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DEPARTMENT OF PHYSICS

THE DESIGN PROBLEMS OF HIGH
APERTURE TRIPLET OBJECTIVES

by

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The Design Problems of High Aperture Triplet Objectives.

ABSTRACT

An investigation has been made of the roots of a certain cubic equation $f(y_{ob}) = 0$, which arises in the theory of the type-111 triplet photographic objective. It has been shown that with the residuals and the parameter values such as might be used in the type 111 triplet, this equation gives three positive roots of which only one leads to a practical solution.

It was shown that if certain parameters which enter into the coefficients of y_{ob} in this cubic equation are given values much greater than is usual in a type 111 objective, a second root of the equation leads to a practical solution. In this way, a new region of triplet solution has been opened up characterised by low powers for the components in the initial thin lens arrangement. It was expected that this region would provide a basis for the development of high aperture objectives.

The general physical principles underlying the achievement of these high values of initial parameters has involved a careful study of the properties of thick meniscus shaped cemented triplet components of negative power.

A procedure for the design of a type 131 objective, which is the simplest form of objective incorporating these principles, has been developed and is described with numerical examples. A study of more complex objectives is needed to exploit the principles which have been opened up in this work. The time available for the investigation has not permitted the study of type 133 and other objectives from this point of view.

DECLARATION STATEMENT

I, K. Edara, hereby declare that, except as stated therein, the Thesis contains no material which has been accepted for the award of any other degree or diploma in any University, and that, to the best of my knowledge and belief, the Thesis contains no copy or paraphrase of material previously published or written by another person, except when due reference is made in the text of the Thesis.

Krishna Mohan Edara

K. EDARA

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It was shown that if certain parameters which enter into the coefficients of y_{ob} in this cubic equation are given values much greater than is usual in a type-111 objective, a second root of the equation leads to a practical solution. In this way, a new region of triplet solution has been opened up characterised by low powers for the components in the initial thin lens arrangement. It was expected that this region would provide a basis for the development of high aperture objectives.

The general physical principles underlying the achievement of these high values of initial parameters has involved a careful study of the properties of thick meniscus shaped cemented triplet components of negative power.

A procedure for the design of a type 131 objective, which is the simplest form of objective incorporating these principles, has been developed and is described with numerical examples. A study of more complex objectives is needed to exploit the principles which have been opened up in this work. The time available for the investigation has not permitted the study of type 133 and other objectives from this point of view.

CHAPTER I.

INTRODUCTION

1.1 REVIEW of the LITERATURE.

Taylor and Lee (1935) discussed the development of the photographic objectives during the previous century and showed how the discovery of the effect of the position of the diaphragm on the third order aberrations directed this development. They expressed the opinion that all high aperture photographic objectives were derived from the original simple Cooke triplet or quadruplet, either by replacing the single component by cemented components, or by splitting a particular component. Lee (1940) explained the necessity for high aperture objectives for studio work and cinematography. He gave a brief history of high aperture anastigmatic objectives including a list of their designers. Kingslake (1940), amongst others, discussed the reduction of the residual zonal aberrations in a lens system, this being a necessary requirement to attain high apertures. He considered the conditions of (i) zero refraction, i.e. $i_{0j} - i_{0j}' = 0$, (ii) normal incidence, i.e. $i_{0j} = 0$, and (iii) aplanatic refraction, i.e. $i_{0j}' - v_{0j} = 0$, which provided the means for reduced values of the third order spherical aberration, coma and astigmatism, and indicated that those principles must enter in some way into the development of high aperture systems. If Kingslake's discussion is generalised it would lead to the statement that to achieve a high relative aperture, the surface contributions to the third order aberration coefficients must be considerably lower than is usual in lens systems of moderate aperture. He also pointed out that the most difficult problem involved was to control the aberrations below the accepted level due to the fact that the tolerances required became more difficult to satisfy as the aperture was increased.

In a very detailed article Merte (1943) reviewed the development of high aperture objectives up to 1943. According to this reviewer the earliest successful lenses belonged to the "Ernostar" series (Fig. 1.1), designed by Bertele, but these were soon superseded by the "Sonnar" lenses developed by the same designer (Fig. 1.2). Excellent objectives of this type were produced having relative apertures of $f/1.5 - f/2$ in focal lengths of 50 - 100 mm. and covering semifields up to 25 degrees. In addition to these, lenses of the "Double Gauss" type (Fig. 1.3) had been extensively developed, resulting in objectives of similar quality.

Kingslake (1944), (Fig. 1.4), explained the importance of glass selection in high aperture anastigmats making special reference to the Ektar $f/1.5$ lens designed by Schade. He stressed the importance of using high refractive indices for the positive elements, to form the collective surfaces in the cemented components. This tended to smooth out the zonal residuals of axial and oblique spherical aberration. The same author (1949), commented on the difficulty of designing a high aperture system, especially when it had to cover a large angular field with a short focal length.

Kaprelian (1949) explained (Fig. 1.5) the necessity of avoiding strongly curved surfaces (other than those which are aplanatic or zero refracting). At such surfaces the angle of refraction, i'_0 , increases rapidly with the height of the ray resulting in the development of very high values of higher order aberrations. He also described in a condensed form how to control aberrations with the help of glass constants and the stop position. He pointed out that commercially produced lenses were more or less limited to the aperture range $f/1.5 - f/2$ because of the practical difficulties of design and production. Apertures greater than $f/1.5$ were only designed for special applications.

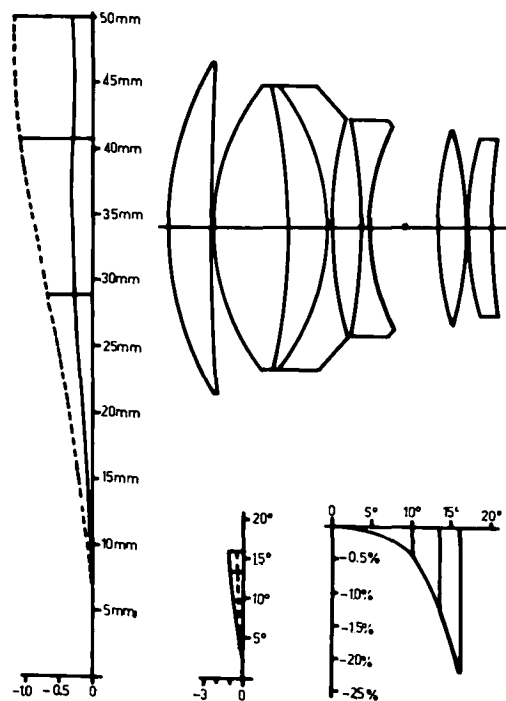


FIG. 1.1 GERMAN PATENT NO. 441 594/1925

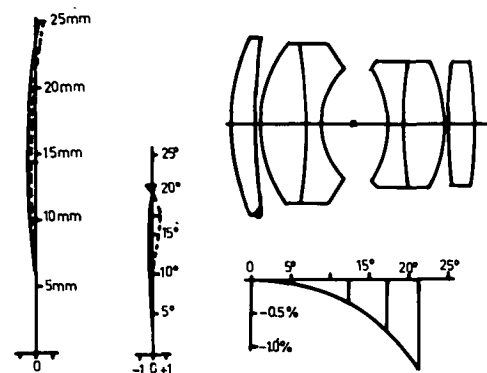


FIG. 1.3 BRITISH PATENT NO. 685 572/1936

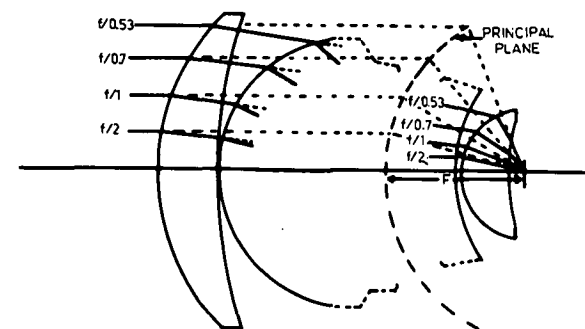


FIG. 15 REFRACTION AT LARGE APERTURES

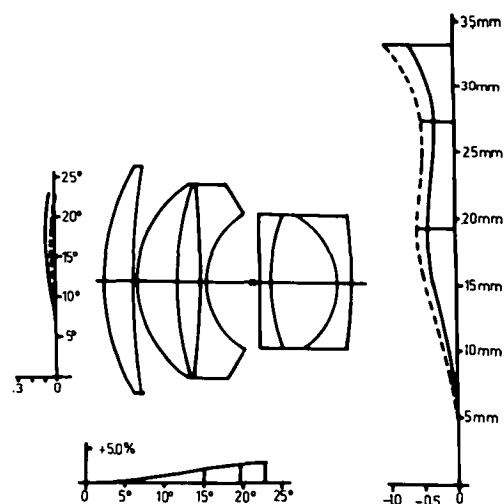


FIG. 1.2 FRENCH PATENT NO. 837 616/1938

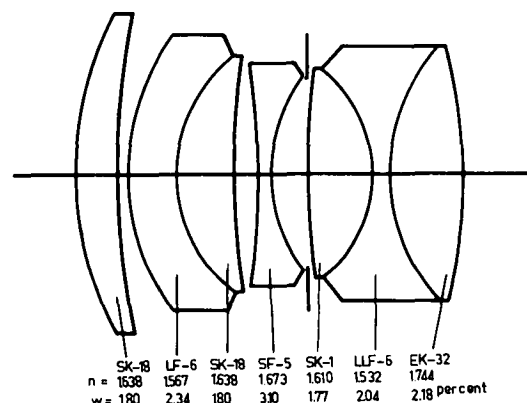


FIG. 1.4 KODAK "EKTAR" f/15

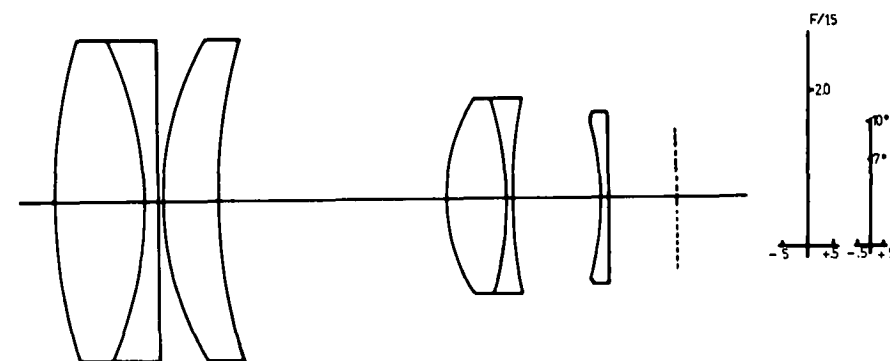


FIG. 16 KODAK PROJECTION EKTAR f/15

Schade (1950) described the modification of the petzval portrait lens (Fig. 1.6) in order to increase the aperture to get better resolving power, contrast, and back focal distance.

This review shows that there are very few papers which explain the physical principles underlying the development of high aperture photographic objectives and none which describes logical methods for their design. The patent literature, of course, contains examples of the construction of such systems, but here again, the physical principles leading to any particular construction are not disclosed.

1.2 THE DESIGN of TRIplet OBJECTIVES.

The purpose of this thesis is to investigate logical methods of developing the design of high aperture systems from the Cooke triplet. In this connection a brief resume of papers on the design of triplets is given.

Taylor (1893) arrived at the triplet construction by considering what happened when the components of an achromatic doublet were separated. He noticed that this separation increased the power of the system without increasing the petzval sum. To control the other aberrations he split the positive element into two parts and placed one on either side of the negative element. He obtained an initial thin lens arrangement for the three powers and two separations by solving five equations based on assumed residual values of spherical aberration, petzval sum, longitudinal and transverse colour and the total power of the system. He then constructed the system and measured the aberrations. Using these measured aberrations he changed the original residuals to obtain the final solution. Thus in view of this assessment the description "optical designing - an art" was justified in the nineteenth century.

After Taylor, Schwarzschild (1905), Kerber (1916), Berek (1930) and Conrady (1960) described methods for the design of triplet objectives, but in all these methods it was necessary to make repeated trials for the determination of some parameter of the system.

Stephens (1948) developed the thin lens analysis of the triplet taking into account both near and infinitely distant object planes. He obtained a solution for prescribed values of the petzval sum, total power, longitudinal colour, transverse colour and the height h_3 , the intersection height of the axial ray at the third lens. He maintained the prescribed values of the Seidel aberrations after thickening the system, and analysed the final design by trigonometrical ray traces. His method, however failed to deal with the control of spherical aberration and provided no guide to the important matter of the selection of glass for the lenses of the system. He believed that the other triplet types, Heliar, Pentac etc. could be generated from the same procedures which he had used for the triplets.

Cruickshank (1956, 58, 60, 68) explained that all triplets with cemented components could be generated from the simple Cooke triplet, or as he called it type 111 triplet or basic triplet (Fig. 1.7). Thus in view of the fundamental importance of the simple triplet, he developed the complete design theory, and examined its properties in a detailed and systematic manner, including glass selection. His method enables any newcomer in this field to design photographic objectives based on the triplet family, methodically. According to him "The triplet objective may be regarded logically as derived from the Wollaston lens by the addition of a compound correcting system comprised of a positive and a negative lens placed in front of the diaphragm". The main function of this

TYPICAL TRIPLET CONSTRUCTIONS

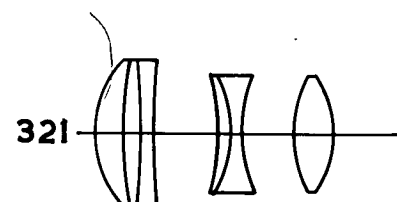
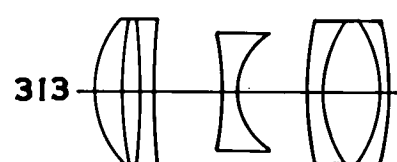
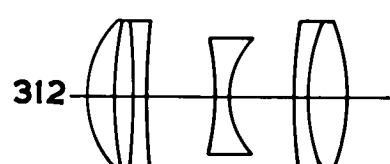
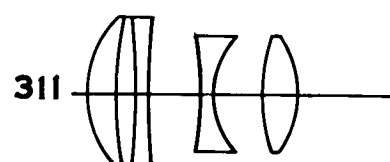
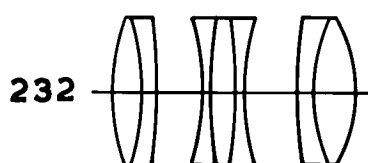
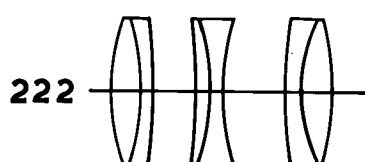
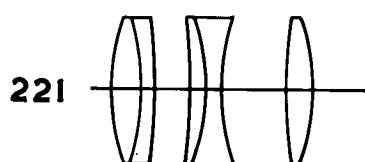
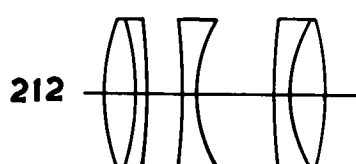
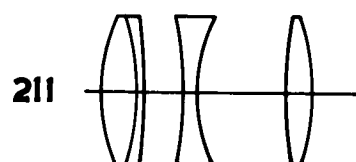
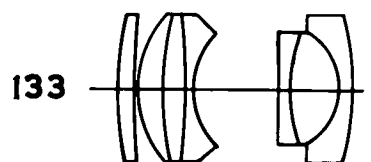
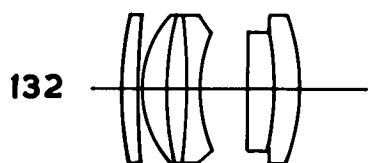
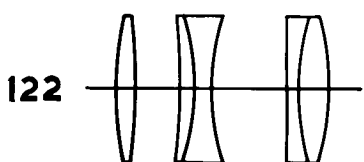
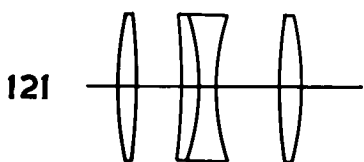
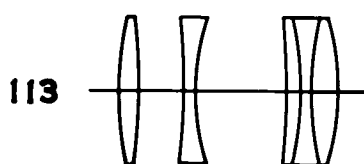
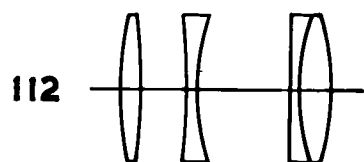
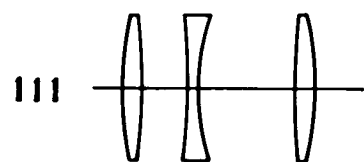


Fig 1.7.

corrector was to introduce aberrations. Thus he replaced the h_s parameter of Stephens by the new parameter χ , the power of the corrector. He also showed (1968) that, for a given set of glasses and prescribed residuals, the spherical aberration, spherical to seventh order could be approximated by a quadratic function of χ , thus:

$$\text{spherical aberration} = B_3\chi^2 + B_2\chi + B_1 + O(9)$$

where B_3 , B_2 and B_1 are constants. The values of χ for which the lens system has a residual spherical aberration up to seventh order of amount R_1 are the roots of the equation

$$B_3\chi^2 + B_2\chi + B_1 - R_1 = 0 \quad (1.1)$$

Cruickshank's initial solution for the three powers and two airspaces was based on prescribed values of χ , petzval sum, total power, longitudinal and transverse colour. He gave the complete design procedure for an infinitely distant object plane and then introduces modifications for finite conjugates.

In 1960 he discussed in detail the general physical principles of the generation of triplets with cemented components. He illustrated this with an example of a Pentac (type 212) objective developed from a typical set of aberration residuals and fictitious glasses. He included graphs showing the variation of the fifth order coefficients of the Pentac system with χ , which indicated very clearly the optimum value of χ . This example illustrated the use of the fictitious glasses in obtaining the initial solution of a triplet system with cemented components and also the use of the Buchdahl (1954) aberration coefficients as a measure of the correction state of the design. It showed clearly the advantage of being able to see the trend of the design using fifth order coefficients.

Cruickshank and Hills (1960) have shown how the aberration coefficients developed by Buchdahl (1954) may be used in the development of the design of an optical system. In this connection the essential feature of these coefficients is that irrespective of order they are obtained by a summation over all the surfaces of the system.

In his 1958 paper Cruickshank developed a cubic equation for the determination of the initial arrangement. He noted that "In general only one root of this equation gives a physically useful solution. There is an important exception, however, in one case in which a second root leads to the possibility of the construction of another group of objectives of high aperture."

In this thesis an account is given of an attempt to investigate this other group of objectives in detail. It was hoped that this would provide a logical starting point for the development of high aperture systems.

CHAPTER II.

2.1 THEORY of the BASIC TRIplet.

It has been stated in the previous chapter that any system of the triplet type has an equivalent triplet, or, as we shall call it, a Basic Triplet. In this section we will summarise the important practical case of the initial design of a triplet photographic objective corrected for an object plane at infinity, as given by Cruickshank (1958). This summary will introduce most of the required notation. The three lens components a, b and c have the glass constants (N_a, V_a) , (N_b, V_b) and (N_c, V_c) respectively. Disregarding the axial thicknesses of the lenses, five variables are required to specify the system initially, namely the powers ϕ_a , ϕ_b , ϕ_c and the separations t_1 and t_2 of the three coaxial lenses. The following five conditions may then be fulfilled: (i) The power of the system shall be unity; (ii) The power of the corrector shall be χ ; (iii) The system shall have a petzval curvature coefficient, $\sigma_4 = R_4$, for the petzval sum; And for the object plane at infinity the system shall have (iv) A residual longitudinal paraxial chromatic aberration, R_c , and (v) A residual transverse chromatic aberration R_t , for an incident pencil of obliquity, v_a , the diaphragm being coincident with the central thin lens b. Using well known relations for systems of separated thin lenses in air these five conditions may be formulated analytically as follows:

$$1/y_{0a} \sum_{j=a}^c \phi_j y_{0j} = 1 \quad (2.1)$$

$$1/y_{0a} \sum_{j=a}^b \phi_j y_{0j} = \chi \quad (2.2)$$

$$1/2 \sum_{j=a}^c \phi_j / N_j = R_4 \quad (2.3)$$

$$1/v_{0c}' \sum_{j=a}^c \phi_j y_{0j}^2 / V_j = R_6 \quad (2.4)$$

$$1/v_{0c}' \sum_{j=a}^c \phi_j y_{0j} y_j = R, \quad (2.5)$$

Where y_j and y_{0j} are the incident heights at the j^{th} component of a principal paraxial ray (b-ray) of obliquity v_a , and an axial (a-ray) paraxial ray respectively, and v_{0j}' is the inclination angle of the axial ray after refraction at the j^{th} component. Since the diaphragm is initially in coincidence with the second thin lens of the system, the incident heights of the principal paraxial ray are such that

$$y_a/y_c = -t_1/t_2; \quad y_{0b} = 0; \quad (2.6)$$

while for the axial ray

$$y_{0b} = y_{0a} - t_1 v_{0a}' = y_{0a}(1 - t_1 \phi_a) \quad (2.7)$$

$$y_{0c} = y_{0b} - t_2 \chi \quad (2.8)$$

A substantial reduction in the symbols specifying the glasses of the system is achieved by writing

$$\begin{aligned} v_a/v_b &= \alpha; & v/v &= \xi; \\ N_a/N_b &= \beta; & N_a/N_c &= \gamma. \end{aligned} \quad (2.9)$$

In addition we can also write

$$2R_6 N_a = P \quad (2.10)$$

$$R_6 V_a = L \quad (2.11)$$

$$R_7 V_a = T \quad (2.12)$$

with this notation equations (2.1) - (2.5) now become

$$\varphi_a + y_{ob}\varphi_b + y_{oc}\varphi_c = 1 \quad (2.13)$$

$$\varphi_a + y_{ob}\varphi_b = \chi \quad (2.14)$$

$$\varphi_a + \beta\varphi_b + \gamma\varphi_c = P \quad (2.15)$$

$$\varphi_a + \alpha y_{ob}^2\varphi_b + \xi y_{oc}^2\varphi_c = L \quad (2.16)$$

$$(1 + T)t_1\varphi_a - t_2\xi y_{oc}\varphi_c = T \quad (2.17)$$

The solution of equations (2.13) - (2.17) is quite easy. Equations (2.13) and (2.14) give at once

$$y_{oc}\varphi_c = 1 - \chi \quad (2.18)$$

and combining this with equations (2.17), (2.6), (2.7) it follows that

$$t_1 = (1 - y_{ob})/\varphi_a \quad (2.19)$$

$$t_2 = \{1 - (1 + T)y_{ob}\}/\xi(1 - \chi) \quad (2.20)$$

Combining equations (2.8) and (2.11) gives

$$\kappa y_{ob} - \xi(1 - \chi)y_{oc} = \chi \quad (2.21)$$

where

$$\kappa = \xi + \chi(1 + T - \xi) \quad (2.22)$$

Subtracting (2.14) from (2.16) yields

$$\varphi_b y_{ob}(\alpha y_{ob} - 1) + \xi(1 - \chi)y_{oc} = L - \chi \quad (2.23)$$

which on combination with (2.21) gives

$$\varphi_b = (\kappa y_{ob} - L)/(1 - \alpha y_{ob})y_{ob} \quad (2.24)$$

Equations (2.18), (2.8) and (2.20) together give

$$\varphi_c = \xi(1 - \chi)^2 / (\kappa y_{ob} - \chi) \quad (2.25)$$

while equations (2.14) and (2.24) give

$$\begin{aligned} \varphi_a &= \chi - y_{ob}\varphi_b \\ &= \{L + \chi - (\kappa + \alpha\chi)y_{ob}\} / (1 - \alpha y_{ob}) \end{aligned} \quad (2.26)$$

If φ_a , φ_b , φ_c are now eliminated from equation (2.15) by means of equations (2.24) - (2.26), we obtain the cubic equation

$$G_3 y_{ob}^3 + G_2 y_{ob}^2 + G_1 y_{ob} + G_0 = 0 \quad (2.27)$$

where

$$G_3 = \kappa^2 + \alpha\kappa(\chi - P) \quad (2.28)$$

$$G_2 = \kappa(P - L - 2\chi - \beta\kappa) + \alpha\{\gamma\xi(1 - \chi)^2 - \chi(\chi - P)\} \quad (2.29)$$

$$G_1 = \beta\kappa(\chi + L) - \{\gamma\xi(1 - \chi)^2 - \chi(\chi - P)\} + L\chi \quad (2.30)$$

$$G_0 = -L\beta\chi \quad (2.31)$$

This cubic equation will have three roots, so that three initial arrangements may be possible. The coefficients can be computed for the given residuals and the roots can be obtained by successive approximation.

From the initial solution, three shapes S_a , S_b , S_c corresponding to powers φ_a , φ_b , φ_c are available to control three characteristics of the system. The analytical theory (Cruickshank, 1968) shows that one can calculate three thin lens shapes S_a , S_b , S_c of the basic triplet to achieve the specified third order aberration residuals R_2 , R_3 , R_5 for coma, astigmatism or flat tangential field condition, and distortion.

It will be seen in general that the third order aberration coefficients in the thickened system σ_2 , σ_3 or $\sigma_6 = 3\sigma_3 + \sigma_4$, will differ from the corresponding specified residuals R_2 , R_3 , R_5 of the thin system. A stage of differential correction is therefore required. In the light of this fact, it is just as effective to introduce arbitrary, but reasonable, shapes in the thin lens system, carry out the standard thickening procedure and use the differential correction method to adjust shapes in order to achieve the prescribed residuals R_2 , R_3 , R_5 in the thickened system.

This method is quite convenient with the above assumed stop position because the shape change at any lens affects only the contributions of that lens to the third order aberration coefficients.

It is essential to prescribe some residuals R_6 and R_7 to minimise the longitudinal chromatic aberration near the 0.7 - 0.8 zone of the full aperture and to correct the transverse chromatic aberration at some field angle using the wavelengths for which the system is to be achromatised. It will be seen in general that the paraxial chromatic aberration residuals R_6 , R_7 of the thin system differ from the corresponding traced values l_{ch} , t_{ch} of the thick system. To achieve the prescribed residuals in the thick system the same as in the thin system, the two parameters L and T are adjusted with differential correction method by forming the derivatives $\partial l_{ch}/\partial L$ and $\partial t_{ch}/\partial T$.

Finally, we adjust the spherical aberration to seventh order for the zone of radius ρ to such a value that the total spherical aberration of the system is suitably corrected. The parameter χ is used for this purpose.

