

*"Tip in three blops of milk...."*

An ethnographic study of the development of mathematical concepts and language in Early Childhood Education.

*Maria*

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## DECLARATION

This research study contains no material which has been accepted for the award of any other degree or diploma in any tertiary institution. To the best of my knowledge and belief, this study contains no material previously published or written by another person, except when due reference is made.

E. Dunn

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## ABSTRACT

It is becoming increasingly obvious that many students taught according to traditional methods do not develop a thorough understanding of the concepts of mathematics. Much of what they can do is mechanically learnt with no links to a body of interrelated concepts, to the web of meanings which lie at the heart of a quantitative view of the world. It would seem that what is lacking in the teaching of mathematics is the teaching of its language. A language is truly learnt when it can be used to communicate ideas to others and can provide a medium for further thought.

The aim of this study was to take elements from the theory of language and of mathematics education and apply them in a classroom setting. The study employed ethnographic methods to record the interactions and the outcomes of the curriculum. This dissertation explores the way the curriculum worked for a group of low achieving students, and attempts to make further suggestions for improving practice.

Eleven six and seven year old children, pupils of a country primary school in Tasmania, were selected by their teachers to participate in this study. They were judged to be poor performers in both language and mathematics. These children were taught as a small group for five weeks, one hour each school day by the researcher. All the mathematical activities were presented in a narrative context, they all had reference to the children's lives, and the work itself was carried out using concrete materials. The classroom climate and general teaching approach was very much like that described by Bickmore-Brand (1993) whose teaching in mathematics followed the language teaching advocated by Cambourne. Children were given practice at speaking about mathematics by frequent audio taping at the conclusion of sessions.

The information collection methods employed were those required of ethnographic studies, including the participant observer, her diary of reflections, children's taped language, children's work and the remarks of their teachers. Individual testing sessions were conducted during the sixth week of the study. The results showed, that compared with three control children, the participating children had learnt many new concepts, and could apply them in similar as well as in challenging new situations. Their taped language improved markedly, as did the children's ability to express themselves mathematically and to understand directions concerning mathematical problems.

The happy atmosphere of the sessions, the good relationships which developed, and the increased competence of the children in both mathematics and language, confirm the choice of the teaching methodology. The problems encountered, the children whose behaviour was at times negative, have provided ideas for further developments.

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# CHAPTER 1

## INTRODUCTION

*"We are all fallible. With enough exposers around, all of us are certain to get caught in the end. The only place completely safe from exposure is the tomb. The only truly successful defensive rigour is rigor mortis."* (Midgley, 1989, p.68)

### CONCERNS OF THE STUDY

The reason for undertaking this study was a concern for making mathematics education more effective during the years children spend in Early Childhood Education. Throughout the study the focus was always on the practical concerns of a classroom teacher.

The study attempted to address two major concerns. One was the problem of developing a curriculum which was "real" and accessible to the children, because it was built on their current concepts and knowledge. Such curricula, when reported in the literature, often rely on teaching very small groups of children. The question was, how can they be best used in a full class? The second concern arose from the observation that many children come to school with a home language which, though English, does not provide an adequate foundation for the communication of the traditional mathematics curriculum. How can the mathematics curriculum address the language needs of children?

Many writers, among them Noddings (1990), have remarked that one of the problems with current advice about teaching is the assumption made about the intensity of interaction possible in a classroom between teacher and student. Labinowicz (1985) could see the problems which might arise from the difference between his small group and teaching a class: "The problems I experienced in teaching only four children will be used as springboards for discussion of the realities of teaching a group of as many as thirty-five children." (p.221) Schoenfeld (1992) also felt that the gap between theory and instructional practices was still too great. He had the constructivist school of instruction in mind.

The best methodologies imply steadiness of mind and a calm spirit, a leisurely atmosphere for the interaction of teacher and child. Such tranquil encounters are to lead to unhurried conversations between teacher and student, where the child's current concepts can be demonstrated or voiced, and the teacher can hear them, record them, and plan strategies designed to develop incomplete concepts further. Such an image brings to mind the tutoring model of instruction not the classroom teaching one. Many current theories advise to take the child's current concepts as the starting point, and suggest methods of teaching which rely heavily on

communication between teacher and student. Since success in communication largely depends on the linguistic skills of both teacher and student, the question of language competence becomes an allied concern. If teaching mathematics is an exercise in communication, and mathematics, like other forms of language is a cultural construct, then teaching and learning mathematics could be viewed as a language learning task. If this is so, we must ask how best to teach language, in this instance the language of mathematics. Many research studies show that family background, especially the type of language used in the family is very important in children's school learning. (Donaldson, 1992; Bradley, 1990) No teacher could rest accepting the situation that a child's family background should forevermore destine that child's educational achievement level. Many studies have demonstrated and lamented the very low level of understanding students have of mathematical language. One may take as an example Davis' (1989) analysis of the "disaster studies", or Martin Hughes' (1987) chapter on "What is the problem?". The implication is that the school failed to teach students the *meaning* of many mathematical words, and hence the concepts the teacher and text books use. If schools and teachers are not doing as well as they should at the moment, how could better results be achieved?

This study attempts to shed light on these two questions: how can a teacher teach on the basis of current concepts in a class of thirty children, and how can teaching consciously address the language gap between home and school. Relying on what seemed the most promising of theoretical and practical advice in the literature, a methodology was worked out for the teaching of mathematics to a group of low achieving six and seven year olds. This is the report of what was planned, what actually happened and what the learning outcomes were. Throughout the study there was special emphasis laid on the teaching and learning of the language of mathematics.

## RESEARCH METHODOLOGY

The study attempted to record the "small facts" of teaching and learning in the hope that some of them might reveal "large issues" (Geertz, 1973, p 23). The researcher carried the dual role of teacher/researcher. This mode of operation fitted well with ethnographic methods. The role was that of the participant observer, central to this method. Eisenhart (1988) stressed the importance of the researcher's deep involvement as an essential condition for good ethnographic work:

"Ethnography depends on the researcher's active and personal involvement in the data collection and analysis; where this involvement is unlikely or impossible, ethnography should not be used."(p.109)

In this case the condition was fulfilled. Eisenhart gave a useful summary of what information collection methods could and should be used by an educational anthropologist. Her list was as follows:

1. participant observation;
2. ethnographic interviewing;
3. search for artefacts;
4. researcher introspection.

In this study the participant observer was the teacher/researcher. The two roles, while they can be played simultaneously, have distinct characteristics and create different demands. Labinowicz (1985) noted,

"Somewhere there is a fine line between interviewing and teaching, with the interviewer and the teacher demonstrating different questioning, listening and responding behaviours" (p.221)

The question about a dual role always has to be, how much of each? In this study the researcher taught the children for one hour a day for five weeks. During this time she was fully a teacher. In that she arranged for information collection strategies to operate during this time, she was also a researcher. In the individual testing sessions conducted during the sixth week, the researcher acted like an interviewer, and the children's responses showed that they noticed and went along with the perceived difference. A "Research Diary" was kept daily, which recorded the intentions of each teaching session as well as the reflections on what actually happened. Diary keeping was part of the researcher role and also the means of "researcher introspection". All these facets of the study would classify it as a typical action research project in the Habermas (1977) tradition.

The plan concerning "ethnographic interviewing" was that the children were to be interviewed at the end of each teaching session, and that these interviews were to be audio taped for more leisurely analysis later. This proved to be too much of a burden on the children's patience at times, so on days when work took longer than expected or the children appeared restless, no taping was done. Sixteen teaching sessions included taping. Twelve individual testing sessions were also taped in full. The twenty eight taping sessions form the information source of "ethnographic interviewing".

Two types of "artefacts" were collected in this study. Children's language was collected through audio taping at the end of the teaching sessions, and during the testing interviews carried out in week six. The testing interviews were fully recorded and transcribed. In this way the language used in the testing sessions, the teacher's language as well as the children's, was preserved in full. The second type of artefact was the children's written or drawn works. These were collected at the end of each session.

Ethnographic methodology was selected for this study, so that a complex situation could be described as accurately as possible with the aim of identifying further issues in need of exploration and clarification. It was accepted that this methodology has its intrinsic limitations. Its results may



not transfer easily to other groups and its lessons may not be generalisable over a notional population. Its value lies in that the picture presented using this methodology has the capacity to tell a story with sufficient life to allow readers to make judgements. It was felt that the information collected for this study should satisfy the requirements of a good anthropological study. Taken as a body of evidence it should form the "thick description" Ryle (1949) demanded of anthropology.

## CHAPTER 2

### THEORETICAL BACKGROUND

*"A scientific theory is an organ we develop outside our skin. This is one of the many reasons why the idea of completely untheoretical, and hence incorrigible, sense-data is mistaken. We can never free observation from the theoretical elements of interpretation." (Popper, 1968, p.163 )*

#### THE PLACE OF THEORY

The practical questions or concerns which inspired this study have already been stated. These were, *how can a teacher teach on the basis of current concepts in a class of thirty children, and how can teaching consciously address the language gap between home and school?* Focus on "current concepts" inevitably brought to mind the twin notions of reality and meaningfulness. Many writers emphasise the need to create a mathematics curriculum which is both "real" and meaningful to the children, one which provides opportunities for active learning. The learner should be enabled, they say, to construct her knowledge actively. The learner "actively constructing her knowledge" brought in questions about the role of action in learning.

A re-examination of the theoretical background seemed to be called for, if the well-worn phrases quoted above were to take on a significance over and above that of mere slogans, and once again have the capacity to guide the development of practice. The theoretical questions allied to the practical ones which were considered for this study were the following.

1. What is the role of action, including play, in developing symbolic behaviour?
2. How are language and meaning related?
3. What is the relationship between language development, reality and culture?
4. What is the nature of mathematics and its language?

The philosophical background is centred around linguistic questions. All human arts, including mathematics, are rooted in language and function through it. It is impossible to consider mathematical language without recalling the larger framework provided by the theory of language. The four questions will now be addressed in turn in the hope of finding further guidance for curriculum development and classroom management.

## THE ROLE OF ACTION, INCLUDING PLAY, IN DEVELOPING SYMBOLIC BEHAVIOUR

One of the perspectives on human knowledge is to see it as the result of evolutionary processes. After much of what each species could do was gradually laid down in the genetic code, evolution took a new turn with the production of individuals who could acquire a repertoire of behaviours in direct response to their specific environment. Blakemore (1977) said this in the course the 1976 Reith lectures:

"Inherited reflexes contain a static description of the events of high probability in the past experience of the species, but learning allows each animal to add a stock of personal secrets to its description of the probabilities of the world." (p.114)

From the inherent behavioural patterns which were genetically encoded, evolution moved to patterns acquired by learning in response to immediate environmental pressures. Here is the beginning of the contrast between tacit knowledge and propositional knowledge, the two dialectical counterparts in Polanyi's philosophy, or the "knowing how and knowing that" of Ryle (1949). In the Introduction to Polanyi's (1969) *Knowing and Being*, Grene writes:

"All knowing.....is orientation. The organism's placing of itself in its environment, the dinoflagellate in the plankton, the salmon in its stream or the fox in its lair, prefigures the process by which we both shape and are shaped by our world, reaching out from what we have assimilated to what we seek." (p. xi)

Much of human knowledge can be attributed to adaptation accumulated over evolutionary history. Human babies are born with skills, and knowledge of the world, the extent of which are only now becoming appreciated. This adaptation is the result of the moment-by-moment confrontation with the world, or as Popper (1990) observed, adaptation to the environment is the result of the "problems to be solved in the task of living." (p.37) Long term adaptational knowledge forms the backdrop for short term observation-based knowledge. The link with evolution highlights the fact that *all* knowledge, whether genetically encoded or individually acquired, is gained by organisms through active learning comprising trial and error.

In the 1960's, psychologists started to appreciate the insights which could be gained from studying animals in their natural environmental and social settings. These studies were conducted not in the logical positivist tradition according to empiricist experimental design, but rather by anthropologists using ethnographic methods. Through these studies the animal world revealed its complex social structures and the communication links which kept such social structures functioning.

For teachers, the ways animals learn hold a fascination and a perceived relevance. Folk pedagogy has always tended to see similarities between training a dog and training children. The rod was thought to be good for the

training of both. Scientific psychology of the experimental kind also left its mark on education. To this day theories and practices in the classroom still tend to rely on the conditioning model of animal training. It is widely, though at times implicitly held, that just as pigeons learn to peck according to some predetermined time or space structure, just as rats can learn their mazes, so children, through extensive repetition of basic skills, can become proficient users of these. The work of B.F. Skinner is still one of the guiding lights behind classroom practices. Even a cursory glance at current classroom methodologies reveals that repetitious work coupled with various "rewards" is constantly used with the aim of instilling competencies.

When educators take a look at the way animal communities teach the communication system of the tribe to the young, they gain a new perspective. For a long time orthodox scientific thinking about animals was that their behaviour was formed by instincts, while human behaviour, by contrast, was formed by learning. What ethnographic studies of animal tribes, of apes, wolves, and so on, revealed was that these animals needed to teach their young much more of the behavioural repertoire than was previously believed. Included in this repertoire was the use of the animals' communication system. Blakemore (1976) pointed to the evolutionary significance of social learning in animals, including humans.

"Just as individual memory has partly released each animal from the immediate restrictions of the genetic code, so the sharing of learned ideas by social animals has added an entirely new dimension to the progress of evolution."(p.116)

The role of the genetically encoded responses to the world, the role of individual learning, and the role of the social dimension of this learning are all extremely important for the classroom teacher.

Because behavioural scientists are taking a new look at the raising of the young, the study of play has become of increasing interest. Peter Reynolds' (1972) chapter on the role of play in the socialisation of young animals, draws a clear parallel between the play of young social animals and language acquisition by children. As he says, "there is reason to believe that the history of man consists largely of a progressive phylogenetic elaboration of the flexibility complex." (p.622) This flexibility is due to human neoteny.

The hypothesis of neoteny says that the human brain even in maturity resembles the brain of young animals. In other words, humans function like baby apes. Reynolds lists some important developments which flow from what he has called the flexibility complex. The main developments are of interest here, because education exploits what increased flexibility has to offer. These developments include the human capacity to learn from the manipulation of objects and from observation; the acquisition of conventional sequences of imitated behaviour patterns; greater reliance on play for social development; a progressive "complexification" of the subculture of the play group; and the acquisition of survival functions by play group subcultures. "These changes are not independent of each other but are synergistically related" (p.623), says Reynolds.

Play takes place in actual social settings, that is with the other members of the tribe around the young. It also takes place against the background of actual events and circumstances. The only aspect absent from the play situation is the real purpose or danger which will accompany the behaviours when fully formed. What play provides the young animal is an understanding of what different behaviours mean in the tribe. Play gives the animal an opportunity to develop an understanding of the symbolic meaning of certain behaviours. Through contextualised action, meaning is created in the animal's mind.

One of the consequences of the empirical research on animal behaviour was, that mechanical behaviours were given primary focus. As Mary Midgley (1978) says, all animal owners and observers have always known that animals can show great flexibility of behaviour, intelligence bordering on the cunning, versatility and sensitivity. Because many animals are capable of using symbolic communication, they can interact meaningfully with others of their own species as well as members of other species, including humans. Animals and humans can and do develop symbolic communication links with each other, most of which depend on gesture and tone. In humans the most complex symbolic meanings are encoded in language.

Symbolic behaviour in the animal tribe serves certain functions: establishing dominance in the hierarchy, signalling aggression, bonding between members of the group, and so on. Functionalist philosophers follow this lead and see the social function of language as its salient characteristic. For example, Putnam (1988) drew attention to the close tie between the semantic meaning of words and their referential function. He expressed this referential function by pointing to the contribution of the environment to meaning. To concentrate on the function of language is to open up the linguistic debate to considerations of the part played by social groups and cultures. Harsanyi (1968) pointed this out in these words.

"In terms of Sir Karl Popper's terminology, the rationalistic approach represents an individualistic methodology while the functionalistic approach represents a form of collectivism. This is so because the rationalistic approach explains people's behaviour in terms of their personal objectives, as well as the strategies and information personally available to them. In contrast, the functionalistic approach explains social behaviour in terms of the functional needs of society as a whole."(p.305)

To invest the environment with meaning is the task of society. Reference is a social phenomenon. As Putnam says, "There is *linguistic division of labour*. Language is a form of cooperative activity, not an essentially individualistic activity." (p.25) Just as a whole animal tribe is needed to give meaning to the symbolic behaviours of their kind, so it is in human societies. Through the routines of care-giving and of play the pliable young are given practice in situational settings to learn the functions of language.

Many professional linguists focus on *how* individual children acquire language. Such analyses can be found in summary form in books like Bolinger (1981) and Yule (1988). But to focus on the individual fails to explain how we come to know meaning. This limitation was emphasised by Goodman (1988) as he maintained that current linguistic theories of Chomsky or Fodor, did not explain how we understand meanings. He pointed out that their theories cannot encompass the understanding of novel sentences and figurative language, because it is the placement of an utterance in a referential background which gives it meaning. The concept of meaning is central to modern linguistic theory.

## THE RELATIONSHIP OF LANGUAGE AND MEANING

Mary Midgley's book, *Wisdom, Information and Wonder: What is knowledge for?* (1989), gives an excellent summary of the philosophical movements of the past five decades in the area of linguistics. By tracing a continuity from Descartes, to Russell and to behaviourist education theories and practices, she shows the development of the most commonly held assumptions many educators still take for granted. This assumption is that the "self [is] an isolated intellect, connected only contingently with a body and a set of emotions." (p.137) Curricula tend to centre around the question of how to put knowledge into the head of a child. The metaphors of reified knowledge, and of the independent intellect or mind, seem to go hand in hand. It is useful to trace the threads of thought which resulted in the virtual hegemony of these metaphors in our beliefs about learning and schooling.

As recently as the 1930's, philosophers attempted to trace the connection between language and knowledge in the one to one correspondence of words and what they signified. Built on Frege's recent development in logic, the 'atoms' (elementary parts) of word and thought were hoped to correspond one to one, in a pattern that could constitute secure *knowledge*. Russell and the young Wittgenstein tried to build on this foundation, but this intellectual edifice could not stand up to investigation. The quest to find the elements of knowledge gave way to the search for the dynamics of meaning. In fact, even in the *Tractatus*, Wittgenstein (1922/1961) was pointing to a way beyond a simple one-to-one correspondence. He saw that ordinary language was not just an embryonic form of specialised, scientific language, but rather it had an integrity of its own: "In fact, all the propositions of our everyday language, just as they stand, are in perfect logical order." (*Tractatus*, 5.5563)

Logical atomism aimed to build language into an explanatory structure to an extent unknown before in philosophy. While the atomistic approach proved to be unworkable, the focus on language remained, and opened up not only new philosophical areas, but also a new range of thinking in pedagogy. Concentration on language and meaning broke the hold of the logical positivists such as Ayre, who in 1946 could still hold that "the

meaning of a statement is the method of its verification". The belief that truth could be found only by the rigours of one kind of scientific inquiry was displaced by concentration on the nature of language and its functioning in the building of meaning.

In *Philosophical Investigations* (23, 27, 43), Wittgenstein (1953/1978), having turned his back on the ideas of the *Tractatus*, promulgated a new set of beliefs concerning the relationship between language and meaning. The three major points were the following.

1. Concepts of knowledge and meaning were not separated from the rest of language.
2. Sense was to be found in the web of meaning surrounding it in language, and wider still, in a form of life.
3. The self was seen as part of a world of others without whom we could not have the concept of self in the first place.

These points place language in a context of use. As Wittgenstein said, "the meaning of a word is its use in the language." (43.)

The empiricists sought the final units of truth and knowledge in experience. From Hume to the young Wittgenstein of the *Tractatus*, it was held that whatever could not be verified, or falsified, by experience was not a fit subject for logical, scientific inquiry. Wittgenstein (1921/1961) says in the concluding paragraphs of *Tractatus*,

"whenever someone else wanted to say something metaphysical, to demonstrate to him that he had failed to give meaning to certain signs in his propositions.....What we cannot speak about, we must consign to silence."(6.53, 7)

Midgley (1989) sees the positive contribution of empiricism as the insistence on taking experience seriously as a source of knowledge: "the ready acceptance of the richness of experience, and a refusal to distort it by a premature intrusion of theory" (p.201). Experience remains an essential ingredient in the acquisition of knowledge and meaning, but the linguistic focus draws attention to the fact that much of human learning takes place through the medium of language.

The importance of the linguistic focus in philosophy could be summarised in the following two points.

1. It is profitable for a culture to concentrate on the ways language is used, so that unnecessary confusions may be avoided.
2. Everyday language, the language of feelings, intuitions and of morality can be seen as legitimate, if it is considered as the use of words within a particular 'form of life'.

This means that one of the most important functions of teachers is to build up the class as a 'form of life', in which language acquisition can take place. The best kind of schooling provides a context in which learning gains meaning. Bloor (1983) summarised the legacy of Wittgenstein as the conviction that logic and truth rest on linguistic conventions and rules, and these are defined according to the situation or context.

The function and the social situation or context provide language with the meaning we attach to it. Part of the function of language in social contexts is to link the consciousness of the group with the physical world. Many cultures think of the physical world as being either wholly or partially the world which has "reality". The next section aims to summarise what some thinkers have said about the links between language, reality and culture.

## LANGUAGE DEVELOPMENT, REALITY AND CULTURE

Together with the adaptational knowledge with which evolution endowed human minds, culture is the other shaping force of consciousness. The experience of the world, seen through the eyes of culture is encoded in language and ritual forms of activity. This thesis was put forward by Sapir (1921) and beautifully summarised in these sentences: "Languages are more to us than systems of thought transference. They are invisible garments that drape themselves about our spirit and give a predetermined form to all its symbolic expression." (p.236) Sapir's student, B.L. Whorf (1956) followed up this thesis by asserting that we construct our world through the medium of the language we have happened to be born in. He believed that different languages actually alter the construction of the reality their speakers experience. For example, he saw the different ways categories are created in different languages as way of altering the perceptions of the speakers:

"And every language is a vast pattern-system, different from others, in which are culturally ordained the forms and categories by which the personality not only communicates, but also analyses nature, notices or neglects types of relationships and phenomena, channels his reasoning, and builds the house of his consciousness." (p. 252.)

Empiricists, tended to concentrate on the physical conditions in which evolution placed its subjects, and largely neglected the cultural and linguistic factors so important to Sapir and Whorf. Goodman (1988) felt that the empiricist emphasis on experience led to some narrow conclusions:

"Empiricism maintains that knowledge depends on experience. This contention, although true enough, may be misleading. For it neglects to mention that the dependence goes both ways - that experience likewise depends on knowledge." (p.5)

Empiricism's explanations are familiar not only to professionals in the scientific areas, but they have also become part of "folk philosophy" among the general population, especially in English speaking countries. Empiricist explanations have a beguiling simplicity. From the *tabula rasa* of Locke, to the experiential nature of learning, from Pavlov to Hull and Skinner, the path is charted by using just one assumption: knowledge depends on experience. The Enlightenment view of human nature was that it is "as regularly organised, as thoroughly invariant, and as marvellously simple as Newton's universe." (Geertz, 1973, p.34) Furthermore, the nature of experience could be understood and defined by a trained outside observer.

But as Goodman and Elgin (1988) say, the relationship between experience and knowledge is mutual.



"In system building we never start from scratch. Inevitably we start with some conception of the objects in the domain and with some convictions about them. These guide our constructions." (p.12)

Even the very youngest of babies, from a few weeks old, seem to have structured ways of organising their experiences. They already start to see what they know. Donaldson (1992) has brought together an impressive body of findings to prove this point. Empiricism could neither extend its explanatory power over large and significant areas of human experience, nor could it account for facts as we know them.

Empiricism has also failed at a philosophical level. As Midgley (1989) recounts, Russell and the young Wittgenstein attempted "the classic task of finding a single, universal, philosophical structure linking human thought to the world, an underlying pattern which would bring the two together at a deep level entirely remote from ordinary experience." (p.139) Their failure is demonstrated, if by nothing else, than by the mature Wittgenstein's *volte-face*. There is a quantum leap of thought between *Tractatus* and *Philosophical Investigations*.

If reality is not 'out there' ready to be experienced by humans and become knowledge, then where or what is it? D'Ambrosio (1986), in his address to the *International Congress on Mathematics Education*, emphasised the interactions of human beings with the reality they perceived around them.

"I am placing *myself* in the position of looking at 'reality' as it is perceived by individuals who use their abilities in the form of strategies, to perform actions which invariably have their uses in modifying reality. I am talking of human behaviour as a cyclic model connecting *reality - individual - action - reality* as characteristic of human beings." (p.2.)

This quote is important in drawing attention to another way humans are connected with the 'world': it is through action. Action not only changes the environment and the reality but indirectly it also changes the individual self. Reality, D'Ambrosio goes on to say, is both 'out there' and 'inside' the self.

