

$$I_s = \frac{24A}{(N+24)\pi} \sin\frac{N\pi}{24} \sin\left(\Phi' - \frac{N\pi}{24}\right), \tag{4.5}$$

$$I_{as} = \frac{24A}{(N-24)\pi} \sin\frac{N\pi}{24} \sin\left(\Phi'' - \frac{N\pi}{24}\right). \tag{4.6}$$

It can be seen that the anti-sidereal amplitude $(I_{as})_{max}$ for one sign of N is the same as the sidereal amplitude $(I_s)_{max}$ for the opposite sign of N. In Figs. 2 and 3, full lines are drawn through values of $(I_d)_{max}$, $(I_s)_{max}$ and $(I_{as})_{max}$ expressed as percentages of $(I_{0d})_{max}$, for values of N in the range $-48 \leqslant N \leqslant +48$. The case of a pure sidereal d.v. at $N = -24$ is prominent as is the case of

a pure anti-sidereal d.v. at $N=+24$. It should be noted that the ratio of $(I_{as})_{max}$ to $(I_s)_{max}$ varies greatly with N.

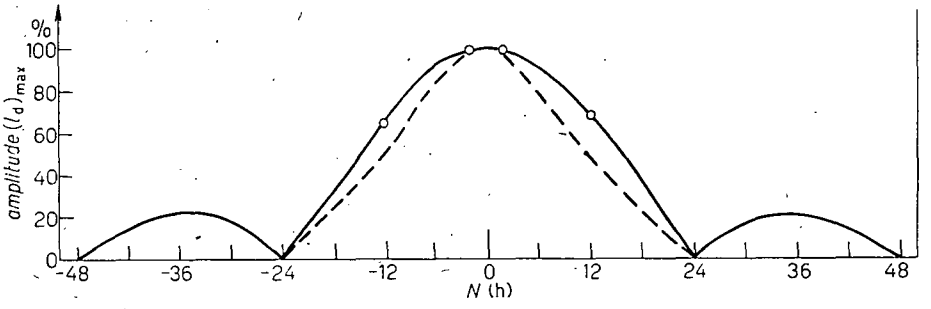

Fig. 2. – The annual mean amplitude of a solar daily variation *vs.* total annual displacement (N) of phase. Full lines: theoretical relationship. Dashed lines: relationship obtained from the « underground » model. Circles: values from a sinusoidal model.

Let us now look at the phase relationships between I_s, I_{as} and I_d for different values of N. For practical reasons we will consider N only in the range

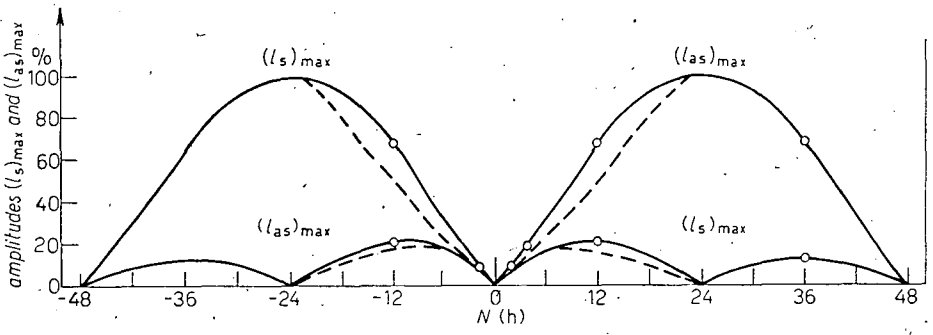

Fig. 3. – The amplitudes $(I_s)_{max}$ and $(I_{as})_{max}$, *vs.* total annual displacement N of phase of a solar diurnal variation. $(I_s)_{max}$ is the amplitude of the annual mean d.v. in sidereal time and $(I_{as})_{max}$ is the corresponding amplitude in anti-sidereal time. Full lines: Theoretical relationship. Dashed lines: Relationship obtained from the « underground » model. Circles: Values from a sinusoidal model.

$-24 \leqslant N \leqslant +24$. First of all then, for $0 < N \leqslant +24$ the amplitude terms of I_d and I_s are positive while that of I_{as} is negative and so it can readily be seen that

$$\Phi'_m = \Phi''_m + \pi = \Phi_m .$$

Again, for $-24 \leqslant N < 0$, the amplitude terms of I_d and I_{as} are positive while that of I_s is negative so that

$$\Phi'_m = \Phi''_m + \pi = \Phi_m + \pi .$$

Facsimile daily variations of intensity with linear displacement of phase have been developed for comparison with the theory from the sinusoidal and « underground » models described in Section 3. In this case the basis of the sinusoidal model is the expression

$$i_d = -1000 \sin 2\pi \left(t - \frac{15}{24} - \frac{NT}{288} \right).$$

The data derived from this expression for a given value of N consist of twelve sets of bi-hourly values, each set representing a monthly mean diurnal variation of intensity. Thus the monthly mean d.v. for month r comprises the twelve values of i_d for $t = 0, 2, 4, 6, \ldots, 22$, where each value is the average of the values for $T = r$ and $T = r - 1$. Analyses were carried out in solar, sidereal and anti-sidereal time for $N = \pm 2, \pm 12$ and $+36$. The amplitude values are shown as circles in Fig. 2 and 3 and, as one would expect, are in close agreement with the theory.

The « underground » model gives a somewhat different result. Facsimile d.v.s were developed for $N = \pm 2$, ± 4, ± 6, ± 8, ± 10, ± 12, ± 18 and ± 22. Dashed lines are drawn through the amplitude points in Figs. 2 and 3 and it is clear that the values from the « underground » model, between the limiting values at $N = 0$ and $N = \pm 24$, are somewhat smaller than those predicted by the theory.

The phase relationships are shown in Fig. 4, where full lines represent the variations of $\Phi_m - \Phi_{0m}$, $\Phi'_m - \Phi_m$ and $\Phi''_m - \Phi_m$ with N in the range $-12 \leqslant N \leqslant +12$. The squares represent values from the sinusoidal model and values from the « underground » model are shown as circles. There is fairly good agreement with

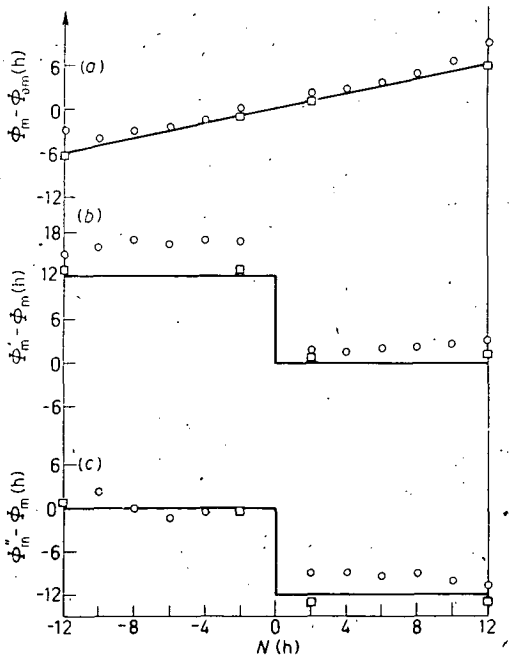

Fig. 4. – The relationships between the annual mean solar (Φ_m), sidereal (Φ'_m) and anti-sidereal (Φ''_m) diurnal maxima for a solar daily variation with linear displacement of phase, *vs.* N, the total annual phase displacement. Φ_{0m} is the time of maximum of the unmodulated d.v. Full lines: Theoretical relationships. Squares: Values from a sinusoidal model. Circles: Values from the « underground » model.

the theory except for a marked discrepancy in the values of Φ'_{in} from the « underground » model for negative N.

When we consider that many reported values of the apparent sidereal d.v. of c/r intensity are of the order of 10% of the solar d.v. or less, it is apparent that a linear displacement of phase of the solar d.v. could have an appreciable contaminating effect. A year to year mean displacement of phase of only two hours would produce a fictitious sidereal d.v. somewhere between 10% and 20% of the solar d.v. Mean secular displacements of phase of two hours or more, per year, do in fact take place in the course of the long term cycle of variation of phase and amplitude of the diurnal variation of c/r intensity at sea level. However, such changes may not be even approximately linear within each year. For example, the secular changes may be the result of year to year variations in the distribution of days when solar daily variations of a certain type are present.

5. – An annual linear displacement of phase of a sidereal daily variation.

It will be assumed throughout the sections concerning the effects produced by modulations of a sidereal d.v. that a genuine sidereal daily variation of cosmic ray intensity is adequately represented in the unmodulated form by

$$i_s = A \sin 2\pi(t' - t'_0) .$$

For an annual linear displacement of phase we have

$$i_s = A \sin 2\pi \left(t' - t'_0 - \frac{NT}{288}\right),$$

where N is the total annual displacement of phase in hours

$$= A \sin\left(\psi' - \frac{N\theta}{24}\right). \tag{5.1}$$

The daily variation expressed in anti-sidereal time is

$$i_{as} = A \sin\left(\psi'' - \frac{N-48}{24}\theta\right). \tag{5.2}$$

The annual mean d.v. in sidereal time is

$$I_s = \frac{A}{2\pi}\int_0^{2\pi} \sin\left(\psi' - \frac{N\theta}{24}\right) d\theta, \; = \; \frac{24A}{N\pi} \sin\frac{N\pi}{24} \sin\left(\Psi' - \frac{N\pi}{24}\right). \tag{5.3}$$

Similarly, in anti-sidereal time

$$I_{as} = \frac{24A}{(N-48)\pi} \sin\frac{N\pi}{24} \sin\left(\Psi'' - \frac{N\pi}{24}\right). \tag{5.4}$$

The amplitudes $(I_s)_{max}$ and $(I_{as})_{max}$ of the annual mean d.v.s are shown as functions of N in the range $-48 \leqslant N \leqslant +48$ in Fig. 5. They are expressed as percentages of the amplitude $(I_{0s})_{max}$ of the unmodulated sidereal d.v.

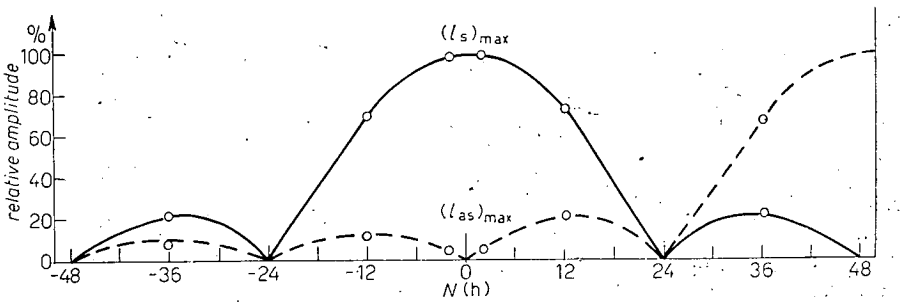

Fig. 5. – The amplitudes of the annual mean sidereal (I_s) and anti-sidereal (I_{as}) diurnal variations associated with a linear displacement of phase of a sidereal diurnal variation, versus N, the total annual phase displacement. The amplitudes are measured relative to the amplitude of the unmodulated sideral d.v.

Facsimile d.v.s were derived from a sinusoidal model by the method described in Section 4. Results from the model were compared with the theory for $N \pm 2$, ± 12 and ± 36. The amplitude values, represented by circles in Fig. 5, were found to be in good agreement with the theory.

Summarising the phase relationships for N in the range $-24 \leqslant N \leqslant +24$ only:

a) *When* $0 < N \leqslant +24$. Since the amplitude term of I_s is positive and that of I_{as} is negative we have

$$\Psi'_m = \Psi''_m + \pi .$$

b) *When* $-24 \leqslant N < 0$. Since the amplitude terms of I_s and I_{as} are both positive we have

$$\Psi'_m = \Psi''_m .$$

In Fig. 6(*a*) the full lines represent Ψ'_m the time of maximum of the annual mean sidereal d.v., as a function of N, in relation to Ψ'_{0m}, the time of maximum of the unmodulated d.v. The circles represent, of course, the corresponding values obtained from the analyses of the model variations. Figure 6(*b*) shows

how the time of maximum of the annual sidereal d.v. is related to the time of maximum, Ψ''_m, of the anti-sidereal d.v. as a function of N. The values obtained from the model indicate that there might be some uncertainty in predicting the time of maximum of, say, the anti-sidereal variation produced by a sidereal d.v. of cosmic ray intensity with known annual linear displacement of phase.

It can be seen that the annual anti-sidereal variation is practically negligible except for very large positive displacements of phase.

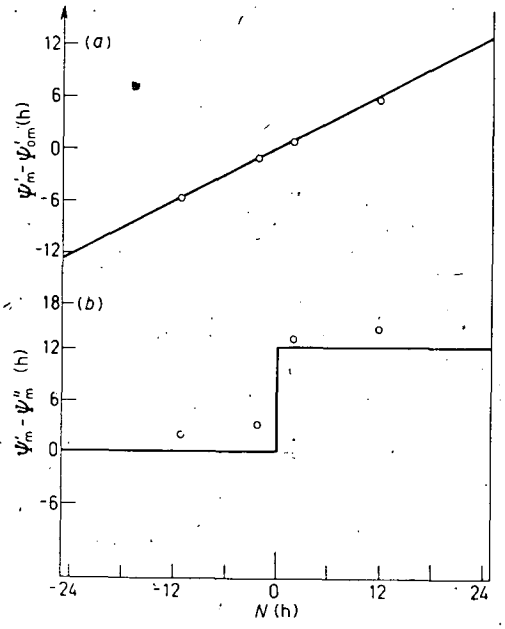

Fig. 6. – The relationships between the annual mean sidereal (Ψ'_m) and anti-sidereal (Ψ''_m) diurnal maxima for a sidereal diurnal variation with linear displacement of phase, *vs.* N, the total annual phase displacement; Ψ'_{0m} is the time of maximum of the unmodulated d.v. Full lines: theoretical. Circles: values from a sinusoidal model.

6. – An annual linear displacement of amplitude of a solar daily variation.

We have

$$i_d = A\left(1-\frac{KT}{12}\right)\sin 2\pi(t-t_0) = A\left(1-\frac{K\theta}{2\pi}\right)\sin\varphi\,, \tag{6.1}$$

where K is the total annual fractional change of amplitude. Similarly, when the solar d.v. is expressed in sidereal and anti-sidereal time respectively, we get

$$i_s = A\left(1-\frac{K\theta}{2\pi}\right)\sin(\varphi'-\theta)\,, \tag{6.2}$$

$$i_{as} = A\left(1-\frac{K\theta}{2\pi}\right)\sin(\varphi''+\theta)\,. \tag{6.3}$$

The annual mean d.v. in solar time is then

$$I_d = \frac{A}{2\pi}\sin\varphi\int_0^{2\pi}\left(1-\frac{K\theta}{2\pi}\right)\mathrm{d}\theta\,.$$

We finally obtain the following expressions for the annual mean diurnal variations in solar time, sidereal time and anti-sidereal time

$$I_d = A\left(1-\frac{K}{2}\right)\sin\Phi\,, \tag{6.4}$$

$$I_s = A\,\frac{K}{2\pi}\sin\left(\Phi'-\frac{\pi}{2}\right), \tag{6.5}$$

and

$$I_{as} = A\,\frac{K}{2\pi}\cos\Phi''. \tag{6.6}$$

For $K<2$ the relationship between the times of maximum of the annual mean d.v.s is

$$\Phi'_m = \Phi_m + \pi = \Phi_m + \frac{\pi}{2}\,.$$

The ratio of amplitudes of I_s and I_{as} is unity and the ratio of the amplitude of I_s to the amplitude of the annual mean solar d.v. is

$$\frac{(I_s)_{max}}{(I_d)_{max}} = \frac{K}{\pi(2-K)}\,.$$

These relationships were found to be in close agreement with the results of analyses of facsimile diurnal variations derived from the expression

$$i_d = 1000\left(1-\frac{T}{24}\right)\sin 2\pi\left(t-\frac{9}{24}\right),$$

i.e.

$$K = 0.5\,,\quad t_0 = \tfrac{9}{24} \quad\text{and}\quad A = 1000\,.$$

A value of $K=0.6$ will give a fictitious sidereal d.v. whose amplitude is equal to about 10% of the amplitude of the solar d.v. That is to say, an annual mean reduction in amplitude of about 30% is sufficient to give rise to an apparent sidereal d.v. whose amplitude is about 10% of the amplitude of the solar d.v. However, although year to year changes of amplitude of, for example, the d.v. of meson intensity at sea level are occasionally as great as 30%, it is doubtful if they are uniformly linear. And, of course, the amplitude changes are generally accompanied by changes of phase as well. For that reason, the following section has been included.

7. – Combined linear changes of phase and amplitude of a solar diurnal variation.

In this case a solar diurnal variation is represented by

$$i_d = A\left(1-\frac{KT}{12}\right)\sin 2\pi\left(t-t_0-\frac{NT}{288}\right),$$

where K is the total annual fractional change of amplitude and N, in hours, is the total annual displacement of phase.

Thus.

$$i_d = A\left(1-\frac{K\theta}{2\pi}\right)\sin\left(\varphi-\frac{N\theta}{24}\right). \tag{7.1}$$

In sidereal time we have

$$i_s = A\left(1-\frac{K\theta}{2\pi}\right)\sin\left(\varphi'-\frac{N+24}{24}\,\theta\right), \tag{7.2}$$

while in anti-sidereal time

$$i_{as} = A\left(1-\frac{K\theta}{2\pi}\right)\sin\left(\varphi''-\frac{N-24}{24}\,\theta\right). \tag{7.3}$$

The expressions for the annual mean diurnal variations are rather cumbersome but may be reduced to the form

$$f(\beta) = r\cos(\beta-\alpha), \tag{7.4}$$

where

$$r=\sqrt{p^2+q^2}\,; \qquad \cos\alpha = p/r\,, \qquad \sin\alpha = q/r\,,$$

$$p=\frac{A}{2R\pi}\left\{\frac{K}{2R\pi}\sin 2R\pi-(K-1)\cos 2R\pi-1\right\},$$

and

$$q=\frac{A}{R\pi}\sin R\pi\left\{\frac{K}{2R\pi}\sin R\pi-\cos R\pi\left(\frac{K}{2}-1\right)\right\}.$$

When

$$f(\beta)=I_d\,, \qquad \beta=\Phi \qquad \text{and} \qquad R=\frac{N}{24}.$$

When

$$f(\beta)=I_s\,, \qquad \beta=\Phi' \qquad \text{and} \qquad R=\frac{N+24}{24}.$$

When

$$f(\beta)=I_{as}\,, \qquad \beta=\Phi'' \qquad \text{and} \qquad R=\frac{N-24}{24}.$$

As an example, let us consider a solar d.v. for which there is a mean annual reduction of amplitude of about 12 % and a mean advance of phase of two hours. A combined secular advance of phase and secular decrease of amplitude of about this magnitude is to be found in the annual mean d.v. of high latitude meson intensity at sea level between 1955 and 1958. See PARSONS (6). Putting $K=0.25$ and $N=+4$ in the expressions for i_d, the annual mean diurnal variations become

$$I_d = 0.836 \cos(\Phi - 0.658\,\pi),$$

$$I_s = 0.133 \cos(\Phi' - 0.697\,\pi),$$

$$I_{as} = 0.189 \cos(\Phi'' + 0.265\,\pi).$$

Both sinusoidal and « underground » models were constructed to give comparisons with the theory for $K=0.25$ and $N=+4$. In each case the model daily variations obtained in Section **4** for a linear displacement of phase with $N=+4$ were multiplied month by month by the factor $(48-T)/48$. The results obtained by all three methods are shown for comparison in Table I. It can be seen that agreement as to relative amplitudes of the maxima is fairly good while the greatest discrepancy in the time of maximum is about two hours.

TABLE I. – *The effect of combined linear changes of phase and amplitude of a solar diurnal variation for* $N=+4$, $K=0.25$ *and* $t_0=0$.

	Annual solar d.v.		Sidereal d.v.		Anti-sidereal d.v.	
	Amplitude % of $(I_{od})_{max}$	Time of Max (h)	Amplitude % of $(I_{od})_{max}$	Time of Max (h)	Amplitude % of $(I_{od})_{max}$	Time of Max (h)
Theory	83.6	0800	13.3	0800	18.9	2100
Sinusoidal model	83.3	0800	12.6	0900	19.3	2200
« Underground » model	80.3	0800	9.7	1000	15.6	2300

8. – An annual linear change of amplitude of a sidereal daily variation.

The characteristics of the resulting annual mean sidereal d.v. are, of course, exactly the same as those derived for the solar d.v. in Section **6**. Thus we

(6) N. R. PARSONS: *Ph. D. thesis* (Hobart, 1959).

have, in sidereal time,

$$i_s = A\left(1-\frac{KT}{12}\right)\sin 2\pi(t'-t'_0)\,,$$

where K is the total fractional displacement of amplitude per year,

$$= A\left(1-\frac{K}{2\pi}\theta\right)\sin\psi'. \tag{8.1}$$

Expressing the sidereal d.v. in anti-sidereal time, we have

$$i_{as} = A\left(1-\frac{K}{2\pi}\theta\right)\sin\left(\psi''+2\theta\right). \tag{8.2}$$

The annual mean d.v. in sidereal time is

$$I_s = \frac{A}{2\pi}\sin\psi'\int_0^{2\pi}(1-\frac{K}{2\pi}\theta)\,\mathrm{d}\theta = A\left(1-\frac{K}{2}\right)\sin\Psi'\,, \tag{8.3}$$

while in anti-sidereal time we get

$$I_{as} = \frac{AK}{4\pi}\cos\Psi''\,. \tag{8.4}$$

The ratio of amplitudes is then

$$\frac{(I_{as})_{max}}{(I_s)_{max}} = \frac{K}{2\pi(2-K)}\,.$$

The phase relationships, for $K<2$, are simply

$$\Psi'_m = \Psi'_{0m} = \frac{\pi}{2}\,,$$

and

$$\Psi''_m = 0\,.$$

This section has been included for completeness of treatment, but naturally a very large linear change of amplitude would be required to produce a significant anti-sidereal variation. For instance, if $K=0.5$, the amplitude of the annual mean d.v. in anti-sidereal time is only about 5% of the amplitude of the annual mean sidereal d.v. which itself is typically only of the order of 0.05%.

9. – A seasonal variation of amplitude of a solar daily variation.

In this and the following three sections we can visualise more clearly what is happening if we make use of the techniques employed in radio theory to analyse amplitude and phase modulations of carrier waves. This is essentially the way FARLEY and STOREY (4) treated the case of an annual variation of amplitude of a solar d.v. We will recapitulate their treatment briefly and then use the same method to deal with some other possibilities.

It is well-known that when a sinusoidal carrier wave is amplitude-modulated with a sinusoidal signal the modulated carrier may be resolved into three components: the original carrier and two sideband components whose amplitudes are equal and whose frequencies equal the carrier frequency plus and minus the modulating frequency. The carrier in the present case is a solar diurnal wave and therefore its frequency is 365 cycles/year. If it is modulated to give an annual variation of amplitude the modulating frequency is 1 cycle/year. Therefore, the modulated carrier may be resolved into the original carrier plus two sidebands, symmetrically disposed about it in frequency, at 366 and 364 cycles/year. These are, of course, the frequencies of a sidereal diurnal wave and an anti-sidereal diurnal wave respectively and we may conveniently call them the sidereal and anti-sidereal frequencies. If we now arrange our modulated solar d.v. in sidereal time, for instance, and obtain the average variation over a complete year we extract the sideband of sidereal frequency and cause the other two components to vanish. Similarly, analysis in anti-sidereal time allows us to extract only the sideband component whose frequency is 364 cycles/year.

If the amplitude modulation is semi-annual, that is to say the modulation frequency is 2 cycles/year, then the sideband frequencies are 363 and 367 cycles/year. Consequently a semi-annual variation of amplitude will not produce an annual mean d.v. in sidereal or antisidereal time. In fact it is evident that no integral multiple of the annual (1 cycle/year) modulating frequency can give rise to sideband components at the sidereal or anti-sidereal frequencies.

Returning to the annual variation, a solar d.v. with annual modulation of amplitude may be expressed as

$$i_a = A\left\{1 + k\cos 2\pi\left(\frac{T}{12} - \frac{T_0}{12}\right)\right\}\sin 2\pi\,(t - t_0)\,.$$

Here k is the maximum fractional change of amplitude and T_0 is the time of year (in months measured from the September equinox) when maximum amplitude occurs.

Then

$$i_d = A\{1 + k\cos(\theta - \theta_0)\}\sin\varphi\,,$$

where

$$\theta_0 = \frac{2\pi T_0}{12}$$

(9.1) $$= A\sin\varphi + \frac{Ak}{2}\sin\{\varphi + (\theta - \theta_0)\} + \frac{Ak}{2}\sin\{\varphi - (\theta - \theta_0)\}\,.$$

Remembering that $\varphi = 2\pi f_1(t - t_0)$, where the frequency f_1 is 1 cycle/day and that $\theta = 2\pi f_2(T/12)$ where the frequency f_2 is $\frac{1}{12}$ cycle/month or 1 cycle/year we can see that the first term on the right-hand side of eq. (9.1) represents the original carrier, while the second and third terms represent the sideband components at the sidereal and anti-sidereal frequencies. Integrating with respect to θ and averaging we readily obtain the following annual mean d.v.s in solar time, sidereal time and anti-sidereal time:

(9.2) $$I_d = A\sin\Phi\,,$$

(9.3) $$I_s = \frac{Ak}{2}\sin(\Phi' - \theta_0)\,,$$

(9.4) $$I_{as} = \frac{Ak}{2}\sin(\Phi'' + \theta_0)\,.$$

This is the well-known case that is generally thought to apply whenever the attempt is made to correct an observed apparent sidereal daily variation of intensity for a spurious component. In fact, the assumption that is made is even more restrictive, namely, that the anti-sidereal effect is produced by a seasonal change of amplitude of the familiar solar diurnal variation of intensity and not of some other secondary daily variation having a different time of maximum.

In a more complete treatment of this case to be given in a following paper it will be shown that the amplitude and phase relationships derived from the « underground » model differ considerably from those predicted by the theory.

10. – A seasonal variation of amplitude of a sidereal daily variation.

The frequency of the carrier in this case is 366 cycles/year. An annual modulation of amplitude produces symmetrical sideband components at frequencies of 365 and 367 cycles/year and therefore, although there is a spurious

component in solar time, there is no component at the anti-sidereal frequency.

In the semi-annual case, the modulation frequency is 2 cycles/year giving rise to a sideband component at the anti-sidereal frequency, 364 cycles/year, and a matching component at 368 cycles/year. Clearly, no other integral values of the modulating frequency can produce effects in anti-sidereal time.

Looking at the semi-annual case in more detail, we may represent the sidereal d.v. by the expression

$$(10.1)\quad i_s = A\{1 + k\cos(2\theta - \theta_0)\}\sin\psi' = A\sin\psi' + \frac{Ak}{2}\sin\{\psi' + (2\theta - \theta_0)\} + \frac{Ak}{2}\sin\{\psi' - (2\theta - \theta_0)\}.$$

The first term on the right-hand side represents the unmodulated carrier, while the second term represents the sideband component at the anti-sidereal frequency. Upon integration with respect to θ the annual mean d.v.s in sidereal and anti-sidereal time are then

$$(10.2)\quad I_s = A\sin\Psi'$$

and

$$(10.3)\quad I_{as} = \frac{Ak}{2}\sin(\Psi'' + \theta_0).$$

This type of modulation of a sidereal d.v. can give rise to an appreciable anti-sidereal variation, the amplitude relationship being

$$\frac{(I_{as})_{max}}{(I_s)_{max}} = \frac{k}{2}.$$

The time of maximum of the annual anti-sidereal d.v. depends on the phase constant of the seasonal variation of amplitude. The phase relationship is

$$\Psi'_m = \Psi''_m + \theta_0 = \Psi'_{0m}.$$

The amplitude and phase relationships were found to be in complete agreement with the results of an analysis of facsimile d.v.s derived from the expression

$$i_s = 1000\left\{1 + \cos 2\pi\left(\frac{T}{6} - 1\right)\right\}\sin 2\pi t',$$

i.e. $A = 1000$, $k = 1$, $T_0 = 12$ and $t'_0 = 0$.

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11. – A seasonal variation of phase of a solar daily variation.

a) *Annual sinusoidal.* The expression for the solar d.v. is

$$i_d = A \sin 2\pi \left\{ t - t_0 - \frac{n}{24} \cos 2\pi \left(\frac{T}{12} - \frac{T_0}{12} \right) \right\},$$

where n, in hours and always positive, is the maximum advance of time of maximum and occurs T_0 months after the September equinox. To simplify the expression we have

$$i_d = A \sin (\varphi - a \cos \alpha),$$

where

$$a = \frac{\pi n}{12} \text{ and } \alpha = \theta - \theta_0 = \frac{\pi T}{6} - \frac{\pi T_0}{6}$$

$$= A \sin \varphi \cos (a \cos \alpha) - A \cos \varphi \sin (a \cos \alpha). \tag{11.1}$$

If we expand $\cos (a \cos \alpha)$ and $\sin (a \cos \alpha)$ in terms of Bessel functions we may then obtain the solar d.v. in the following form (*e.g.* see GOLDMAN (7)):

$$\begin{aligned} i_d = A[&J_0(a) \sin \varphi - J_1(a) \cos (\varphi + \alpha) - J_1(a) \cos (\varphi - \alpha) - \\ &- J_2(a) \sin (\varphi + 2\alpha) - J_2(a) \sin (\varphi - 2\alpha) + \\ &+ J_3(a) \cos (\varphi + 3\alpha) + J_3(a) \cos (\varphi - 3\alpha) + \dots]. \end{aligned} \tag{11.2}$$

We can see from eq. (11.2) that the phase-modulated carrier may be represented by a component at the carrier frequency plus an infinite number of pairs of sidebands at plus and minus integral multiples of the modulating frequency, and that the first two sidebands are at the sidereal and anti-sidereal frequencies. Therefore, this type of modulation of a solar d.v. must give rise to daily variations in sidereal and anti-sidereal time.

Upon integration, we obtain for the annual mean d.v. in solar time

$$I_d = A J_0 \left(\frac{\pi n}{12} \right) \sin \Phi. \tag{11.3}$$

The corresponding annual mean d.v.s in sidereal and anti-sidereal time can be obtained from eq. (11.2) by integration, after replacing φ by $\varphi' - \theta$ and $\varphi'' + \theta$

(7) S. GOLDMAN: *Frequency Analysis, Modulation and Noise* (New York, 1948).

in each case, giving finally

$$I_s = AJ_1\left(\frac{\pi n}{12}\right)\sin\left(\Phi' - \theta_0 - \frac{\pi}{2}\right), \tag{11.4}$$

and

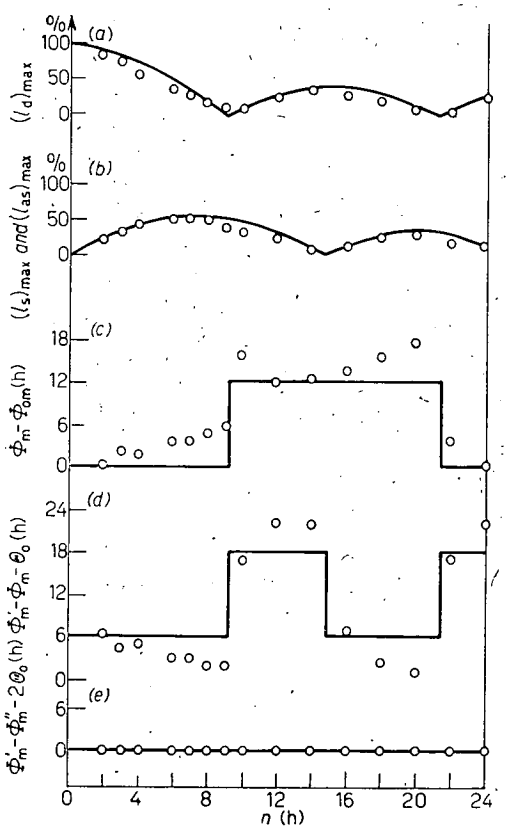
$$I_{as} = AJ_1\left(\frac{\pi n}{12}\right)\sin\left(\Phi'' + \theta_0 - \frac{\pi}{2}\right). \tag{11.5}$$

The Bessel functions are periodic in « a » and are therefore positive or negative depending on the value of « a ». Consequently, the amplitudes of I_d, I_s and I_{as} are proportional to the moduli of the appropriate Bessel functions, and we have

$$(I_d)_{max} = A\left|J_0\left(\frac{\pi n}{12}\right)\right|,$$

and

$$(I_s)_{max} = (I_{as})_{max} = A\left|J_1\left(\frac{\pi n}{12}\right)\right|.$$



The amplitudes as functions of n in the range $0 < n \leqslant 24$ are shown as full lines in Fig. 7(a) and 7(b).

When $n \sim 9.2$ and when $n \sim 21.4$ the function $J_0(\pi n/12)$ changes sign while $J_1(\pi n/12)$ changes sign when $n \sim 14.8$. Consequently there are reversals of phase at these points as shown in Fig. 7(c), 7(d) and 7(e), where the full lines represent the phase relationships as functions of

Fig. 7. – The amplitude and phase relationships for the annual mean solar (I_d, Φ_m), sidereal (I_s, Φ_m') and anti-sidereal (I_{as}, Φ_m'') diurnal variations produced by a solar d.v. with an annual variation of phase, *vs.* n, the maximum phase displacement. Φ_{0m} is the time of maximum of the unmodulated d.v. and Θ_0 is the phase constant determined by the time of year when the maximum displacement of phase occurs. Full lines: theory. Circles: values from the « underground » model.

n in the range $0 < n \leqslant 24$. It suffices to state here that in the sub-interval that is perhaps of greatest interest, *i.e.* $0 < n \leqslant 9.2$, we have

$$\Phi'_{\mathrm{m}} = \Phi''_{\mathrm{m}} + 2\theta_0 = \Phi_{\mathrm{m}} + \theta_0 + \frac{\pi}{2}.$$

The « underground » model described in Section 3 was phase-modulated to obtain sets of simulated daily variations with seasonal displacement of phase. The following procedure was used. The unmodulated « underground » daily variation may be represented by the function $i_{\mathrm{d}} = f(t - t_0)$. We now have to construct $f(t - t_0 - t_{\theta n})$, where $t_{\theta n} = (n/24) \cos 2\pi((T/12) - (T_0/12))$ and like t and t_0, is in units of days.

We arbitrarily select 2 400 hours on the 31st of December as the time of maximum displacement of phase for the model. This means that $T_0 \sim 3.3$, in months. Expressing $t_{\theta n}$ in hours, for a number of days after the 31st of December it remains within half an hour of its maximum value for a given value of n. This can be readily calculated, since if $t_{\theta n}$ is expressed in degrees it changes at the rate of very nearly 1° per day (360 degrees in 365 days). A complete month of 30 days may now be divided up into groups of days for each of which $24 t_{\theta n}$ is a constant number of hours to the nearest half an hour. If, for example, we put $n = 24$ the maximum displacement of phase is 24 hours on the 31st of December. For the first twelve days of January $24 t_{\theta n}$ is within half an hour of 24 hours, for the eight following days it is within half an hour of 23 hours, for the next nine days it is within half an hour of 22 hours and for the last two days it is within half an hours of 21 hours. We neglect the two days for which $t_{\theta n} = 21$. It can be seen that the number of days for which $24 t_{\theta n}$ is 24, 23 and 22 respectively, are approximately in the ratio 3 : 2 : 2. We can now construct the weighted mean daily variation for January from three arrangements of the « underground » model, namely with the time of maximum advanced 24, 23 and 22 hours respectively. We proceed in a similar way to construct monthly mean d.v.s for the other months of the year. The twelve-monthly sets of weighted mean values of the daily variations may then be arranged in the usual way to give *a*) the annual mean solar daily variation, *b*) the annual mean apparent sidereal effect and *c*) the anti-sidereal variation. In a similar manner sets of values are obtained for other values of n.

Models were constructed for $n = 2$, 3, 4, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 22 and 24. For comparison with the theory the various amplitudes and phase relations are shown in Fig. 7 as circles. The amplitudes are in good agreement with the theory, suggesting that the theory is adequate to predict the sidereal and anti-sidereal amplitudes for known seasonal changes of phase. There is also agreement for the relationship between Φ'_{m} and $\Phi''_{\mathrm{m}} + 2\theta_0$, as seen in Fig. 7(*e*). However, there is marked disagreement in the prediction of the

variation of the time of maximum, Φ_m, of the annual mean solar d.v. with n. The model studies indicate that there is a tendency for the annual mean solar diurnal maximum to advance gradually with increasing values of n. If this is so, then a solar diurnal variation which exhibits a seasonal variation of phase will also show an annual secular displacement of phase which may change from year to year with changing values of n, even if these values are quite small.

It should be noted that a seasonal displacement of phase with maximum displacement of only two hours is sufficient to cause a spurious sidereal d.v. whose amplitude is 25% of the solar daily variation.

b) Semi-annual sinusoidal. We can very easily obtain the phase-modulated diurnal variation for this case by putting 2α for α, 4α for 2α, and so on, in eq. (11.2). It can be seen at once that the modulated carrier has no sideband components at the sidereal or anti-sidereal frequencies. Therefore we conclude that this type of modulation of a solar d.v. gives rise to no annual mean daily variation in sidereal or anti-sidereal time.

12. – A seasonal variation of phase of a sidereal daily variation.

a) Annual sinusoidal. The expression for the modulated sidereal diurnal variation i_s is the same as that for i_d in eq. (11.2) if ψ' is substituted for φ. The carrier frequency is now 366 cycles/year and we can see that there is a sideband component $-AJ_1(a)\cos(\psi'-\alpha)$ at the solar frequency and another sideband $-AJ_2(a)\sin(\psi'-2\alpha)$ at the anti-sidereal frequency.

The annual mean d.v.s in sidereal and anti-sidereal time can readily be found after integration of $AJ_0(a)\sin\psi'$ and $-AJ_2(a)\sin(\psi''-a\alpha)$ with respect to θ.

We then have

$$I_s = AJ_0(a)\sin\Psi' \tag{12.1}$$

and

$$I_{as} = -AJ_2(a)\sin(\Psi'' + 2\theta_0)\,. \tag{12.2}$$

The amplitudes of I_s and I_{as} are proportional to the moduli of the respective Bessel functions:

$$(I_s)_{max} = A\,|J_0(a)| = A\left|J_0\left(\frac{\pi n}{12}\right)\right|.$$

$$(I_{as})_{max} = A\,|J_2(a)| = A\left|J_2\left(\frac{\pi n}{12}\right)\right|.$$

The full lines in Fig. 8(*a*) and 8(*b*) represent $(I_s)_{max}$ and $(I_{as})_{max}$ as functions of the maximum displacement n, of time of maximum of the sidereal d.v. for $0 < n \leqslant 24$. They are, as usual, expressed as percentages of the amplitude $(I_{0s})_{max}$ of the unmodulated d.v. The corresponding phase relationships are shown in Figs. 8(*c*) and 8(*d*), We have, for example, in the sub-interval $0 < n \leqslant 9.2$:

$$\Psi'_m = \Psi''_m + 2\theta_0 - \frac{\pi}{2} = \Psi'_{0m},$$

where Ψ'_{0m} is the time of maximum of the unmodulated d.v.