The main advantage of this basic triplet theory is that the above procedure can be applied to design any triplet of the basic triplet family shown in Figure 17 of Chapter I.

2.2 A STUDY of the ROOTS of the CUBIC EQUATION.

The equation (2.27) is of the form $f(y_{0b}) = 0$, where $f(y_{0b})$ is a cubic polynomial. The coefficients of the polynomial are complicated functions (see equations 2.6 to 2.31) of the glass constants and the residuals of the system. The analysis of the dependence of the roots of this equation on these quantities is therefore a complex problem. It is fairly obvious that certain of these quantities have a greater range of variation than others. Among these are α , P and ξ . The ratio $V_a/V_b = \alpha$ can be varied quite widely. A doublet as the first component may have an effective V-value (see section 3.1) anywhere from 40 to 300 or more quite easily, while a negative doublet or triplet for the central component may have an effective V-number as low as 10. It is clear, then, that α can be varied continuously in practical systems. The ratio $V_a/V_c = \xi$ can also be varied continuously in the same way by replacing the last component by a cemented doublet or triplet, but the range of variation is more limited. The ranges of variation for β and γ , ratios of refractive indices, are much more limited still. We shall find that the parameter P can be increased considerably beyond the ordinary value (about 0.6), which it has in normal type 111 triplets. It should be remembered that $P = 2R_4N_a$, where R_4 is the third order petzval curvature coefficient in the thin lens system. The replacement of thin lenses by thick ones may often result in a reduction of the petzval sum, especially if meniscus and thick cemented components are used. It is therefore not unpractical to consider values of P up to about twice the value which is usual in the type 111 triplets, provided the thick lens replacements are designed with a reduction of petzval sum in mind. This point is considered further in Chapter III.

In Fig. (2.1), curve (1) is the graph of the polynomial $f(y_{0b})$, and its intersections with the y_{0b} axis give the three roots of the equation $f(y_{0b}) = 0$. The curve has been drawn for values of the residuals such as might be used in the design of a type 111 triplet. In particular, P has the value 0.60. Of the three positive roots, it is clear that only the second root $y_{0b} = 0.84$ provides the basis for a practical system. The first root $y_{0b} = 0.045$ would lead to undesirably high powers for the components; while according to equations (2.19), (2.20), the third root, y_{0b} greater than unity, would give negative values for both air spaces.

Curves (2), (3) and (4) in Figure (2.1) are similar graphs of $f(y_{0b})$ corresponding to the values $P = 0.8, 1.0, 1.2$ respectively. An interesting feature is that an increase in P results in the reduction of the value of the third root of the equation and raises the question as to whether this third root can be less than unity. Curve (4) shows that increasing P to the value 1.20 does not produce the desired result, for in this case the curve falls below the y_{0b} axis. The cubic equation then has only one real positive root $y_{0b} = 0.02$, which is not of practical use. A third root of the equation less than unity cannot be obtained simply by increasing P for this set of residuals. Figures (2.2) and (2.3), which are drawn for the cases in which the power χ of the corrector system has values of -0.2 and $+0.2$ respectively, show that the variation of this parameter with these residuals and glass constants does not alter the situation significantly.

We consider next the effect of the variation of the glass parameter α in the range 1.4 to 5.0, say. This could mean, for example, that we are allowing the effective V-number of the glass of the central component to be reduced

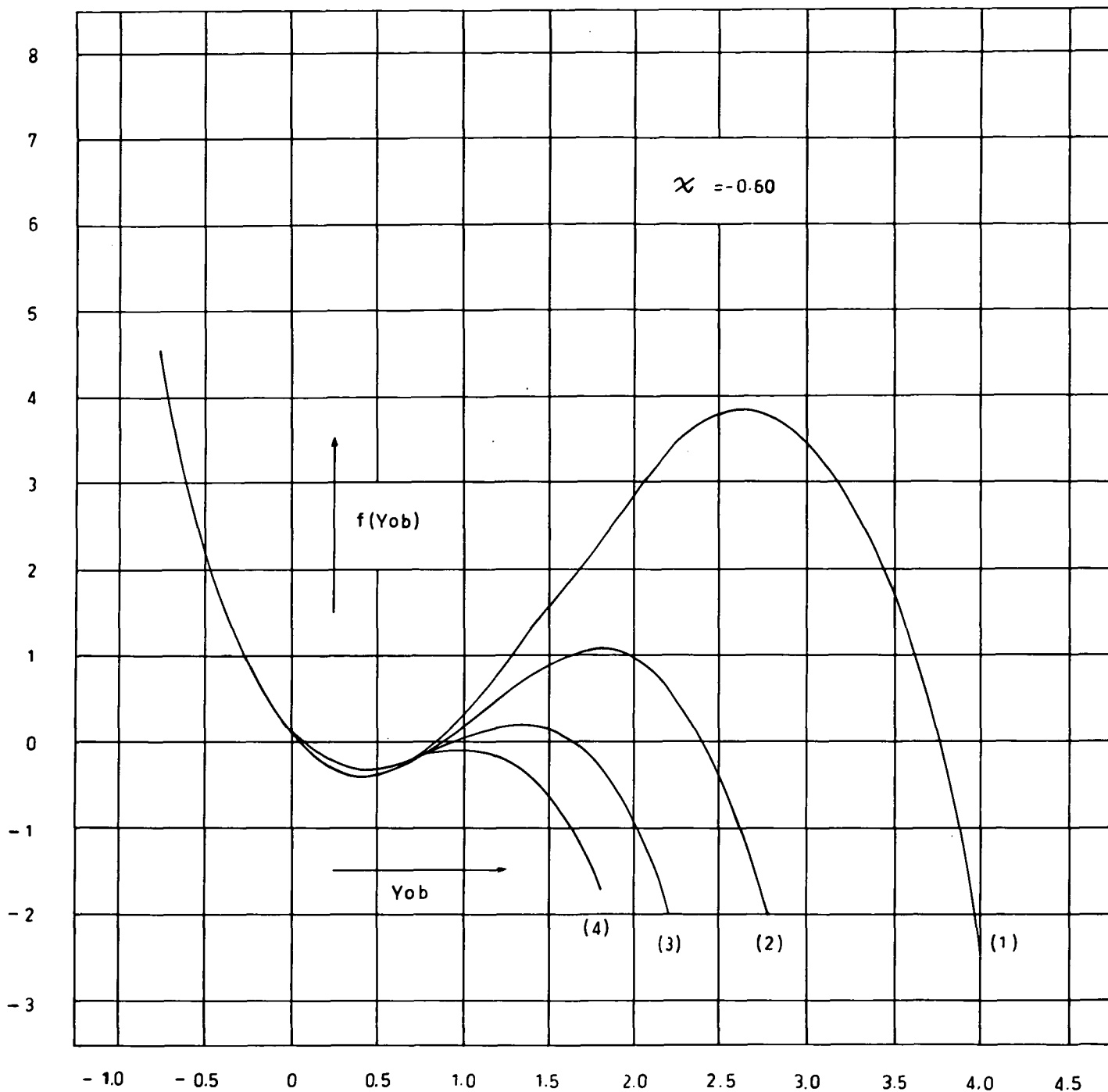


FIG (2.1)

The effect of the variation of P on f(yob) Vs yob curves in type 111 triplets

GLASSES:

SK 6	1.6163	56.11
BASF10	1.6541	38.87
SK 6	1.6163	56.11

GLASS CONSTANTS:

$\alpha = 1.4435$, $\beta = 0.9772$, $\xi = 1.0$, $\tau = 1.0$.

RESIDUALS:

$L = 0.1$, $T = 0.0$.

(1) $P = 0.6$, (2) $P = 0.8$, (3) $P = 1.0$, (4) $P = 1.2$.

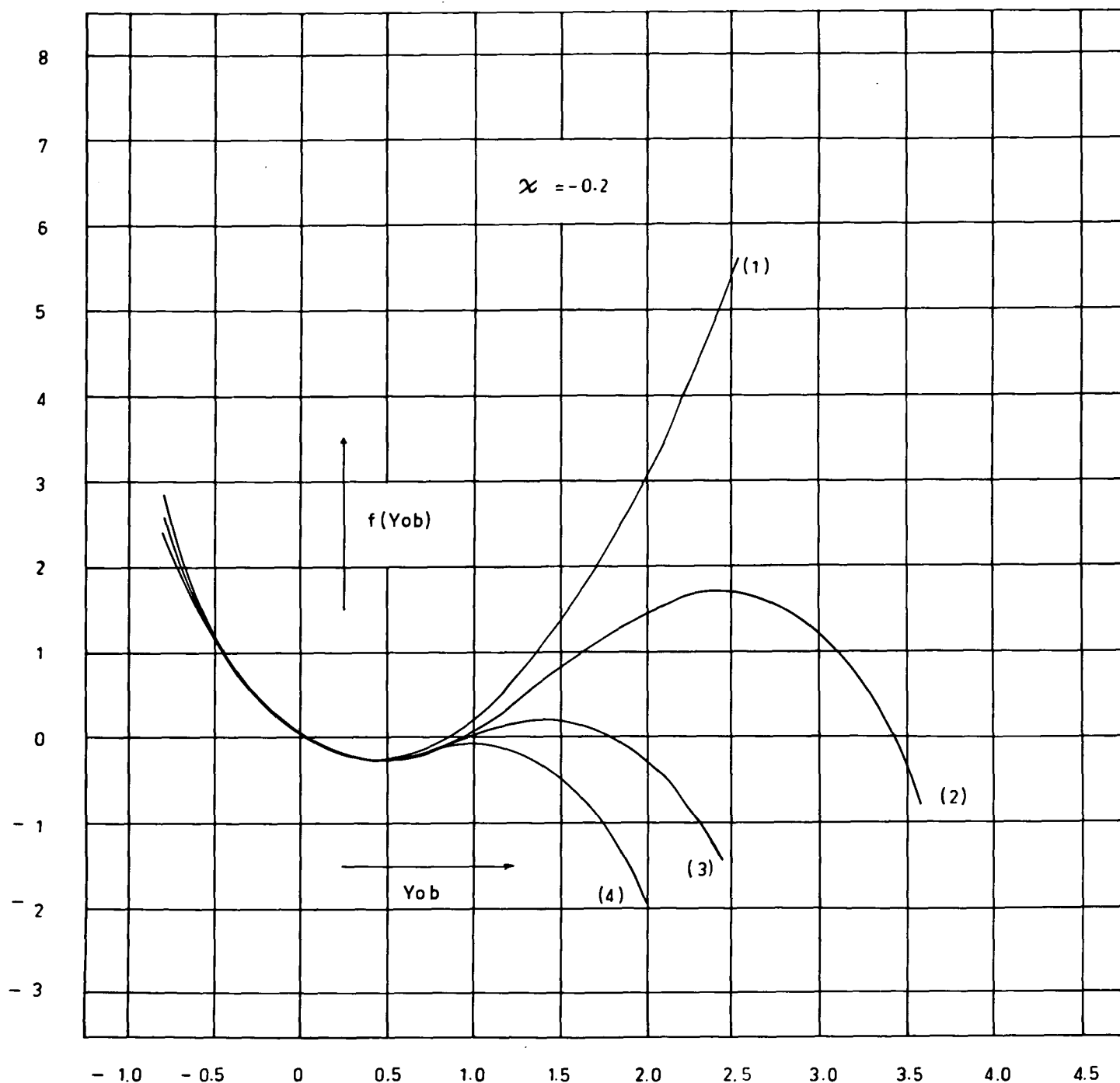


FIG (2.2)

The effect of the variation of P on f(yob) Vs yob curves in type 111 triplets

GLASSES:

SK6	1.6163	56.11
BASF10	1.6541	38.87
SK6	1.6163	56.11

GLASS CONSTANTS:

$\alpha = 1.4435$, $\beta = 0.9772$, $\xi = 1.0$, $\tau = 1.0$.

RESIDUALS:

L = 0.1, T = 0.0.

(1) P = 0.6, (2) P = 0.8, (3) P = 1.0, (4) P = 1.2.

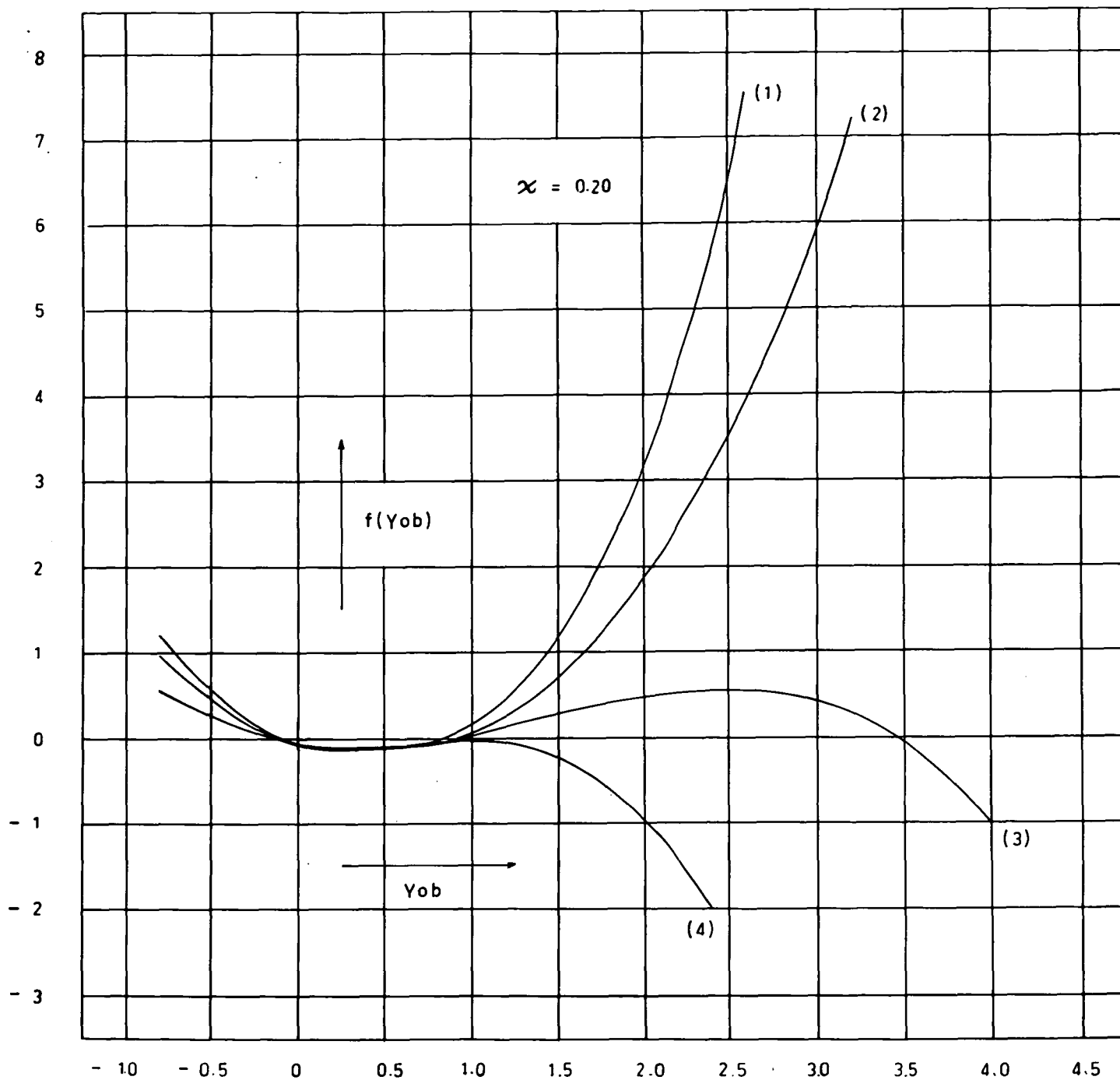


Fig (2.3)

The effect of the variation of P on $f(yob)$ Vs yob curves in type 111 triplets

GLASSES:

SK 6	1.6163	56.11
BASF10	1.6541	38.87
SK 6	1.6163	56.11

GLASS CONSTANTS:

$\alpha = 1.4435$, $\beta = 0.9772$, $\xi = 1.0$, $\tau = 1.0$.

RESIDUALS:

$L = 0.1$, $T = 0.0$.

(1) $P = 0.6$, (2) $P = 0.8$, (3) $P = 1.0$, (4) $P = 1.2$.

progressively. Such a variation could be achieved practically if the single negative component were replaced by a suitable negative doublet or triplet.

In Fig. (2.4(a), curve (1) shows the result of increasing α from its earlier value of 1.44435 to 3.00000, P having the value 1.10. The equation $f(y_{ob}) = 0$ now has three real roots between 0 and 1, and the possibility of a new basis for a practical lens system arises. Curves (2) and (3) show the graphs of $f(y_{ob})$ for the cases in which α has the values 3.5 and 4.0 respectively. Investigation of the initial thin lens solution corresponding to the new third root is not very encouraging, for, as Figure 2.4(b) shows, the power of the first component and the first air space are both negative. These will not provide a practical lens system.

Fig. 2.5(a),(b), show similar curves resulting from changing the value of χ from -0.6 to -0.2. From Figure 2.5(b), it can be seen that the first component now has a small positive power and the first air space is now also positive, but large. This suggests that further investigation of the effect of the increase χ should be made.

With this in mind, χ was increased to +0.20 and the corresponding curves are represented in Fig. 2.6(a),(b). It can be easily observed from Fig. 2.6(b) that the initial solutions have become much more practical and offer promise of a reasonable basis for a lens system.

Because of the important part played by the parameter α , it is more informative to present the results obtained so far in graphs in which α is the independent variable.

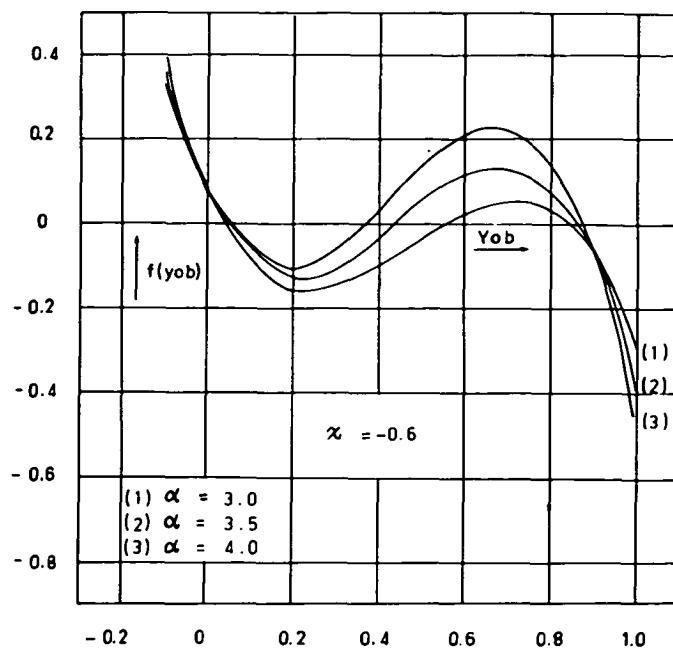


Fig 2.4(a)

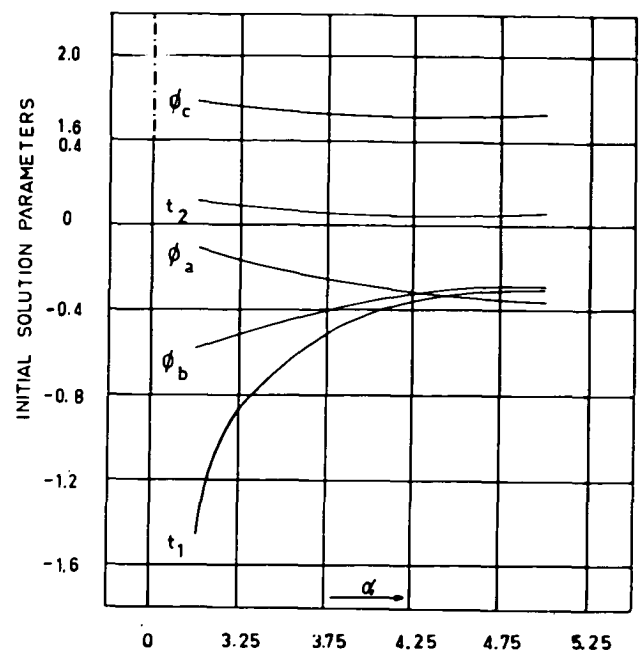


Fig 2.4(b)

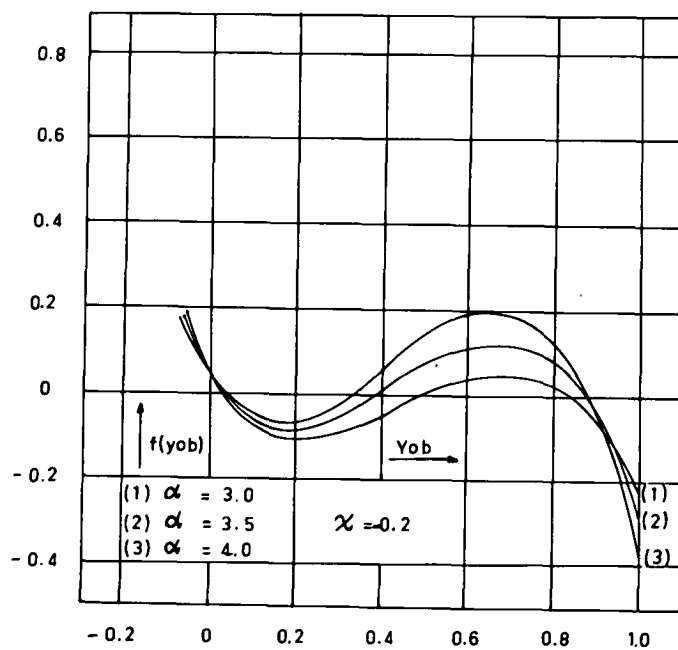


Fig 2.5(a)

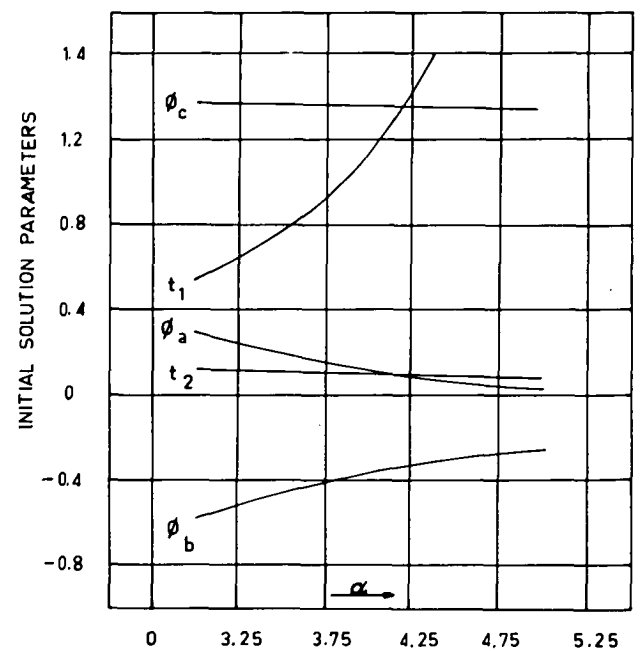


Fig 2.5(b)

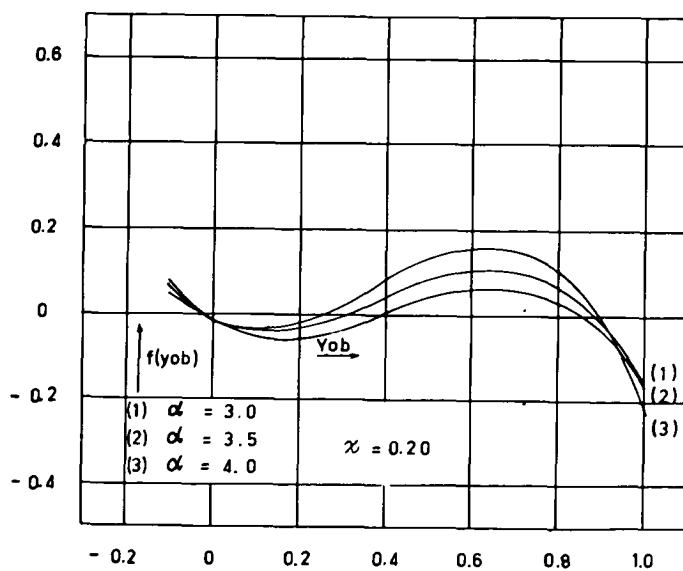


Fig 2.6(a)

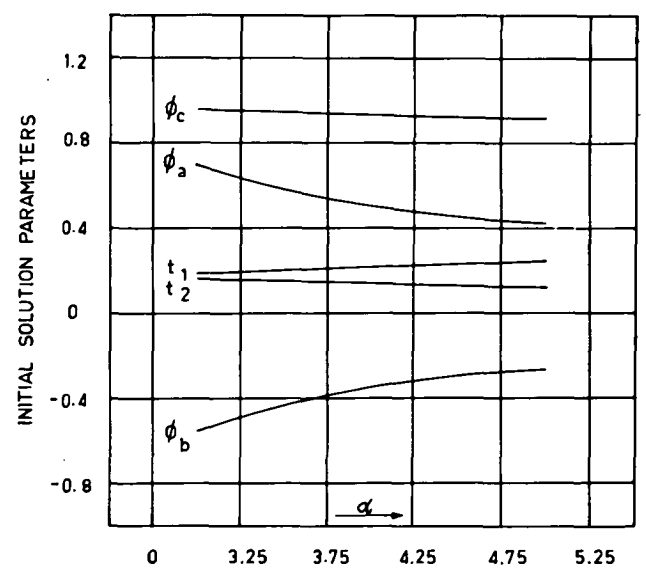


Fig 2.6(b)

Fig. 2.7(a) shows the roots of the cubic equation (2.27) as a function of α , for different values of P . Curves (1) and (1)' are drawn for the case in which $P = 0.6$. These curves show the variation with α of two of the roots. The remaining root which has a value close to zero and has no practical significance is not shown. If a triplet is designed with $P = 0.6$ then the root of the cubic equation on which it is based must lie on the curve (1) because this is the only useful root below unity. Curves (2) and (2)' show the corresponding roots for the case in which $P = 1.0$. It can be observed from curve (2)' that the values of the roots represented in this branch are much reduced compared with the values summarised in curve (1)'. On curve (2)' however, all the values still lie above unity. If P is increased to 1.05 it becomes clear that a change has occurred in the solution of the cubic equation, resulting in the roots lying in the continuous curve (3), provided $\alpha \geq 2.15$. In curve (3) for $\alpha \geq 2.15$ we obtain two positive roots for each value of α and these both have values below unity. The smaller of these two roots belongs to the general conditions represented in curves (1) and (2), on which the design of the traditional triplet is based, while the larger must correspond to a set of new conditions which may lead to another class of triplet solutions. The effect of increasing P beyond 1.05 can be seen in curves (4), (5) and (6), which are drawn for the P values 1.1, 1.15 and 1.2 respectively. The new conditions represented in the upper portions of these continuous curves will be the main subject of further investigation.