"We consider reality as both environmental, which comprises the natural and the artificial, and as a pure intellectual, emotional, psychic, cognitive reality, which is the very intimate abstract reality of ideas. Thoughts are part of a reality which affect all individuals in a very intimate way, as well as emotions. The individual is not alone but is also part of a society. Reality is also social." (p.3)"

Here we have three aspects of reality: the environmental 'outside' reality, the cognitive and emotional 'inside' reality and the social reality of a person in a social context. Social reality is the chief source for the complex cognitive structures through which humans perceive their world. These cognitive structures are encoded in language. Goodman and Elgin (1988) put it this way,

"The systems we build guide our constructions. Statements are judged true or false according to their fit with the system we have built up. Our goal in constructing a verbal system is to organise a domain in a way that enables us to formulate increasingly many interesting and accurate statements about it." (p. 12)

It is part of human thought to be able to remember, to plan and to conjecture. According to Donaldson (1992) and her evidence, young children from the age of nine months can construct a "what may happen" scenario of their experiences. So the "real" is not restricted to the actually present whether inside or outside. As Popper (1990) said, what is real may be only present theoretically, as mere possibility.

"Propensities, like Newtonian attractive forces, are invisible and, like them, they can act: they are *real*. We therefore are compelled to attribute a kind of reality to mere possibilities, and especially to weighted possibilities, and especially to those that are as yet unrealised, and whose fate will only be decided in the course of time, and perhaps in the distant future." (p.18)

Popper (1979) described his scheme of the three worlds humans inhabit:

"We call the physical world 'world 1', the world of our conscious experiences 'world 2', and the world of the logical contents of books, libraries, computer memories, and suchlike 'world 3'." (p.74)

All these worlds, taken together, form the 'real world'. As far as we know, only human language has the power to open up all these worlds, including the world of propensities. Because language is a cultural product, reality for humans is also a cultural product. A part of our reality is human nature itself. Geertz (1973), emphasised the shaping force of culture on what we consider to be human nature.

"Undirected by culture patterns - organised systems of significant symbols - man's behaviour would be virtually ungovernable, a mere chaos of pointless acts and exploding emotions, his experience virtually shapeless." (p.46)

In *Acts of Meaning*, Bruner (1990) gives three reasons why any consideration of human knowledge, especially symbolic knowledge, should consider the messages which come from anthropology.

1. It is man's participation *in* culture and the realisation of his mental powers *through* culture that make it impossible to construct a human psychology on the basis of the individual alone.
2. Given that psychology is so immersed in culture, it must be organised around those meaning-making and meaning-using processes that connect man to culture. By virtue of participation in culture, meaning is rendered *public* and *shared*.
3. Human beings do know about human nature and they have collected such wisdom in "folk psychology". Folk psychology, like cultural psychology, is especially interested in situated action and not in behaviour.

Culture is the wide context in which human interactions take place, which supplies the structure for them and imbues them with meaning. As Bruner says, culture imposes its meaning on human action.

"It does this by imposing the patterns inherent in the culture's symbolic systems - its language and discourse modes, the forms of logical and narrative explication, and the patterns of mutually dependent communal life." (p.34)

Bishop (1991) makes full use of these ideas in his work showing that teaching and learning mathematics can be conceptualised as the novice's enculturation into the special cultural mode, into the special language, which is mathematics. He repeatedly stresses the importance of context. In 1985 he wrote: "if there is one thing to be learned from research into social aspects of mathematics education it is that the *context*, and the *situation*, are all important."

The classroom is a representative portion of the cultural context. As D'Ambrosio said, the world and the self are both culturally constructed and the two are interlinked. This places heavy responsibility on teachers. Rosaldo (1989) begins his book with this quotation from Adrienne Rich's "Invisibility in Academe":

"When someone with the authority of a teacher, say, describes the world and you are not in it, there is a moment of psychic disequilibrium, as if you looked into a mirror and saw nothing." (p. ix)

It is frightening to contemplate how often this must happen in most classrooms. Teachers have the power to define the worth of their students, because they are given the privileged role to define reality in the classroom. It is part of the teacher's role to know what is right and what is wrong, and what counts as true knowledge. Among other things teachers may define is the nature of mathematics.

## THE NATURE OF MATHEMATICS AND ITS LANGUAGE

Mathematics has been regarded in the past as the "language of the Universe", the unalterable expression of some ultimate reality, and its products, the insights and the processes of mathematics, as being immutable and true. Mathematical truths were thought to have but one correct definition. As against this view of mathematics, Imre Lakatos (1976), in *Proofs and Refutations*, stated:

"Mathematical activity is human activity. Certain aspects of this activity - as of any human activity - can be studied by psychology, others by history. Heuristic is not primarily interested in these aspects. But mathematical activity produces mathematics. Mathematics, this product of human activity, 'alienates itself' from the human activity which has been producing it. It becomes a living, growing organism, that *acquires a certain autonomy* from the activity which has produced it; it develops its own autonomous laws of growth, its own dialectic." (p.146)

Seeing mathematics as the product of what mathematicians do, leads the mathematics teacher into new ways of speaking about and teaching the subject. Recognising that mathematical language is a cultural product, just as all language is, leads the teacher to find ways of teaching that language to novices of the language community. The acquisition of this language, and its correct usage is the aim of mathematics teaching. The language of mathematics is both a means of communication and an instrument of thought (Kaput, 1988). Ernest (1991) lays stress on the point that it is only through the acquisition of this language that participation becomes possible in the 'form of life' which is the discipline of mathematics.

Many writers about mathematics make the distinction between what some have called the 'relational' and 'representational' knowledge of mathematics (French and Nelson, 1985). To function as a mathematician one must *know* about mathematics and also *do* mathematics. Steen (1990) links them with the language of mathematics in this way:

"By examining many different strands of mathematics, we gain perspective on common features and dominant ideas. Recurring concepts (eg., number, function, algorithm) call attention to what one must know in order to understand mathematics; common actions (eg., represent, discover, prove) reveal skills that one must develop in order to do mathematics. Together, concepts and actions are the nouns and verbs of the language of mathematics." (p.8)

Language is central to mathematics. It is interesting to reflect on the fact that often the distinction is made between those children who are good at language and those who are good at mathematics. Possibly it would be more useful to see language as a basic requirement and the forms of language as different types of discourses or "registers". Ernest (1991) contends that there can be no participation in the mathematical discourse without proficiency in the language of mathematics:

"A key stage in the cycle of mathematical creation is the internalization, that is the inner subjective representation, of objective mathematical and linguistic knowledge. Through the learning of language and mathematics inner representations of this knowledge, including the corresponding rules, constraints and criteria are constructed. These permit both subjective mathematical creation, and participation in the process of criticizing and reformulating proposed (ie., public) mathematical knowledge." (p.44)

Language is used in specific and novel ways in mathematics. At first these new ways of using language may appear strange to children, but eventually they become a workable vehicle for further thought. Skinner (1990) gives a useful summary of the nature of mathematical language as it is used in the Early Childhood classroom.

1. Mathematics is a language of abstractions. In speaking of 4 dogs and 4 cats both the 4's are the same. The child sees or mentally pictures four cats and four dogs, the images while different, have a number of things in common including fur and tails and whatnot, but the fourness of the images is an abstraction which only gradually becomes apparent. The use of number words without an accompanying noun may increase the difficulty of mathematical calculations for children as old as seven. Asking the question of what is four and three is not the same as asking about four children and three dogs in a billycart.
2. Mathematics is a language which compresses a wide range of experiences:  $4-3=1$  stands for millions of circumstances. A number sentence has no context.
3. Mathematical language focuses heavily on conventions and written symbols, most of which have no concrete relationship to what they represent:  $4-3=1$  but the same entities differently ordered do not mean the same:  $3-4$  is not 1. Skinner gives many examples of conventional and idiosyncratic ways of using mathematical symbols.

4. Mathematical language uses familiar words in unfamiliar ways: shapes have "faces" which do not smile, "take-aways" are not pizzas, "difference" between 3 and 4 is not a discussion of their different shapes or appearances.

To be able to use the register of mathematics, that is being able to take part in mathematical discourse, is to be able to use the meanings of mathematics.

"Learning to speak, and more subtly, learning to *mean* like a mathematician, involves acquiring the forms and the meanings and the ways of seeing enshrined in the mathematics register." (Pimm, 1987, p.207)

It is part of the process of becoming a mathematician to learn to speak like one. If the children in our classes see a role for themselves as mathematicians, they will be more ready to learn the language of mathematics.

Teachers have to realise that teaching mathematics is to teach its language, if the focus is on "mathematics as a way of knowing", as Brissenden (1988) put it. Having built his view of mathematics on the theories of Lakatos, Brissenden comes to the same conclusions as those espoused in this study.

"I believe that teachers would do best to think of mathematics as a way of knowing, about ourselves and about the world - a creative activity, carried on by people, and involving meaning." (p.222)

The idea that teaching mathematics is like teaching a language has been explored further by drawing parallels between teaching mathematics and teaching a foreign language (Borosi and Agor, 1990).

Language is the symbol system used to communicate about the nature of reality within a group. Even though mathematics is a system of conceptual structures, it arises from and can influence the physical world. Skemp (1989) says:

"The power of mathematics in enabling us to understand, predict, and sometimes to control events in the physical world lies in its conceptual structures - in everyday language, its organised network of ideas." (p.90)

These ideas are expressed through a system of symbols which is mathematics in its recorded form, and the tool for its use. Skemp goes on to say,

"Though the power of mathematics lies in its knowledge structures, access to this power is dependent on its symbols. Hence the importance of understanding the symbolism of mathematics."

The symbol structure embodies the system of logic which was involved in the *development* of mathematics (Esty, 1992). Knowledge of the symbol system is the passport for all those who wish to contribute to the future development of the subject, and for those who want to use what it has to offer. The symbol system is the basis for the discourse of mathematics.

Concentration on the linguistic and therefore the cultural aspects of mathematics is characteristic of the philosophers of science who have been most quoted here: Popper and Lakatos. It should be noted that not everyone holds these philosophers in high esteem. David Stove (1982) called Popper and Lakatos, together with Kuhn and Feyerabend, the "four modern irrationalists". Stove used the term "irrationalist" in its full pejorative sense. He rejected the approach of these successors of Hume, who have tried to inject scepticism and cultural relativity into scientific knowledge. Viewing mathematics in the "irrational", culturally relative way, can be very helpful to teachers facing thirty individual cultural products with their own particular interpretations of reality. This does not have to mean, however, that teachers end up with no freedom to devise mathematics curricula independent of the children's background. If a teacher is to be successful she has to have belief in the concepts she is trying to communicate. The history of mathematics has a legitimate claim on the mathematics curriculum which is to be balanced with the needs of children.

Cultural relativity is also limited by the claims of the physical environment. Lakatos not only admitted, but actually argued, that cultural relativity is not a reason for a sane person for walking out the window of a forty story skyscraper. He fully acknowledged that reality is to be found in the environment with which we interact to gain both knowledge of the world and knowledge of ourselves. Even though our view of reality is both formed by, and therefore limited by, the frameworks given us by our perceptual system developed through evolutionary processes, and by our system of concepts we inherit through language and culture, both the interaction with the environment, and the environment itself have a dignity and reality of their own.

Empiricists and pragmatists like Stove, keep reminding the philosophical community of the importance of the physical environment in forming and shaping knowledge. They also stress that humanity's accumulated wisdom is encoded in the tradition of the disciplines. Their viewpoint should be born in mind to balance the currently dominant philosophy of cultural relativism.

Balanced thinking is also essential for teachers seeking advice about educational methodology. Philosophers are useful when they argue a point in its pure form, like the irrationalists against the rationalist, in Stove's case. Teachers, on the other hand, are useful when they can develop a methodology which helps children not only to take their part in the society into which they have been born, but also have the freedom and skills to critique that same society. In mounting the study of this dissertation it was hoped that by keeping the best of theoretical advice in mind a satisfactory methodology for the teaching of mathematics may be worked out.

## SUMMARY OF THEORETICAL BACKGROUND

This reconsideration of the philosophical background was helpful to the researcher in giving back meaning to the well worn phrases of educational advice. In order to plan the activities of the study all the points discussed in the theoretical background were taken into consideration. There were many methodological suggestions found which were implied in the theories. The most important suggestions were as follows.

- The learning activities of the children in the classroom should parallel the "play" element important in children's lives. Play should be used as the medium for the active involvement of children in their physical and conceptual environment.
- The activities should be "real" in the children's conceptual world. This does not only mean the use of concrete materials and problems with accessible narrative structures, but also utilising situations which are capable of interacting with the developing self of the learner.
- The language of mathematics should be rooted in real learning situations, which have the function of endowing both the activities and the materials used with meaning.
- Through the use of language, tacit, personal knowledge is to be transmuted into the explicit, social domain. To achieve the aim of having children who can use language in their thinking about mathematics, the language has to have its origins in, and it must function through, the child's social world. The language of the mathematics classroom must therefore have organic links with the physical world, the self of the learner, and function in a meaningful social setting.

At the planning stage of the study, it was felt that a curriculum which was centred around situations the children understood and appreciated as problems, would give the mathematics its "real" quality. The problems were to be worked out through activities the children already understood and could use as the bases of games and play. The language was to be built up gradually, closely linked with the actual situations in which the children found themselves. Frequent audio taping was to provide practice in the expressive use of mathematical language.

Naturally, the major question asked in the study was, how successful would be the teaching structured around the insights which flow from the theoretical background. There were, however, three other questions, based on evaluation, which could only be answered after the study took place.

1. How well were the mathematical concepts contextualised?
2. How well were individual children's developmental levels catered for?
3. How well balanced were the agendas of teacher and children?

Since answers to these questions could only be found by reflecting on the experiences of the study, they would be raised after the description on the study itself and form the body of the "Discussion" section, Chapter 5. The reader will find that because these questions of reflection raised new issues there would be a need to examine some additional supporting literature.



## CHAPTER 3

### THE STUDY

*"The search is increasingly conceived, not as the effort to understand something which is itself of great importance, but rather as the accumulating of information which is guaranteed to be correct, almost regardless of its content. Although everybody knows that some information actually is trivial."*

(Midgley, 1989,p.128)

#### SETTING THE SCENE

This chapter describes how the study was set up, how it was conducted and evaluates the teaching sessions. Since the aim of the study was to examine the kind of curriculum which would come from a careful consideration of the theoretical advice available, it is important to judge how well theory and practice were linked in the course of this study.

If the theoretical questions, and implied answers, point to a more considered form of mathematics teaching, then it must be possible to incorporate all the advice into mathematics curricula which are perceived as both real and practical by the students. In fact, if the advice is based on the "true" nature of language and mathematics, it should be able to be delivered relatively easily and effectively. The proof of any theory must lie in the practice based on it.

To what extent could children's language use in and about mathematics be improved if the teaching they received was at once structured, concrete and presented in contexts meaningful to the children? If extensive practice of language was built into the teaching situation would children acquire the language more readily? If an experienced teacher followed all the advice, would it mean that many of the problems of learning and classroom management were solved? If practice still has problems then it is possible that there is a need to examine the theoretical bases.

The study was devised to trace the incorporation of new words and concepts into children's expressive and receptive language use. It was hoped that the process of the study would benefit the children involved. It was partly for this reason that it was decided to have an experimental group of children who were identified by their teachers as poor performers in both language and mathematics.

### *Contact with the school*

The Principal of a rural Primary School was contacted for permission to conduct the study in the Infant section of the school. The Principal put the researcher in touch with the Assistant Principal in charge of that section. The Assistant Principal kindly agreed to support the project, and undertook to inform teachers, select the children and to organise a timetable.

### *Time, place and people*

The teaching sessions unfortunately had to fit in with other commitments, which meant that three sessions a week were run between two o'clock and three o'clock in the afternoon, and two sessions were held between eleven and twelve in the morning. This time table worked as smoothly as can be reasonably expected in the usual hustle and bustle of ordinary school life.

Due to crowded conditions in the school, the teaching sessions took place in different areas each day of the week depending on what rooms were available. Two Kindergarten rooms, the staff room, an open area, and the medical room were all used during each week.

The time constraints meant that the children were sometimes lacking in discipline by the afternoon, and some of them appeared tired and listless. The younger ones especially showed some resentment at being asked to do "formal" work, since they were used to more relaxed afternoon schedules.

Because the study had no home room, the sort of materials which could be used were limited by the need to carry all resources from room to room. The one hour time constraint also defined the projects the children could undertake, so the projects were designed to be self-contained activities which could be finished within the hour. When the group came together we did "maths"; there were no opportunities for linking different subject areas and looking at the concepts in a wider framework. The whole curriculum was packed up in the cardboard box of resources. Since the project had no home-room, no display of work was used as a means of reflection or as a means of inducing pride in achievements.

While the time and place conditions, and consequently the curriculum, may have been less than ideal, the support of the Assistant Principal and of the classroom teachers could not have been more readily and sensitively given. They listened to successes and woes, encouraged the researcher and the children; they could always be relied on to say just the right words of admiration when children showed them work.

### *Children and teacher*

Four teachers of grades 1 and 2 were asked to identify children in their classes who were the poorest performers both in language and in mathematics. Eleven children were listed by the teachers, six boys and five girls. Only three of the children were from grade one, the others were from three different grade 2 classes. Two of the boys were frequently absent from school, and during the study they only attended the teaching sessions three times each out of the possible 25 hours. These children were not tested at the end of the study, and hence they were excluded from the study group. To protect their right to privacy all the names of the children were changed for this report of the study.

For the sake of comparison, three new children were selected for the testing sessions which were conducted during the sixth week. Teachers from the three grade 2 classes were asked to select one child each who was only marginally better at mathematics and language than the children they selected for the study. These three children, John, Danny and Abel, were selected to represent the knowledge which might have been gained from the curriculum focus in each of their classes rather than from the experimental teaching sessions. The three of them were given the same test as the others.

The size of the group, 9-11 children, was decided upon as a number which would be large enough to generate the kinds of teacher-pupil and pupil-pupil interactions characteristic of a normal classroom, but small enough to enable some monitoring of language use to take place. Intensive studies which try to chart individual children's learning most often concentrate on very small numbers of children, who are taught and interviewed individually, or in twos or threes. One of criticisms often levelled at some modern theoretical advice on mathematics teaching, for example that given by constructivists, is that the suggested methodology is too difficult to use in a class of thirty children. Having a larger group in this study, but not a whole class, was designed to be a kind of compromise, which would allow more realistic suggestions to emerge for whole class instruction.

The teacher/researcher has worked in education for sixteen years. Some of this time has been spent in research and some in classroom teaching. She has taught six, seven and eight year old children for the past eight years.

### *Teaching Methodology*

While preparing to teach the group in this study, there were a number of methodological theories taken into practical consideration. Included among these were the constructivist school of von Glasersfeld (1991), the insistence on "rich context" by Hans Freudenthal (1991) and his phenomenologist colleagues, and the advice of Pimm (1987) on teaching mathematical language.

Based on the belief that Polanyi (1969) was right when he said: ".....all knowledge is *either tacit or rooted in tacit knowledge....a wholly explicit knowledge is unthinkable*"( p. 144), the teaching sessions were built up day-by-day so that first the children would take part in activities, that is *do* things, and then were given practice and encouragement to make that tacit knowledge explicit. The topics to be covered and the type of tasks the children were given were decided upon by the teacher.

Children were selected on the basis of being poor at both language and mathematics in order to test the theory that the use of structured, concrete materials help the development of both language and mathematical knowledge. Lowenthal (1990) pointed to the conundrum that to develop syntactic structures a child needs good language background, to gain this background she needs good syntactic props. Building on the work of Jakobson (1956), Lowenthal emphasised the need to break through this by linking syntax with the real world of sounds and with real objects. As he says, "A good communication system is needed by the children in order to build their own representations of the environment, but one must note that every communication system is based on a subjacent logic." (Lowenthal, 1990, p. 198)

Communication systems have their roots in logical activities society engages in. The verbal expression of communication is the superstructure built on logical activity "games". According to Lowenthal, one of the best ways to make this logic demonstrable in the classroom, is to use non-verbal communication devices, or NVCD's. A good NVCD has these advantages.

- As they are concrete objects they avoid the need to rely on verbal communication as the sole representation device.
- As parts of a formal system they can only be grouped in logical and often in a predetermined stepwise fashion, and therefore they can be used to demonstrate and teach logic.
- In the formal system the logic of reasoning is accomplished; this can be done in a large number of different ways.
- The concrete representation of the formal system carries the syntax on a concrete level and within technical constraints.
- Manipulation of the small elements, such as blocks, toy animals, make it attractive.

Concrete mathematical systems are also useful in that by watching the way children manipulate these objects, the teacher can gain a more accurate insight about their current concepts. Van den Brink (1984) stresses that NVCD's allow the development of higher order mathematical communication and logical thought. "[S]uch a 'non-verbal communication device' favours the development of structured communication, allowing very young children to learn how to build a convincing verbal argument." (p.264) Since an understanding of the child's current conceptual world is of great interest to teachers who wish to build on these, some way of externalising the concepts becomes essential. The teacher can also use NVCD's to demonstrate concepts she has in mind.

Most educational theorists stress that teaching must be based on the child's present concepts. This legacy of Piaget is equally endorsed by the constructivists like von Glasersfeld (1991), by the cognitive theorists like Brissenden (1988), Skemp (1979) and Labinowicz (1985), phenomenologists like Marton and Neuman (1990), by the followers of various problem solving approaches, including Polya's (1962), by the advocates of real life mathematics such as Mellin-Olsen (1987), as well as by those educators who think teaching is a process of enculturation (Bishop, 1988, 1991).

This emphasis on the individual child's concepts make many teachers feel that the translation of the suggested methodologies to the teaching of a class is altogether too difficult. In this study the children were given their tasks as a class. They performed these tasks either individually or in groups, and only the monitoring and the final testing session was designed to take children one-by-one. The types of activities done in this study could be done in a full class, especially if small groups were habitually used. While recognising the difficulties entailed in translating from small experimental groups to a normal class, it was still hoped that the findings of the study would fit with the real constraints of teaching a full class.

The topics dealt with and the kinds of tasks suggested to the children were selected to represent the Wiskobas approach inspired by Freudenthal (1991) and extensively used by van den Brink (1984, 1993). This approach consists of using context-rich situations to present mathematical problems, in order to allow the children to use their 'real world' knowledge. By using some narrative context for all the tasks, it was hoped that the "boundary conditions" for the problem were made clear to the children, as Clements and Ellerton (1991) say:

"There is an important sense, then, in which stories that establish boundary conditions for action in real-life problematic situations can provide interesting contexts in which children can develop their powers of deductive reasoning." (p.30)

Schoenfeld (1991) recommended the contextualised approach in these words:

"The richness of the tasks, contexts, and student activities that van den Brink describes stands in stark contrast to the sterility of the traditional context-free curricula." (p.293)

Schoenfeld felt that the important question was how to move from this kind of "real" curriculum to a relatively context-free abstract mathematical curriculum. This question can be best answered through actual classroom practice. The answer demonstrated in this study is the same as that shown to work for van den Brink. The process used is one of gradual formalisation of the problems and of the solutions required in tackling different aspects of the "real life" problem. The process is discussed in the section "Building a network of concepts".

The way the problems were presented to the children also fits the problem solving approach of Polya (1962). Groves and Stacey (1990) summarise the problem solving approach by saying that it succeeds in linking young children's reality with mathematical concepts if the children tackle tasks,

for which there appear to be no solutions or obvious means of finding them and the solutions involve some form of mathematics. These conditions operated every session.

It seemed important in this study to allow children time to face the problems posed, and to give them the opportunity to identify the various aspects of the problem situation. Children were encouraged to ask more questions as well as give more answers. As Cobb (1990) says, the context of a problem is the subject's own construction of the situation, including his or her own purposes and intentions. It takes time and experience with and within the situation to build up an appreciation of which purposes and intentions may play a part in a given context. Constructivists stress that the child's solution to a problem is always rational, even though it may be incomplete. In this study children were given repeated experience with the same problem in order to have the opportunity to refine solutions.

Both constructivists and cognitive theorists place heavy emphasis on the use and development of mathematical language. For example, Wheatley (1991) categorically states that "In order to do mathematics and science, students and teachers must learn how to carry on a scientific discussion" (p.19). Pimm (1987) alerted his readers to the many difficulties, and hidden agendas in the communication system of the mathematics class. Among many other researchers who found deficits in children's understanding and use of mathematical language, Hardcastle and Orton (1993) showed that of their year 8 students only 39% could give acceptable definitions for some common mathematical terms. On the whole these authors did not think that students understood what their teachers were talking about. They said, "clearly the evidence suggests that helping pupils with the language of mathematics should be an important part of mathematics teaching." (p.14). As Ellerton's (1986) girl student exclaims:

"I don't like maths that much and sometimes I find it hard and difficult. Sometimes I don't understand what things mean." (p. 144)

It was one of the aims of this study to help children learn the language involved in the tasks.

In order to achieve better language learning, children were asked to tape record a report on the day's activity and on their solutions. The teacher used questioning as a means of repeatedly using certain mathematical terms. Children were expected to develop both in their receptive and in their expressive use of mathematical language.

