# AN INVESTIGATION

# OF

# ADULT LEARNING OF MATHEMATICS

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This paper contains no material which has been accepted for the award of any other degree or diploma in any tertiary institution and to the best of my knowledge, contains no material previously written by another person, except where due reference is made in the text of the paper.

## CONTENTS

## ABSTRACT

This case study investigated some of the factors facilitating the learning of mathematics by adult students. In particular, the research looked into the use of visualising strategies related to work place experiences, and real-world referents, to facilitate the understanding and use of algebraic variables, and algebraic notation and conventions. The subjects were six male and two female students, chosen from the engineering and science students at the Burnie campus of the North West Coast Regional College of TAFE. They were mature age students, with reasonable work experience, and all had finished school-based study of mathematics in Grade 10. Four of them had a trade background, and four had had no formal training since leaving school. Background factors were investigated with a written questionnaire, followed by an interview. Basic mathematical competence was measured using the ACER Mathematics Profile Series Operations Test, direct visualising ability was measured with Betts Questionnaire on Mental Imagery, and spatial ability was measured with the ACER Mathematics Profile Series Space Test Unit IV. A twenty item algebra test investigated the students' meanings and use of algebraic letters, and their facility with algebraic notation and conventions. Following error analysis, the subjects were interviewed to obtain insights into their cognitive processes, particularly where errors had occurred. Results of analysis of the first three test instruments revealed high levels of mathematical competence for these students, but no correlation between the two tests of visual and spatial abilities. The TAFE students' results on the algebra test were consistently higher than results recorded for similar research with a wide range of students in both England and Australia. Evidence from interviews suggested that the TAFE students' greater facility with algebraic concepts could be related to their experience of mathematics with real-world referents, both in the workplace and in exposure to applied mathematics in other subjects. Less able students appeared inhibited in moving beyond a concrete base to more abstract relationships, but the able students successfully transcended their real-world referents and were able to interpret and manipulate variables logically and consistently.

# CHAPTER 1

## INTRODUCTION

Adult learners returning to the study of mathematics essentially proceed through the same learning sequence as school-age students studying similar material. However, a large number of factors influence the learning processes of such mature students. These include age, maturity, life-stage, personal circumstances, background experience including work experience, and past school experience, especially in school learning of mathematics.

This study investigated whether some of these factors facilitated the learning of mathematics by adult-learners. In particular, evidence was sought of the use by these students of strategies developed in their workplace or everyday lives to deal with mathematical problems. Background variables were also explored to discover any common factors which might have influenced the mathematical learning of these mature-age students.

Cognitive science, and in particular the theory of constructivism, offered insights into the ways in which people understand and learn mathematics. The review of the literature in chapter two presents a summary of much of the research into the cognitive processes involved in studying mathematics. This research mainly involved school-age students, with very little written on the related learning processes of adults returning to the study of mathematics. There was however, a large body of research on the competence people display in dealing with mathematical tasks occurring in their everyday situations. Analyses of this competence saw it derived from strategies deeply embedded in context, with a strong visualising component.

Biggs and Collis (1982, 1991) theory of the structure of outcomes of the learning process, SOLO, provided a theoretical perspective for understanding the use of visualisation in mathematical tasks. Using a neo-Piagetian approach, they related outcomes of school-based learning to successive modes of cognitive operation. Of particular interest to this discussion was the way in which they described the **ikonic mode**, which essentially involves visualisation and context based meaning, which they saw developing in strength and complexity throughout life. Evidence was sought of the use by the mature-age students, of ikonic mode or context-based strategies to facilitate their understanding of algebraic variables and algebraic notation and conventions.

In this study, the term **visualisation** is used in the sense of image-forming. The images can vary from the true ikonic or detailed "rich images", to image schemata or structures, with large abstract components.

**Algebra** in the context of this research refers to generalised arithmetic, and **variables** to the use of alphabetical letters to represent a specific number, or numbers, or any number on the real number continuum, depending on the context.

Watson, Campbell and Collis (1993) define categories of ikonic responses:

" (i) **Images**: Reporting visual images related to the problem...

(ii) **Reality, beliefs etc.**: The use of real world experience which appears to have some practical relationship to the problem.