From the values of y_{0b} on the continuous curves (3), (4) and (5) are computed the powers ϕ_a , ϕ_b , ϕ_c and the separations t_1 , t_2 of the basic thin triplet, using the appropriate equations from (2.19) to (2.26). The results of these computations are plotted in figures 2.7(b) to 2.7(e). The interesting feature of Fig. 2.7(b) and 2.7(c) is the great reduction in the powers of the

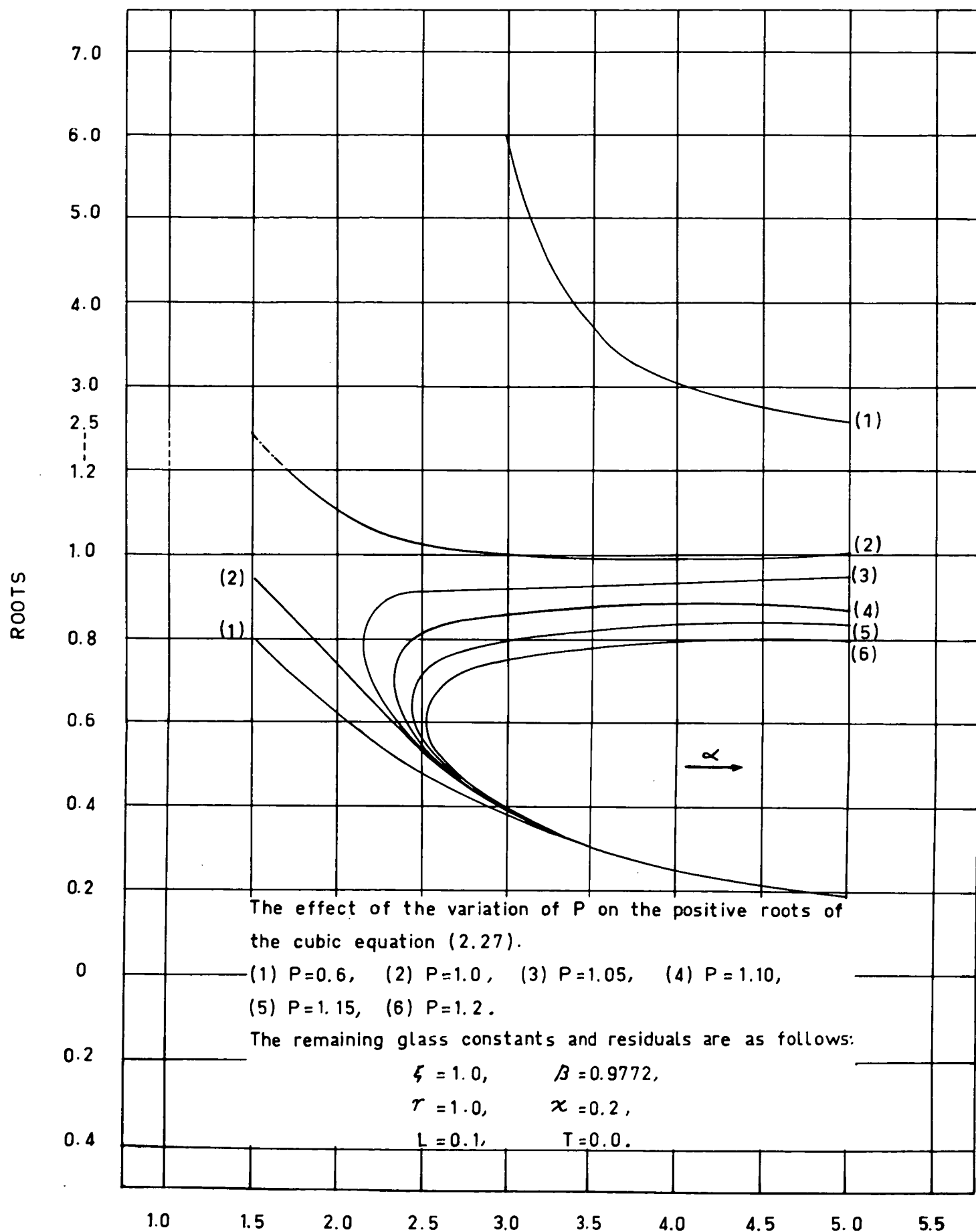


Fig 2.7(a).

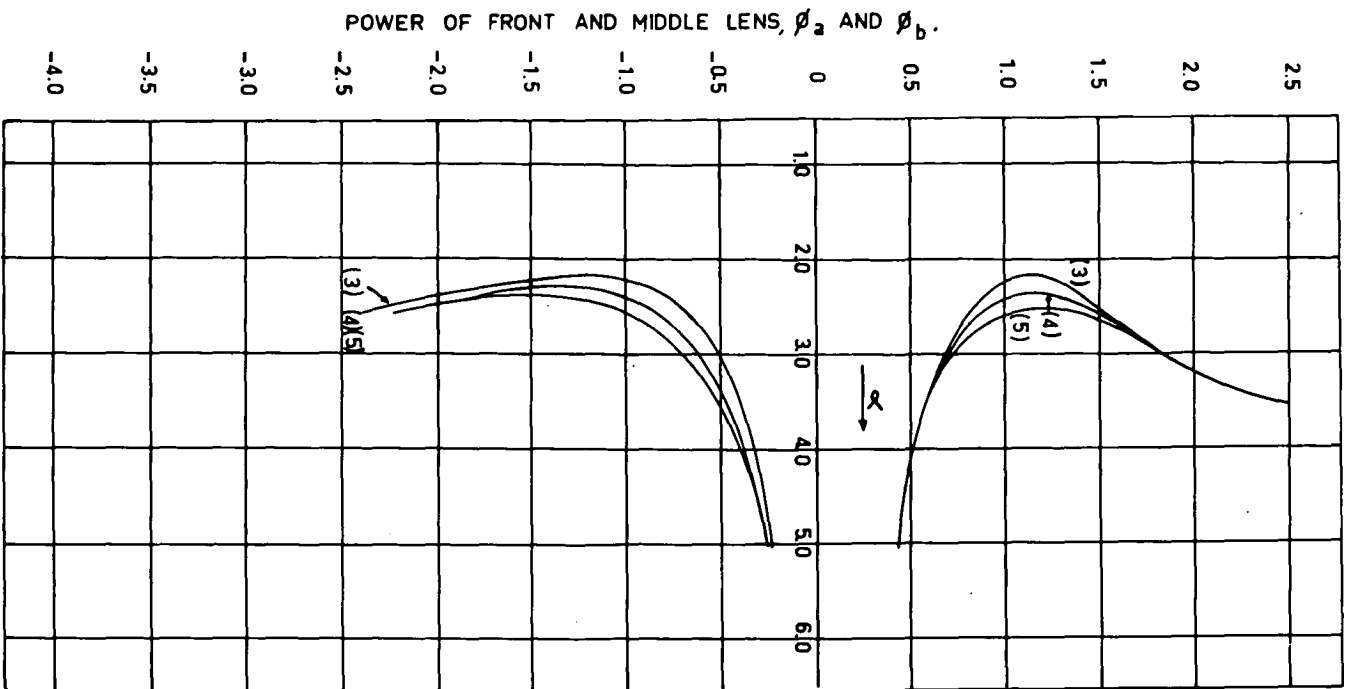


Fig 2.7(b)

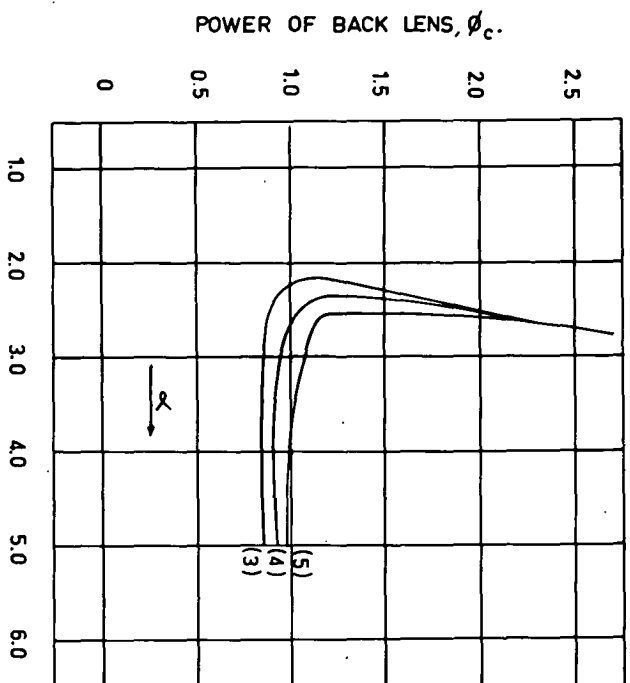


Fig 2.7(c)

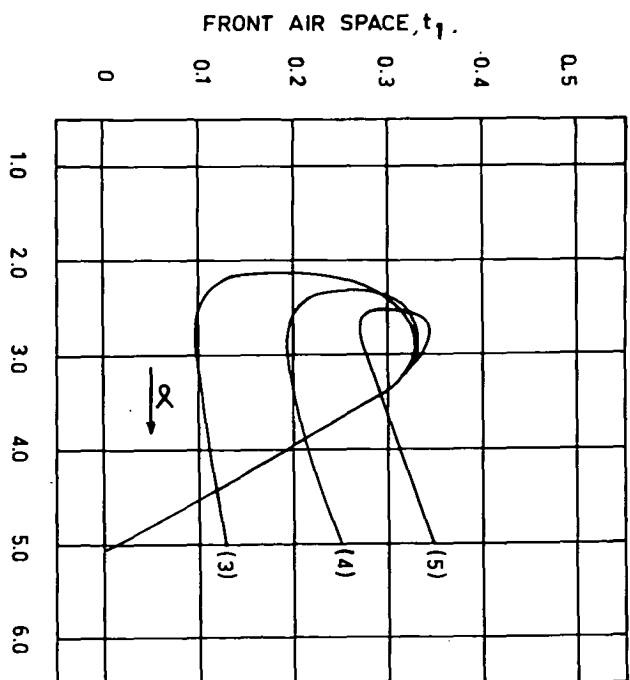


Fig 2.7(d)

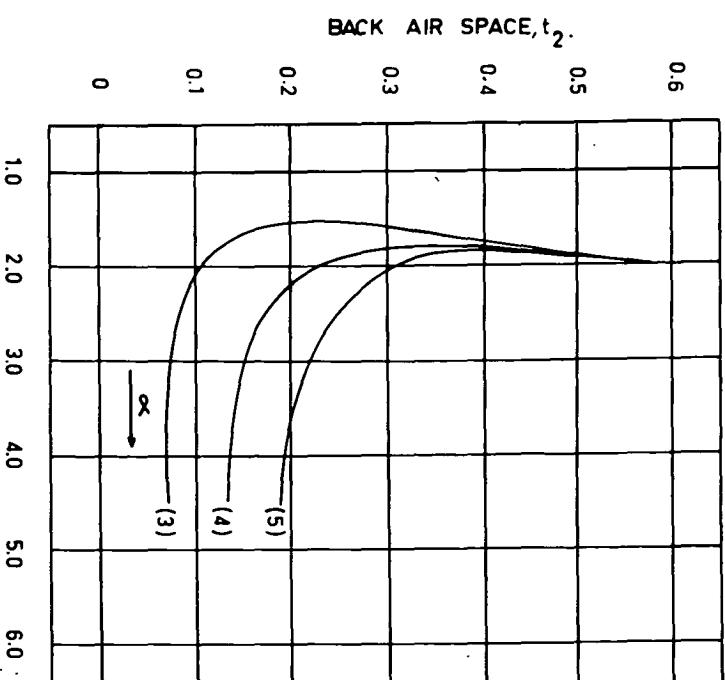


Fig 2.7(e)

The variation of Initial Solution parameters correspond to the value of the roots on curves (3), (4), and (5) in fig (2.7a).

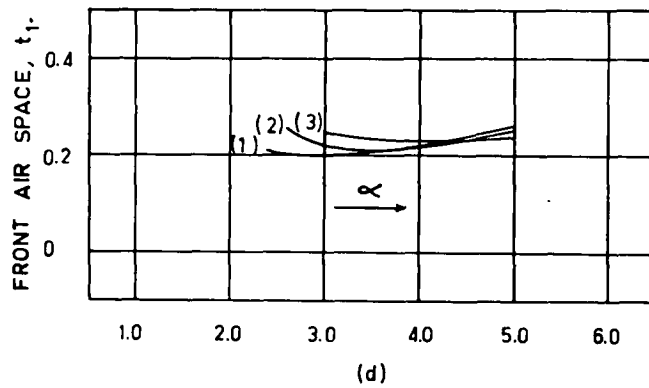
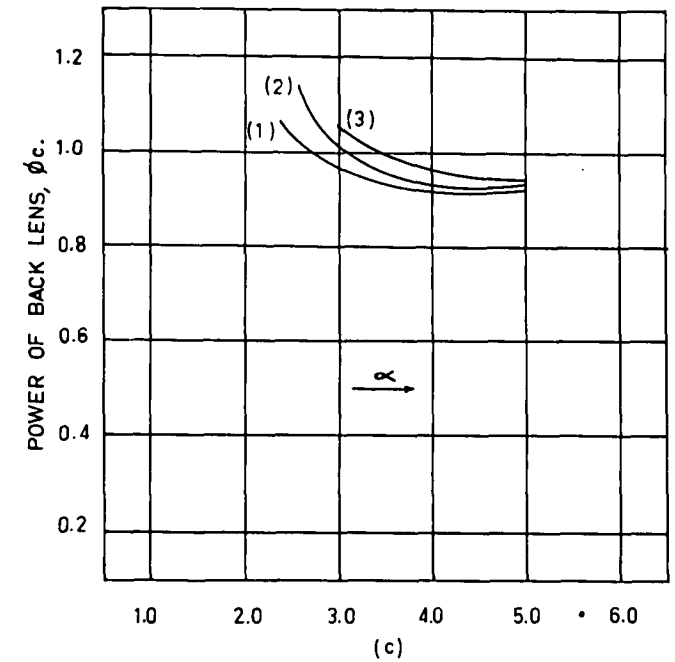
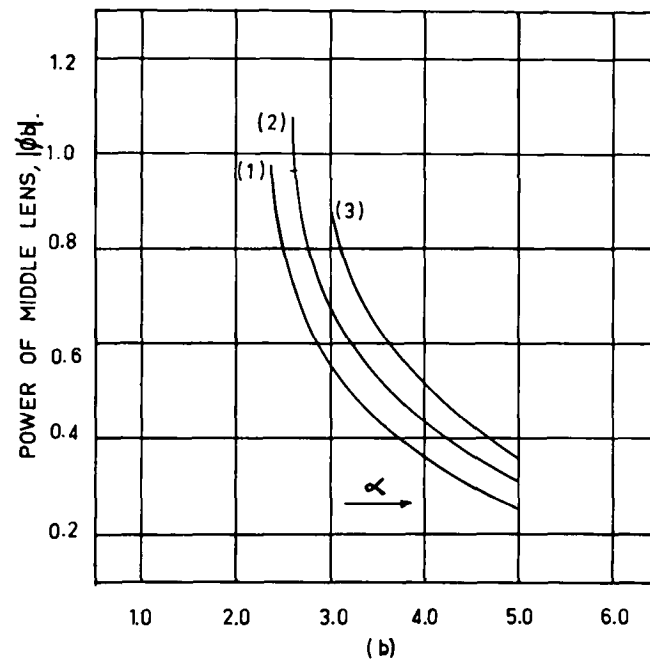
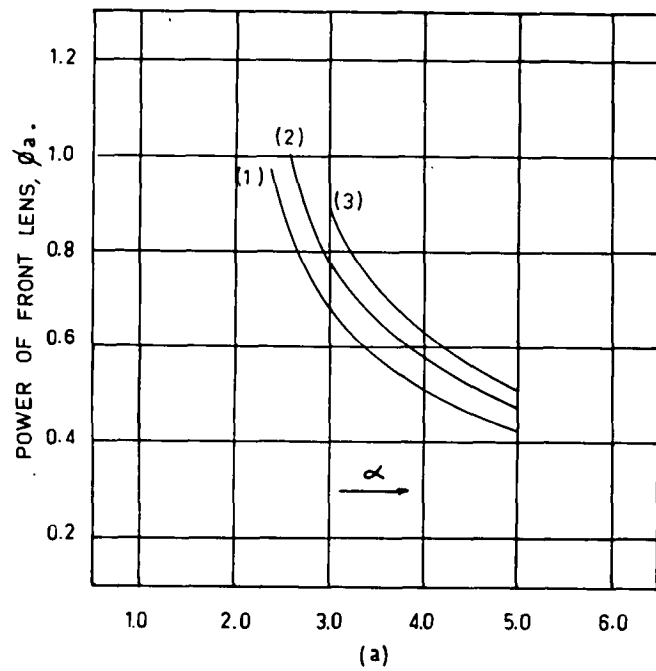
components corresponding to these y_{ob} -values. The curves of t_1 and t_2 which have lower slopes are associated with y_{ob} -values in the upper portion of curves (3), (4) and (5) in Fig. 2.7(a). All these results look promising for the existence of a new class of solutions, having much lower powers than in traditional triplets.

The coefficients of the cubic equation (2.27) are not only functions of χ , P and α considered so far, but also of ξ , β , γ , L , T . Although the ranges of variation of these are more limited, the effects of their variations must be investigated. In doing so we shall limit our considerations to the roots represented by the upper portions of the continuous curves.

Figures 2.8(a) to 2.8(e) show the variations of the powers ϕ_a , ϕ_b , ϕ_c and the separations t_1 , t_2 , respectively, regarded as functions of α for different values of ξ for the case in which $P = 1.10$. Curves (1), (2), and (3) are drawn for the ξ values 1.0, 1.2 and 1.4 respectively. These figures show that the powers of the components change considerably with increase of ξ , especially ϕ_a and ϕ_b , whereas the values of the separations t_1 and t_2 do not vary significantly. The minimum value of α for which solutions are possible increases with ξ .

We consider next the effect of the variations of the glass parameters $N_a/N_b = \beta$ and $N_a/N_c = \gamma$. This could mean, the generation of fictitious glasses (see Chapter III, Section 3.1) in the replacement of single components by cemented components.

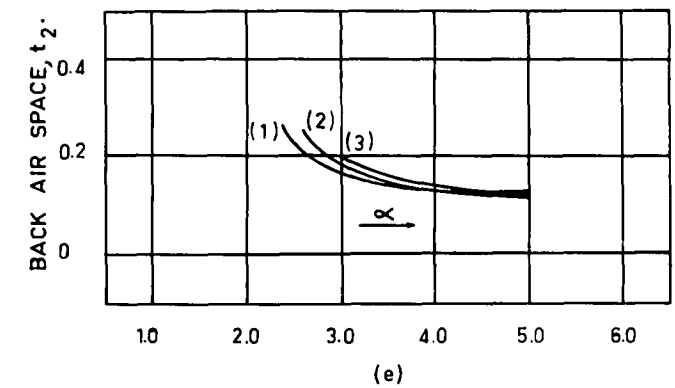
In Figures 2.9(a) to 2.9(e) curves (1), (2) and (3) are drawn for different values of β , namely 0.7772, 0.8772, 0.9772. These are similar to Figures 2.8(a) to 2.8(e), and show that increasing β will increase the thin lens separations and the power ϕ_c much more than the powers ϕ_a and ϕ_b .

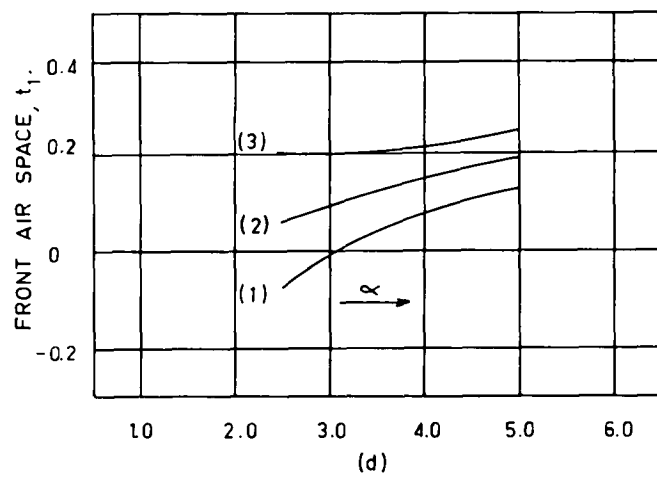
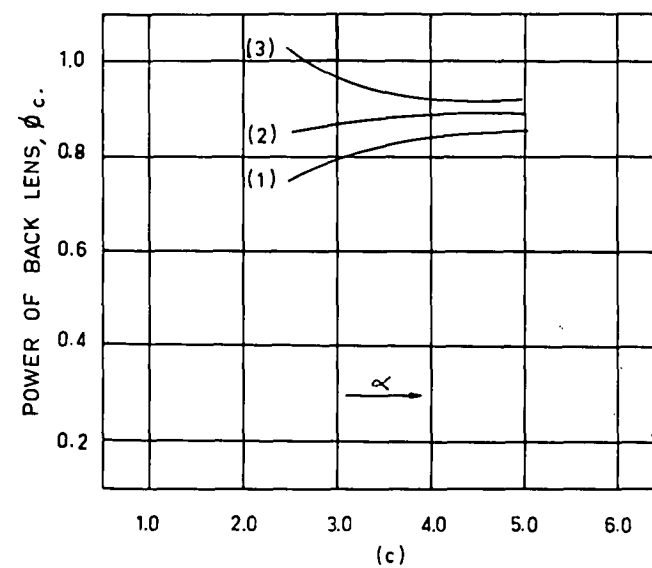
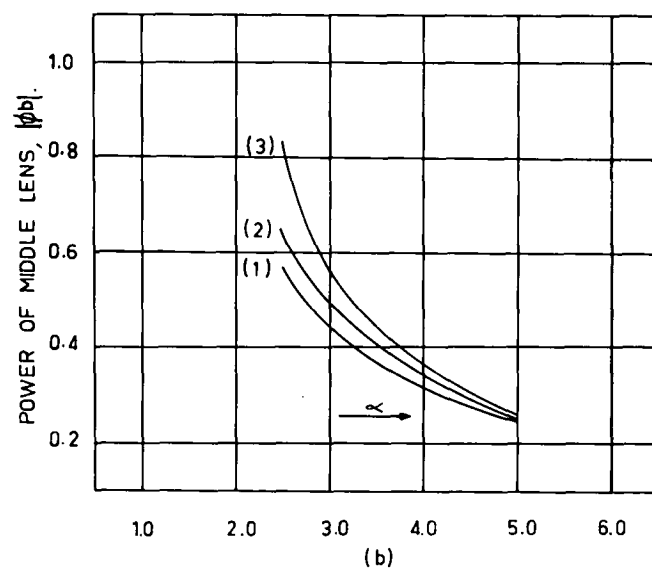
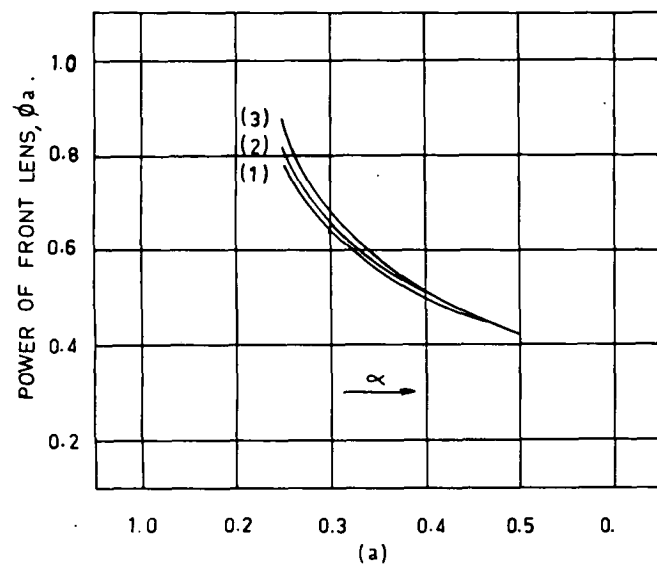


The effect of the variation of ξ .
 (1) $\xi = 1.0$, (2) $\xi = 1.2$, (3) $\xi = 1.4$.
 The remaining glass constants and residuals are
 as follows:

$\beta = 0.9772$,	$\gamma = 1.0$,
$P = 1.1$,	$\alpha = 0.2$,
$L = 0.1$,	$T = 0.0$.

fig (2.8).