Each week a different topic was covered, but during the five teaching weeks new concepts were built on ones used in previous weeks. The topics were selected by the teacher. A full list of the activities can be found in Appendix 1. Every effort was made to find out what the children already knew and to build on their concepts. This was done during the time the children were working on the assigned tasks. No formal, deliberate testing of the children's concepts was undertaken prior to teaching. Nel Noddings (1990) pointed out that the relative decision making-spheres of teacher and students has remained unspecified by constructivist theories. No

constructivist has ever said to what extent, what percentage of time, should the self-discovery of concepts be the methodology adopted in a constructivist classroom. In this study the teacher decided on the activities, but the children were encouraged to interpret the activities and to offer solutions based on their current understandings.

### *Curriculum and classroom management*

Each day, the hour's work was structured the following way.

- The children were acquainted with the task for the day.
- The teacher spoke about and often demonstrated some aspect of the task in a brief ( 5-10 minutes) whole group session.
- Children undertook the task, and were given individual attention and help in this.
- Each child spoke about his or her solution, and this was recorded on audio tape.
- Written work was collected and kept for the purposes of monitoring the children's development.

The curriculum was as follows.

#### *Week 1. RACING*

The problem presented to the children was to lay out racing tracks which would make the results of the race fair. Day by day they were expected to incorporate more of the conventions of a racing track. On the first day they raced marbles and spools and found that at least a starting line and a finishing line had to be agreed upon. By day three, they laid out tracks which were all of equal width, a width comfortable for the racers, and the starting and finishing lines were perpendicular to the tracks. On day four, they measured out 20 and 50 metre tracks for themselves for running races.

#### *Week 2. LONGITUDINAL PATTERNS*

Starting with a fairly free session of colouring "snakes", the children were asked to observe more and more structure through which the relationship of visual patterns and number groups was explored. They completed colour patterns as well as action patterns. Their most ambitious action patterns were set out with invented symbols standing for each action. This was done on the playground and the resulting pattern could be used as a kind of hopscotch game.

#### *Week 3. SHARING*

This week's work touched on sharing out a set number of items as well as cutting up rectangular and circular shapes into equal portions. The week concluded with a party where sweets and cakes had to be shared out among the ten party goers.

#### Week 4. SUBTRACTION

During this week the focus was on subtraction, using games and concrete aids. We played a bus game and did some familiar take-away problems, using the story of getting on and off buses for framework. Some of the work was presented within the context of a known story, such as the "Old woman who lived in a shoe". In this problem, the children drew a picture of the old woman, but as she could only fit 10 children in the shoe, they had to put the rest into the garden to live under a tree. They could choose any number of children between ten and twenty.

#### Week 5. COOKING

Taking only five children at a time, we made jelly on Monday with the first group and then took the second group on Tuesday. Wednesday and Thursday we made scones, again only taking five children at a time. Friday we talked about cooking and made inclines with Lego. Lego was available most of the week as a fill-in activity for the inevitable waiting periods in the cooking schedule.

#### Week 6. INDIVIDUAL TESTING

Taking one child at a time, they were tested on five tasks which corresponded with the five weeks' teaching topics. Three additional children were also tested to act as "controls" for the teaching group. All testing sessions were audio taped for the purposes of analysis and evaluation.

### EVALUATION OF TEACHING SESSIONS

During the six weeks a daily teaching diary was kept. This evaluation draws on the entries of the teaching diary and deals specifically with the quality of the teaching sessions. The children's learning achievements will be assessed in Chapter 4.

The first week went well. The children settled down relatively quickly to the routines of the group and seemed to learn something every day. During the first week most things seemed successful, but by no means all. For Thursday (24/6/93) the diary records:

*There is a need to find time for small group work, so that children can benefit more from their work. There were too many people in a small space today and I had the afternoon grumps anyway. We have had rain for about six days non-stop, it seems!*

The second week, the one dealing with longitudinal patterns, was enjoyed by the children. Most of them produced thoughtful work, but Joe and James, the two first graders, started to show reluctance to engage in the activities and even more of a reluctance to complete them. The diary entry



for the 30/6/93 indicate a growing sense of difficulty, if not of unease, regarding these two children.

*Joe found it hard to name the colours and kept losing track of his pattern. He could count to 24 with a little help - 20, 21....- but he seems to lack confidence to do a good job. He therefore seems reluctant and as his teacher says"*

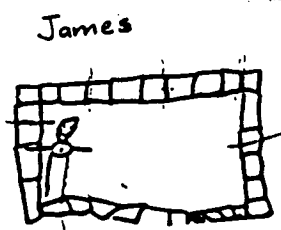
The following is the taped conversation with James at the end of this same session:

**James** We have 24 squares and chopped them into three's.

**E.D.** Did you get it right?

**James** It was hard to put 24 blocks in a square.

James decided do the pattern in a block, not longitudinally, and got muddled, as shown in Figure 1.



**Figure 1.**  
**James' pattern set out in a block**

Both these children must have left this session on patterns with a sense of failure, though they were praised for their attempts.

On the Friday of the second week, 2/7/93, there were only four children in the group, because the others were on an excursion. Three of them had a lovely time creating, recording and enacting their action patterns, but it is worth noting the diary entry regarding Joe.

*There were only four children present, and Joe had enough and went back to his classroom. He hated to be the only boy in the small group. A very enjoyable activity session, but Joe's decision to go back to class might have been taken more seriously. It is possible that it was a measure of his increasing alienation from the activities which are possibly getting too hard for him.*

The following week was dedicated to "Sharing". On the Monday (5/7/93) the children were given a list of sweets their Mums had bought and they had to share it out among their friends. The list was written on a large piece of cardboard and read out at least four or five times. The children were given counters to do the sharing. The list was as follows.

*Share out between four people: 5 chocolates, 8 jelly beans, 4 Mars bars and 12 smarties.*

This task proved to be too difficult for a number of the children and nearly impossible for Joe and James. The diary records the following.

*The task was far too hard for the group. Only two children completed the whole task, Michael and Kim. Most found it difficult to work from a written information sheet, and too much to remember without recourse to some memory jogger. This taping session convinced me that to tape all ten children everyday was not profitable. I obviously found the stress of having to 'get through' them all both a time strain and a strain on discipline. Just four children a day on a rotational basis would be a lot more useful, and I could let them talk longer without harassing and interrupting them.*

On subsequent days the children were given other sharing activities which were easier. It was difficult at times to pitch the tasks at the children's ability level, because not enough was known about their previous experiences or their usual performance levels.

During week four the topic was subtraction. To some extent it was an easy topic for the children because it was very much like "school maths". Many of the activities were obviously familiar to them. On Thursday (15/7/93), however, these were three of the taped contributions:

**James** I don't like drawing patterns, they are too hard. And writing maths on paper, writing sums on paper ....

**E.D.** How do you feel about working in this group?

**James** [I feel] still good, but I don't want to do it.

Luke spent most of his time on "fixing" a toy truck in the resource box. This was either an avoidance activity or an act of kindness towards the teacher. It was hard to decide which.

**Luke** I fixed the truck with two pieces with sticky tape and it now can tip all the stuff out.

**E.D.** What did you do to fix it?

**Luke** I found the piece that broke off.

**E.D.** Great! How about your pattern?

**Luke** I was gonna have 10-2, no, 10-1 ...(We got a handful of sticks and he counted out 10. Luke then recited his pattern.) 10-2=8, 8-1=7, (Confused).....7-3=4, 4-4=0.

**E.D.** That's a very interesting pattern, Luke! Let's do it together!

**E.D. and Luke** 10-1=9, 9-2=7, 7-3=4, 4-4=0.

Joe found it very difficult to follow a pattern. He decided it was all a little too hard and wanted a cuddle. The entry in the diary concerning Joe was,

*Basically all he did was work avoidance. He did write the sum  $3-3=0$ . I asked him to count to ten into the tape recorder. His confidence in counting is increasing daily.*

Was Luke also showing signs of strain, and was he engaging in a form of passive resistance? The evaluation at the end of the day read:

Some of the children were restless, and unsettled, especially, James, Joe and Luke. They found the task too difficult, at first, I think, and decided to do something different. Out of the whole group, only Kim, Michael and Sarah showed that they were at home with the form of the written algorithm for subtraction, such as  $10-2=8$ . Melinda could use it with many slips, [Figure 2.] but some of the others were hampered by it rather than helped.

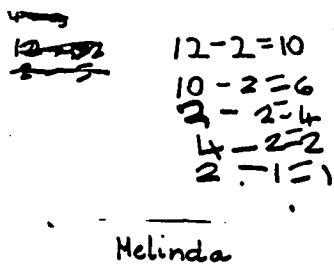


Figure 2.  
Melinda's number pattern

A difficult session may not necessarily be a wasted one. After the session it became apparent that the some of the children learned a good deal. The diary records,

After the session I went to see Mrs B., one of the teachers, to praise Kim's work. [Figure 3.]She said that she had noticed a great upswing in Kim's confidence level in both maths and language. Michael did not want to be left out and offered to recite his pattern, which he did faultlessly ( $20-4=16$ ,  $16-4=12$ ,  $12-4=8$ ,  $8-4=4$ ,  $4-4=0$ ).

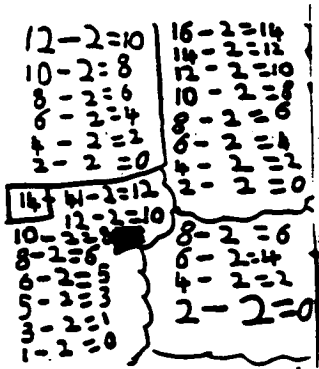


Figure 3.  
Kim's successful pattern

*Then Luke volunteered his pattern, and reeled it off correctly ( $10-1=9$ ,  $9-2=7$ ,  $7-3=4$ ,  $4-4=0$ ).*

*Both Mrs B. and I were astonished at the children's accuracy, which obviously showed an appreciation of the number patterns involved.*

The fact that Luke learned a lot, could remember it all, and was proud of his achievements showed that while he might have found it hard to keep to designated tasks all the time, he was involved and was learning new skills and concepts.

Friday turned out to be another difficult day. The activity was the construction of a decreasing pattern, done with small, coloured paper squares and glue. The diary entry reads:

*Some of the children decided that they did not want to do the activity and some even declared that they did not want to come to these sessions any longer (James, Joe, Luke). The only person who got the idea and really did the task successfully was Michael.*

It was then decided to take groups of only five children a day, and to do cooking with them. Building with Lego blocks was used as a fill-in activity. On the Monday of the following week, the cooking and building started. At the commencement of the session the whole group was called together, and was told that only five children could cook at any one time, so now five would have to go back to their classes, and have their turn next day. The five who missed out that day set up a universal "Aah! I don't want to go!" and James, who seemed the most difficult child at times, said: "Please let me stay! I don't mind that I can't cook today. I'll be quietly sitting in the corner building with Lego! Please, please let me stay!" Joe was equally devastated, in fact they all left most reluctantly. Their reluctance indicated that basically they were happy to be participating in these sessions. While there were some difficulties and some frustrations, these were not seen as important enough by the children to affect them for long. During this week one of the evaluations reads:

*Today's session was very good. The children were cooperative and disciplined. There is not a hint of reluctance from either James or Joe. Melinda was completely immersed in the Lego play. I would guess it is an activity which could be extended to supply her with some of the concepts and logic skills she seems to lack at the moment. James' and Joe's teacher reports great improvement in these children's performance in language, as well as in maths.*

In the light of the outcomes in learning, and the very good relationships which finally developed in the group, it is hard to say what could have been done differently. Obviously, with better information about the children's knowledge and skills, the five weeks' work could have been more closely tailored to their levels of development. Yet at the beginning of each year teachers have to start teaching before they can get to know their children. During these initial weeks all classroom teachers are in the same position as the teacher of the study. Information accumulates slowly as teacher and children get to know each other. If there is a methodological moral to the story, then it must be that life in the classroom can never be totally plain

sailing with the variety of ages and personalities which have to negotiate for *lebensraum*.

There were also some other lessons to be learned from the evaluations. One was the value of flexibility. It was important to feel that whatever the plans for a teaching segment, there was also sufficient flexibility to allow for the inevitable changes to be made. Not all the children were taped after each session, and not all the children attended every day during week five. These changes were made reluctantly, but they were necessary for the success of the program because they made the class less stressful for both children and teacher. The second lesson was that each child's developmental level had to be taken into consideration when planning activities for the group. The children's developmental levels may affect both their cognitive, and their social competence. The problems experienced in these sessions by some of the children, especially James and Joe, were largely due to developmental differences. The issue of developmental stages will be taken up again in Chapter 5.

## CHAPTER 4

### LEARNING OUTCOMES

*"We feel as if we had to penetrate phenomena: our investigation, however, is directed not towards phenomena, but, as one might say, towards the 'possibilities' of phenomena."  
(Wittgenstein, 1953/1978, 90.)*

This chapter presents the evidence collected concerning what nine children learnt during the five weeks of teaching. Using the advantages of the ethnographic methodology, and respecting its limitations, individual children's learning in mathematics and in language is documented and discussed. Attention is focused both on the processes of learning and on learning outcomes.

#### BUILDING A NETWORK OF CONCEPTS

The teaching sessions were designed in such a way that the concepts regarding a certain aspect of mathematics were built up during one week's consecutive activities. The first two weeks will be used here to illustrate the way concepts built up over the course of four or five sessions.

The main focus of "Racing" in Week 1, was a geometrical one. The emphasis was on drawing up a measured course, where the tracks assigned were of the same width, as well as a sensible width for the kind of racing they were designed to accommodate.

The children were expected to think about the following considerations when setting up a racing track.

1. There is a need to define the starting line and the finishing line so that the *length* of the track be an agreed one.
2. There is a need to mark out the tracks all of equal *width* so that all the contestants would have an equal amount of room, a sensible amount for their requirements.
3. There is a need to match the number of tracks to the number of contestants.
4. There can be a need to measure the length of the course and/or the width of the tracks by some conventional means, such as a metre ruler or a trundle wheel.

Looking at the children's taped reflections for the whole week, it is possible to identify and to count their mention of these different points, and to see them take more and more concepts into consideration. Table 1 shows this accumulation of concepts regarding "Racing".

**Table 1.**  
**Accumulation of concepts - Racing**

DAYS	Social behaviour	Task	Winning	Start	Finish	Tracks	Fit	Measure
21/6/93 9 children	4	1	4	5	4	-	-	-
22/6/93 7 children	-	1	-	5	6	5	6	1
23/6/93 10 children	1	1	1	9	9	6	7	7

The headings in this table attempt to capture the following concepts and children's concerns:

*social behaviour* - concern with the rules of the games, and with desirable conduct during the activities;

*task* - recounting what the task involved without necessary reference to the purpose of the task, for example, "We had to draw lines on the carpet";

*winning* - concentrating on the outcome of the games;

*start* - recognising the need to have a starting line defined;

*finish* - recognising the need to have a finishing line, and/or mentioning both finishing and starting lines in order to define length of course;

*tracks* - mentioning the need to have a separate track for each participant;

*fit* - recognising that the tracks have to fit the participants and that the fit may be achieved by simple comparison;

*measurement* - using some form of conventional unit of length to express either length of course or width of tracks.

The record of what one child said on tape at the end of four sessions shows how individual children built more and more considerations into the activities. Basically the activities were the same for all four days of this week, only the kind of racing and the dimensions of the required tracks varied. Michael was one of the more able members of the group. He was in grade 2, and received Special Education assistance in the language area. Whatever his problems with written language, he was a capable and enthusiastic user of oral language. He could talk fluently and coherently. He performed well in this group, much to his classroom teacher's amazement. The competition, however, was of a different calibre from that which operates in a heterogeneous classroom.

The collection of Michael's taped contributions at the end of the first week's daily teaching sessions show how the concepts involving a racing track built up for Michael.

(21/6/93):

**Michael** We had to make our own plan on a piece of paper, then we raced and I won two of them. We done a picture of us doing the caterpillar race. (*Concepts: task, winning*)

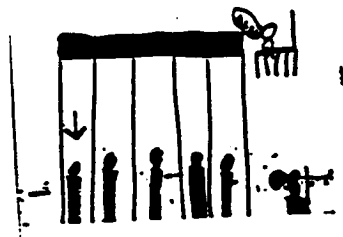
(22/6/93)

**Michael** We had to get sticky tape and we put it on the carpet and we had to put four lines and a finish line and a starting line, and we had races on the tracks made. If you go over the line it is out and you've got to start again and the first one across the finish line is the winner. And we made the tracks by ourselves.

**E.D.** How wide did we make the tracks?

**Michael** For the people to crawl through.

(*Concepts: task, rules, start-finish, counted tracks and mentioned the number without prompt, compare and fit*) Figure 4 shows Michael's drawing done of this day's track.



**Figure 4.**  
**Michael's racing track**

(23/6/93)

**Michael** We had to get sticky tape and we had to get the sticky tape to stick to the rope onto the carpet.

**E.D.** Tell me about the race...

**Michael** We had to put down the balls....

**E.D.** What did we draw first?

**Michael** The finish line and the start line.

**E.D.** How did we do that?

**Michael** With string and sticky tape.

**E.D.** How wide were our tracks?

**Michael** 32 metres.

**E.D.** 32 centimetres?

**Michael** Because that's a fair game.

**E.D.** What makes 32 centimetres a fair game?

**Michael** It fits the people.

(*Concepts: task, fairness, start-finish, tracks, conventional measurement, number remembered- wrong unit name, fit*)

(24/6/93)

**Michael** (His contribution to group discussion) You have to measure it to make sure it is the right measure and you have to make up the line for the all people. (*Match numbers, fit, measure*)



A similar development of concepts can be traced during the second week which was concerned with longitudinal patterns. The main theme explored was that patterns can cut up the number line in different ways. The build up of concepts started with creating a repeating number pattern expressed in a variety of ways, such as visually or in action patterns, and culminated in the use of a predetermined number pattern dividing a longer sequence.

**Table 2.**  
**Accumulation of concepts - Pattern**

DAYS	Repeating	Expression	Made of units	Infinity	Number link	Division
28/6/93 9 children	7	7	-	-	-	-
29/6/93 8 children	7	1	8	3	4	-
30/6/93 7 children	5	-	6	3	8	-
1/7/93 8 children	7	-	7	-	6	6
2/7/93 3 children	3	-	3	1	2	3

The headings in Table 2 are shorthand for the following concepts:  
*repeating* = a pattern may repeat a block or unit of numbers;  
*expression* = a pattern can be expressed in a variety of ways;  
*made of units* = the section which repeats can be considered a unit;  
*infinity* = a number of children, but especially Natalie was fascinated by the idea that patterns, like numbers can potentially go on forever;  
*number link* = a visual or action pattern can be expressed by numbers;  
*division* = a pattern unit can be used to segment a longer sequence.

Natalie's teacher has taught her for two years, but she is still puzzled by Natalie's difficulties with both mathematical and language concepts. Natalie certainly seemed to find many of the activities during these six weeks very difficult. As far the patterning activities went, however, she was in her element. During the testing session, she solved the patterning problem in one try. She might find mathematics difficult, but she certainly showed a strength in thinking visually. Here is her sequence of taped reflections from Week 2.

(28/6/93)

**Natalie** First we had to get a piece of paper and we had to draw our pattern.

**E.D.** How long was your piece of paper?

**Natalie** 1 metre [long].

**E.D.** Would you like to read your pattern?

**Natalie** Purple, red, green, green... purple, red, green, green... (Does stamp, clap, nod, nod action pattern.)

*(Concepts: task, pattern recurs, it can be expressed in a number of ways, conventional measure correctly remembered, complex pattern successfully linked with numbers.)*

(29/6/93)

**Natalie** We had to have a piece of paper and glue your pattern on and when you finished your pattern you had to put on a stick and just keep going on... and if we had a long, long, long, long piece of paper you could keep going on and on and on and on forever..... just like numbers.

**E.D.** Would you like to read your pattern for the tape recorder?

**Natalie** Orange, red, yellow, blue, orange,....

*(Concepts: Pattern units, recurring nature of patterns, potential infinity, link with number systems. )*

(1/7/93)

**Natalie** We got a piece of paper and then we got some blocks and counted 24 and then we chopped it into numbers. Mine is eights.  
1,2,3,4,5,6,7,8,..1.2.3....

*(Concepts: Pattern units, recurring nature of patterns, link between patterns and numbers, numbers can be divided by patterns of numbers, visual appreciation that  $3 \times 8 = 24$ .)*

(2/7/93)

**Natalie** First had to get a piece of chalk and then we made a pattern and when we finished we had to do a line, and you had to keep going on and on and on and on....

**E.D.** What was your number pattern?

**Natalie** Mine was a five pattern.

**E.D.** Would you like to read it, and do the actions if you like.

**Natalie** Clap, clap, blink, hop, jump

**E.D.** How long was your activity snake?

**Natalie** Fifty steps, five activities in each block...ten blocks.

*(Concepts: Pattern units, recurring nature of patterns, potential infinity, link between patterns and numbers, numbers can be divided by pattern blocks, relationship between numbers, remembers  $5 \times 10 = 50$  in semi-abstract form.)*

A similar record could be given for the remaining weeks, but these records may be enough to show how the activities could be used to refine concepts. The process of building an ever more complex pattern of concepts is at the heart of teaching. Some curricula build concepts in an atomistic manner, taking a number of simple concepts sequentially and combining them to support a more complex one. The curriculum used in this study built more complex concepts by helping the children question a complex situation with increased rigour. The evidence in this section demonstrated the process of accumulation of concepts day-by-day. At the end of each session during the week, the children mentioned more concerns, and their written work also showed that they were trying to solve more complex problems. The study's favoured method of concept building seemed to have been very successful.

## INDIVIDUAL CHILDREN'S LEARNING

Now we turn to individual children's learning and consider the results of the testing sessions held during the sixth week. During these sessions every attempt was made to see if the children really had an understanding of the concepts. Some of the questions were rephrased a number of times. Riley, Greeno and Heller (1983) have shown that "children's failure to show understanding of a concept on one task should not be taken as firm evidence that they lack understanding of the concept." (p.191.) The sessions attempted to collect information about what, and how much the children *retained*, and how well they could *use* what they had learned in novel problem solving situations. All the problems used in testing were new to the children. It was also significant that they were alone in the testing situation, while all the teaching sessions were based on group work. The test activities were as follows.

1. Draw a one metre track on the floor for two toy cars to race on.
2. Make a 3 pattern of 12 blocks. The child was given eight of one colour and four of the other.
3. Share 15 sweets (unifix blocks) among three people, who were represented by two puppets and a toy koala.
4. Subtraction: There are nine children waiting for the bus (unifix blocks) and the bus comes (box) but there is only place for seven children. How many have to stay and wait for the next bus? Can you tell what sum (number sentence) goes with this story? The more capable children were given the problem in a more difficult form: Some children got on the bus and two are left. How many got on the bus? Can you tell what sum (number sentence) goes with this story?
5. Tell the story of how to make scones. The children were given a flour packet, bowl, nest of measuring cups, spoon, butter container, milk carton, half a lemon, rolling pin and a glass to serve as reminders of the cooking process.

These activities, or problems, were very similar to the ones faced during the teaching sessions but they were not identical with them, and some were novel. For example the question about the construction of a three pattern out of a finite set of blocks was quite new to the children.

It was important to try to judge how much of what the children could do came from the teaching sessions of this study and how much from their classroom curriculum. During the time of the study, one class was concentrating on patterns, and another on subtraction. It would have been wrong to presume that the children brought no knowledge and experience from their classrooms. To try to judge how much they might have picked up in the classroom, as opposed to the study's 23 hours of small group sessions, three children were selected by their teachers to provide some comparison. These children were selected because their teachers judged them to be just a little better in mathematics and language than the children in the study. One child was selected from each of the three participating grade two classes, John, Danny and Abel, to represent the knowledge which might have been gained from the curriculum focus in

each. Table 3 summarises the results of the children in the study and those of the three children selected for comparison. The information used to create this Table is the twelve children's performance in the testing sessions. A full transcript of these sessions is given in Appendix 2.