(iii) **"Aha" experience**: A sudden, apparently unbidden "insight" into the structure of the problem...

(iv) **Diagrams**: The use of Venn diagrams, graphs, etc. to solve a problem ... has a strong visual component which may be classified as ikonic" (p. 50).

Finally, the term **adult-learner** refers in this context to any student whose secondary schooling is complete, and who no longer participates in compulsory or formal schooling. Students at TAFE colleges, regardless of age, are regarded as adult learners, since their enrolment and participation in TAFE courses is voluntary.

Chapter three describes the method followed in the research. A case study was undertaken of eight students who were studying mathematics in engineering and science courses at the Burnie campus of the North West Regional College of TAFE [NWRCOT]. These students were chosen on the basis of their representativeness of two major groups within the science and engineering student population. The common characteristics of both groups included: mature-age; schooling, including study of mathematics, finished at Grade 10; and considerable work experience. The students in group one, the "trade group", had a trade background, including apprenticeships in their respective trades. The students in group two, "non-trade group", had no formal training or education since leaving school.

Over a period of four months, with the students' consent, relevant data were collected on their backgrounds, visualising abilities, general mathematical operational abilities, and their understanding and interpretations of algebraic concepts. This was accomplished by the completion of a background questionnaire, followed by interviews. Tests were used to measure visualising abilities and general mathematical abilities. Finally, the students completed an algebra test, designed to investigate their understanding of the use of algebraic variables and notation, together with their interpretation of simple algebraic equations. Additional interviews followed up questions and areas in which students experienced difficulties. These interviews allowed for deeper probing of students' approaches to algebraic items. The interviews were audiotaped and later transcribed, in order to provide exact records. In addition, notes were made of any relevant classroom interactions.

The collected data were organised and analysed to discover any particular strategies used in learning mathematics, and in particular use made of context-based or ikonic mode strategies and the relationship between these and background experience in the workforce. A summary and discussion of results for the tests used, and for each question in the algebra test is given in chapter four. The TAFE students' results for each algebra question were compared to the results obtained for school-age students in the related research (Kuchemann, 1981; Booth, 1984; MacGregor, 1991; & Quinlan, 1992). Again, an attempt was made to relate notable differences between the two sets of results to the workplace and life experiences of the older students.

Final conclusions and possible implications of the research are given in chapter five. Overall, the research indicated a higher level of mathematical competency in the TAFE students. This appeared to be linked to a strong sense of context and reality-based understanding of mathematics. Even when dealing with abstract algebraic concepts, this well-developed awareness of meaning appeared to facilitate consistent, logical application of algebraic principles. In several of the weaker students, however, this linking of algebraic concepts to a concrete reality appeared to inhibit their ability to deal with more abstract expressions.

There were limitations to the research. Especially in a case study, some degree of observer bias is almost unavoidable. In addition, the research participants were students in the researcher's class, and the teacher-student relationship may have impacted on the students' responses, and on the researcher's perception of those responses. However, it was felt that these limitations were more than offset by the strong bond and degree of trust between the researcher and the students involved. This allowed for free and very open exchanges in the interview situation, where the students discussed their past experiences of school and work. In addition, the students appeared relaxed and unembarrassed when discussing their mistakes in the algebra test. It was felt the generally supportive nature of the mathematics class facilitated this openness.

McIntyre (1993) highlighted several important features which he felt made a case study approach particularly appropriate for research into adult education . He emphasised the importance of describing the learner perspectives as they are structured in social and institutional contexts. The wider application of any conclusions drawn from this study, however, was limited by the small size of the sample.

It is possible that the results of the study, in terms of the successful learning of mathematics by these mature students, may have been more related to their age and maturity. Additionally, low retention rates beyond Grade 10 in these rural communities may have resulted in a higher proportion of intelligent students entering the workforce, and returning to study at a later date. This may have affected the internal validity of the study, although this effect was minimised as far as possible, by choosing students with a range of abilities.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Introduction

Constructivism is seen as a useful cognitive theory in the context of mathematical learning, and an initial summary of some of the current literature on this topic is given. In particular, the neo-Piagetian approach of Biggs and Collis (1982, 1991) in their theory of the structure of the observed outcomes of learning [SOLO] is given in some detail, as their description of the ikonic and concrete symbolic stages, and the notion of "bottom-up" facilitation of higher order learning was of great relevance to this study.