In Section **11** the method of development of facsimile diurnal variations with seasonal variation of phase is fully described. The same method was employed in this case except that, instead of the « underground » model, the following cosine model was used:

$$i_s = 1000 \cos 2\pi \cdot \left\{ t' - \frac{5}{24} - \frac{n}{24} \cos \frac{2\pi T}{12} \right\},$$

i.e. $A = 1000$, $t'_0 = \frac{5}{24}$ and $T_0 = 0$.

Facsimile daily variations were developed for each of the following values of n: 2, 4, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 22 and 24. In Fig. 8 the circles represent values obtained from the model. It was perhaps to be expected that the amplitudes obtained from the model would be in agreement with the theory but the equally good agreement in phase was rather surprising in view of the discrepancies found in Section **11** when the « underground » model was used. It gives support to the view that the theory in Section **11** based on a simple sinusoidal repre-

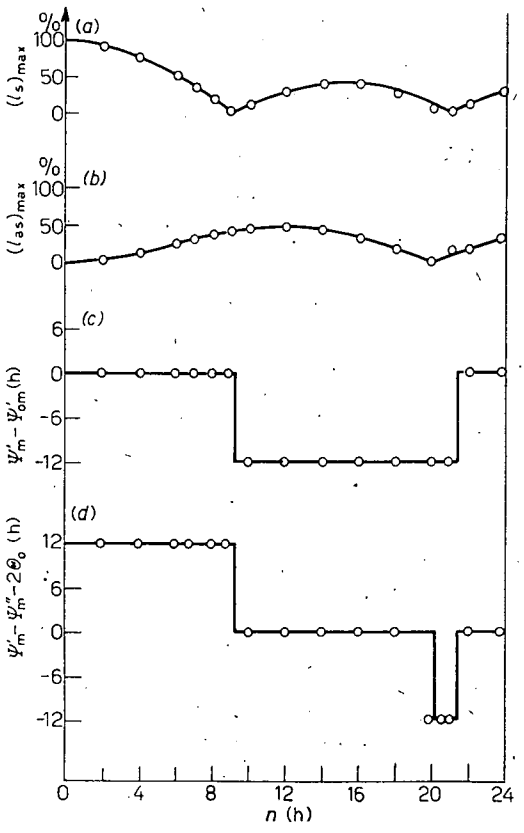


Fig. 8. – The amplitude and phase relationships between the annual mean sidereal (I_s, Ψ'_m) and anti-sidereal (I_{as}, Ψ''_m) diurnal variations for a sidereal d.v. with an annual variation of phase, *vs.* n, the maximum phase displacement. Ψ'_{0m} is the time of maximum of the unmodulated d.v. and the phase constant Θ_0 is determined by the time of the year when maximum displacement of phase occurs. Full lines: theory. Circles: values from a sinusoidal model.

sentation of the solar d.v. of cosmic ray intensity was not adequate to predict the phase relationships accurately.

Clearly, the relationship between I_s and I_{as} varies greatly with n. A rather bizarre example is that of a sidereal d.v. with a maximum seasonal displacement of phase of about nine hours. There would be no annual mean sidereal daily variation but the anti-sidereal effect would be quite appreciable.

b) *Semi-annual sinusoidal.* Referring again to eq. (11.2), we find that the expression for the modulated sidereal d.v. in this case can be easily obtained by substituting ψ' for φ, 2α for α, 4α for 2α and so on. The component at the carrier frequency is, of course, $AJ_0(a)\sin\psi'$ as for the case of an annual modulation of phase but the sideband component at the antisidereal frequency is now $-AJ_1(a)\cos(\psi'-2\alpha)$. There is no sideband at the solar frequency. The annual mean d.v.s are found in the usual manner and thus we have

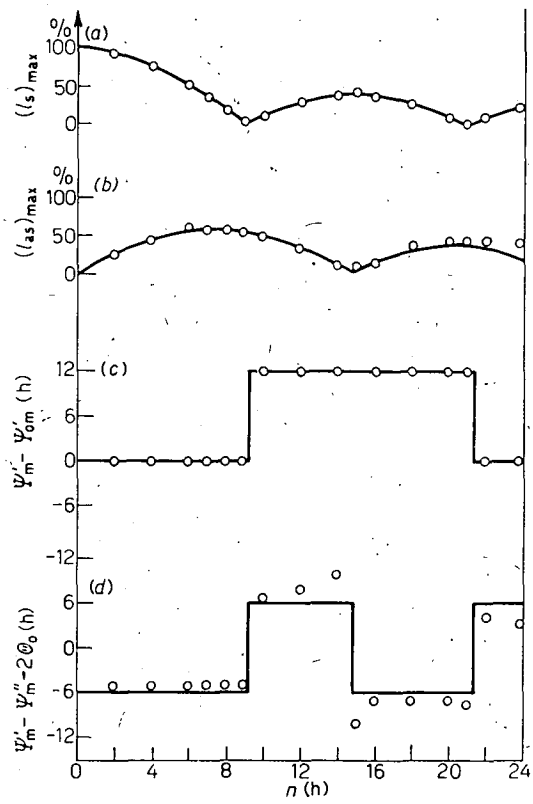


Fig. 9. – The amplitude and phase relationships between the annual mean sidereal (I_s, Ψ'_m) and anti-sidereal (I_{as}, Ψ''_m) diurnal variations for a sidereal d.v. with semiannual variation of phase, *vs.* n, the maximum displacement of phase. Ψ'_{0m} is the time of maximum of the unmodulated d.v. and the phase constant Θ_0 is determined by the time of the year when maximum displacement of phase occurs. Full lines: theory. Circles: values from a sinusoidal model.

$$(12.3)\qquad I_s = AJ_0(a)\sin\Psi'$$

and

$$(12.4)\qquad I_{as} = AJ_1(a)\sin\cdot\left(\Psi'' + 2\theta_0 - \frac{\pi}{2}\right).$$

The phase relationships, as functions of n in the range $0 < n \leqslant 24$, are represented by full lines in Fig. 9(*c*) and 9(*d*). Phase reversals accompany changes of sign of the amplitude terms $AJ_0(\pi n/12)$ and $AJ_1(\pi n/12)$, therefore we note here merely that in the sub-range $0 < n \leqslant 9.2$ we have

$$\Psi'_m = \Psi''_m + 2\theta_0 - \frac{\pi}{2} = \Psi'_{0m}.$$

Using the method outlined in Section **11** *a*), the cosine model

$$i_s = 1\,000 \cos 2\pi \left(t' - \frac{5}{24} - \frac{n}{24} \cos \frac{2\pi T}{6} \right)$$

was developed to obtain facsimile diurnal variations with semi-annual variations of phase for the following values of n: 2, 4, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22 and 24. The various amplitude and phase relationships are shown in Fig. 9 where the circles representing values obtained from the model analyses may be compared with the theoretical effect. It can be seen that there is good overall agreement although there are certain minor discrepancies in the evaluation of the time of maximum, Ψ''_m, of the anti-sidereal daily variation when the two methods are compared.

13. – Curvature associated with the eleven-year cycle of intensity variation.

As CHAPMAN and BARTELS have pointed out [8], if a daily mean intensity follows a uniform cyclic variation with time a daily variation due to curvature will be found, and this cannot be eliminated by the usual linear adjustment for secular change. For example, convex curvature at the intensity maximum in the 11-year cycle of cosmic ray intensity variation would give rise to a diurnal maximum at midday. The presence of the midday maximum would be independent of the length of day used or of the chosen time of commencement of the day. Therefore it would be found in local solar time, universal time, local sidereal time and in local anti-sidereal time. If the same curvature persisted over a complete year the same maximum would also be found in the annual mean daily variation. (Towards the intensity minimum, of course, concave curvature would produce a phase reversal and the diurnal maximum would occur at the beginning of the day). Accordingly it is, important to determine whether or not curvature may give rise to a significant spurious sidereal effect.

The long-term cosmic ray intensity variation associated with the 11-year cycle of sunspot activity may be perhaps rather imperfectly represented by the function

$$I = I_0(1 + K \cos\theta) , \tag{13.1}$$

where $\theta = 2\pi T/(11 \times 365)$, T being time in days measured from the date of maximum intensity, and where K is the maximum fractional increase of in-

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[8] S. CHAPMAN and J. BARTELS: *Geomagnetism*, vol. 2 (London, 1951), p. 547.

tensity in the cycle. If eq. (13.1) represents the intensity at the beginning of day T the intensity at midday is

$$I_m = I_0\{1 + K\cos(\theta + \theta_c)\},$$

where θ_c is the angular equivalent of half a day. The intensity at the end of the day is then

$$I_e = I_0\{(1 + K\cos(\theta + 2\theta_c).$$

The linear secular adjustment at midday is

$$\Delta I = \frac{I - I_e}{2} = \frac{KI_0}{2}\{\cos\theta - \cos(\theta + 2\theta_c)\}.$$

Therefore, the relative amplitude due to curvature is

$$C = \frac{I_m + \Delta I - I}{I} = \frac{K\cos(\theta + \theta_c)(1 - \cos\theta_c)}{1 + K\cos\theta} \sim \frac{K\cos\theta}{1 + K\cos\theta}(1 - \cos\theta_c).$$

The maximum value of the amplitude is

$$C_{max} = \frac{K}{1 + K}(1 - \cos\theta_c). \tag{13.2}$$

Now, for an 11-year cycle.

$$\theta_c = \frac{\pi}{11 \cdot 365} = 0.782 \cdot 10^{-3} \text{ rad.},$$

and therefore

$$C_{max} = 0.3 \cdot 10^{-6}\,\frac{K}{1 + K}.$$

It follows that the daily variation due to curvature associated with the 11-year cycle would be somewhat less than 10^{-3}% no matter what value was assigned to K. Further than this, there is recent evidence that the 11-year cycle may not constitute a continuous uniform process of intensity variation. FORBUSH [9] and LOCKWOOD [10] for instance, have found that in some cases, long-term changes of cosmic ray intensity seem to be connected with Forbush

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[9] S. E. FORBUSH, Jr.: *Journ. Geophys. Res.*, **63**, 651 (1958).
[10] J. A. LOCKWOOD: *Journ. Geophys. Res.*, **65**, 19 (1960).

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decreases. This suggests that the 11-year variation may contain a contribution which proceeds in a series of steps of varying magnitude.

We may conclude, then, that if the spurious annual sidereal effect due to curvature exists at all, it is quite insignificant.

14. – Discussion.

Table II summarises the phase and amplitude relations between I_s, I_{as} and I_d for the various types of modulation of a solar daily variation considered in this paper, except for the case of a combined linear displacement of phase

TABLE II. – *Annual mean sidereal and anti-sidereal daily variations produced by a modulation of a solar daily variation. Phase and amplitude relations.*

Type of modulation	Range of N, K, n and k	Phase relations	$\frac{(I_{as})_{max}}{(I_s)_{max}}$	$\frac{(I_s)_{max}}{(I_d)_{max}}$
Linear displacement of phase	*a*) $0 < N \leqslant +24$ *b*) $-24 \leqslant N < 0$	$(\tau'_m = \tau''_m + 1200 = \tau_m)$ (*) $(\tau'_m = \tau''_m + 1200 = \tau_m + 1200)$ (*)	$\frac{N+24}{N-24}$	$\frac{N}{N+24}$
Linear change of amplitude	$K < 2$	$\tau'_m = \tau''_m + 1200 = \tau_m + 0600$	1	$\frac{K}{\pi(2-K)}$
Seasonal variation of amplit. *a*) Annual *b*) Semi-annual.	 All k All k	 $\tau'_m = \tau''_m + 4T_0$) (*) $(\tau'_m = \tau_m + 2T_0)$ (*) No effect	 1 (*) No effect	 $k/2$ (*) No effect
Seasonal variation of phase (*a*) Annual (*b*) Semi-annual	 $0 < n \leqslant 9.2$ All n	 $\tau'_m = \tau''_m + 4T_0$ $\cdot(= \tau_m + 2T_0 + 0600)$ (*) No effect	 1 No effect	 $\frac{J_1[(\pi n)/12]}{J_0[(\pi n)/12]}$ No effect

(*) Model analyses indicate that this relation is only a rough guide.

$\tau_m = 24t_m$ = time of maximum of annual solar d.v. in local time;

$\tau'_m = 24t'_m$ = time of maximum of annual sidereal d.v. in local sidereal time;

$\tau''_m = 24t''_m$ = time of maximum of annual anti-sidereal d.v. in local anti-sidereal time.

T_0 = time of maximum seasonal positive change of phase or amplitude, in months measured from 22nd of September.

and amplitude. The symbols τ $(=24t)$, τ' and τ'' have been introduced so that the phase relationships can be expressed in hours. The case of a seasonal variation of phase has been summarised for n in the range $0 < n \leqslant 9.2$ only;

for $n \sim 9.2$ this gives a total seasonal phase change of about 18 hours, which seems a reasonable upper limit in the light of present knowledge of seasonal changes of phase of the solar d.v. of cosmic ray intensity.

Table III summarises in a similar fashion the phase and amplitude relations between I_s, I_{as} and I_{0s} when a sidereal daily variation has been modulated in any one of the ways outlined in the paper.

TABLE III. – *Annual mean sidereal and anti-sidereal daily variations produced by a modulation of a sidereal daily variation. Phase and amplitude relations.*

Type of modulation	Range N, K, n and k	Phase relations	$\frac{(I_{as})_{max}}{(I_s)_{max}}$	$\frac{(I_s)_{max}}{(I_{0s})_{max}}$
Linear displacement of phase	$0 < N \leqslant +24$ $-24 \leqslant N < 0$	$\tau'_m (= \tau''_m + 1200) = \tau'_{om} + N/2$ (*) $(\tau'_m = \tau''_m) = \tau'_{om} + N/2$ (*)	$\frac{N}{48 - N}$	$\frac{24}{\pi} \sin \frac{N\pi}{24}$
Linear change of amplitude	$K < 2$	$\tau'_m = \tau''_m + 0600 = \tau'_{om}$	$\frac{K}{2\pi(2-K)}$	$1 - K/2$
Seasonal variation of amplit. (*a*) Annual (*b*) Semi-annual	 All k All k	 No effect $\tau'_m = \tau''_m + 2T_0 = \tau'_{om}$	 No effect $k/2$	 1 1
Seasonal variation of phase (*a*) Annual	$0 < n \leqslant 9.2$	$\tau'_m = \tau''_m + 4T_0 + 1200 = \tau'_{om}$	$\frac{J_2[(n\pi)/12]}{J_0[(n\pi)/12]}$	$J_0 \frac{(n\pi)}{12}$
(*b*) Semi-annual	$0 < n \leqslant 9.2$	$\tau'_m (= \tau''_m + 4T_0 - 0600) = \tau'_{om}$ (*)	$\frac{J_1[(n\pi)/12]}{J_0[(n\pi)/12]}$	$J_0 \frac{(n\pi)}{12}$

(*) Model analyses indicate that this relation is only a rough guide.

$\tau'_m = 24t'_m$ = time of maximum of annual sidereal d.v. in local sidereal time;

$\tau''_m = 24t''_m$ = time of maximum of annual anti-sidereal d.v. in local anti-sidereal time;

T_0 = time of maximum seasonal positive change of phase or amplitude, in months measured from 22nd of September.

It should be emphasized that the tables refer to two very distinct classes of events. On the one hand, a modulation of a solar daily variation which gives rise to a variation in anti-sidereal time must also be responsible for a spurious daily variation in sidereal time. But in the other category, summarized in Table III, a modulation of a sidereal daily variation may be capable of producing an appreciable daily variation in anti-sidereal time but of course there is no corresponding spurious sidereal effect.

Let us now suppose we have obtained significant observations of an apparent annual mean sidereal d.v. of cosmic ray intensity. If we find in addition that the daily variation in anti-sidereal time is effectively nonexistent, then we can be reasonably certain that the whole of the daily variation in sidereal time is of sidereal origin. However, before deciding this, we have to satisfy ourselves *a*) that the result has not arisen fortuitously through the presence of irregularities in the solar daily variation at various times of the year and *b*) that it has not been produced by a solar d.v. of constant amplitude exhibiting a uniform retardation of phase at the rate of two hours each month (*i.e.* the situation described in Section **4** for $N=-24$). Possibility *a*) can be tested by arranging the monthly mean vectors of the solar d.v. on a harmonic dial and noting whether the end-points progress systematically in an anti-clockwise manner. If the counting statistics are not good enough to enable this to be seen, then it remains to be found if the result is reproducible from year to year. Possibility *b*) on the other hand, simply cannot be distinguished from a genuine unmodulated sidereal d.v. and is only rejected on the grounds that it is rather implausible.

Suppose again that we find an observed apparent annual mean sidereal d.v. to be accompanied by a significant daily variation in anti-sidereal time. Then

1) The generator of the anti-sidereal daily variation has to be identified.

2) If the anti-sidereal d.v. has its origin in a modulation of a solar diurnal variation, the amplitude and phase of the spurious sidereal component have to be estimated and the component vector subtracted from the total apparent sidereal vector.

As the tables show, observations of the annual mean daily variations I_s and I_{as} alone will not even allow us to say whether the anti-sidereal d.v. was generated by a modulation of a solar d.v. or of a sidereal d.v. We have to remember also that I_d, the annual mean of a modulated solar d.v., is not necessarily the usual solar daily variation and, if present, may not be easily observed. Generally speaking, there seems to be little hope of identifying the type of modulation involved unless monthly mean daily variations of high statistical accuracy are available. One method of analysis to deal with this problem and which requires accurate values of the monthly mean daily variations has been developed to interpret the cosmic ray data obtained at the underground station near Hobart. It will be presented in a following paper.

When the generator of an observed anti-sidereal d.v. has been identified as one of the types listed in Table II, the spurious sidereal vector can be readily deduced from the phase and amplitude relations given in the Table. For example, let us suppose that the generator happens to be a seasonal variation of phase of a solar daily variation. The spurious sidereal d.v. can be estimated in either of two ways:

1) *From the characteristics of the solar d.v.* Let the time of maximum, τ_m, of the annual mean solar d.v. be, say, 1800 hours local time and the amplitude be 0.05%. If the maximum displacement of phase is 4 hours ($n = 4$) occurring at about the March equinox ($T_0 = 6$), the amplitude and time of maximum of the annual mean spurious sidereal d.v. will then be

$$(I_s)_{max} = \frac{J_1(\pi/3)}{J_0(\pi/3)} \cdot 0.05 \sim 0.03\% ,$$

and

$$\tau'_m = \tau_m + 2T_0 + 0600 = 1800 + 1200 + 0600 = 1200 .$$

As noted in the summary for this case, this phase relation yields only a rough estimate of τ'_m. Values of $J_0(\pi n/12)$ and $J_1(\pi n/12)$ as functions of n can be obtained from Fig. 7(*a*) and 7(*b*).

2) *From the characteristics of the anti-sidereal d.v.* The amplitude of the spurious sidereal d.v. is obtained simply as

$$(I_s)_{max} = (I_{as})_{max}$$

and if the observed time of maximum, τ''_m, of the anti-sidereal d.v. is, say, 1000 hours local anti-sidereal time, a better estimate of τ'_m (see Fig. 7(*e*)) than that given in 1) above would be

$$\tau'_m = \tau''_m + 4T_0 = 1000 + 2400 = 1000 .$$

15. – Conclusion.

In the past, the presence of an annual mean anti-sidereal daily variation of cosmic ray intensity has been dealt with as though it were produced by a seasonal change of amplitude of the solar daily variation. In this paper some of the various other ways of generating an anti-sidereal daily variation have been outlined, not the least of them being a modulation of a genuine sidereal variation. Rather than give a comprehensive treatment the intention has been to illustrate, by means of a number of simple cases, the following two essential points:

Firstly, the presence of an observed annual mean d.v. in anti-sidereal time is not necessarily linked with a fictitious d.v. in sidereal time. We have seen, in fact, that in a number of simple cases a modulation of a genuine sidereal diurnal variation can give rise to an annual d.v. in anti-sidereal time. The most important cases are probably the seasonal variations of phase and of amplitude of a sidereal d.v. It seems reasonable to expect to observe such phenomena, particularly if, at some time of the year, the sun tends to transit the source of a sidereal anisotropy as viewed from the observing station.

Secondly, the phase and amplitude relationships between I_d, I_s and I_{as} vary greatly with the type of modulating mechanism involved and may depend critically on the parameters pertaining to a particular case. It can be seen that there is a wide range of possibilities, even if we consider only the various combinations of the types of modulation processes that we have already examined. Therefore, before attempting to deduce the phase and amplitude of the fictitious sidereal d.v. that may correspond to an observed annual d.v. in anti-sidereal time it may be necessary to determine rather precisely the type of modulating process that gave rise to the observed effect and then to calculate the phase and amplitude relationships appropriate to that case. This generally means that month by month changes in phase and amplitude of the cosmic ray daily variation need to be known accurately and that, therefore, high counting rates are necessary.

The theory has been compared with results from analyses of daily variations constructed from models. As a result it seems that, on the whole, the method of integrating simple sinusoidal functions to obtain the annual mean d.v.s gives results that should be in quite good agreement with those to be found experimentally. However, as noted in the summaries, the method probably does not predict some of the phase relationships as accurately as it does others.

* * *

The author gratefully ackowledges a number of useful discussions with other members of the Cosmic Ray group at Hobart and in particular those with Dr. K. G. McCracken and Mr. Paul Edwards.

RIASSUNTO (*)

Si delinea un trattamento teorico della variazione giornaliera annua siderale apparente e della variazione giornaliera antisiderale associate con i seguenti fenomeni: *a*) Uno spostamento lineare annuale della fase di una variazione giornaliera solare. *b*) Uno spostamento lineare annuale di una variazione giornaliera siderale. *c*) Cambiamento lineare annuale dell'ampiezza di una variazione giornaliera solare. *d*) Cambiamenti lineari combinati della fase e dell'ampiezza di una variazione giornaliera solare. *e*) Un cambiamento lineare annuale dell'ampiezza di una variazione giornaliera siderale. *f*) Una variazione stagionale dell'ampiezza di una variazione giornaliera solare. *g*) Una variazione stagionale dell'ampiezza di una variazione giornaliera siderale. *h*) Una variazione stagionale della fase di una variazione giornaliera solare. *i*) Una variazione stagionale della fase di una variazione giornaliera siderale. *j*) La curvatura associata al ciclo undecennale di variazione dell'intensità. Alla teoria seguono analisi di imitazioni delle variazioni giornaliere, opportunamente modulate.

(*) *Traduzione a cura della Redazione.*

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PAPER 3

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Cosmic Ray Observations at 42 m w.e. Underground at Hobart, Tasmania

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(ricevuto il 3 Luglio 1961)

Summary. — A cosmic ray observatory and associated recording equipment located at 42 m water equivalent underground are described. The average primary proton energy corresponding to this depth is about 300 GeV. Data from two large vertical meson telescopes have been used to determine the atmospheric correction coefficients, and the results are compared with calculated values. The observed coefficients are:

Total barometer coefficient (95% fiducial limits).	$= -(0.655 \pm 0.042)\%/\text{cm}$
Partial barometer coefficient	$= -(0.591 \pm 0.044)\%/\text{cm}$
Negative temperature coefficient	$= -(0.459 \pm 0.250)\%/\text{km}$
Positive temperature coefficient	$= +(0.020 \pm 0.010)\%/°\text{C}$
Positive 3-fold temperature coefficient	$= +(0.028 \pm 0.009)\%/°\text{C}$

The low value for the positive temperature coefficient suggests that the attenuation length for π-mesons in the upper atmosphere is at least 125 g cm^{-2}. The effect on the underground intensity of a large change in the water content of the soil above the recorder is discussed.

1. – Introduction.

For the investigation of cosmic ray intensity variations of primary origin, it is desirable to determine the magnitude of the variations as a function of primary particle energy. For this purpose use is often made of data obtained

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at various latitudes, involving vertical primary cut-off energies in the geomagnetic field ranging up to about 15 GeV. In order to extend the observations to higher mean energies, it is convenient to locate the detecting equipment below ground so that the less penetrating secondary particles are filtered out. Results obtained in this way are particularly suitable for comparison with those from nearby ground level stations, because the particles detected in each case have traversed practically the same regions of the exosphere and atmosphere. On the other hand, recorders located at different latitudes detect particles that have traversed substantially different regions of the earth's environment; this may lead to undesirable influences of a local nature when the energy dependence of transient cosmic ray intensity changes is being studied.

With the above points in mind, a small cosmic ray observatory was installed in a disused railway tunnel about 6 miles from the main cosmic ray station at Hobart.

In this paper we shall present a brief description of the underground observatory and the meson telescopes, together with the results of a correlation analysis carried out in order to obtain atmospheric correction coefficients. Transient intensity variations have been observed on a number of occasions since the telescopes commenced operation in 1957; discussion of these will be deferred until later papers.

2. – Location of observatory and description of equipment.

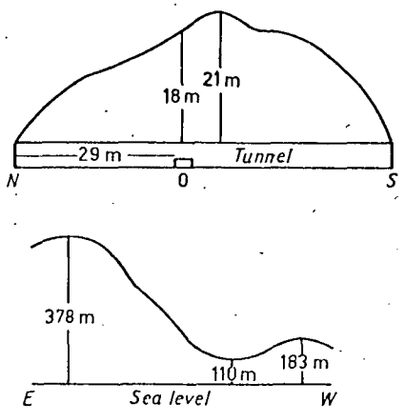

Fig. 1. – Vertical sections through the tunnel at Hobart, Tasmania, in the N-S and E-W directions. The surface of the land above O, the cosmic ray observatory, is 110 m above sea level.

The observatory is located in a disused railway tunnel approximately 6 miles north-east from Hobart; the geographic co-ordinates are 42.8° S latitude, 147.5° E longitude. The tunnel runs in a north-south direction below a saddle joining two hills, as shown in Fig. 1. The equipment is at an altitude of 110 metres above sea level. The absorber vertically above the equipment amounts to approximately 4 200 g cm^{-2} of shale and clay, or 42 metres water equivalent (m w.e.). It is not possible at present to give accurate values for the amount of absorber as a function of zenith and azimuth angles, but we have ascertained that this is a minimum in the vertical direction.

The cosmic ray equipment consists of two vertically directed triple coincidence Geiger counter telescopes, together with recording apparatus. The counter trays are each of 1 m^2 sensitive area and the distance between the top and bottom trays is 50 cm. In detail, the telescopes and electronic circuits are similar to those operated elsewhere by the Hobart cosmic ray group, as described by PARSONS (1). The counting rate of each telescope is about 34 000 counts per hour, data from the two telescopes together having a statistical standard deviation of $\pm$ 0.38 % for hourly values and $\pm$ 0.07 % for daily mean values. Hourly totals are recorded photographically, while for short-term data a chart recorder is used. Mains voltage stabilisers are used to provide a constant AC supply to the equipment. The temperature inside the underground observatory remains within about five degrees of 13 °C throughout the year without thermostatic control. Intermittent operation of the telescopes began in May 1957, and practically continuous records are available from October 1957.

3. – Energy of particles detected.

The particles detected by the telescopes are predominantly muons of high energy. In order to penetrate the atmosphere followed by 42 m w.e. of absorber, these must have an energy at production in excess of 15 GeV (GEORGE (2)); however, due to the multiplicity of production in the upper atmosphere, the corresponding primary particle energies are much higher. DORMAN (3) states that from available evidence, the muon energy at production is about one tenth of the primary proton energy. Assuming a differential energy spectrum of the form $E^{-2.7}$ for the protons concerned, this leads to the conclusion that the average energy of the primary particles responsible for the counting rate observed at 42 m w.e. is about 350 GeV. The low energy cut-off of 150 GeV suggested by Dorman's factor of 1.0 will in practice not be sharp due to fluctuations in the meson production multiplicity. This may lower the average energy somewhat, so that the above figure should probably be reduced to 300 GeV. For comparison, the average primary proton energy corresponding to particles detected by standard I.G.Y. neutron monitors and

(1) N. R. PARSONS: *The Design and Operation of ANARE Cosmic Ray Recorder « C »*, Australian Nat. Antarctic Res. Expeditions Interim Report no. 17, Melbourne (March 1957).

(2) E. P. GEORGE: *Progress in Cosmic Ray Physics*, J. G. WILSON (Ed.), chap. 7 (Amsterdam, 1952).

(3) L. I. DORMAN: *Cosmic Ray Variations*, State Publishing House for Technical and Theoretical Literature (Moscow, 1957). Translation Tech. Doc. Liason Office, Wright-Patterson Air Force Base (Washington, 1958).

meson telescopes operated near sea level at high geomagnetic latitudes is about 7 and 30 GeV respectively.

Thus, in the underground measurements, we are concerned with effects due to primary protons of energy about an order of magnitude above the minimum energy required to enter the earth's atmosphere vertically at the equator. About 10% of the cosmic ray muons near sea level arise from this portion of the primary spectrum and are capable of penetrating to 42 m w.e.

4. – Variations of atmospheric origin.

We have followed the usual practice of assuming that the meteorological effect on the cosmic ray meson intensity is adequately described for most purposes by the relation

$$\frac{\mathrm{d}I}{I} = \beta_{IB.HT}\,\mathrm{d}B + \beta_{IH.BT}\,\mathrm{d}H + \beta_{IT.BH}\,\mathrm{d}T\,,$$

where B is the barometric pressure at sea level, H is the height of the assumed mean pressure level for production (the 100 mb level is used here) and T is the temperature in the neighbourhood of the production level (we have taken this as the mean temperature in the pressure interval (100÷200) mb). Accordingly, $\beta_{IB.HT}$ is the partial barometer coefficient, $\beta_{IH.BT}$ the negative temperature coefficient and $B_{IT.BH}$ is the positive temperature coefficient. The total barometer coefficient will be referred to as β.

The various coefficients were found experimentally in two series of 4-fold correlation analyses, using selected data from the years 1958 and 1959. Daily mean values of cosmic ray intensity were correlated with daily mean values of barometric pressure and the values of H and T extracted from radiosonde data, supplied by the Hobart Bureau of Meteorology. All days on which it was known that pronounced anomalous variations of the primary cosmic ray intensity had occurred were rejected. In the analysis for 1958, the cosmic ray results from a single telescope were used and a total of 202 selected days of data was available. By comparison, the accuracy of the coefficients for 1959 was considerably greater since the combined data from both telescopes were available for most of the 230 days selected. A large secular change of cosmic ray intensity that was clearly not correlated with secular changes of B, H or T, (evidently part of the 11-year cycle of intensity variation) was eliminated before using the data.

In a recent criticism of the use of the least squares method of regression analysis, TREFALL and NORDÖ [4] have pointed out that errors of measurement of the radiosonde data may influence the regression coefficients considerably. We have therefore used the estimated error variances and covariances (extracted from Table II and Table III of their paper) of the height H and the temperature T at the 100 mb level to take account of the systematic errors of measurement of the height of the 100 mb level and the temperature in the (100÷200) mb interval at Hobart. The ratios between the total variances and estimated error variances are shown in Table I. All the observed values of

(4) H. TREFALL and J. NORDÖ: *Tellus*, **11**, 4, 467 (1959).

TABLE I. – *The ratios of the total variances to the estimated error variances for H and T at Hobart.*

	T ((100÷200) mb)	H (100 mb)
1958	5.87	5.77
1959	6.21	9.15

the regression coefficients involving H and T have been corrected for these errors. The partial barometer coefficient was practically unaffected, the negative temperature coefficient was increased by 15%, the positive temperature coefficient $\beta_{IT.HB}$ by 33% and $\beta_{IT.B}$ by 20%. Only the corrected values are quoted below, under the headings for the various coefficients, and, unless otherwise stated, the observed values of the coefficients are the weighted means of the 1958 and 1959 values.

4·1. *The total barometer effect.* – The coefficient β was found to have a weighted mean value of $\beta = (-0.651 \pm 0.041)\%/\mathrm{cm\ Hg}$ (95% fiducial limits). TREFALL [5] has obtained the total barometer coefficient as a function of cut-off energy at sea level for energies up to about 11 GeV. As well as considering the effect of pure mass absorption, he has taken into account the contribution from μ-e decay arising from the dependence of the mean life of the μ-meson on the distribution of energy loss in traversing the atmosphere. It appears that the latter contribution is the dominant one at low cut-off energies (less than about 500 MeV), but at 12 GeV (equivalent to 15 GeV at production) the barometer coefficient is due almost entirely to the effect of mass absorption. Extrapolation of Trefall's curve to 12 GeV gives a theoretical value of $-0.57\%/\mathrm{cm\ Hg}$. Our observed value is significantly greater than this. However, the result has been obtained from a wide angle recorder and no account has been taken of the dependence of the barometer coefficient on zenith angle.

(5) H. TREFALL: *International Conference on Cosmic Radiation*. National Institute for Scientific Investigation (Mexico, 1958).

BARNOTHY and FORRO [6] have investigated this problem with a telescope at sea level using different thicknesses of lead absorber and there is an indication from their work that at high cut-off energies the barometer coefficient increases with zenith angle.

4·2. *The partial barometer effect.* – The following value for $\beta_{IB.HT}$ was found:

$$\beta_{IB.HT} = (-0.591 \pm 0.044)\%/\text{cm Hg} \quad (95\%\ \text{fiducial limits}).$$

The usual day to day range of pressure at Hobart is about 1.5 cm Hg and the seasonal range is about 1 cm, so that the partial barometric effect contributes appreciably to variations of the underground intensity.

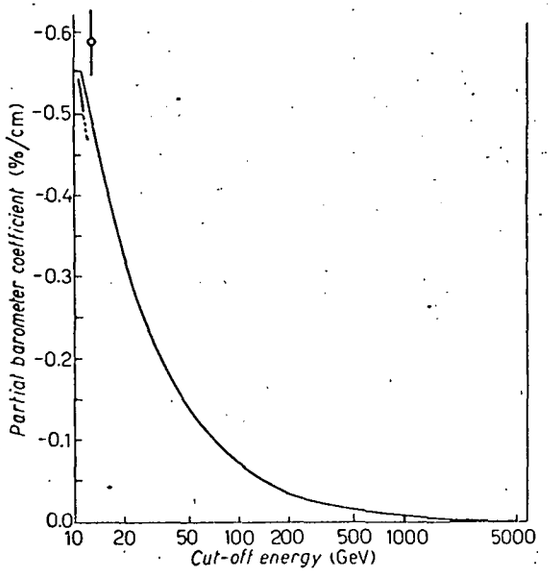

Fig. 2. – The calculated partial barometer coefficient as a function of cut-off energy at sea level. The overlap with a similar curve calculated by Trefall for lower energies is also shown. The error tails on the value observed underground at Hobart are the 95% fiducial limits.

Once again, as TREFALL has shown, the effect of μ-e decay must be taken into account when calculating $\beta_{IB.HT}$ for low cut-off energies, but in our case it appears that the contribution from this process can be neglected (*e.g.* see Trefall's Fig. 2 [5]). Using an expression that is identical with that used by TREFALL, but for a slightly different μ-meson spectrum at production, the partial barometer coefficient has been calculated as a function of cut-off energy at sea level in the range $(10 \div 5)\cdot 10^3$ GeV. In Fig. 2 the difference between our calculated values and Trefall's, where they overlap, is seen to be very small. On the other hand, the observed value is significantly greater than either estimate. Bearing in mind that the barometer coefficient may increase with zenith angle at high energies, our result indicates that the theoretical values of $\beta_{IB.HT}$ for vertical incidence are probably not greatly in error.

(6) J. BARNOTHY and M. FORRO: *Zeits. f. Phys.*, **100**, 742 (1936).

4·3. *The negative temperature effect.* – The value of the negative temperature coefficient was found to be

$$\beta_{IH.HT} = (-0.459 \pm 0.250)\%/\text{km} \quad (95\%\ \text{fiducial limits}).$$

The range of H (100 mb) from day today is usually about 0.3 km and the seasonal range is about 0.5 km. Therefore, fluctuations of H (100 mb) have very little influence on the underground intensity and, for most purposes for which daily mean values of intensity are used, may be neglected.

The negative temperature coefficient is thought to be due solely to the effect of variations in the path length on the probability of μ-meson decay.

One has to be careful, when calculating a theoretical coefficient, to choose a model atmosphere which conforms to the regression equation from which observed values are obtained. TREFALL has demonstrated this clearly in his theories of the meteorological effects. He has calculated the dependence of the negative temperature effect on cut-off energy for two cases:

i) in which the model conforms to the equation

$$\frac{\mathrm{d}I}{I} = \beta_{IB.H}\,\mathrm{d}B + \beta_{IH.B}\,\mathrm{d}H\,.$$

Accordingly, temperature variations are assumed to be uniform through the atmosphere;

ii) in which the model conforms to eq. (1). In this case the stratospheric temperature T is constant. The marked difference between the calculated case i) and case ii) values at low energies comes from a different distribution of energy losses along the path of the μ-meson for each case. The distinction becomes relatively unimportant at high energies.

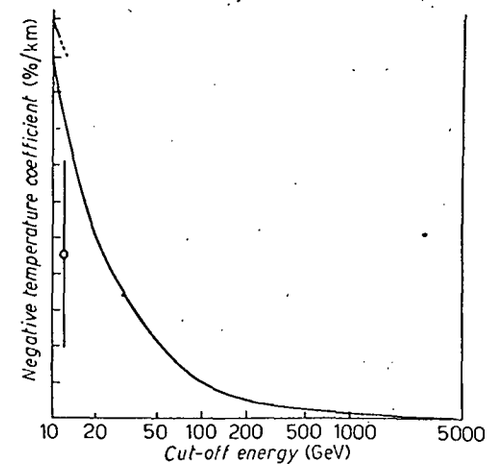

Fig. 3. – The calculated negative temperature coefficient as a function of cut-off energy at sea level. Also shown is the overlap with a similar curve calculated by Trefall for lower energies. The error tails on the weighted mean value observed underground at Hobart are the 95% fiducial limits.

As seen in Fig. 3, our observed value of $\beta_{IH.HT}$ is considerably less than our estimate. The curve shows the calculated value of the coefficient as a function of sea-level cut-off energy in the range $(10 \div 5)\cdot 10^3$ GeV. It has been obtained by partial dif-

ferentiation of the integral momentum spectrum at sea level with respect to H. In deriving the integral momentum spectrum the expression $F(\varepsilon) = f(E)\exp[-k]$ has been used, relating the differential energy spectrum at sea-level, $F(\varepsilon)$, to the differential energy spectrum at production, $f(E)$, where E is the energy at production of a μ-meson which has the energy ε on arrival at sea-level. In the evaluation of $k = (Hm_\mu c/\tau_0 E')$ the loss of meson energy in traversing the atmosphere is neglected, and consequently E' has been given the value E. Here H is the meson production height, c the velocity of light, m_μ the μ-meson rest mass, τ_0 its mean lifetime at rest, and $\varepsilon < E' < E$. It can be seen that our curve and that for Trefall's case ii) differ only slightly where they overlap.