**Table 3**  
**Testing results by task and concepts**

	Experimental group								Controls			
Concepts	M.	K.	L.	S.	N.	Me.	Ja.	Joe	R.	J.	D.	A.
<i>Racing</i>												
start	3	3	3	3	3	3	3	3	3	x	x	x
finish	3	3	3	3	3	3	3	3	3	3	x	x
tracks	3	3	3	3	3	3	3	3	3	x	x	x
fit	3	3	3	3	3	3	3	?	x	x	x	x
measuring	3	3	3	x	?	x	3	x	x	x	x	x
1 metre	100	100	100	90	100	80	100	80	90	30	25	200
<i>Pattern</i>												
repeating	3	3	3	3	3	3	3	3	3	3	3	3
visual	3	3	3	3	3	weak	3	3	3	x	x	x
numerical	3	3	3	3	3	3	3	weak	3	3	3	x
no. of tries	4	2	1	2	1	3	3	6	1	x	x	x
<i>Sharing</i>												
dealing	str	est	str	t/e	t/e	str	est	t/e	str	str	est	t/e
"altogether"	3	3	3	3	3	3	3	3	3	x	3	3
"each"	3	3	3	3	3	3	3	3	3	x	3	3
formal lang.	-	-	-	-	-	-	-	-	f	-	f	-
<i>Subtraction</i>												
relation of numbers	3	3	3	weak	3	3	3	3	3	weak	3	3
formal lang.	f	f	s/f	f	s/f	s/f	f	f	f	s/f	f	s/f
<i>Cooking</i>												
ingredients	3	3	3	3	3	3	3	3	3	x	x	x
quantity	3	3	3	x	x	3	x	x	x	x	x	x
sequence	3	3	x	3	3	3	x	x	3	x	x	x

M = Michael, K = Kim, L = Luke, S = Sarah, N = Natalie, R = Rachel,  
Me = Melinda, Ja = James, Joe, J = John, D = Danny, A = Abel

3= good or adequate concept;  
x = no, or inadequate concept;  
weak = weak connections;  
est. = done by estimation;  
f = formal language;

?= impossible to judge on evidence;  
100 = 100 centimetres;  
str. = workable strategy used;  
t/e = done by trial and error,  
s/f = semi-formal language.

Looking at the results in Table 3 one might gain some immediate impressions. Considering the tasks one-by-one, it is interesting to see the performance differences between the children in the study, and those selected for the purpose of comparison.

In the task of constructing a racing track in response to the first question which was, "Draw a one metre track on the floor for two toy cars to race on", we see that all the study children retained the knowledge that it is important to mark out the length of the course, to have tracks for the individual participants of the race and to have tracks of a width which suits the participants. The ability to use conventional measuring techniques brings out differences between the children, which will be discussed as an indication of their social maturity, in the section on "Developmental levels". In spite of the fact that the one metre ruler and the trundle wheel were both lying on the floor in full sight, four of the study children ignored them. Natalie used the metre ruler to get a neat, straight line, but was unsure how long the resulting line was. Even so, it would seem that their mental image of what a one metre should look like was fairly accurate, even if they were not fully conscious that they possessed such a mental image. Two of the comparison children seemed to have a 30 cm school ruler as their image for this length, while Abel seemed to have gone for "long" when he heard the measurement, "one metre". Of the other possible considerations, John drew a finish line, but all the three tracks looked like a section of roadway with the dotted line down the middle. It is also noteworthy that Abel did not understand that the two cars were to *race each other*. In his interpretation a "race" seemed to have had more in common with the expression of "racing about", that is, running about without organised structure.

None of the comparison children solved the patterning problem. The problem was, "Make a 3 pattern of 12 blocks with eight blocks of one colour and four of the other". They were asked for a *three* pattern and that is what they produced: four sets of three, without any regard to the colours of the blocks. Of the study children Melinda got sidetracked into action patterns, and Joe kept on losing the idea that both colours and numbers had to play a part in the solution. Once Melinda could suppress her first impulse to clap and blink and click, she actually did the task quite easily in three tries. Left to himself without adult encouragement, Joe would have abandoned the task after about three tries. All the study children, including Joe, recognised at once that the nub of the problem was that there were "not enough yellow blocks". The comparison children did not seem to be able to isolate the problem.

The task of sharing fifteen sweets between three people was a great deal more familiar to the comparison children than the racing track and the patterning tasks were. They performed much the same way as the study children. Three of the study children used an unstructured, trial-and-error dealing system, like Abel did. Only John confused the meanings of the expressions of "each" and "altogether". Cummins (1991) noted that the use of "altogether" as an expression of joint ownership is difficult for children to use correctly and that many have trouble with "each" as well. However,

she also found that visual representation of the situation often helped in the selection of the correct terminology. Visual and dramatic enactment did not seem to help John.

In the sharing task no oral language was specifically asked for, but to the question "How many did we have altogether?" Rachel answered: "Fifteen....because three groups of five equals fifteen." Danny obviously saw the whole sharing situation in terms of numbers, as he did the subsequent subtraction problem. He dealt out the blocks quickly by grouping, and when asked how many he had altogether, he replied without pause, "Fifteen". He was asked, "How did you work it out so quickly? You didn't even have to count them." Danny answered, "Five plus five equals ten, and ten plus five equals fifteen".

In the subtraction problem Sarah seemed to see little relationship between the story and the "sum" she was asked for. To the question "Can you make up a sum that goes with 9, 7 and 2?", she answered "Three plus four equals seven."

E.D. Good! Can you use all the numbers? Start with nine... make it a take-away sum....

Sarah Nine take away five equals four!

E.D. What about the story we've just had? What would be the sum to go with that?

Sarah Nine take away seven equals two.

This is the kind of interaction implied in the "weak" connection between the numbers of the story problem. John switched from nine to eight without any reason, indicating that the number was of no great importance to him. Joe estimated that "three or four" children went on the bus when two got left behind out of the nine. Such a wild guess did not count as an "estimate" and he was judged to have perceived little connection between the numbers in the story.

Language was scored semi-formal if the answer was still fully or partially embedded in the story. The question asked was "Can you make up a sum that goes with this story?" If the answer was something like Melinda's "Seven got on the bus and two waited for the next bus" the language was classified as semi-formal. Only answers like James' counted as formal, "Nine, take away two is seven." It is worth noting that Danny used mathematical language not as an end product of the concrete problem situation, but as an aid to solution. When asked how he managed to find the answer to the problem without evident thought, he answered, "Well, nine take away two equals seven." He said it not just as a justification but also as a self-evident fact. Danny's grasp of formal language was more secure than that of any of the study children.

It would be blatantly unfair to compare study children's information about cooking with that of the control children. After all they did not make scones during the previous week. All the study children identified the ingredients correctly, and most had a good grasp of the sequence of activity involved in

baking scones. It is interesting to note that those children who ignored the measuring task in the "Racing" problem were by and large those who could not recall the quantities used in the recipe.

Considering that the control children were judged by their teachers to be better, more competent performers than the study children, there is no doubt that the study children learned a great deal during the five teaching weeks. On the whole, the comparison children did not know what was expected of them in the Racing track and the Patterning problems. The Sharing and the Subtraction problems were familiar to all the children from the classroom, and the three performed about as well as the study children. Danny had a better grasp on these problems than the study children, while John was worse than any of the others in Sharing, and weak in Subtraction. It is gratifying to think that the study children would have performed a lot like John or Abel without the five weeks of teaching. While some of the teaching sessions had their difficult moments, the results seem to have been worth the problems and the occasional disappointments.

## LANGUAGE GAINS

The children were taped after sixteen teaching sessions, as well as during the testing sessions. They were asked to talk about what the session was all about or "to explain to the tape-recorder" about the written or drawn work they had done. Seeing that the children were taking turns at the tape-recorder and must have felt the pressure of time, and of taking turns, there is no way of regarding what they said as a full record of what they could have said given the peace and tranquillity of the testing sessions during Week 6. No attempt was made to collect information for any kind of before-and-after comparison, but some of the narratives can be compared. The longest narrative from all the children is in recounting how to make scones during the testing session. It would have been useful to be able to compare the narratives of the study children with those of the control children. Unfortunately this could not be done, because the comparison children could not really make anything of the task of explaining how to make scones. Their low narrative scores, and their general lack of knowledge regarding the baking process made the idea of using their contribution for the purposes of comparison totally unreasonable. These problems left language *gains* very difficult to judge. Because of these difficulties, other, less direct ways needed to be used if this gain was to be assessed at all.

During the teaching sessions the teacher's questions indicated that there were three areas of special concern. These were,

1. the *correct sequencing* of tasks were performed or problems solved;
2. the *quantities* or *numbers* involved;
3. the *reasons* or *conditions* which had to be taken into consideration.

Keeping these three linguistic, logical and mathematical concerns in mind it was possible to score the children's narratives. Table 4. gives a summary

of results. Unsolicited correct information in any of these categories scored 2 points, and those which were made in response to a question, scored 1 point. Any answers which were overtly or by implication mentioning reasons or conditions were counted separately and appear marked with a "C". Two taping sessions were used for comparison. On the Thursday of the first week, all the children taped their reflections on making tracks for racing. These contributions were scored according to the system above. The same was done to their narrative on "How to make scones", which was recorded during the testing sessions. The two sets of scores, side by side, give some idea about the relative performance of each child in the study. In some ways it is possible to compare the two sets of scores as well, because the process of marking out the tracks with string and sticky tape was about as complex as baking scones, and involved as much measurement. Potentially, they could have gained similar scores on the two occasions. The pressures of taking turns, and the hustle and bustle of the group taping session as opposed to the unhurried atmosphere of the individual testing session has to be born in mind when making any comparisons.

**Table 4.**  
**Children's narrative scores in two activities**

Names	Tracks	Cooking	Change in rank
Luke	19+3C	40+1C	-2
Joe	16+1C	32	-5
Melinda	15+1C	45+2C	+1
Michael	13+2C	53+3C	+3
Kim	13+1C	39+1C	+1
Rachel	11+1C	25	-3
James	10+1C	25+1C	-1
Sarah	7	36+1C	+3
Natalie	1	35+2C	+4

Changes in the rank order seemed to offer a way of assessing if some children have benefited more than others from the program. Rank order was calculated on the narrative scores including those separated in the Table as meriting a "C" designation. While all the children show gains in their narrative scores, those whose commitment seemed to flag were those who had fallen behind in rank. Changes in rank order seemed to reflect the child's commitment to the program rather than any true change in their relative achievement levels.

Conditional or reason statements were indicated separately because they seemed to imply a slightly more sophisticated kind of thinking than that which sequential narrative requires. The statements which qualified for this classification are listed below to give some feel for the type of language which was judged to imply either causative thinking or a regard for reasoned necessary conditions.



**Michael:** "...and made circles with it, so we could have our scones.."  
 "We rolled it out until it was nice and flat..."  
 "And then put them in the oven, and when they were all ready we brang them out..."  
**Kim:** "...we rolled it out not too thin"  
**Luke:** "We pushed it (the glass) down really hard and turned it around and then it would break where it was..."  
**Sarah:** "We twisted it around so it would cut it.."  
**Natalie:** "We had to put the glass right near the edge..."  
 "We put them in the oven to make them cook."  
**Melinda:** (Which is the real cup?) "The big one which has '1 cup' on it."  
 "We rolled it flat but not too thin...."  
**James:** "We brought them around to the classrooms and only the people who made them got them."

Another possible way of seeing how the children developed in fluency and confidence, not to mention relevance, was to collect one child's tapes from the beginning of the study to the end. It seemed at the beginning of the study that every utterance had to be elicited by a question, and still their answers were mostly in telegraphic style. It was gratifying to start to get answers which relied less and less on questioning.

Melinda's tapes were chosen as an example, because they showed her responses becoming more mathematical and less reliant on questioning. Melinda was a quiet, diffident child, who had extreme problems in both language and mathematics. She received Special Education assistance in language. Her performance in the two narratives which were scored, the track making and the cooking ones, indicate that she had linguistic strengths which might be used to build up her performance level. The following records demonstrate her increasing confidence in using mathematical language.

21/6/93

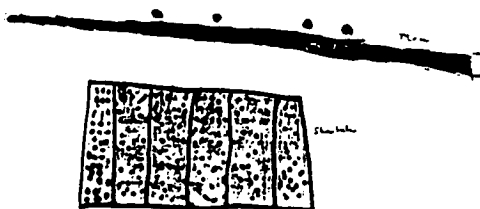
**Melinda** Drew the caterpillars. We had people caterpillars.  
**E.D.** What were the important things you had to think about?  
**Melinda** Sharing....We had to all have enough room to race.  
 (*Concepts: relationships, task*)

22/6/93

**Melinda** We made it with sticky tape onto the carpet. We rolled the white balls along the ground.  
**E.D.** How did we make them roll?  
**Melinda** We blew them with straws.  
**E.D.** How did we get the tracks even?  
**Melinda** We made the sticky tape the same amount.  
**E.D.** Why did we make them that wide?  
**Melinda** Because you had to go as wide as the person.  
 (*Concepts: compare and fit*)

Figure 5. shows Melinda's plan for a racing track which she drew at the beginning of the session, and her sketch of the track after its completion.

Melinda



**Figure 5.**  
**Melinda's plan done before the activity**  
**and her sketch of the completed track.**

23/6/93

**Melinda** We put the string down

**E.D.** Can you tell the tape recorder how you made the tracks?

**Melinda** We done the first, then the second, then the third, then the fourth one, (sides of tracks) then we put sticky tape on them.

**E.D.** When did we put down the starting and the finishing lines?

**Melinda** First!

**E.D.** How did we get the tracks even?

**Melinda** We measured them.

**E.D.** How wide did you make them?

**Melinda** 32.

**E.D.** Get the tape measure and show me what 32 cm looks like! Good! Natalie and Melinda go and check if they really have 32 cm between each line. (One of the tracks was in fact 36 cm wide as the two girls found out. They said that this made the race unfair.)

**E.D.** Why did we make the tracks 32 cm?

**Melinda** So that the people could fit.

*(Concepts: start-finish, conventional measurement-without unit name, fit)*

24/6/93 (Part of Group Two's response.)

**E.D.** How long was your running track?

**Melinda** Fifty metres.

**E.D.** What did you have to draw to make it a track?

**Melinda** Finishing line.

30/6/93

**E.D.** How long is your pattern?

**Melinda** Twenty four

**E.D.** What was the number pattern you put into twenty four?

**Melinda** I had patterns of six.

**E.D.** Did that work out?

**Melinda** Yes it worked out exactly.

E.D. Let's read it!

**Melinda** (Reading pattern with teacher): 1, 2, 3, 4, 5, 6, stick, 7, 8, 9, 10, 11, 12, stick, 13, 14, 15, 16, 17, 18, stick, 19, 20, 21, 22, 23, 24, stick.

(Concepts: Pattern units, recurring nature of patterns, link between patterns and numbers, numbers can be divided by patterns, remainder/no remainder)

1/7/93

**Melinda** We got some chalk and we done a line and then we done a pattern: stamp, stamp, clap, clap blink, blink, click, click, (stick), stamp, stamp.....

E.D. How long was your activity pattern?

**Melinda** I got up to 30 steps and then I done some more

E.D. How many activities did you have in each pattern block?

**Melinda** Eight! One on each side of the line. (They were drawn in pairs.)

(Concepts: Pattern units, recurring nature of patterns, link between patterns and numbers, numbers can be divided by patterns blocks)

5/7/93

**Melinda** Five chocolates and one was left over.....none (of the 8 jellybeans) was left over ...

E.D. How many jellybeans did each person get?

**Melinda** Two (each)

E.D. What did you do to the chocolate that was left over?

**Melinda** I crossed one out...

(Concepts: Share equally, put aside remainder, share different groups separately)

6/7/93

**Melinda** I've got eight squares.

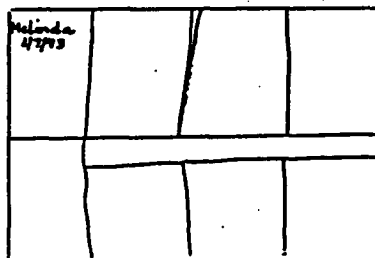
E.D. How many should you have?

**Melinda** Ten.

E.D. How many do you have on each side?

**Melinda** Four.

(Concepts: To share an object it needs to be cut up, the size of the pieces matters, they have to be equal) Figure 6 shows Melinda's work for this task.



**Figure 6.**  
**Melinda's attempt to cut a cake  
into ten equal pieces.**

20/7/93

**Melinda** We made jelly and played with Lego. I wrote 100. and I haven't finished it yet....I am making a house.

**E.D.** Did you take out 100 pieces of Lego?

**Melinda** (Shakes head)

**E.D.** No, you just wanted to be sure that you had enough, didn't you?

*(Concepts: task, quantities can be written in numerals)*

28/7/93

**E.D.** Take that box and see what's in it. What does that all remind you of?

**Melinda** We made...we done some cooking...

**E.D.** What did we cook with all these things?.....

**Melinda** Some scones...

**E.D.** Tell the tape-recorder...

**Melinda** The first thing was we put flour in the bowl...

**E.D.** How much flour?...

**Melinda** Three cups...

**E.D.** Which is the real cup?

**Melinda** The big one which got 1 cup on it....and then we put the butter on...

**E.D.** How much butter? .

**Melinda** One big spoonful and we got some milk and tipped it in...

**E.D.** How much milk?

**Melinda** Three...

**E.D.** Three what? .

**Melinda** Three blops....we squeezed the lemon in. We got it with our hands and squeezed it together, and then one person, Joe, one person put their hand in, and mixed it with their hand...and then we putted it on the board and we rolled it,.... and then we stirred it...

**E.D.** After we rolled it?...

**Melinda** After we rolled it we stirred it...we rolled it flat but not too thin, and we got the cup..

**E.D.** ...the glass..

**Melinda** ...glass, and we put the cup upside down and turned it around and made them come out...then we put them on the tray, then we put them in the oven....that's all..

**E.D.** What did we do when we took them out of the oven?...

**Melinda** We took it around to people in the classroom, and gave some to the teachers, and to the people who helped to make them. *(Concepts: Quantities of ingredients are important, correct measuring instruments/units have to be identified and used, some processes are sequential)*

It is important to remember that the children had their work in front of them when they taped their reflection on what they had done during the session. It was felt that to have something tangible to show and to talk about would help children to find what to say. This might have worked to some extent but it also may have stopped the flow of words, because the children could *show* what they had done and often felt that there was no need for words. Hardcastle and Orton (1993) found that blindfolding students as they report on mathematical activities forced them to use more

precise language. In this case, however, it was felt that not having something demonstrable with them might completely dry up the flow of words.

So far the focus has been on expressive language, on the production of language by the children. Their growth in understanding the teacher's language, their receptive language, needs to be similarly documented. Ease of understanding helps the development of good social relationships. In this case learning the teacher's language meant that the children could respond appropriately to it.

The contrast between the amount of explanation each testing task needed for the study children as opposed to the three control children highlights receptive language learning. An analysis of the number of prompts necessary for each child to complete the test task shows the difference five weeks' working together made to the study children's understanding of the language used. The only prompts counted were those which were necessary for the *completion of the task*, and for the *clarification of expectations*, not for the correct solution to be achieved. Those prompts which were concerned with the accuracy of the task were not counted. Neither were instances of misunderstandings between teacher and child. Specific instances of misunderstandings will be discussed in the section "Adult-child communication". Table 5 gives a summary of the results.

**Table 5.**  
**Number of prompts needed for the**  
**clarification of testing tasks**

Tasks	Experimental group							Controls				
	M.	K.	L.	S.	N.	Me.	Ja.	Joe	R.	J.	D.	A.
Track	0	0	0	0	1	1	0	3	0	1	2	3
Pattern	0	0	0	1	0	3	0	1	0	2	3	4
Sharing	0	0	0	0	0	0	0	0	0	2	0	1
Subtraction	0	0	0	1	0	1	0	0	0	1	0	2
Cooking	0	0	0	0	0	0	0	0	0	2	0	1

There were 12 prompting questions asked of the nine children who participated in the study (average of 1.3 questions per child) and 24 of the three children who did not take part in the teaching sessions (average of 8 questions per child). Clearly, there was a development of shared expectations and understandings between the teacher and the children during the five weeks, which did not happen for the other three children. Some of the growth in mutual understanding resulted from shared activities and experiences, and some from learning the teacher's mathematical language. In this situation, experience and language could not be separated.

## ADULT - CHILD COMMUNICATION

A number of workers in the field of adult-child communication warn that because of the vague way language is used in everyday life it is possible for adults and children to use the same words and mean quite different things (Lowenthal, 1990). In this study, one of the aims was to create circumstances which were conducive to checking meanings. On the whole the approach seemed to work. The results presented in the previous sections show how well the teacher and the children came to understand one another. The smooth flow of the individual testing may be taken as evidence for the quality of communication built up during the previous five weeks. This section, while presenting some of the many successes of communication, will look at the few failures in greater detail, because often failures can point to issues which are still problematic.

Some children built up very good communication links with the teacher. Michael was one of these children. The smallest hint was enough to remind him of some aspect of the task he may have omitted. He finished the teacher's sentences and showed impatience when he felt that directions went on for too long.

**E.D.** Do they start here or here, where do we go? What's this line? (Michael immediately writes an "F" on one end and an "S" on the other.) I see, that's the finish line and that's ....

**Michael** ...the start.

He indicates that the teacher's well meant hints are not necessary. He knows what needs to be considered:

**E.D.** It has to use up all those blocks!

**Michael** Yeah, yeah! Start again!....Yellow, blue, yellow, blue, yellow, blue....

Michael interrupts as directions are given to show that he already understands the task:

**E.D.** What I'd really like you to do is to use these things to remind you and to tell...

**Michael** ...how we made scones!

Similarly, Kim is very quick to perceive the "hidden message" in a question:

**E.D.** Where is the starting line?

**Kim** Silly duffer, I am. This is the starting line and.....(this is) the finishing line.

Sarah had a great deal more trouble to achieve common meanings with the teacher:

**Sarah** I'll need the ruler.(Got metre ruler and drew a line, and because of the ruler used she had a 1 metre line) Two gifrent [different?] one metres.

**E.D.** What does one metre look like?

**Sarah** It's long.

**E.D.** How long is it? How are you going to find out how long is one metre?

**Sarah** With that. (Points to trundle wheel.) (She correctly sets up starting point, starts the wheel while counting steadily under her breath. She stops on the click.) Eleven!

**E.D.** Eleven what?

**Sarah** Eleven metres.

**E.D.** Well, we only wanted a one metre track. How long is one metre, how are you going to find that out? How did you find out that was eleven?

**Sarah** (She does the whole operation again, but does not start correctly and when the click is heard she has only got up to six in her counting.) Six!

**E.D.** So how are you going to make a 1 metre track?

**Sarah** Do it a bit more longer.

**E.D.** All right.

**Sarah** (She extends both lines by about another 20 centimetres. "Measures" them again.) Six!

**E.D.** So you think that's a 1 metre track?

**Sarah** Yes.

Sarah's most used, maybe the only, mathematical procedure is counting. It certainly caused confusion when she used counting as a measuring device. This confusion may, or may not be regarded as purely linguistic, but it has a linguistic root. For Sarah the word "measure" means counting. "Working out" seems to be another synonym for counting, as the next exchange shows, which took place as she was considering the subtraction problem about the children getting left behind by the bus.

**E.D.** Do you want to have all the people back, or do you just want to work it out?

**Sarah** (She reaches out for the two "buses".) I'll work it out...1,2,3,4,5,6,7,8,9! (Counts them. This is her "working out".)

The next misunderstanding is about the making up of the "number story" with the three numbers which featured in the story. Sarah does not realise that the expectation is that all the numbers should be put into one number sentence, and uses them more or less randomly.

**E.D.** Can you make up a sum that goes with the numbers 9, 7, and 2?

**Sarah** Three plus four equals seven.

**E.D.** Good! Can you use all the numbers? Start with nine. Nine....make it a take-away sum...

**Sarah** Nine take-away five equals four!

**E.D.** What about the story we have just had? What would be the sum to go with that?

Melinda also has problems with some of the words used. The one which seems to be causing difficulty here is the word "sum". Is it the same as "some"? Melinda is not sure.

**E.D.** Is there a sum that tells about this story? Nine...

**Melinda** and went on the bus and seven people left.

**E.D.** Right! This a take away sum, isn't it? How would you say the take away sum? '

**Melinda** 'Cause you take away some and there is less left.

Joe solved his problem with some superfluous "marshmallows" after sharing out, by offering them to the teacher. Certainly an unexpected move.

**Joe** (He takes away all the extras and leaves four each.) Now they all've got four! See? 1,2,3,4; 1,2,3,4; 1,2,3,4!

E.D. There are still some left over! Can you share out those too?

Joe Here you are, you have them....

E.D. No, give them to the puppets!...

It is worth noting that the three new children asked many questions of clarification, which were not necessary for the others. The following exchange took place as Abel attempted to get the patterning problem clarified.

E.D. Here are some blocks, and I'd like you to make a snake pattern, a three pattern, which uses up all those blocks.

Abel How do you mean?

E.D. The pattern has to go in threes.

Abel Like, you have a pattern there, another pattern there, and another pattern there?

E.D. Well, no. Every bit of the pattern has to have three things in it, and just go until you've used up all those blocks and made a long snake.

Abel You mean like this.. or..

E.D. Just put it together and see if it makes a pattern.

Danny attempts to shift the responsibility for the solution onto the teacher.

E.D. All right, try again! (Tries yyy, bbb, y...)

Danny There needs to be more yellows!

E.D. I can see your problem, but you can't have any more yellows! See if you can do it with the blocks you've got.

Danny You do it!

E.D. No! (Laugh) I am the teacher! I don't have to do it!

Danny Is there a way?

E.D. Yes there is a way.