Since the research involved adult students, some of the current literature on adult development and adult learning is presented. The literature on mathematical competence in everyday life had especial relevance in the context of adult learners returning to the study of mathematics, usually after wide experience in the workforce.

Literature on the cognitive aspects involved in learning mathematics, and in particular the use of visualisation abilities, gave a theoretical perspective on the strategies observed being used by the adult participants in the study. A major review is given of current writings on cognitive obstacles to the understanding of mathematics, with special emphasis on difficulties experienced by beginning learners of algebra. Although the literature concentrates on school-age students, the similarity of learning material and universal nature of many of the cognitive obstacles encountered, gave credibility to extending the application of the results from much of the research, to adult students.

## 2.2 Learning theory

### 2.21 Constructivism

Many mathematics educators today see constructivism as a theory of learning which offers a cognitive perspective particularly suited to the understanding of the learning processes in mathematics (Kilpatrick, 1987 cited in Schoenfeld, 1987).

Schoenfeld (1987) described the constructivist approach, " ... we all build our own interpretive frameworks for making sense of the world, and we then see the world in the light of these frameworks. What we see may not correspond to 'objective' reality" (p. 22). Everything we see is an interpretation. Silver (1987) emphasised the invention aspect of learning; new knowledge is largely constructed by the learner. However, he noted that new information was not simply added to the store of information, but connected and built into structures already present.

This idea is developed by Hiebert and Lefevre (1987). They defined **conceptual knowledge** as "knowledge that is rich in relationships" (p.6). It grows through the assimilation of new material into appropriate knowledge networks i.e. through meaningful learning. **Procedural knowledge** they defined as made up of "the symbol representation system of mathematics and the rules, algorithms or procedures used to solve mathematical tasks" (p.6). Such procedural knowledge involves either conventional symbol manipulations, or strategies and procedures for solving mathematical problems. These operate on concrete objects, visual images or other non-standard symbols.

Their essential theme was that "Procedures that are learned with meaning are linked to conceptual knowledge" (Hiebert & Lefevre, 1987, p. 8). In this sense, rote learning involves knowledge of procedures that are tightly linked to surface features; such learning does not generalise easily. However, if conceptual and procedural knowledge are linked, more powerful learning occurs and procedures are used much more effectively.

Dienes (1963), cited by Biggs and Telfer (1987), supported this notion. He viewed the formation of concepts through a process of abstraction from the concrete to the increasingly symbolic. The more varied the experience, the more powerful the concept that is formed. Biggs and Telfer linked this to the central idea of meaningful learning in their description of generic coding, which connects learning with previous knowledge at many levels, and involves a high degree of abstraction.

### 2.22 SOLO theory

In an essentially constructivist view of learning, the SOLO theory of the structure of observed outcomes of learning, developed by Biggs and Collis (1982), provided a useful theoretical perspective for understanding levels of cognitive development in relation to students' responses to school-based tasks. Based on the Piagetian idea of stages of development, they proposed a series of hierarchical skill structures. These are grouped into sets of levels or stages, which incorporate skills of increasing complexity and involve the ability to cope with an increasing degree of abstraction. Biggs and Collis distinguished between the **mode** of functioning, which is determined by the level of abstraction involved, and the complexity of the **structure** of the response within a mode.

The theory proposed five modes of intelligence development, and described the typical ages at which operation in a particular mode begins. These are summarised below.

1) Sensorimotor (from birth).

2) Ikonic (from 18 months).

3) Concrete symbolic (from six years).

4) Formal (from 16 years).

5) Post-formal (from 18 years).

Within each mode, five levels of response are identified, with movement from a one-dimensional response to multistructural responses and eventually to relational and extended abstract responses (Biggs & Collis, 1982, 1991).

In this study, the research was concerned in particular, with the ikonic, concrete symbolic and formal modes. Collis and Romberg (1991) summarised the characteristics of these three modes in the following way.