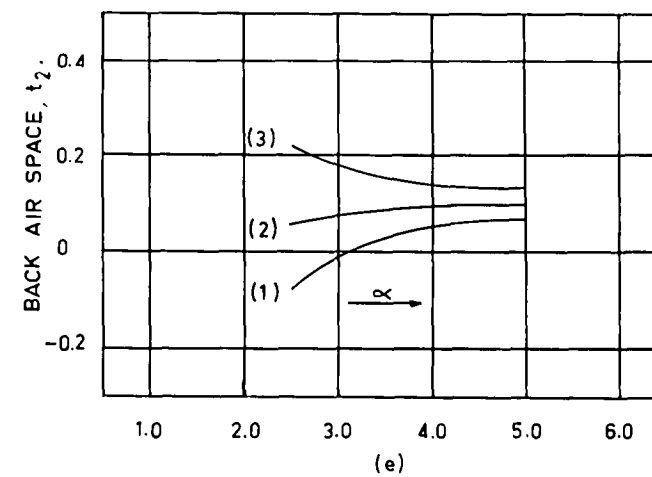


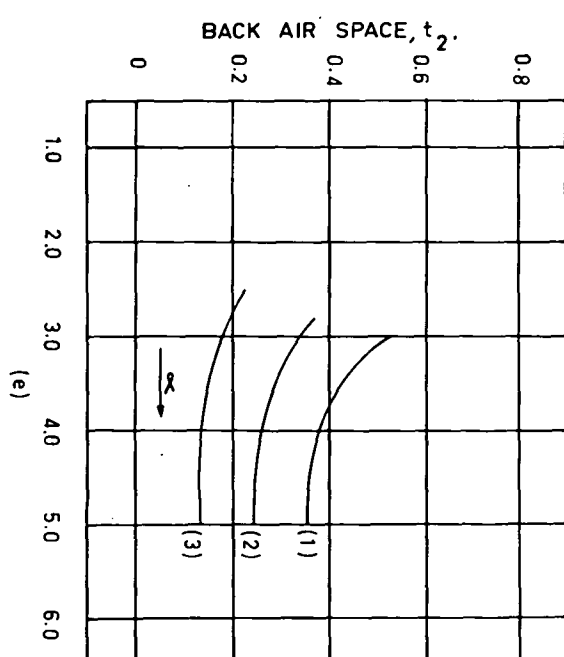
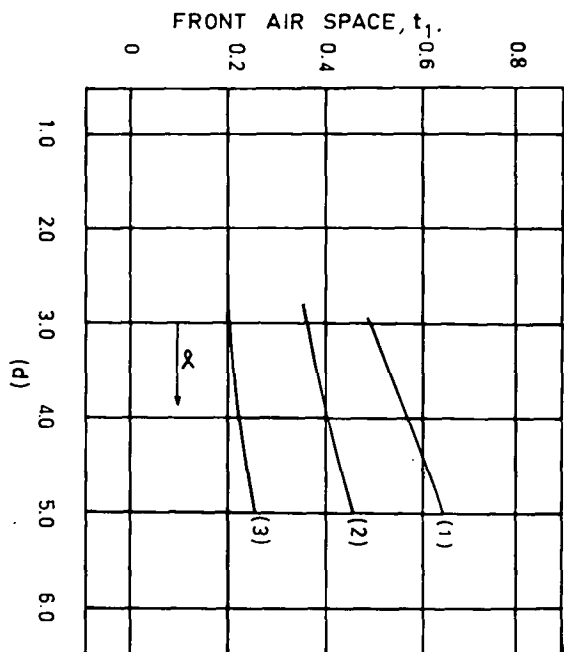
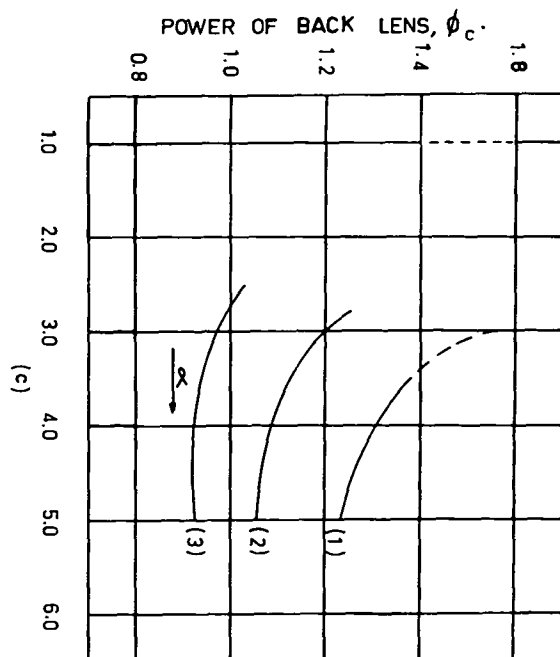
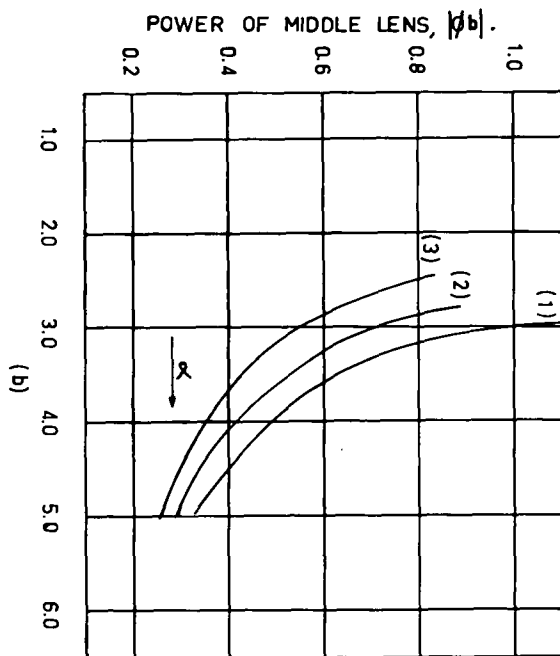
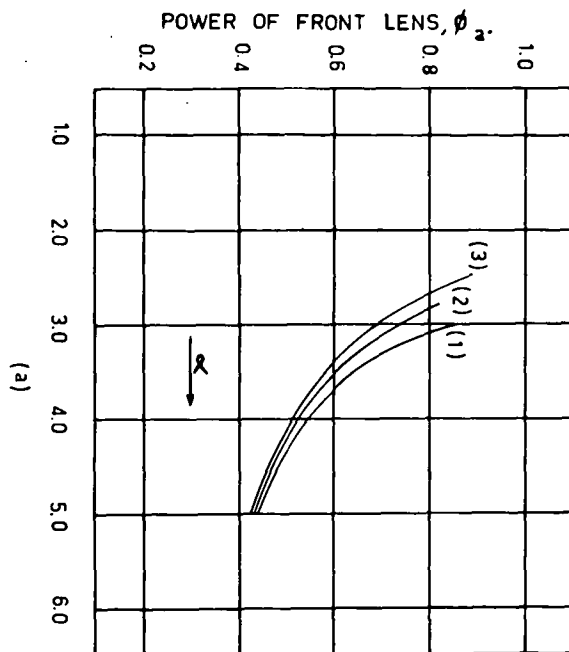


The effect of the variation of β .
 (1) $\beta = 0.7772$, (2) $\beta = 0.8772$, (3) $\beta = 0.9772$.
 The remaining glass constants and residuals are as follows:

$$\begin{aligned} \xi &= 1.0, & \tau &= 1.0, \\ P &= 1.1, & \chi &= 0.2, \\ L &= 0.1, & T &= 0.0. \end{aligned}$$

fig (2.9)

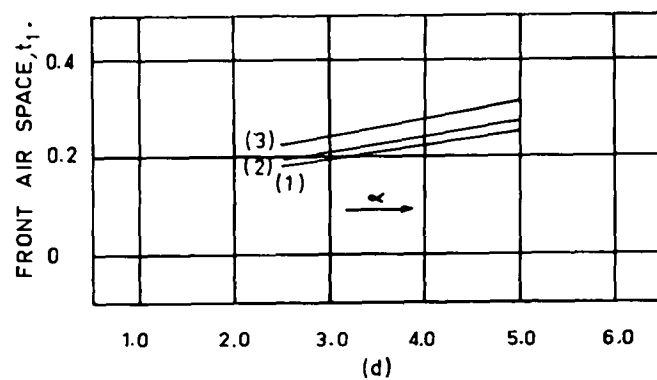
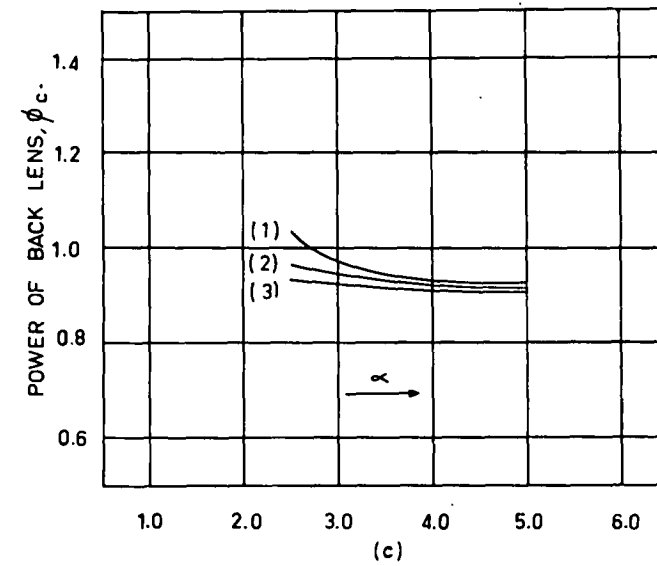
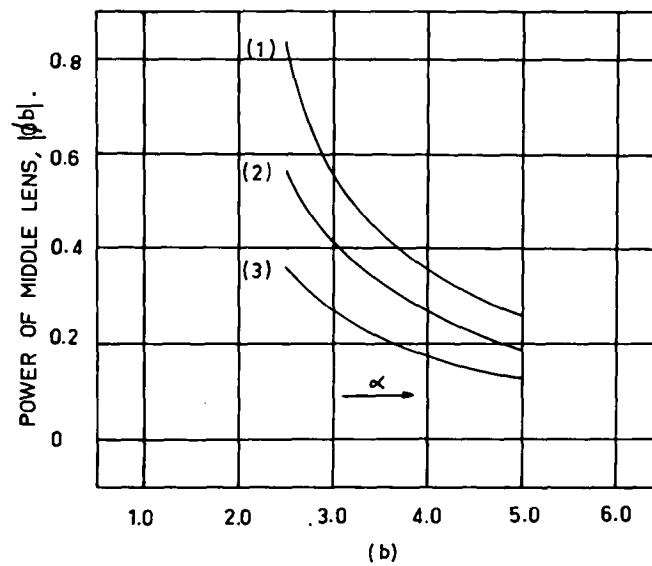
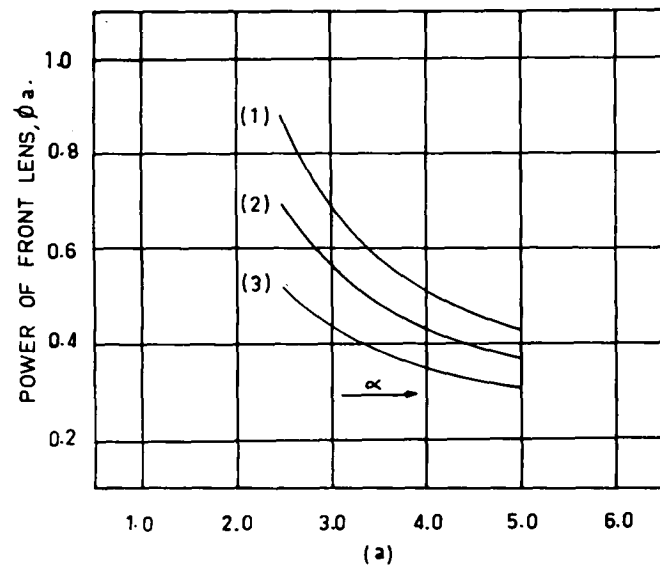




The effect of the variation of τ ,
 (1) $\tau = 0.8$, (2) $\tau = 0.9$, (3) $\tau = 1.0$.
 The remaining glass constants and residuals are
 as follows:

$\mathcal{L} = 1.0$, $\mathcal{R} = 0.9772$,
 $P = 1.1$, $\mathcal{X} = 0.2$,
 $L = 0.1$, $T = 0.0$.

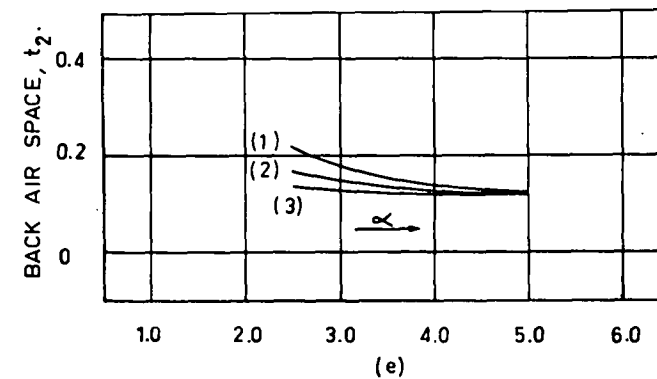
fig (2.10)

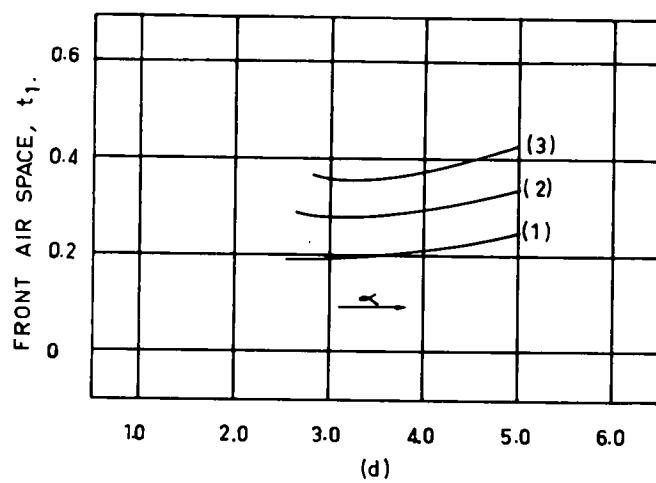
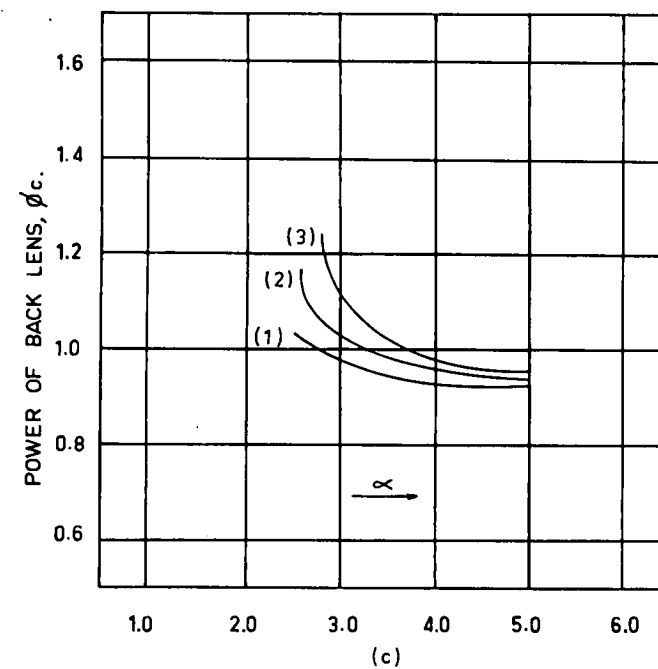
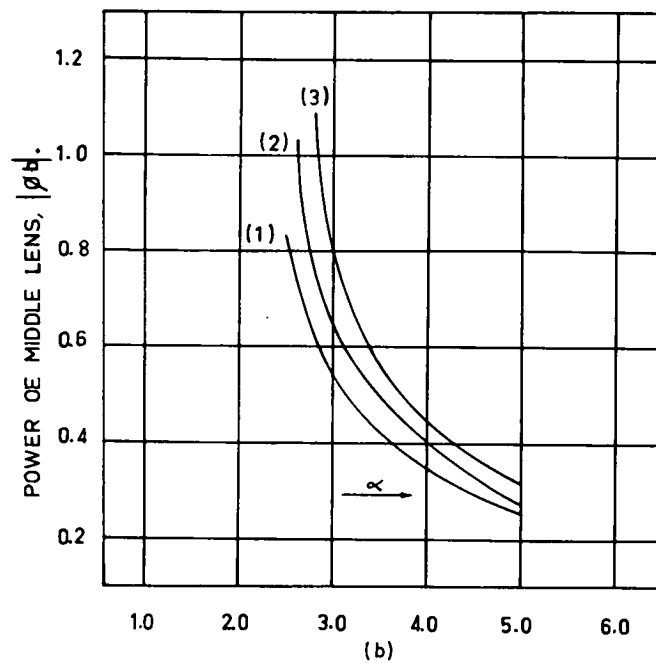
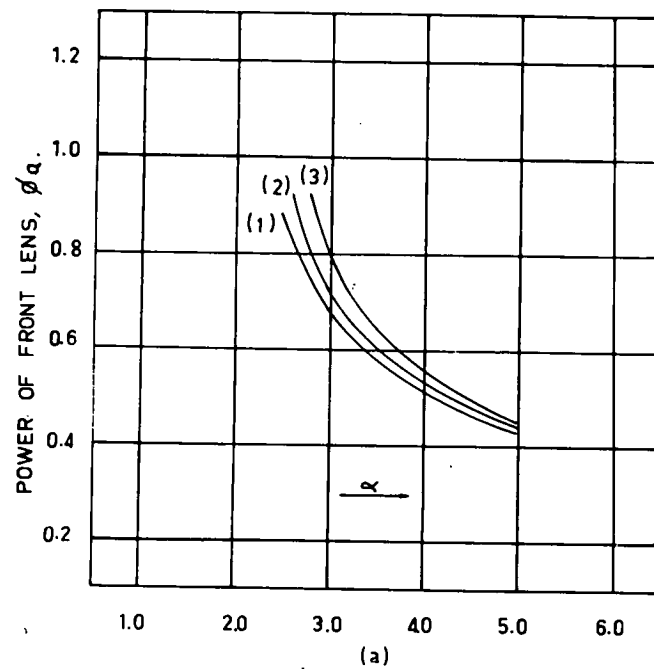


The effect of the variation of L .
 (1) $L=0.1$, (2) $L=0.3$, (3) $L=0.5$,
 The remaining glass constants and residuals are
 as follows:

$$\begin{aligned} \xi &= 1.0, & \beta &= 0.9772, \\ \gamma &= 1.0, & P &= 1.1, \\ \alpha &= 0.2, & T &= 0.0. \end{aligned}$$

fig (2.11)





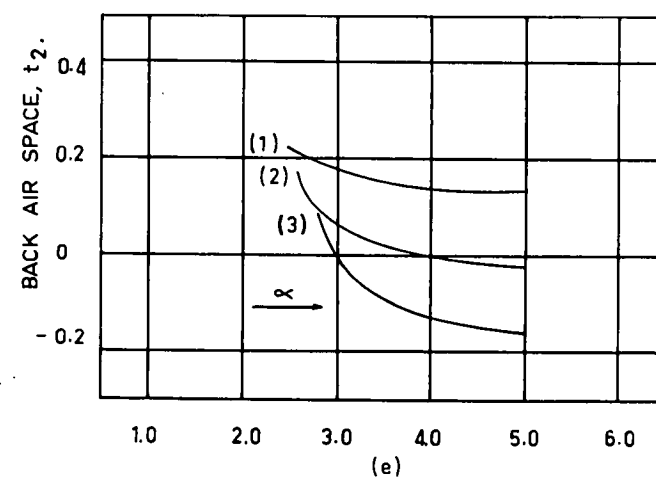
The effect of the variation of T .

(1) $T=0.0$, (2) $T=0.2$, (3) $T=0.4$.

The remaining glass constants and residuals are as follows:

$$\begin{aligned} \xi &= 1.0, & \beta &= 0.9772, \\ \tau &= 1.0, & \alpha &= 0.2, \\ P &= 1.1, & L &= 0.1. \end{aligned}$$

fig (2.12)



Similar curves (1), (2), (3) in Figures 2.10(a) to 2.10(e), drawn for the values $\gamma = 0.8, 0.9, 1.0$ respectively, show that the increase of γ will decrease the values of all the parameters of the initial solution. The reductions in the airspaces and in the powers $|\phi_b|$ and ϕ_c are more noticeable than the reduction in ϕ_a .

The parameters L and T cannot be chosen as degrees of freedom in the same way as the glass constants because R_s and R_t are to be prescribed to control the chromatic aberrations of the system. However, variations of these are considered at this stage because the values of L and T in the new class of triplets may differ considerably from the corresponding values in the type 111 triplet objectives on account of the presence of compound components.

In Figures 2.11(a) to 2.11(e) the curves (1), (2), (3) for which L has the values 0.1, 0.3, 0.5 respectively show the resulting change in the powers and the separations. These do not call for any particular comment. Figures 2.12(a) to 2.12(e) show the effects of the increase of T . The curves (1), (2) and (3) are drawn for T values of 0.0, 0.2 and 0.4. It can be seen that the main result of increasing T is an increase of the first airspace, t_1 , together with a substantial decrease of the second airspace, t_2 , which has even become negative in two of the cases considered. It will appear subsequently that advantage can be taken of this effect of T in a very interesting manner.

CHAPTER III.

GENERAL PROPERTIES OF THE NEW CLASS OF OBJECTIVES

3.1 SOME PROPERTIES of CEMENTED TRIPLETS of NEGATIVE POWER.

In his discussion of the triplet family of objectives, Cruickshank (1958, pp27-29) has shown that any thin lens (ϕ, \bar{N}, \bar{V}) of power ϕ and with glass constants \bar{N} and \bar{V} in the basic triplet can be replaced by a set of thin lenses $(k_1 \phi, N_1, V_1), (k_2 \phi, N_2, V_2) \dots (k_n, V_n, N_n)$ in contact, the replacing group having the same power, the same petzval sum and same paraxial chromatic aberrations as the singlet it replaces, provided that

$$k_1 + k_2 + \dots + k_n = 1 \quad (3.1)$$

$$k_1 / N_1 + k_2 / N_2 + \dots + k_n / N_n = 1 / \bar{N} \quad (3.2)$$

$$k_1 / V_1 + k_2 / V_2 + \dots + k_n / V_n = 1 / \bar{V} \quad (3.3)$$

This means that it is possible to design a cemented component, which is equivalent as regards power, petzval sum, and the paraxial chromatic aberrations to a thin lens having within a certain range any value of \bar{N} , and, independently thereof, any value of \bar{V} . The artificial glass refractive index generated in this way is known as a fictitious refractive index and the \bar{V} -number produced is called the effective V-number.

In order to maintain the same power, the same petzval sum, and the same chromatic aberrations as the singlet it replaces and to obtain variation over a considerable range, for the important parameter α in the new class of thin lens initial solutions discussed in the previous chapter, we replace the central thin lens $(\phi_b, \bar{N}_b, \bar{V}_b)$ in the basic triplet by a set of three thin lenses $(k_1 \phi_b, N_1, V_1), (k_2 \phi_b, N_2, V_2), (k_3 \phi_b, N_3, V_3)$ in axial contact. Then the equations (3.1) - (3.3) become

$$k_1 + k_2 + k_3 = 1 \quad (3.4)$$

$$k_1/N_1 + k_2/N_2 + k_3/N_3 = 1/\bar{N}_b \quad (3.5)$$

$$k_1/V_1 + k_2/V_2 + k_3/V_3 = 1/\bar{V}_b \quad (3.6)$$

The three unknowns k_1, k_2, k_3 for a given set of glasses can be calculated for any value of \bar{V}_b and \bar{N}_b , i.e. for any value of α and β from equations (3.4) - (3.6). The values are given by

$$K_1 = \Delta_1/\Delta; \quad K_2 = \Delta_2/\Delta; \quad K_3 = \Delta_3/\Delta$$

where

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1/\bar{N}_b & 1/N_2 & 1/N_3 \\ 1/\bar{V}_b & 1/V_2 & 1/V_3 \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1/N_1 & 1/\bar{N}_b & 1/N_3 \\ 1/V_1 & 1/\bar{V}_b & 1/V_3 \end{vmatrix},$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1/N_1 & 1/N_2 & 1/\bar{N}_b \\ 1/V_1 & 1/V_2 & 1/\bar{V}_b \end{vmatrix},$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1/N_1 & 1/N_2 & 1/N_3 \\ 1/V_1 & 1/V_2 & 1/V_3 \end{vmatrix}.$$

Multiplying the power ϕ_b by k_1, k_2, k_3 in turn, we obtain the powers of the individual components of the replacing triplet.

To avoid an excessive number of air-glass surfaces, we may decide that the components of this triplet should be cemented together. When the shape factor of one component is arbitrarily selected, the shapes of the remaining components are then fixed by the cementing condition. There is thus only one shape factor for the compound component. We shall use the shape of the front component of this cemented triplet to specify the shape factor of the whole component.

It will be remembered that we increased the value of P in the thin basic triplet system, assuming that the replacement of a thin component by a thickened meniscus cemented component may often lead to a reduction of the petzval sum of the whole objective. We study, therefore, the variation in the value of the petzval curvature coefficient σ_4 of a cemented negative triplet with change of shape.