Danny Do you have to use up all of them?

E.D. All of them. The colours are not in a pattern, are they? Do you think you could change it so that the colours make a pattern too?

Since none of the three comparison children solved the patterning problem there was plenty of opportunity for exchanges like the ones given here.

The solution of the racing track problem was also peppered with crossed communications. While the study children were quick to respond to hints about the beginning and the ending of the track, the three others missed the allusions given them.

E.D. Where are we going to line up our cars so that we can race?

Danny Here..

E.D. And where are we going to?

Danny Here..(Danny points but is not putting any more lines down.)

E.D. Where are we going to put the cars? (John places them at the end, but does not add any more lines.)

While everybody else drew a straight line for the track, Abel's was a curving, twisting two metre long line. Another surprise! He also seemed to have had a different idea from that of the teacher about what racing involved.



**E.D.** I was going to race the two cars to see which was the faster. What were you going to do?

**Abel** .I was just going to push mine along...(He does so along the track. We try the two cars alternately.)

John got the words "each" and "altogether" mixed up in the Sharing task:

**E.D.** How many did they each get?

**John** (Counts the lot.) Fifteen...

Abel was asking a lot of clarifying questions about the terminology surrounding the Subtraction task:

**E.D** Here is the bus and here are some children waiting for the bus in their blue uniforms....

**Abel** ....and they went....

**E.D** .Hang on! I'm doing the story and you're doing the maths..

**Abel** The what?

**E.D** The maths, the sums...

**Abel** What's a sum?..

**E.D** The numbers..

The ease with which children learn to cope with new language presented to them seems one of the most important factors in their teachers' judgement of their ability. Teachers tend to underestimate the capabilities of those children who do not understand them (Donaldson, 1992). A lack of skills in receptive language seems to be the salient characteristic of low achieving children. Ability in language reception is the same as the ability to gain information from instructions, and to follow them. Therefore it is no wonder that receptive language plays an important role in determining the quality of relationships in the classroom.

## **SUMMARY OF LEARNING OUTCOMES**

The children showed impressive competence in building more and more complex concepts as they perceived the demands of the tasks in greater detail. They also became more fluent in discussing their work. During the individual testing sessions the children in the study performed much better on some tasks than the control children who were judged to be marginally more able by their teachers. They remembered the tasks from the teaching sessions well, and were able to do similar tasks with ease. They were also able to do tasks which were new to them, like the patterning task, which was definitely an extension of what they had done before.

As the weeks of the study wore on the children became more vocal, and were able to put together longer narrative sequences. They used and understood more mathematical terms. Communication about tasks and the mathematics involved became much easier.

At the completion of the study, when the results were assembled and considered, it was felt that the study's teaching aims had been, by and large, achieved. The children learnt many new concepts, consolidated some with which they were already familiar from the classroom, they started to speak more about the mathematics they were doing, and grew in confidence. Their teachers spoke about their improved performance in both mathematics and language. The evidence assembled for the "Learning Outcomes" section strongly suggests that when children are taught mathematics in contexts which are familiar to them, using concrete objects as aids to thought and to communication, they will learn mathematics in a meaningful way. It was found that the meaningfulness of the mathematical situations helped the development of both receptive and expressive mathematical language, though some misunderstandings still occurred.

If it seems that failures were unduly highlighted in the "Learning Outcomes" sections, it was done deliberately, because successes merely confirm what is already known, while failures point to new issues and to yet unsolved problems. It could be said that to those who wish to improve their practice failures are more precious than successes.

## CHAPTER 5

### DISCUSSION

*"Self-reflection leads to insight due to the fact that what has previously been unconscious is made conscious in a manner rich in practical consequences: analytic insights intervene in life, if I may borrow this dramatic phrase from Wittgenstein."*

(Habermas, 1977, p.23)

*"...social actions are comments on more than themselves; .... Small facts speak to large issues....." (Geertz,1973, p.23)*

The results of the study seem to justify the researcher saying that the learning situation had many beneficial effects on the children's learning, but the ethnographic research methodology does not allow an isolation of different aspects of the situation. Such an isolation of aspects would be seen as neither logical nor desirable. While the ethnographic method of research is not suited to an analysis of the teaching-learning situation, it is well suited to the promotion of researcher reflection, a point stressed by action research theory.

The daily research diary recorded the niggling problems, recounted worries about sessions which did not work as well as anticipated and told of children who did not always do their best. In the diary there are also accounts of tangled lines of communication and of strained relationships between teacher and children. For the purposes of this chapter, three evaluative, reflective questions were isolated for consideration.

1. How well were the tasks contextualised?
2. How well were the individual children's developmental levels catered for?
3. How well were the teacher's and the children's agendas balanced?

These three interrelated questions will be now considered in turn, using the support of some relevant literature. It was inevitable in this work, that the literature used to shape the study would need to be supplemented when the questions raised by the study itself were being considered. This chapter will introduce this additional material in the course of the discussion

#### HOW WELL WERE THE TASKS CONTEXTUALISED?

In the theoretical section of this dissertation reasons were given why the social context, and this includes the linguistic context, is so important for the learning of mathematics. Meaning, it was said, is created through the interaction of the individual's needs and purposes with the physical and

the social world. Interaction with the social world is heavily reliant on the cultural medium of language. Language functions not only as a medium of communication but also as a tool for thinking.

Before we can turn to the experiences of the study it is necessary to look how the concept of context has been used by different writers in the field. Their thoughts create a background from which some of the events of the study can be appreciated and understood. The first point to be noted is that "context" is not a unitary entity which is either present or absent. In the study, the tasks were always presented within the context of a story, a story all the children could appreciate and which gave the tasks a narrative context. Yet, as we have seen, some of the children, even though they could certainly understand the problem in its narrative context, showed reluctance to put in the effort necessary to find good solutions and to complete tasks.

Sometimes it is through reading curriculum material designed for a different culture that the realisation hits the reader that there are other, unstated contextual variations which go unnoticed within our own culture. Reading the curriculum segment on number lines designed for Russian children and reported by Venger and Gorbov (1993) would make most Australian teachers of six year olds feel apprehensive. The political context of this study, by "political" meaning the system of interpersonal power relations, is that the teacher tells the children what to do, how to do it, in a "repeat after me" manner. Furthermore, the exercises themselves are not embedded in any purpose other than the one stemming from the teacher's idea of what children need to know. Context is only supplied by the narrative embedding of the task, for example, "going mushroom gathering" over the number line. Reading this article one cannot escape the realisation that there are levels of context operating over and above the narrative context of the task. These levels of context together make up the total context for any given learning situation.

The levels of context may be labelled as different "domains", following Greeno's (1991) metaphor of the "conceptual domain". It is possible to identify the political domain, the social domain, the conceptual domain and finally the domain of the immediate task. All these domains may operate in any one mathematics class. Lyotard (1984) said that all disciplines have their narratives, through which crises in the axiomatic foundations are overcome. The true flavour of any real situation, such as the teaching situation, can only be appreciated if the narratives, the stories, of all four domains are told and explored in turn.

Human activity, including learning takes place in a total context, including the political, the social, the conceptual and that of the specific task. Some of the reasons for this may be found in Dunn's (1988) work on the development of the child's social interactions. The child will take part in social situations as a *protagonist*, an active agent, even before he or she can tell or retell the narrative of those situations. Dunn says,

"They communicate shared beliefs about the way life should be lived and about relationships between the members of their world in a variety of subtle and not-so-subtle ways. To become a *person* - a member of that complex world - children must develop powers of recognizing and sharing emotional states, of interpreting and anticipating others' reactions, of understanding the relationships between others, of comprehending the sanctions, prohibitions, and accepted practices of their world." (p.5)

In order to take part, the child has to have some understanding, albeit on a tacit level, of the nature of the situation as a political and social reality. Mellin-Olsen (1987), Bishop (1988, 1991) and Popkewitz (1988) have all written extensively about the political context of mathematics education. Phenomenologists like Freudenthal (1991), Marton (1981, 1983, 1990) and van den Brink (1984, 1988, 1993) have long stressed the importance of having socially understood and valued content within the mathematics curriculum. Those who emphasise the cultural component of language and learning also see the social context as decisive for learning (Lave, Murtaugh, and de la Rocha 1984; Dreyfus and Dreyfus, 1986; Bergeron and Herscovics, 1988).

As learning takes place in the public domains of politics and social interaction this implies that children need to explore learning situations in order to determine what kinds of roles they might play as protagonists. Learning uses all the "objective materials of the common culture" to build up its patterns (Geertz, 1973). These objective materials may be physical objects, behaviour patterns such as ritual actions, or symbolic actions such as the use of language. The social dimension of learning is highlighted by many, from Vygotsky (1962) to Bruner (1977, 1983). "The infant's principal 'tool' for achieving his ends is another person", said Bruner. The foundation of Mead's (1934) thesis was that self realisation through social interaction is the basic motivating force of humans. Child protagonists assess the social situation on the basis of how it will benefit the self. Through understanding ourselves we have the power to predict the behaviour of others who may be influential for good or for ill in the social situation we encounter. In Blakemore's (1977) words,

"If evolution has given us internal awareness, it must presumably have had survival value. One advantage that consciousness provides us is the ability to make predictions about the behaviour of *other* people, or even of other animals. It supplies a set of rules for relating emotional states to external events....."(p.37)

After the political and the social contexts, the participant may explore the conceptual domain (Greeno, 1991) in which the task or actual interaction is situated. Lukacs (1971) maintained that the conceptual domain is but a subset of the social one. Its origin is in the cultural history of the social group. Lukacs said that there is a "structural identity" between mind and society. The logical structure of formal concepts is not universal, but rather it is itself a product of history. In spite of close links between the social and the conceptual domains, it is possible to look at the conceptual domain in

isolation from the social one. Many educationists do this when they focus on the importance of seeing specific tasks as being part of an overarching conceptual world. Such educationists may express their concerns by stressing the importance of cognition (Baroody and Ginsburg, 1990), learning for understanding, intelligent learning or meaningful mathematics learning.

Finally, the task itself needs to be contextualised. Briars and Larkin (1984) use actions, Venger and Gorbov (1993) gather mushrooms, van den Brink (1988) plays the bus game, and so on.

Keeping these points in mind, it can be said that when the children in the study entered the novel situation of learning with the teacher, they assessed it primarily with the aim of discovering what positive roles the situation may have had to offer them. The quality of their participation depended on their assessment of the situation, and the quality of participation in its turn affected their learning.

There is no doubt that the way different children interpreted the contexts of the teaching sessions had a great deal to do with their outcomes. On the level of the political context the study was identical with the school. There was a teacher who told you what to do, where you had to be and for how long. Some of the children had problems at this level. They felt that this context did not offer positive roles for them. Children who had problems with the politics of schooling, also had problems of involvement in the study.

Joe and Luke were seen as "problem children" by their teachers, as they did very little work, they did it reluctantly, and lacked discipline. James was reported to be better adjusted to the school as an institution, but the interpersonal/social context did not seem to suit him. He was seen as a child who always wanted to be a winner, who could not bear to be seen to have failed at a task. He would avoid a task rather than expose any weaknesses.

During the five weeks of the study, these three children, Joe, James and Luke, had compliant and non-compliant days, which were on the whole independent of the topics and the tasks involved. The difficulties the teacher/researcher experienced with these children was identical with those their classroom teachers reported. These children did not see the study as offering them new and more promising political/social contexts, but merely reproduced those aspects of schooling which these children had already found non-productive for self enhancement.

One of the major concerns of the study was to give children a curriculum which was "real" to them. In spite of this aim, not all the topics had the same relevance to the children's conceptual world. The children's reactions to the tasks suggested that the four teaching weeks were not identical in "realness". Racing, cooking and building were perceived as more real than sharing, in spite of the fact that all the sharing activities were clustered

around the theme of a party. Pattern making was perceived as less real than the first three, and subtraction less real than pattern making. Sarah and Rachel, Melinda and Natalie only did some of the activities because they wanted to please the teacher, but they did not understand the meaning of the activity. The activities lacked meaning in that these children could not make the connection between themselves and their possible roles within them. Only Michael and Kim could see all the activities as meaningful in offering them a positive role to play.

All the tasks were presented in a narrative context. This made them accessible to all the children. The narrative context enabled children to understand the problem and to retain its focus for the duration of the activity. In spite of this, and in spite of the concrete materials used, some of the children, on some days, were less than enthusiastic about doing them well. Mushroom gathering may be fine in its own place but it is only a partial solution to the problem of engaging all children in a meaningful mathematics curriculum.

The lessons gained from this study were that

- all the children benefited from contextualising the tasks in a narrative form;
- some of the children needed to have tasks which had purposes they already understood and appreciated, such as cooking;
- some of the children relied on the social context to help them come to grips with the tasks, in other words, they worked to please the teacher;
- some of the children found the political context of the school burdensome, and made every attempt to show their opposition to it.

In conclusion, it was found that to contextualise mathematics curriculum it is necessary to consider all the levels of context, if all children are to find self-enhancing roles for themselves. Different topics and tasks were perceived differently by individual children. Part of what the art of teaching implies is the juggling of the content of the curriculum with the possible contexts which can be managed within a classroom, and within our system of schooling.

## **HOW WELL WERE INDIVIDUAL CHILDREN'S DEVELOPMENTAL LEVELS CATERED FOR?**

Another way of looking at the results of the study is to see what the needs of the children were which they tried to fulfil through participation. These needs had not only social, but also developmental bases. In a class of nine children a number of developmental levels were represented. In a class of thirty this problem is ever-present for all teachers. Working with the teaching group of only nine children created ideal conditions for the observation of developmental levels. This section will attempt to structure the observed developmental levels into a hierarchy.

Following Piaget there have been many attempts to identify the stages of children's cognitive development. Kamii (1985) felt that children reached stages where they could engage with problems of greater complexity and abstractness. Van Hiele (1986) built his system on "levels of thinking", which is similar in many ways to Biggs and Collis' (1982) SOLO taxonomy. The problem with having a system of stages which concentrates on cognitive strengths is that it ignores the well documented finding that children's success at cognitive tasks is heavily context determined.

Feeling that something different was required if the context dependence of thinking was to be taken into proper consideration, Donaldson (1992) identified four largely successive modes of thinking. For the four modes she took as the basis of differentiation the *time frame* the child was able to consider. She thought that in the *point mode* the child was chained to the here and now. In *line mode* an appreciation of a personal past and a personal future emerged. In *construct mode* the child was able to appreciate that the time line was not necessarily personal but existed outside the child's immediate experiences. In *transcendent mode* the young person frees herself from any restriction of time and can consider questions and issues independently of time and place constraints.

The results of this study seemed to fit a framework a little like that of Donaldson. The observed behaviours and results could be fitted into a fourfold developmental hierarchy. Donaldson used children's developing consciousness of time frames, but in this study it was the children's developing appreciation of an *ever widening social contexts and their own place in these* which seemed to determine their understandings, and allowed or precluded participation. Taking the social focus helped to link the learning contexts with the children's learning outcomes. One way of thinking about these stages, was to imagine different "worlds" the child may be inhabiting, something like the three worlds of Popper. The metaphor of the kinship circle was attractive because it drew on an evolutionary vision of human development. It made sense to refer to humanity's million year history and see its heritage both in the individual and in today's cultures. The boundaries of each world are defined by the child's experience and imagination. New worlds are appropriated by gradually taking part in them. In this framework, the four stages of Donaldson appear in new guises.

### 1. *The world of physical contact*

In this world the child sees herself as a being with physical and emotional needs. Her purpose is to have these needs filled. Vygotsky (1962) observed that the human infant from the very first is a being with intentions and purposes; an observation which has been well substantiated in studies such as Dunn's (1988). There has already been discussion of the evidence that even newborn babies see the world through some preferred structures. They make the world which they are inhabiting. They also have a role in this world. They have a role of being the one to be cared for, fed, entertained, cuddled. They are born with this role, and come equipped to



play it so charmingly that the rest of the community, mostly their doting kin, falls in with it and complies.

*When asked how long his pattern was Joe replied, "Give me cuddle".*

This may be seen as a typical answer from a two year old.

## *2. The totally shared world of immediate family*

In this world it is assumed that the communicators share an experiential world and are face to face when they talk to each other. Language development in young children seems to depend on a care-giver speaking for the infant, paraphrasing those intentions and observations the child is yet unable to express. It is a world of gestures, allusions, and of a special vocabulary only understood within the close circle of people sharing this small world.

*When asked about the width of her track Kim replied : "About this wide" - - (showing with hands) .*

This may be seen as a typical answer from a four year old.

## *3. The world shared with kin, the world of the subculture*

From about the age of two and a half to three years, the child constructs the larger world of kin. She now realises that not all allusions will be understood, because many situations and experiences might not have been shared. However, the basic assumptions of the subculture are still there to make communication easier. Precisely because all experiences have not been shared, that the child can find a new role, that of the story teller. By the narration of life's events this larger world is knit together. Now there is need for a more standard vocabulary, and many more referential markers in the narrative if communication is to be effective. The assumption is still that communication takes place face-to-face.

*When Alan was asked about the width of the track he replied, "Big".*

This may be seen as a typical answer from a four year old.

## *4. a. The world of strangers, the world of the culture*

In the world of strangers even less can be taken for granted. It becomes necessary to try to find out about the stranger's world to make communication possible. One of the roles available in this world is that of the giver and taker of information. To be successful in this world multiple roles have to be played, and conventional reference points need to be observed. Because the role carries prestige, those children who enter this world earliest are thought to be "bright". They are seen, and therefore see themselves, as being helpful to the community. As Donaldson said those children who come to school being able to comprehend the *words* of strangers, without context are those teachers think of as "intelligent".

*Sarah did not have the confidence to operate in this world. When asked about the width of her track she replied: "I don't remember".*

*James gave it a go. When asked about the width of his track he said: "One centimetre.."*

This may be seen as a typical answer from a five or six year old.

*Luke succeeded in the world of strangers. When asked about the width of his track he said: "Thirty centimetres, no.. thirty two centimetres."*

This may be seen as a typical answer from a seven to nine year old.

#### *4.b. The world of issues and problems, the world of intercultural and paracultural contexts*

The community not only has members, their interactions and events, it also has problems, discusses issues, evaluates behaviour, and so on. The roles in this world are those of the clarifier of issues, the judge of rights and wrongs, the finder of new solutions, and the intermediary between "tribes".

*Natalie spoke of her pattern and pointed out its potentially infinite nature: "We had to have a piece of paper and glue your pattern on and when you finished your pattern you had to put on a stick and just keep going on... and if we had a long, long, long, long piece of paper you could keep going on and on and on and on forever..... just like numbers."*

This remark might have been made by a person of any age.

If these worlds correspond to developmental levels, how is it possible to find an example of each in the language of ten six and seven year olds, who were judged to be "backward" by their teachers? It would seem that people do not grow out of the simple, immediate worlds, but rather learn to become citizens of multiple worlds. Furthermore, research into children's behaviour in specific contexts shows how quickly children find out about the social rules of the new situation and take their part in them (Kendrick, 1986 study of hospitalised children, quoted in Dunn, 1988, p.172). Different contexts allow children to move into a new way of operating, a new way of looking at a situation and their own role within it.

How much explanatory power do these hypothetical "levels" have? It seems that the idea of the "four worlds" seen as an expanding social circle can help teachers to match children and problems, because it enables

- a classification of mathematical problems on the basis of the social context they imply;

- an identification of the worlds a child is already comfortable in.

Success at learning might be predicted by seeing how well the implied world of the problem matches developmentally available worlds of the child. It is useful to look at a classification of the tasks in the study to see into which world they best fitted.

Broadly speaking the mathematical tasks in this study can be classified as follows.

1. The world of physical contact was represented by such activities as playing with equipment, running about, chatting, chanting, interaction with others, and observing social rules of physical well-being.

The activities of racing during the first session could be interpreted as belonging in this world: rolling spools, and balls without a defined track, crawling on one's stomach in a caterpillar race are immediate, physical

tasks. A participant who saw them only as world 1 tasks, could not be differentiated from one who saw them with added meaning.

2. The totally shared world of immediate family was the context for activities such as devising games with rules, and measurements if these were necessary, which were agreed only within the immediate group, and had no necessary reference points outside this group.

Luke's account of the second racing session shows that in his mind roles and rules dominated.

*"We got some sticky tape and we stuck it to the carpet and we made some spaces and we had a beginning person and an end person these people made sure that the ball didn't go over the tracks. Next time we had another rule that when we blew the ball out of the tracks we had to start all over again."*

3. The world shared with kin, the world of the subculture operated when the children obligingly did the tasks the teacher asked of them. In the subculture, it was acknowledged that the teacher had a right to devise tasks which did not seem to have an immediate meaning, but children were nevertheless expected to do them.

Michael interpreted the racing task in this way. Notice the expression "we had to" indicating compliance.

*"We had to get sticky tape and we put it on the carpet and we had to put four lines and a finish line and a starting line, and we had races on the tracks made. If you go ever of the line it is out and you've got to start again and the first one across the finish line is the winner. And we made the tracks by ourselves."*

Joe's account is also in this vein.

*"We had to measure the line and we had to put a dot where number 32 was, and we had to put a straight line instead of a wobbly line. If we had a wobbly line the ball would go out of the track and the person would have to start all over again. James went out of the track and he started all over again and he still won."*

4. The world of strangers, of issues and problems, the world of intercultural and paracultural contexts operate when people do mathematical tasks according to rules and patterns which originate outside a person's own experiential world; when they use conventional units of measurement and formal mathematical processes. In this world there is a need to communicate with people who have different life stories.

None of the children were at home in this fourth world, and therefore could not understand the rules of its discourse. Notice that measurement unit names kept disappearing from their accounts, because the children

could not see the use for them. When they were used, they were used only in a world 2 or 3 sense as a 'chant' or 'what you have to do'. Because the children's use of the unit names depended on memory and not on understood use and meaning, they were often disregarded and easily forgotten.

Turning to the individual children themselves, what do we find? Joe was the most "immature" child in the group. The multiple 'worlds' explanation would allow us to say that Joe could only function comfortably in worlds 1 and 2. For him the world where the teacher could invent tasks which had nothing to do with him personally did not make sense. It did not offer him a role, let alone a satisfying role. During the second week, the one concerned with patterns, he ignored the direction to make a colour pattern and ended up doing something idiosyncratic with arrows, which he then could not explain.

The record for the following day says:

*Joe found it hard to name the colours and kept losing track of his pattern...he seems to lack confidence to do a good job. He therefore seems reluctant and as his teacher says "lazy".*

And the day after that:

*Basically all Joe did was work avoidance.*

It is obvious that the tasks asked of him had no meaning for him. At the beginning of the experimental time he tried to become the teacher's baby in a true world 1 sense, and when he found that there was not enough time, opportunity or inclination to fulfil this wish, he gave up on both the teacher and the situation.

Most of the other children in the group were just able to see that there were some rewards, and some fun to be got out of activities teachers devised. They could operate in the world of the school. Unfortunately, to be successful does not depend purely on good will. Simply wanting to be part of the subculture is not enough. For the majority of these children it was the meaning of the school subculture which kept slipping from grasp in spite of their wish to participate. Sarah always had the greatest good will and the eagerness to do the right thing, but she often failed to see why certain tasks needed to be performed, and what answers they might yield. A good example of her lack of appropriateness was the incident when she attempted to use the trundle wheel in conjunction with her own counting.

James also had problems with the demands of the tasks and the kind of situations which were forced on him. He enjoyed running around, racing and winning. During weeks three (Sharing) and four (Subtraction) he became increasingly disenchanted, in spite of the fact that he could do the tasks set. Finally, he rebelled against the paper and pencil tasks which were meaningless to him.

Luke was one of the two or three most able students in the group. Yet he too opted out during week four. He found a more useful role in being the one who "fixed" the teacher's little wooden truck. Living in world 3 exhausted Luke, he became "world-weary". He retreated back to world 2, where you do little personal favours for members of your family and are thought clever.

The point to note about the explanatory power of the "worlds" theory is that it is not the tasks which are being discussed. Most of the tasks were framed in terms of some immediate and familiar story line and had concrete material supplied for the doing of them. Much of the material was "really real" such as sweets, cakes, real games, cooking ingredients and Lego blocks. In spite of this, a number of children did not function willingly or well some of the time. What was lacking was their personal involvement and role taking in the situations demanded of them.

An attempt was made in this study to present tasks which suited all developmental levels, in that "the children could perform at their own level". The last is a well worn phrase in curriculum development but it has little value, if the child cannot make the translation for him or herself. It is part of the teacher's task to present problems at all levels, and let the children choose for themselves. A session on sharing might have had the following activities in it.

1. Set out a Teddy bears' picnic with an adult helper.
2. Make take-home parcels of "sweets" for your friends, who are in your group.
3. Take a packet of mixed sweets prepared by the teacher, and find out what would be the best number of children you could invite to your party, if all the sweets were to be shared evenly and as few as possible left over.
4. Take handfuls of blocks, share them out between (2, 3, 4, 5, and so on.) people and write the number stories that tell about the sharing. Identify some mathematical rules which govern sharing tasks.