For the energies we are concerned with, the mean level of production of mesons is probably higher that the 100 mb level (BARTON (7)). It may be that the choice of the reference level and of T (100÷200) mb as the stratospheric temperature in the neighbourhood of production are the main reasons why the coefficient $B_{IH.BT}$ is so much lower than the theoretical estimate. The question of the effect of choice of reference levels and stratospheric temperatures on the negative temperature coefficient at Hobart is at present being investigated.

4·4. *The positive temperature effect.* – The weighted mean value of $\beta_{IT.BH}$ was found to be

$$\beta_{IT.BH} = (0.020\,2 \pm 0.010)\,\%/°C \quad (95\,\%\ \text{fiducial limits}) .$$

The day to day range of temperature of the (100÷200) mb level is approximately 8 °C and the seasonal range is about the same. Therefore variations of stratospheric temperature have only a slight influence on the underground intensity. The magnitude of the effect is about the same as that due to variations in height of the 100 mb level.

In his theoretical treatment of the positive temperature effect due to π-μ decay TREFALL (8) has used a model atmosphere in which temperature variations are independent of atmospheric depth. Therefore the temperature coefficient appropriate to this model is assumed to be the partial 3-fold coefficient $\beta_{IT.B}$ obtained from the regression equation

$$\mathrm{d}I = \beta_{IB.T}\mathrm{d}B + \beta_{IT.B}\mathrm{d}T . \tag{2}$$

For such a model Trefall shows that there are three contributions to the temperature coefficient. The first is a negative effect of μ-e decay due to the change in survival probability which accompanies the change in height of an assumed reference level in the vicinity of the mean level of production of mesons. The second is a smaller second-order effect of μ-e decay depending on the displacement of the mean level of production relative to the reference level. The third contribution is Duperier's positive temperature effect of π-μ decay caused by competition between absorption and decay of π-mesons. The total contribution from μ-e decay as a function of cut-off energy for the equipment may be calculated from Trefall's eq. (5) and used as a correction so that the net positive temperature effect of π-μ decay can be obtained from observed values of $\beta_{IT.B}$.

The positive temperature effect of π-μ decay is a function of λ_π, the attenuation length of π-mesons. Trefall's calculated values of the effect as a function of cut-off momentum (Fig. 2 (5)) are rather insensitive to the value of λ_π for the low cut-off energies applicable to cosmic ray intensity measurements at sea level, and also for the very high cut-off energies which apply to measurements at extreme depths underground. However, as he points out, his calculations show that measurements at moderate depths underground should enable us to determine from the positive temperature effect whether the value of λ_π is comparable with the collision mean free path for π-mesons, or with the attenuation length, λ_p, of the primary proton component. The curves in Trefall's Fig. 2 for each of these cases and for the case where $\lambda_\pi = 2\lambda_p$ have been redrawn in our Fig. 3 (where the positive temperature effect is expressed as a function of cut-off energy instead of momentum). Also reproduced from his figure is Trefall's estimated value of the π-μ effect using the positive temperature coefficient obtained by MACANUFF (9) at a depth of 60 m w.e. underground. MACANUFF found a positive temperature coefficient $\beta_{IT.B} = (+0.055 \pm 0.005)\,\%/°C$, where T was the temperature of the (100÷300) mb layer. A total correction for μ-e decay of +0.054 %/°C gave the net positive temperature effect of π-μ decay as +0.109 %/°C.

Our data were re-analysed in conformity with eq. (2) and annual values of $\beta_{IT.B}$ of $(+0.049\,7 \pm 0.022)\,\%/°C$ and $(+0.024\,3 \pm 0.010)\,\%/°C$, were obtained for the years 1958 and 1959 respectively. The errors given are the 95 % fiducial limits. The weighted mean value was $(+0.028\,6 \pm 0.009)\,\%/°C$. The correction for the principal μ-e decay effect calculated from Trefall's eq. (5), where the cut-off energy at production was taken as $E_0 = 142\,mc^2$, was found to be so close to the value Trefall quoted for $E_0 = 155\,mc^2$ that the same total correction of +0.054 %/°C has been used. This finally gave a weighted mean value of $(+0.082\,6 \pm 0.009)\,\%/°C$ for the net positive temperature effect of π-μ decay at 42 m w.e. underground, as shown in Fig. 3.

(7) J. BARTON: *Proc. Phys. Soc.*, A **67**, 637 (1954).
(8) H. TREFALL: *Proc. Phys. Soc.*, A **68**, 625 (1955).
(9) J. W. MACANUFF: *Thesis* (University of London, 1951).

BARTON (7) has criticized MacAnuff's choice of the temperature of the (100÷300) mb layer as the temperature variable T in the regression analysis. Since there are contributions to the positive temperature effect at all levels of the atmosphere a weighted mean temperature, such as the T_{eff} used by some investigators (see TREFALL (5)), is possibly the best one to use. BARTON has pointed out that the use of $T_{(100\div300)\,mb}$ as the temperature variable leads to an under-estimation of the atmospheric temperature variation and would make MacAnuff's coefficient too large by a factor of ~ 2. Since the temperature variations tend to be greater at the higher levels of the atmosphere our use of $T_{(100\div200)\,mb}$ as the temperature variable should also tend to make our estimate of the positive temperature effect too large but might give a better estimate of the effect than if we had used $T_{(100\div300)\,mb}$.

It would seem from our results and those of MACANUFF that the attenuation length λ_π, of π-mesons is at least 125 g cm^{-2}, and may be considerably larger.

In addition to the meteorological influences we have described we have evidence that variations in the amount of water in the soil above the tunnel can produce significant changes of cosmic ray intensity underground. A striking example of this occurred at the end of April 1960 when phenomenal rains caused severe flooding in Southern Tasmania, following several months of very dry weather. The total rainfall recorded near the tunnel from February 1st to April 21st was 3.1 in., whereas 9.5 in. fell during the 22nd, 23rd and 24th of April. Between the 21st and 23rd of April the pressure-corrected underground intensity showed a gradual decrease of 0.8% (±0.07% standard error) and remained at this reduced level until the end of April. Over the same period the vertical meson intensity at sea level in the Hobart laboratory remained at a constant level after the usual corrections for atmospheric

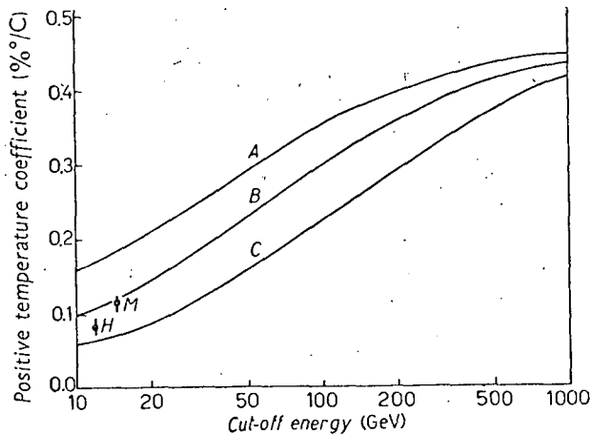

Fig. 4. – The positive temperature effect due to π-μ decay calculated by Trefall for A: $\lambda_\pi = 62.8$ g cm^{-2}; B: $\lambda_\pi = 125$ g cm^{-2} and C: $\lambda_\pi = 250$ g cm^{-2}. The value M has been derived by TREFALL from the positive temperature coefficient $\beta_{IT.R}$ observed by MACANUFF, corrections having been made for 1st and 2nd order effects of μ-e decay. The value H from Hobart has been derived in the same way. The error tails on the Hobart value are the 95% fiducial limits.

changes had been applied. Using an intensity-depth relationship published by CLAY (10) it was found that a reduction of the counting rate by 0.8% at 42 m w.e. underground would indeed correspond to an increase of the absorber by approximately 10 in. water equivalent.

Variations in the moisture content of the material above the tunnel are naturally rather difficult to assess. They must, in particular, seriously complicate the observed seasonal changes of the daily mean underground intensity. However, apart from rare occasions such as the one quoted, day to day changes of the absorber due to rainfall would generally not produce intensity variations greater than the standard error of observation (± 0.07 %).

We conclude from the foregoing that the variations in atmospheric structure have a very small influence on the underground cosmic ray intensity. In this respect underground observations have an advantage over ground level meson intensity measurements which are often considerably influenced by changes in atmospheric structure. However the mass absorption effect is appreciable and in many cases the data must be corrected for this effect.

* * *

Financial assistance for this project was received from the Commonwealth Scientific and Industrial Research Organization and the Australian Academy of Science.

(10) J. CLAY: *Rev. Mod. Phys.*, **11**, 128 (1939).

RIASSUNTO (*)

Descriviamo un osservatorio per raggi cosmici e le relative apparecchiature di registrazione collocati sottoterra a 42 m a.e. L'energia media dei protoni primari in corrispondenza di questa profondità è circa 300 GeV. Abbiamo usato due grandi telescopi verticali per mesoni per determinare i coefficienti di correzione atmosferica, ed abbiamo confrontato i risultati con i valori calcolati. I coefficienti osservati sono:

Coefficiente barometrico totale (limiti di fiducia 95%)	$= -(0.655 \pm 0.042)$%/cm
Coefficiente barometrico parziale	$= -(0.591 \pm 0.044)$%/cm
Coefficiente di temperatura negativo	$= -(0.459 \pm 0.250)$%/km
Coefficiente di temperatura positivo	$= +(0.020 \pm 0.010)$%/°C
Coefficiente triplo di temperatura positivo	$= +(0.028 \pm 0.009)$%/°C

Il basso valore del coefficiente di temperatura positivo suggerisce che la lunghezza di attenuazione per i mesoni π nella parte superiore dell'atmosfera è almeno 125 g cm^{-2}. Discutiamo l'effetto sull'intensità sotterranea di una forte variazione del contenuto di acqua nel suolo al di sopra degli apparecchi di registrazione.

(*) *Traduzione a cura della Redazione.*

PAPER 4

R. M. JACKLYN
1° Ottobre 1963
Il Nuovo Cimento
Serie X, Vol. 30, pag. 40-76

The Apparent Sidereal Daily Variation of Cosmic-Ray Intensity at 42 m w.e. Underground at Hobart, Tasmania.

I. – Results of Observations in 1958.

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Summary. — The apparent sidereal daily variation of cosmic ray intensity underground at Hobart during 1958 is examined. There is a large annual mean apparent sidereal effect, and a comparable anti-sidereal effect, associated with pronounced month by month changes in phase and amplitude of the solar daily variation. A technique is devised which permits the daily variation (d.v.) in sidereal time to be examined month by month, uncomplicated by the presence of certain background components, including for instance, a solar daily variation of constant phase and amplitude. An attempt is made to interpret the apparent sidereal effect on the assumption: *a*) that it is entirely spurious, arising from a solar-time modulation of the solar daily variation; *b*) that there is a genuine part to the effect, consisting of a sidereal d.v. of constant phase and amplitude, and a spurious part, arising from an annual variation of amplitude of the solar d.v., produced by a solar-time mechanism; *c*) that the effect is entirely genuine, arising from a sidereal-time modulated sidereal daily variation. The theory for each case is tested against the observations by means of models. When various simple possibilities are considered, it is found that the evidence clearly favours assumption *c*). The hypothesis which gives best agreement with the observations is that a solar d.v. of constant amplitude and phase is accompanied by a sidereal d.v. with semi-annual variation of amplitude and phase. In that case, the unmodulated sidereal d.v. would have amplitude 0.09% and RA ~ 07.00 h, suggesting that the source of the anisotropy may be in the direction looking outward along the proposed spiral arm magnetic field.

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1. – Introduction.

Following the development of large cosmic ray counter telescopes at about the time of the I.G.Y., it has become possible to observe fluctuations of meson intensity underground for moderately high energies of parent primaries, with recorders that have greatly improved counting statistics. We should therefore be able to examine the apparent sidereal daily variation in some detail and with perhaps better prospects than formerly of deciding whether or not there is a genuine sidereal anisotropy among primary protons whose energies are not less than about 10^{11} eV.

We have already seen (1) that the annual mean daily variations of cosmic ray intensity in sidereal time (I_s) and in anti-sidereal time (I_{as}) may conceivably arise in a great variety of ways, embracing, with I_d, the annual mean d. v. in solar time, a wide range of phase and amplitude relationships. We cannot deduce solely from observed values of I_s, I_{as} and I_d just how the annual mean apparent sidereal effect has come about. In fact even if we had supporting evidence which allowed us to determine in detail how the solar daily variation had been modulated throughout the year we would still have to contend with what is perhaps the crux of the problem of the sidereal effect at moderate energies: any daily variation in solar time may be regarded as having its origin in some kind of sidereal daily variation and conversely any sidereal daily variation may be regarded as having arisen from some kind of solar d. v. The simplest example to illustrate the first part of this proposition is that of an apparent solar d. v. of constant phase and amplitude. This may conceivably have been produced by a sidereal daily variation of constant amplitude and having a time of maximum which becomes progressively later in sidereal time at the uniform rate of two hours each month. Conversely, an apparent sidereal daily variation of constant amplitude and phase can be produced by a solar d. v. of constant amplitude and having a time of maximum which becomes progressively earlier at the uniform rate of two hours each month. (In a practical situation we are more likely to find that this type of d. v. is superimposed as a small modulating component on a larger solar d. v. of constant phase and amplitude.)

Evidently, then, we can always find at least two kinds of hypotheses consistent with the observations, namely, a solar type, postulating a spurious effect in sidereal time, and a sidereal type, postulating that the whole effect in sidereal time is of sidereal origin. A mixed hypothesis may have to be considered as well, presupposing that part of the effect in sidereal time is genuine and that the rest is of solar origin. It may well be that all three will

(1) R. M. JACKLYN: *Nuovo Cimento*, **24**, 1034 (1962).

account completely for the observed modulations of the daily variation. In such circumstances, failing any other criterion for selection, we would have to accept the hypothesis which was the simplest one consistent with the observations.

Now, in many instances, it should be possible to distinguish between a solar-time modulation of a solar d. v. and a true sidereal effect by comparing simultaneous observations from cosmic ray recorders which (*a*) scan different directions in space and/or (*b*) are located in different hemispheres (*e.g.*, see ELLIOT's experiment (2)). But an investigation of that sort has its own problems, involving, possibly, various differences between recorders, a lack of uniformity of energy response at different locations and the greater risk of introducing to the experiment the effects of more than one source of sidereal anisotropy. In any event the design of a suitable inter-comparison experiment will depend to some extent on the type of hypothesis we wish to test.

It is therefore advisable as a first step to find out which hypotheses are consistent with the observations from a single recorder at one place. To make such detailed distinctions we require continuous records of high statistical accuracy. It is also desirable to employ a technique for separating out the modulation terms of the daily variation from unwanted terms of constant phase and amplitude and for isolating one type of modulation from another if more than one type happen to be present at the same time. In particular we would wish to be able to separate a modulation specified in sidereal time from one specified in solar time. In this paper a rather simple method is described which meets these requirements for certain types of modulation of the daily variation. If we use some such method to process a year's data we can then determine some of the modulation parameters, as, for example, the times of the year when maximum displacements of phase and amplitude occur and the range of displacements of phase and amplitude.

There are various ways of testing each chosen hypothesis in detail to find out which one is in closest agreement with our observations. Models of a type described earlier are used in the present study, providing a flexible means of expressing the theory in practical form. They are especially useful for studying the effect of modulating a nonsinusoidal type of daily variation.

Inevitably the daily variation is subject to irregular changes of phase and amplitude, apart from those due to the usual random fluctuations of the counting rate. Such changes could somewhat mask the characteristics of the more uniform modulations that we are interested in, although they may not contribute greatly to the annual mean effect. Fortunately, the irregularities appear to have a proportionately smaller effect on the intensity at 42 m w.e. than they do at sea level and, as we shall see, there seems to be good reason to believe that

(2) H. ELLIOT and D. W. N. DOLBEAR: *Journ. Atmos. and Terr. Phys.*, **1**, 205 (1951).

the daily variation underground in 1958 was not affected to a great extent by irregularities of that sort. A more serious hindrance to the kind of detailed analysis being undertaken here is presented by what may be called the « scale of duration » of a given process of modulation—that is, the period of time over which a given type of modulation of the daily variation remains substantially the same. If the period is less than a year then any analysis on an annual basis making use of the phase and amplitude relationship between I_d, I_s and I_{as} is no longer practicable. Again, 1958 seems to have been a favourable year in this respect but it will be shown in paper II that for some time thereafter the modulation process changed rapidly to forms with a scale of duration of rather less than a year.

2. – The D_r variations.

If we wish to study modulations of the solar daily variation month by month, unwanted terms of constant phase and amplitude may sometimes be eliminated by subtracting the annual mean d. v. from each of the monthly mean d. v. s. This has been done occasionally in the past. However the method is not suited to our purposes if only for the reason that it causes a solar d. v. with linear month by month displacement of amplitude to transform to a solar d. v. with an annual variation of amplitude and a semi-annual reversal of phase. This is one of the difficulties that may be avoided if, instead of subtracting the anuual mean d. v., we subtract the monthly mean d. v. for month $r+6$ from the mean d. v. for each month r. Thus if the mean d. v. in solar time for month r is V_r we obtain a set of averaged difference variations, D_r, defined as

$$D_r = \frac{V_r - V_{r+6}}{2}.$$

One of the most useful consequences of transforming from V_r to D_r variations is that a solar d. v. of constant phase and amplitude vanishes, while a sidereal d. v. of constant phase and amplitude is unimpaired by the transformation. It also turns out that if we wish to distinguish between the presence of specified solar and sidereal d. v. s. in the data when one or both of them happen to be modulated in a simple manner, the transformation from a V_r to a D_r d. v. offers definite advantages. For example, a solar d. v. with semi-annual variation of amplitude or phase vanishes in the D_r form of d. v., while a sidereal d. v. similarly modulated, and expressed in solar time, is unaffected when transformed. Table I sets out the results of transforming a solar d. v. from a V_r to a D_r variation when it has been modulated in various simple ways. A similar list for a modulated sidereal d. v. is set out in Table II.

TABLE I. – *The transformation from a solar diurnal variation to a solar D_r variation.*

Characteristics of solar V_r variation	Characteristics on transformation to solar D_r variation
1. Constant phase and amplitude $V_r = A \sin\varphi$	Vanishes $D_r = 0$
2. Linear displacement of amplitude $V_r = A\left(1 - \frac{K\theta}{2\pi}\right)\sin\varphi$	Constant phase and amplitude $D_r = \frac{AK}{4}\sin\varphi$
3. Linear displacement of phase $V_r = A \sin\left(\varphi - \frac{N\theta}{24}\right)$	Linear displacement of phase $D_r = A \sin\frac{N\pi}{48}\sin\left(\varphi - \frac{2n\theta + \pi(N-24)}{48}\right)$
4. Annual sinusoidal variation of amplitude $V_r = A(1 + k\cos\theta)\sin\varphi$	Semi-annual variation of amplitude and reversal of phase $D_r = Ak\cos\theta\sin\varphi$
5. Annual sinusoidal variation of phase $V_r = A\sin(\varphi - a\cos\alpha)$	Semi-annual variation of amplitude and reversal of phase $D_r = A\sin(a\cos\alpha)\sin(\varphi - \pi/2)$
6. Semi-annual sinusoidal variation of amplitude $V_r = A(1 + k\cos 2\theta)\sin\varphi$	Vanishes $D_r = 0$
7. Semi-annual sinusoidal variation of phase $V_r = A\sin(\varphi - a\cos 2\alpha)$	Vanishes $D_r = 0$

$\theta = 2\pi T/12$, where T is in months measured from 22nd September.
$\alpha = \theta - \theta_0$ where θ_0 is a phase constant.
Definitions of these and other symbols used are given in an earlier paper (1).

It should be emphasised that transformations are always made in solar time. The result, in sidereal time, is simply the average d. v. for months r and $r + 6$.

When the observations have been arranged in the form of D_r variations in solar time a graphed plot of a sequence of monthly mean D_r d. v. s. may itself clearly indicate how the daily variation has been modulated. Remembering that a set of six consecutive monthly mean D_r d. v. s. (comprising, of course, a complete year's data) is not independent of the preceding or following set, any one set may now be rearranged in sidereal and anti-sidereal time for inspection of the modulations of the daily variation in those respective

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TABLE II. – *The result, when expressed in sidereal time, of transforming a sidereal diurnal variation in solar time to a solar D_r variation.*

Characteristics of sidereal diurnal variation (i_s)	Characteristics, in sidereal time, after transforming to solar D_r variation (D_{rs})
1. Constant phase and amplitude $i_s = A \sin\psi'$	Unaffected $D_{rs} = A \sin\psi'$
2. Linear displacement of amplitude $i_s = A\left(1 - \frac{K\theta}{2\pi}\right)\sin\psi'$	Linear displacement of amplitude $D_{rs} = A\left(1 + \frac{K(\pi - 2\theta)}{4}\right)\sin\psi'$
3. Linear displacement of phase $i_s = A \sin\left(\psi' - \frac{N\theta}{24}\right)$	Linear displacement of phase $D_{rs} = A \cos\frac{N\pi}{48}\sin\left(\psi' - \frac{N(2\theta + \pi)}{48}\right)$
4. Annual sinusoidal variation of amplitude $i_s = A(1 + k\cos\alpha)\sin\psi'$	Constant phase and amplitude $D_{rs} = A \sin\psi'$
5. Annual sinusoidal variation of phase $i_s = A \sin(\psi' - a\cos\alpha)$	Semi-annual variation of amplitude (for $a \leqslant \pi/2$) $D_{rs} = A \cos(a\cos\alpha)\sin\psi'$
6. Semi-annual sinusoidal variation of amplitude $i_s = A(1 + k\cos 2\alpha)\sin\psi'$	Unaffected $D_{rs} = A(1 + k\cos 2\alpha)\sin\psi'$
7. Semi-annual sinusoidal variation of phase $i_s = A \sin(\psi' - a\cos 2\alpha)$	Unaffected $D_{rs} = A \sin(\psi' - a\cos 2\alpha)$

$\theta = 2\pi T/12$, where T is in months measured from 22nd September.
$\alpha = \theta - \theta_0$, where θ_0 is a phase constant.
Definitions of these and other symbols are given in an earlier paper (1).

time-systems. From such sets we also calculate the annual mean d. v. s., D_s and D_{as}, so named to distinguish them from I_s and I_{as} obtained directly from the V_r variations.

3. – The observations, 1957-1958.

The underground laboratory near Hobart has been described elsewhere (3). Two vertical semi-cubical telescopes placed in a disused railway tunnel beneath

(3) A. G. FENTON, R. M. JACKLYN and R. B. TAYLOR: *Nuovo Cimento*, **22**, 285 (1961).

approximately 42 m w. e. of shale and clay were put into operation in July, 1957 and reliable observations of the daily variation were obtainable from the beginning of October. However, until the end of 1958 when a set of somewhat defective counters was replaced, one of the telescopes was used solely as a reference for counting efficiencies and occasionally to fill in gaps in the records from the other, so that in effect results were available from a single telescope only.

The basic data for daily-variation studies comprised hourly values of intensity corrected to a standard pressure of 28 in. of mercury using a total barometer coefficient, $\beta = -1.651\%/\text{in}$. The following useable results were excluded. Centred on the day of maximum depression of intensity during a Forbush decrease observations were rejected for a number of complete days that depended on the severity of the decrease. Whole days were also rejected when more than two consecutive hours of data were missing. From the remainder, the mean solar daily variations (V_r) were then obtained month by by month in the form of percentage departures of bi-hourly mean values from the monthly mean intensities, after adjustment had been made for secular changes. Individual bi-hourly deviations from the monthly mean intensities for a single telescope had standard errors (SE) of estimate $\pm 0.07\%$.

The amplitudes and times of maximum of the annual mean d. v. s. in solar, sidereal and anti-sidereal time for 1958, and their SE's, were obtained from the sums of the first and second harmonics of best fit:

Solar: $(I_d)_{max} = (0.126 \pm 0.01)\%$, $\tau_m = 1500$ local solar time;

Sidereal: $(I_s)_{max} = (0.090 \pm 0.01)\%$, $\tau'_m = 0730$ local sidereal time;

Anti-sidereal: $(I_{as})_{max} = (0.052 \pm 0.01)\%$, $\tau''_m = 1000$ local anti-sidereal time.

There was, then, a large and highly significant apparent sidereal effect. Unfortunately, no clue to its origin is to be found from the annual mean d. v. s I_d, I_s and I_{as} alone.

In Fig. 1 the V_r variations of bi-hourly values are shown month by month from October, 1957 to December, 1958. For convenience the d.v.s are plotted over two full days. Some kind of annual variation in phase and amplitude is evident. Maximum amplitude (0.25%) occurs during January-February and again during May-June-July while the daily variation almost disappears during September-October-November. The latest maxima (at 1800 local time) occur during February-March and the earliest (at 1300) during August-September.

The modulation of the daily variation takes on a rather different aspect if we transform from the V_r to the D_r type of d.v. Before doing this, considerable smoothing of the V_r variations may be effected by averaging the monthly mean bi-hourly deviations progressively in groups of three to form sets

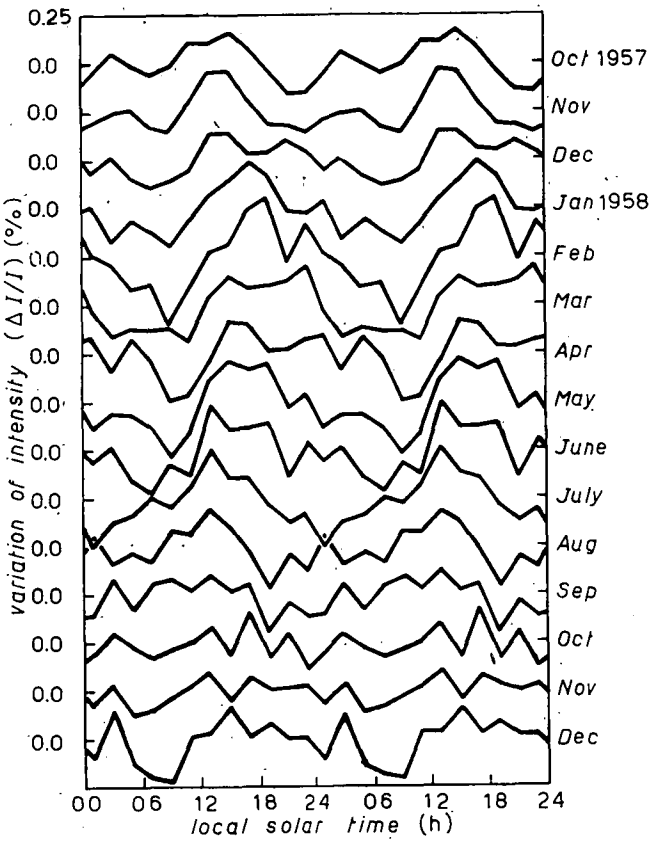

Fig. 1. – The monthly mean-pressure-corrected solar daily variation at 42 m w.e. underground, Hobart, from October, 1957 to December, 1958. The mean bi-hourly deviations for each month are plotted over two full days.

of twelve six-hour running means (RMs). Curves of the daily variation to fit the RMs may be drawn quite readily by eye. The amplitudes of these curves somewhat underestimate the true amplitudes but on the other hand the statistical uncertainties of amplitude are clearly less than if they had been obtained by harmonic analyses of the unsmoothed bi-hourly figures. Another advantage is a considerable saving of time which becomes important later on when large numbers of these amplitudes have to be obtained.

In Fig. 2 the D_r variations have been arranged in sidereal time over two-day intervals month by month from October, 1957 to December, 1958. Remembering that any six curves form an independent set, comprising a complete year's data, it is evident that these results could be ascribed to a sidereal daily variation with a semi-annual variation of amplitude and phase. The earliest

maxima are found in January and July and the amplitude minima are in December and June. It should be noted that there has evidently been no important change in the character of the modulating mechanism between the end of 1957 and the end of 1958.

Having some idea of the kinds of hypotheses that might be advanced to explain the apparent sidereal effect we now proceed to test various hypotheses in detail. All the basic information required to do this is presented graphically in Fig. 3. Although precise statistical limits cannot be assigned to the amplitudes of the curves drawn through the annual RM's in Fig. 3*a*, 3*b* and 3*c*, the uncer-

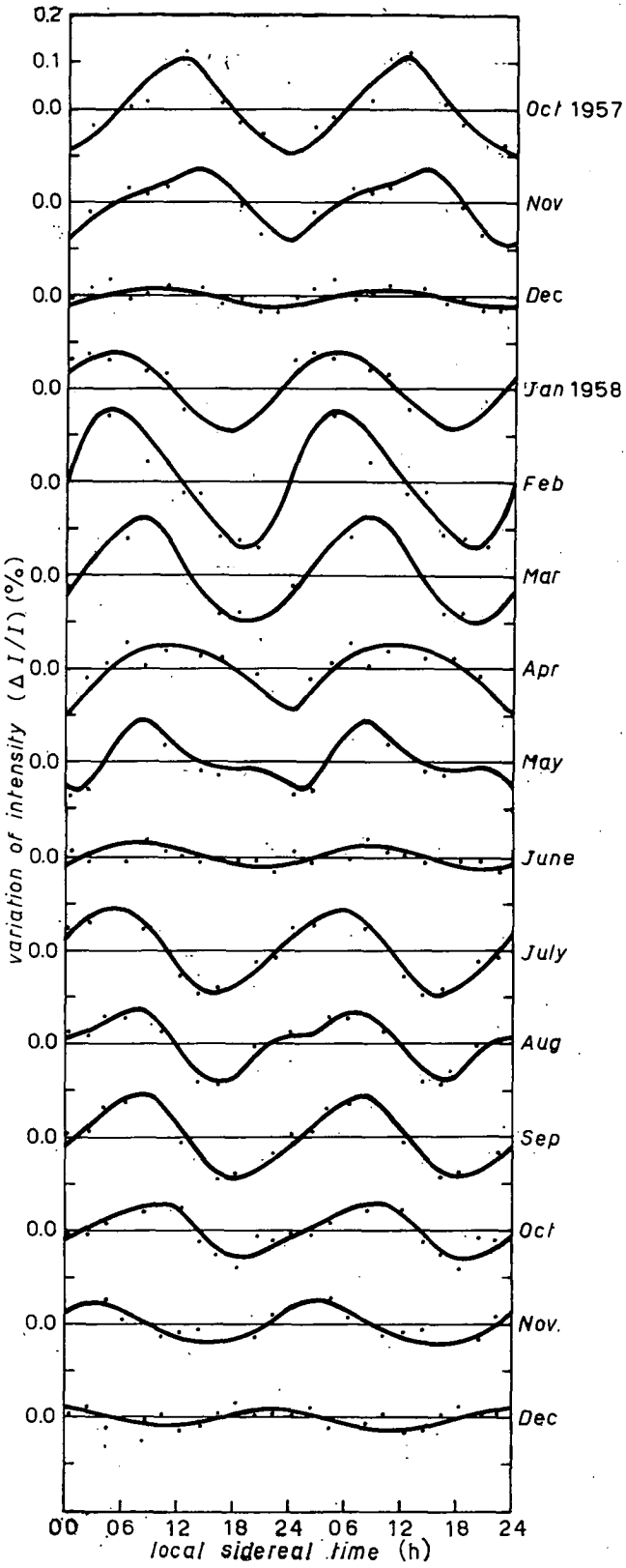

Fig. 2. – The monthly mean daily variations of D_r in sidereal time at 42 m w.e. underground, Hobart, from October, 1957 to December, 1958, obtained from 6 h running means and plotted over two full days.

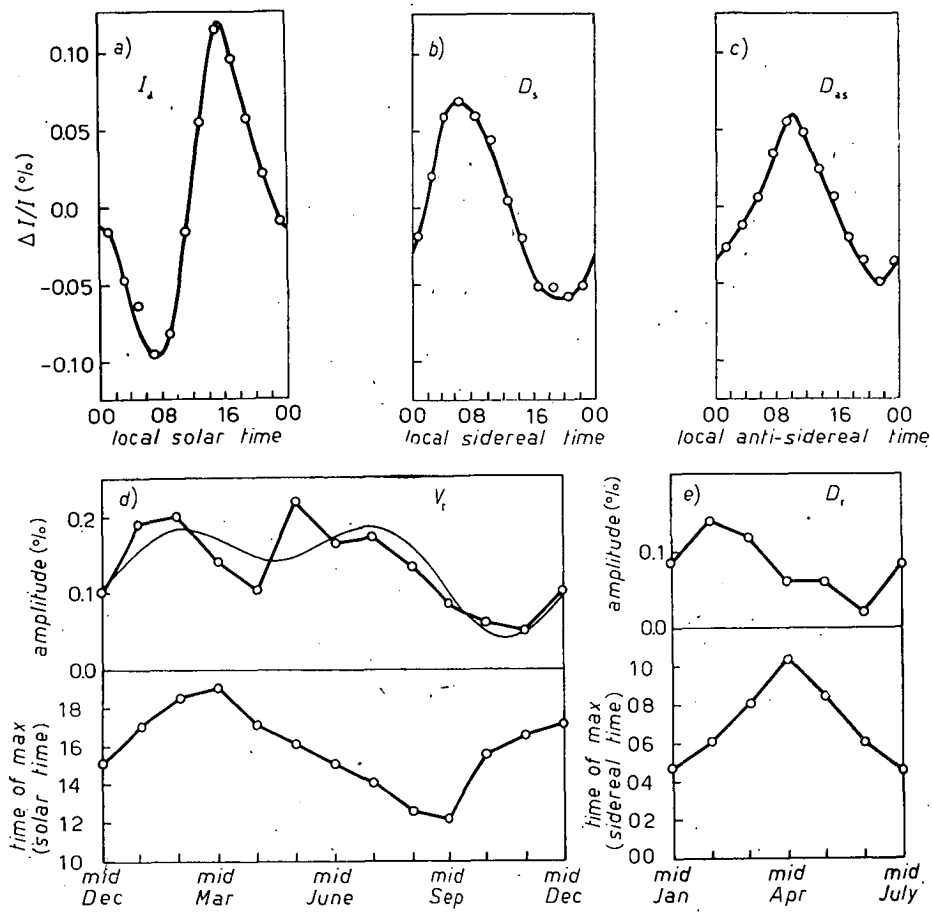

Fig. 3. – Characteristics of the daily variation at 42 m w.e. underground, Hobart, in 1958. The circled points are obtained after conversion of the observed bi-hourly mean deviations of intensity to 6 h running means. *a*) The annual mean daily variation in solar time. *b*) The annual mean difference, D_r, variation in sidereal time. *c*) The annual mean D_r variation in anti-sidereal time. *d*) Month-by-month mean values of amplitude and phase of the solar daily variation. A curve of best fit to the amplitude values is also shown. *e*) Month-by month mean values of amplitude and phase of the D_r daily variation in sidereal time.

tainties of amplitude must be very small. The annual mean D_r d.v. in sidereal time is the average of the set of six D_r d.v.s shown in Fig. 2 for the period January to June, 1958. The monthly mean amplitudes and times of maximum shown in Fig. 3*d* are obtained from the results shown in Fig. 1 after conversion to 6-hour RMs. The curve of best fit to the amplitude variations, comprising the sum of the first and second harmonics, has also been drawn in. It is important to know whether the amplitudes in March-April were indeed

significantly less than in January-February and May-June. Therefore, monthly mean amplitudes were also obtained by harmonic analyses of the unsmoothed bi-hourly figures using the sums of the first and second harmonics. They were seen to follow the pattern of variation of the RM amplitudes closely but at a somewhat higher level. The difference in amplitude between January-February and March-April was found to be 0.07 % while between March-April and May-June it was 0.08 %. The SE of estimate of amplitude for each pair of months was ± 0.02 %. Therefore, the amplitude minimum in March-April would have to be regarded as genuine even if we had not used smoothed values. The D_r amplitudes and times of maximum in sidereal time shown in Fig. 3*e* are obtained directly from Fig. 2.

As mentioned earlier, there are three kinds of hypotheses to examine. In the first place we may consider the apparent sidereal effect to be entirely spurious. Hypothesis $I(a)$ supposes that the effect is produced by an annual sinusoidal variation of phase and amplitude of the solar daily variation. In a slightly more complicated form of this hypothesis we suppose (I(b)) that the solar d.v. comprises two independent components—a solar daily variation of constant phase and amplitude and a sinusoidal solar d.v. with a sinusoidal annual variation of phase and amplitude. In other words, in this case the apparent sidereal effect would be produced by a solar d.v. whose phase and amplitude characteristics did not depend in any way on the observed annual mean solar d.v.

The second kind of hypothesis is that the apparent sidereal effect is partly genuine but contains a spurious component of solar origin. The simplest possibility worth examining here is (II(a)) that a sidereal daily variation of constant phase and amplitude is present together with the ordinary solar d.v. which has an annual sinusoidal variation of amplitude. Many studies of the sidereal effect in the past have been centred on this possibility and it was with this in mind that FARLEY and STOREY [4] devised the technique of analysis in anti-sidereal time. It can easily be shown by summing the vectors on a harmonic dial that the genuine sidereal d.v. superimposed on the amplitude-modulated solar d.v. will give rise to a solar d.v. with an annual variation of amplitude and phase. In turn we should expect to find in the D_r form an apparent sidereal d.v. with some kind of semi-annual variation of amplitude and phase. A more flexible variant of this hypothesis is (II(b)) that there is, together with a sidereal d.v. of constant phase and amplitude, a solar d.v. of constant phase and amplitude plus a separate sinusoidal solar d.v. with a sinusoidal annual variation of amplitude. The amplitude-modulated solar d.v. in this case does not have to conform to the observed annual mean solar d.v. in shape and in time of maximum. It will be shown that the matter of

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[4] F. J. M. FARLEY and J. R. STOREY: *Proc. Phys. Soc.*, A **67**, 996 (1954).

shape of the modulated solar d.v. is a very important one if we wish to predict the effects of the modulation in sidereal and anti-sidereal time.

Finally, it is possible that the apparent sidereal effect is entirely genuine. In hypothesis III(*a*) the possibility of a sidereal daily variation with semi-annual modulation of amplitude and phase is put forward, as suggested by the phase and amplitude changes shown in Fig. 3*e*. Consequently a sidereal d.v. with combined annual and semi-annual modulation of phase, hypothesis III(*b*), must also be considered, since when expressed in the D_r form, it is indistinguishable from III(*a*).

The phase and amplitude characteristics of I_d, D_s and D_{as} derived from Fig. 3*a*, 3*b* and 3*c* are set out in Table III. The first requirement of any specific model will be to reproduce these characteristics as nearly as possible.