In Figure 3.1, curve (1) is drawn for the case in which

$$(1) \quad \phi = -1, \quad \bar{N}_b = 1.8381, \quad \bar{V}_b = 19.4233$$

In this figure, we have not plotted σ_4 itself, but the related quantity $2\sigma_4 \bar{N}_b$, which is the value of the parameter P , for this component. The curve shows that as S increases the value of P becomes more negative. Similar curves (2) and (3) are drawn for the cases

$$(2) \quad \phi = -1, \quad \bar{N}_b = 1.8381, \quad \bar{V}_b = 16.6486$$

$$\text{and } (3) \quad \phi = -1, \quad \bar{N}_b = 1.8381, \quad \bar{V}_b = 14.5676$$

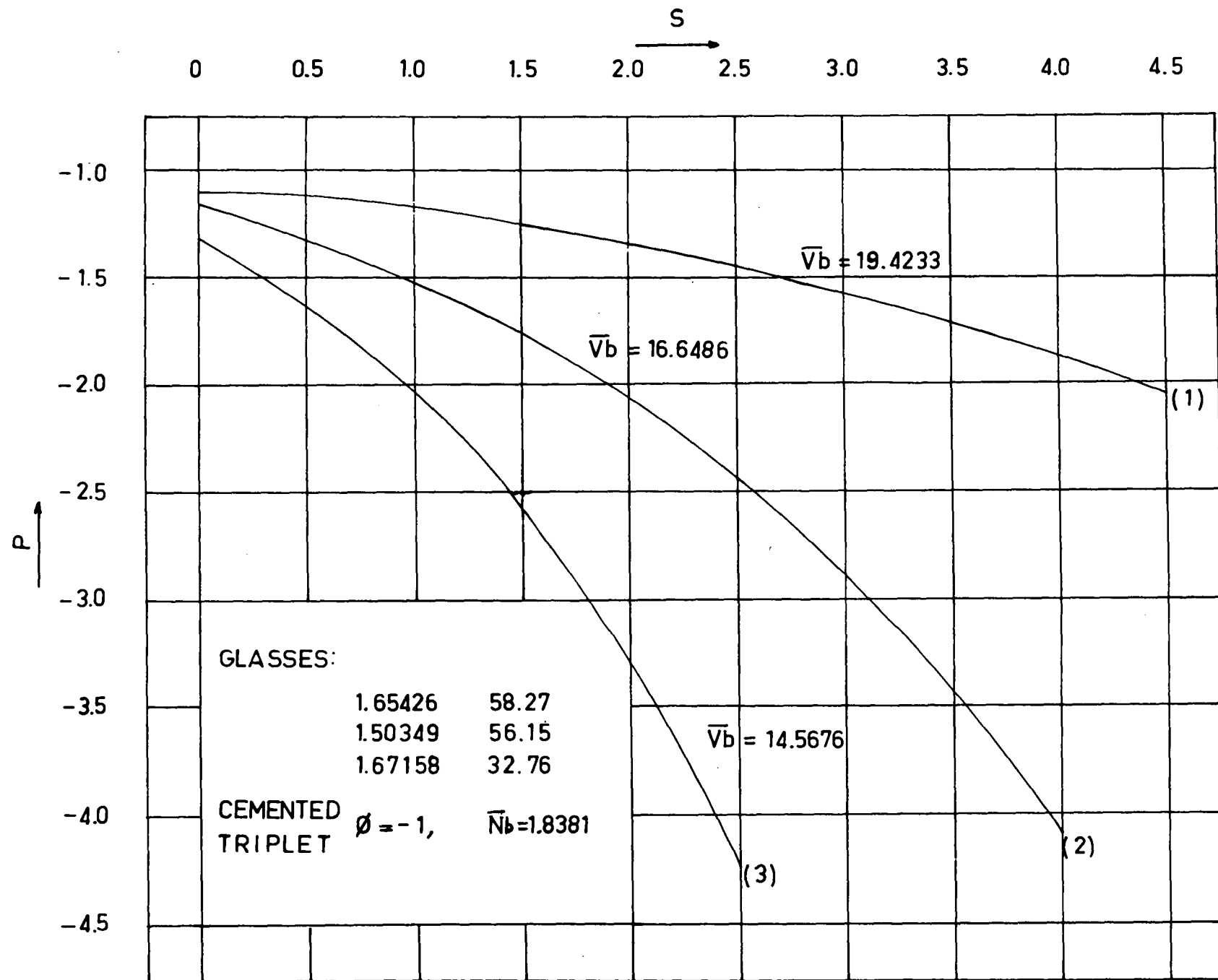


Fig 3.1

respectively, and these curves show P to be changing more rapidly with S than curve (1). It becomes clear, then, that the negative contribution which the thickened cemented triplet makes to the value of the parameter P can be substantially increased by change of its shape and by reduction of its effective V-number \bar{V}_b . Our next consideration is to know the particular surface or surfaces of this compound triplet, responsible for the reduction of P . It is observed that the last curvature of this compound triplet turns out to be deep with increase of shape and produces very high negative values of P . The P values of the last surface corresponding to the case (2) in Figure 3.1 are computed, and these results are represented by curve (3) in Figure 3.2. The curve (2) in Figure 3.2 is the same as the curve (2) in Figure 3.1 which is drawn for comparison.

The use of a cemented triplet having a positive shape factor and low value of \bar{V}_b as the central component in a triplet objective results in reduction of the P -value of the whole objective, because the P -value is unaffected by the object position. This fully justifies our earlier assumption that larger values of P used in the basic triplet initial solution could lead to useful practical solutions.

We will now study the variation of the locations of the principal points of the cemented negative triplet. To do this, the principal point distances are computed for the components given in cases (1) and (2) of Figure 3.1, and the results of these computations are plotted in Figure 3.3 as functions of S . For case (1), curves (1), (1)' represent the variation of first and second principal point distances with increase in S . The effect of decreasing \bar{V}_b can be seen in similar curves (2), (2)' drawn for case (2). All these curves show that the first principal point distance is always greater than the second principal point distance for a given value of \bar{V}_b and S .

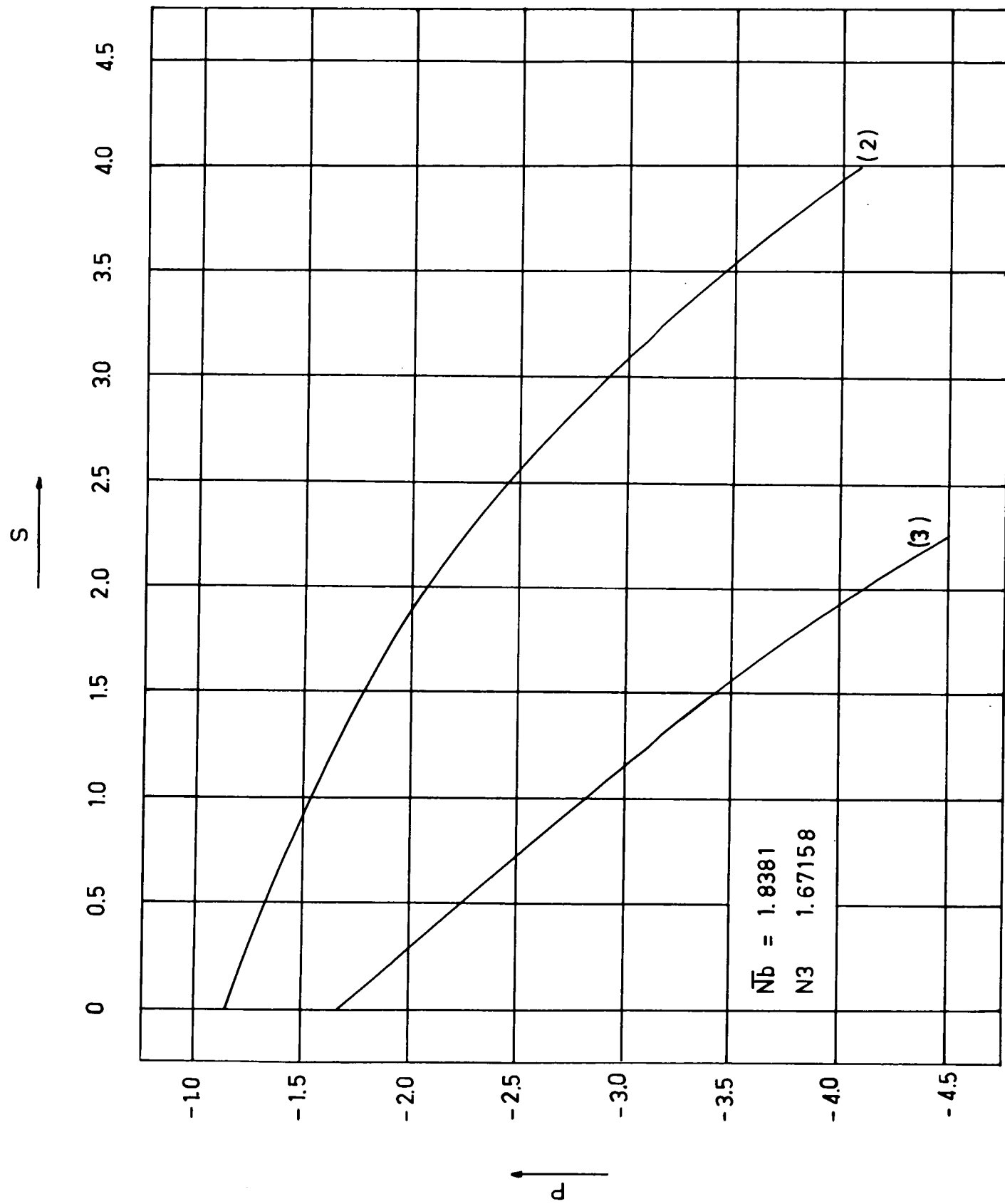


Fig 3.2

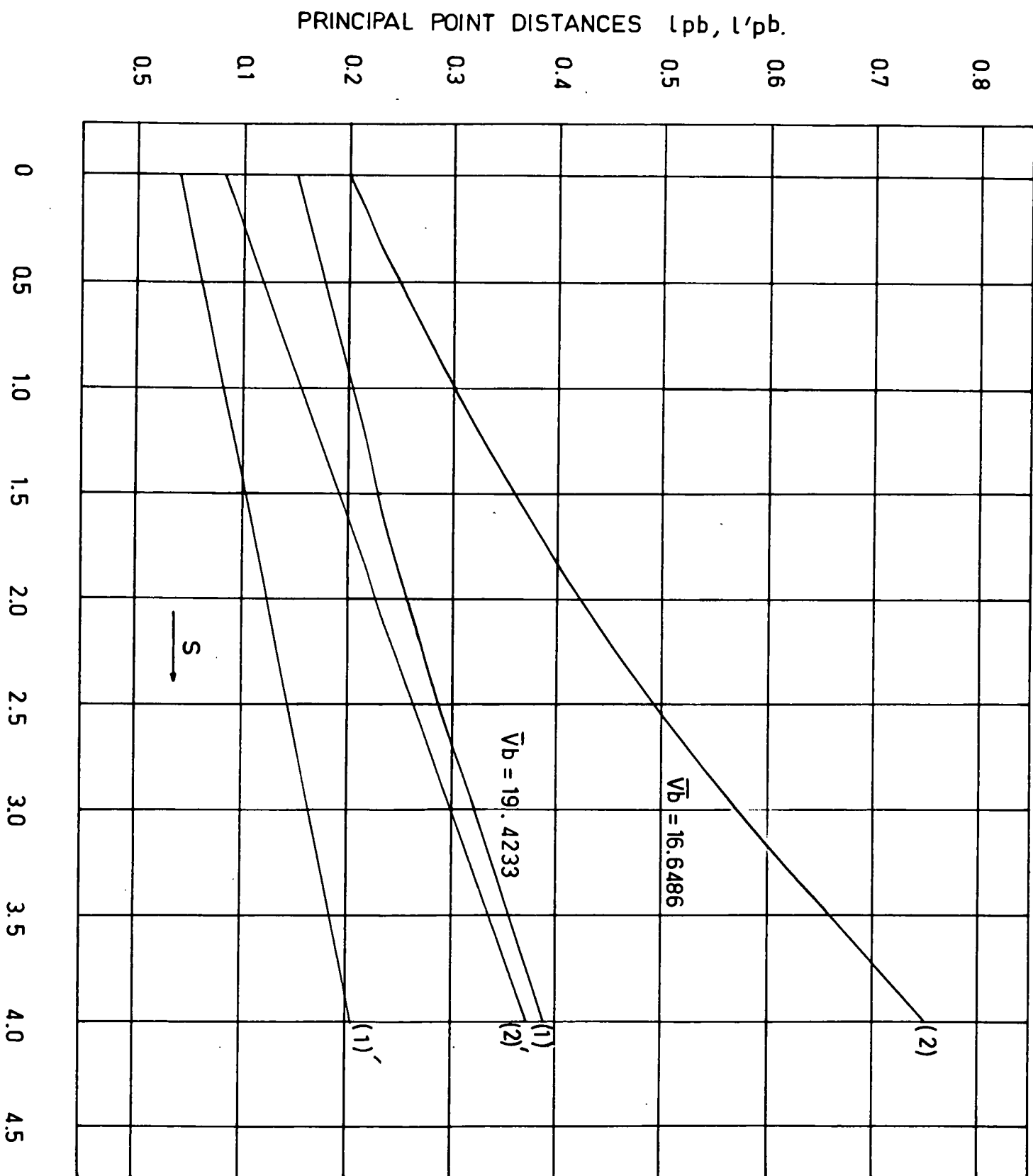


Fig 3.3

3.2 SUMMARY of the NEW CLASS of TRIPLETS.

The new region discussed in Chapter II was arrived at in the initial solutions of the basic triplet by increasing the normal values of the parameters α , P , and χ . Increasing α means decreasing the V-number of the central component of the basic triplet for a given V-number of the first component. The only way to fulfil this condition for a given V-number of the first component of the basic triplet is to replace the central thin lens of the basic triplet by a compound component.

In order to maintain the same power, the same petzval sum and the same chromatic aberrations as the singlet it replaces, one has to replace the central thin lens of the basic triplet by three thin lenses in axial contact.

The large values of P used in the initial solution will be reduced to an acceptable level in the actual system of the whole objective if a positive meniscus shape and low \bar{V}_b of the central compound component are maintained. This central compound triplet with positive meniscus shape will have its principal points at a considerable distance from its vertices. It is clear, then, that these triplet objectives with a thick cemented central triplet must have, in the initial arrangement, a high positive value of t_1 to accommodate this central component, and a low or even negative value of t_2 to maintain a compact objective. The trend of the distribution of principal points in the class of triplet objectives are illustrated in Figures 3.4(a) and 3.4(b).

While we investigated the initial solutions in the last Chapter, we noticed that a high positive value of t_1 and a low or even negative value of t_2 could be obtained by increasing the parameter T . It follows, therefore, that the required positive value of t_1 and low value of t_2 can be provided in

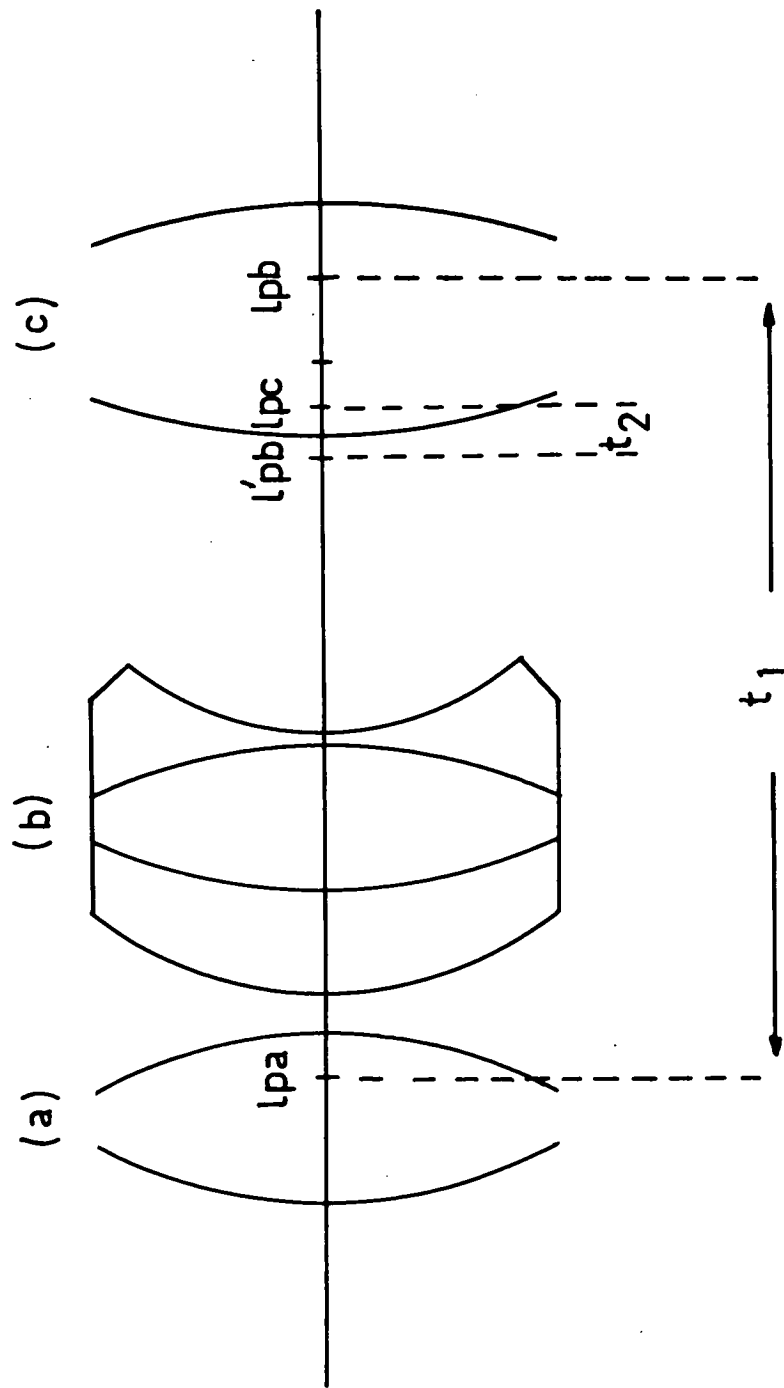


Fig 3.4(a)

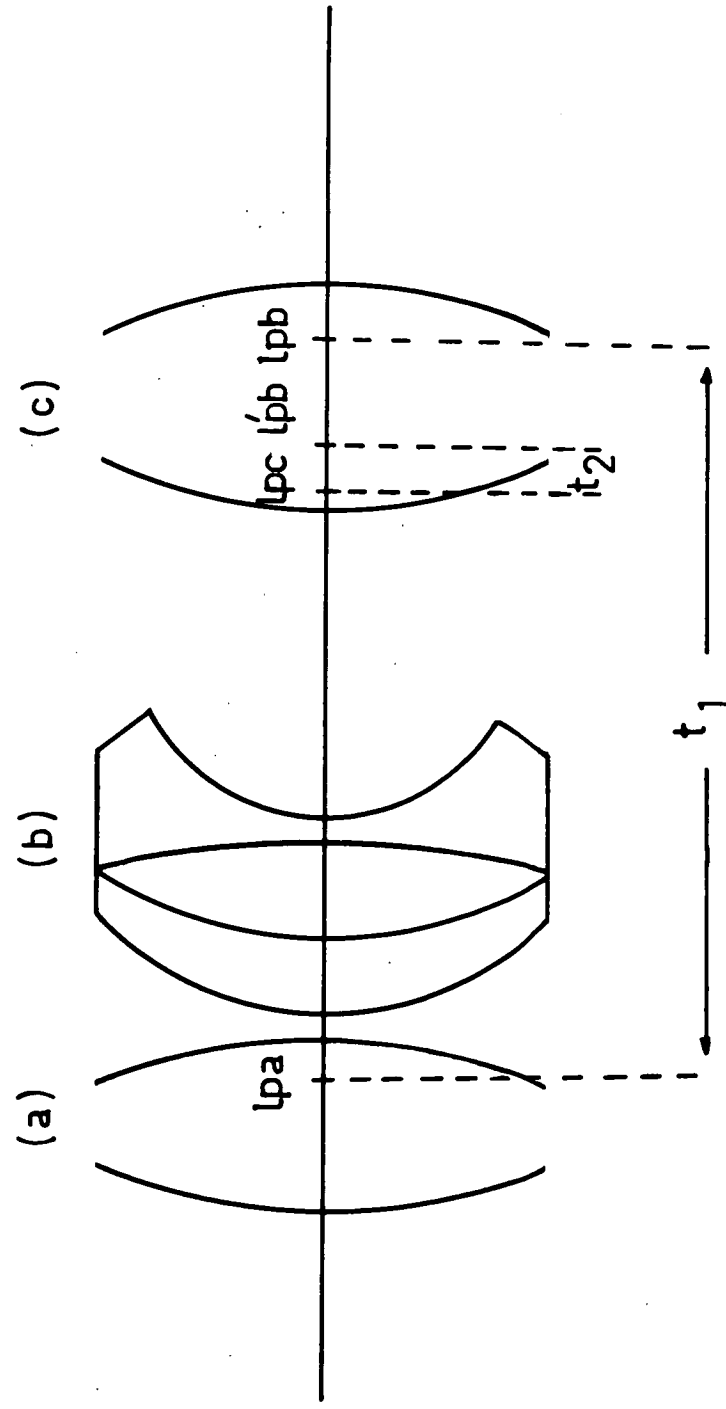


Fig 3.4(b)

this class of objectives by giving an appropriate positive value of T in the initial solution. But the next question is whether this value of T is consistent with control of the transverse colour.

We begin to see now how the important parameters α , P and T are interrelated to control the over all optical characteristics of this class of objectives. With a thorough understanding of the effects of these parameters, together with the effects (studied in the second chapter) on the initial solutions of the other parameters, we are in a position to progress towards practical solutions for this class of objectives.

The design of this class of objectives should start then with the selection of reasonably high values of α , P , χ , T , along with moderate values of the other parameters. This should give a suitable initial arrangement with large front air space and small rear air space. Next, the central component in the basic triplet is replaced by a compound negative triplet and this should give possible triplet objective belonging to this new group, i.e. this group starts with the type 131 triplet. The remaining triplets of this family could be 132, 133, 231, 232, ... etc. The replacement of components other than the central component by compound components provides more degrees of freedom and one can expect better performance with increased aperture and wide field, compared to the optimum 131 triplet.

In the next Chapter, we try to develop the practical application of these ideas by considering the design of 131 triplet objectives.

CHAPTER IV

AN INVESTIGATION OF THE DESIGN OF A 131 TRIplet

4.1 DESIGN PROCEDURE.

We begin with the five selected glasses (N_a, V_a) , (N_1, V_1) , (N_2, V_2) , (N_3, V_3) , (N_c, V_c) , the residuals R_4, R_6, R_7 corresponding to P, L, T and the parameters α, β, χ which will lead to a solution lying in the new region suggested in Figure 2.7 of Chapter II. Along with these, the residuals R_2, R_3 and R_5 are selected (normally zero values at the initial stage) and the basic triplet initial solution parameters $\phi_a, \phi_b, \phi_c, t_1, t_2$ are computed by using the appropriate equations (2.19) - (2.26) of Chapter II. Having done this, the glass constants \bar{N}_b, \bar{V}_b can be evaluated from equations $\bar{N}_b = \beta/N_a, \bar{V}_b = \alpha/V_a$. Using these constants, the three constants k_1, k_2, k_3 will be calculated by solving three simultaneous equations (3.4) - (3.6) of Chapter III. Then the three thin lens powers $\phi_{b1}, \phi_{b2}, \phi_{b3}$ are calculated from the equations $\phi_{b1} = k_1 \phi_b, \phi_{b2} = k_2 \phi_b, \phi_{b3} = k_3 \phi_b$. These power calculations complete the replacement of the central thin lens of the basic triplet by three thin lenses in axial contact, i.e. the basic thin triplet is converted into a thin 131 triplet having the same initial residuals and values for χ, P, L, T as the basic triplet.

The next step is to compute the thin lens curvatures of the 131 triplet for a set of three arbitrary shapes S_a, S_b, S_c . The thin system is then thickened, introducing the prescribed axial thicknesses d_2, d_4, d_6, d_8 of the five components, using Cruickshank's (1968) standard thickening procedure, which keeps the total and the individual powers of the three thick components a, b, c equal to the total and the individual powers of the corresponding thin

components. The two standard paraxial rays, i.e. the axial paraxial ray ($y_{01} = 1, v_{01} = 0$) and the principal paraxial ray ($y_1 = p, v_1 = 1$) for the allotted stop position (fixed initially at the first principal point of the central compound), for an object plane at infinity are traced through the system. From the results of these two traces, we compute the third order aberration coefficients $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ and, for convenience, $\sigma_6 = 3\sigma_3 + \sigma_4$. The next step is to adjust the three available shapes S_a, S_b, S_c so as to make $\sigma_2 = R_2, \sigma_3 = R_3$ or $\sigma_6 = R_3, \sigma_5 = R_5$, using the standard Cruickshank (1968) differential correction method based on the following equations:

$$\frac{\partial \sigma_2}{\partial S_a} \Delta S_a + \frac{\partial \sigma_2}{\partial S_b} \Delta S_b + \frac{\partial \sigma_2}{\partial S_c} \Delta S_c = R_2 - \sigma_2 \quad (4.1)$$

$$\left. \begin{aligned} \frac{\partial \sigma_3}{\partial S_a} \Delta S_a + \frac{\partial \sigma_3}{\partial S_b} \Delta S_b + \frac{\partial \sigma_3}{\partial S_c} \Delta S_c &= R_3 - \sigma_3 \\ \frac{\partial \sigma_6}{\partial S_a} \Delta S_a + \frac{\partial \sigma_6}{\partial S_b} \Delta S_b + \frac{\partial \sigma_6}{\partial S_c} \Delta S_c &= R_3 - \sigma_6 \end{aligned} \right\} \quad (4.2)$$

or

$$\frac{\partial \sigma_5}{\partial S_a} \Delta S_a + \frac{\partial \sigma_5}{\partial S_b} \Delta S_b + \frac{\partial \sigma_5}{\partial S_c} \Delta S_c = R_5 - \sigma_5 \quad (4.3)$$

This is essentially an iteration process and each iteration brings the aberration coefficients closer to the desired residual values. This method is very convenient because the same procedure can be applied to any triplet of the basic triplet family.

After adjusting the shapes, the paraxial traces for the wavelengths for which the system is to be achromatised are computed through the system and the paraxial chromatic aberrations, l_{ch} , and t_{ch} , of the thick system are adjusted to have the prescribed values R_6 and R_7 . This is done by the differential correction method explained in Chapter II.

The next stage of the design will be to make the spherical aberration to seventh order for the zone of radius ρ equal to the prescribed residual R_1 thus satisfying the equation

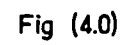
$$^* \sigma_1 \rho^3 + \mu_1 \rho^5 + \tau_1 \rho^7 = R_1 \quad (4.4)$$

It is clear from Cruickshank's equation (1.1), mentioned in the review of the literature, that the spherical aberration of the basic triplet is very closely a quadratic function of χ . At this point, this relationship is assumed to be valid for any triplet derived from the basic triplet because χ determines the power distribution between the two positive components of the triplet, whether the components are single or compound. All these steps are now organized into a single computing programme, which is described in the following section.

4.2 DESCRIPTION of the PROGRAMME.

The main programme can be best understood from the block diagram (4.0). It starts by reading the refractive indices of the selected glasses, the axial thicknesses of the lenses, the initial values of α , β , P , χ , ρ and the residuals R_1 , R_2 , R_3 , R_5 , R_6 , R_7 . With these input values it computes the

* Throughout this problem Buchdahls optical aberration coefficients are used.



initial solution corresponding to the new region. Then it replaces the central thin lens b of the basic triplet by three thin lenses in axial contact and thickens the system in accordance with the prescribed axial thicknesses, using a given set of arbitrary shapes. The two standard paraxial rays are traced with the computed position of the paraxial entrance pupil corresponding to a given position of the stop. The third order aberration coefficients are evaluated next, and this is followed by a discrimination which tests whether $\sigma_2 = R_2$, $\sigma_5 = R_3$, and $\sigma_6 = R_4$. If any one of these conditions is not achieved, small changes are made in the shapes S_a , S_b , S_c and the approximate derivatives $\partial\sigma_i/\partial S_j$ ($i = 2, 5, 6$ $j = a, b, c$) are calculated. The shapes are then adjusted iteratively until the required conditions are achieved. In practice, this takes approximately 5 or 6 iterations. Should the conditions not be achieved within 10 iterations, the programme alters the value of α and begins again. If the required conditions cannot be satisfied within the prescribed α range, then the value of β changes, and, with the initial value of α , restarts the programme. By this means, sample solutions are explored within the complete α and β ranges prescribed in the programme.