During the study, the first two days of each new topic were immediate but soon the children were required to look at the topic in more structured ways. Some of the children could not build concepts at the rate the study curriculum required of them. Looking back, one can see that Joe should have been given problems which had been deliberately framed in a way that was not only concrete, but also dealt only with Joe's immediate world. James should have had a curriculum which though contained problems of a social nature, made him the hero of all the adventures, and so on. Even though the teacher accepted and praised the work the children did, some of them felt that they had not made sense of the problem.

It is likely that there would have been fewer behaviour and involvement problems if the task for the day had been presented on at least four levels of immediacy. The expanding social circle model possesses not only explanatory powers but also gives some guidance for the construction of

more accessible mathematics curricula. The challenge is there to construct such a curriculum for the use of all classroom teachers, but especially those who have a wide range of ages and maturity levels to cater for.

## HOW WELL WERE THE AGENDAS OF TEACHER AND CHILDREN BALANCED?

Classroom life consists of a process of continuous negotiation of agendas. The teacher would like to present concepts and tasks she thinks the children should do, but some of the children may not be able to understand what is required. The teacher may be anxious to see some project completed, but some of the children may refuse to put in the required effort. The relative burden of responsibility for the quality of the outcomes can also be subject to negotiation. The question of negotiating differing agendas seems to be missing from research studies of mathematics teaching. Possibly because most studies deal with small numbers of willing children, issues of conflict do not even get reported, yet in the classroom they loom large. It was one of the aims of this study to reproduce classroom life albeit under more manageable conditions. It is therefore not surprising that balancing the teacher's and the children's agendas became an important aspect of the curriculum and of classroom management.

The previous two sections suggested that all the different *levels of context* in which mathematical problems and activities were situated had relevance to the problems' accessibility. The children's *stages of development* also seemed to determine what kind of problems they could imaginatively enter, solve and gain satisfaction from. It is a complex task for a teacher to take all these variables into consideration.

Some of the best advice about mathematics teaching arrives at what seems to be a methodological impasse. Evidence overwhelmingly favours some form of constructivist model of learning. From Piaget to Vygotsky, to von Glasersfeld and to the group of Cobb, Yackel and Wood, evidence mounts to show that each learner is forced to construct his own learning if the learning is to be more than a recitation of others' facts. Careful consideration of the relative responsibilities of teachers and those of students in the mathematics class offers one way out of the impasse. It could be said that constructivism tells us what the *students* should be doing in a good mathematics class. Von Glasersfeld (1990) stressed that the knower is responsible for his constructions. This has to be so. In the final analysis no one else can bear the responsibility for another person's learning, not even for the learning of young children. If the construction of concepts is the responsibility of students, what are the proper responsibilities of the teacher?

The obvious answer is that the teacher's responsibility is to *help* children develop concepts, but of course this still begs the question of how best to do this. In this study the focus of attention was on the kind of problems the

children were to solve, the activities they were to engage in, and the ways mathematical language was to be acquired. The teaching methodology employed in this study operated on the premise that the teacher was responsible for the selection of mathematical concepts to be considered, for the activities and tasks to be undertaken and for the necessary physical resources. The teacher left the children relatively free to put their own constructions on how to perform the tasks and what they were to gain from these in terms of concepts. It is hard to judge from constructivist literature whether mathematical concepts are meant to emerge from the children themselves as radical theory suggests, or they are to be introduced by the teacher, as Cobb, Yackel and Wood imply in their studies of early childhood mathematics. It could be said that many of the concepts used in racing came from the children. The topic of subtraction, on the other hand, was included in the study's timetable, because it was felt to be useful to see how such a traditionally teacher-centred topic would fare among the study children.

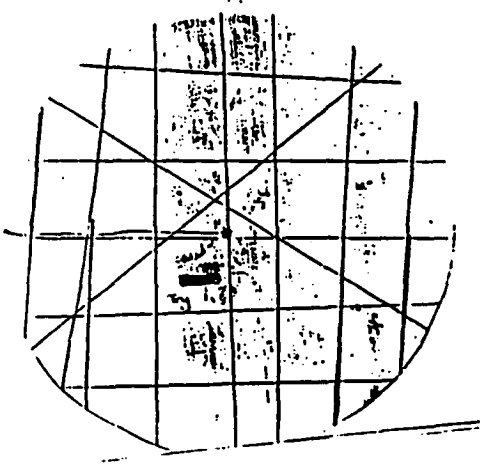
The question of an appropriate curriculum cannot be resolved by deciding whether child or teacher centred curricula provide better learning environments. It would seem that the answer must be in the balance of the relative rights and responsibilities of teacher and children. Because the teacher as an adult, furthermore, an adult in a role of authority, has more power than her students, it is her responsibility to initiate processes of negotiation which open up the questions of roles and influences.

In the role of the classroom manager, it is the teacher's responsibility to create favourable political, social, and cognitive contexts for learning. She should be responsible for the following aspects of the curriculum.

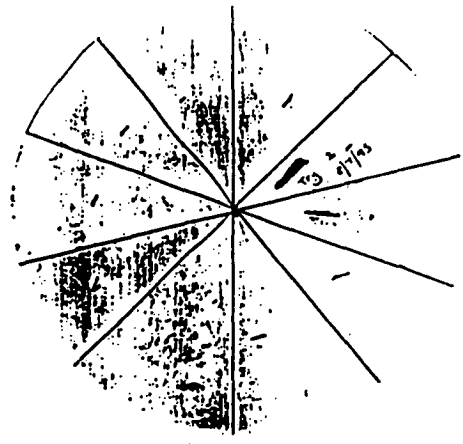
1. She should create situations where the children have a role in the world of mathematics, a role which brings prestige. She should highlight and strengthen the place of mathematics in contexts they understand.
2. She should be ready to hear what the child has to say, and suggest ways of checking out current concepts. She should be aware of the sort of concepts children use. Some of them appear so strange, that most adults miss the point. Teachers cannot afford to do so.
3. It is part of her organisational tasks to put children in touch with each other's solutions, and the solutions of the masters of mathematics.
4. She should present children with exciting problems, in the shape of interesting situations where the problems to be solved, and the best solutions are mathematical ones.
5. Her curriculum should be so structured as to help integrate mathematical concepts into the life of the child. For example, patterning may be found in mathematics, science, crafts, literature, visual arts, music, movement, dance, and so on.
6. She should introduce children to materials and techniques which may help them to check and develop their mathematical concepts.
7. She should provide children with interesting means, including physical resources, for developing and practising mathematical understandings and skills.

Teaching situations can be structured in such away that children may respond to those aspects which make sense within their own world. Furthermore, problems can be selected which have all the immediacy of a baby's world built in to them, yet have the potential to be considered in wider contexts by those children who are more mature. By having children work in groups, individuals can grow in appreciation of more mature approaches to problem situations by seeing others' work. While it is the student's responsibility to learn, it is the teacher's responsibility to offer activities to all the children appropriate to their level of development.

A clash of agendas occurs every time a child says, by behaviour or words, that "I will not tackle tasks beyond my comfort level" and the teacher insists that he should "give it a try". The child's refusal is nearly always a cry for more support, more help. The teacher's insistence is a way of trying to make the child responsible for his own learning, to learn to face possible failure or reap the reward of his success. It is difficult to decide at any given moment whether to help or to insist on a further try. In this study Michael ended up crying in frustration when he was asked to have another go at dividing his "cake" into ten portions. His delight knew no bounds when his second attempt was obviously very successful, as Figures 7 and 8 show.



**Figure 7.**  
Michael's first attempt at  
dividing a cake into ten  
equal pieces



**Figure 8.**  
Michael's second attempt  
at the same task

In Michael's case the situation turned out to be a positive one for both child and teacher, but some of the teacher's expectations appeared unreasonable to James and Joe. While there were no violent conflicts, at times there was an underlying feeling of a battle willed. The teacher wanted Joe to work and he refused. The teacher wanted James to put more time and effort into his work and he refused. The most constructive way of dealing with such situations would have been to give these children more support. But even in the small group of ten or eleven children James and Joe could not be given the total, undivided attention they were seeking. It is possible that a more thorough knowledge of the children would have enabled the teacher



to judge more accurately when it would be beneficial to push the children further, and when to back off. At times these situations were handled well but at times they could have been done better.

Racing and cooking were the most successful activities in this study, in that most children could participate and gain meaning from them. This is not the same as saying that subtraction, the most abstract group of activities, should not have been tackled. Joe and James could have done their problems in play situations, and not be required to produce written records and then the week on subtraction would have been less stressful. James specifically objected to the pencil-and-paper nature of the tasks required when he said: "I don't like drawing patterns they are too hard. And writing maths on paper, writing sums on paper." The teacher's expectations seemed to have been inappropriate for some of the children, at some of the tasks.

## **SUMMARY OF DISCUSSION**

The chapter addressed three questions which arose from the experience of conducting the study. Through systematic reflection on practice, as action research theory suggests, it was possible to isolate further issues and new directions for improvement. While it was useful to see to what extent and for which children the teaching methodology of the study worked well, it was also important to reflect on what did not work out quite as was hoped or planned.

The study succeeded in contextualising the mathematical problems and situations the children were expected to work on. All the problems were contextualised on the level of the tasks, by providing narrative contexts for them. The problems were also contextualised at the level of concepts by being presented in situations which were conceptually complex and which mattered to the children and made a difference in their lives. The political context of the study and the social context of the teacher-pupil relationships was the same as those operating in the school. Those children who had not found the school to be a congenial learning place did not find the study group much better.

The study attempted to cater for the different learning needs of children operating on different developmental levels by presenting problems which could be tackled at a variety of conceptual levels. This strategy was successful most of the time, but there were also instances when the children could not reinterpret the task in such a way as to suit their own level of development. A system of viewing contexts in a hierarchical way was developed using the experiences of the study. It is hoped that this system would help in the future to classify problem situations, and to match children and problems more accurately.

The third question of the Discussion dealt with the relative responsibilities of teachers and children regarding learning, and touched on the need of resolving potential conflicts of interest. It was suggested that it was more productive, as well as more realistic to accept that the teacher should retain responsibility for the topics of mathematics to be studied, for the kinds of tasks to be done, and for arranging curriculum resources. Radical constructivism contributed the thought that the learner was responsible for the quality of her constructions. In the study this was followed in that while the children's work was accepted by the teacher, the child was also encouraged to reflect on the quality of the solution she offered. It needed to be emphasised that the management task of the classroom teacher was already so complex and demanding that she should only take on those responsibilities which properly belong to her. In the study itself the teacher felt justified in taking on the responsibilities she did, but added to these a more carefully worked out strategy on the basis of the developmental levels of children would have improved classroom management.

When all the ingredients of a curriculum work well the classroom hums. Carefully negotiated political and social contexts result in harmony. Contextualised tasks mean that the children know what they are doing, and do it happily. The relative responsibilities of teachers and students, when worked out in a balanced way, safeguard the individual's need to construct knowledge and also open the way for tradition to play an appropriate role in the shaping of the curriculum. No wonder that teaching is considered an art with all the considerations which have to be born in mind and built into teaching if all the children are to learn successfully.

## CHAPTER 6

### CONCLUSION

*"Theology and Christian Morals, thought old Avril grimly, all of them neatly locked up in great books to the making of which there was no end. If only it were true. If only anyone could tell anybody else anything. If only one could know by being told."*  
(Margery Allingham, 1952, p.185)

This study was based on the thesis that communication is more than language, and language is more than the spoken word. It was understood that spoken language without anchorage in social interactions and without reference to the physical world, can float over the heads of the students who hear it. The teacher's voice can become little more than background noise, like the radio in the kitchen, or the Muzak in the supermarket. The thrust of the study was to make the language used in the classroom, meaningful and easy to learn.

In order to create an atmosphere conducive to learning, there were attempts made to give the children active roles in the learning process. The problems were so structured that the quality of the children's solutions affected the quality of their lives, albeit in minor ways. A racing track which was not laid out equitably, resulted in unfair races. Sharing of sweets and cakes for a party, unless done properly, would have resulted in an unfair distribution. Through the perceived need to guard their rights, the children become involved protagonists in the world of the problem.

The development of mathematical concepts, in terms of increasing formalisation, was achieved by drawing attention to more and more detailed aspects of the problem situation. Some of the children were ready to explore the problems to the point of formalisation where such things as specialised language and conventional measurement units became important to use. Some others, especially the two six year olds, became restless when they could no longer see the relevance of the problem to themselves.

The results of the study in terms of learning outcomes was impressive. All the children learned a great deal, and could use more formal mathematical language both expressively and receptively. The kind of classroom management employed in the group of nine to eleven children, was such as could be used with a class of thirty children. While the children were expected to make sense of their activities as the theory of constructivism would suggest, the teacher remained in charge of the topics for discussion and also suggested the activities for the group.

The main source of discipline problems seemed to arise from the teacher's expectations that all the children should attempt to deal with the problems in increasingly detailed and formal ways. A theoretical framework, like that of the expanding social world of the child, discussed in chapter 5, would have given guidance as to the nature of the problem situation each child might find engaging. As it was, some of the children ended up quietly, and at times not so quietly, sabotaging some sessions.

It was found that it was possible to work within the political framework of a school and still improve mathematics teaching by contextualising mathematical problems in such a way that they matched the social and cognitive world of the child. In part, it was the narrative structure of the problem which helped to achieve this. The language of mathematics was taught successfully by building logical structures into both the physical and the social environment. It was also found that it was possible to share the responsibilities of the classroom in such a way that the children's interests and the traditionally handed down wisdom of the discipline were both served.

The value of this study to the teacher/researcher rested in the issues which became more clearly defined, in the new insights into the problems which emerged, and in new directions developed for future improvements. Accepting that the study was valuable to the researcher herself, raises the question how a study such as this can be used by others to improve mathematics education. Since an ethnographic study is not generalisable statistically to a defined population, what processes need to operate to have the findings of the study affect its readers? The answer is not very different from the way literature changes its readership. By presenting a work containing many details honestly reported, readers may be able to link the experiences of the study to their own. They may be able to confirm insights, try some ideas or methods and explore any new directions suggested here. Something like generalisation can take place through the universality of human experience. Many classes contain a waif of a child like Joe, a little toughie like James, a shrinking violet like Melinda, a thoughtful Kim, and an eager-beaver like Michael. There are also thousands of teachers who are trying to develop more meaningful mathematics curricula. There is a great deal to be learnt from the experiences of others.

This study represents a small step towards a future mathematics class which gives all children the opportunity to work on problems appropriate to their developmental understandings. A curriculum which succeeded in making all children valued protagonists in the mathematical life of the class would herald an exciting new chapter in education.

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## APPENDIX 1.

### List of activities:

#### *Week 1- Racing*

- Day 1. Racing beads and spools without any defined structure. Need for starting and finishing lines recognised. Children's "caterpillar" race. Need to have enough room in the tracks so that no one would be kicked.
- Day 2. Race of ping-pong balls blown with short straws. Tracks laid out with masking tape. Emphasis on width of tracks.
- Day 3. Ping-pong ball race in tracks laid out with string. Straighter tracks and emphasis on measurement of length and width using metre ruler. Measurement for "fairness", tracks perpendicular to starting line.
- Day 4. Running races outside. Use of trundle wheel to mark out a 20 and a 50 metre race course. Necessary length of starting and finishing lines to accommodate all the runners. Repeated use of a metre ruler to measure more than one metre.

#### *Week 2. - Longitudinal patterns*

- Day 1. Task: Colour in a 1 or 2 metre strip of paper with regular pattern of stripes for a snake.
- Day 2. Task basically same as yesterday but using coloured paper to glue onto strip of cardboard, sticks used to mark end of pattern units.
- Day 3. Same task as yesterday but children had to decide on the numerical value of their pattern unit, and follow it through.
- Day 4. They had to decide on the numerical value of their pattern unit, and divide 24 by this unit.
- Day 5. Using chalk a pattern of actions drawn on the asphalt of the playground. Used a little like a hopscotch. Children's own inventions, eg: hop, hop, jump, clap, click, hop, hop, etc

#### *Week 3. - Sharing*

- Day 1. Activity: Share out 5 chocolates, 8 jelly beans, 4 Mars bars, 12 smarties between four people. Pencil and paper drawings, with available counters.
- Day 2. Shared out a handful of "jellybeans" among three friends. Had paper plate which was divided into portions by texta line. Wrote division sum on plate and the children read it to their classroom teachers.
- Day 3. Cut up either circle or oblong piece of cardboard into equal portions in preparation to cutting up cake.
- Day 4. Party. Some children shared out various sweets onto paper plates. Others cut up a round and a long cake as equally as was possible.

#### *Week 4. - Subtraction*

- Day 1. Played bus game. Children superfluous to number called out "fell off" the bus. Did take-away sums with unifix blocks sitting in a circle.
- Day 2. Made up own subtraction stories and drew them, then wrote the sum to go with story.
- Day 3. Picture of the "Old woman", who had any number of children 10-20, but only 10 fitted in the house the others had to live in the garden.

Day 4. Laid out pattern of blocks, sticks and bottle-tops. Whole pattern unit had to be taken away at each time. For example:12-3-3-3-3=0

Day 5. Using paper squares and glue make a decreasing pattern. For example:5,4,3,2,1; or 10,8,6,4,2; etc.

*Week 5. - Cooking*

Day 1. Jelly making. Investigation of measuring jug and measuring cup. Following the instructions to use 450 mls of water.

Day 2. Jelly making. Investigation of measuring jug and measuring cup. Using cup measure instead of trying to use mls.

Day 3. Baking scones. Children did all the measuring and other processes themselves. Identified 1 cup measure out nest of measuring cups.

Day 4. Baking scones. Children did all the measuring and other processes themselves. Identified 1 cup measure out nest of measuring cups. Cooking directed by a child from previous day's group.

Day 5. Recording experience of baking scones. Building inclines with Lego.

*Week 6. - Testing*

Children taken individually to do five tasks, in order to test concepts in the areas covered. Whole session audio-taped.

## APPENDIX 2.

### Testing sessions - Individual assessments

#### *The tasks given for testing*

1. Draw on the floor a one metre track for two toy cars to race on.
2. Share 15 sweets (unifix blocks) among three people; represented by 2 puppets and a toy koala.
3. Make a 3 pattern of 12 blocks. The child was given eight of one colour and four of the other.
4. Subtraction: There are nine children waiting for the bus (unifix blocks) and the bus comes (box) but there is only place for seven children. How many have to stay and wait for the next bus? In a more difficult form: Some children get on the bus and two are left. How many got on the bus? Can you tell the sum that goes with this story?
5. Using flour packet, bowl, cup, spoon, butter container, milk carton, half a lemon, rolling pin and a glass to jog memory, tell the story of how to make scones.

### Transcript of testing sessions

#### Order of testing:

Michael, Kim, Luke, Melinda, Sarah, Natalie, James, Joe, Rachel; and the three "Control" children: John, Danny, Abel.

These transcripts record all the verbal interactions between the child and the teacher. The interactions are presented here as continuous prose, with the child's contribution printed in bold lettering. Observations of behaviour are recorded in parentheses if they are essential to the meaning of the conversation.

#### Michael's testing session

We'll start with the first things we did a long time ago. Come here, Michael, and have a look at these things if you want to, if you think they will remind you of what we did. (Showing a bundle of Michael's work.)

1. The first thing I'd like you to do is, here on this carpet, could you please make up a track to race these two cars on. A two car track, one metre long. Here is some chalk, you can use anything else you like. A metre racing track, on this carpet. How would you do that?

(Trying chalk) **It doesn't show up.** It'll show enough, it'll be fine.

(Michael is drawing the tracks free-hand) **Oops!** (The line is too crooked for his liking.) What's that line that you're making? There are all these things you might want to use. (Indicating metre ruler and trundle wheel.) It has to be one metre. (Michael gets the trundle wheel) **I'm going to put down one metre so I can start my car track.** (He carefully starts with the starting triangle and measures out to the first click. Marks it with chalk.) **Now I'm**

going to make another track. Right, we need two tracks for the two cars. I'll start from here...(Draws second track, and looks up) ...**Right?** That looks fine. So...Where do we...What do we do now? **Get the cars...put them on the track and race them up.** Do they start here or here, where do we go? What's this line? (Michael writes an "F" on one end and an "S" on the other.) I see, that's the finish line and that's ....**the start.** Oh! All right! Are we ready to race, then? Ready.....**steady, go!** (Racing noises) **Try again! Ready, steady, go!** (Racing noises) **I won!** (*Concepts: start, finish, correct conventional measurement, fit*)

2. Put those things back and let's see the next thing! This is a really tricky one! Come here onto the carpet and then the tape-recorder will be able to hear you better.

There are some blocks in here.. **Yes?....**You remember all the patterns we did? Well, I would like you to make a pattern with those blocks, which uses up all those blocks, a sort of a three (3) pattern. **Like that?...**(Making a b-y-b chain.) See how it works out. Is it a three pattern? **Yes...one, two, three...**(adding y-b-y) **one, two...wait there...one, two, three...one, two...wait there!** (Continues until there are four blue blocks left.) It has to use up all those blocks! **Yeah, yeah! Start again!....Yellow, blue, yellow, blue, yellow, blue....**You might try a different sort of pattern... **yellow, yellow, blue...**See if it will work for you.. (Changes to b-b-y) **Blue, blue, yellow...blue, blue, yellow...**Did that work? **Yep! Read it! Blue, blue, yellow...blue, blue, yellow...blue, blue, yellow...blue, blue, yellow! Right! Excellent!** (*Concepts: patterns repeat, patterns have elements, visual patterns can stand for number systems* )

3. Now the next thing .. It is a sharing one. (sets out dolls) Once upon a time there was a boy a man and a koala. And they have got some marshmallows. (Pours out white blocks.) And they ask you to share out all those marshmallows between them. **You get one, you get one, and you get one..**(giving them out one by one) How many did they get each? (Counting silently with pointing) **Five!** (Checking) **One, two, three, four, five... one, two, three, four, five... one, two, three, four, five!** How many did they get altogether? **Fifteen** Excellent sharing! Put those marshmallows back! **They are only unifix's!**(*Concepts: sharing means to divide into equal parts, can be done by counting elements* )

4. (Getting next task) Here is a bus and here are some children in their uniforms. **Blue uniforms!** Guess what! **What?** All these children are waiting at the bus stop and the bus comes along, but... guess what! **What?** There are only places for seven children. ...and the rest of them have to wait for the next bus. **Here are my children!** Are you ready? **Yep! Toot-toot!** Only seven people are allowed to get in! **One, two, three, four, five, six, seven.. and a driver...**You can't have a driver, because it will not work out... you'll have to have an imaginary driver. How many children got left behind for the next bus? **Two!** How many did you have altogether? (Silent counting with pointing) **Nine! Did I do that one good too?** You did wonderfully well! Is there a sum you could do with 9, 7 and 2? **Yes....nine....two plus seven**

leaves....altogether make....nine!....Two plus seven equals nine! What's another sum we can make? Nine....take away...seven equals two! I know another one with using those two (boxes). There was a big box of children going along....and a few tumbled out....that much...(tips out three)...three tumbled out and then there was six! Check and see! I'll show you! See? One, two, three, four, five, six! Six children left! Wonderful! It was a very good sum! I told you! *(Concepts: Two addends can be calculated by counting on, addition and subtraction are linked, there is a special language for sums)*

5. And now for the very last one, can you get all those things ...in the basket. What's this? A flour bag, milk carton, butter carton, a lemon and cups and spoon... we used that (these) to make our scones!

What I'd really like you to do is to use these things to remind you and to tell... how we made scones! We got flour and got the spoon, to put the flour in.. How much flour? Three...Three what? Three cups...Which cup did we use? Which is the real cup? This one...no it wasn't...this one it was! We got the flour and we put one in there....one?... Three. What next? Milk? I don't know...Stirred it up first... Stirred up what? The flour...and...butter, it was. How much butter did we put in? Two...Two what?...Two spoonfuls. We mixed it up with our hands and we put in milk...How much?...One, no two cups and then we had to get it...the melon..(lemon) and squeezed it in....and then tip some more milk in and squeezed it all up in our hands, until it was a big round shape, and then we tipped it out, and then we got the rolling pin and rolled it out until it was nice and flat, and then we got a cup, and made circles with it, so we could have our...scones. And then put them in the oven and when they were all ready we brang them out and put some butter...there was cheese ones...some just had butter and there was jam on some, and then we shared them out. That was wonderful, Michael! *(Concepts: Quantities of ingredients are important, correct measuring instruments/units have to be identified and used, some processes are strictly sequential)*

### **Kim's testing session**

1. Can you draw a track for the two cars to race on? Make it a one metre track.

(Took metre ruler and drew three lines, parallel, checking with her fingers that the two tracks were equally wide.)

What are you doing now?

Putting in arrow at the starting line to show the cars that is the way they have to go down. Where is the starting line? Silly duffer, I am. This is the starting line and..... the finishing line.