TABLE III. – *Amplitudes and times of maximum of the annual mean d.v.s.* I_d, D_s *and* D_{as} *for 1958, derived from Fig. 3.*

Type of d.v.	Amplitude (%)	Time of maximum (h)
Solar, I_d	$0.120\, I_{dm}$	1530 local solar time, τ_m
Sidereal, D_s	$0.070\, D_{sm}$	0630 local sidereal time, τ'_{Dm}
Anti-Sidereal, D_{as}	$0.060\, D_{asm}$	1000 local anti-sidereal time, τ''_{Dm}

Generally speaking, the method of analysis will be the same as that used in a previous paper (1). As far as possible the same symbols will be used. We first of all develop the theory for a given hypothesis, based on sinusoidal modulations of sinusoidal diurnal variations. From the theory we obtain certain constants to apply to the development of model d.v.s, which we can then rearrange for comparison with our observations in exactly the same way as we would the experimental data. We can approximate even more closely to the conditions of experiment in some cases by making the unmodulated model d.v. conform to the nonsinusoidal type of d.v. that we actually observe. As we shall see, the modulation of a nonsinusoidal, instead of a sinusoidal, d.v. can sometimes give rise to a remarkably different result.

4. – Possibility of an entirely spurious effect.

4·1. *Hypothesis I(a). An annual sinusoidal variation of amplitude and phase of the solar daily variation.* – An unmodulated sinusoidal d.v. is represented by

$$i_d = I_{0dm} \sin \varphi \,, \tag{4.1}$$

where $\varphi = \varepsilon - \varepsilon_0 = (\pi/12)(\tau - \tau_0)$, ε_0 and τ_0 being the phase constant expressed in angular measure and in hours respectively.

When annual sinusoidal modulations in phase and amplitude are impressed upon it the d.v. becomes

$$i_d = I_{0dm}(1 + k\cos(\theta - \theta_a))\sin\left(\varphi - \frac{\pi n}{12}\cos(\theta - \theta_p)\right), \tag{4.2}$$

where $k =$ the maximum fractional change of amplitude, $n =$ the maximum displacement of phase, in hours, $\theta = \pi T/6$ where T is in months measured from 22nd September, $\theta_a =$ the annual amplitude constant, $\theta_p =$ the annual phase constant.

Now, our problem is to determine values of the constants I_{0dm}, ε_0, k, n, θ_a and θ_p to give us a daily variation having as nearly as possible the characteristics shown in Fig. 3. One way of going about this is to obtain estimates of k and n from Fig. 3*d* and then determine the values of I_{0dm}, ε_0, θ_a and θ_p that are required to give us the annual mean characteristics set out in Table III. This we will call method *A*. We now determine the relationships between the constants and the characteristics of the annual mean d.v.s theoretically.

In simpler form the daily variation on any given day is

$$i_d = I_{0dm}(1 + k\cos\alpha_a)\sin(\varphi - a\cos\alpha_p), \tag{4.3}$$

where

$$\alpha_a = \theta - \theta_a, \quad \alpha_p = \theta - \theta_p \quad \text{and} \quad a = \frac{\pi n}{12}.$$

Six months later the d.v. is

$$(i_d)_{+6} = I_{0dm}(1 - k\cos\alpha_a)\sin(\varphi + a\cos\alpha_p).$$

We then arrive at the following expression for the D_r variation $(i_d - (i_d)_{+6})/2$

$$\frac{D_r}{I_{0dm}} = -\sin(a\cos\alpha_p)\cos\varphi + k\cos\alpha_a\cos(a\cos\alpha_p)\sin\varphi. \tag{4.4}$$

If $\sin(a\cos\alpha_p)$ and $\cos(a\cos\alpha_p)$ are expanded by means of Bessel functions we obtain an expression for D_r/I_{0dm} from which terms of sidereal frequency ($\varphi' = \varphi + \theta$) may be extracted, after the manner shown in an earlier paper [1]. The annual mean d.v. in sidereal time, D_s, comprises the sum of these terms, and we have finally

$$D_s = I_{0dm}(p_s\cos\Phi' + q_s\sin\Phi') = I_{0dm}r_s\cos(\Phi' - \varrho_s), \tag{4.5}$$

where

$$p_s = -J_1(a)\cos\theta_p - \frac{J_0(a)k}{2}\sin\theta_a - \frac{J_2(a)k}{2}\sin(\theta_a - 2\theta_p)\,,$$

and

$$q_s = -J_1(a)\sin\theta_p + \frac{J_0(a)k}{2}\cos\theta_a - \frac{J_0(a)k}{2}\cos(\theta_a - 2\theta_p)\,.$$

Similarly, extracting terms of anti-sidereal frequency ($\varphi'' = \varphi - \theta$) the half-yearly mean d.v. in anti-sidereal time is

$$D_{as} = I_{0dm}(p_{as}\cos\Phi'' + q_{as}\sin\Phi'') = I_{0dm}r_{as}\cos(\Phi'' - \varrho_{as})\,, \tag{4.6}$$

where

$$p_{as} = -J_1(a)\cos\theta_p + \frac{J_0(a)k}{2}\sin\theta_a - \frac{J_2(a)k}{2}\sin(2\theta_p - \theta_a)\,,$$

and

$$q_{as} = J_1(a)\sin\theta_p + \frac{J_0(a)k}{2}\cos\theta_a - \frac{J_2(a)k}{2}\cos(2\theta_p - \theta_a)\,.$$

To obtain agreement with the observed annual mean d.v.s we require $D_{asm}/D_{sm} = 0.857$ and $\tau''_{Dm} - \tau'_{Dm} = 0330$ (see Table III), so that in eq. (4.6) we need to put $r_{as} = 0.857 r_s$ and $\varrho_{as} = \varrho_s + 52° 30'$. This enables us to express p_{as} and q_{as} in terms of p_s and q_s and finally leads to the following two equations which may be solved for θ_a and θ_p when k and n are known:

$$1.522\,p_s - 0.680\,q_s + 2J_1(a)\cos\theta_p = 0\,, \tag{4.7}$$

and

$$0.680\,p_s - 0.478\,q_s - 2J_1(a)\sin\theta_p = 0\,. \tag{4.8}$$

Suitable values for k and n are found by inspection of Fig. 3*a*.

From the lower graph of the figure we see that the maximum displacement of phase of i_d must be approximately three hours, so we put $n = 3$. The maximum fractional displacement of amplitude, k, is not quite so easily estimated by inspection of the upper graph because of the apparent anomaly in March-April. But a reasonable figure, giving a range of ± 0.092 about a value approximately equal to the annual mean amplitude would be $k = 0.77$. We insert these values of k and n into the expressions for p_s and q_s and then solve eqs. (4.7) and (4.8) for θ_a and θ_p: This was done graphically and, from the various possible solutions, the values appropriate to our case were

$\theta_a = 157°$ and $\theta_p = 150°$. This means that the day of maximum amplitude would be 28th February while 21st February would be the day of maximum advancement of phase. (Referring to Fig. 3*d* it would appear that the predicted time of maximum amplitude is rather too early in the year).

We immediately get p_s and q_s and consequently the following values for the remaining constants, I_{0dm} and ε_0:

$$I_{0dm} = 0.139\,\%\,, \qquad \varepsilon_0 = 163°\,42'\,, \qquad i.e. \qquad \tau_0 = 1054\,.$$

Let us now look at the values of amplitude and phase of the annual mean solar d.v., I_d, predicted by the constants we have obtained. If we expand the expression for i_d (eq. (4.3)) using Bessel functions we find that the terms of solar frequency are $J_0(a)\sin\varphi$ and $-J_1(a)k\cos(\theta_p - \theta_a)\cos\varphi$. Therefore the annual mean d.v. is

$$(4.9) \qquad \frac{I_d}{I_{0dm}} = J(a)\sin\Phi - J_1(a)k\cos(\theta_p - \theta_a)\cos\Phi = \\ = p_d\cos\Phi + q_d\sin\Phi = r_d\cos(\Phi - \varrho_d)\,.$$

Putting in the values we have found for the constants, we have

$$r_d = 0.893 \qquad \text{and} \qquad \varrho_d = 107°\,55'\,.$$

This leads to the following values for the amplitude and phase:

$$I_{dm} = 0.124\,\%\,, \qquad \tau_m = 1806\,.$$

From Table III we see that the observed amplitude (0.120 %) is fairly close to that predicted but that the observed annual time of maximum (1530) differs rather markedly from the predicted figure.

We may test our hypothesis in detail against the results shown in Fig. 3 by developing facsimile d.v.s which embody the constants we have just found. The basis of the model is the observed annual mean pressure-corrected solar daily variation of vertical intensity underground, averaged over the three years 1958-60 inclusively. It is clearly nonsinusoidal. A description of this model and of the method of impressing modulations of phase and of amplitude upon it are to be found in an earlier theoretical paper (1). Combined annual modulations of phase and amplitude were not considered. To obtain such modulated d.v.s month by month it is necessary to multiply the phase term by the mean amplitude term for each value of t_θ, where t_θ is the number of days for which the time of maximum is constant to the nearest half an hour (see Section **2** of the earlier paper). The weighted means of these com-

bined terms for each month are then calculated. It is a rather tedious process and almost all the calculations must be carried out afresh for any other pair of values of θ_a and θ_p. It was found in fact that the same result was achieved, for the rather small range of variation in phase that concerns us, if the mean amplitude term for each month was multiplied by the weighted mean phase term.

Facsimile V_r variations with annual variation of phase and amplitude were prepared for $\theta_a = 157°$, $\theta_p = 150°$, $n = 3$ and $k = 0.77$. The amplitude (I_{0dm}) and time of maximum (τ_{0m}) of the unmodulated solar d.v. were adjusted so that when the D_r variations in sidereal time were averaged the right values were obtained for the amplitude and phase of D_s. Thus, much of the discrepancy in the phase and amplitude characteristics of the model annual mean d.v.s will be taken up in the annual mean solar d.v., I_d. The various annual mean characteristics of the model are set out in Table IV.

TABLE IV. – *Amplitudes and times of maximum of the annual mean daily variations* I_d, D_s *and* D_{as}, *derived from a facsimile nonsinusoidal solar d.v. with annual variation of phase and amplitude, where* $n=3$, $k=0.77$, $\theta_a=157$, $\theta_p=150$, $I_{0dm}=0.154$ *and* $\tau_{0m}=1700$.

Type	Amplitude (%)	Time of maximum (h)
Solar, I_d	$0.135\, I_{dm}$	1945 local solar time, τ_m
Sidereal, D_s	$0.069\, D_{sm}$	0630 local sidereal time, τ'_{Dm}
Anti-sidereal, D_{as}	$0.058\, D_{asm}$	1100 local anti-sidereal time, τ''_{Dm}

Clearly, the model requires the maximum of the annual mean solar d.v. to be larger and to occur much later than is actually observed.

The dashed lines in Fig. 4 represent the results from the model using method A. There is, as there must be, some kind of agreement with the observations in general. However, as indicated in the theory, there are serious discrepancies, the most important being, perhaps, the disagreement in phase and amplitude shown by the annual mean solar d.v. (Fig. 4*a* inset). The annual V_r variations of amplitude of the model are also in poor agreement with the observed variations (Fig. 4*a*). If we regard the March-April minimum as an irregularity unrelated to the overall modulation mechanism, a sinusoidal annual variation of amplitude cannot account for the high level of amplitude from January to July. Again, the amplitude minimum of the model occurs several months too early.

We now consider briefly another way of presenting hypothesis I(*a*). In method B we choose suitable values for k, n, θ_a, θ_p, I_{0dm} and τ_{0m} by inspection of the observed V_r variations shown in Fig. 3*d*. We then note how well a model which employs these constants agrees with the observations in various other respects. We thus begin with a model that gives as reasonable a fit as possible with the observations in solar time, whereas in method A our model

was designed to give some degree of fit in sidereal and anti-sidereal time as well as in solar time. As we did then, we choose $n = 3$ and $k = 0.77$. If we regard the time of the amplitude minimum to be not earlier than mid-October, then mid-April is about the earliest date we can give for the amplitude maximum, making $\theta^a = 204°$. It is clear that the time of maximum advance of phase must be approximately mid-March, giving us $\theta_p = 173°$. Again, from Fig. 3*d* we observe that an assumed sinusoidal variation of amplitude would be symmetrical about a value approximately equal to the annual mean amplitude. The value chosen for I_{0dm} was 0.11 %, slightly less than the annual mean. The observed seasonal change of phase is symmetrically displaced about a value which is close to the annual mean (Fig. 3*d*). Therefore we put $\tau_{0m} = 1530$. The dotted lines in Fig. 4 show the result of constructing a model based on these constants. It appears from Fig. 4*a* and 4*b* that agreement with the phase and amplitude variations of V_r is about as good as one could expect on this hypothesis. But such a fit requires the time of maximum of

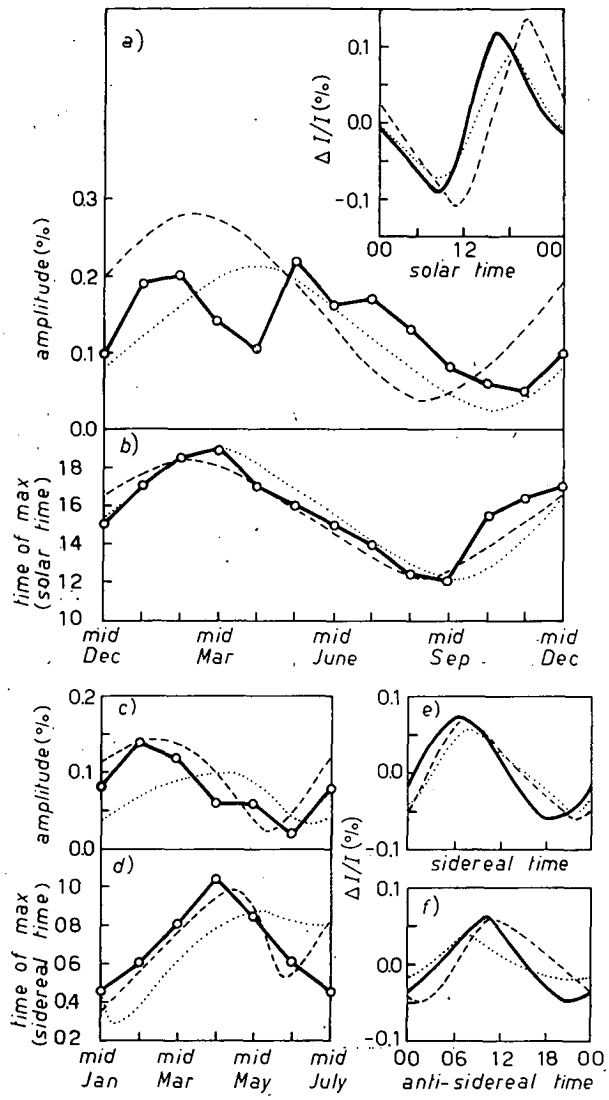

Fig. 4. – Hypothesis I(*a*). The results of model analyses compared with the observed characteristics of the daily variation. Dashed lines: method A. Values of k and n are estimated from the observations. The model is designed to give the correct values for the amplitudes and times of maximum of D_s and D_{as}. Dotted lines: method B. Values of k, n, θ_a, θ_p, I_{0dm} and τ_{0m} are estimated from the observations.

I_d to be somewhat later and the amplitude to be considerably less than that observed (Fig. 4*a* inset). In sidereal and anti-sidereal time the degree of correspondence with the observations is very poor, (Fig. 4*c* to 4*f*).

4·2. *Hypothesis I(b). A sinusoidal solar d.v. with a sinusoidal annual variation of amplitude and phase plus a solar d.v. of constant phase and amplitude.* – Our aim is to find a sinusoidal solar d.v. which, when modulated to give an annual sinusoidal variation of amplitude and phase, satisfies the relationships

$$\frac{D_{asm}}{D_{sm}} = 0.857 \qquad \text{and} \qquad \tau''_{Dm} - \tau'_{Dm} = 0330 ,$$

for suitable values of k, n, θ_a and θ_p. When these constants are applied to a sinusoidal model we can then easily compose another facsimile solar d.v. of constant phase and amplitude such that the two d.v.s combine to give us an annual mean solar d.v., I_d, identical with that observed. Thereby we obtain the first important requirement of a model, complete agreement with all the observed annual mean values of phase and amplitude set out in Table III.

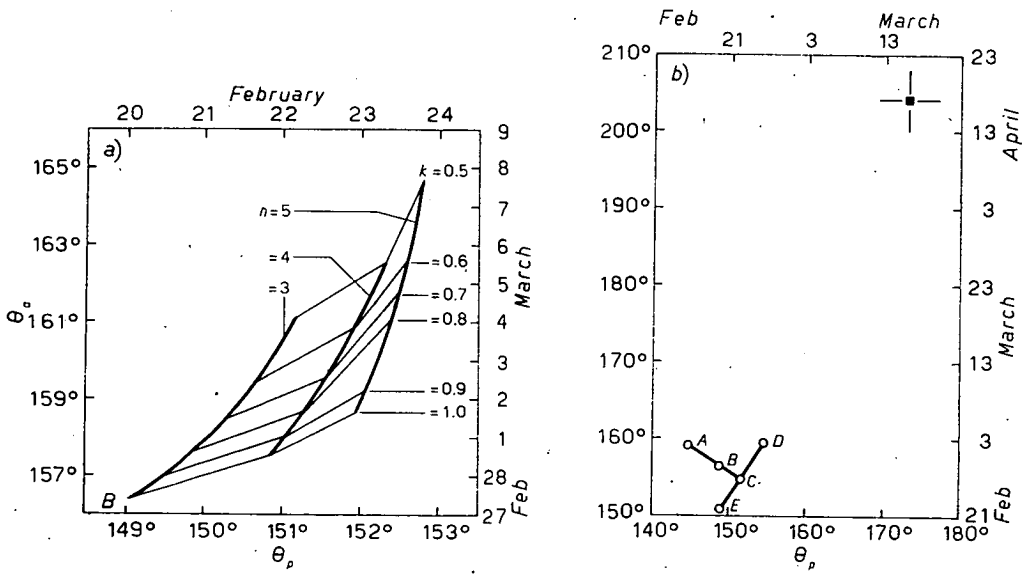

Fig. 5. – (*a*) Values of θ_a and θ_p for which a sinusoidal solar d.v. with annual sinusoidal variation of amplitude and phase satisfies the relations $D_{asm}/D_{sm} = 0.857$ and $\tau''_{Dm} - \tau'_{Dm} = 0330$, for various values of k and n. (*b*) Variations in position of the point B (reproduced from (*a*) above) – (1) As the ratio D_{asm}/D_{sm} changes from 0.75 (*A*) to 0.95 (*C*) when $\tau'_{Dm} - \tau''_{Dm} = 0330$ and (2). As $\tau'_{Dm} - \tau''_{Dm}$ changes from 0300 (*D*) to 0400 (*E*) when $D_{asm}/D_{sm} = 0.95$. The starred square represents the approximate values of θ_a and θ_p estimated from the observations for 1958.

In the first place, then, we want solutions of eqs. (4.7) and (4.8), giving us values of θ_a and θ_p corresponding to suitable values of k and n. Such solutions, as functions of n and k, are shown in Fig. 5*a*. It is at once apparent that the times of the year when maximum amplitude and maximum advance of phase occur, vary by no more than a few days over a wide range of values of k and for values of n in the range $n=3$ to $n=5$. In Fig. 5*b*, the point B (the solution for $n=3$ and $k=1$) is reproduced from Fig. 5*a* while the line ABC shows how the solutions for $n=3$ and $k=1$ change as D_{asm}/D_{sm} varies from 0.75(A) to 0.95(C), while the phase relation $\tau''_{Dm}-\tau'_{Dm}=0330$ remains constant. The line ECD shows the effect of varying the phase relation $\tau''_{Dm}-\tau'_{Dm}$ between the values 0400(E) and 0300(D) while $D_{asm}/D_{sm}=0.95$ remains constant.

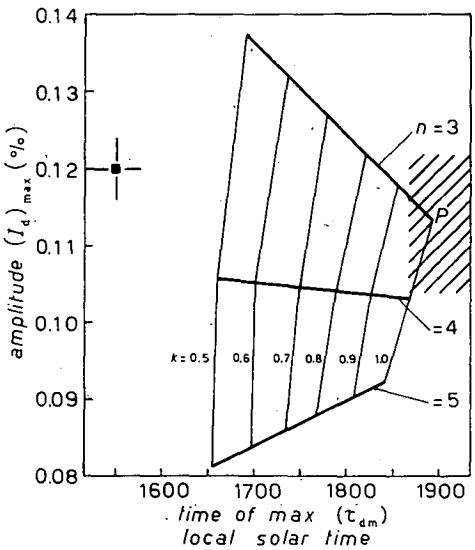

Fig. 6. – Values of the amplitude I_{dm} and time of maximum τ_{dm} of the annual mean sinusoidal solar d.v. with annual sinusoidal variation of amplitude and phase satisfying the relations $D_{asm}/D_{sm}=0.857$ and $\tau''_{Dm}-\tau'_{Dm}=0330$ when $D_{sm}=0.070$ and $\tau'_m=0630$. The starred square represents the annual mean solar d.v. for 1958, in amplitude and phase. The hatched area embraces all positions of the point P that correspond to values of D_{asm}/D_{sm} between 0.75 and 0.95 and values of $\tau''_{Dm}-\tau'_{Dm}$ between 0300 and 0400.

Therefore, even allowing a fair margin of error in our estimates of the phase and amplitude relationships shown in Table III, it is clear that a modulated sinusoidal solar d.v. must have maximum amplitude at some time between the third week of February and the end of the first week of March while the maximum advance of phase must occur between about the middle and the end of February. On the other hand, the starred point in Fig. 5*b* indicates that if the observed annual variations of amplitude and phase of V_r are considered to be basically sinusoidal, the times when maximum amplitude and maximum advance of phase occur would seem to be somewhere near 17th April and 16th March respectively.

Again, in Fig. 6, the annual mean amplitude, I_{dm}, and the time of maximum, τ_{dm}, of the modulated sinusoidal solar d.v. are shown as functions of solved values of k and n, when we are also given the amplitude and time of maximum of, say, D_s, the annual mean D_r d.v. in sidereal time. The starred point represents the amplitude and phase of the observed annual mean solar d.v.

The hatched area indicates a zone of variation in position of the point P that corresponds to the previously defined latitude of variation in estimate of D_{asm}/D_{sm} and $\tau''_{Dm} - \tau'_{Dm}$. Figure 6 shows clearly that the observed effects in sidereal and anti-sidereal time can only be ascribed to the modulation in phase and amplitude of a sinusoidal solar d.v. if the time of maximum, τ_{dm}, is some hours later than that of the observed annual mean solar d.v. This was one of the difficulties experienced with hypothesis I(a) when the whole effect was attributed to a nonsinusoidal solar d.v. with annual variation of phase and amplitude. However, in the present case, we overcome that difficulty by inventing a suitable unmodulated solar-time component which, when added to the modulated sinusoidal d.v., gives us the required annual mean values for I_{dm} and τ_{dm}.

The basis of a model was therefore a modulated sinusoidal component, whose characteristics were determined by solved values of θ_a, θ_p, n, k, I_{0dm} and τ_{0m}, plus an unmodulated component obtained as follows. If the observed annual mean solar d.v. is $I_{d\ observed}$ and the annual mean facsimile d.v. derived from the modulated sinusoidal component is $I_{d\ sinusoidal}$, a balancing term I_{dc} is defined by algebraic subtraction so that

$$I_{dc} = I_{d\ observed} - I_{d\ sinusoidal}.$$

Thus I_{dc} becomes the unmodulated d.v. to be added to each monthly mean d.v. developed from the modulated sinusoidal model. This gives a set of facsimile V_r variations that is considerably influenced by the presence of the unmodulated component. However, in the D_r form this component vanishes and therefore contributes nothing to the variations of D_r in sidereal and anti-sidereal time.

Models were prepared using various sets of solved values of the constants. The dashed lines ($n = 4$, $k = 1.0$) and dotted lines ($n = 4$, $k = 0.5$) in Fig. 7 show the results for two cases that appeared to give as good agreement with the observations as could be obtained on this hypothesis. The annual mean d.v. in solar time is not shown since the addition of I_{dc} causes the annual mean d.v. of the model to agree identically with the observed d.v.

It turns out that I_{dc} is a daily variation with two maxima, the first between about 0200 and 0500 local time, and the later one between about 1400 and 1500. The relation between the amplitudes of the two maxima depends greatly upon the value of k used in the modulated d.v. Therefore I_{dc} by no means approximates to a true semidiurnal wave.

In spite of the fact that the facsimile d.v.s I_d, D_s and D_{as} agree precisely with the observed d.v.s both in phase and amplitude, hypothesis I(b) fails to account for the peculiar month by month variations of amplitude of the solar d.v. (Fig. 7a). It was found that the positions of the calculated maxima

and minima were not perceptibly affected by the choice of n in the range $n=3$ to $n=5$ and of k in the range $k=0.5$ to $k=1.0$. In other words, as Fig. 5 and 6 indicate, the characteristics of the calculated curves of Fig. 7*a* are remarkably stable for given values of D_{asm}/D_{sm} and $\tau''_{Dm}-\tau'_{Dm}$.

Another rather serious lack of agreement is illustrated in Fig. 7*d* where the observed semi-annual variation of phase of sidereal D_r is seen to be symmetrical about an April maximum in the first half of the year. In the case of the model, the form of the semi-annual variation of phase is saw-toothed, with a phase reversal taking place sometime between May and June.

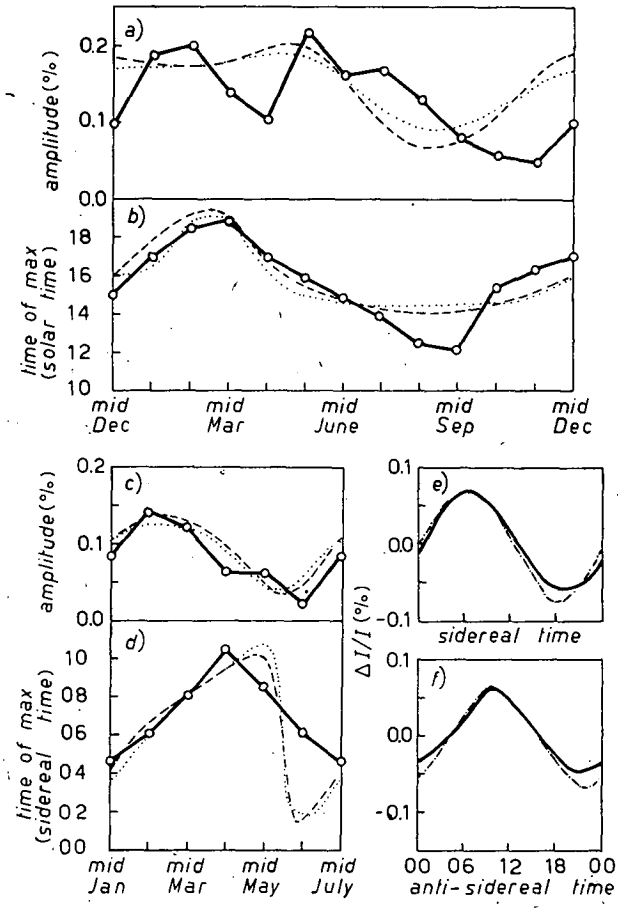

Fig. 7. – Hypothesis I(*b*). The results of model analyses compared with the observed characteristics of the daily variation reproduced from Fig. 3. The model is designed to give the correct values for the amplitude and time of maximum of D_s and D_{as} and to reproduce I_d identically. Dashed lines: $n=4$, $k=1.0$. Dotted lines: $n=4$, $k=0.5$.

5. – Possibility of a mixed effect.

5·1. *Hypothesis II(a). An annual sinusoidal variation of amplitude of the solar daily variation plus a sidereal daily variation of constant phase and amplitude.* – Earlier considerations of this case have been centred on the assumption that the solar daily variation may be adequately represented by a sine

wave. The amplitude-modulated d.v. is then

$$(5.1) \qquad I_d = I_{0dm}(1 + k\cos(\theta - \theta_a))\sin\varphi\,,$$

$$(5.2) \qquad = I_{0dm}\left[\sin\varphi + \frac{k}{2}\sin(\varphi + (\theta - \theta_a)) + \frac{k}{2}\sin(\varphi - (\theta - \theta_a))\right].$$

On the right-hand side of eq. (5.2) there is a term at each of the solar, sidereal and anti-sidereal frequencies. Accordingly, the annual mean d.v.s are:

$$(5.3) \qquad I_d = I_{0dm}\sin\Phi\,,$$

$$(5.4) \qquad I_s = I_{0dm}\frac{k}{2}\sin(\Phi' - \theta_a)\,,$$

$$(5.5) \qquad I_{as} = I_{0dm}\frac{k}{2}\sin(\Phi'' + \theta_a)\,.$$

From expression 4, Table I, it can be seen that I_s and I_{as} must be identical with their counter parts D_s and D_{as}, in the D_r form of the daily variation.

Analyses of facsimile d.v.s based on a sinusoidal model precisely confirm the following simple phase and amplitude relations, derived from eqs. (5.3), (5.4) and (5.5):

$$(5.6) \qquad \begin{cases} \dfrac{I_{asm}}{kI_{0dm}} = 0.5\,; & \tau_m - \tau_m'' = 2T_a\,, \\ \dfrac{I_{asm}}{I_{sm}} = 1.0\,; & \tau_m' - \tau_m'' = 4T_a\,. \end{cases}$$

Note that the annual mean amplitude I_{dm} may be replaced by I_{0dm} in this case. The symbol T_a, equivalent to θ_a, represents the time when maximum amplitude occurs, measured in months after 22nd September.

In a practical situation we would have observed values of I_{dm}, τ_m, I_{asm} and τ_m''. Therefore the phase and amplitude of the spurious d.v. in sidereal time can be immediately deduced from the relationships given above. Then, the vector representing the spurious sidereal d.v. in phase and amplitude may be subtracted from that which represents the observed apparent sidereal d.v. The residual vector represents the true sidereal d.v. of constant phase and amplitude.

The simple vector method of deducing the true sidereal effect no longer applies if the amplitude modulation is being impressed on a nonsinusoidal solar d.v. Furthermore, the various phase and amplitude relations are considerably affected if we use a nonsinusoidal model, as we shall now see. Our model is based as usual on the observed annual mean solar d.v. underground,

pressure-corrected and averaged over the years 1958-60. The dashed lines in Fig. 8 show the annual mean phase and amplitude relationships derived from this model and expressed as functions of T_a. The relationships predicted by the theory are shown as straight lines while the circles represent results obtained from the sinusoidal model.

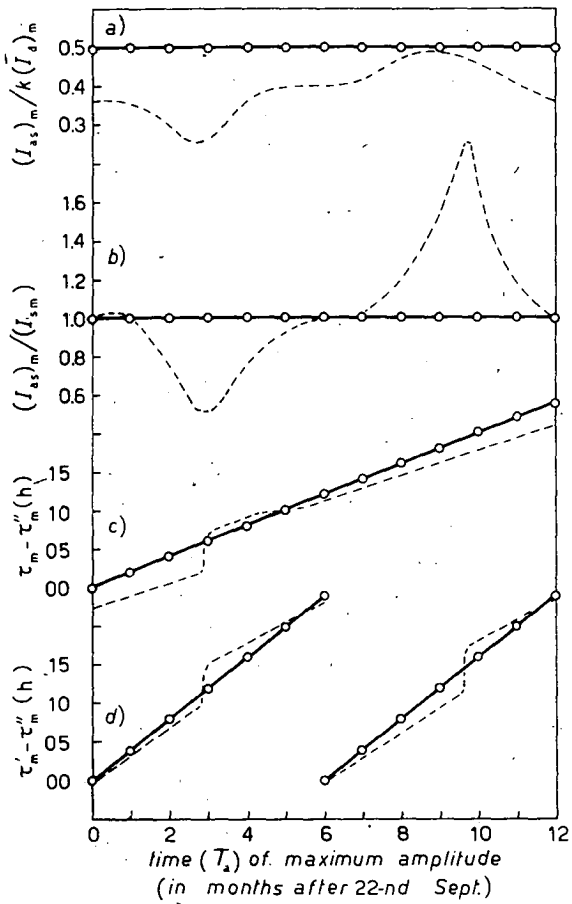

Fig. 8. – A solar daily variation with annual sinusoidal variation of amplitude. The predicted phase and amplitude relationships between I_d, I_s and I_{as} are shown as functions of T_a, the time of the year when maximum amplitude occurs. Full lines: theory. Circles: sinusoidal model. Dashed lines: « underground » model.

Clearly, in the nonsinusoidal case, the dependence of the amplitude ratios on T_a and the nonlinear dependence of phase relationships, greatly complicates the task of predicting the spurious sidereal effect. It becomes necessary to work out new phase and amplitude relations for every solar d.v. whose shape differs from that of the model described here. In the present instance, however, we can identify the shape of the unmodulated facsimile d.v. with the basic solar d.v. underground in 1958, since, holding to our hypothesis, we assume the basic unmodulated d.v. for that year to be identical with the annual mean d.v. Alterations in the shape of the annual mean d.v. over the years 1958, 1959 and 1960 were only slight.

The dependence of $\tau_m - \tau''_{Dm}$ on T_a, shown as the dashed line in Fig. 8*c*, will give us the value of T_a which relates to the 1958 observations. From Table III we see that $\tau_m - \tau''_{Dm} = 0530$, so that $T_a \sim 2.9$. It happens that this is a critcal value of T_a since at this point the results from the nonsinusoidal model show maximum departure from the sinusoidal case. The coincidence may be accidental, but it is interesting to speculate that it may

indicate that there is a connection between the mechanism which causes the solar daily variation to be nonsinusoidal and the mechanism which gives rise to the apparent sidereal effect.

Since we know T_a, D_{asm} and I_{dm}, we can determine k from Fig. 8*a*. This gives us $k = 2$.

A modulated nonsinusoidal model may now be prepared with the following values of the constants:

$$I_{0dm} = I_{dm} = 0.120\,, \qquad \tau_{0m} = \tau_m = 1530\,, \qquad T = 2.9 \qquad \text{and} \qquad k = 2\,.$$

This model ensures that I_d and I_{as} have the right phases and amplitudes. The model also gives rise to some kind of annual mean spurious sidereal d.v. which we shall refer to as $I_{s\,(model)}$. The « true » sidereal d.v. of constant phase and amplitude, $I_{s\,(true)}$, is obtained by subtracting $I_{s\,(model)}$ from $I_{s\,(observed)}$ algebraically. Finally, we prepare a model for each month, comprising the model just described plus $I_{s(true)}$ expressed in solar time for that month. $I_{s(true)}$ has no influence on the characteristics of I_d or I_{as}.

The results of the model (in the D_r form, where appropriate), are shown

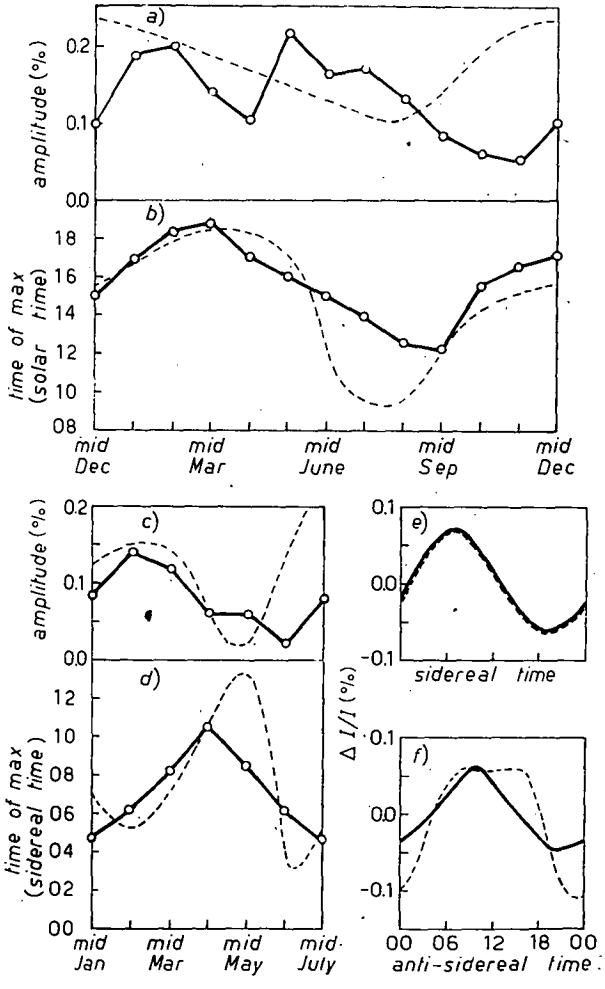

Fig. 9. – Hypothesis II(*a*). The results of analyses (dashed lines), based on the « underground » model, compared with the observed characteristics of the daily variation reproduced from Fig. 3. The model is designed to give the correct amplitude and time of maximum of D_{as} and to reproduce I_d and D_s identically.

as dashed lines in Fig. 9. It is evident that agreement with the observations in detail is generally poor and particularly so in the case of the month by month variations of V_r (Fig. 9*a*). Where there is complete agreement (Fig. 9*e*), it has been artificially created in the process of determining $I_{s(\text{true})}$. Figure 9*f* shows the transition form of the phase of the facsimile d.v. I_{as} when $T_a \sim 2.9$ (viz. Fig. 8*c*).

The value of $k = 2$ required by the model poses the problem of inventing a mechanism which will modulate the solar d.v. in amplitude to such a degree that at some time in the year a phase reversal takes place.

The amplitude of $I_{s(\text{true})}$ was found to be 0.144 % and the time of maximum 07.15 local sidereal time.

5·2. *Hypothesis II(b). A solar d.v. of constant phase and amplitude together with a sinusoidal solar d.v. with a sinusoidal annual variation of amplitude and a sidereal d.v. of constant phase and amplitude.* – This hypothesis permits of a fairly simple theoretical treatment when the daily variations in sidereal and anti-sidereal time are expressed in the D_r form, enabling us to get rid of the nonsinusoidal solar d.v. of constant phase and amplitude. Essentially we use the vector methods outlined at the beginning of Section 5·1, but the treatment is complicated by the fact that the amplitude and phase of the unmodulated sinusoidal d.v. are unknown.

The expression for the daily variation in sidereal time will contain terms of both sidereal and solar origin. To make these distinctions clear some new symbols must be introduced. The term D_s refers to the total observed annual mean D_r d.v. in sidereal time. The spurious component is D_{ss} (identical with I_s in eq. (5.4)) and the true sidereal component of constant phase and amplitude is D_{ts}. Expressing the annual mean d.v.s in the cosine form:

$$D_s = D_{sm} \cos(\Phi' - \varrho_s),$$

$$D_{ss} = I_{ssm} \cos(\Phi' - \varrho_{ss}) =$$

$$= D_{asm} \cos(\Phi' - \varrho_{as} - 2\theta_a), \qquad \text{see relations (5.6)},$$

and

$$D_{ts} = D_{tsm} \cos(\Phi' - \varrho_{ts}).$$

The vectors specifying the phases and amplitudes must satisfy the relation

$$\boldsymbol{D}_s = \boldsymbol{D}_{ss} + \boldsymbol{D}_{ts}.$$

If the vector triangle is drawn on a harmonic dial we obtain the following

sets of relationships by projection at right angles:

$$D_{sm} \cos \varrho_s = D_{tsm} \cos \varrho_{ts} + D_{asm} \cos (\varrho_{as} + 2\theta_a) , \tag{5.7}$$

and

$$D_{asm} \sin (\varrho_{as} - \varrho_s + 2\theta_a) = D_{tsm} \sin (\varrho_s - \varrho_{ts}) . \tag{5.8}$$

The unknowns are D_{tsm}, ϱ_{ts} and θ_a. We now indicate how D_{tsm} and ϱ_{ts} may be expressed in terms of a single unknown.