The next discrimination tests the condition $l_{ch} = R_5$, after tracing the paraxial rays in the prescribed colours. If $l_{ch} \neq R_5$, then the approximate derivative $\partial l_{ch}/\partial L$ is calculated and L is adjusted until the condition is achieved. Similarly, T is adjusted until $t_{ch} = R_7$. After emerging from these iterations, the fifth and seventh order spherical aberration coefficients are computed. Then if the flag integer $G \neq 1$ the remaining fifth order aberration coefficients are computed and printed out.

If $G = 1$, the programme proceeds to compare the spherical aberration to seventh order with the prescribed residual R_1 . If $\text{sph} \neq R_1$, then the approximate derivative $\partial \text{sph} / \partial \chi$ is calculated and χ is adjusted until $\text{sph} = R_1$. The fifth order coefficients for this system are then computed and printed out and the constructional data of the system are punched out on a separate output to serve as the input data of the standard programme for seventh order coefficients available in the design section.

4.3 A NUMERICAL EXAMPLE.

To give some idea of the operation of the programme, a brief account is given of a typical numerical example. In the input data, we give the refractive indices and V-numbers of the selected glasses, and values for the parameters α , β , P , χ , and such of the residuals R_1 to R_7 (excluding R_4) as are necessary to ensure that the solution obtained will belong to the "new region" which is of interest. The axial thicknesses for the various components are also assigned. The chosen values are:

$N_s = 1.65426$	$V_s = 58.27$	$\alpha = 3.2$	$R_1 = 0.000184$	$d_s = 0.08$
$N_1 = 1.65426$	$V_1 = 58.27$	$\beta = 0.84$	$R_2 = 0$	$d_1 = 0.08$
$N_2 = 1.50349$	$V_2 = 56.15$	$P = 1.20$	$R_3 = 0$	$d_2 = 0.08$
$N_3 = 1.67158$	$V_3 = 32.76$	$\chi = 0.3$	$R_5 = 0$	$d_3 = 0.02$
$N_c = 1.67341$	$V_c = 46.82$		$R_6 = 0.001734$	$d_4 = 0.08$
			$R_7 = 0.008580$	

The value of ρ for which the sph aberration will be computed is $\rho = 0.2222$ and the stop is assumed to coincide with the first principal point of the central component.

The programme begins by solving for the initial thin lens arrangement of the objective, i.e. the "basic triplet", giving the powers and separations as

$$\begin{aligned}\varphi_a &= 1.227706 & \varphi_b &= -1.38852 & \varphi_c &= 1.151832 \\ t_1 &= 0.343648 & t_2 &= 0.185640\end{aligned}$$

In the next stage, the central negative component is replaced by three thin lens in axial contact according to equations (3.4) - (3.6). The k-values obtained are

$$k_1 = -0.591023 \quad k_2 = -1.2971148 \quad k_3 = 2.888138$$

and the powers of the three components become

$$\varphi_{b1} = 0.820647 \quad \varphi_{b2} = 1.801070 \quad \varphi_{b3} = -4.010237$$

Assuming arbitrary shapes S_a , S_b , S_c for the components of the thin 131 triplet

$$S_a = 1.5 \quad S_b = 2.5 \quad S_c = 1.0$$

The programme computes thin lens curvatures and this thin system is then thickened and adjusts shapes by differential correction method until $\sigma_2 = R_2$, $\sigma_6 = R_3$, and $\sigma_8 = R_3$. This takes five iterations.

After adjusting the shapes, the programme computes the paraxial longitudinal chromatic aberration l_{ch} , compares it with R_6 and by variation of L adjusts the system until $l_{ch} = R_6$. This takes two iterations. In a similar manner, a further two iterations serve to adjust T to such a value $t_{ch} = R_7$. At this point, the coefficients μ_1 , τ_1 are calculated and the spherical aberration to seventh order is calculated for the zone of $\rho = 0.2222$, this is compared with R_1 and χ is adjusted until $sph = R_1$. The shapes of the three lenses and χ value at this stage are

$$S_a = 1.6514 \quad S_b = 1.7379 \quad S_c = 0.5373 \quad \chi = 0.425.$$

The specifications of the system corresponding to these shapes is punched out and is given below:

C	d	N
2.487622	0	1
0.663353	0.08	1.65426
1.225344	0.090555	1
0.371941	0.08	1.65426
-2.896428	0.08	1.50349
2.737914	0.02	1.67158
1.301541	0.232867	1
-0.42678874	0.08	1.67341

The aberration coefficients up to fifth order are calculated next, and later the seventh order aberration coefficients using another programme. The coefficients for this system are shown below:

$\sigma_1 = 0.5130$	$\mu_1 = -5.6687$	$\tau_1 = -87.922$
$\sigma_2 = 0$	$\mu_2 = -12.334$	$\tau_2 = -177.250$
$\sigma_3 = -0.0971$	$\mu_3 = -8.1418$	$\tau_3 = -132.760$
$\sigma_4 = 0.2927$	$\mu_4 = -12.2230$	$\tau_4 = -178.410$
$\sigma_5 = 0$	$\mu_5 = -3.8008$	$\tau_5 = -57.458$
$\sigma_6 = 0$	$\mu_6 = -8.9351$	$\tau_6 = -236.87$
	$\mu_7 = -4.0952$	$\tau_7 = -144.180$
	$\mu_8 = -2.8231$	$\tau_8 = -126.460$
	$\mu_9 = -1.2525$	$\tau_9 = -59.846$
	$\mu_{10} = -2.3146$	$\tau_{10} = -7.5763$
	$\mu_{11} = -0.79982$	$\tau_{11} = -48.978$
	$\mu_{12} = -1.6290$	$\tau_{12} = -50.413$
		$\tau_{13} = -8.3296$
		$\tau_{14} = -26.433$
		$\tau_{15} = -6.214$
		$\tau_{16} = -4.211$
		$\tau_{17} = -1.139$
		$\tau_{18} = 2.673$
		$\tau_{19} = -1.329$
		$\tau_{20} = -2.543$

The total time required for the Elliott 503 machine for this run, excluding the seventh order aberration coefficients, is just under three minutes.

4.4 SURVEY of TYPE 131 PROPERTIES.

In high aperture systems, the aberration which mainly limits aperture of the system is the spherical aberration. We have assumed in section 4.1 that the spherical aberration is a quadratic function of χ in this class of objectives. To test this assumption, the computing programme was used to calculate the spherical aberration coefficients C_1, μ_1, τ_1 , for a number of values of χ . The results are plotted in Figures 4.1 and 4.2. In the former, the variations of the separate coefficients with χ are shown, and in the latter the total spherical aberration to seventh order is plotted for three different zones of the apertures, namely $\rho = 0.25 (f/2)$, $0.227 (f/2.2)$, $0.208 (f/2.4)$. In figure 4.3, the spherical aberration curve for the $f/2.2$ zone, has been redrawn (curve (1)), and is compared with curve (2), which is drawn for the same zone on the assumption that spherical aberration is a quadratic function of χ . The coefficients are B_3, B_2, B_1 of the quadratic are determined in the usual way by computing for three different values of χ . The good agreement between the curves (1) and (2) over the main region of interest shows that this assumption is reasonably valid in these systems also. This is how Cruickshank's introduction of χ is so useful in photographic objectives.

Returning to Figure 4.2, it is evident that for a certain range of maximum apertures it is possible to select values of χ for which the total spherical aberration is corrected. In this class of objectives, therefore,

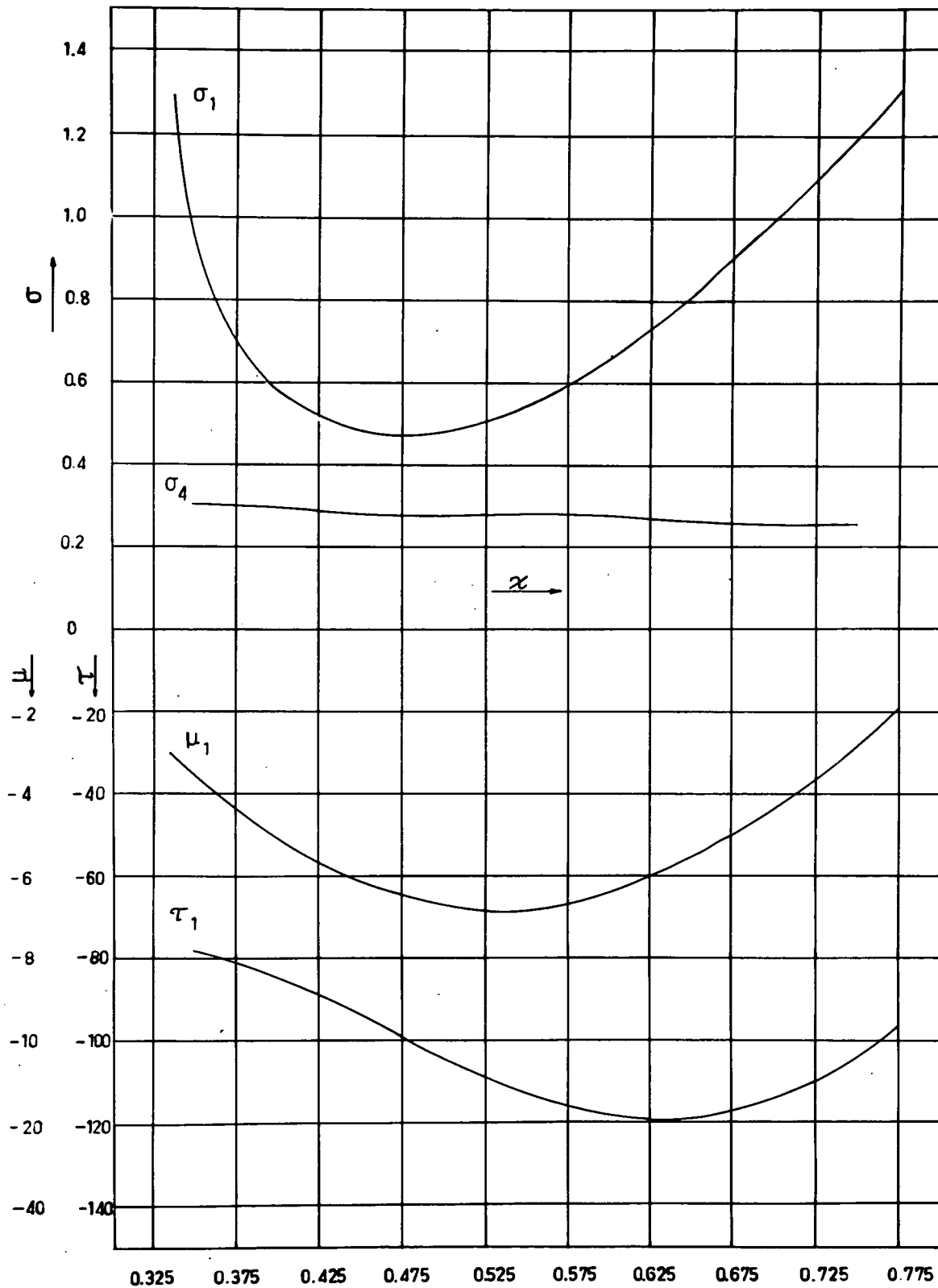


Fig (4.1)

$$\begin{aligned}
 \alpha &= 3.2 & \beta &= 0.84 \\
 \xi_1 &= 1.2448 & \gamma &= 0.9986 \\
 L &= 0.1 & T &= 0.5 \\
 P &= 1.20 \\
 \sigma_2 &= 3\sigma_3 + \sigma_4 = \sigma_5 = 0.
 \end{aligned}$$

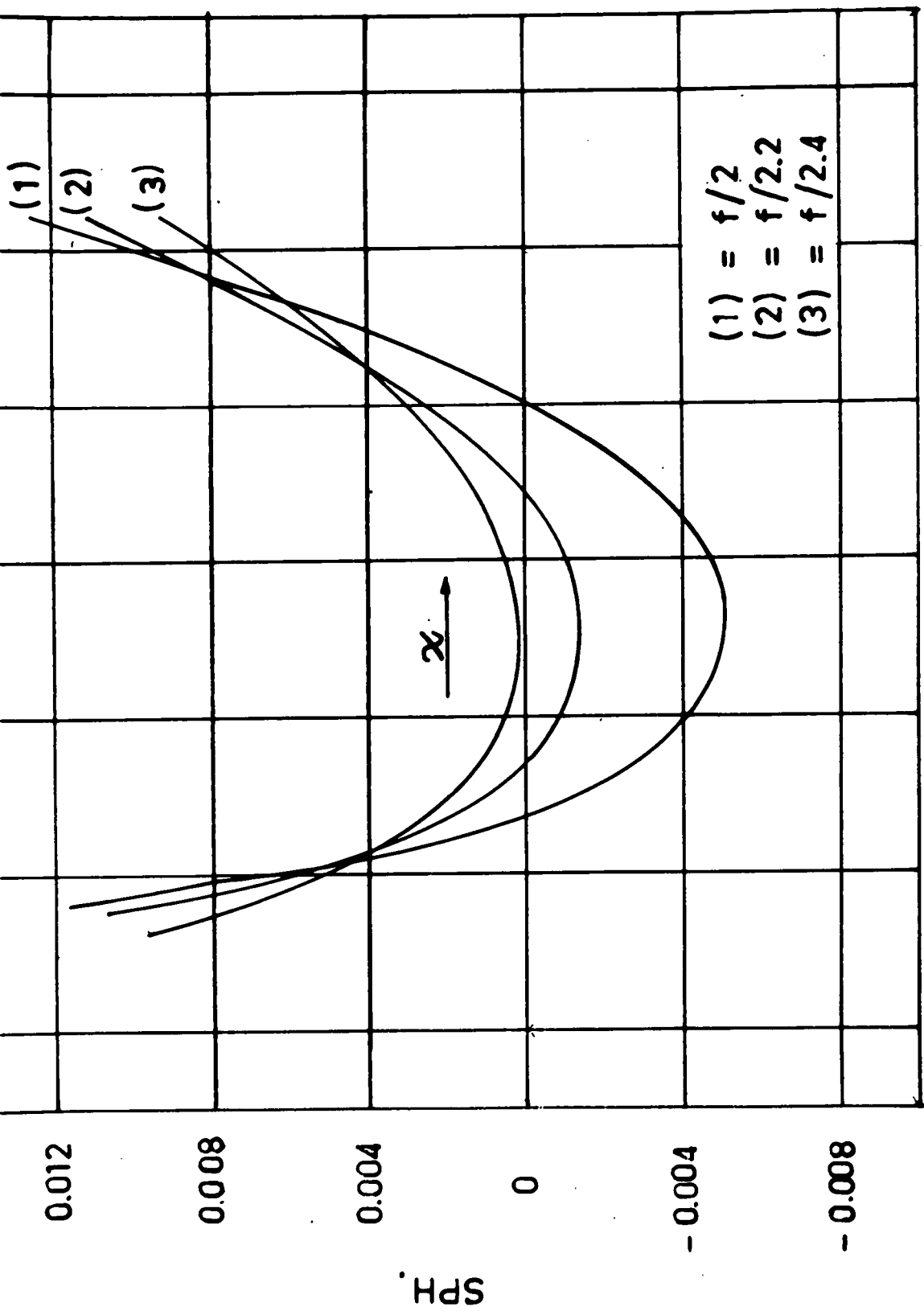


Fig (4.2)

$$\alpha = 3.2 \quad \beta = 0.84$$

$$\chi = 1.2446 \quad \gamma = 0.9886$$

$$L = 0.1 \quad T = 0.5$$

$$P = 1.20$$

$$\sigma_2 = 3\sigma_3 + \sigma_4 = \sigma_5 = 0.$$

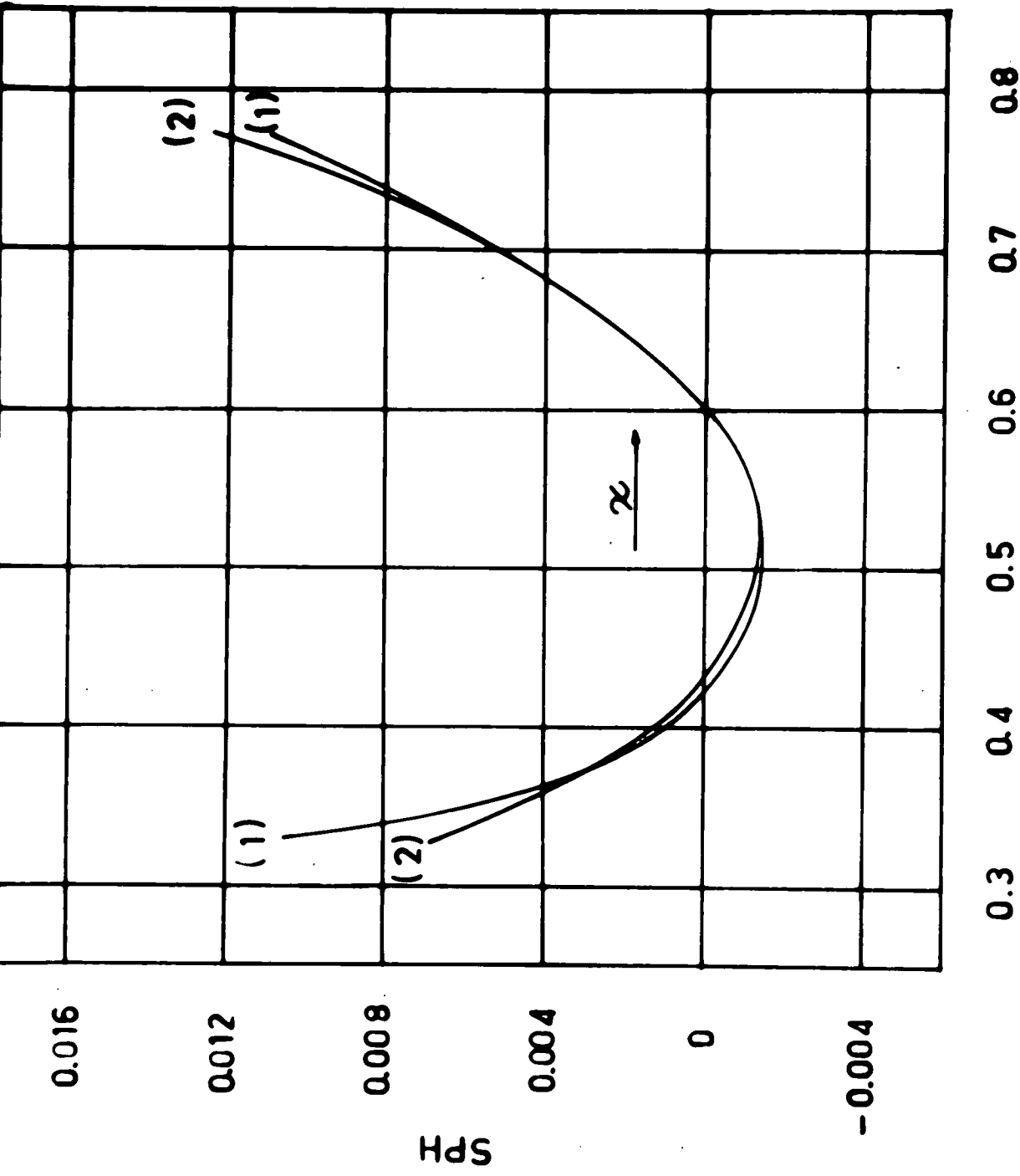


Fig (4.3)

$$\begin{aligned} \mathcal{I} &= 3.2 & \beta &= 0.84 \\ \mathcal{I}_1 &= 1.2446 & \gamma &= 0.9986 \\ L &= 0.1 & T &= 0.5 \\ P &= 1.20 \\ \sigma_2 &= 3\sigma_3 + \sigma_4 = \sigma_5 = 0. \end{aligned}$$

we have the familiar situation of the left-hand and right-hand solutions determined by particular values of χ . The question arises as to which of the two available solutions is the better. Inspection of Figure 4.1 shows that the zonal aberrations will be less for the left-hand solution because of the comparative values of the coefficients. The decision to give preference to the left-hand solution should rest, however, on a wider basis of information than the spherical aberration only. In other words, the general performance of the systems corresponding to the two solutions should be compared. To do this, we have drawn in Figure 4.4 the variations of all the fifth order coefficients as a function of χ as given by the computation using the programme.

It can be seen that the coefficients of linear coma, μ_2 , μ_3 , and the coefficients of cubic astigmatism, μ_4 , μ_6 , have fairly large values, and it is obvious that these aberrations will limit the useful field of the objectives. From the fact that these coefficients are slightly less in magnitude for the value of χ corresponding to the left-hand solution, it becomes quite clear that the left-hand solution is to be preferred rather than the right-hand solution.

For the time being, we can restrict our considerations in the further investigation of the left-hand solution only. For the aperture $f/2.2$, Figure 4.2 shows that the left-hand solution corresponds to $\chi \approx 0.425$, is a reasonable value to correct spherical aberration, and this is based on the original value of $\alpha = 3.2$. At this point, consideration is given to the effect on the spherical aberration curve of small variations in α . Using the programme again, we obtain results which are plotted in Figure 4.5. It is clear from this Figure that the spherical aberration curve as a whole rises with increase of α

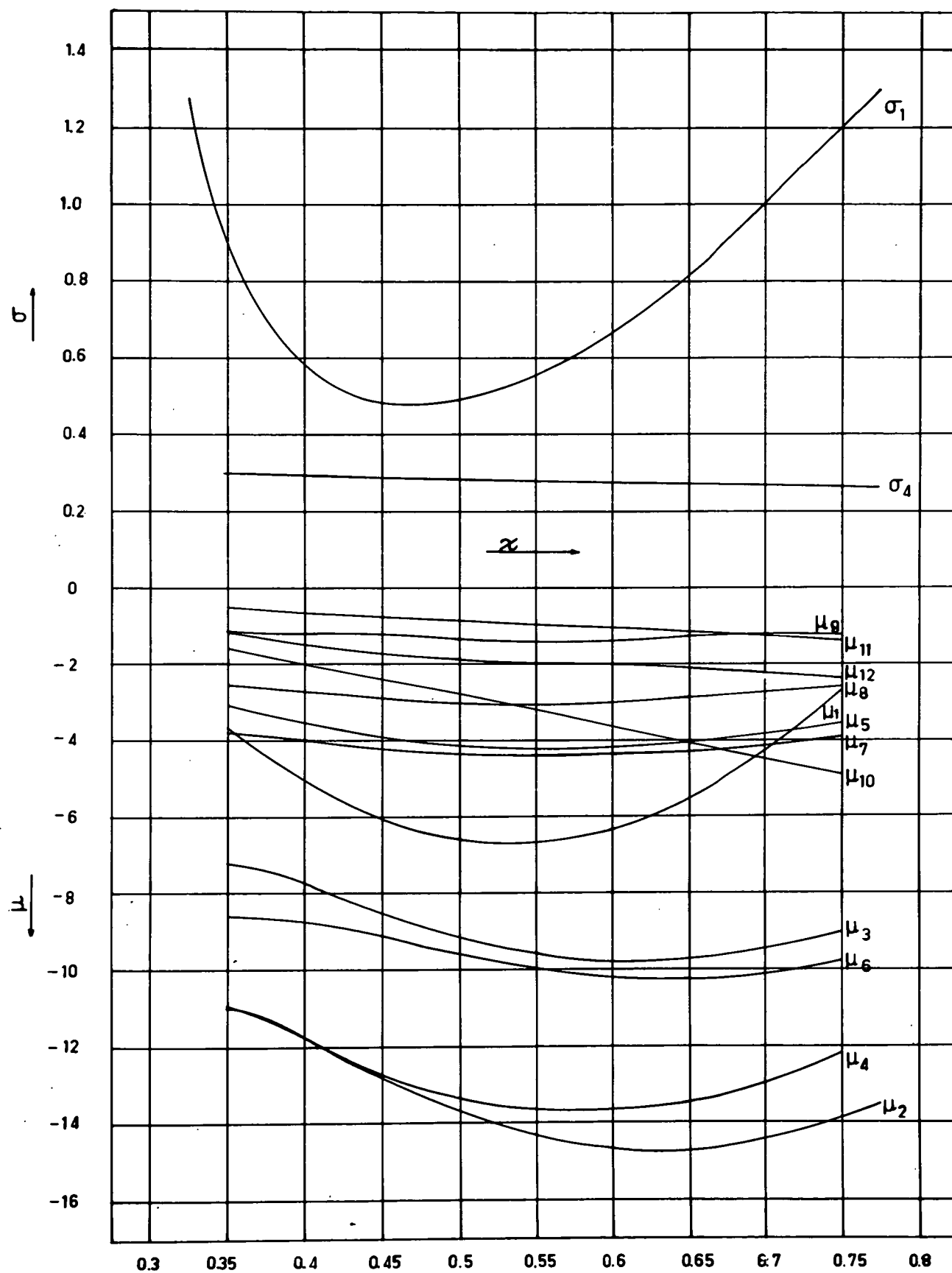


Fig (4.4)

$$\begin{aligned}
 \alpha &= 3.2 & \beta &= 0.84 \\
 \xi &= 1.2446 & \gamma &= 0.9996 \\
 L &= 0.1 & T &= 0.5 \\
 P &= 1.20 \\
 \sigma_2 &= 3\sigma_3 + \sigma_4 = \sigma_5 = 0.
 \end{aligned}$$

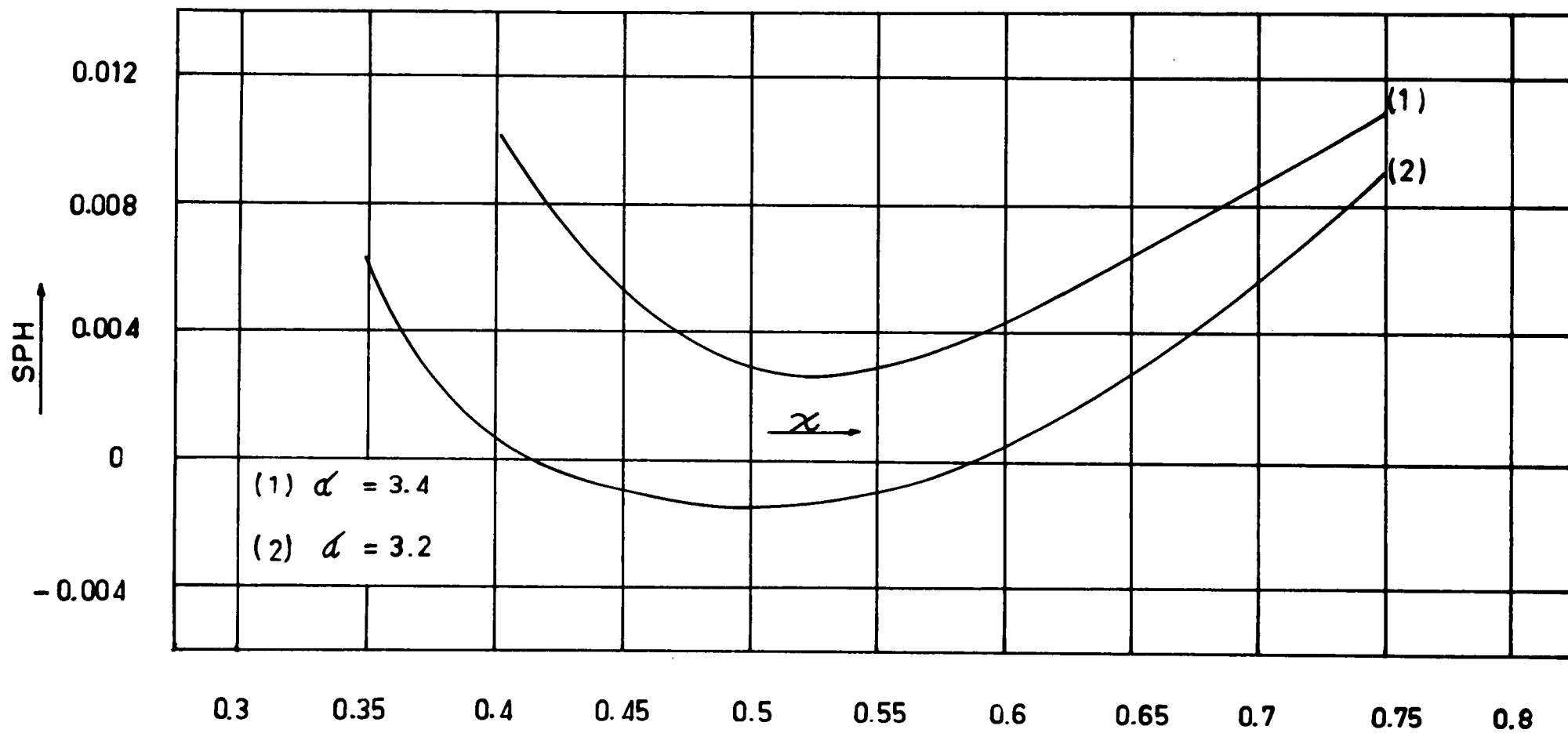


Fig (4.5)

and effects very profoundly the value of χ for which spherically corrected left-hand solutions are obtained. It also means that there is a limit beyond which we cannot increase α if we are to obtain spherically corrected solution at a maximum prescribed aperture.