All right let's race! (They race twice)

It's a really beautiful track and you have done a really neat job of it too.

*(Concepts: start, finish, correct conventional measurement, fit)*

2. Remember we did lots of patterns. All our snakes are over there.

In this box we have lots of unifixes and I'd like you to make a three pattern that uses up all those blocks.

All right, read it for us. **Blue, yellow, blue, yellow, blue...no!...**(Changes pattern) **Blue, blue, yellow; blue, blue, yellow; blue, blue, yellow; blue, blue, yellow!** You're happy? **Yep!** (*Concepts: patterns repeat, patterns have elements, visual patterns can stand for number systems* )

3. Get those three people down and I'll get you some marshmallows to share out between them.

They are having a party and they have to share out their marshmallows between them. You can be a sharer. They say: "Dear Kim please come and share out our marshmallows or we will end up with a fight!" (She counts them out silently.) So what happened?

**They all got five each!** How many was there altogether? **Ten.** Count them back into the box! **1,2,3,4,5,6,7,8,9,10,11,12,13,14,15....**How many? **Thirteen....Thirteen?....**(Giggle) **Fifteen....Fifteen,** and each of them got...**five each!** Thank you, a good job! (*Concepts: sharing means to divide into equal parts, can be done by counting elements* )

4. Now here comes the bus! Once upon a time there were some children waiting for the bus in their blue uniform. How many children? (Counts silently, picking them up into her hand) **Nine!...Nine children.** Put them down and let them wait for the bus. ....Along came the bus...but only seven children could get into the bus the others had to wait for another bus. **1,2,3,4,5,6,7** How many children got left behind? **Two!** Is there a sum you could make up about 9, 7 and 2? Let's have a look again: there were nine children....how many got on the bus? **Can't remember...Two!** How many? **Seven!** and that left.. **two!** So what is the sum you could make up about it? **Nine children got on the bus....No!....Nine people standing at the bus stop, and the bus came along and only allowed seven on and that left two.** Is there a sum you could....**I just told one.** That was a story. Can you just say the sum that goes with it? (Exasperated sigh) **Nine...Nine children, take away seven, equals two....two left for the next bus.** That was excellent! Tricky Michael did another sum. There are nine children in this bus and they went along, and they went along and there is a huge big bump, and three children fell out of the bus....**and that left...six! Three and three and three is nine!** Well done! (*Concepts: Two addends can be calculated by counting on, addition and subtraction are linked, there is a special language for sums, numbers can be grouped*)

5. The last thing is the red bowl. Take out all these things and see what they remind you of. What did we do with all these things? **Made scones.** Can you tell the tape-recorder how we made the scones?

**Well, we got the red bowl, and put the flour in...How much flour?...Three cups of flour..**(Picks out the correct cup) **we mixed it up with butter, two spoonfuls...we mixed it up with our hands...milk in it....one cup of milk, ...with...lemon...we mixed it up with our hands again, and we got the big board and then we got the rolling pin and we rolled it out not too thin, and then we got this little glass and we went backwards and forwards and we**

put it on some trays and baked it in the oven..... Then we pulled them into halves and we put butter on them and then we took them around the classes. Thank you very much! (*Concepts: Quantities of ingredients are important, correct measuring instruments/units have to be identified and used, some processes are strictly sequential*)

### Luke's testing session

(He was the first to actually look at his bundle work from the previous five weeks, which was offered to the others as well.)

1. I want you to make a track for the two cars to race, and I want it to be a one metre track. **What with?** You can just use chalk on the carpet right here. (Draws three lines from one edge of the carpet square to the other.) Is it a one metre track? What are you doing? **I am measuring if it is a one metre track.** (Using metre ruler) Is it? **No.** Fix it so it is one metre. What's this line that you've got here? **It is the line where the starting is ...you can get a big run up.** Where do we have to get to? **To the finishing line.** Do you want to have a go? .....I won that one!

Could you check with the trundle wheel if it is really a metre? How would you do that?

**You'd have it on the arrow what says start, and go until it clicks** (tries trundle wheel) Was it really 1 metre? **Yes.** (*Concepts: start, finish, correct conventional measurement, correct use of measuring instruments.*)

2. Here are some blocks and I'd like you to make a three pattern that uses up all those blocks.... **A three pattern?....** That uses up all those blocks. Is that going to work?...*(Just keeps going until all the blocks are used.)* Read it to the tape-recorder. **Yellow, blue, blue; yellow, blue, blue; yellow, blue, blue; yellow, blue, blue.** Beautiful! Let's break it up so that nobody will see your pattern. (*Concepts: patterns repeat, patterns have elements, visual patterns can stand for number systems*)

3. The next job is a sharing job. Here is a boy and a man and a koala having a party. The koala comes up to you and says "Dear Luke, we're in terrible trouble because we can't share out our marshmallows. Would you please help us? Please share out our marshmallows!" (Shares them out silently) "Thank you, please tell us what you've done." **I gave you all five each.** "How many did we have altogether, I don't even know that." **Fifteen.** "Are you sure it's fair?" **Yes..** "Thank you Luke!" (*Concepts: sharing means to divide into equal parts, can be done by counting elements*)

4. Here is a bus and here are some children in their blue uniforms waiting at the bus stop. Find out how many children are waiting... **Nine...** Along comes the bus, but unfortunately, only seven can get on the bus, and the rest have to wait for the next bus. I want to know how many children wait for the next bus....**2,3,4,5,6,7 and two more have to wait for the next bus...** Is there a sum you could make up with 9,7 and 2? **There were nine blocks, seven got on one bus and there were two waiting for the next bus, and then they got on and then there was none.** Can you make a take-away sum too?



**There were nine children waiting for the bus, they all got on but then there were allowed only seven and two got kicked off.** (*Concepts: Two addends can be calculated by counting on, addition and subtraction are linked*)

5. What do all these things remind you of? **Cooking scones.** would you like to tell the tape-recorder how we cooked scones? **We buttered the tray with butter and then we made some scone pastry ...** How? **We had a one metre cup, and we filled it up to water and we put it all in and we seen if it was one metre and it was.** Then what did we do? **We put it in the jug and then we boiled the jug and then while we were waiting we put in the jelly mix...Ah! That's how we made jelly. That's when we boiled the jug. that's not how we made scones....**(Looks sheepish) **That's all right! That's fine!** You were quite right about how we found out which was the real one cup. That was important, wasn't it? So we got the red bowl and then what did we do? What about the scones? **When the jug was boiled we put it in the mixture and then we stirred it up, and it sounds like this.**(Stirs noisily for the tape-recorder's benefit) So what were some of the things we needed to make scones? **Milk, flour, butter, a big cup, buttered trays, lemon....**What did we do with all this stuff? **We used it to make scones and jelly....**You remember the jelly much better than you remember the scones, don't you? What did we put in the red bowl first when we made scones? **The jelly flavour, what come in the packet, in a little box....** and then we put some water in the cup and we found out if it was a proper cup, and it was so we filled it up with water and we put that in the jug and boiled it and we took it over to the bowl and it was hot water then we ....Did we drink it as it was?... No, we stirred it up and then we took it over to the staff room and we put it in the fridge.....

And here is how made scones! How did we, Luke?

**We had a bowl, some flour, some butter, some milk, some lemon, a cup. We put in three cups of flour, in the proper cup, and then we buttered the trays and we had a little cup of milk and after we finished we put a little bit of milk (on top) and put it in the microwave, before that we mixed it with our hands, with the butter....** How much butter?....**Three spoonfuls....we....put some butter on the trays, and then we took it out of the red bowl and then we put it on a big chopping board, and we rolled it up flat....we had a glass and we put it down by the edge and we turned....** put it in the scone mix and we pushed it down really hard and turned it around and then it would break where it was and take it out and get it and put it on the buttered tray.....and then we baked it....**And I got two!.. and I broke them up and I got four!**(*Concepts: Quantities of ingredients are important, correct measuring instruments/units have to be identified and used, some processes are sequential*)

### **Sarah's testing session**

1. The first thing I'd like you to do is to make a track for two cars to race along and I'd like it to be a 1 metre long track. You can draw with chalk on the carpet, if you want. One metre long track for two cars to race on. You can use whatever you like. **I'll need the ruler.**(Got metre ruler and drew a line,

and because of the ruler used she had a 1 metre line) **Two gifferent (different?) one metres.** What does one metre look like? **It's long.** How long is it? How are you going to find out how long is one metre? **With that.(trundle wheel)** (She correctly sets up starting point, starts the wheel while counting under her breath. She stops on the click and says) **Eleven!** Eleven what? **Eleven metres.** Well, we only wanted a one metre track. How long is one metre, how are you going to find that out? How did you find out that was eleven? (She does the whole operation again, but does not start correctly and when the click is heard she announces) **Six!** So how are you going to make a 1 metre track? **Do it a bit more longer.** All right. (She extends both lines by about another 20 centimetres. "Measures" them again and says:) **Six!** So you think that's a 1 metre track? **Yes.** Do you want to race the cars, or do you want to do something else? Where do we line up? **At the starting.** You going to draw one? **And the finishing line.** (Draws both) Two turns at racing. (*Concepts: start, finish, tracks, attempt at using conventional measurement*)

2. Remember we made all those snake patterns? This job I'd like you to do is a bit like that, here are some blocks, and I'd like you to do a three-pattern that uses up all those blocks. **Blue, yellow, blue....** (inaudible) Is this going to use up all the blocks? **No.** All right, have another go. It has to be a three pattern.....**That worked out!** Tell the tape-recorder what you tried and then how it all worked out. **Before I couldn't work it out, and this is my pattern.** Read it! **Blue, blue, yellow; blue, blue, yellow; blue, blue, yellow; blue, blue, yellow.** Are you happy with that? (Nods) (*Concepts: patterns repeat, patterns have elements, visual patterns can stand for number systems*)
3. There was a man and a boy and a koala. They don't know how to share things. Here are a lot of marshmallows... **Yum!** ....and they need you to share out for them. **One, two, three for the boy, two for the man, two for the koala, two for the boy and another two for the man, two for the koala, one for the boy....oops! that one didn't get any!** Poor little boy! Let's have another go. **Three for the boy, three for the man, three for the koala, two for the boy, two for the man, two for the koala.** That was beautiful! How many did each person get? **Five, five, 1,2,3,4,..five!** How many marshmallows altogether? (Counts) **Fifteen.** (The koala says a "thank you" speech) (*Concepts: sharing means to divide into equal parts, can be done by counting elements*)

4. This is yet another job. This is a story too. Once there were some children waiting at the bus stop in their blue uniforms. And along came the bus ...But not all the children could get on. Two children got left behind and they had to wait for the next bus. So the question is how many children got onto the first bus? (She joins two unifixes to make one person...it looks more like a person now, but of course she ends up with a puzzling half a person.) **Three and one left over....I see!** Could you pretend that each of the blocks is a person? I know that they are a bit chunky that way, but you could just pretend they are chunky people. (Giggle) **1,2,3,4,5,6,7!** Here comes the next bus.. **Two!** How many people were there before the buses came? **I forgot to count.** Do you want to have all the

people back, or do you just want to work it out? (She reaches out for the two buses and says) **I'll work it out...1,2,3,4,5,6,7,8,9!** (Counts them. This is her "working out".) Can you make up a sum that goes with the numbers 9, 7, and 2?) **Three plus four equals seven.** Good! Can you use all the numbers? Start with nine. Nine....make it a take-away sum...**Nine take-away five equals four!** What about the story we have just had? What would be the sum to go with that? **Nine take-away seven, equals two.** Good! (*Concepts: Sum is the putting together of two (or more) addends; numbers can be used in an abstract way to represent situations*)

5. What does this all remind you of. **Of cooking....of cooking scones.**

Could you tell the tape-recorder how to make scones just by using these things to remind you? **We got the flour and we put the flour into the bowl...How much flour did we use? One cup...Can you find the one cup that we have to use? This is the right cup....we put the butter into the flour...three cups....**(Holding up spoon) What did we use this for? **For the butter...one cup... we put a bit of lemon into the butter, no, into the milk....How much milk did we use?...Two cups...and we used our hands like that...and then we put in the milk....and then we rolled it out...and we put a little bit of flour on the board... and then some people made circles with the glass...we twisted it around so it could cut it...we putted it ...we got some butter and we put the butter on the tray and we put the scones on there, and we put them in the oven. And some people made some cheese scones...How did we do that? We had some left and we had some cheese and we had some dough left over and we put some milk in there and some butter and some flour and that and we rolled it out and made the cheese scones.** (*Concepts: A "cup" is a way of measuring volume, some processes have set sequence*)

### Natalie's Testing Session

1. The first thing I'd like you to do is to make a track for two cars to race along and I'd like it to be a 1 metre long track. You can draw with chalk on the carpet, if you want. One metre long track for two cars to race on. you can use whatever you like. **Do I have to do the starting line?** Yes, that would be good. Start with the starting line. (As she draws with the metre ruler:) Right, would you like me to hold it? **Yes, please! ....Did Joe come?...No** he hasn't been yet. **It's easy to get off, isn't it?** (The chalk off the carpet.) Yes, there is no problem. So, where are the tracks for these two cars? **Only one though! One track.** Better make another one then. ....**One....**And where is the one for the yellow car? **Well it's really black.** It was yellow once...**then it turned to black.** (giggle) (Under the chipped yellow paint the car shows black) **I did not know where to come until Sarah told me....**(Looking at the two tracks, and pointing to the wider track) **This one is bigger...**We might have to let it go this time... So where are we racing to? **Down to the ...finish line! Where should I put it?** Where one metre finishes, I suppose. Where is that? **Here or here?** How can you tell when something is one metre? **At the end of the ruler.** Is this a one metre ruler? **I don't know!** (But she draws a finish line at the end of her marks, which are in fact one metre long.) Right!

**Can you drive one too?** Certainly, I'd like to race you. **You put the wheel on the ...starting line? On the marks, get seady(!), go!** (*Concepts: start, finish, even fit, attempt at using conventional measurement*)

2. Remember we made all those snake patterns? This job I'd like you to do is a bit like that, here are some blocks, and I'd like you to do a three-pattern that uses up all those blocks. **All of them? Aha! Blue, blue yellow; blue, blue, yellow; blue, blue, yellow; blue, blue, yellow.** Are you happy with that? (She nods smiling, checks her pattern again and beams at me.) That was a quick one! (*Concepts: patterns repeat, patterns have elements, visual patterns can stand for number systems*)

3. This one is a sharing one. Remember one week we did a lot of sharing. we had our party at the end of that week. And this is a party too. At the party there were a man and a girl and a koala. They don't know how to share things. Here are a lot of marshmallows.....and they need you to share out for them. O.K. **One for koala, one for pirate, one for the little girl; one for koala, one for pirate, one for the little girl...Oops! Two for koala, two for pirate, two for the little girl....another two for koala, another two for pirate...another two for... girl...one for koala...Ah! Some are left over!** Something is not quite right yet! I know! **Three for koala, three for pirate, three for the girl, three for koala, three for pirate, not right!** There must be a way of doing it so they all get the same amount! **Four for koala, four for pirate, four for girl...Ah!...Aha! Five for koala, five for pirate, five for girl!** (Gives a satisfied smile. Koala speaks:) "How many did we each get now, tell me again?" Five. "How many did we have altogether before we started?" 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15! **Do we have to do it again?** (Hopefully) Certainly not! **Ah!** (Disappointed)(*Concepts: sharing means to divide into equal parts, can be done by counting elements, note the hit-and-miss sharing strategy*)

4. This is yet another job. This is a story too. Once there were some children waiting at the bus stop in their blue uniforms. **They were in a line..**(Putting them in columns of two) Of course they were in a line, they were very well behaved children. How many people were waiting for the bus? 1, 2, 3, 4, 5, 6, 7, 8, 9. And along came the bus ...But this bus can only take seven people so how many were left for the second bus? **Two.** Good! Now have a think about what we have just done. We had ...how many children? **Nine.** And we drove away with how many? **Seven....**and there were left ...**two...** So can you think of a sum which goes with these numbers...**There were nine children, take away three...seven ..seven got into that bus .... take away seven pigs...seven children! Pigs! Seven children...equals two.** (*Concepts: the special language of sums can be applied to situations*)

5. Take that box and see what's in it. What does that all remind you of? **Cooking.** What did we cook with all this stuff? **Scones.** Do you think you could tell the tape-recorder how to cook scones? **First we had to get milk, and then a spoon, and then the butter and the lemon, and then we had to ...keep ... some....you put all these little cups things out and we had to**

choose which one we were going to use, and we used the biggest one. That's right! So that's the real cup, we'll take the others away. Right! Get the red bowl and start making pretend scones. **Put the milk in....**What's this thing?....**Self-raising flour...**What did we start our scones with?... **Self-raising flour.** How much self-raising flour did we put in? **Oops! Half a cup.** Then what? **Two teaspoons...**Table spoons they call it....**Two table spoons of butter...** put the lemon in?....I don't know....squeeze it in...put the milk in...How much milk are you going to put in? ....**Half a cup...**We got the spoon...did we?...**No, we mixed it with our hands, (miming) and then tipped it all out, put flour on the board, and then rolled it, and then we got the glass and made the shapes. We had to put the glass right near the edge. We got some other people to do the same, ...and then we put some butter on the top, and we put ....we put them on the tray, and then we took.... we put them in the oven to make them cook...and while they was cooking we played with the Lego's...And when they came out of the oven what did we do? ...We put...we left some butter... and we put some butter and some jam on them and we took them around to the classes....Good! Finished?** (Concepts: A "cup" and a "spoon" are ways of measuring volume, some processes have set sequence)

### Melinda's Testing Session

1. The first thing I'd like you to do is to make a track for two cars to race along and I'd like it to be a 1 metre track. You can draw with chalk on the carpet, if you want. One metre long track for two cars to race on. You can use whatever you like.

How long is one metre? (A stray metre ruler was lying in the middle of the small room where the testing took place. Melinda drew a free-hand line which just happened to end at the ruler and happened to point at the 80 cm line. When the question was asked, she looked at what she had done, and said:) **Because it is up to 80.** But you can move that. So how are you going to find out if the track you drew is one metre? **To see if the cars will fit inside there.** (Considering *width* of track. She then drew the starting and the finishing lines and wrote "S" and "F" next to them. The length of her track was pure guess work to which she paid no attention. However, she drew about a metre long track) **Starting .....and finishing line.** That's great! (Racing) (Concepts: start, finish, even fit)

2. Remember we made all those snake patterns? This job I'd like you to do is a bit like that, here are some blocks, and I'd like you to do a three-pattern that uses up all those blocks. **Clap, clap, stamp, click,...**Three pattern...it has to use up all those blocks...**blink, blink...**A three pattern has to have three things in it....(Breaks off three blocks:) **Clap, clap...**(then joins the whole lot up and starts to read it:) **Clap, clap....**Read the colours, love...**Blue, yellow, blue, yellow, yellow....**Is that a pattern? **Yes....**(She changes the whole stick) **Is that finished? Does it make a pattern? Read it! Blue, blue, yellow, blue, yellow, yellow, blue, blue, yellow, blue, blue, blue.** That doesn't really make a pattern. It doesn't happen all over again like a pattern should. Can you fix it so that it will go happening all over again, like a patterns should?

.....Now I can see a pattern...Is that working now? Read it to the tape-recorder .... **Blue, blue, yellow; blue, blue, yellow; blue, blue, yellow; blue, blue, yellow!** Did that work? (Smiling) **Yes! That worked! That was good, excellent!** (*Concepts: A pattern has elements, recognition of repeating pattern*)

3. This one is a sharing one. Remember one week we did a lot of sharing. we had our party at the end of that week. And this is a party too. At the party there were a man and a girl and a koala. They don't know how to share things. Here are a lot of marshmallows.....and they need you to share out for them. **I don't know...I wasn't here at the party, I was sick...** You'll be able to help them....(She shares out one by one) Tell me what you did, Melinda? **I gave.. I put 1, and then 2, and then 3, and then four, and then five.** "How many did we all get?" **Five each.** "How many did we have altogether before you shared it out, Melinda?" (Counts under her breath) **Fifteen.** (*Concepts: sharing means to divide into equal parts, can be done by counting elements, sharing strategy by ones*)

4. This is yet another story. Once there were some children waiting at the bus stop in their blue uniforms. And along came the bus, and some children got on, and two children got left behind and they had to wait for the next bus. So the question is how many children got onto the first bus? (Tips it all out.) How many children were waiting for the bus? **Two were waiting...**Now, at the beginning...Find out ....(Counts) **Ten.** ... How many? Shift them a bit when you count them...**1, 2, 3, 4, 5, 6, 7, 8, 9... Nine!** Nine children, some got on the bus and that left two of them waiting for the next bus. How many got on the bus? (Counts the bus load) **Seven got on the bus and two waited for the next bus.** Is there a sum that tells about this story? **Nine... and went on the bus and seven people left.** Right! This is a take away sum, isn't it? How would you say the take away sum? **'Cause you take away some and there is less left.** (Note the sum=some confusion.) (*Concepts: Check on quantifiable events by counting*)

5. Take that box and see what's in it. What does that all remind you of? **We made...we done some cooking...**What did we cook with all these things?..... **Some scones...**Tell the tape-recorder...**The first thing was we put flour in the bowl...**How much flour?...**Three cups...**Which is the real cup? **The big one which got 1 cup on it....and then we put the butter on...**How much butter? **..One big spoonful, ... and we got some milk and tipped it in...** How much milk? **Three...Three what? We tipped in three blops.** We squeezed the lemon in, we got it with our hands and squeezed it together, and then one person, Joe, one person put their hand in, and mixed it with their hand...and then we putted on the board and we rolled it,... and then we stirred it...After we rolled it?...After we rolled it we stirred it...we rolled it flat but not too thin, and we got the cup ... the ... glass ... glass, and we put the cup upside down and turned it around and made them come out...then we put them on the tray, then we put them in the oven....that's all. What did we do when we took them out of the oven?...**We took it around to people in the classroom, and gave some to the teachers, and to the people**

**who helped to make them.** (*Concepts: Quantities of ingredients are important, correct measuring instruments/units have to be identified and used, some processes are sequential*)

### **James' Session**

1. The first thing I'd like you to do is to make a track for these two cars to race along and I'd like it to be a 1 metre track. You can draw with chalk on the carpet, if you want. One metre long track for two cars to race on. You can use whatever you like. (Immediately picks up metre ruler) Would you like me to hold it for you? **No..** Remember we made lots and lots of tracks...What's that line? **Start.....and finish.** How do you know that it is a metre? **'Cause I done it with the ruler.** (Racing) (*Concepts: start, finish, tracks, correct conventional measurement*)

2. Remember we made all those snake patterns? This job I'd like you to do is a bit like that, here are some blocks, **I get the other ones?** (boxes) No, no! They are for some other jobs.) ...and I'd like you to do a three-pattern that uses up all those blocks. **Blue, yellow, blue, yellow, blue, yellow, blue, yellow, blue, blue, blue, blue.** Did that work out? **Yes...No, because it hasn't got another yellow there.** That's right. Have another go! A three pattern...**Blue, blue, blue, yellow, yellow, yellow, blue, blue, blue, yellow.....blues are still left over.** Have another go! **Yellow, blue, yellow, blue, ..... That's not going to work out either! No.....Yellow, blue, blue; yellow, blue, blue; yellow, blue, blue; yellow, blue, blue!** How do you feel about that one? **Yep!** That one turned out didn't it? That was really, really good! (*Concepts: patterns repeat, patterns have elements, visual patterns can stand for number systems.*)

3. This one is a sharing one. Remember one week we did a lot of sharing. we had our party at the end of that week. And this is a party too. At the party there were a man and a boy and a koala. **Are they puppets?** Yeah! **How do they work?** You get your finger in there and the other fingers in here. Will you talk to them? "Hello, we're having a party. Will you share for us please?" **Yes.** "Thank you, James!" They don't know how to share things. Here are a lot of marshmallows.....and they need you to share out for them. Tell the tape-recorder what you are doing. **I am putting five on each...for each person...**How did you work out that each of them is going to get five? **Because I counted them first before I done them..** So how many were there? (Counts them under his breath) **Fifteen.** Fifteen and you shared it out and you gave each of them.... **five.** (The man puppet thanks him.) That was very quick, James. You are very clever at sharing. (*Concepts: sharing means to divide into equal parts, can be done by counting elements, sharing strategy by equal groups and estimation*)

4. This is a sort of a story too. Once there were some children waiting at the bus stop in their blue uniforms. And along came the bus, ...(putting them into line) How many children were waiting? **Nine..** and some children got on, and two children ..**Their mothers are picking them up...** That's right,

but only one mother is coming to pick them both up. Here she comes. So the question is how many children got onto the first bus? **Eight or seven.** Well, how many? **Seven...** Would you like to check? (Offers box/bus) **Seven!** How did you work it out? **There was nine and then we took two away...we took seven away on the bus...I was thinking , of a sum like one I done at home.**(defensively) If you wanted to say a sum that goes like that, what would that sum be? Nine, seven gone, and two left... **Nine, take away two is seven..** That's right, or nine take-away seven is ...**Two.**

Do you want to do a really tricky sum that Michael made up?...No...Go on! Have a go!...There was a big bus of children going along ...on a very bumpy road, and a few tumbled out... that much...(tips out three)..three tumbled out ...How many are still in the bus? **Six...** Check to see...**One, two and two and two is six.** Well, done! you're just as clever as Michael and you're only in grade one! (*Concepts: Subtraction makes the quantity less, numbers can be used in an abstract way to represent situations, note estimation strategy before counting*)

5. As for the last thing! Take that box and see what's in it. What's all that? **Lemon, bag, spoon, cup, cup, and butter.** What does it all remind you of? **The scones...**The scones! Do you think you could tell the tape-recorder how to make scones? **We had a rolling-pin, a bowl, and a bag and the milk.....**What was in this bag? **Raising flour.....self-raising flour....right!....lemon, butter, cups and cup.** We put the self-raising flour in the ...How much self-raising flour did we put in? **A cup.. and tipped it in.. and we got the milk and we tipped it in...How much milk? One cup...then we mixed it around and we got the butter and we put in it...How much butter did we put in? A spoon.. We got the cup...we got the rolling pin and we were rollin' it out and we got the cup and we put....we dug the cup in and we made some round scones...and then...then we got a sticker and went to our classroom...Didn't we bake the scones?...Yes...So once we had the round shapes what did we do? **Baked the scones. And then we put them on a tray and then we brought them around to the classrooms, and only the people who made them got them.** We didn't have enough for everyone, did we? **No.** (*Concepts: Ingredients are important, some processes consist of a series of actions*)**

### Joe's Session

This is some work like the work we've done in the last few weeks...We never done this work...(picking up one of the puppets used in the sharing task with a cheeky smile) No, no, not that!!