From the observed semi-annual variation of the monthly mean D_r d.v. (Fig. 2), it is known that the total D_r d.v. for individual days must be reduced virtually to zero twice a year. Therefore θ_s is defined as the day, reckoned in angular measure from the 22nd of September, when the D_r d.v. is first reduced to zero. It now suffices to say that D_{tsm} and ϱ_{ts} may be replaced by the single unknown θ_s, using simple vector relationships, and that on substitution in eqs. (5.7) and (5.8) we finally have

$$D_{sm} \cos \varrho_s = 2D_{asm} \cos (\varrho_{as} + 2\theta_a) + D_{asm} \cos (\varrho_{as} + 2\theta_s) , \tag{5.10}$$

and

$$2 \sin (\varrho_{as} - \varrho_s - 2\theta_a) = \sin (\varrho_s - \varrho_{as} - 2\theta_s) . \tag{5.11}$$

Putting in the observed values for D_{sm} (0.070), D_{asm} (0.060), ϱ_s (97° 30′) and ϱ_{as} (150°), we can solve for θ_s and θ_a. This was done graphically, giving as solutions appropriate to our case:

$$\theta_s = 40° \ (T_s = 1.3 \text{ months}) \qquad \text{and} \qquad \theta_a = 143° \ (T_a = 4.8 \text{ months}) .$$

This result immediately leads to all the other constants we require. Table V lists the predicted constants of the annual mean d.v.s involved in the apparent sidereal effect according to our hypothesis, consistent with the observed phase and amplitude characteristics given in Table III.

TABLE V. – *Characteristics of the annual mean d.v.s.* $I_{d\ sinusoidal}$, I_{ss} *and* I_{ts} *predicted by the theory for hypothesis II(b) corresponding to the observed characteristics of* I_d, I_s *and* I_{as} *listed in Table III.*

Type of d.v.	Amplitude (%)	Time of maximum (h) (in local time according to type)
Solar diurnal ($I_{d\ sinusoidal}$)	$0.120/k$ (I_{0dm})	1940 (τ_d)
Spurious sidereal (I_{ss})	0.060 (I_{ssm})	0500 (τ'_{ss})
True sidereal (I_{ts})	0.027 (I_{tsm})	1010 (τ'_{ts})

Note that although the product $k\ I_{0dm}$ is known it has not been necessary to obtain k or I_{0dm} specifically. In fact, it can be easily shown that the V_r variations in solar time are determined uniquely without having to know either k or I_{0dm}.

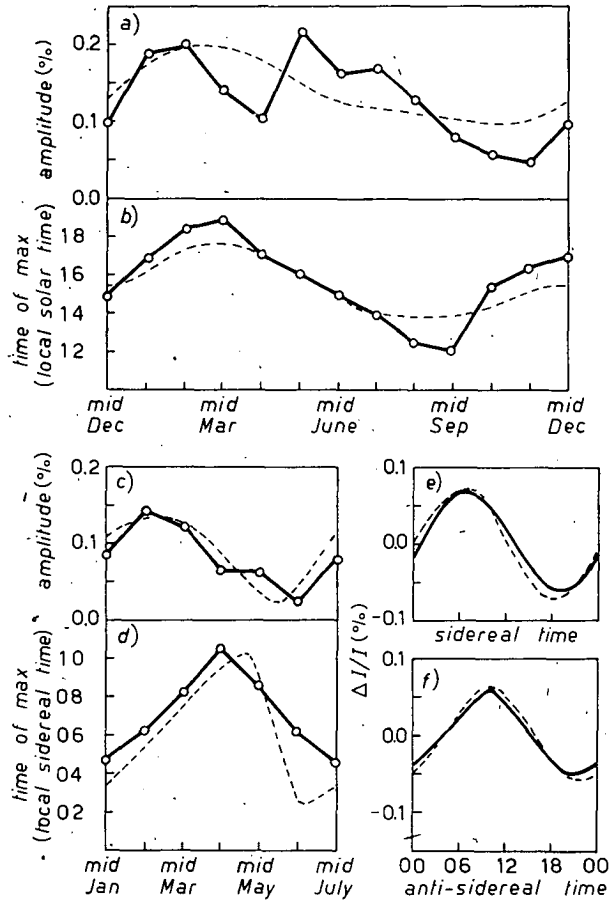

Fig. 10. – Hypothesis II(b). The results of model analyses (dashed lines) compared with the observed characteristics of the daily variation reproduced from Fig. 3. The model is designed to give the correct amplitudes and times of maximum of D_s and D_{as} and to reproduce I_d identically.

Facsimile V_r variations were prepared as the sum of three components as follows, using constants listed in Table V:

$$D_r = 0.120 \cos\frac{\pi}{6}(T - 4.8) \cos\frac{\pi}{12}(\tau - 19.7),$$

$$i_s = 0.027 \cos \frac{\pi}{12} (\tau + 2T - 10.2) ,$$

I_d = observed annual mean solar daily variation averaged over 1958-1960 .

The results from the model are shown as dashed lines in Fig. 10. As to be expected, a greatly improved agreement in detail with the observations indicates that this hypothesis is to be preferred to hypothesis II(*a*). Again, however, the observed variations of amplitude of V_r present difficulties and are rather poorly represented by the dashed curve in Fig. 10*a*. The model also presents an unsymmetrical semi-annual variation of phase of the D_r d.v. in sidereal time (Fig. 10*d*) and this is not in accord with the observations.

6. – Possibility of a pure sidereal effect.

6·1. *Hypothesis III(a). A sidereal daily variation with semi-annual variation of amplitude and phase together with a solar daily variation of constant amplitude and phase.* – A sinusoidal sidereal d.v. with semi-annual sinusoidal variation of amplitude and phase is represented by:

$$(6.1) \qquad i_s = I_{0sm}(1 + k \cos (2\theta - 2\theta_a)) \sin \left(\psi' - \frac{\pi n}{12} \cos (2\theta - 2\theta_p)\right) =$$

$$= I_{0sm}(1 + k \cos 2\alpha_a) \sin (\psi' - a \cos 2\alpha_p) .$$

The symbols k, n, a, θ_a, θ_p, α_a and α_p have their usual meanings, referred to modulations in sidereal time.

If i_s is expressed in solar time and converted to the D_r form it is found that the V_r and D_r d.v.s are identical. Therefore, we may regard the following theoretical treatment as applying either to the V_r or the D_r forms of the daily variation.

Our problem is to find values of k, n, θ_a and θ_p consistent with all the relevant observations. To do this we estimate k and n directly from Fig. 3 and then determine values of θ_a and θ_p for the chosen values of k and n such that the observed annual mean phase and amplitude relationships listed in Table III are satisfied. First of all, then, we find out how k, n, θ_a and θ_p are related theoretically to the annual mean d.v.s I_s and I_{as}.

We expand i_s and express $\sin (a \cos 2\alpha_p)$ and $\cos (a \cos 2\alpha_p)$ in terms of Bessel functions as we did in the case of the modulated solar d.v. in Section 4·1. We find that this expanded form of i_s contains components at the sidereal and anti-sidereal frequencies and at a variety of other frequencies as well. The sum of components at the sidereal frequency gives us the annual mean

d.v. I_s, as follows:

$$I_s = I_{0sm}(p_s \cos \Psi' + q_s \sin \Psi') = I_{0sm} r_s \cos(\Psi' - \varrho_s) , \tag{6.2}$$

where

$$p_s = - kJ_1(a) \cos(2\theta_a - 2\theta_p) ,$$

and

$$q_s = J_0(a) .$$

Similarly, the annual mean d.v. in anti-sidereal time, I_{as}, is the sum of components at the anti-sidereal frequency and we have

$$I_{as} = I_{0sm}(p_{as} \cos \Psi'' + q_{as} \sin \Psi'') = I_{0sm} r_{as} \cos(\Psi'' - \varrho_{as}) , \tag{6.3}$$

where

$$p_{as} = \frac{J_0(a)\, k}{2} \sin 2\theta_a - J_1(a) \cos 2\theta_p + \frac{J_2(a) k}{2} \sin(2\theta_a - 4\theta_p) ,$$

and

$$q_{as} = \frac{J_0(a) k}{2} \cos 2\theta_a + J_1(a) \sin 2\theta_p - \frac{J_2(a) k}{2} \cos(2\theta_a - 4\theta_p) .$$

As before, the observed characteristics of I_s and I_{as} (Table III) require $I_{asm}/I_{sm} = 0.857$ and $\tau''_m - \tau'_m = 0330$ so that we need to put $r_{as} = 0.857 r_s$ and $\varrho_{as} = \varrho_s + 52° 30'$ in eq. (6.3). We can now express p_{as} and q_{as} in terms of p_s and q_s and finally obtain two equations similar to (4.7) and (4.8) which may be solved for θ_a and θ_p after inserting values for k and n. From inspection of Fig. 2 and 3*e* it was decided to put $k = 0.8$ and $n = 3$. The solutions for θ_a and θ_p appropriate to our case were then found to be

$$\theta_a = - 27° 30' \quad (T_a = - 0.97) \qquad \text{and} \qquad \theta_p = 22° \quad (T_p = 1{:}20) .$$

These values give maximum amplitude of i_s in late August and late February while maximum advance of phase would occur at the end of October and at the end of April. These times are in good agreement with the observations.

Various other solutions were considered. It was found that realistic solutions were only available for $n = 3$ (to the nearest integral value) and $k \geqslant 0.8$ (to the nearest tenth). The model for $n = 3$ and $k = 1.0$ gave a slightly worse fit than for $n = 3$ and $k = 0.8$.

A facsimile sinusoidal sidereal d.v. with semi-annual variation of amplitude and phase, based on the constants $k = 0.8$, $n = 3$, $\theta_a = - 27° 30'$ and $\theta_p = 22°$

was developed. The amplitude I_{0sm} and time of maximum τ'_{0m} of the unmodulated model d.v. were determined by normalizing the annual mean d.v., I_s model, so that it agreed exactly in amplitude and phase with I_s observed. To obtain the V_r variations a facsimile solar d.v., I_{dc}, of constant phase and amplitude had to be added to each monthly mean d.v. of the sidereal model when expressed in solar time for that month. It should be mentioned here that when i_s (eq. (6.1)) is expanded it is found that there are no components at the solar frequency. Therefore the annual mean solar d.v. should contain no contribution from the modulated sidereal d.v. This means that the solar d.v. of constant phase and amplitude that we require must be the annual mean solar daily variation for 1958. The latter, therefore, became the facsimile d.v., I_{dc}.

The results from the model are shown as dashed lines in Fig. 11. There is good overall agreement with the observations but the result of greatest interest is the form of the variation of amplitude of the facsimile V_r d.v. in Fig. 11*a* where the curve of best fit has also been drawn in for comparison. Although the picture which the model presents is of a damped oscillation of amplitude this is clearly the most acceptable description of the character

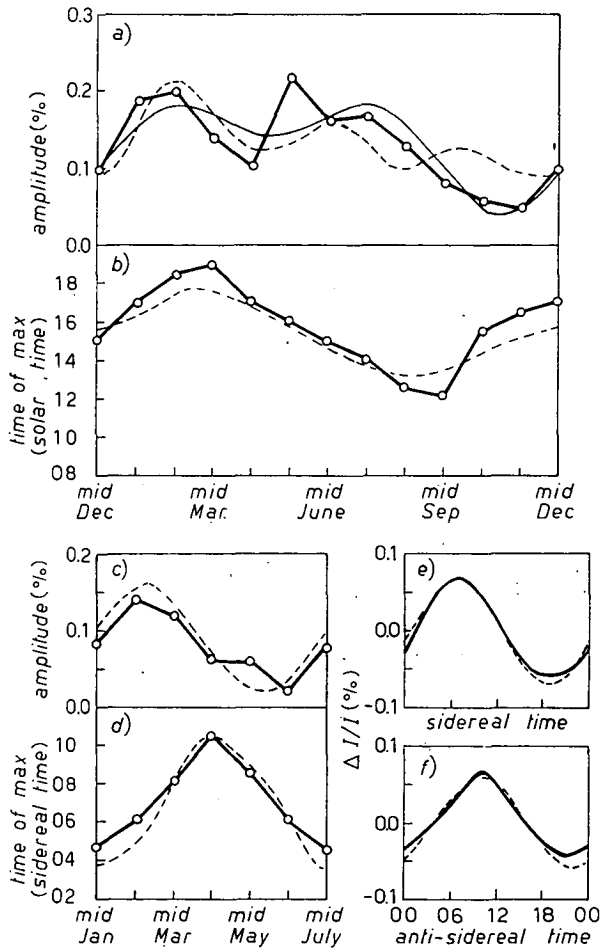

Fig. 11. – Hypothesis III(*a*). The results of model analyses (dashed lines) compared with the observed characteristics of the daily variation reproduced from Fig. 3. The values of the sidereal constants k and n used in the model are estimated from observations and the model is designed to give the correct values for the amplitudes and times of maximum of D_s and D_{as} and to reproduce I_d identically. In (*a*) the curve of best fit to the amplitude variations is also shown.

of the amplitude changes up to the end of August that has been found so far. On this hypothesis the pronounced October-November minimum is anomalous as indeed it has been on any of the other hypotheses put forward. However, an irregularity at this time of the year is rather likely, for, as will be shown in paper II, the modulation characteristics changed greatly and relatively quickly after 1958.

The model results are naturally in good agreement with the observed semi-annual variation of phase of the sidereal D_r d.v. as shown in Fig. 11*d*. Of those tested, hypothesis III(*a*) is the only one which predicts phase variations of the right character. (See Fig. 4*d*, 7*d*, 9*d* and 10*d*).

From the model the following values were obtained for the amplitude (I_{0sm}) and time of maximum (τ'_{0m}) of the predicted unmodulated sidereal daily variation:

amplitude, $I_{0sm} = 0.082\,\%$;

time of maximum, $\tau'_{0m} = 0700$ local sidereal time.

For comparison, I_{0sm} and τ'_{0m} may be calculated directly from the theory by inserting the values found for k, n, θ_a, θ_p, $I_{sm(observed)}$ and $\varepsilon'_{sm(observed)}$ in eqs. (6.2), (6.3) and (6.4), remembering that $I_{sm} = I_{0sm} r_s$ and $\varepsilon'_m = \varepsilon_0 + \varrho_s$ where ε_0 is the angular equivalent of τ'_{0m}. This gives us $I_{0sm} = 0.079$ and $\tau'_{0m} = 0730$. Therefore the model and the practical methods of analysis associated with it produce results that agree quite closely with the predictions of the theory.

Finally, if we obtain the phase and amplitude of the observed annual mean sidereal d.v. in the D_r form from harmonic analysis of the unsmoothed bi-hourly values, we get the following result from the sum of the first and second harmonics, after making the appropriate corrections for smoothing:

$$I_{sm} = 0.080 \pm 0.01\,, \qquad \tau'_m = 0630 \pm 0030\,.$$

Based on these two figures, our best estimates of the amplitude and time of maximum of the unmodulated sidereal d.v. are then:

amplitude, $I_{0sm} = (0.094 \pm 0.01)\,\%$;

time of maximum, $\tau'_{0m} = (0700 \pm 0030)$ local sidereal time.

The limits, as usual in this paper, are the standard estimates of error.

6·2. *Hypothesis III(b). A sidereal daily variation with combined annual and semi-annual variation of phase together with a solar daily variation of constant amplitude and phase.* – The modulated sidereal d.v. may be represented by

$$(6.4) \qquad i_s = A_0 \sin\left(\psi' - a_1 \cos 2(\theta - \theta_1) - a_2 \cos(\theta - \theta_2)\right) =$$

$$= A_0 \sin(\psi' - a_1 \cos 2\alpha_1 - a_2 \cos 2\alpha_2)\,,$$

where $a_1 = \pi n_1/12$ and $a_2 = \pi n_2/12$, n_1 and n_2 being the maximum semi-annual and annual displacements of phase.

As mentioned in Section 2 the D_r form of a daily variation in sidereal time is the average of the d.v.s for months r and $r+6$ so that we have

$$i_s(D_r) = A_0 \cos(a_2 \cos\alpha_2) \sin(\psi' - a_1 \cos 2\alpha_1). \tag{6.5}$$

This is a daily variation in sidereal time with a semi-annual variation of amplitude and phase (for $a_2 \leqslant \pi/2$).

Therefore we have to be very careful to distinguish between hypotheses III(a) and III(b) since they both lead to the same result in the D_r form. It is convenient to use this fact as a starting point for comparison. That is to say, we equate the expression $i_s(D_r)$ in eq. (6.5) with its counterpart $i_s(D_r$ or $V_r)$ in eq. (6.1), giving us $a_1 = a(n_1 = n)$, $\theta_1 = \theta_p$ and

$$A\cos(a_2\cos(\theta - \theta_2)) = I_{0sm}(1 + k\cos 2(\theta - \theta_a)). \tag{6.6}$$

From eq. (6.6) the conditions at maximum and minimum amplitude (*i.e.* when $\cos 2(\theta - \theta_a) = \pm 1$) give us

$$A_0 = I_{0sm}(1+k)\,,$$

$$\theta_2 = \theta_a - \frac{2m+1}{2}\pi \qquad m = 0,\ 1. \tag{6.7}$$

and

$$a_2 = \arccos\frac{1-k}{1+k},$$

From these relationships the values of the constants k, n, θ_a, θ_p and I_{0sm} required to give best fit for hypothesis III(a) can now be used to determine the values of n_1, n_2, θ_1, θ_2, and A_0 for hypothesis III(b) that will give exactly the same agreement with the observed D_r variations. We then construct a model for hypothesis III(b) based on eq. (6.4) that will give precisely the fit shown in Fig. 11*c*, 11*d*, 11*e* and 11*f*. However, the V_r variations in solar time predicted by this model must necessarily differ from those shown in Fig. 11*a* and 11*b*. The reason for this becomes clear when eq. (6.4) is expanded to give

$$i_s = A_0 \cos(a_2\cos\alpha_2)\sin(\psi' - a_1\cos 2\alpha_1) - \tag{6.8}$$

$$- A_0 \sin(a_2\cos\alpha_2)\cos(\psi' - a_1\cos 2\alpha_1)\,,$$

Letting A be the first term on the right-hand side of eq. (6.8) and B the other

term we note that term A is in fact $i_s(D_r)$ of eq. (6.5), equivalent to $i_s(D_r$ or $V_r)$ of eq. (6.1). Therefore in the model analysis hypothesis III(b) is distinguished from III(a) by the contribution of the additional term B, in the sidereal d.v., to the V_r variations in solar time.

Term A contains sidebands at the sidereal and anti-sidereal frequencies but not at the solar frequency. An important characteristics of term B is that although it has no sidebands at the sidereal or anti-sidereal frequencies it does have one (I_{d_s}) at the solar frequency, as follows:

$$I_{d_s} = - A_0 J_1(a_2) J_0(a_1) \cos(\Psi + \theta_2) - A_0 J_1(a_2) J_1(a_1) \sin(\Psi + 2\theta_1 - \theta_2) .$$

This means that the observed annual mean solar d.v. I_d must contain the term I_{d_s} due to the modulated sidereal d.v. Therefore I_d will be influenced by changes in the degree of modulation of i_s.

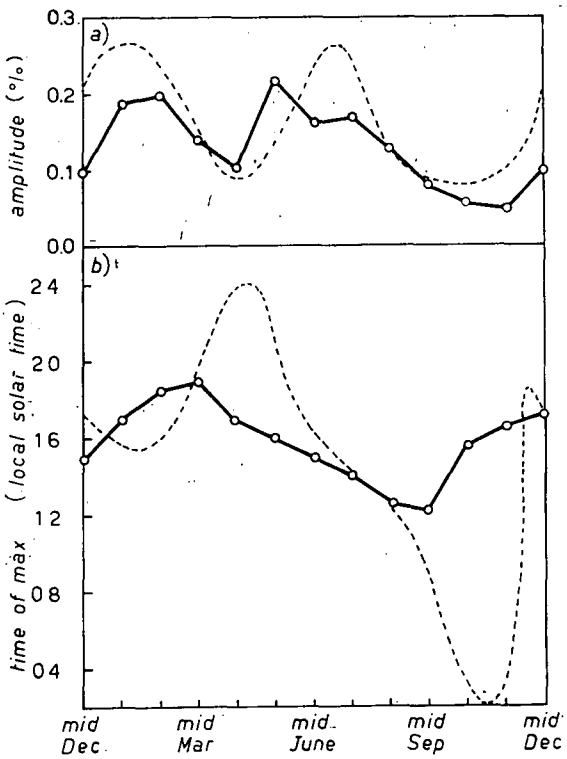

Fig. 12. – Hypothesis III(b). The results of model analyses (dashed lines) compared with the observed characteristics of the V_r variations reproduced from Fig. 3. The model is designed to give the same results in sidereal and antisidereal time as are shown in Fig. 11c to 11f for hypothesis III(a).

The results of the model analysis for hypothesis III(a) shown in Fig. 11 refer to a sidereal d.v. with semi-annual variation of amplitude and phase where the amplitude factor is $k = 0.8$ and the maximum semi-annual displacement of phase in hours is $n = 3$. The equivalent model for hypothesis III(b) requires the same value $n_1 = 3$ for the maximum semi-annual displacement of phase while the value of the maximum annual displacement of phase has to be $n_2 = 5.5$. Thereby both models give the same result, shown in Fig. 11c to 11f, for the D_r variations in sidereal and anti-sidereal time. However, hypothesis III(b) gives this result for either of the two values 63° or 263° of the phase constant θ_2: In Fig. 12 the V_r variations of the model in solar

time are shown only for the case $\theta_2 = 263°$ since the fit is much poorer in the other case.

It is clear that the addition of term B, distinguishing the model results of Fig. 12*a* and 12*b* from those of Fig. 11*a* and 11*b*, has worsened the agreement with the observations, especially with respect to the seasonal changes of phase. No improvement was effected when other solutions for θ_1 and θ_2 were used, corresponding to the small range of values available for k and n.

The sideband component at the solar frequency, I_{ds}, was found to have amplitude 0.04 % and time of maximum 2200 local solar time. The removal of I_{ds} has virtually no effect on the amplitude and time of maximum of the model solar d.v., which is the observed annual mean d.v. for 1958, but broadens and reduces the minimum considerably.

The amplitude, A_0, of the unmodulated sidereal d.v. would be $I_{0sm}(1 + k)$ and in the case of our model it was found to have the value 1.476 % (verifiable by substituting for I_{0sm} and k from hypothesis III(*a*)). Therefore, other things being equal, if the modulation disappeared, the amplitude of the observed annual mean sidereal d.v. should be more than double what it was under conditions of modulation. The shift in the time of maximum would be slight however—an advance of about half an hour.

It may be worth noting that the difference between the model results and the observed V_r variations shown in Fig. 12 can be largely removed if the solar d.v. of constant phase and amplitude is replaced in the model by a solar d.v. with semi-annual variation of amplitude and phase. This is found by subtracting the monthly mean values of i_s (model) expressed in solar time from the observed monthly mean V_r variations in solar time, and noting the characteristics of the residual d.v. There are many complex possibilities of this kind which need not be considered seriously at present when a simpler alternative is adequate.

7. – Discussion.

If we consult Fig. 4, 7, 9, 10, 11 and 12, which summarize the results for each hypothesis, it is evident that I(*a*) and II(*a*) are untenable. Of the others, hypothesis III(*a*) is clearly in best agreement with the observations. Moreover, it is a relatively simple possibility in comparison with I(*b*) and II(*b*). It requires a solar d.v. identical with the observed annual mean d.v. to be modulated by a sinusoidal sidereal component (amplitude 0.09 %, time of maximum 0700 sidereal time) with semi-annual variation of amplitude (maximum increase 80 % in late Febbruary and August) and semiannual variation of phase (maximum advancement 3 h at the end of April and October). It

provides a virtually complete description of the observed apparent sidereal and anti-sidereal effects (Fig. 11*c*, 11*d*, 11*e* and 11*f*). It is also evident that this type of hypothesis has the capacity to explain the observed month-by-month changes in phase and amplitude of the solar daily variation, with the exception perhaps of the October-November minimum.

Hypothesis III(*b*) gives an unsatisfactory result unless a considerably more complex version is used. As to whether this is needed or not may become clear from the study of long-term changes in the modulation characteristics of the sidereal effect.

The more disturbing aspect of each of the other hypotheses is the implication that the V_r variations of amplitude are dominated by large irregularities. Such irregularities would undoubtedly contribute to the sidereal effect whereas it is apparent from Fig. 2 in particular that the sidereal effect is produced by remarkably systematic modulations of the daily variation.

Again, the hypothesis for a spurious effect that is to give closest agreement with the observations must obviously be rather complex. It would have to be equivalent to a solar d.v. with an annual variation of phase and with a damped annual oscillation of amplitude or with amplitude changes which followed the curve of best fit, either of these giving us the right results in sidereal and anti-sidereal time.

If we tentatively accept hypothesis III(*a*) as the correct one it may be of interest to comment briefly on the predicted direction of anisotropy. At 0700 local sidereal time (RA) the vertical semi-cubical telescope at Hobart (geographic latitude 43° S) views approximately outwards along the spiral arm in which we are situated. (It should be pointed out that the direction of maximum radiation sensitivity for this type of vertical telescope is not along the axial direction but along directions at an angle of approximately 25° to the axis defining a conical surface symmetrical about it, viz. PARSONS (5).) DAVIS (6) gives the local direction inward along the spiral arm, based on an idealized model of Munch and Greenstein, as declination=35° N and RA=20 h, giving us declination = 35° S and RA = 08 h along the outward direction.

DAVIS predicts that the galactic magnetic field lines probably follow along the direction of the spiral arm with local deviations of about five or ten degrees. Now, it has been suggested (3) that the mean energy of the primary protons responsible for the meson flux at the depth of 42 m w.e. is about 300 GeV. This is a very approximate figure but serves to show that we are concerned for the most part with primaries of only moderate energy. The corresponding mean gyroradius in the presence of the galactic magnetic field of estimated

(5) N. R. PARSONS: *Ph. D. Thesis* (Hobart, 1959).
(6) L. DAVIS: *Phys. Rev.*, **96**, 743 (1954).

strength 10^{-5} G is of the order of 6 a.u. Even if the field strength is in fact nearer 10^{-6} G, the direction outward along the galactic field lines seems to be the best and perhaps the only likely one from which to expect a source of sidereal anisotropy in the Southern hemisphere at these energies.

It is proposed to postpone further discussion of the physical implications of hypothesis III until we have seen, in Part II, how the modulation characteristics change with time.

8. – Conclusion.

Having examined the basic premises of a completely spurious effect, a mixed effect and a pure sidereal effect, we have found for each of these a particular hypothesis which accurately accounts for the observed phase and amplitude relationships between the annual mean d.v.s I_d, I_s and I_{as}. There is no doubt that more complex hypotheses of each type could be found which would also completely account for the month by month changes in the characteristics of the daily variation, both in solar and sidereal time. However, our concern for the moment has been to find the simplest hypothesis which will predict these changing characteristics with reasonable accuracy. In both these respects, hypothesis III(a), relating to a pure sidereal effect, is clearly the most acceptable of those that have been tested. On the other hand it seems most unlikely that the apparent sidereal effect is entirely spurious.

If hypothesis III(a) is tentatively accepted as the correct one, it means that we have been observing a genuine sidereal daily variation with semi-annual modulation of amplitude and phase. The predicted amplitude of the unmodulated sidereal d.v. is quite large (0.09 %), being comparable with the amplitude of the annual mean solar daily variation underground. The time of maximum intensity (RA $\sim$ 0700) suggests that the source of the anisotropy may be in a direction looking outwards along the spiral arm and therefore in the local direction of the proposed galactic magnetic field. On this hypothesis it would be of great interest to know how the amplitude and phase modulations come about. It would also be interesting to know how the modulation characteristics change with time and if there is any substantial evidence for an excess in the cosmic ray flux from the direction inwards along the spiral arm.

RIASSUNTO (*)

Si esamina la variazione giornaliera siderea apparente dell'intensità sotterranea dei raggi cosmici ad Hobart durante il 1958. Si ha un grande effetto siderale medio annuo apparente ed un comparabile effetto antisiderale, associati con rilevanti cambiamenti mensili della fase e dell'ampiezza della variazione solare giornaliera. Si escogita una tecnica che permette di esaminare mese per mese la variazione giornaliera (d.v.) nel tempo siderale, senza le complicazioni derivanti dalla presenza di alcune componenti di fondo, comprendenti, per esempio, una variazione solare giornaliera di fase ed ampiezza costante. Si tenta di interpretare l'effetto siderale apparente nelle ipotesi: *a*) che esso sia del tutto spurio, nascendo da una modulazione col tempo solare della variazione solare giornaliera; *b*) che vi siano nell'effetto una parte genuina, costituita da una variazione siderale giornaliera di fase ed ampiezza costante, ed una parte spuria, proveniente da una variazione annua dell'ampiezza della variazione solare giornaliera, prodotta da un meccanismo del tempo solare; *c*) che l'effetto sia del tutto genuino, nascendo da una variazione siderale giornaliera modulata nel tempo solare. In ciascun caso si confronta la teoria con le osservazioni tramite modelli. Avendo considerato varie semplici possibilità, si trova che le prove sono chiaramente in favore dell'ipotesi *c*). L'ipotesi che dà la migliore concordanza con le osservazioni è che una variazione solare giornaliera di ampiezza e fase costanti sia accompagnata da una variazione siderale giornaliera con variazione semi annua di fase ed ampiezza. In tal caso, la variazione siderea giornaliera non modulata avrebbe un'ampiezza del 0.09% ed RA $\sim$ 0700 h, e questo suggerisce che la sorgente dell'anisotropia possa essere in direzione esterna lungo il proposto campo magnetico del braccio di spirale.

(*) *Traduzione a cura della Redazione.*

PAPER 5

R. M. JACKLYN
16 Aprile 1965
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The Apparent Sidereal Daily Variation of Cosmic-Ray Intensity at ~ 40 m w.e. at Hobart, Tasmania. II—Results of observations from 1958 to 1962.

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University of Tasmania - Hobart

(ricevuto il 26 Ottobre 1964)

Summary. — Changes in the characteristics of the daily variation of vertical cosmic-ray intensity underground at Hobart since 1958 have been such as to confirm the view that a genuine sidereal component exists. The essential additional evidence since 1958 has been *a*) that a substantial apparent sidereal effect continues to be observed even when seasonal modulation responsible for sideband phenomena is at a low level, and *b*) that the total daily variation towards the end of 1961 exhibited changes in phase that were consistent with the dominating influence of a sidereal component. It is shown that the phase anomaly was to be expected on the basis of the observed long-term changes in the amplitudes of the solar and apparent sidereal daily variations. The time of maximum of the first harmonic of the observed sidereal effect, averaged over the five years 1958 to 1962, is 0608 ± 0015 local sidereal time. There is also a significant second harmonic which appears to be in phase with the first. Making provisional allowance for the deflection of primaries in the earth's magnetic field it seems that the Right Ascension of the direction of maximum intensity of the primaries responsible for the first harmonic observed at Hobart would not be more than about 0700, with a year to year variability of about one hour.

1. – Introduction.

This is the second paper in a series of two, relating to the apparent sidereal daily variation of cosmic-ray intensity observed at Hobart with vertical counter telescopes underground. An analysis of the results obtained in 1958 has been presented in paper I (1). It will be noted that in the title of the present paper

(1) R. M. JACKLYN: *Nuovo Cimento*, **30**, 40 (1963).

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a different value for the depth underground has been used. Although the amount of shale and clay vertically above the equipment has been estimated at ~42 m w.e. (2), a recent assessment of the mean effective absorber for the vertical semi-cube, based on the observed ratio of intensities above and below ground, gave a value closer to 40 m w.e. Therefore the latter figure will be used from now on to denote effective depth underground for the vertical telescopes.

In a consideration of the annual average of the daily variation (d.v.) expressed in sidereal time the essential analytical problem is to distinguish between the two components that may be present, namely, a genuine component orientated in sidereal time and a spurious, or modulation, component. Given a single year's data it is usually not possible to discriminate between these two possible contributions to the total sidereal effect by considering merely the characteristics of the annual mean daily variations in solar, sidereal and anti-sidereal time. During 1958, however, seasonal changes of amplitude and phase of the total daily variation in solar time were so pronounced as to provide additional well-defined characteristics against which various hypotheses could be tested. Accordingly, from detailed consideration of the ability of each hypothesis to account for the month by month changes in the character of the daily variation as well as for the phases and amplitudes of the annual mean d.v.'s, it was concluded that the whole phenomenon could be attributed to a genuine sidereal component with semi-annual variation of amplitude and phase, superimposed on a relatively constant solar-time component. This explanation was found to be the simplest one which could account for all the observed characteristics with a reasonable degree of accuracy. On the other hand it seemed most unlikely that the sidereal effect was spurious.

Thus there was indirect evidence in 1958 for a strongly modulated sidereal component of the daily variation. It is the purpose of this paper to show, in the first place, that the substantial changes in the characteristics of the daily variation that have occurred since 1958 have been such as to confirm that a genuine sidereal component of the daily variation exists, having a time of maximum which so far has not varied outside the range 0500 to 0800 local sidereal time (LST). Secondly, direct evidence for the existence of a genuine sidereal component will be presented. This has already been reported briefly elsewhere (3), but it is hoped to show here in rather more detail how the direct and indirect types of evidence at Hobart confirm one another.

The phrases « solar component », « sidereal component », « spurious sidereal daily variation », « apparent sidereal daily variation » and « seasonal modula-

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(2) A. G. FENTON, R. M. JACKLYN and R. B. TAYLOR: *Nuovo Cimento*, 22, 285 (1961).

(3) R. M. JACKLYN: *Proceedings of the International Conference on Cosmic Rays* (Jaipur, India, 1963).

tion » should perhaps be clarified. In this paper the « solar component » means that part of the total daily variation which is actually generated in solar time. The « sidereal component », or genuine sidereal d.v. is likewise that part of the total daily variation which is generated in sidereal time. The « spurious sidereal daily variation » is, of course, of a totally different nature and has no day to day connection with sidereal time. It originates in systematic modulation of the solar component and appears as a sideband term when the total daily variation expressed in sidereal time is integrated over a complete year to give us the « apparent sidereal daily variation ». The latter may therefore consist of a spurious part and a genuine part. « Seasonal modulation » refers specifically to either the solar or sidereal component of the total daily variation, unless otherwise stated. It should not be confused with seasonal modulation of the total daily variation itself, such as would occur even if the latter consisted simply of a constant solar component and a constant sidereal component.

2. – Changes in the characteristics of the annual mean daily variation since 1958.

Month by month mean values of the intensity of mesons at sea level, Hobart, and of neutron intensity at Mt. Wellington, compensated as far as possible for variations of atmospheric origin, indicate that a minimum of intensity in the 11-year cycle probably occurred here some time in 1958. Since then the intensity has gradually increased. It has also increased underground, where the mean energy of primary protons to which the telescopes respond is now thought to be about 200 GeV ([4]). It has already been found at Hobart and other places that Forbush decreases occur at moderate depths underground, so that it is evident that primaries of moderately high energy are appreciably influenced by solar disturbance activity. It is perhaps not surprising, then, that in recent years there have been progressive changes in the characteristics of the daily variation underground.

In order to detect the long-term trends the following procedure was used. The daily variation of six-hour running mean intensity values was averaged over the first complete year available underground at Hobart, the year centred on March, 1958. The phase and the amplitude of the curve drawn through the average intensity deviations were noted. The process was repeated for the year centred on April, 1958 and for successive months thereafter, so that a sequence of annual moving averages of amplitude and phase was obtained.

([4]) A. G. FENTON: *Proceedings of the International Conference on Cosmic Rays* (Jaipur, India, 1963).

In Fig. 1 the sequence of amplitudes is shown for the daily variations arranged separately in solar time, sidereal time and antisidereal time. In each case significant changes have taken place. It can be seen (Fig. 1*a*) that the amplitude of the d.v. in solar time has declined to a minimum, centred on December, 1961, this value being about 40% of the average amplitude for the year 1958. Now, it is evident, when Fig. 1*a* and Fig. 1*b* are compared, that in the vicinity of minimum amplitude of the solar d.v. the amplitude of the apparent sidereal d.v. increased to a maximum and for some time exceeded the amplitude of the solar d.v. It should be appreciated that this was the only occasion when it did so over the whole period of observation from 1958 to 1962. It follows that if the annual apparent sidereal effect can be attributed entirely to a genuine sidereal component of the daily variation (*i.e.*, if it is not largely a spurious phenomenon) then the sidereal component should have dominated the daily variation towards the end of 1961. Evidence for this will be discussed in the following Section.

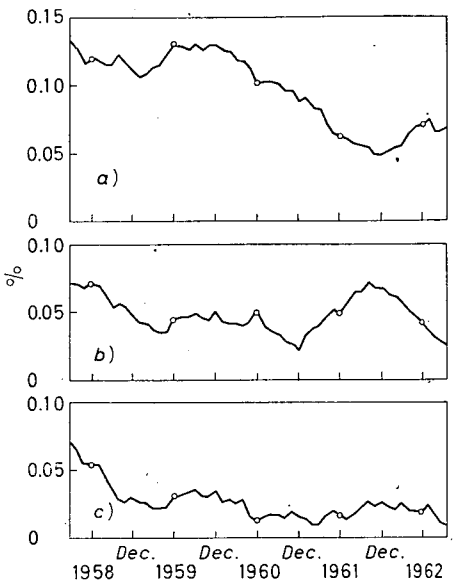

Fig. 1. – Twelve-month moving averages of amplitudes of the daily variations in *a*) solar, *b*) sidereal and *c*) antisidereal time. The dates refer to the centre months and the circles indicate the conventional annual mean values.

Before discussing the long-term changes in amplitude of the antisidereal effect it might be as well to mention briefly the significance of observations arranged in antisidereal time. An appreciable annual mean daily variation in antisidereal time is a sideband phenomenon and is produced by systematic modulation of one or more of the components of the daily variation. Essentially it provides a means of detecting certain types of seasonal modulation ([5]). However, a more elaborate analysis is required if we wish to know which of the components of the daily variation has been modulated.

It was shown in paper I that the large antisidereal effect in 1958, comparable in magnitude to the annual mean d.v.'s in solar and sidereal time, could be reasonably attributed to semi-annual modulation of a genuine sidereal component. Whether or not this interpretation was correct, it is clear (Fig. 1*c*)

([5]) R. M. JACKLYN: *Nuovo Cimento*, **24**, 1034 (1962).

that the degree of modulation declined quite rapidly after 1958 and in fact was probably already declining during that year. From 1960 onwards it would seem that the amplitude of the antisidereal effect had settled down to a minimum of $\sim 0.015\,\%$, less than 20 % of the value averaged over the first year of observation. At this low level the antisidereal effect produced by systematic modulation would be difficult to distinguish from a contribution arising from the irregular fluctuations to be expected in the daily variation from month to month.