In earlier chapters, it was stressed that higher values of α are desirable as they give low powers for the components and greater reduction of the petzval sum in this class of objectives. We have reached a point, however, at which it is evident that there is complicated interrelation between the parameters α , χ , and ρ in their effects on the correction of the spherical aberration of the system.

The more general effects of the variation of α are shown in Figures 4.6 and 4.7 where the variations of σ_1 , σ_4 , and the fifth order and seventh order aberration coefficients are shown.

It is evident from these figures that once again the coefficients of linear coma μ_2 , μ_3 , τ_2 , τ_3 and the coefficients of astigmatism μ_4 , μ_6 , τ_4 , τ_6 have considerable magnitudes. In addition, σ_1 rises rapidly with increase of α , and this will lead to rapidly increasing zonal aberration. The real problem in covering a moderate aperture is to balance spherical aberration with reasonable zonal residuals and at the same time reduce the comatic and astigmatic coefficients so that a reasonable angular field may be covered.

Another important point to be considered at this stage in Figure 4.6 is the variation in the value of σ_4 with α . In the previous chapter, we came to the conclusion that the petzval sum will reduce (i) with decrease of \bar{V}_b , i.e. with increase of α for a given V-number of the first component, and (ii) with increasing positive value of the shape of the central component.

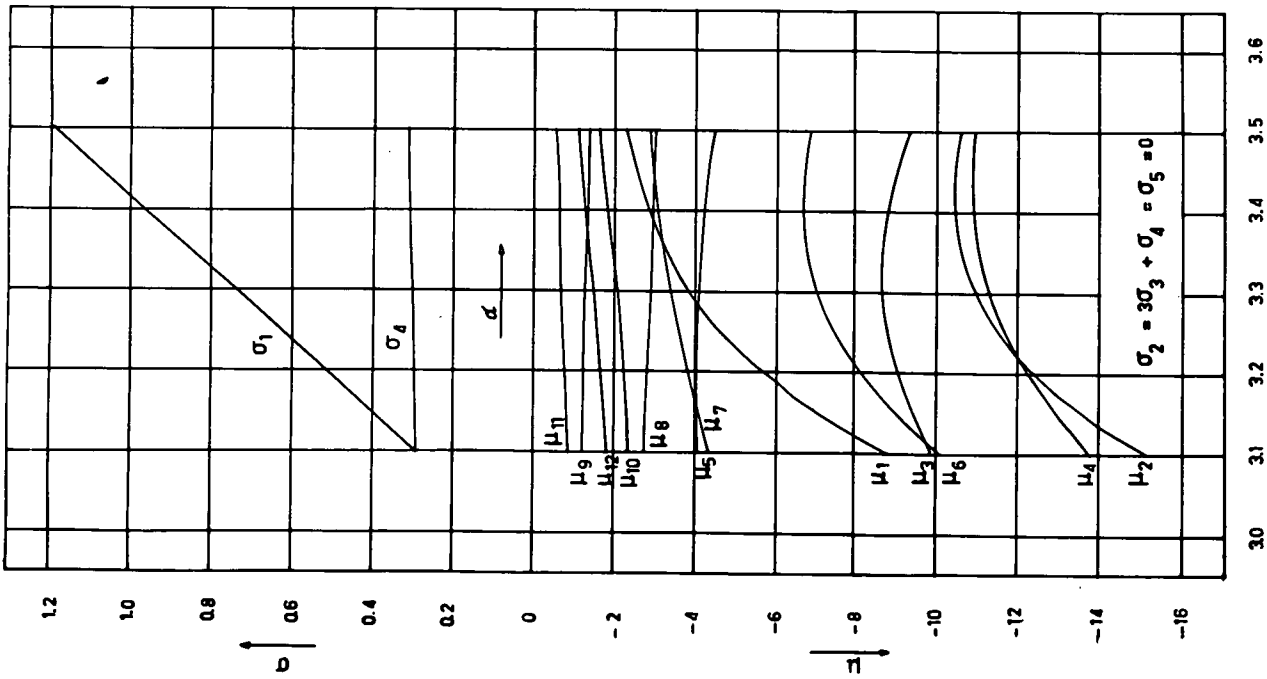


Fig 4.6(a)

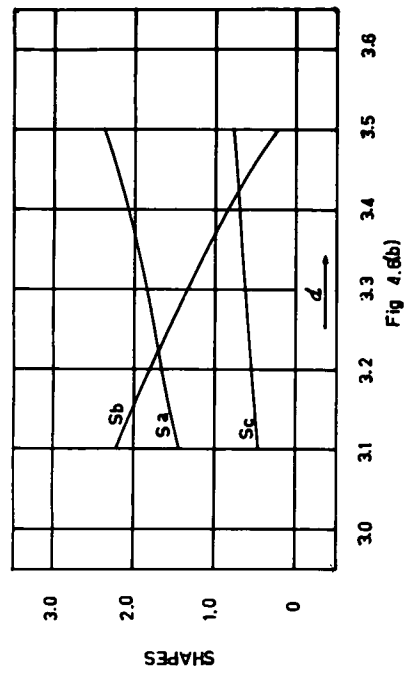


Fig 4.6(b)

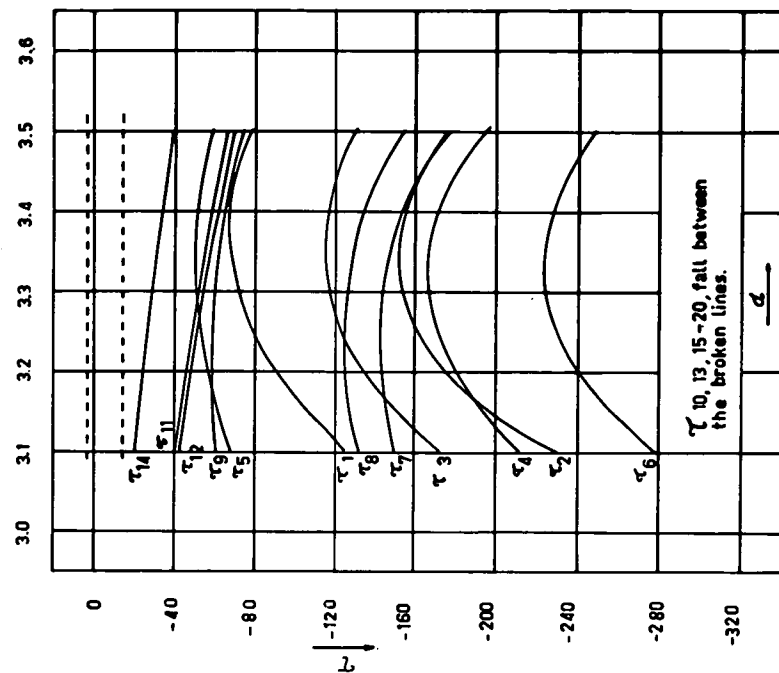


Fig (4.7)

It appears from Figure 4.6 that the σ_4 value is practically unaffected by variation of α . If anything, it is increasing slightly with increase of α . The reason for the near constancy of σ_4 is that the reduction in σ_4 due to the increase in α is compensated by a reduction in the shape of S_b with increasing α . This becomes apparent from Figure 4.6B where the shape changes in the components as calculated in the programme are plotted as functions of α . What is observed in the actual problem is that with increase of α , the shape of the central compound decreases, and its negative contribution to the petzval sum decreases in amount. At the same time, the shape S_a of the first component increases with increase of α resulting in a reduction of its positive contribution to σ_4 , and thus these two shape changes compensate each other, leaving the total σ_4 practically unaffected.

Returning to Figures 4.6 and 4.7, it is clear that the fifth and seventh order aberrations, being of the same sign, will augment each other. This suggests that positive third order residuals are inevitable. To balance the effects of these negative coefficients, it is necessary in the first place to observe the effects of the introduction of positive third order residuals. Beginning with $R_2 = 0.1$, the same programme was used again to compute the new values of the fifth order aberration coefficients and the results are plotted in Figure 4.8. It can be seen from this Figure that there is a considerable improvement in fifth order, spherical, comatic and astigmatic coefficients, but this is at the expense of an increase in σ_1 .

Next, in addition to $R_2 = 0.1$, a residual $R_5 = 0.15$ was given in the programme data and the results are presented in Figure 4.9. This Figure presents the useful information that there is an overall improvement in fifth order coefficients of coma and astigmatism, and there is a reduction in the

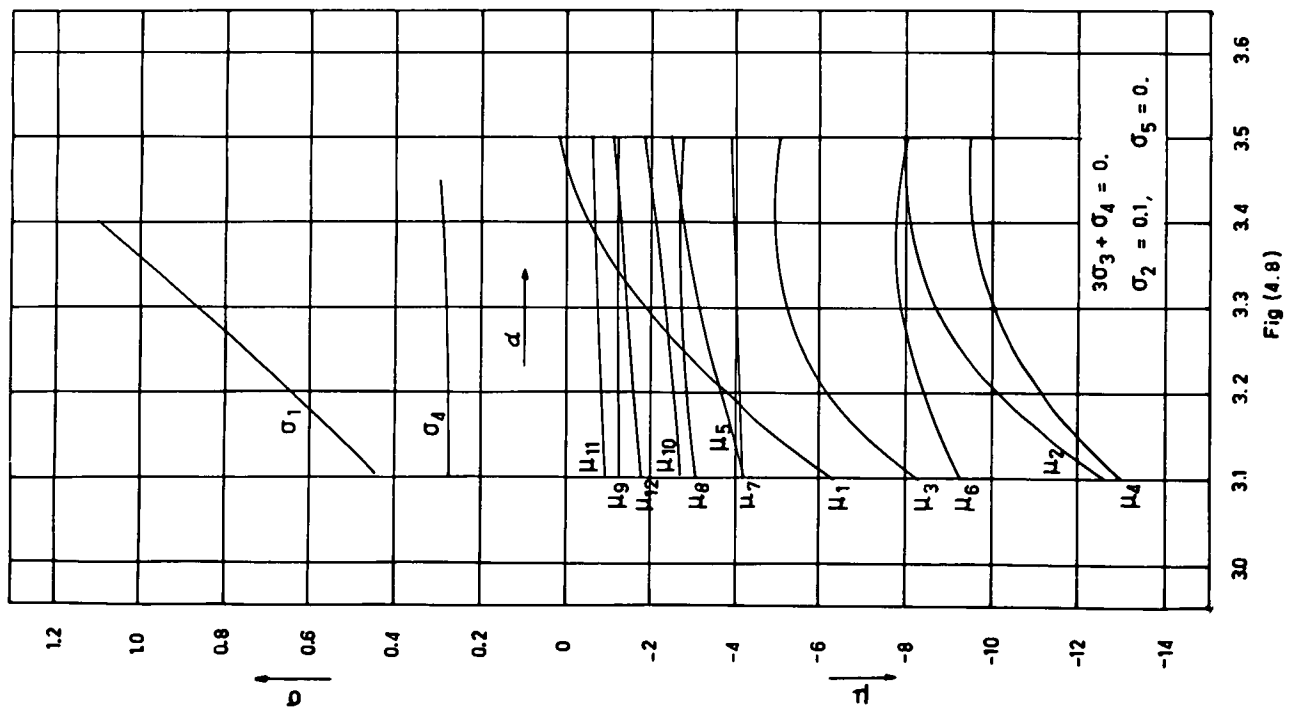


Fig (4.8)

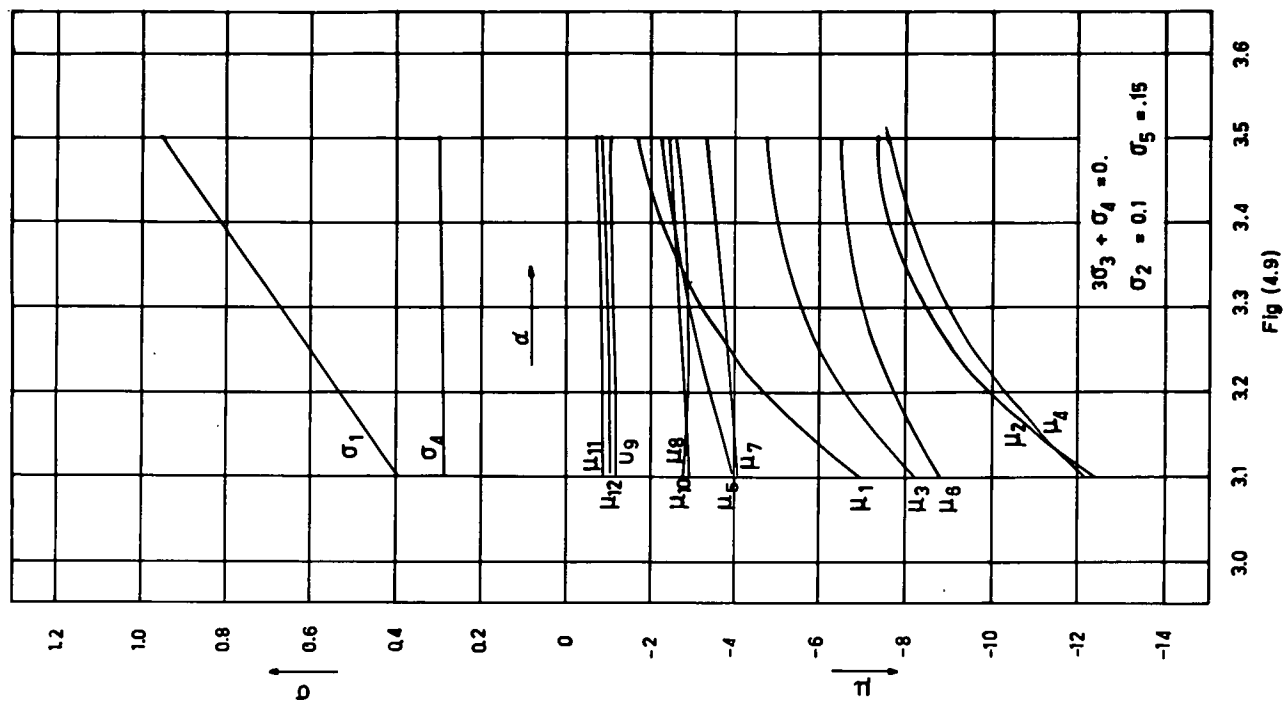


Fig (4.9)

value of σ_1 . It is observed that the introduction of the R_2 , R_3 residuals has had little effect on the seventh order aberration coefficients and it is not possible to do much with these coefficients.

The remaining third order residual to be considered is that for σ_3 . Up to this point, we have made $3\sigma_3 + \sigma_4 = 0$. In Figure 4.10, we show the result of making the mean field flat in the third order rather than the tangential field, i.e.

$$2\sigma_3 + \sigma_4 = R_3 = 0 \quad (4.5)$$

It is evident that the fifth order comatic, and astigmatic coefficients, are increased, but there is a significant decrease in the value of σ_1 . Since this is accompanied by more negative value of μ_1 , the spherical aberration balance can only be obtained for apertures lower than those used previously. The off-axis performance is worsened. In the next run with the programme, the adjustment $4\sigma_3 + \sigma_4 = R_3 = 0$ was made, giving σ_3 about -0.07. The general results of this change are shown in Figure 4.11 and indicate a general improvement in the values of the fifth order coefficients. A further run with the adjustment $3\sigma_3 + \sigma_4 = 0.15$, thus making σ_3 about -0.03, gives the results shown in Figure 4.12. It follows from this study of the effects of the variation of the value of σ_3 on the solutions that in the type 131 objective it is difficult to obtain good spherical and comatic correction at a high aperture, together with flat anastigmatic field. If we are prepared for the mean focal field to be curved, the opportunity for developing a system with an aperture of $f/2$ would be considerably enhanced. One way remains of improving the situation on the axis a little, and that is a further increase in the third order distortion residual, R_5 . This means that the

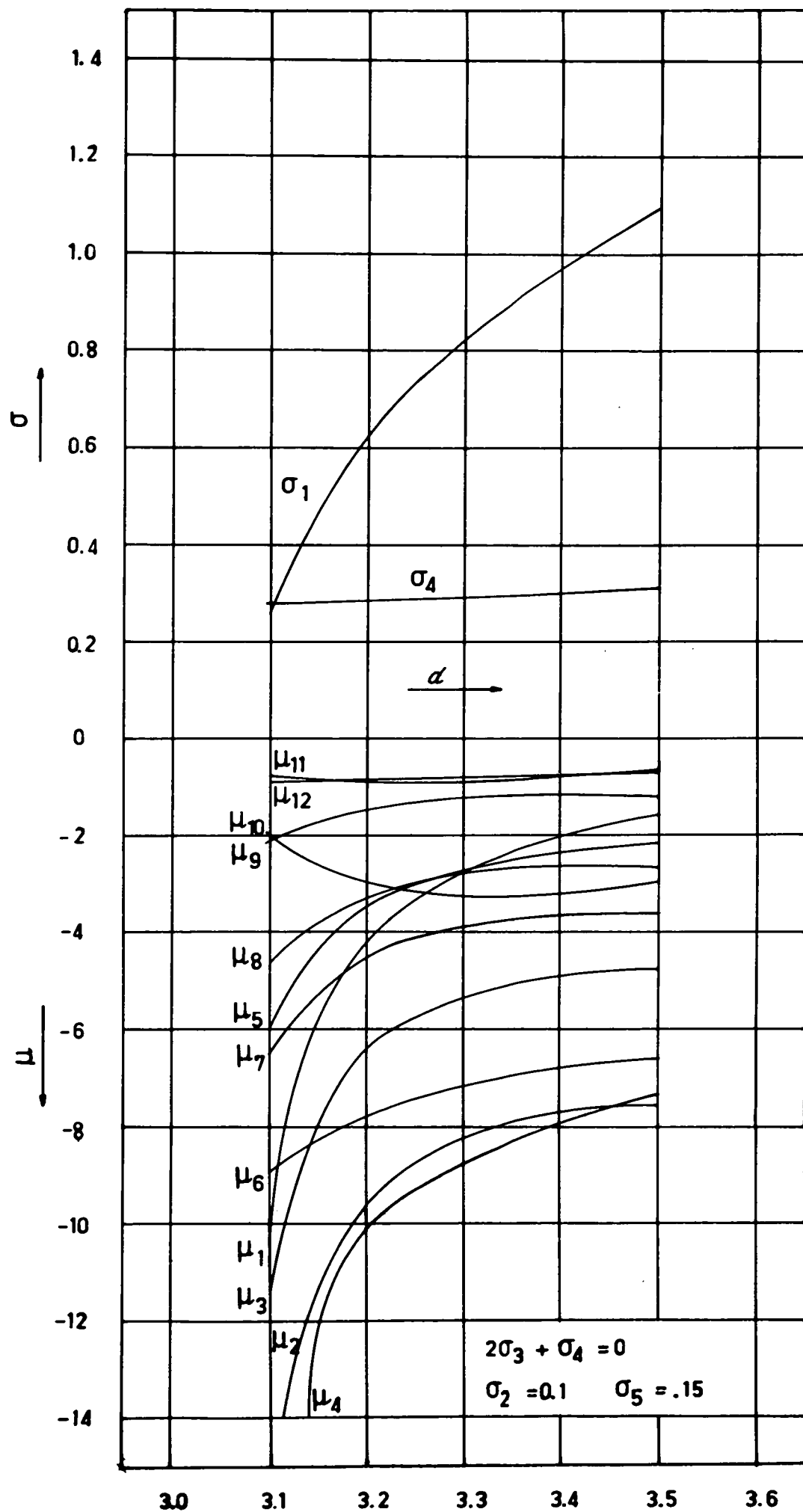


Fig (4.10)

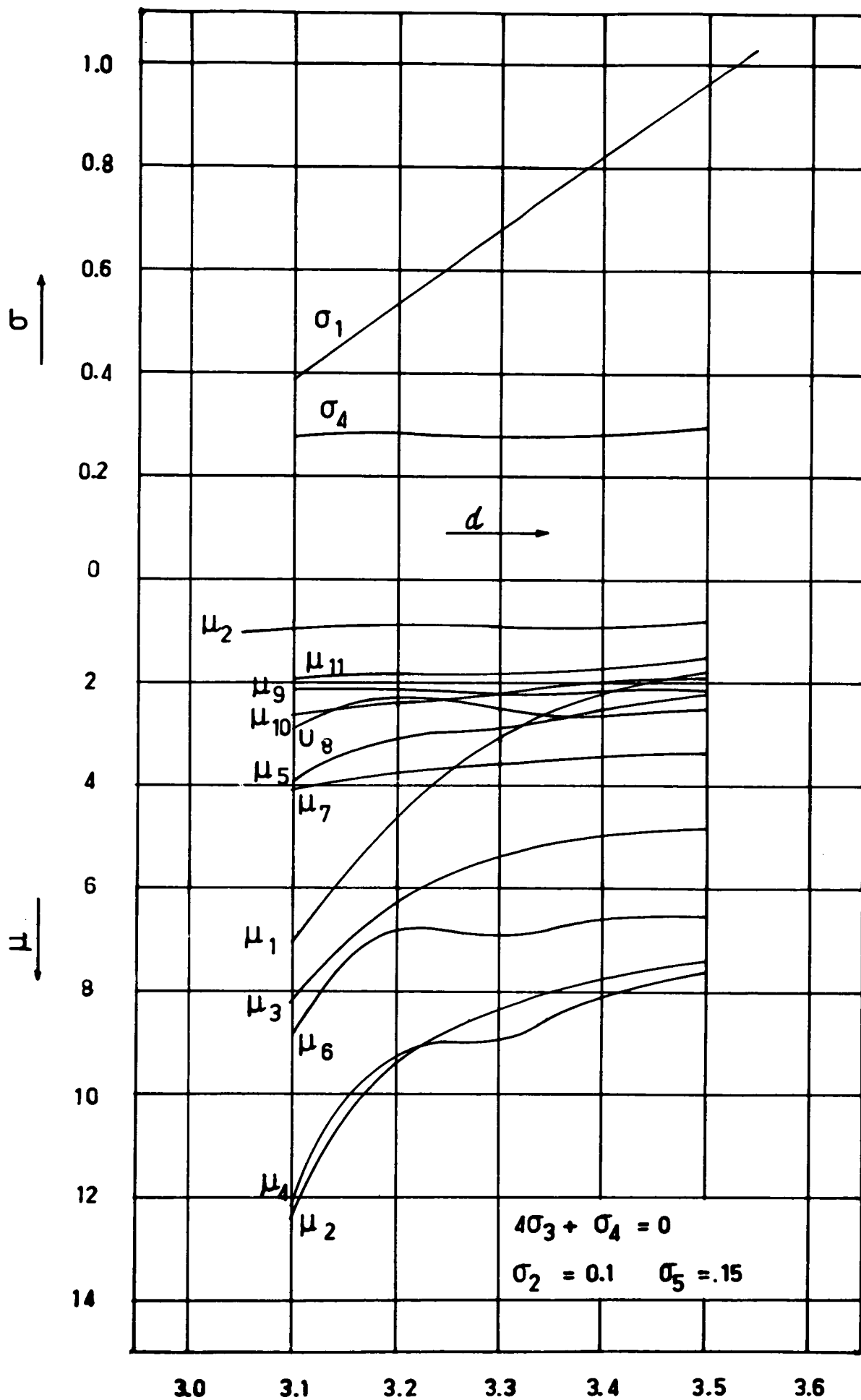


Fig (4.11)