1. The first thing I'd like you to do is to make a track for these two cars to race along and I'd like it to be a 1 metre track. You can draw with chalk on the carpet, if you want. One metre long track for two cars to race on. You can use whatever you like. (Draws the track free hand-the track is about 80 cm long) Is that all? I don't know is that a metre? How long is a metre? **How do we get it off?** We can just rub it off when we have finished..... All right! Where are the cars going to line up? (He immediately draws a starting and a finishing line)**A starting line... and a finish line.** (Racing) What is this, Joe?



**A ruler.** How long is this ruler? **One hundred.** Yep, it says 100 on the end, doesn't it? 100....centimetres, isn't it? And what is this thing? (Holding up trundle wheel) .....A "trundle wheel", it's called. What does it do? **It counts..** That's right, it counts. (*Concepts: start, finish, tracks*)

2. Now let's get onto the next thing. **What's in it?** I'll show you in a minute! **Dice!** No, not dice, blocks. Remember another week, we made lots and lots of snake patterns? ..and I'd like you to do a three-pattern that uses up all those blocks.....(Stops after picking up three blocks) It has to be a pattern all the way to the end. **We haven't got much yellow!** Haven't got enough yellows for that! You were going to ....just read what you've got there Joe, because the tape-recorder can't see what you've done. **Yellow, yellow, blue; yellow, yellow, blue!** Then what happened? you couldn't finish...why? **Because we didn't have enough yellows.** So break it up and start again! Have another go! **Yellow, blue, yellow, blue, yellow, blue, yellow, blue, ...**What happened now? **We never had enough yellows...**Still didn't have enough yellows! Let's have another go!...Ah! (Reluctant, obviously, left to his own, he wouldn't try again.) **b,y,b,...**so there is your three pattern, **b,y,b,...**right! **Mm... y,y,y,b...** What happened? **There are some left...**Let's have another go , so there are none left. **But how?** Well, have another think and another try! **Oh! b,b,b,y,y,y,b,b,....**Ooops! It didn't work. Have another go! **But how can we?** It can be done. Truly, I promise you. ....You've tried that one before...(Makes a pattern which starts with b,b,y, and rolls away from it all in evident frustration).. have a look at that bit and have a think about it...**That's a three..** Yes that's a three pattern. **But that's b,b,y...**Would that work? **No...**(while already making it)**b,b,y,b,b,y,b,b,y,b,b,y,b,b,y.** Did that work? **Yes...**(giggle) **I'll break it! ....Can I put the lid on?** Sure, can! (*Concepts: patterns repeat, patterns have elements, visual patterns can stand for number systems, note number of tries required*)

3. Remember one week we did a lot of sharing. we had our party at the end of that week. And this is a party too. At the party there were a man and a boy and a koala. **So these are two children and that's father.** Here are a lot of marshmallows.....and they need you to share out for them. **He gets two, he gets two, he gets two; he gets two, he gets two, he gets two; .....he gets two, and he gets one.....**"I am going to cry, because I haven't got as many as other people!" (Checks by counting) Do they all have the same number? **Four he's got, and he's got six, and he has five.** "Oh, it's terrible, I am going to cry!" (Boy puppet cries) (He takes away all the extras and leaves four each) **Now they all've got four! See? 1,2,3,4; 1,2,3,4; 1,2,3,4!** "There are still some left over! Can you share out those too?" **Here you are you have them....**No, give them to the puppets!...**You get one and you get one and you get one!** Oh? Did that work out better? **Yes.** How many did they each get? (Counts under breath) **Five!** Is it all right now? **What are you gonna do?...** (Boy puppet thanks the giggling Joe.) (*Concepts: equal partition, some hit and miss and some counting strategies*)

4. This is a sort of a story too. Once there were some children waiting at the bus stop in their blue uniforms... **I'll stand them up. They're all jumping around....**How many children are waiting? **Nine..** Along came the bus and some children got on, and two children...waiting for their mummies...How many children got on the bus? **Three...four...** Let's have another go! How many children were waiting? **Nine!...**Along came the bus and some children got on, and two children...got left, how many got on the bus? **Eight!** Would you like to check? **Seven....Seven!** So it went like this: nine children waiting, seven went into the bus, and there were...**Two left.** Is there a take-away sum that goes like this story? **Nine...take-away seven leaves two!** Well done! This was excellent! (*Concepts: Subtraction makes the quantity less, numbers can be used in an abstract way to represent situations, note estimation/guessing strategy before counting, there is a special language used for sums*)

5. As for the last thing! Take that box and see what's in it. Unpack it! **This has nothing in it.** No, it's only pretend...What does it all remind you of? **We made scones!...**Would you like to tell the tape-recorder how we made scones? **We put in some flour...How much flour did we put in? A big cupful...**(Choosing the correct cup measure) We chose that cup didn't we, because that's a real cup, the others are only half cups and quarter cups...**Then we crushed it with our hands...with what? with our hands.. we had to put the hard butter in...How much butter did we put in? Here is the butter...Spoon...Can't remember...You'll just have to guess...** (puts in one spoonful) one spoonful do you think? **Yep...Pushed up our sleeves...and we mixed it with our hands...milk in it...we put some lemon into the milk and then we tipped it in and then we crunched it ...we rolled it ...In the bowl? Noo! We took it out...and rolled it until it was... flat. Then we got a cup...(mimes cutting out scones) then we put the scones on a tray, ...we shared them around the classes...Didn't we bake them? Yes! Put them into the oven...and when they came out...we shared them around all the classes.** Very good! (*Concepts: Ingredients are important, a "cup" is a measure of volume, some processes consist of a series of actions*)

### **Rachel's Testing Session**

1. The sorts of things I'd like you to do are the things we have done the last few weeks. The very first thing I'd like you to do is to make a track for these little cars to race on. Here is some chalk, and other things are behind me, and if you make a 1 metre long track I'll race you. **I draw all bumpy, I can't draw straight lines...**Well, use a ruler and then it will be straight. (Draws a 70 cm track for both cars, 10 cm wide) Where are we going to put our cars? Oh, you're not finished yet. (She puts on a starting line and a finish line) **Finish line...start...(Race)** You've done a good job, Rachel! (*Concepts: start, finish, tracks, fit*)

2. Remember one week we did lots and lots of snake patterns? **Yes...Well,** here are some blocks and I'd like you to make a snake pattern. I'd like it to be a pattern of three, and it has to use up all these blocks. **If I do three**

lots...That's up to you...**Blue, yellow, blue; blue, yellow, blue; blue, yellow, blue; blue, yellow, blue!** Are you happy with that? **yep...**I am happy too.  
*(Concepts: patterns repeat, patterns have elements, visual patterns can stand for number systems)*

3. This one is a sharing one. Remember one week we did a lot of sharing. we had our party at the end of that week. And this is a party too. At the party there were a man and a girl and a koala. They don't know how to share things. **Did you make them?** This one I made but I bought the others. Here are a lot of marshmallows.....and they need you to share out for them. "Dear Rachel please share out these marshmallows so we wouldn't have to have a fight about it! Thank you!" (Shares out one by one) How many did they each get? **Five..** How many did we have altogether? **Fifteen.. because three groups of five equals fifteen.** Thank you, Rachel, please pack them up! **How come you have written 15 on that?** I had to write something on the lid to tell me that is the sharing box. All the boxes look the same....Thank you, it was lovely sharing! *(Concepts: sharing means to divide into equal parts, can be done by counting elements, sharing strategy by equal groups and estimation, use of abstract language)*

4. Once there were some children waiting at the bus stop in their blue uniforms. How many people were waiting for the bus?(Puts them in line) **1, 2, 3, 4, 5, 6, 7, 8, 9...Nine were waiting...**all in a line like good children should.. and along came the bus...Toot, toot! and some children got on the bus and two children stayed there to wait for their mummy to come...So, how many children got on the bus? **Nine, well, there was nine altogether,.....can you work it out?...No.** Would you like to check? **Seven...**Seven got on the bus and two stayed behind. **Yes...Eight and nine there** (pointing to the waiting 'children') That was a good thing to say! Is there a sum you could make up to go with the story? **Nine....take away two is seven.** Listen to this sum that Michael made up. There were nine children on the bus and it went on a really bumpy road...and three children fell out of the bus...**Yayk!** So how many children are still on the bus? **Fi..Six...Why? Because if you...because three and...three lots of three leaves six.** Check to see if you were right. **Six! That was good!** It was excellent!  
*(Concepts: Sum can be found by counting on from first addend, numbers can be used in an abstract way to represent situations, note grouping strategy, some accurate use of the special language of sums)*

5. Now for the very last thing. Would you like to unpack everything in this box? What does it all remind you of? **The scones that we made.** Would you like to tell the tape-recorder how to make scones? You can pretend making them if you like. **I forgotten how to make them.** Just look at the things, that will help you to remember. **We had the bowl and we put the flour in...**How much flour? What did we use to measure the flour? **A cup...**Which one? **That one** (holds up 1 cup measure) How much flour did you put in the red bowl? **Two cups.** What's next? **The milk.** How much? **Three cups...**Which one? **This one** (holds up 1/3 cup) **You can tell because it has the three on it.**

One ...two...three.. put a little bit of lemon in it. The butter...How much butter?... A table spoon....We squashed it all with our hands...We got it out and we rolled it... Really thin? No! We got the cup to make the shapes. We had to put them on a ...tray, but we had to put butter on as well...cooked them in the oven, ...When we took them out of the oven what did we do? Let them cool and cut them and we put butter on them. Then what? We made some cheese ones. ...We squashed all the dough that was left in there in the red bowl and made some cheese ones...we put some cheese in there...and made some cheese scones. Thank you very much, it was wonderful! (Concepts: Ingredients are important, a "cup" is a measure of volume, some processes consist of a series of actions)

### John - Control 1.

1. You see these cars? I'd like to race them but we need to have a racing track. Do you think you could draw a 1 metre long racing track for the two cars to race? A 1 metre long racing track for the two cars to race?

(Draws track of about 30 cm, with dotted line down the middle, like a roadway, no starting or finishing line) Where are we going to put the cars? (Places them at one end) Are you sure it is one metre? **Not sure...** (Draws the finishing line) Are you ready to race? (Racing) What's the line you put on the end? **The finish..** (Concepts: finish line)

2. There are some blocks in here. What I'd like you to do is to make a three pattern that uses up all those blocks. Do you think you could do that? Is that three pattern? **Yes.** (Places the threes under each other in four rows. None of the other children have done this, because they were asked for a 'snake pattern', a term well understood) Is it going to use up all those blocks?

bbb  
ybb  
yyy  
bbb

The colours are not in a pattern, are they? Do you think you could change it so that the colours make a pattern too? **yyb; yyb...**(Stops and changes pattern: **byb, byb, byb, byb** in the same formation as above)**Blue, yellow, blue; blue, yellow, blue; blue, yellow, blue; blue, yellow, blue.** Very, very well done! (Concepts: A number pattern repeats - 3,3,3,3)

3. These puppets are having a party, and they have a lot of marshmallows to share out. They are not very good at sharing and they hope that you would share out the marshmallows for them. Here they are. (Shares out one by one, makes a mistake and corrects it.) So what happened? How many did they each get? (Counts the lot) **Fifteen...**There were fifteen altogether. How many did each person get? **Five, five, five.** Thank you; very good sharing! (Concepts: sharing means to divide into equal parts, can be done by counting elements, strategy by one's, note confusion of "altogether" and "each")

4. Here is another story. It's about some children. They are waiting for the bus in their blue uniforms. Do you want to find out how many children are waiting? **Five...**(separating a group) They are all waiting. How many? **Nine...**Along comes a bus, and some children are getting on the bus. These children got on the bus...and those are the children waiting for their mummies. How many children got on the bus? Can you work that out without counting? .....**Eight...**You think, eight. Check and see!...**1,2,3,4,5,6,7. Seven!** Is there a sum you can think of that goes with 9, 7 getting on the bus and two more waiting....**seven go on the bus and two go with their mums.** Thank you very much, that was good too. You are very clever. (*Concepts: Subtraction makes the quantity less, note estimation/guessing strategy before counting*)

5. This is really unfair! While we were working together, we made some scones and these were some of the things we used. Have you ever made scones? **N-n-n!** You've never made scones! **My big sister made one of these ones...**Well, have a look at these cups. Which one do you think is a cup to use when your recipe says "take one cup of flour" which cup would you use? (Separates nest of measuring cups and looks at them. Chooses the 1/3 cup. Puts in one scoop of flour with the 1/3 cup.) What is this? **Butter..** How would you get the butter out of there? (Picks up spoon) Spoon...How many spoonfuls would you put in there? **One...**Now what? What's in that? **Milk..**How much milk are you going to put in? **One.** One what? **One blob...**Now what are you going to do? ....Hard to tell isn't it? (Adds glassful of..) What's that? **Water...Water, right...** Put it out ...**on a bench...** and get the.. **roller...Cut it...** Cut it into shapes? (He cuts it into circle, square, triangle, oblong...) **Put it on a tray... put it in the oven...wait until it's hot...**Thank you! (*Concepts: Ingredients, combination of ingredients, some process*)

### **Danny - Control 2.**

1. I've got two little cars here and a piece chalk and some other things there too, you might like to use, (indicating ruler and trundle wheel) to draw a 1 metre track for the two cars to race along. How long is one metre? **Ummm...**Just guess and draw the track. **Here up to here?** (Indicating about 20-30 cm. Draws two parallel lines with dotted line down the centre)) Fine. Where are we going to line up our cars so that we can race? **Here...**And where are we going to? **Here...**(pointing, but not putting any more lines down) (Racing) I don't know where the finishing line is so I don't know where we're heading.....(Points) **Will we get into trouble for writing on the mat?** No, you can rub it out really easily.

2. You know how you can get a long, long snake pattern with all sorts of colours, here are some blocks, what I want you to do is to think up a three pattern that uses up all these blocks. ....Give it a try! (Does b,y,b,y, until runs out of yellows) **My mistake...**All right, try again! (Tries yyy, bbb, y...) **There needs to be more yellows!** I can see your problem, but you can't have any more yellows! See if you can do it with the blocks you've got. **You do it!** No!

(Laugh) I am the teacher! I don't have to do it! **Is there a way?** Yes there is a way. **Do you have to use up all of them?** All of them. You have to use up all the blocks and have a three pattern. Is there a way of doing it? Nnnn. Can I show you? There are different ways of doing it, but there is blue, blue, yellow,---keep going on that one. (He completes task) **How many ways is there?** Well, they are all much the same, John did one like this, not in a snake but you can put it into a snake: b,y,b; b,y,b...All right, that was a good try!

(Concepts: A number pattern repeats - 3,3,3,3)

3. The next thing is a party! At this party there are the koala, the man and the boy! "Danny, would you please share out the marshmallows, because we're not good at sharing. **These look like blocks...**I am sorry!...(Immediately gives five to all the puppets) How many did they each get? **Five.** How many was there altogether? **Fifteen.** How did you work it out so quickly? You didn't have to count them. **Five plus five equals ten and ten plus five equals fifteen.** Well done. It was excellent and very quick too. (Concepts: sharing means to divide into equal parts, can be done by counting elements, note grouping strategy)

4. This is a sort of a story too. Once there were some children waiting at the bus stop in their blue uniforms... ..How many children are waiting? **Nine..** Along came the bus and some children got on, and two children...waiting for their mummy...How many children got on the bus? **Seven.** How did you work that out? **Well, nine take-away two equals seven.** (Stating it as self-evident fact) One of the kids made up this sum. All the nine children are in the bus, and it goes on a really bumpy road, and some children fall out of the bus. These children fall out of the bus (showing three) how many children are still in the bus? **Six.** How can you tell that? **Well, nine take-away three is six.** That's great! (Concepts: Use of number facts, confident use of mathematical language)

5. (Made scones with grandmother, he said)Look, we used all these things to make scones. (Identifies all the things). How would you make scones? (Chooses 1/2 cup as the "cup") **One cup of flour, butter,** How much butter?--**-teaspoon...Put some milk in...**How much? **All of it...**The whole carton? (1 litre) **Yep...Lemon...Then you stir..**(with the end of the rolling pin) Use the spoon to stir with! (as the whole pretend mixture spills out of the upset real bowl)**Tip it all out and flatten it...Then get a cup and put it on to make it into scones...**Yes, that's right. Where are you going to put the scones? **Tray...**Here is a tray (holds out flat palms) **Put it in the oven...wait until they're ready...put some jam and cream on them.** I enjoyed your pretend cooking! **Did I get it right?** You certainly did!

### Abel - Control 3.

1. The very first thing I'd like you to do is to get yourself a piece of chalk, here it is, and here are two little cars, and I'd like you to draw on the carpet a track, a one metre track, for the two cars to race along. **How long is that?**

How long is one metre? (Starts drawing a curved line) Please make it straight. A straight one metre. (Draws at least a two metre track) **How big's the cars?** (Makes track to fit *one* car, and then puts dots down the middle) **Are you allowed to draw on the carpet?** Yes, because you can just rub this sort off very easily. Are you ready to race? **Yes.** Where are you going to put your car? Where are we going to start it off? **I'd say there, start there.** (One car goes on the track, there is nowhere for the other to go) There is nowhere for this car to go. I was going to race the two cars to see which was the faster. What were you going to do? **I was just going to push mine along...**(and does so along the track. ("Racing" as in "Stop racing about"?)) (We try the two cars alternately) (*Concepts: some concern for measurement*)

2. Here are some blocks, and I'd like you to make a snake pattern, a three pattern, which uses up all those blocks. **How do you mean?** The pattern has to go in threes. **Like, you have a pattern there, another pattern there, and another pattern there?** Well, no. Every bit of the pattern has to have three things in it, and just go until you've used up all those blocks and made a long snake. **You mean like this.. or..** Just put it together and see if it makes a pattern. **Oh no! I did it wrong!....** Hang on! Before you break it up, could you read the pattern into the tape-recorder? **B, y, b, b, b, y, y, y, b, b, b, b.** Now have another go! **b, y, b, y, b, y, b, y...and the rest's just...**(Does another version) **What about this one? Is that work out all right?** What do you think? Read the three pattern, show me where the threes are, **Three blues, three yellows, three blues.....**You haven't got enough yellows to keep going, have you?.... Keep going, it can be done! Try to get the same thing going over and over again...Just read it in the tape-recorder what you've done **bbb,yyy,bbb ....**If you wanted to finish this pattern what would you need next? **Yyy, bbb...**But you haven't got that. .... Do you want to keep going or do you want me to give you a hint? **Give me a hint!** All right! I will give you a hint. What if you made your threes out of some blues and some yellows? **Like that?** (holds up byb) Yes?.....See what happens if you keep on going with that pattern. **No, it's not gonna work...Keep going!** (As he slips from byb to yby): So what is your three again? **Blue, yellow, blue...blue! yellow, blue...blue, yellow, blue, blue, yellow, blue!** (Great smile) Happy? **Yes.** Well done! **Can we listen to this?** When we've finished we will listen to the tape. (*Concepts: A number pattern repeats - 3,3,3,3*)

3. When you go to parties you have to share out your sweets and things, don't you? **Yes...**Well, these three people are having a party, a koala, a man and a boy. They have to share out their marshmallows. **What are marshmallows?** They're a sort of sweets, but these are only pretend marshmallows this time. They ask you to share out the sweets so there would be no fighting (Counting under breath) **1,2,3,4,5,6, six for the boy, 1,2,3,4,5,6, six for the man ....**Only three for the poor koala, who is crying. Have another go! **Three for the boy and three for the man and three for the koala, and ....**(adding one each) **four for the boy and four for the man and four for the koala, and...five for the koala, five for the man and five for the boy.** Did that work out? **Yes!** How many did you have altogether? (Counting under breath)...**Fourteen.** And how many did they each get?

**Five!** Wonderful! (*Concepts: sharing means to divide into equal parts, can be done by counting elements, note estimated groupings*)

4. Here is the bus and here are some children waiting for the bus in their blue uniforms....**and they went...**hang on I am doing the story and you are doing the maths...**The what?** The maths, the sums...**What's a sum?**...The numbers..So here comes the bus, and all these children get on the bus and the two are waiting for their mummies. How many children got onto the bus? ...Do you know how many children you had to start off with? **Ten?**.... Count them.. **1,2,3,4,5,6,7,8,9** ...**Nine!**...**Almost!** You were close!...Let's put these children on the bus again....How many children got on the bus? (Counting on fingers, starting with nine and bending down two) **There were...was nine on the bus..nine waiting...I mean nine waiting and the bus left two, so there 1,2,3,4,5,6,seven! Seven went on the bus.** Check and see if you were right...**1,2,3,4,5,6,7!** Well done! and I liked the way you worked it out on your fingers too. Would you like to have a really tricky sum that Michael made up? Do you want to do a really tricky sum that Michael made up? There was a big bus of children going along....on a very bumpy road, and a few tumbled out... that much...(tips out three)...three tumbled out ...How many are still in the bus? (Again using fingers, starting with nine) **That's three** (bending them down)...**and six are on the bus.** Well done, very good!

(*Concepts: Interchangeability if parts of subtraction sums*)

5. Have you ever made scones? **No...**Well. would you like to have a guess at it? Here are all the things we used, they might help you to guess. **But I've made pies and a birthday cake.** Well just pretend to make one of those with these things and tell me how you do it! See what are the things that we've got...**Flour, milk, cup,**(for the glass) **bowl things** (a nest of measuring cups) ...They're called measuring cups, **a big bowl, a roller, spoon, butter and lemon.** What did you say you've made? A pie and cakes...I don't mind if you tell me how you made one of those things....you need the same sorts of things...I suppose you also need eggs...we have pretend eggs...**When are we going to really cook?** You'll have to ask your teacher that. With the others we made jelly and scones and I've even made marshmallows, real ones, but I've finished cooking now...**Where are they now?** We've eaten them, my kids and I ate them. Let's have a try at pretending to make whatever you like....try to guess what you might do...**Got any pretend eggs?** Yes, here is an egg, ....break it into the bowl....how many eggs would you like? **Two...**two eggs, ...**mix it with a spoon,** (picks up 1 cup measure) Why did you pick up that one and not one of the others? **Because they are too small...**If your recipe said "Take one cup of flour" which cup would you choose? **One cup of flour?** I'd choose that...(picks up 1/3 cup) but you want the big one this time...**Yeh...**(spoons 'flour' into cup)...**Need about two more** (spoonfuls to fill cup)...**one, two...put that over there, and now milk...**you still have the flour in the cup...(tips it in with eggs and stirs vigorously...**that is that, and then a bit of milk..**How much milk?...**Oh, just a little bit...** What's a little bit? **About a cupful of this...**(picks up 1/4 cup) All right....**What's this here for?** (lemon)..You can use it if you want to..(picks up butter container) How



much butter are you going to use? That cup? What are the numbers on that cup? **A one and a two...**Now what? **Put it in the oven...**Not in my bowl! The bowl will melt! **Oops!** What are you going to do with the rolling pin? **I'm going to use it *after* it...**(after having it in the oven)...I see...Well, let's pretend it is a microwave oven then it is all right to put plastic things in it...**Yes, it is a microwave...**Have you got a microwave at home? **Yes...We have a *real* one at home!..**Well, we'd better set it then...**It's cooked now!**