The annual moving averages of phase are shown in Fig. 2. It is significant that the time of maximum of the daily variation in solar time (Fig. 2*a*) has not changed over the period of observation in spite of the fact that the amplitude during 1961-1962 was only about half the value which prevailed during 1958-1960. It is concluded that if the solar daily variation underground consists of one part which varies from year to year and another part which is constant then these two contributions must be approximately in phase. The presence of a constant annual component due to daily variation of atmospheric temperature might be suspected, since the underground data have only been corrected for pressure changes. Now, it is important for the interpretation of the sidereal effect that the solar d.v. should not contain a large contribution due to atmospheric temperature variations. The latter are known to exhibit seasonal changes of amplitude, so that the well-known sideband phenomena, namely a spurious annual mean d.v. in sidereal time and an antisidereal effect of about the same amplitude, might be significant. DORMAN (6) and many others have discussed the possible influence of the daily variation of atmospheric temperature on cosmic-ray intensity. KUZ'MIN (7) has carefully examined such a contribution to the daily variation of intensity at ground level and has found that the maximum intensity due to atmospheric temperature would occur near midnight. This is almost opposite in phase to the maximum of the total solar daily variation and therefore it would seem that any such daily

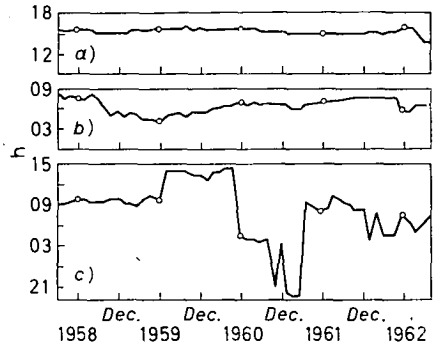

Fig. 2. – Twelve-month moving averages of times of maximum of the daily variations in *a*) solar, *b*) sidereal and *c*) antisidereal time. The dates refer to the centre months and the circles indicate the conventional annual mean values.

(6) L. I. DORMAN: *Cosmic Ray Variations* (Moscow, 1957).

(7) A. I. KUZ'MIN: *Žurn. Èksp. Teor. Fiz.*, **28**, 614 (1955). [Translation *Sov. Phys. JETP*, **1**, 560 (1955)].

variation of atmospheric origin underground at Hobart must be very small. There are various other indications of this. For example, it was noticed from observations at Hobart over the years 1958 and 1959 that the amplitude of the pressure-corrected annual mean solar d.v. underground was about two-thirds of the amplitude at sea level. On the other hand, the Duperier type of negative temperature coefficient derived from underground observations was only about 10%, and the much smaller positive temperature coefficient about 50%, of the sea-level values. This means that the contribution of variations of atmospheric temperature to the daily variation underground would be expected to be about one fifth that at sea level and therefore the effects of modulation of the temperature contribution would be negligibly small. In fact, at some places they may even be negligible above ground, since BERCOVITCH has recently estimated that the amplitude of the temperature effect at Deep River is only about 0.02% with a seasonal range of 50% ([8]). This would give rise to a spurious sidereal effect of amplitude $\sim 0.005\%$ above ground. Finally, even if the temperature effect was not small, it is difficult to understand how seasonal modulation of that type of daily variation could have fallen off since 1958 by the factor of five indicated by the antisidereal effect.

Although the time of maximum of the apparent sidereal d.v. has varied from time to time within the range (0500 ÷ 0800) LST (Fig. 2*b*) there is no firm indication of a long-term trend towards later or earlier maxima. Let us consider the implications of this, supposing that the annual mean d.v. in sidereal time consists of a genuine sidereal component and a spurious contribution due to a seasonally modulated solar component. Now, we know that seasonal modulation during 1961-1962 must have been at a low level, much lower than in 1958. Therefore it is highly probable that a genuine sidereal component was responsible for most, if not all, of the observed sidereal effect during 1961-1962. The overall constancy of phase of the apparent sidereal d.v. would then require that any spurious component be approximately in phase with the genuine sidereal component if the spurious effect was to have been large in 1958. Altogether, rather narrow limits are set for the existence of a spurious sidereal effect. Virtually what is required is seasonal modulation of that part of the solar daily variation which was responsible for the constant day-time maximum and which was presumably of extra-terrestrial origin. The sideband component at the sidereal frequency so produced would have to be approximately in phase with the genuine sidereal component of the apparent sidereal d.v. The time of maximum of the sideband component is partly determined by the time of

([8]) M. BERCOVITCH: *Proceedings of the International Conference on Cosmic Rays* (Jaipur, India, 1963).

the year when seasonal modulation is at a maximum so that the attainment of all the required phase relationships is connected with a degree of improbability. However, a more serious objection is to be found when one attempts to use an hypothesis of this type to explain the observations in 1958. In paper I the simplest hypothesis of the kind we would need, hypothesis II(a), was found to be quite unacceptable. A more sophisticated version, hypothesis II(b), would require an undesirable phase difference of almost $\pi/2$ between the spurious and true sidereal components, and moreover would be unable to account for the observations in 1958 in several important respects (see Fig. 10 of the paper).

The alternative explanation that is offered for the modulation phenomenon is that essentially the sidereal and not the solar component was being modulated in 1958. The rather simple hypothesis put forward, III(a), showed that a possibility of this kind would be quite capable of accounting for the observations. In relation to the long-term trends the sidereal-modulation hypothesis has the attraction that a lessening of the degree of modulation, or even the complete disappearance of it, would have no effect on the annual mean solar d.v. and only a very small effect on the annual mean d.v. in sidereal time. This is what the evidence suggests. That is to say, the long-term changes in the solar and apparent sidereal d.v.'s do not seem to be closely following the changes in the antisidereal effect.

The instability of phase of the antisidereal effect from 1960 onwards is illustrated in Fig. 2c. To some extent this variability was to be expected because of the difficulty of estimating the phase when the amplitudes were so small. Usually two or more maxima appeared to be present and the impression gained was that the total effect was made up of several elements which varied separately from time to time. A small residual antisidereal effect seems to be inevitable, with contributions from irregular changes and from minor systematic modulations of the components of the daily variation.

However the modulation phenomenon is interpreted, it seems clear from examination of the annual running mean d.v.'s that a large part at least of the apparent sidereal effect would have to be attributed to a genuine daily variation in sidereal time, having a time of maximum somewhere in the range 0500 to 0800 LST.

3. – Changes in the monthly mean total daily variation since 1958.

Month to month variations of amplitude and phase of the monthly mean total daily variation underground are shown in Fig. 3. Each pair of values of amplitude and phase has been estimated from the curve drawn through plotted

bi-hourly intensity deviations each of which was a six-hour running mean value. Thus, although considerable smoothing of purely statistical fluctuations has been effected, a degree of distortion of amplitude and phase is involved. Therefore Fig. 3 is only intended to indicate systematic time-variations of the characteristics, not the absolute values.

The large seasonal variations of amplitude and phase shown for 1958 have been dealt with in paper I, where the 1958 portion of this Fig. 3 appears as Fig. 3*d*. It is evident that there was a much smaller degree of seasonal variation in 1959 and 1960. It seems that this came about partly because of a tem-

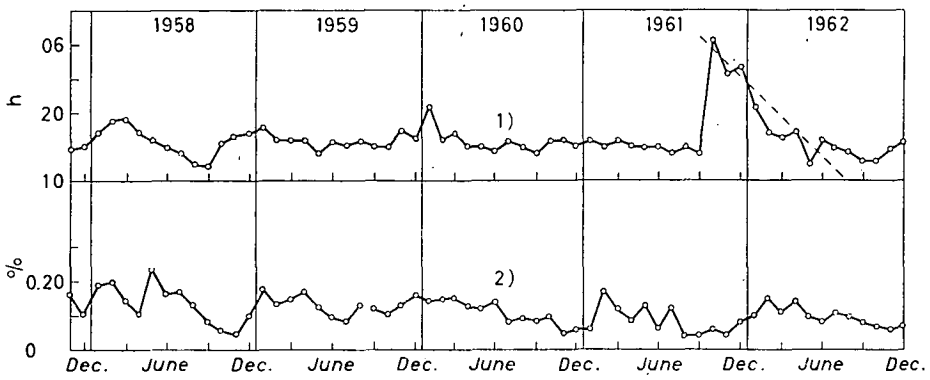

Fig. 3. – Monthly mean values of 1) local time of maximum and 2) amplitude of the daily variation in solar time. The dashed line represents the solar time equivalent of 0700 local sidereal time.

porary decline in the amplitude of the sidereal effect relative to the amplitude of the solar d.v. and partly because seasonal modulation of the sidereal component had weakened.

We now come to the remarkable displacements of phase which occurred towards the end of 1961. There had previously been occasional signs of two maxima in the daily variation and in September 1961 two maxima were clearly present. In October a maximum at 1500 solar time could be discerned but was dominated by another maximum at 0700. In succeeding months a single maximum prevailed and became progressively earlier at the rate of approximately two hours each month, such as would be characteristic of a daily variation generated in sidereal time. In fact it is evident from the dashed line in Fig. 3, representing the solar time equivalent of 0700 local sidereal time, that the dominant component of the total daily variation for about six months

from October onwards behaved in the manner of a genuine sidereal d.v. with a maximum at 0700 LST.

It is most important to note that the phase anomaly in 1961, while providing direct evidence for a genuine sidereal component in its own right, was also the result to be anticipated from the indirect evidence which was discussed in Sect. 2. As we saw in that Section, if a genuine sidereal component were responsible for the apparent sidereal effect it should have become dominant in the total daily variation towards the end of 1961. In simplest terms what appears to have happened is shown schematically in Fig. 4. If we represent the solar and sidereal components by first harmonics in solar time and if (pre-October) the solar component is the greater of the two, the resultant vector representing the total daily variation will exhibit an annual variation of amplitude and phase on the harmonic dial (Fig. 4*a*), as the sidereal vector rotates uniformly anticlockwise. However if (October and after) the sidereal component is greater than the solar component the resultant vector will also rotate uniformly anticlockwise (Fig. 4*b*), the time of maximum becoming earlier at the rate of two hours each month. The actual situation is of course more complicated than this, particularly because of the presence of a large second harmonic in the solar d.v. Therefore the following algebraic method is preferred for a more realistic and detailed simulation of the observed changes in 1961.

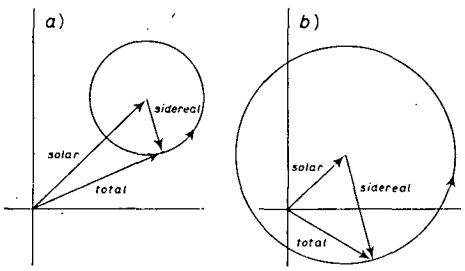

Fig. 4. – Harmonic dials in solar time indicating the annual variation of the first harmonic of the total daily variation when the amplitude of the first harmonic of the sidereal component is *a*) less than and *b*) greater than the amplitude of the first harmonic of the solar component.

The characteristics of the total daily variation in 1961 may be reconstructed month for month by means of a facsimile d.v. based on our two-component hypothesis. A monthly mean solar component may be derived from the observed annual mean solar d.v., adjusted to give monthly mean d.v.'s. which decrease uniformly in amplitude from 0.09 % in January to 0.05 % in December, thus conforming with the secular trend of amplitude shown in Fig. 1*a*. To a first approximation the sidereal component may be considered as a sine wave with its maximum at 0700 LST and adjusted in amplitude to produce a uniform increase from 0.033 % in January to 0.076 % in December. Thus the total daily variation in solar time is represented by the algebraic sum of the facsimile components, from which its month by month characteristics and the annual mean d.vs. in solar, sidereal and antisidereal time can be derived. In Fig. 5 the results from the model are shown as full lines while the observed

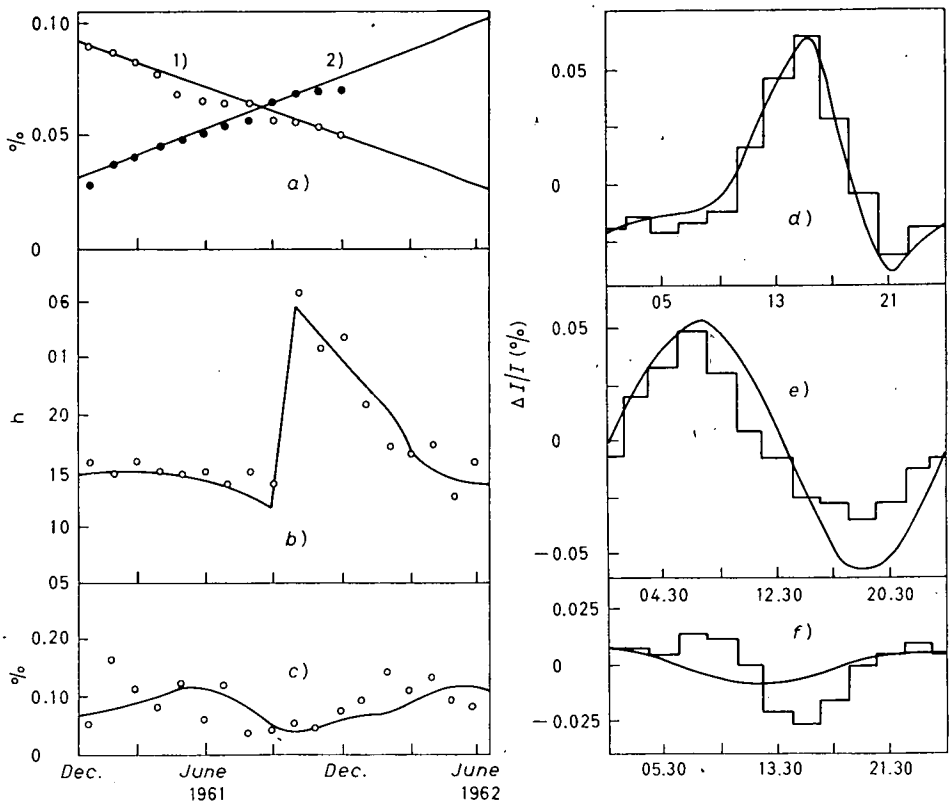

Fig. 5. – The reconstructed characteristics (full lines) of the daily variation, relating to the period of the phase anomaly, compared with the observed characteristics (circles, dots and histograms). *a*) The annual moving averages of amplitudes in 1) solar and 2) sidereal time. *b*) Monthly mean values of the time of maximum of the daily variation in solar time. *c*) Monthly mean values of amplitude of the daily variation in solar time. *d*), *e*) and *f*) The annual mean daily variations in *d*) solar, *e*) sidereal and *f*) antisidereal time.

values are represented by the histograms and open circles. The model correctly predicts the phase anomaly (Fig. 5*b*) and conforms substantially with all the other characteristics of the daily variation in 1961.

4. – The average apparent sidereal effect.

The average apparent sidereal daily variation of bi-hourly pressure-corrected intensity deviations (not running means), averaged over the five years 1958 to 1962, is represented by the histogram in Fig. 6. The characteristics

of the first and second harmonics of best fit and of their sum (the full line in the figure) are given in Table I.

It should be mentioned that the underground observations are recorded only in solar time and are subsequently arranged in sidereal time month by month, in steps of two hours referred to the centre of each month. This introduces smoothing of amplitude, to correct for which the observed amplitudes have to be multiplied by 1.0116. The correction is negligible in the case of the results presented in the Table.

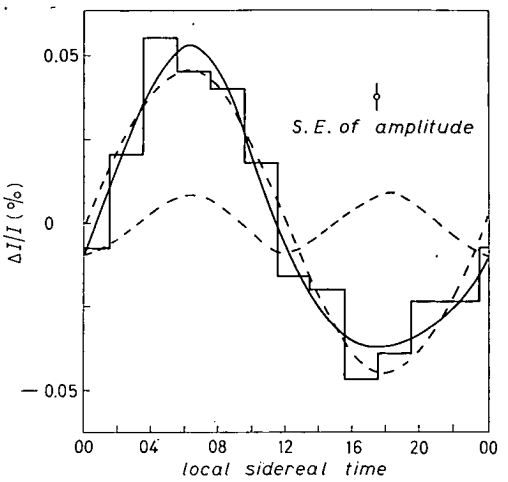

Fig. 6. – The observed daily variation in sidereal time averaged over the years 1958 to 1962 inclusive. The dashed lines are the first and second harmonics and the full line is their sum.

The average observed time of maximum can be said to be about 0600 LST whether we consider only the first harmonic or the total curve of best fit. The fact that the first and second harmonics are in phase, and there is evidence from individual years that this phase relationship is characteristic, is of great importance for the interpretation of the apparent sidereal effect and will be discussed elsewhere.

TABLE I. – *The apparent sidereal daily variation of bihourly intensity deviations, averaged over the years 1958 to 1962 inclusively.*

	Amplitude %±S.E.	Time of maximum Local sideral time±S.E.
First harmonic	0.045±0.003	0608±0015
Second harmonic	0.008±0.003	0612±0125
Sum of first and second harmonics	0.054	0610

As we have seen, there seems to be little justification for supposing that the average sidereal d.v. contains a significant spurious component. On the other hand, if it is entirely genuine, strong modulation in 1958 will have probably introduced a retardation of phase in the long-term average, but hardly more than a small fraction of an hour. Therefore on the evidence it is

concluded that the time of maximum of the unmodulated d.v. must also be about 0600 LST, with a mean year to year variability of perhaps an hour.

To transform from the observed time of maximum to the Right Ascension (R.A.) of the direction of maximum intensity of the primaries at asymptotic distances it is necessary to determine the asymptotic acceptance cone that is appropriate to the anisotropy. The cone of acceptance cannot be worked out without making assumptions as to the configuration of the anisotropy and the rigidity-dependence of the anisotropic primaries. Therefore it is desirable to have some rudimentary prior knowledge concerning the dependence of the observed sidereal effect on the directions of viewing and rigidity responses of cosmic-ray recorders in various parts of the world. All that can be said at this stage is that, whatever the configuration, unless the mean response of the underground telescope to the anisotropic primaries is much less than 100 GeV, the mean asymptotic direction of viewing will be located not more than about 15° East of the meridian. In that event, the R.A. of the direction of maximum intensity of the primaries would not be more than about 0700, with a year to year variability of an hour or so.

5. – Summary of the evidence.

5·1. *The evidence for a genuine sidereal daily variation.* – It may be convenient to summarize briefly here the evidence that we now have for the existence of a genuine sidereal daily variation underground at Hobart.

Three lines of evidence have been examined. In the first place, it was shown in paper I that the well-defined characteristics of the daily variation in 1958 could be attributed to the presence of a genuine sidereal component exhibiting strong seasonal modulation, superimposed on a relatively constant solar component. In any event it seemed unlikely that the large apparent sidereal effect could have been spurious.

Secondly, the long-term trends of phase and amplitude of the daily variations in solar, sidereal and antisidereal time showed that even when seasonal modulation was at a low level there was a large apparent sidereal d.v., presumably due to a genuine sidereal component. Indeed, from various points of view the underground observations tended to refute the idea that there could have been any appreciable spurious contribution to the total sidereal effect.

Finally, there was evidence of a more direct nature. The relative trends of amplitude of the daily variations in solar and sidereal time indicated that if the sidereal effect was genuine a phase anomaly in the total daily variation should occur late in 1961. The phase anomaly did occur at the time predicted and explicitly corroborated the indirect evidence, both for the existence of a genuine sidereal component and for a time of maximum between 0500 and 0800 LST.

5·2. *Seasonal modulation.* – The amplitude of the antisidereal effect is a convenient index of seasonal modulation as far as annual variations of the solar component and semi-annual variations of the sidereal component are concerned.

It was so large in 1958 that seasonal modulation during that year had to be considered as a factor of critical importance for the interpretation of the apparent sidereal effect. It necessarily introduced some doubt as to whether the apparent sidereal d.v. was itself a seasonal modulation phenomenon. However in later years the antisidereal effect declined to such an extent that it became difficult to say if systematic modulation existed at all. At such a low level it no longer had an important bearing on the evidence for the existence of a genuine sidereal component. It has remained at a low level up to the year ending December 1963, at least, so that it is perhaps following the 11-year cycle of solar activity. In that case, comparison of seasonal modulation of the daily variation underground between northern and southern latitudes at sunspot maximum might show more decisively whether a solar component or a sidereal component is being modulated. Of the three possibilities we have examined here—modulation of the sidereal component, modulation of the atmospheric contribution to the solar component and modulation of the extra-terrestrial contribution to the solar component—seasonal modulation of the sidereal component seems to be the only one which would be compatible with the changes in the various characteristics of the daily variation that have taken place between 1958 and 1962.

6. – Conclusion.

From examination of seasonal variations, secular trends and anomalous phase changes it is submitted that the underground observations at Hobart provide coherent evidence for a genuine sidereal component of the daily variation. The proposed sidereal component has been shown to be compatible with the relevant characteristics of the total d.v. over a wide range of their variation and to enable the changes in the characteristics in their different categories to be connected in a natural sequence. In fact, if there were no sidereal component it is difficult to see how the various phenomena could be reconciled with one another.

On the assumption of a genuine sidereal effect, the average R.A. of the direction of maximum intensity of the anisotropic primaries is tentatively placed between 0600 and 0700. It seems that it could not have been more than 0700 unless the mean rigidity of the anisotropic primaries to which the underground telescopes were responding was much less than the mean rigidity of response to the isotropic background (believed to be about 200 GV).

In another paper this Hobart result will be considered in relation to observations of the apparent sidereal effect that have been obtained elsewhere and will be compared with results from telescopes inclined to the North and South of the zenith at the underground station. Until then it would be premature to discuss possible sources of the apparent anisotropy. However, the spiral arm magnetic field is clearly a possibility to be considered, its direction viewed from Hobart being the outward one whose R.A. is thought to be about 0800 and declination about 35° S. Another source orientation of interest might be the normal to the plane of the ecliptic which in the southward direction has R.A. 0600 and declination 66°30′ S. To distinguish experimentally between the various possibilities it is essential to determine the declination as well as the R.A. of the direction of maximum intensity of the primaries. The dependence of the amplitude of the sidereal effect on latitude is involved here, so that statistical accuracy of a rather high order is called for.

If the sidereal effect is essentially genuine it follows that from time to time there are also genuine variations of an hour or so in phase (Fig. 2*b*) and quite large fluctuations in amplitude (Fig. 1*b*). However it may not be necessary to invoke a variable source mechanism to account for these changes since it may be found that they derive independently from modulation of the intensity anisotropy as the primaries traverse the interplanetary medium.

Finally, we must also conclude that there is no indication at present that the amplitude of the sidereal effect is smaller at sunspot maximum, or indeed that the amplitude and sunspot number are simply correlated.

RIASSUNTO (*)

I cambiamenti nelle caratteristiche della variazione giornaliera della intensità verticale dei raggi cosmici sotto terra ad Hobart a partire dal 1958 sono stati tali da confermare l'idea che esista una genuina componente siderale. Le ulteriori prove essenziali dopo il 1958 sono state *a*) che un sostanziale effetto siderale apparente si continua ad osservare anche quando la modulazione stagionale responsabile degli effetti collaterali ha un livello basso e *b*) che la variazione giornaliera totale verso la fine del 1961 presentò cambiamenti di fase che erano in concordanza con l'influenza predominante di una componente siderale. Si mostra che ci si doveva attendere un'anomalia di fase sulla base dei cambiamenti a lungo termine osservati nelle ampiezze delle variazioni giornaliere solare e siderale apparente. Il tempo del massimo della prima armonica dell'effetto siderale osservato, mediato sui cinque anni dal 1958 al 1962, è 0608±0015 tempo siderale locale. C'è anche una significativa seconda armonica che sembra in fase con la prima. Tenendo provvisoriamente conto della deflessione dei primari nel campo magnetico terrestre sembra che l'Ascensione Retta della direzione di intensità massima dei primari responsabili della prima armonica osservata ad Hobart non debba essere superiore a circa 0700, con una variazione di anno in anno di circa un'ora.

(*) *Traduzione a cura della Redazione.*

PAPER 6

Evidence for a Genuine Sidereal Daily Variation of Cosmic Ray Intensity at ~40 m.w.e. Underground at Hobart in 1961.

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Two vertical semi-cubical telescopes at the Hobart underground laboratory have been operating continuously since the beginning of October, 1957. When the pressure-corrected records for the first complete year were analysed, a highly significant apparent sidereal effect was found, the amplitude of the sum of first and second harmonics being (0·09 ± 0·01 SE)% and the time of maximum being 0730 local sidereal time (LST). This was accompanied by a daily variation of comparable amplitude in anti-sidereal time. Over the same period the daily variation in solar time exhibited very marked seasonal variations of amplitude and phase. After testing various hypotheses it was tentatively concluded that the modulation of the solar d.v. had been produced by a genuine sidereal daily variation which had itself experienced appreciable semi-annual modulation of amplitude and phase. A description of the analysis of these results for 1958 is in press.

In subsequent years the situation changed quite remarkably and it is proposed to present elsewhere a full account of these longer-term aspects of the observations. It suffices to mention here that, as Fig.1 shows, two significantly stable characteristics over this period were the times of maximum of the solar d.v. and of the apparent sidereal d.v. These facts in themselves suggest that if the conclusions drawn from the observations in 1958 were correct the daily variation in 1961 should have contained a genuine sidereal component with a maximum somewhere between 0500 and 0800 LST. It is now proposed to present direct evidence for this from examination of the monthly mean values of the daily variation.

The sequence of monthly mean values of phase and amplitude of the daily variation in solar time is shown in Fig. 2 for the period 1958 to 1962. The values for each month have been obtained by inspection of the daily variation of bi-hourly deviates expressed in the form of six-hour running means. It is submitted that this figure depicts the strong

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influence of a sidereal component in two greatly differing situations —— in 1958, when there were the large seasonal fluctuations, and in 1961, when there was the phase anomaly to which we now refer. There was a displacement of the time of maximum in

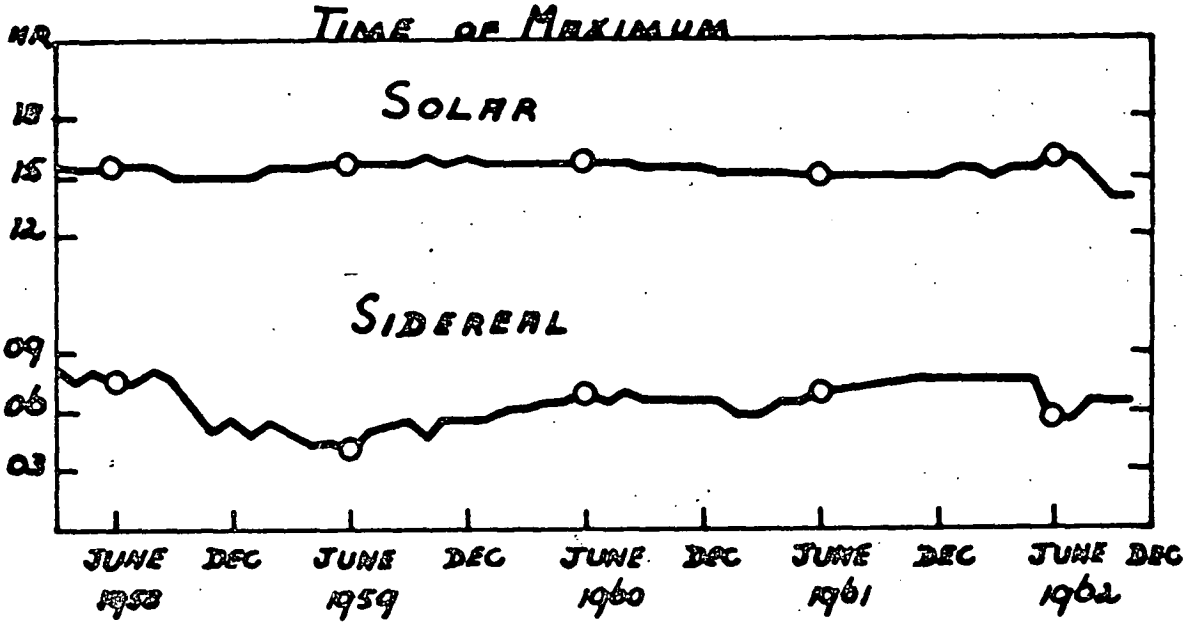

Fig.1. Twelve-month moving averages specified at the centre-months, of the solar daily variation in local time and of the apparent sidereal daily variation in local sidereal time. The circles represent the conventional annual mean values.

October, 1961 from 1500, the prevailing value, to 0700 local solar time. Thereafter the daily variation exhibited progressively earlier maxima in the manner of a genuine sidereal d.v. with a maximum at 0700 LST, as indicated by the dotted line in the figure. It should be emphasised that at any other time of the year this type of phase anomaly would not have

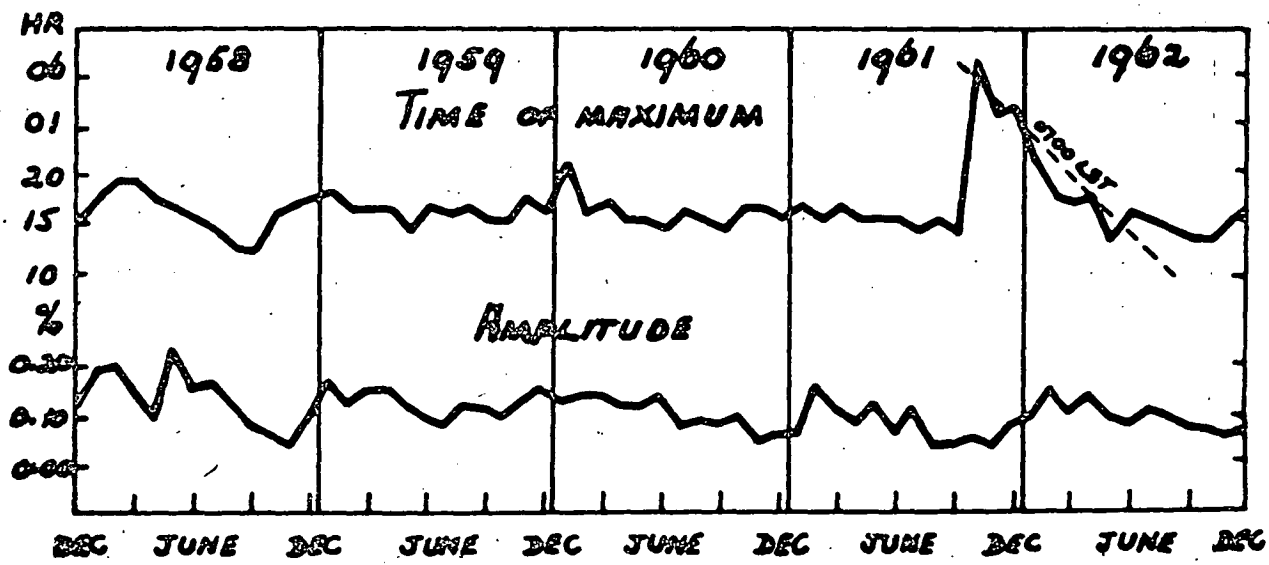

Fig. 2. Monthly mean values of time of maximum (in local solar time) and amplitude of the daily variation of vertical cosmic ray intensity underground at Hobart.

conformed with a sidereal maximum at 0700 LST. If the same sequence of events had commenced in March, for example, it would have conformed with a sidereal maximum at 1900 LST.

It becomes clear that the sidereal component would predominate late in 1961 when one examines the month by month values of annual running mean amplitude of the solar and apparent sidereal d.v.s over this period. The circled points in Fig. 3(a) represent the annual mean amplitudes of the solar d.v. for the months on which they are centred while the dots represent the corresponding amplitudes of the apparent sidereal d.v. If we identify the apparent sidereal effect with a genuine sidereal component of the daily variation it is evident that the sidereal amplitude was increasing linearly while the solar amplitude was decreasing, and in such a manner that the former would predominate after the crossover early in September, 1961. Although linear trends of the annual mean amplitudes were not maintained beyond the December-centred values they would apply to the individual monthly mean components for a further six months, as indicated by the full lines in the figure.

An attempt was made to reconstruct the characteristics of the daily variation in 1961 by means of facsimile d.v.s. The monthly mean solar time component was represented by the observed annual mean solar d.v. adjusted in amplitude for each month so as to produce a uniform decrease in amplitude from 0·09% in January to 0·05% in December. The sidereal component was represented by a sine wave with a maximum at 0700 LST and adjusted in amplitude to produce a uniform increase from 0·033% in January to 0·076% in December. Monthly mean d.v.s in solar time and annual mean d.v.s in solar, sidereal and anti-sidereal time were derived from the sum of the facsimile components and then compared with the observations. It was clear that the hypothesis on which the models were founded accounted substantially for the observations in 1961. In Fig.3(b) the predicted (full lines) and observed phases of the monthly mean d.v.s are shown.

Since the estimates of phase and amplitude based on six-hour running averages must introduce a certain amount of systemmatic bias, a best estimate for the annual mean sidereal daily variation in 1961 was obtained by harmonic analysis of the twelve unsmoothed bi-hourly values. The amplitude of the first harmonic was (0·040 ± 0·008 SE)% at 0610 ± 0050 LST, of the second harmonic was 0·010% at 0600, and of the sum of first and second harmonics was (0·050 ± 0·008 SE)% at 0600 ± 0040 LST. It would follow that the maximum amplitude in June, 1962 would have been ~0·100%.

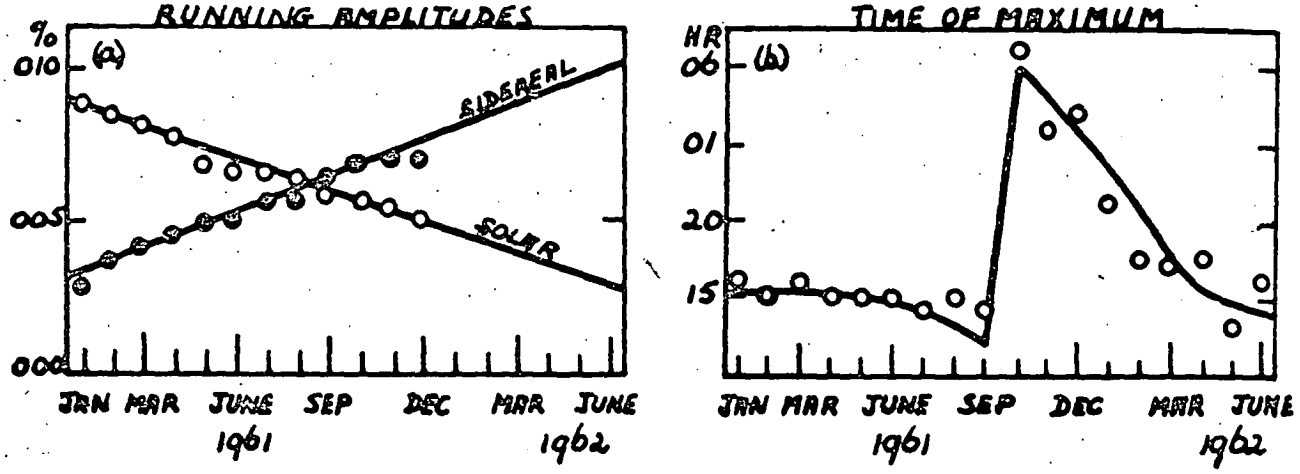

Fig. 3. Comparison between the observed characteristics of the daily variation underground and model (full lines).
(a). The annual running mean amplitudes in solar and sidereal time.
(b). The times of maximum of the monthly mean d.v.s in local solar time.

If we tentatively suppose that the sidereal effect at Hobart (geographic latitude 43°S) is, in the main, evidence for a maximum of primary cosmic ray intensity in a direction looking out along the proposed local spiral arm magnetic field (dec.~35°S, RA~0800 LST), it appears that the observed amplitude would be about 60% of the amplitude of the intensity anisotropy. This takes into account the radiation sensitivity pattern for the telescope, a cosine dependence of the anisotropic intensity on direction measured away from the direction of the spiral arm field, the coordinate conversion from the celestial to the terrestrial frame of reference and the assumption (perhaps not justified) that the primaries which are responsible for the sidereal effect underground are not appreciably deflected in the earth's magnetic field. This would lead to a value of about 0·16% for the amplitude of the intensity anisotropy corresponding to the maximum sidereal amplitude so far experienced at Hobart.

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THE UPPER LIMITING PRIMARY RIGIDITY OF THE COSMIC RAY SOLAR ANISOTROPY

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Summary

A method of determining the upper limiting rigidity of the solar diurnal variation of the cosmic ray primaries in free space is described. It involves a comparision of the response to the anisotropy of neutron monitors at sea level and of meson telescopes underground. Making use of the model for the free-space first harmonic proposed by Rao, McCracken, and Venkatesan, the annual average value for the upper limiting rigidity (R_u) in 1958 is estimated to have been 95 GV with an error of estimate of about 10–20 GV. Changes in the observed annual mean daily variation between 1958 and 1962 indicate that R_u may have decreased by about 20–40 GV over this period, but a more refined analysis is needed to confirm this.

Changes in R_u could also influence a sidereal daily variation as observed at the Earth. Evidence is presented for such an effect, from the observations underground at Hobart.

It is shown that the annual average pressure-corrected solar diurnal variation observed at a depth of 40 m.w.e. must be largely under the control of the solar anisotropy. The most significant evidence presented comes from observations in three different directions underground at Hobart during 1961 and 1962. The first harmonics of the solar daily variations observed in these directions have been found to be broadly compatible with the model for the anisotropy proposed by Rao, McCracken, and Venkatesan.

We also present the results of an investigation of the response of an underground telescope to generalized free-space first and second harmonics, for different approximations to the geomagnetic field and for small and large sets of arrival directions.

I. INTRODUCTION

Recent theories of modulation of galactic cosmic rays within the interplanetary medium (Ahluwalia and Dessler 1962; Parker 1964; Axford 1965) have indicated that charged particles approaching the Earth's orbit with gyro-radii greater than about 1 a.u. would be too energetic to take part in the cosmic ray solar anisotropy, which appears to be responsible for most of the observed pressure-corrected solar daily variation at the Earth. Consequently it has been predicted that the upper limiting rigidity R_u (the rigidity above which the solar daily variation in free space vanishes) would have been approximately in the vicinity of 10^2 GV at the recent maximum of solar activity when the average interplanetary magnetic field strength at the Earth's orbit was a few gammas. At solar minimum R_u should be somewhat smaller, depending on the nature of the minimum. This solar-cycle dependence should be reflected in changes in amplitude and phase of the solar daily variation.

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It is also reasonable to suppose that R_u might be the threshold rigidity for observations at the Earth's orbit of any sidereal anisotropy of the charged primaries, and there could be a significant relationship between the solar and sidereal daily variations because of this common dependence on R_u. In short, there are a number of reasons why experimental determinations of R_u are needed at various stages of the solar cycle, in addition to the more fundamental one, that it is one of the constants that characterize the region of modulation. It is proposed to describe here a method of determining annual average values of R_u within reasonably narrow limits and to report a provisional value for 1958, the year of minimum cosmic ray intensity in the present cycle. This follows an earlier estimate of 200 GV, to within a factor of two, given by Rao, McCracken, and Venkatesan (1963), based on the Hobart underground observations in 1958. The present method depends on the fact that changes in the value of R_u would have a much greater effect on the solar daily variation observed underground than on the daily variation observed with neutron monitors at high latitude sea level stations.