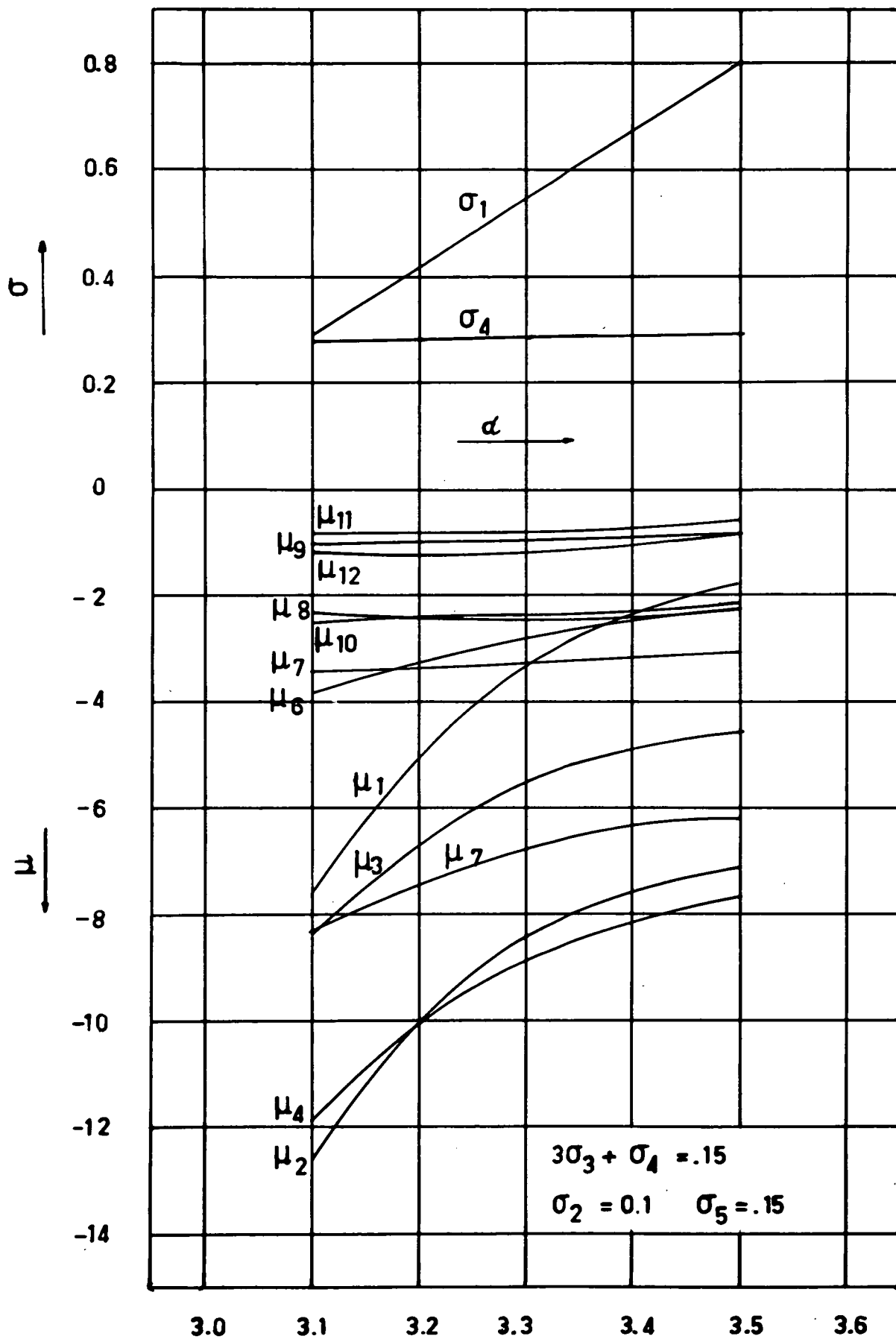
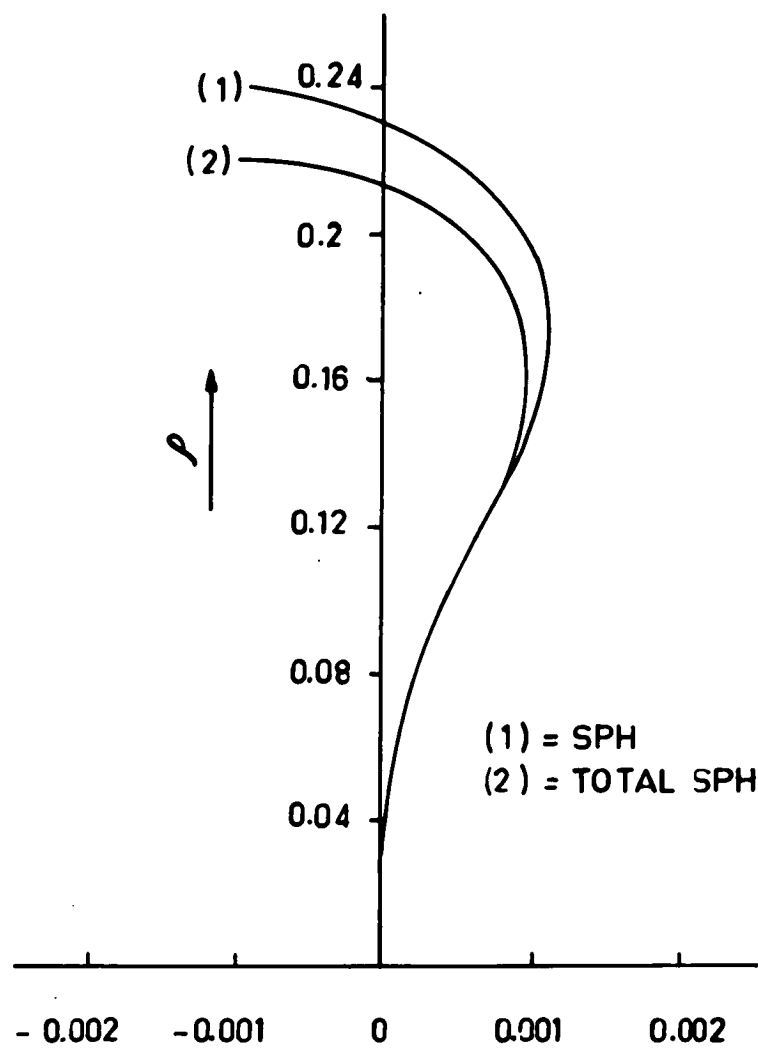


Fig (4.12)

distortion is quite deliberately allowed to increase, but, in any case, since the angular field to be expected is not very great the effects of the increased distortion will not be serious. As a compromise, then, R_s is allowed to attain the value of 0.2. This device has probably been used by Bertele and others because Lee (1940) mentions that the triplet objectives with central compound components suffer from distortion. At this stage, further investigations were made on the effects of small changes in P and β , but nothing really significant resulted from this. It would seem that the general results summarised in the last few figures indicate the general possibilities inherent in type 131 objectives. Figure 4.13 shows a check of the spherical aberration of the system, curve (1) showing the spherical aberration based on the calculated values of α_1 , μ_1 , τ_1 , while curve (2) shows the results obtained by a full ray trace. The agreement is of the order which one can expect, and curve (2) indicates that a system might be reasonably expected to have a good performance on axis, at an aperture $\rho = 0.22$, i.e. $f/2.25$, or thereabouts. Figure 4.14 shows the spot diagrams of this system at the relative aperture $f/2.25$ on axis and at 4° off axis. It is obvious that the field covered for this objective will be quite small. This prompted an investigation of what might happen if the aperture of the system is decreased a little. In Figure 4.15, spherical aberration curves for the e-line, both predicted (curve (1)) and traced (curve (2)) are shown for a system having a relative aperture about $f/2.5$. In Figure 4.16, the spot diagrams up to 12° off axis are shown for this system and they indicate clearly that the type 131 construction is not capable of being pushed to much higher apertures than this. Comparison of Figures 4.14 and 4.16 shows the effect of trading away a little aperture in order to gain field coverage.



Fig(4.13)

$V = 0^\circ$

$V = 4^\circ$

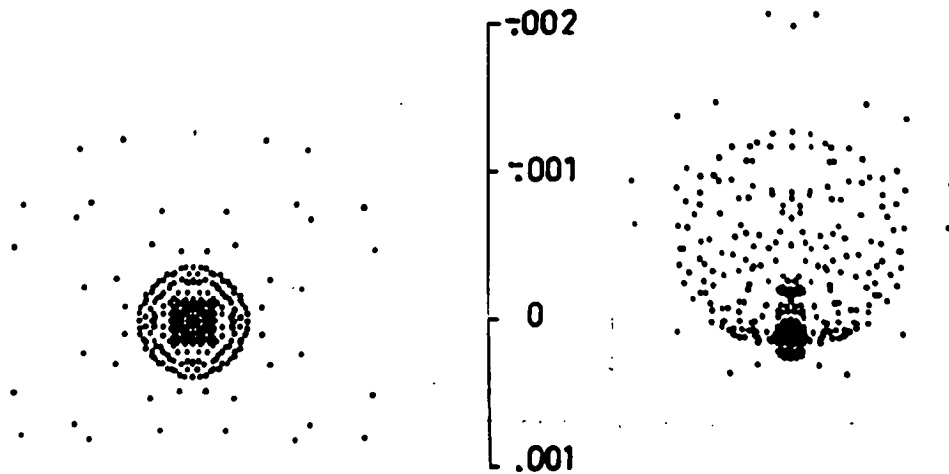


Fig (4.14)

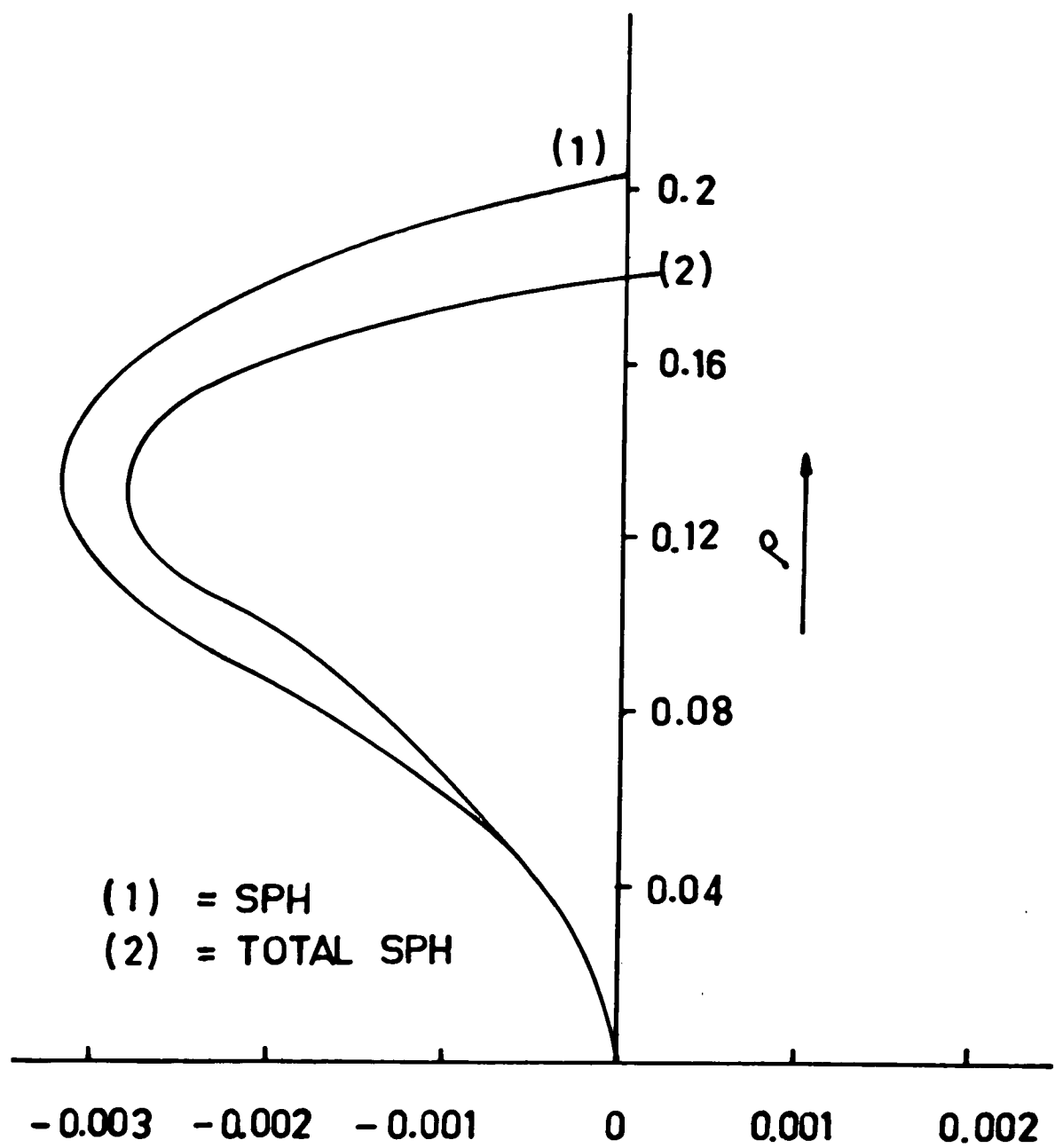
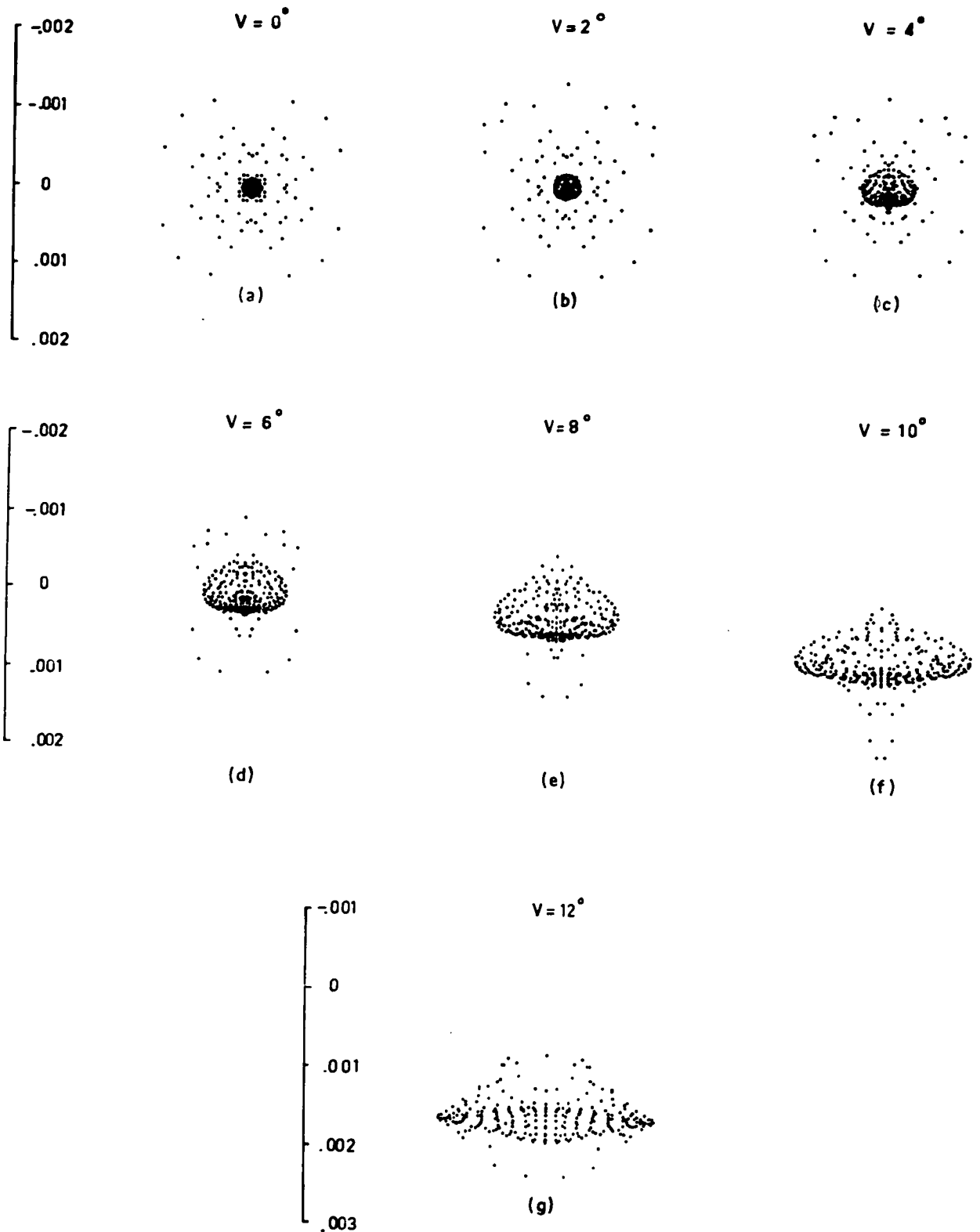


Fig (4.15)



$X = -0.002$

Fig (4.16)

TOTAL SPH. IN C.E.F LINES

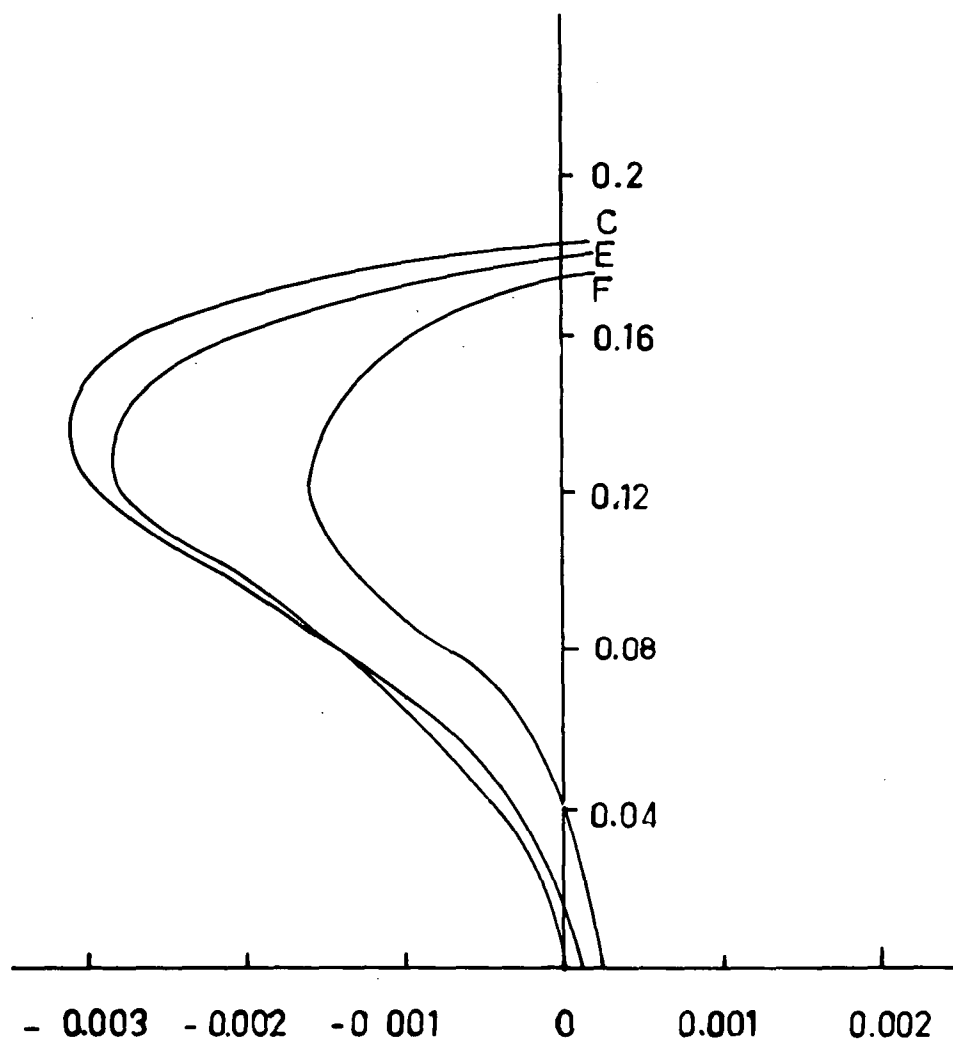


Fig 4.17(a)

DISTORTION IN C.E.F LINES

ρ

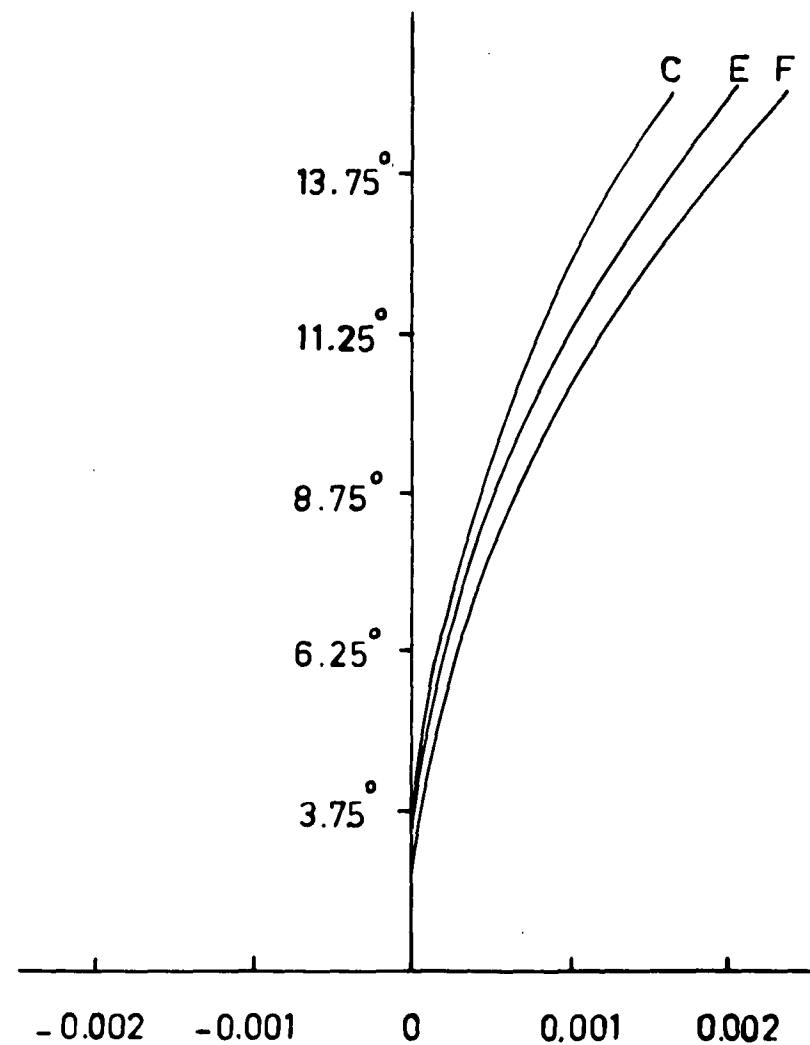


Fig 4.17(b)

So far, the question of chromatic correction of the objective has not been considered, in particular a high value of T has been used without any real investigation of the resulting transverse chromatic aberration. In Figure 4.17A, spherical aberration curves are plotted for the C, E, F lines and in 4.17B the distortion curves are plotted for the same lines. It is seen from these Figures that neither of these aberrations are in any sense out of control. Small adjustments of L and T would suffice to bring the C and F lines closer together and thus decrease the chromatic aberrations still further.

4.5 CONCLUSIONS.

We have faced three difficulties in controlling the aberrations in the investigation of the design of 131 triplet objectives to achieve high aperture with a reasonable field. The first difficulty was in balancing the spherical aberration with low zonal aberrations. The second one was in reducing the magnitudes of the comatic coefficients, and the third hurdle was in controlling the astigmatism. The first two difficulties can be eliminated to a reasonable extent, subject to the limitations of the type of the construction with low powers. The control of astigmatism to cover a reasonable field can be obtained only with low petzval sum in the triplet. If one goes to the second Chapter, it is clear that we developed a new region which gives low powers with high values of α , P , χ in the initial solution, and we proved later that the high value of α is practicable by replacing the central component by a compound component. In the next Chapter, we showed that the high value of P used in the thin system can be reduced to a very low and practical value, with positive meniscus shape and

with low \bar{V} of the compound negative triplet, i.e. we developed the physical principles to meet the requirements of high aperture systems, which also cover a reasonable field. Accordingly, we achieved an aperture of $f/2 - f/2.2$, in an objective which covers a narrow field. The field limitation is due to high petzval sum value present in the system. This means we could not obtain a final solution with reasonable positive meniscus shape, i.e. the last curvature of the compound triplet is not deep enough to cause substantial reduction in the total petzval sum. This shows that the power distribution in this compound triplet is not in the proper proportion to obtain the proper shapes. This means that the glass selection was not good enough to get the proper power distribution which decides the proper shape. In this compound triplet, it is not only the fourth curvature that matters, but also the cemented third surface curvature because this third surface with deep negative curvature contributes high amounts of negative third order spherical, comatic, and astigmatic coefficients, and these aberrations will enhance with the high negative magnitudes of fifth and seventh order coefficients, resulting in the system suffering from the over-corrected aberrations. This is what happened in the investigation of the design of 131 objectives with the selected glasses mentioned in section 4.3. This reveals the fact that no high aperture system with moderate field coverage can be designed with large third order surface aberration coefficients. Under these circumstances, one has to increase the refractive index of the central component of the compound component to have smaller curvature. The amount of increase must be judged by the magnitudes of the power. One has to change the refractive index of the last component in a similar fashion to obtain the final solution with deep fourth curve. Thus the glass selection is also very important, especially in the compound component.

The real problem in getting a good solution at the final stage lies in understanding the properties of the surfaces involved in the objective to be designed. Besides the glass selection, in 131 triplet the field will be limited by coma, and distortion because this objective cannot have symmetry with respect to aperture stop. The distortion can be eliminated to a reasonable extent at the cost of little increase in q values, provided one is prepared for longer objectives, i.e. the distortion is very sensitive to the length of the back air space. This is what Kingslake (1940, p.65) mentions about the triplets with central compound components, that "His (Bertele) plan was to weaken the rear face by taking power from the rear to the front, where it was added in the form of an approximately aplanatic meniscus-shaped lens immediately following the normal front lens. The resulting distortion was then balanced out by increasing the rear surface". Probably in the 131 one can reach aperture of $f/2$ with an angular field coverage $\pm 15^\circ$ with the proper glass selection.

If one satisfies in his design the conditions developed in the second and third chapters (low powers, low petzval sum value ... etc.), in the final solutions these objectives give excellent central definition with moderate field coverage.

A study of more complex objectives is needed to exploit the principles which have been opened up in this work. The writer commenced the detailed study of the 133 objective. It had been hoped originally that his stay in Hobart would afford an opportunity to make a thorough study of this objective, but the necessity of returning to India has not permitted him to do this.

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