1. The ikonic mode is intuitive; it provides qualitative insights into a problem.

2. The ikonic mode is concerned with forming images or visualisations of situations. It is related to real world experience.

3. Communication in the ikonic mode is in everyday language.

4. The ikonic mode is the affective mode.

5. Much social functioning is based in the ikonic mode.

The concrete symbolic mode has the following characteristics.

1. This mode enables the concrete world to be interpreted through symbolic systems, such as the written language and the signs and symbols of mathematics. However, there is a clear link between the symbols systems and the real world.

2. The elements of thought for the concrete symbolic mode include mathematical concepts. However, although abstract, mathematical concepts dealt with in this mode are still directly related to the empirical world.

3. Communication in the concrete-symbolic mode is via propositional statements within a logical system.

Both these modes of functioning clearly maintain links with real-world experience; they are concrete or reality based.

In contrast, functioning in the formal mode is concerned with truly abstract elements and systems.

1. The formal mode involves higher-order abstractions where the symbols and operations can rarely be related directly to an empirical reality.

2. Competence in this mode implies an understanding of first principles underlying a particular discipline.

3. In the formal mode, rules and relationships can be operated upon directly to produce all the logical transformations.

4. Whether arising out of physical reality or based upon mathematical conventions, the formal mode implies a systematic view of all constituent factors or relationships (Collis & Romberg, 1991).

Each mode develops throughout life; there is not a linear progression through the learning modes, but **multi-modal** functioning is possible in a learning situation (Watson, Campbell, & Collis, 1993; Biggs & Collis, 1991; Collis & Biggs, 1991). Biggs and Collis amplified their theory to include the ideas of 'top-down' facilitation of lower order learning, and 'bottom-up' facilitation of higher order learning. 'Top-down' facilitation involves activation within one or more higher modes to enhance performance in a skill in a lower mode e.g. the sensorimotor mode. 'Bottom-up' facilitation occurs when lower levels are invoked to augment learning at a high level mode (Biggs & Collis, 1991).

The description of this latter type of learning strongly supported the main thesis of this study, that learning with meaning involves learning which is rich in relationships.

**2.221 Ikonic mode**

Research has shown visualisation to be an important element in the approach of adults to mathematical tasks in their everyday lives (Scribner, 1986; Lave, Murtaugh & de la Rocha, 1984; Nunes, Schliemann & Carraher, 1993). The function and operation that Biggs and Collis (1991) described for the ikonic mode provided an effective theoretical basis for understanding this use of visualisation, and were important for the purposes of this study.

Collis, Watson and Campbell (1993) supported the idea of the continuing development of the ikonic mode into adulthood to "...the intuitive thinking displayed in such areas as aesthetics, mathematics and science ... The latter two ... appear to make regular use of this mode in problem solving. There appears to be an interaction between modes which enables the typical ikonic mode strategies to be applied to symbolic ways of representing reality" (pp. 4-5).

In general, research in several areas offered a reasonable basis for utilising the ikonic mode concepts for understanding adult competence in mathematical tasks.

### 2.23 Adult cognitive development beyond formal operations

When adult development beyond the post-formal stage was considered, most writers saw a strict Piagetian stage construction as inappropriate. However, there was general recognition of hierarchical levels of development in adulthood distinct from the life phases described by Levinson (1978) and cited in Callan (1992). Sternberg (1984), Kohlberg and Armon (1984), Fischer, Hand and Russell (1984), and Richards and Commons (1984), all proposed relatively different theoretical constructs for the type and number of additional cognitive developmental levels or stages in post-formal development. However, there was a common recognition that the ability to perceive and understand abstract relations was central to intelligence, and that this ability usually increased with age.

A major constraint noted in the studies by Fischer, Hand and Russell (1984), and Commons, Armon, Richards and Schrader (1989), was that such development in adults was much more closely tied to environment than early development, and higher modes would only be reached with sufficient experience in the relevant context.

Fischer, Hand and Russell (1984) quoted research by Roberts (1981), Shatz (1977), and Siegler (1981), and cautioned on the use of performance on a single task, or very complex tasks, to draw conclusions about cognitive developmental stages. Their research showed that the complexity of a problem may lead a person to function far below his or her capacity.