Towards the end of 1960 two inclined telescopes began operating at the Hobart underground site, in addition to the two vertical semi-cubes. Their axial directions were 30° north of the zenith and 45° south of the zenith respectively, in the plane of the geographic meridian. The whole arrangement would permit a simultaneous scan of the southern sky along three broadly overlapping asymptotic latitude strips. Some results from the three directions during 1961–62 are presented here, partly to indicate how R_u may have changed since 1958, and also to demonstrate that the first harmonics observed underground at about 40 m.w.e. must have been produced in the greater part by an extraterrestrial anisotropy.

It should be emphasized that the following treatment concerns only the first harmonic of the solar anisotropy. However, it does appear, notably from the work of Sarabhai and his associates (Sarabhai and Subramanian 1963*a*; Subramanian 1963), that a second harmonic must be considered when attempting to describe some of the characteristics of the anisotropy as a whole. There is also evidence that the second harmonic is greater at higher rigidities. We wish to make the point here that, although the free-space daily variation may not be completely described by a first harmonic as Rao, McCracken, and Venkatesan had tentatively concluded, they have shown that the first harmonic has certain definite characteristics that enable an upper limiting rigidity to be worked out for it quite simply. In this instance what is true for the first harmonic might reasonably be expected to apply to the free-space daily variation as a whole.

It is also important to note the argument (Sarabhai and Subramanian 1963*b*) that annual-averaged daily variations fail to provide a realistic picture of the solar anisotropy because the daily variations observed on individual days tend to fall into distinct groups that are attributed to solar anisotropies of differing character. Thus, although on the annual average the index β of variation spectrum may be almost zero, it seems that it is not characteristically zero on individual days. On the other hand, annual averages of upper and lower limiting rigidities and of the amplitude constant of the anisotropy are not open to misinterpretation in the same way. It appears that the two methods of approach are complementary and should lead to

conclusions that are compatible, provided the limitations of each method are understood. Perhaps the greatest single advantage of using an annual average of the solar daily variation is that the influence of a sidereal anisotropy is thereby completely averaged out.

II. Outline of a Method of Determining R_u

The first harmonic of the pressure-corrected daily variation observed underground in 1958 can now be related to the solar anisotropy of the primaries with some degree of confidence. In the first instance, Rao, McCracken, and Venkatesan, making careful allowance for the asymptotic cones of acceptance of particle detectors, were able to delineate the free-space first harmonic of the anisotropy in 1958, at least so far as to account for the manner in which the first harmonic of the observed annual mean daily variation of neutron intensity depended on the geographic location of the detector. Since adequate coupling coefficients that could be applied to underground detectors of charged particles were not available then, the authors could make only a rough estimate of R_u. However, recent calculations have indicated that underground telescopes have a mean energy of response that is lower than had been previously thought. Specifically, Fenton (1963) has worked out the yield functions for the pions that are thought to be responsible for the μ-meson intensity at the underground depth of 40 m.w.e. Thereby he has obtained the differential coupling coefficients, which give the fraction of the observed counting rate underground due to primaries of any given energy E. He has found that the coupling coefficients are similar to those derived by Mathews (1963) from an empirical response curve extrapolated from latitude–intensity data. Fenton calulates the mean energy of response to be about 200 GeV, and that less than 10% of the counting rate is due to primaries of energy < 50 GeV. From such information the cone of acceptance technique enables the first harmonic underground to be estimated as a function of R_u, in relation to the given free-space first harmonic.

Rao, McCracken, and Venkatesan could find no convincing evidence from the neutron data that a semi-diurnal component of extraterrestrial origin existed. Accordingly they specified the anisotropic component of intensity in free space for primaries of rigidity R, and for a very narrow angle detector, as being simply a first harmonic $\alpha_1 I_0(R)\cos\lambda\cos(\psi-\psi_0)$, where α_1 is the amplitude constant, $I_0(R)$ is the omnidirectional differential intensity of primaries, λ is the ecliptic latitude of the detector, ψ is the direction of viewing in the plane of the ecliptic, measured eastward of the Earth–Sun line, and ψ_0 is the direction of maximum intensity. Initially the expression given by them also contained the factor R^β, denoting the dependence of amplitude on rigidity, but one of their most important findings was that the annual average amplitude was independent of rigidity in 1958, so that $\beta = 0$. On transforming to rotating terrestrial coordinates it can be shown (Jacklyn 1963*a*) that the corresponding free-space solar diurnal variation is, to a sufficient degree of approximation,

$$\Delta I(R)/I_0(R) = \alpha_1 \cos\delta\cos(\phi-\phi_0), \tag{1}$$

where δ is the geographic latitude of viewing, ϕ is the time of day relative to noon, in angular measure (or longitude east of the observer's meridian at noon), and

ϕ_0 ($\simeq \psi_0$) is the time of maximum intensity. In this context, free space, or the asymptotic region, is the region of modulation situated beyond the effective limits of the Earth's magnetic field.

Now, the fraction of the observed first harmonic underground that is due to anisotropic primaries of rigidity R will differ considerably from $\Delta I(R)/I_0(R)$, partly because at any instant the primaries involved do not approach the Earth's magnetic field from a single direction (δ, ϕ) but from a cone of directions that will allow them to arrive at the meson production level within the solid angle of viewing of the recorder, after penetrating the field. The cone is the asymptotic cone of acceptance of which the concept and general method of application to an anisotropy have been fully outlined by Rao, McCracken, and Venkatesan. The present treatment differs somewhat in detail from theirs and is perhaps only appropriate when dealing with high energy primaries. The cone of acceptance is divided up into N segments specified by intervals of asymptotic latitude. These intervals can be conveniently defined so that each segment of the cone contributes about the same amount to the fraction of the counting rate that is due to primaries of the given rigidity. The contribution $v(R)$ to the first harmonic of the daily variations underground from primaries of rigidity R is then approximated by the summation

$$v(R) = \alpha_1 \frac{Y_R}{N} \sum_{n=1}^{N} A_{Rn} \cos \delta_{Rn} \cos(\phi - \phi_0 + \phi_{Rn}). \tag{2}$$

In this expression Y_R is the differential coupling coefficient and A_{Rn} is an amplitude reduction factor determined by the manner in which the contributions to Y_R from the nth segment of the cone are distributed in asymptotic longitude. An individual contribution will be defined more precisely below as the "differential radiation sensitivity" $I(R, \omega)$. The distribution also determines an effective longitude of viewing ϕ_{Rn} (east of the observer's meridian) with respect to a first harmonic of the anisotropy. The mean asymptotic latitude δ_{Rn} is obtained from distribution in asymptotic latitude of the contributions to Y_{Rn} from the nth segment of the cone.

The total first harmonic v is estimated as

$$\begin{aligned} v &= \sum_{R=R_c}^{R_u} v(R) \\ &= \alpha_1 \sum_{R=R_c}^{R_u} \sum_{n=1}^{N} \frac{Y_R}{N} A_{Rn} \cos \delta_{Rn} \cos(\phi - \phi_0 + \phi_{Rn}), \end{aligned} \tag{3}$$

where R_c is the lowest primary rigidity to which the recorder can respond, provided it is greater than R_L, the lower limiting rigidity of the anisotropy. In practice the vector form of $v(R)$ is more useful, being denoted by

$$\mathbf{v}_R = |v_R| \boldsymbol{\rho}_R = \frac{\alpha_1 Y_R}{N} \sum_{n=1}^{N} A_{Rn} \cos \delta_{Rn} \boldsymbol{\rho}_{Rn}, \tag{4}$$

where ρ_{Rn} is the phase angle on the harmonic dial. In turn, the total first harmonic

vector is

$$\mathbf{v} = |v|\boldsymbol{\rho} = \sum_{R=R_c}^{R_u} |v_R|\boldsymbol{\rho}_R. \tag{5}$$

Considerable use is also made of the relative harmonics $\mathbf{v}_R/\alpha_1$ and $\mathbf{v}/\alpha_1$, whose amplitudes are denoted by B_{1R} and B_1 respectively.

If one substitutes for v with a reliable observed first harmonic (notably one that is as free as possible from contributions of atmospheric origin), the free-space amplitude α_1 may be estimated from equation (3) for a given value of R_u. It will be shown that, in relation to the underground observations, α_1 is a steeply varying function of R_u when R_u is in the vicinity of 100 GV. On the other hand, when the form of equation (3) that applies to a high latitude neutron monitor is worked out and v is replaced by the first harmonic of an observed daily variation of neutron intensity, one finds that the variation of α_1 with R_u flattens off as R_u increases above 100 GV. Therefore, if R_u is thought to be about 200 GV, the neutron monitor observations can be used to determine the free-space amplitude α_1 regardless of the exact value of R_u. This is what Rao, McCracken, and Venkatesan have done. However, the result is less precise if R_u is closer to 100 GV. In general it is more profitable to find the intersection of the curve specifying α_1 as a function of R_u, relating to observations of high latitude neutron intensity (curve B in Fig. 3), with the corresponding curve relating to simultaneous observations of meson intensity underground (curve A in Fig. 3). The sharp intersection gives the values of both α_1 and R_u that apply to the observations. This is the method that shall be used in estimating R_u for 1958.

III. Method of Determining $\mathbf{v}/\alpha_1$ as a Function of R_u

To calculate the response underground to a first harmonic of the anisotropy we determine sets of values of A_{Rn}, δ_{Rn}, and ϕ_{Rn} for selected rigidities so that by curve-fitting they may be read off as functions of R. It should be noted that A_{Rn}, δ_{Rn}, and ϕ_{Rn} are constants of the detector at the given location and may be used to determine the response to any free-space first harmonic, either solar or sidereal. Corresponding quantities A_{2Rn}, δ_{2Rn}, and ϕ_{2Rn} relating to a second harmonic may be obtained by the same procedures, and so on for higher harmonics.

Consider the μ-mesons, due to isotropic primaries of rigidity R, which arrive at the detector within a small solid angle ω_r specified by zenith angle Z_r and azimuth a_r. They constitute a fraction of the total counting rate that is known as the differential radiation sensitivity $I(R, \omega_r)$ given by

$$I(R, \omega_r) = Y_R F(\omega_r) \cos^n Z_r.$$

The factor $F(\omega_r)$, the geometric sensitivity (see Parsons 1957), is the fraction of the total counting rate apportioned to ω_r by virtue of the geometry of the telescope, while $\cos^n Z_r$ expresses the well-known zenith angle dependence of intensity. The primaries of rigidity R that are responsible for $I(R, \omega_r)$ will have approached the Earth's magnetic field from directions within some asymptotic volume element $(\Omega_r)_R$ specified by latitude $(\delta_r)_R$ and longitude $(\phi_r)_R$. Such asymptotic coordinates are usually found either by computing the trajectories of the primaries outward

from the direction (Z_r, a_r) at the geomagnetic location of the recorder (see McCracken, Rao, and Shea 1962), or else from observations of the deflections of charged particles in a physical simulation of the geomagnetic field (see the terrella experiments of Brunberg and Dattner 1953). When $I(R, \omega_r)$ and the associated asymptotic coordinates $(\delta_r)_R$ and $(\phi_r)_R$ have been calculated for each of the volume elements ω_r that make up the solid angle of the recorder, the distribution of the fractional μ-meson intensity due to primaries of rigidity R can be determined with respect to both asymptotic latitude and asymptotic longitude of the primaries. The distribution with latitude can be simply divided up amongst N latitude intervals giving equal contributions to the differential counting rate, thereby defining the N segments of the total asymptotic cone. The mean latitudes δ_{Rn} are obtained from the latitude distributions within the individual segments. The distribution with respect to longitude within each segment, when applied to a first harmonic $\cos\phi$, provides the amplitude reduction factor A_{Rn} and the longitude displacement angle ϕ_{Rn}. However, it should be noted that the distributions themselves are derived on the assumption of isotropic primaries and an average cosmic ray spectrum.

When the constants A_{Rn}, δ_{Rn}, and ϕ_{Rn} are applied to a particular model for the anisotropy, the important simplification in the present treatment is that the mean value δ_{Rn} replaces a latitude distribution. This suggests that the latitude intervals specifying the segments of the asymptotic cones should be small. Several factors are involved in the choice of intervals, one of them being the manner of variation of the longitude distribution with respect to asymptotic latitude. As far as the underground vertical semi-cube is concerned, the final result does not seem to depend at all markedly on the number of latitude intervals used.

IV. Application to the Cosmic Ray Detectors

(a) *The Vertical Semi-cube Underground*

The solid angle of the semi-cube was divided up into 576 elements (ω_r) of dimensions 7·5° in azimuth and 5° in zenith angle. In calculating the values of $I(R, \omega_r)$, Fenton's (1963) coupling coefficients and the geometric sensitivity characteristics calculated by Parsons (1957) were used. Both $n = 2{\cdot}0$ and $n = 2{\cdot}2$ were tried for the $\cos^n Z$ zenith angle dependence of intensity, but for all practical purposes they each led to the same final asymptotic distribution.

Since about 90% of the counting rate underground appears to be due to primaries of energy > 50 GeV, it was considered that a centred dipole was a sufficient approximation to the real geomagnetic field for calculations of the asymptotic coordinates $(\delta_r)_R$ and $(\phi_r)_R$. (Calculations using a more complex representation of the field tended to confirm this view and will be mentioned again below.) Consequently, the particle deflections were estimated from the diagrams of Brunberg and Dattner (1953) for the geomagnetic southern latitude of 50°. Considerable interpolation was necessary and for this reason it was found more convenient to express the asymptotic latitude and longitude data in the form of deflections in zenith angle and azimuth.

At the higher rigidities the differential cones of acceptance change rather slowly with rigidity, and therefore it was decided to work them out only for the three most

important rigidities, namely, 50 GV (a practical lower limiting rigidity of response), 150 GV (near the mean rigidity of response), and infinite rigidity (providing an asymptote for the response constants).

Each cone of acceptance was initially divided up into three segments defined by intervals of asymptotic geographic latitude. For example, it was found that, of the 50 GV primaries contributing to the counting rate, one-third came from the latitude range 90° S. to 46° S., one-third from the range 45° S. to 28° S., and the remainder from the range 27° S. to 20° N. These high, middle, and low latitude intervals were found to be almost exactly the same at the higher rigidities.

The mean latitude δ_{Rn} was worked out for each segment from the distribution $I(R, \omega_r)$ versus $(\delta_r)_R$, after the data had been grouped at 3° intervals of latitude. The distribution $I(R, \omega_r)$ versus $(\phi_r)_R$ was based on longitude intervals of 20° and provided the amplitude constant A_{Rn} and phase constant ϕ_{Rn} when a first harmonic $\cos\phi$ was impressed on it. In addition, constants A_R, δ_R, and ϕ_R were obtained for

TABLE 1

ASYMPTOTIC CONSTANTS* FOR THE VERTICAL SEMI-CUBE UNDERGROUND AT 50° S. GEOMAGNETIC LATITUDE

Primary Rigidity (GV):	50			150			∞		
	A_{Rn}	δ_{Rn}	ϕ_{Rn}	A_{Rn}	δ_{Rn}	ϕ_{Rn}	A_{Rn}	δ_{Rn}	ϕ_{Rn}
High latitude range (H)	0·84	−56°	25°	0·75	−60°	12°	0·74	−62°	0°
Mid latitude range (M)	0·86	−36	35	0·86	−38	10	0·86	−41	0
Low latitude range (L)	0·91	−13	33	0·90	−15	12	0·89	−18	0
Total cone	0·87	−34	30	0·83	−39	11	0·83	−39	0

* The constants specify the differential response to a free-space first harmonic at the given primary rigidities.

the total (undivided) cone. Values of the asymptotic constants for individual segments and for the total cone are given in Table 1 for each of the three rigidities. It can be seen that (*a*) above 150 GV the cone of acceptance does not change with increasing rigidity except for a gradual displacement towards the observer's meridian, and that (*b*) the distribution $I(R, \omega_r)$ versus $(\phi_r)_R$ changes only slightly with latitude, as would be expected.

Curves of fit, from which A_{Rn}, δ_{Rn}, and ϕ_{Rn} could be read off against rigidity R, were drawn using the results from Table 1. The vectors $\mathbf{v}_R/\alpha_1$, relating to the free-space model of Rao, McCracken, and Venkatesan, were then estimated in accordance with equation (4) as the mean sums of the individual vectors $(\mathbf{v}_R)\mathrm{H}/\alpha_1$, $(\mathbf{v}_R)\mathrm{M}/\alpha_1$, and $(\mathbf{v}_R)\mathrm{L}/\alpha_1$. Finally, the total first harmonic vector $\mathbf{v}/\alpha_1$ was estimated as the sum of differential vectors $\mathbf{v}_R/\alpha_1$ for different values of R_u.

Now, it was found that if the total vectors $\mathbf{v}/\alpha_1$ were derived from undivided cones of acceptance, characterized by the constants given in the last line of Table 1, they were practically indistinguishable from those calculated from cones that had been divided up into three segments. This greatly simplified the procedure for estimating the response of the vertical semi-cube to free-space harmonics. However,

as we shall see below, such a drastic simplification would not be justifiable in the case of the south-pointing cube underground.

Curves of fit were drawn through the total-cone values A_R, δ_R, and ϕ_R given in Table 1 and are shown in Figure 1. These curves were used in all subsequent calculations requiring values of the asymptotic constants as functions of R.

It is of interest at this stage to enquire whether the asymptotic constants would have been very different (*a*) if approximations to the geomagnetic field other than a centred dipole had been used, and (*b*) if the solid angle of the semi-cubical telescope had comprised fewer and larger volume elements. We also note how the

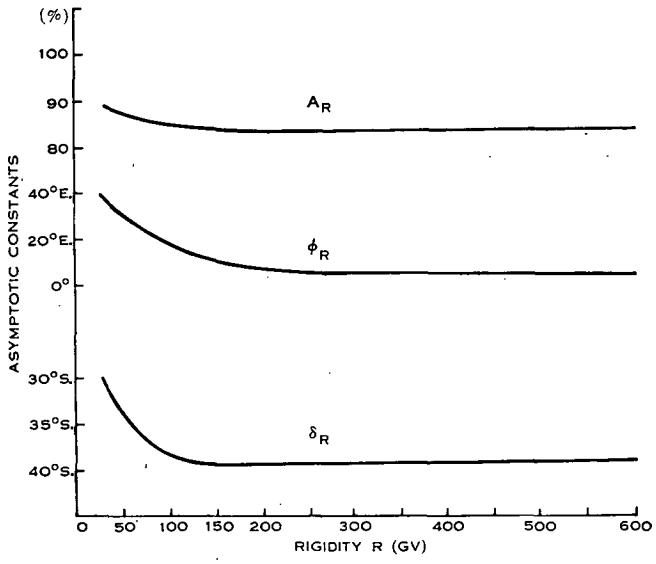

Fig. 1.—The differential asymptotic constants of response to a diurnal variation of the primaries in free space for a vertical semi-cubical telescope at a depth of 40 m.w.e. at $\lambda = 50°$ S.

response to a free-space second harmonic would be affected. It was decided to compare three representations of the field: the centred dipole; the Finch and Leaton sixth-order field (Finch and Leaton 1957), as used by Rao, McCracken, and Venkatesan; and the same field ignoring second-order terms and above (approximating an off-centred dipole). In the last two cases the facilities of an Elliott 503 computer were used in the determination of the asymptotic coordinates. For each representation of the field we tried volume elements ω_r of small size (about 10^{-2} sr, employed throughout the present paper) and of large size (about 10^{-1} sr, centred on the azimuth 0°, 45°, 90°, . . . 315° and zenith angles 5°, 10°, 20°, 30°, 40°, 50°, and 60°, 56 elements making up the solid angle of the telescope). Consequently, with three representations of the geomagnetic field and a small and a large number of arrival directions available we were able to compare six sets of the constants δ_R, A_R, ϕ_R, A_{2R}, and ϕ_{2R}, the last two constants characterizing the differential response to a generalized second harmonic of an anisotropy. Attention was confined to a primary rigidity of 50 GV,

the lowest effective rigidity of response, where the greatest discrepancies would be expected. The first result was that the estimates of δ_R, ϕ_R, and ϕ_{2R} were essentially unaffected. On the other hand, it can be seen from Table 2 that the estimated values of the amplitude factors A_R and A_{2R} tended to be smaller when a large number of arrival directions was used, and larger when more complex representations of the Earth's magnetic field were used. The differences relating to the volume elements could depend on the particular arrival directions that are chosen, as well as on the number. We conclude that it is generally advisable to use a large number of arrival directions, particularly for the estimation of the second harmonic constant A_{2R}. It also appears that the centred dipole field, and specifically the results of the terrella experiments of Brunberg and Dattner, should suffice for present calculations of the response underground to a free-space first harmonic, introducing uncertainties of not more than 2 or 3% in estimates of A_R.

TABLE 2

ESTIMATES OF RELATIVE AMPLITUDES FOR DIFFERENT GEOMAGNETIC FIELD REPRESENTATIONS AND FOR SMALL AND LARGE NUMBERS (N) OF ARRIVAL DIRECTIONS

Values are estimated for the response of a vertical semi-cube at Hobart to first and second harmonics of an anisotropy in the intensity of 50 GV primaries

Geomagnetic Field Representation	First Harmonic A_R		Second Harmonic A_{2R}	
	$N = 56$	$N = 576$	$N = 56$	$N = 576$
Brunberg and Dattner (1953), centred dipole	0·893	0·874	0·633	0·589
Finch and Leaton (1957), first-order terms only	0·915	0·899	0·676	0·635
Finch and Leaton (1957), sixth order	0·915	0·915	0·704	0·663

In curve 1 of Figures 2(a) and 2(b) the relative amplitude B_1 and the phase angle ρ_1 of the estimated relative first harmonic v/α_1 are shown as functions of the upper limiting rigidity R_u. The phase angle ρ_1 is measured east of the direction of maximum intensity of the free-space first harmonic, so that negative values denote earlier times of maximum. The curves 1 demonstrate that changes in R_u have a much greater effect on the estimated amplitude of the first harmonic than on the phase. For example, if R_u decreases from 200 to 100 GV the amplitude decreases by 50% while there is a phase shift to an earlier time of maximum of only about $\frac{1}{2}$ hr.

(b) *The Inclined Telescopes Underground*

There is an average of about 35 m.w.e. of material absorber within the viewing cone of the north-pointing telescope. This has been estimated from the difference in counting rate between the inclined cube and the vertical semi-cube, having regard to their respective radiation sensitivity patterns. The axial direction of the telescope was set 30° north of the zenith, the equivalent latitude being 13°S. In the axial direction, the telescope views upwards along the magnetic field at Hobart and at the

same time this represents a low latitude of viewing. It is for this reason that the asymptotic acceptance cones are relatively compact, centred only slightly east of the observer's meridian, and are not strongly dependent on rigidity, while the distribution of $I(R, \omega_r)$ with asymptotic longitude is virtually independent of asymptotic latitude. Consequently the constants A_R, δ_R, and ϕ_R can be computed from the undivided cones of acceptance with rather less loss of accuracy than in the case of the vertical semi-cube. Following the same procedures as in Section IV(a) above, we determine B_{1N} and ρ_{1N} as functions of R_u. The result is shown in curves 2 of Figure 2.

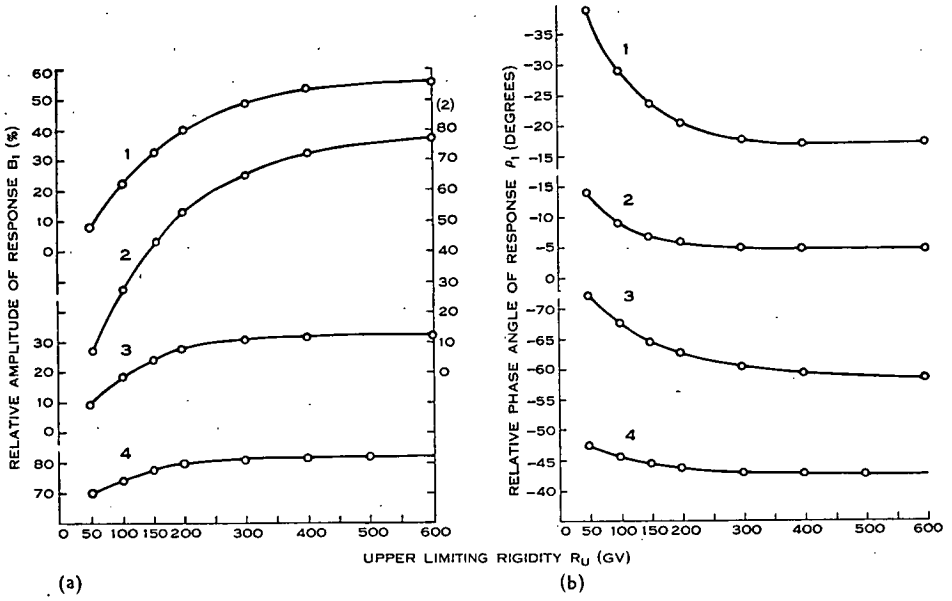

Fig. 2.—(a) The estimated amplitude of the solar diurnal variation, relative to the amplitude of the average free-space first harmonic, and (b) the estimated direction of the solar diurnal variation, measured east of the direction of maximum intensity of the free-space first harmonic, versus upper limiting rigidity for

1, a vertical semi-cube at a depth of about 40 m.w.e. at Hobart;
2, a cube inclined 30° north of the zenith at a depth of about 40 m.w.e. at Hobart;
3, a cube inclined 45° south of the zenith at a depth of about 40 m.w.e. at Hobart; and
4, a high latitude neutron monitor at sea level.

Open circles indicate calculated values. The model for the free-space first harmonic is that proposed by Rao, McCracken, and Venkatesan.

It suffices to note here that, in comparison with the response of the vertical telescope, the amplitude of the first harmonic in the north-pointing direction depends more steeply on R_u, while the phase is rather less dependent on R_u.

The south-pointing cube underground has an average of about 42 m.w.e. of earth and shale absorber within its field of view. The axial direction of the telescope is 45° south of the zenith and this corresponds to a geographic latitude of 88° S. The situation is the reverse of that in the north-pointing direction. Directions of arrival tend to be transverse to the magnetic field and at higher zenith angles are accessible from all 360° of asymptotic longitude. The cones of acceptance are centred far to the

east of the observer's meridian and change very markedly with primary rigidity. The cones were divided into high, middle, and low latitude segments from which the constants A_{Rn}, δ_{Rn}, and ϕ_{Rn} were derived, leading finally to the evaluation of B_{1S} and ρ_{1S} versus R_u, shown in curves 3 of Figure 2. It is clear that the transition from low to high axial latitudes of viewing with a cubical telescope is accompanied by the following changes in response to a first harmonic of the anisotropy:

(1) The amplitude increases less steeply with increasing R_u. This is reflected in the potentially useful relationship B_{1S}/B_{1N} versus R_u, which would enable R_u to be determined independently of α_1, given a simultaneous pair of observations in the north and south directions. Unfortunately the statistical inaccuracies in the observed ratios are far too great for this method to be of any use at present.

(2) There is a pronounced shift to later times of maximum and a greater dependence of the time of maximum on R_u.

(c) *A High Latitude Neutron Monitor*

Rao, McCracken, and Venkatesan have calculated the diurnal relative amplitudes B_1 and the times of maximum for the neutron monitors (see their Table 1) over a rigidity range from near cut-off to 500 GV. In effect then, they have put R_u equal to 500 GV. The actual value was of little consequence in their work, provided it was not much less than about 200 GV. In what follows we show approximately how B_1 and ρ_1 are affected as R_u decreases from 500 to 50 GV.

For rigidities > 50 GV the difference of response between a high latitude neutron monitor and the underground semi-cube is due essentially to the very great differences in the coupling coefficients. The differences in the cones of acceptance are relatively minor. Therefore, since a refined treatment for a neutron monitor is unnecessary at this stage, it suffices to apply the same values of A_R, δ_R, and ϕ_R at the higher rigidities as have been worked out for the semi-cube. Accordingly, the differential first harmonic vectors $\mathbf{v}_R/\alpha_1$ were estimated from equation (4) for the rigidities 75, 125, 175, 250, 350, and 450 GV as they were for the semi-cube except that the values of Y_R were those calculated by Dorman (1957) for a high latitude monitor. The amplitude and the time of maximum of the estimated relative first harmonic v/α_1 for Mt Wellington are given by Rao, McCracken, and Venkatesan in their Table 1 and are the values for $R_u = 500$ GV. Estimates of v/α_1 for lower values of R_u were therefore obtained by successive subtraction of the differential vectors $\mathbf{v}_R/\alpha_1$ from the total vector specified in their table. Thereby the values of B_1 and ρ_1 versus R_u that are shown in curves 4 of Figure 2 were obtained. It can be seen that the estimated relative amplitude begins to fall off rather significantly as R_u decreases below about 150 GV, but of course much less steeply than the relative amplitude underground. A comparison is given in Table 3.

V. 1958 Evaluation of α_1 and R_u

(a) α_1 *versus* R_u *from the Underground Observations at Hobart*

The relative amplitude B_1 in curve 1, Figure 2(*a*), is $|v|/\alpha_1$, where $|v|$ is the amplitude of that part of the observed first harmonic that is caused by the primary

anisotropy. Given a value of $|v|$ for 1958, estimates of the free-space amplitude α_1 for different values of R_u can be obtained directly from this curve.

The amplitude of the first harmonic of the annual mean pressure-corrected daily variation underground in 1958 was ($0{\cdot}095 \pm 0{\cdot}008$ S.E.)% and the time of maximum was (1712 ± 20 min) local solar time. It appears that if there were contributions to the first harmonic from daily variations of atmospheric temperature they must have been very small. This is discussed in a paper on the sidereal effect (Jacklyn 1965). We also present evidence below that the diurnal variations observed in three different directions underground are compatible with the model for the free-space first harmonic put forward by Rao, McCracken, and Venkatesan. It is concluded that the result in 1958 was at least very largely of extraterrestrial origin. With $|v| = 0{\cdot}095\%$, we obtain curve A in Figure 3, giving the estimate of α_1 versus R_u from the underground observations in 1958. The dashed lines give the limits of error arising from the S.E. of estimate of $|v|$.

TABLE 3

ESTIMATED CHANGES IN RELATIVE AMPLITUDE B_1 OF THE FIRST HARMONIC AS UPPER LIMITING RIGIDITY R_u DECREASES

R_u	$\Delta B_1/B_1$ (%) 250–200	200–150	150–100	100–50 (GV)
Neutron intensity at Mt Wellington	−1	−2	−4	−8
Meson vertical intensity underground at Hobart	−10	−18	−32	−68

(*b*) α_1 *versus* R_u *from Neutron Monitor Observations*

Having virtually assumed R_u to be 500 GV, Rao, McCracken, and Venkatesan obtained $0{\cdot}4\%$ as their estimate of α_1 for 1958, this being the average of individual estimates from the records of 17 neutron monitors distributed globally. Looking at their Figure 12, for $\beta = 0$, $g(\lambda) = \cos\lambda$, we see that the end-points of all but three of the individual estimates of the free-space vector are grouped within a circle of radius $0{\cdot}067\%$, this being 3σ, where σ is the S.E. of estimate of amplitude of the diurnal variation at a single station. It seems that, if we regard the radius of the circle as being about twice the S.E. of estimate of α_1 from a single station, it would fairly take account of other random errors. Thus the estimate of $0{\cdot}4\%$ would have a S.E. of about $0{\cdot}008\%$. Clearly, if we use the averaged estimate of α_1, in the product $\alpha_1 B_1$ we have a much more accurate indication of the amplitude of the first harmonic at Mt Wellington to be expected from the model than would be given by the actual observation at the station. From Table 1 of their paper, we see that B_1 for Mt Wellington is $0{\cdot}813$, so that the estimated amplitude at that station is ($0{\cdot}325 \pm 0{\cdot}006$)%. Their Figures 9 and 12 show that the observed amplitude at Mt Wellington was in fact very close to this estimate. Proceeding as in Section V(a) and substituting $0{\cdot}325\%$ for $|v|$, we obtain estimates of α_1 for different values of R_u from curve 4, Figure 2(a). The result is shown as curve B in Figure 3. It indicates how the estimate

of free-space amplitude at a high latitude station, derived from world-wide observations of neutron intensity in 1958, would depend on the value assigned to R_u. The errors due to counting rate statistics would be very small and are not shown. The intersection of curves A and B gives values of R_u and α_1 that would satisfy the observations of the solar diurnal variation both underground at Hobart and at the neutron monitor stations, in relation to the model. The intersection value of R_u is 95 GV with an error tail due to counting rate statistics of perhaps 5 GV. The free-space amplitude is about 0·43%, slightly higher than the estimate given by Rao, McCracken, and Venkatesan.

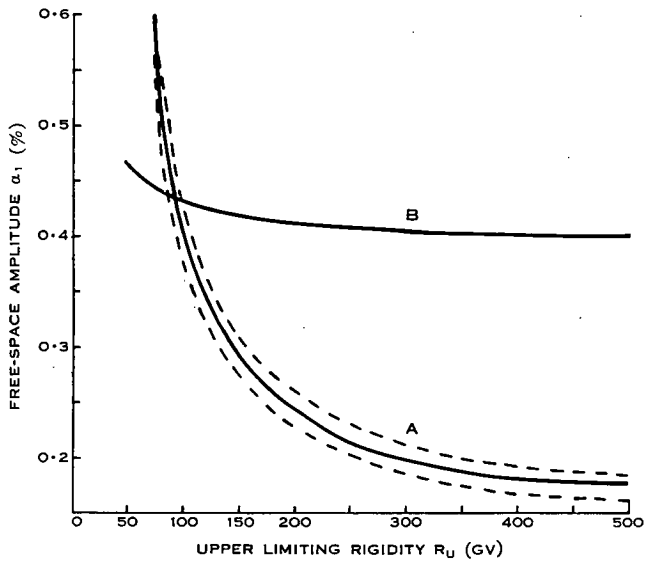

Fig. 3.—Estimates of the amplitude of the average free-space solar diurnal variation of the primaries in 1958 for different values of the upper limiting rigidity based on: A, observations with a vertical semi-cubical telescope at a depth of 40 m.w.e. at Hobart; B, the estimated solar diurnal variation of neutron intensity at a high-latitude sea level station, deduced from world-wide observations of neutron intensity. The average values of R_u and α_1 for 1958 are given by the intersection of the two curves. All the estimates derive from the model for the free-space first harmonic proposed by Rao, McCracken, and Venkatesan. The dashed lines represent the standard errors of estimate of individual points on curve A.

This result only holds if the average free-space first harmonic is independent of rigidity. Supposing the free-space amplitude to vary as R^β, Rao, McCracken, and Venkatesan showed that β was $0{\cdot}00 \pm 0{\cdot}05$, averaged over 1958. Therefore we have to find out how our estimates of α_1 and R_u would be affected if $|\beta|$ was about 0·05. Accordingly, the curves A and B of Figure 3 were recalculated, starting from equation (3) which now contains the factor R^β, for $\beta = +0{\cdot}05$ and for $\beta = -0{\cdot}05$. Essentially the result was that in each case curve A became much steeper for values of R_u below about 150 GV while curve B was displaced towards lower values of α_1 for $\beta = 0{\cdot}05$ and towards higher values for $\beta = -0{\cdot}05$. The intersection values of α_1 and R_u are shown in Table 4. Note that we now have to refer to α_1 as the free-space amplitude constant since it only becomes the free-space amplitude itself when $\beta = 0$.

From these considerations it appears that the error in estimation of R_u would be about 15 GV. This might even take care of uncertainties in the calculation of curve B that do not apply to curve A. These uncertainties will be mentioned in the Discussion, but it is clear that they would affect the determination of α_1 rather than R_u.

TABLE 4

ESTIMATES OF THE UPPER LIMITING RIGIDITY AND THE FREE-SPACE AMPLITUDE CONSTANT FOR THREE VALUES OF THE RIGIDITY DEPENDENCE INDEX

Values are estimated for the first harmonic of the solar anisotropy in 1958

Rigidity Dependence Index β:	$-0{\cdot}05$	$0{\cdot}00$	$+0{\cdot}05$
Upper limiting rigidity R_u (GV)	105	95	90
Free-space amplitude constant α_1 (%)	0·50	0·43	0·38

VI. 1961–62. Results from Three Directions Underground

The four telescopes that have been installed in a disused railway tunnel near Hobart (geographic latitude 43° S.) have already been mentioned in Section I, but it may be useful to summarize here the main features of the arrangement during 1961–62. While the tunnel itself is not quite in the north–south direction, the axes of the inclined telescopes were accurately aligned in the plane of the geographic meridian.

TABLE 5

CHARACTERISTICS OF THE UNDERGROUND TELESCOPES AT HOBART DURING 1961–62

Telescope Type:	North Single Cube	Vertical Duplex Semi-cube	South Single Cube
Inclination of axis to the zenith	30°N.	0°	45°S.
Approximate mean asymptotic latitude of response	17°S.	39°S.	60°S.
Effective absorber excluding atmosphere (m.w.e.)	35	40	42
Particles/hr	16700	70000	8700

Each telescope registered triple coincidences, and individual trays were of 1 square metre sensitive area. Other characteristics of the telescopes are tabulated in Table 5. At the end of 1962 the north-pointing telescope was elongated to give 20° half angle of aperture and the axis was inclined 70° north of the zenith.

The pressure-corrected first harmonic vectors averaged over the two calendar years 1961 and 1962 are shown in Figure 4. The radius of each error circle is 2σ so that even in the south-pointing direction the first harmonic is significant. To show that the vector distribution was broadly consistent with an extraterrestrial origin for the diurnal variation underground, it was tested against the model for the anisotropy on the assumption that $\beta = 0$ and neither α_1 nor ψ_0 had changed appreciably since 1958. Estimates of the first harmonics to be expected in the three directions were

calculated as functions of R_u. The results are shown as dashed lines in the figure. It can be seen that the phase differences between V, N, and S are compatible with the model although the observed time of maximum for N is later than expected. The amplitudes are also those to be expected from the model but they indicate that the average value of R_u must have been about 70 GV during 1961–62 if the other constants had not changed. Alternatively, if α_1 alone had changed, it must have decreased by about 40% since 1958. The estimates of α_1 versus R_u that were derived from each result demonstrate the interrelationships more clearly (Fig. 5). It is quite clear that the same estimate is given by each of the three independent experiments, involving detectors of different geometry and greatly different effective latitudes of scan.

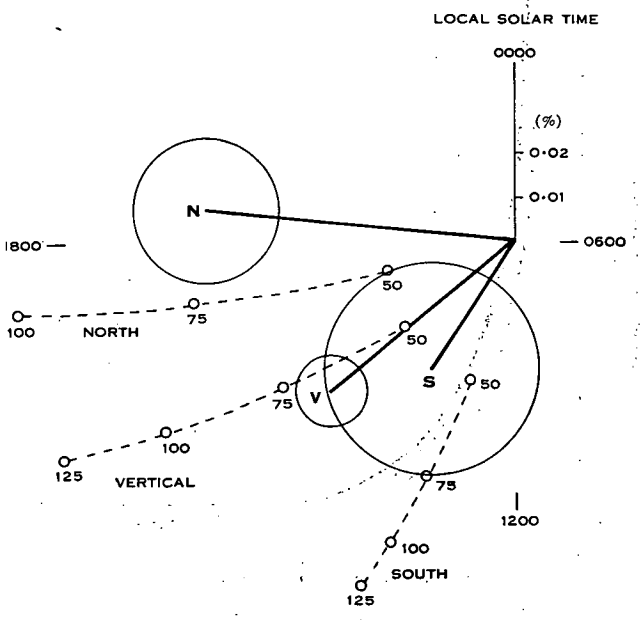

Fig. 4.—The first harmonic vectors of the pressure-corrected daily variations observed in the north (N), vertical (V) and south-pointing (S) directions underground at Hobart, averaged over the years 1961 and 1962. The radius of each error circle is 2σ. The dashed lines give estimates of the first harmonics to be expected in each of the three directions, for different values of upper limiting rigidity. Some values of interest are shown in GV. The estimates are based on the model for the free-space solar diurnal variation of the primaries proposed by Rao, McCracken, and Venkatesan.

VII. 1958–62. Some Results from Other Places

The large decrease of amplitude of the solar diurnal variation that has taken place at the Hobart underground site since 1958 was confirmed by results from the vertical semi-cubical telescopes located at a depth of 40 m.w.e. at Budapest (geographic coordinates 47·5°N., 18·9°E., geomagnetic latitude 46·4°N.). The only complete years available from Budapest were 1959 and 1961. The comparison with Hobart for these 2 years, given in Table 6, shows unmistakable agreement except for the differences in time of maximum in 1961.

Also of relevance are some interesting results reported by Sarabhai and Subramanian (1963*a*). They have found that the peak-to-peak amplitude of the total solar daily variation (sum of first and second harmonics) of neutron intensity at a low latitude station (Huancayo) had decreased by about 35% between 1958 and 1961–62, while at a high latitude station (Churchill) it had decreased by about only 5%. There were also interesting shifts of the times of maximum and minimum to earlier hours at Huancayo but not at Churchill. The results from a cubical meson telescope at Churchill were between those from the two neutron monitors. Sarabhai

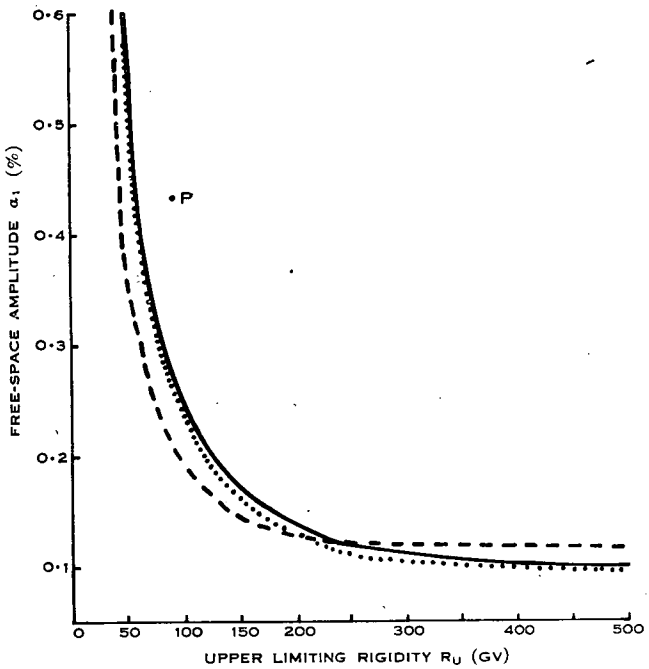

Fig. 5.—Estimates of the amplitude of the free-space solar diurnal variation of the primaries, averaged over 1961 and 1962, for different values of R_u and for $\beta = 0$, based on observations underground at Hobart with (*a*) the vertical semi-cubes, full line; (*b*) the north-pointing cube, dotted line; (*c*) the south-pointing cube, dashed line. The point P represents the intersection of curves A and B for 1958 (Fig. 3).

and Subramanian suggested that the observed changes could have come about if β had become increasingly negative or if R_u had decreased, but that a decrease in the lower limiting rigidity R_L could not have been the main cause. Also, their results do not seem to be consistent with a large decrease in the amplitude constant α_1. This would have produced similar decreases of total amplitude at all stations, bearing in mind that the part played by the second harmonic in their observations is unknown. Finally, changes in the galactic spectrum that might have been expected over the period (see Mathews and Kodama 1964) would clearly not have been responsible for an effect that seems to increase with increasing primary rigidity of response.

VIII. Relevant Changes in the Apparent Sidereal Effect observed Underground

Every year since experiments commenced at the Hobart underground site a significant daily variation in sidereal time has been observed. A general account of the results obtained over the period 1958–62 inclusive has been published recently (Jacklyn 1965). It is submitted in that paper that most, if not all, of the sidereal effect observed at the depth of 40 m.w.e., is genuine. Figure 6 shows the annual running averages of the amplitudes of the total daily variations in solar and sidereal time. As noted in Sections VI and VII, the amplitude in solar time has decreased considerably over the period and it is evident that most of the change took place after December 1960. It seems that the amplitude in sidereal time also decreased somewhat after 1958, but that it began to increase again during 1961 and passed

Table 6

First harmonics of mean pressure-corrected solar daily variation of vertical intensity at Budapest and Hobart during 1959 and 1961

Values were obtained from semi-cubical telescopes at a depth of 40 m.w.e.

	1959		1961	
	Amplitude (%)	T_{max} (local solar time)	Amplitude (%)	T_{max} (local solar time)
Budapest*	0·1114	1640	0·0389	1554
S.E.	±0·0054	±12 min	±0·0054	±34 min
Hobart†	0·1062	1638	0·0389	1408
S.E.	±0·0058	±12 min	±0·0058	±34 min

* Geomagnetic latitude 46° N.

† Geomagnetic latitude 50° S.

through quite a conspicuous maximum at the same time as the amplitude of the solar daily variation passed through a minimum, and a few months before the daily intensity reached a maximum value. Over a period of about 6 months centred on the turning points of the two amplitude curves the genuineness of the sidereal effect was strikingly demonstrated; the time of maximum of the monthly mean daily variation in solar time changed from 1500 to 0700 solar time and became earlier month by month at about the rate of 2 hr each month, in the manner of a sidereal daily variation expressed in solar times. In other words, the sidereal component of the daily variation was dominating the solar component over this period. Accounts of this phase anomaly have been given elsewhere (Jacklyn 1963*b*, 1965). Now, the point we wish to make here is that the changes in amplitudes of the solar and sidereal components over this period were probably not negatively correlated by chance but were connected through opposite dependences on changes in R_u. On the one hand, a decrease in R_u is a decrease in upper limiting rigidity leading to a decrease of amplitude of the solar component. On the other hand, it is a decrease in threshold rigidity leading to an increase of amplitude of the sidereal component. The effect on the sidereal component (assumed to be observable over a very wide range of rigidities above R_u) would not be expected to be very great until R_u had decreased to values below 100 GV,

where the differential response of the underground detector is greatest. Calculations based on a simple model for the sidereal anisotropy show that the observed increase of amplitude could be explained if R_u had decreased from, say, 85 to 50 GV over the period of increase, although it might be necessary for the spectrum of variation (for the sidereal effect) to be somewhat negatively dependent on rigidity.

If, alternatively, the changes in amplitude of the solar component were primarily due to β becoming increasingly negative on the average, then there would be no simple explanation for the correlated changes in amplitude of the sidereal component.

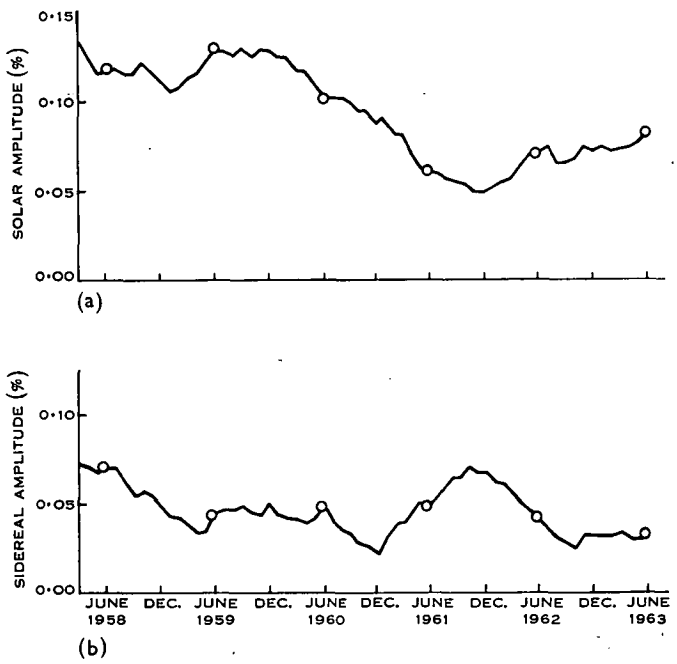

Fig. 6.—Long-term changes observed with the vertical semi-cubical telescopes at the Hobart underground station; month by month annual running averages of (*a*) amplitude of the solar daily variation, (*b*) amplitude of the apparent sidereal daily variation. The dates refer to the central months and the open circles indicate the conventional annual mean values.

IX. Discussion

The fact that the amplitude, but not the phase, of the first harmonic of the solar daily variation underground changed so greatly between 1958 and 1962 is itself evidence that the observed daily variation at this depth must be very largely of extraterrestrial origin. Of greater significance, however, is the evidence that the amplitudes and times of maxima in all three directions underground are compatible with the model for the free-space first harmonic derived from the neutron monitor data. In addition to this there were indications in 1961 that simultaneous changes in the solar and apparent sidereal daily variations were connected. Overall, it seems reasonable to assume that the observed pressure-corrected first harmonic at 40 m.w.e.

is almost entirely due to the anisotropy, a prerequisite for the present method of determining R_u. We have seen that the method depends essentially on the fact that the differential coupling coefficients (Y_R) for the underground semi-cube differ very greatly from the coupling coefficients that apply to a high latitude neutron monitor. This is the main reason for the difference between curves A and B in Figure 3. Moreover, the nature of the difference is such that quite sizeable errors in curve B will not greatly affect the estimation of R_u.

It would seem that more accurate knowledge of the deflections of high energy particles in the Earth's magnetic field, improved estimates of the radiation sensitivity patterns of the underground telescopes, or a more refined treatment of the asymptotic cones of acceptance are not likely to affect curve A very much. The reliability of the coupling coefficients is a more important factor. It seems unlikely that the coefficients used here would be greatly in error since the values worked out by Fenton (1963) are in reasonable agreement with Mathews's (1963) results obtained by empirical methods. The coefficients are only strictly applicable at vertical incidence where the meson threshold energy at production, for detection at 40 m.w.e. underground, is 15 GeV. However, in the case of the vertical semi-cube they should apply quite well up to the limiting zenith angle of about 60°, where the meson threshold energy has only increased to 18 GeV.

The accuracy of estimation of curve B, relating to the Mt Wellington neutron monitor, should be sufficient for this provisional determination of R_u. However, in a detailed study of year to year changes in R_u, α_1, and β, other quantities would be important at the low rigidities of response. It is evident from the work of Mathews and Kodama (1964), for example, that the differential coupling coefficients at rigidities below 15 GV must change considerably during the cycle of solar activity. Therefore, at the least, it might be necessary to use different average cosmic ray spectra at solar maximum and solar minimum when analysing the neutron monitor data.

It is conceivable that β, the index of the variational spectrum, changes with increasing rigidity, although *a priori* there is no reason to suppose that this happens. Indeed, Rao, McCracken, and Venkatesan showed that any trend of this kind in 1958 would have been difficult to detect since β was found to be within the limits $0{\cdot}0 \pm 0{\cdot}05$ at least over the rigidity range 1–100 GV. To provide against the possibility, however, it would be an advantage to have additional estimates of α_1 versus R_u from detectors whose mean primary rigidities of response were between those of neutron monitors and the telescopes at 40 m.w.e. Unfortunately, as the amount of material absorber decreases, the atmospheric negative temperature effect in particular contributes increasingly to the daily variation of meson intensity. At many places, daily variations of atmospheric structure are difficult to estimate and may vary greatly from place to place (see Kane 1963). One way to avoid this problem is to make use of results from the crossed telescopes that operate at Hobart, Mawson, and a number of other places. Even then such data may prove to be of limited value until the differential responses of inclined telescopes at sea level to primaries of rigidity > 15 GV are more reliably known.

So far it has been tacitly assumed that R_u represents both the upper limiting rigidity of the solar anisotropy and the threshold rigidity for observation of a sidereal

effect. This is not necessarily true of course. In the case of the sidereal anisotropy we are interested to know the rigidity above which primaries have access to the Earth's orbit from some particular direction in space, without having been appreciably scattered. On the other hand, if we accept Parker's recent theory (Parker 1964) that the solar diurnal variation is produced essentially by streaming of cosmic rays with an azimuthal velocity of rigid rotation with the Sun, then, as he points out, two conditions for the upper limiting rigidity must be satisfied. R_u must be such that (*a*) the gyro-radius is less than 1 a.u., so that the guiding centre approximation holds for the motion of particles along the spiralling interplanetary field, and (*b*) the gyro-radius is small enough to allow the particles to be scattered several times beyond the orbit of the Earth as they approach, so that a density gradient of intensity would be largely destroyed. In these circumstances there are clearly several possibilities whereby R_u could differ from the threshold rigidity for observation of a sidereal effect. Nevertheless, it seems reasonable to assume as a starting point that the two are not very different and that changes in R_u would be accompanied by similar changes in the sidereal threshold rigidity, but perhaps not vice versa. Moreover, judging from the evidence for a sidereal effect obtained from neutron monitor data by Conforto and Simpson (1957) at the previous sunspot minimum, the average threshold rigidity for observation should usually exceed the lower limiting rigidity of the sidereal anisotropy.

X. Conclusion

The pressure-corrected solar daily variation at 40 m.w.e. appears to be very largely under the control of the primary anisotropy, as evidenced by the nature of the year to year variations of the first harmonic observed at Hobart and Budapest in the vertical direction and by the characteristics of the first harmonic observed in three different directions underground at Hobart. It would follow that a component of atmospheric origin must be small and this is supported by other evidence, which is discussed in an earlier paper (Jacklyn 1965).

The provisional estimate of the average value of R_u in 1958 is 95 GV, this being the average primary rigidity above which a first harmonic of the anisotropy would become unimportant. The uncertainty of 10–20 GV in the estimate refers only to the method of measurement and has nothing to do with any inherent variability in R_u.

It appears that changes in the annual average solar diurnal variation between 1958 and 1962 would have been effected mainly through one or more of the constants R_u, R_L, α_1, β, and ψ_0 of the average free-space first harmonic, and through the average primary spectrum from which the coupling coefficients are derived. All these quantities may have varied to some extent, but it is interesting to note that only a decrease in R_u (of about 20–40 GV) could by itself have brought about the observed decrease in amplitude of the solar diurnal variation underground, the negatively correlated effects in solar and sidereal time underground in 1961, and the type of change observed in the solar daily variation of neutron intensity at Huancayo relative to that at Churchill.

A more refined analysis of long-term changes should take into account the variability of each of the parameters specifying the free-space first harmonic. It

would also be necessary to clarify the part played by the second harmonic. Some of the factors (e.g. changes in R_L) should only affect the daily variation of neutron intensity, while others (e.g. changes in R_u) produce the most noticeable effects underground. Therefore a comprehensive comparative study of year to year changes in the daily variation of neutron intensity and of the underground meson intensity should permit the most important of these factors to be identified and estimated. A project of this type is being undertaken at Hobart. To achieve the desired result many years of continuous and reliable data are needed. Of particular importance are the records from the few neutron monitors at the very high and very low latitudes.

XI. Acknowledgments

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PAPER 8

EVIDENCE FOR A TWO-WAY SIDEREAL ANISOTROPY IN THE CHARGED PRIMARY COSMIC RADIATION

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FOR several years now the sidereal daily variation of intensity of cosmic ray mesons at the Hobart underground laboratory has been closely investigated[1,2]. In common with earlier investigations of this kind the objective has been to obtain decisive evidence either for or against a maximum of intensity among the charged parent primary particles from some fixed direction in space. The most important observational problem has been the possibility that the annual observed daily variation of meson intensity in sidereal time may contain a large spurious component, due to seasonal modulation of the solar daily variation of intensity. Therefore it will be necessary to outline the problem here briefly and show how peculiarly relevant it is to the observations. However, it is because of the distinctive nature of the observations and of the spurious effect that might be expected that it has been possible to design an experiment which makes it possible to decide whether the observed sidereal daily variation is in the main genuine or spurious. The first results of this experiment are presented in this article and confirm the earlier indications that there is a genuine two-way sidereal anisotropy among the charged cosmic ray primaries.

Although the charged primaries in the energy range 10^{10}–10^{12} eV are thought to be on the average very nearly isotropic in the vicinity of the Earth's orbit, a solar anisotropy has been detected[3], the intensity maximum as observed from the Earth having a fixed direction relative to the Earth–Sun line. It produces an observed solar daily variation whose average amplitude may be as great as 0·5 per cent. It might seem that if a comparable sidereal anisotropy existed among the charged primaries it could also be detected by arranging cosmic ray data in sidereal time. Such analyses have indicated that if there is a genuine sidereal component in the observed daily variation the amplitude must be less than about 0·1 per cent. Consequently, if a sidereal component exists it must usually be superimposed on a much larger solar component of the daily variation. In the simplest case this observational difficulty is overcome by considering annual averages: a solar component of constant amplitude and phase would be completely averaged out if the data were arranged in sidereal time, say month by month, and summed over a complete year. Unfortunately, the solar component may vary considerably from time to time and in particular may exhibit seasonal modulations of amplitude and phase. To illustrate the serious difficulties of interpretation that follow from this, let us consider a solar diurnal variation on which is impressed an annual sinusoidal variation of amplitude. The unmodulated diurnal wave can be regarded as a carrier, of frequency one cycle per day or 365 cycles per year. Amplitude modulation of one cycle per year produces matching sideband components of frequency 364 cycles per year and 366 cycles per year. The latter is the sidereal frequency, there being 366 sidereal days in a complete year. It follows that if a solar daily variation with impressed annual modulation of amplitude is arranged in sidereal time and averaged over one year, we observe an annual daily variation in sidereal time that is a completely spurious sideband effect. The time of maximum, or phase, of this spurious sidereal daily variation depends on the month of the year when the solar daily variation has maximum amplitude[4,5]. Therefore, on our assumption that maximum amplitude occurs during a particular season of the year, the phase of the spurious annual sidereal effect observed in the northern hemisphere will differ by 12 h from that observed in the southern hemisphere. This 12 h phase shift should hold at least approximately in practice even if the seasonal modulation of the solar component is rather complex. We shall see how important it is to be aware of this problem of the spurious effect when interpreting the observational data, although the material presented here relates in the main to daily variations of intensity underground, where seasonal changes of the solar component that are thought to be due to contributions of atmospheric origin should be considerably smaller than they are above ground.

Late in 1957 two semi-cubical vertical counter telescopes were installed at a depth of 40 m.w.e. (metres water equivalent) underground near Hobart (42·9° S geographic)[6]. Continuous records have been obtained of the intensity of mesons, the mean counting rate being about 70,000 particles per h. (It has been calculated that more than 90 per cent of this counting rate is due to primary protons of energy greater than 50 GeV and that the primary energy of maximum response is about 100 GeV)[7]. The time of maximum of the annual mean daily variation in sidereal time that has been obtained each year from the pressure corrected data has remained within the limits 0500–0700 local sidereal time[2]. Seasonal modulation of the solar component of the daily variation observed at this site appeared to have fallen to a very low level in the later years, so that from this point of view there was reason to believe that the sidereal effect might be genuine. Therefore it was of interest to compare the Hobart underground results with observations from similar equipment at a similar location in the northern hemisphere. Data from two (and later, three) vertical semi-cubical telescopes located at a depth of 40 m.w.e. at Budapest (47·5° N geographic) were obtained from the World Data Centre *WDC* 2 for the complete years 1959 and 1961. Fig. 1 shows that over the period the times of maximum of the apparent sidereal effect at the two places differed by approximately 12 h, whether judged by the first harmonics of best fit to the intensity deviations, or the sum of the harmonics, since the first and second harmonics are in phase. We have seen that this is what one would expect from a spurious sidereal effect generated by seasonal modulation of the solar component—a 12 h phase difference at places in opposite hemispheres. However, it is worth noting at the outset that this conclusion is not in accord with the evidence that the effect was still present when seasonal modulation was low. More than this, two cosmic ray phenomena of a somewhat different character that have been reported in recent years support the view that the sidereal effects shown in Fig. 1 are genuine. It may be of interest to describe them briefly, before proceeding to the more recent experimental results from Hobart.

In an investigation of monthly average daily variations obtained from neutron monitors and counter telescopes in the northern hemisphere during 1954, a year of minimum solar activity, Conforto and Simpson[8] found that over a period of some 6 months, the time of maximum expressed in solar time was becoming progressively earlier at the rate of approximately 2 h each month, in

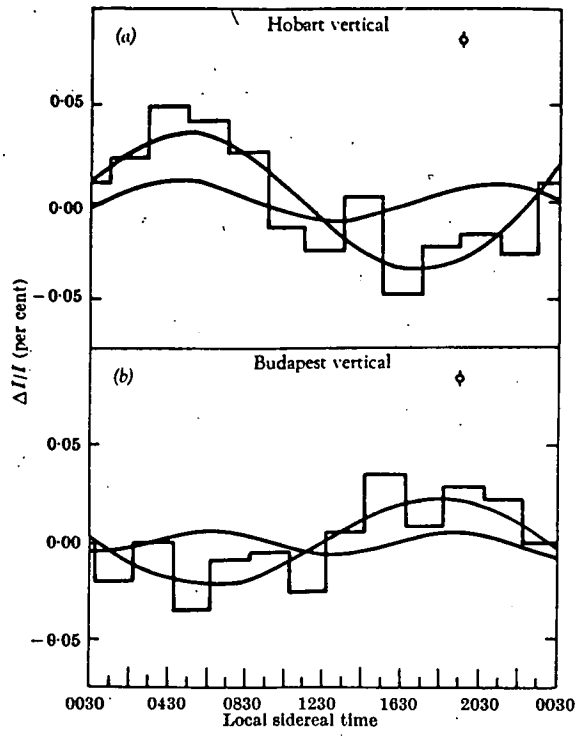

Fig. 1. The observed daily variation in sidereal time, averaged over the years 1959 and 1961, of pressure-corrected meson intensity from vertical semi-cubical telescopes at a depth of approximately 40 m.w.e. (*a*) At Hobart, scanning asymptotic latitudes in the vicinity of 39° S. geographic; (*b*) at Budapest, scanning asymptotic latitudes in the vicinity of 44° N. geographic. The error tails shown are the *S.E.*'s of amplitude associated with the first and second harmonics of best fit

the manner of a genuine sidereal daily variation with a maximum at approximately 2000 local sidereal time; however, it is most important to note that in their terminology this would be 0800 local sidereal time. As Baliga and Thambyahpillai[9] have pointed out, the authors had synchronized solar and sidereal time at the March equinox instead of the September equinox, the customary date, so that 12 h must be added to their published sidereal times of maximum to give the correct values. (This error also appears in a notable summary of the 1954 phase anomaly presented by Dorman[10]). The phenomenon was not detected in the southern hemisphere at that time, although the high counting rate data from the Hobart vertical meson telescope at sea level was carefully examined for the effect by Baliga and Thambyahpillai and by Parsons[11]. However, in 1961 a phase anomaly did occur at the Hobart underground site when the amplitude of the solar daily variation had reached a very low level, the maximum of the monthly mean daily variation in solar time becoming progressively earlier at the rate of 2 h each month, in the manner of a sidereal component with a maximum at 0700 local sidereal time. This time, so far as I am aware, the phenomenon was not detected in the northern hemisphere. In other words, it seems so far to have occurred in only one or the other hemisphere. This point will be reconsidered later. The event at Hobart has been described more fully in earlier publications[12]. Fig. 2 compares it with a typical example of the phase anomaly observed at northern latitudes in 1954, and we note the 12 h phase difference between the respective apparent sidereal maxima.

If, as it seems, the phase anomalies had been produced by genuine sidereal components of the daily variation, there would be no conflict of evidence with the annual apparent sidereal effects observed underground in 1959 and 1961 (Fig. 1) and with the longer term trends which indicated that the effect at the Hobart site was genuine. The inference would be that the direction of maximum intensity of the primaries observable in the northern sky differed from that observable in the southern sky by 180°, that is, a two-way sidereal effect exists. On the other hand, if the argument that the whole effect is spurious were to be sustained, the required seasonally modulated solar component of the daily variation would naturally have to be such as to resemble a genuine sidereal daily variation. To explain the phase anomalies, this would have to be a truly extraordinary seasonal phenomenon—a solar component, of constant amplitude, with a time of maximum which progressed anti-clockwise from noon in midwinter to midnight in midsummer. Nevertheless the evidence so far does not expressly forbid such a possibility.

What is now needed is a simple experimental result exhibiting the 12 h phase shift but explicitly ruling out an explanation based on seasonal changes of the solar daily variation. The first results from such an experiment are reported here.

In October, 1963, a narrow-angle telescope was installed at the Hobart underground site. It consisted of three standard metre-square trays in triple coincidence, the separation between the two extreme trays being approximately 10 ft., giving a total geometric aperture of 36°. The telescope was inclined at an angle of 70° to the north of zenith, so that the latitude of viewing along the axis was 27° N. geographic. Since the telescope responds in the main to primaries of rigidity greater than about 100 GV, deflexions of the trajectories of the primaries in the Earth's magnetic field would be no more than about 5°, so that one could be confident that the asymptotic cone of viewing was located in the northern hemisphere. On the other hand, the asymptotic cones of viewing for the vertical semi-cubes at the underground site are centred at about 39° S. geographic latitude and not more than about 15° E. of the meridian, depending somewhat on the nature of the primary phenomenon to which the telescopes are responding. It is concluded that the vertical and the narrow-angle inclined telescopes view within about 10° of each other in longitude when deflexions of the primary protons in the Earth's magnetic field are taken into account. The reader will now appreciate the purpose of this joint vertical-inclined experiment—to find out whether the apparent sidereal effect obtained from the inclined direction, scanning latitudes in the northern hemisphere, differs by about 12 h from the effect observed with the

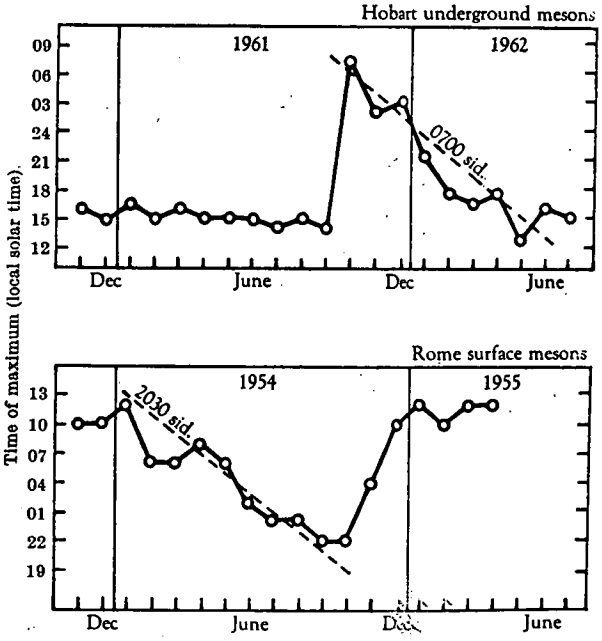

Fig. 2. Monthly averages of the time of maximum, in solar time, of the pressure-corrected daily variation of vertical meson intensity at the Hobart underground site (1961–1962) and at Rome (1954). The dashed lines represent the solar time equivalents of 0700 local sidereal time and 2030 local sidereal time as shown

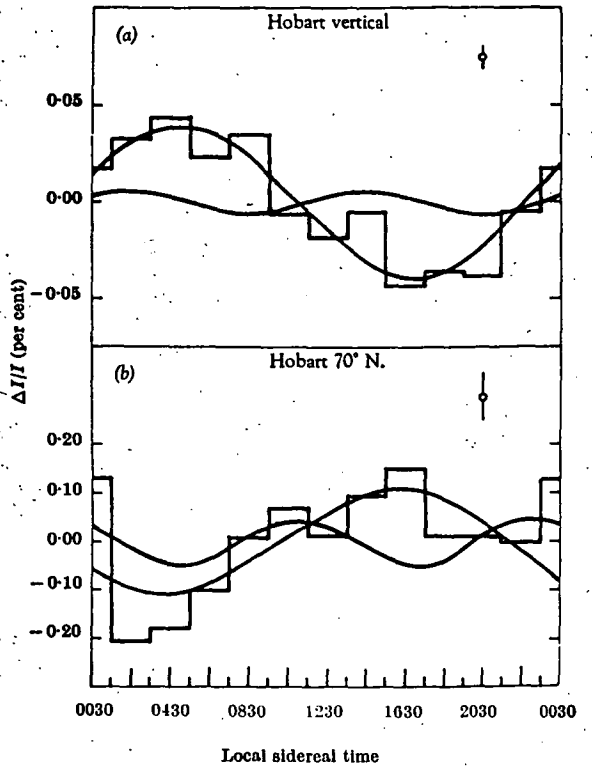

Fig. 3. The observed daily variation in sidereal time, averaged over the year 1964, of pressure-corrected meson intensity at a depth of approximately 40 m.w.e. at Hobart, (*a*) from two vertical semi-cubical telescopes, scanning asymptotic latitudes in the vicinity of 39° S. geographic, (*b*) from a narrow angle telescope inclined 70° to the north of zenith, scanning asymptotic latitudes in the vicinity of 20° N. geographic. The error tails shown are the *S.E.*'s of amplitude, associated with the first and second harmonics of best fit

vertical telescopes, scanning mid-southern latitudes. A 12-h phase difference could not be produced at the one place by any local (that is, atmospheric) process of seasonal modulation, usually thought to be responsible for spurious sidereal effects.

The histograms relating to the first year of operation, 1964, are shown in Fig. 3 with their first and second harmonics of best fit. It is evident that the sidereal daily variation in the vertical direction was much the same in 1964 as it was during 1959 and 1961. In the inclined direction the amplitude of the first harmonic is statistically significant at the 95 per cent level and the time of maximum differs by approximately 12 h from the vertical result. The respective times of maximum are in fact compatible with the Hobart–Budapest findings (Fig. 1) and with the values derived from the phase anomalies (Fig. 2).

We have seen that this latest result, while remaining to be confirmed over several years of observation, was the one to be expected from a sequence which as a whole indicated the existence of a two-way sidereal anisotropy in the primary radiation. The type of anisotropy is depicted schematically in Fig. 4. However, the second objective of the experiment was perhaps the most important one. It now seems scarcely possible that the sidereal effect could be spurious. (Elsewhere we shall also conclude this from considerations of the observed harmonics.)

It is of interest to note that the inward direction of the galactic magnetic field would be expected to pass overhead at about latitude 35° N. at 2000 local sidereal time, while the outward direction (looking towards the periphery of the spiral arm) would pass overhead at about latitude 35° S. at 0800 (ref. 13). These times roughly agree with the observed times of maximum of the sidereal effect. Corrections for deflexions of the primary trajectories in the Earth's magnetic field would improve the agreement. If it should happen that the galactic magnetic field is in fact the seat of adiabatic motions of the charged primaries

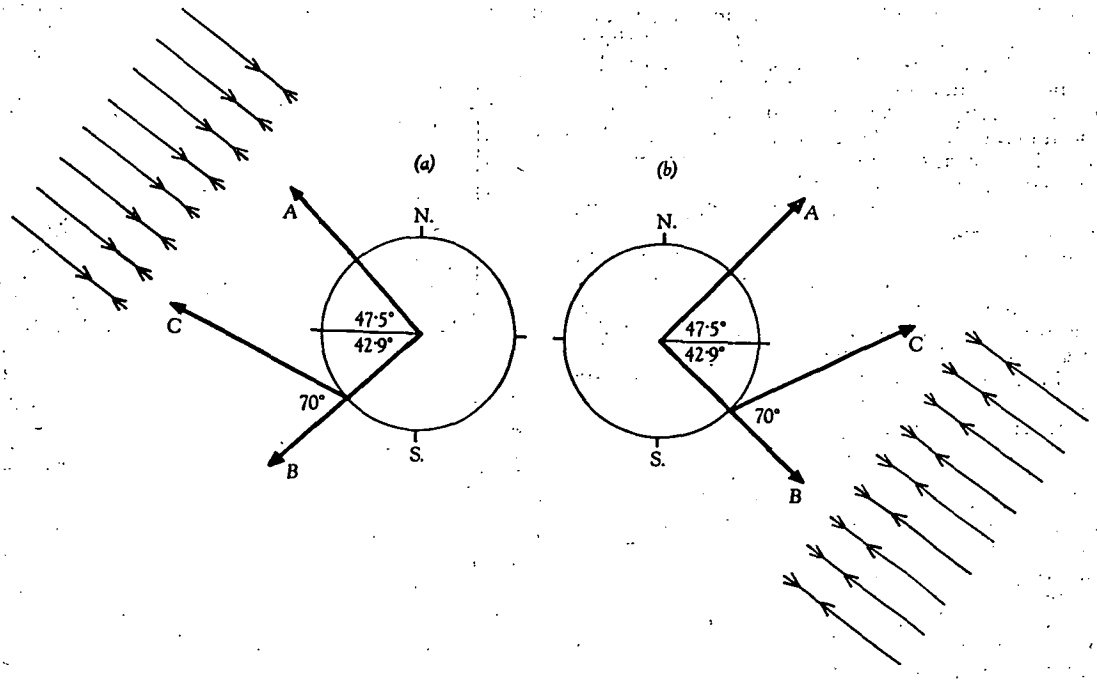

Fig. 4. Schematic diagrams showing the geographic headings of the vertical telescopes underground at Budapest (*A*) and the vertical (*B*) and inclined (*C*) telescopes underground at Hobart, (*a*) at the local sidereal time T h, (*b*) at the local sidereal time $T+12$ h. The parallel lines indicate primary intensity maxima both backwards and forwards along some fixed direction, having declination d, which passes locally overhead at the latitude d in the northern hemisphere at time T and at the same latitude in the southern hemisphere at time $T+12$

in both the backward and forward directions and that these are being detected as intensity maxima at the Earth, then the declination of the direction of the anisotropy should be that of the galactic field in the vicinity of the solar system. An estimate of the declination from cosmic ray intensity measurements will be presented elsewhere. It is based on a simple empirical model for the anisotropy and makes use of observations obtained in other directions in the plane of the meridian at the Hobart underground station.

It should be pointed out that in the model both the first and second harmonics of the sidereal daily variation have physical meaning. It is clear, for example, that at some latitude of viewing between Hobart and Budapest only a semi-diurnal effect should be observed, and that if the primary intensity maxima in opposite directions were equal, the first harmonic would disappear when the asymptotic cone of viewing was centred on the equator.

Finally, it is emphasized that although the first harmonic now being observed underground at Hobart is much larger than at Budapest (Fig. 1), this appears to be only temporary. Long-term trends of the apparent sidereal effect in the ion chamber data from Cheltenham (38·7° N. geographic) and Christchurch (43·5° S. geographic) indicate that the amplitudes were much larger in the northern than in the southern hemisphere during the early 1950's. This could be the main reason why a phase anomaly was detected only in the northern hemisphere in 1954 and only in the southern hemisphere in 1961. As to the changes in the relative amplitudes with time, the following suggestion is offered for what it is worth: when a sidereal effect is viewed through the region of interplanetary solar modulation, asymmetries in the modulating region with respect to the plane of the ecliptic might be responsible for different amplitudes in the northern and southern hemispheres. The amplitude differences would change from time to time as the asymmetry itself changed.

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SYMMETRY OF THE TIME AXIS

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Interactions are governed by laws of physics that may be conveniently divided into two classes. Laws of the first type, such as those of energy and momentum conservation, must be rigorously obeyed in each interaction. These laws show no preferred direction of time, for reversal of the sign of t wherever it appears in the equations does not alter their validity. In the second class we have laws of a statistical character which dictate the probable outcome of many interactions considered as a whole. In this class we include the second law of thermodynamics and statements concerning the cross-sections for reactions. It is these laws which, while not distinguishing between any direction in three-dimensional space, nevertheless have an asymmetric behaviour with respect to the time axis. Thus, for example, the cross-section for a forward reaction in general differs from that of the reversed process. This is not because of any difference in the strengths of the two interactions, but arises as a result of the density of final states factors. Similarly, entropy is well known to vary in a systematic fashion with time.

Why the otherwise perfect symmetry of the space-time framework should be flawed by a restricted class of laws and in one dimension only has long been a subject of discussion. In this article I shall try a somewhat different approach to the problem. The question asked is: "If the time axis were perfectly symmetric in respect of *all* the laws of Nature, how would the world appear to us?"

The effect of imposing such a symmetry is best illustrated by taking a particular interaction as an example. Let us therefore consider the coupling between the neutron state and the $(p+e^-+\bar{\nu})$ state. The world lines of the particles involved in any such interaction may adopt the general configuration of Fig. 1 or Fig. 2. If the two time directions are indistinguishable there is no reason why either configuration should be favoured. Nevertheless, our observations do indicate a preference for one particular pattern of world lines, say those of Fig. 1, and these we refer to as representing the decay of a neutron, while Fig. 2 shows the creation of a neutron from a triple collision between a proton, electron and anti-neutrino. Once given such an asymmetric pattern of world lines, a direction in time can be defined, and laws formulated which describe the asymmetry in terms of the density of final states factors. Discussions on the symmetry properties of time generally do not seek to deny the existence of such laws and asymmetries, but centre rather on the nature of the contingency that allowed our world to choose between two equally possible sets of world line configurations.

The line of thought we are pursuing here is that no arbitrary choice has been made: both patterns of world lines are equally allowed at all times. This means that in addition to the neutron states with which we are familiar, there exist others such that the transitions from them to the $(p+e^-+\bar{\nu})$ states occur with the greater probability in the opposite direction of time, as in Fig. 2. So that we may distinguish between the two particle states, we introduce the concept of intrinsic time sense. Thus all particles besides carrying basic properties, such as mass, charge, isotopic spin, baryon number, and intrinsic parity, are also to be further sub-divided according to whether they have a positive or negative intrinsic time sense. So far as we know, all matter of which we have experience belongs only to one time sense category. Matter of the opposite time sense we shall designate as "faustian". Interactions between particles in faustian states would be identical in every respect to interactions between our own particles, except that their time asymmetry would be reversed.

Perhaps it should be emphasized that particles of a reversed time sense are not to be confused with particles moving backwards in time. This latter phrase is custom-