### 2.24 Approaches to learning and motivation

Writers on learning theory saw the motivation for learning and the approach taken to it as two important factors which could significantly effect learning outcomes. Biggs and Telfer (1987) exemplified these ideas.

**Intrinsic** motivation, springing from the task itself, appeared to produce the most successful learning. **Achievement** motivation was seen to have a positive or negative effect. High-need achievers tended to thrive on competition and focused on results rather than learning. However, low-need achievers wished only to avoid failure and became disaffected very easily from the learning process (Biggs & Telfer, 1987).

Biggs and Telfer discussed the negative effects on the motivation and achievements of low ability students, when there was streaming into groups with homogeneous ability. They postulated that the greatest generator of intrinsic motivation, attribution of success to ability, was taken away from these students. This had relevance to the present study, since anecdotal evidence and experience indicated that many adult learners of mathematics have experienced low success or failure in school-based learning of mathematics.

Learning theories distinguished between **surface** approaches to learning, which emphasised reproduction of facts, data or skills, and a **deep** approach which looked for understanding and meaning, and involved an intrinsic interest in the task or subject. The **achieving** approach concentrated on making the best use of time and energy, and tended to identify learning with results. Studies showed that optimal results of learning were associated with **deep-achieving** approaches (Biggs & Telfer, 1987).

Deep, rather than surface approaches to learning, appeared to characterise mature-age students. Biggs and Telfer cited studies by Lawrence, Dodds and Volet (1983), and Biggs (1986), which showed mature age students of average ability displaying a greater degree of metacognitive activity than very intelligent senior school students.

## 2.25 Fluid and crystallised intelligence

Biggs and Telfer (1987) referred to the theory of fluid and crystallised intelligence proposed by Cattell (1971) and Horn (1979). This theory defined fluid intelligence as `pure' potential, highest at birth and declining with age and/or brain damage. Research had shown that with learning experiences, acculturation and schooling, fluid intelligence changed into `crystallised intelligence' . Studies had demonstrated that crystallised intelligence continued increasing into old age (Biggs & Telfer, 1987).

## 2.16 Learning theory - Conclusions

Writers generally found a constructivist point of view useful in describing cognitive development. In particular, the enlargement of such ideas in the expanded neo-Piagetian theories of stage development appeared to give valid and reliable descriptions of changes in cognitive style in response to environmental cues, with age and differing environmental influences.

There was disagreement however on the exact nature of cognitive development in adults beyond the formal stage described by Piaget, although all concurred on increasing intellectual development with age. Fischer and co-workers quoted the work of several authors which suggested caution in the selection of tasks and instruments used in assessing such cognitive development.

The available literature proposed differing approaches to learning and motivation, between young and mature-age students. Research appeared to support the idea that adult learners generally were more intrinsically motivated and adopted a deeper approach to their learning than their younger counterparts.

## 2.3 Adult life-stage theory

### 2.21 Lifespan development theories

The literature on adult psychology contained many theories of lifespan or life-stage development, such as Levinson's life stages (Callan, 1993), Riegel's dialectical model (Squires, 1993), and Mezirow's theory of perspective transformation (Tennant, 1993). They have been useful in revising notions of adult life as one of unchanging stability, and in giving insights into the nature of adult development and learning generally. Such theories were seen as pivotal in traditional theories of andragogy, whose best known exponent is Malcolm Knowles (1990).

The theories typically identified critical 'ages', 'stages' or 'life events' which presented the adult with developmental tasks and choices. Andragogical theory saw such life situations as prompting task-centred and intrinsically motivated learning. (Knowles, 1990).

Of relevance to the participants in this study, were the stages of self-evaluation described by Callan (1992) citing Levinson (1978). These writers described the midstage review (around thirty years of age), and at forty years, a second stage of review, when questioning and choices about career typically arose. In addition, these years may have involved significant alterations to personal circumstances, such as parenthood. Terry (1992) cited Miller (1980), Hobbs (1965), and Hobbs and Cole (1976), on the large body of research into the major stress dimensions on the new parents. These included physical demands, emotional demands, marital strains and lifestyle changes. Such factors could be expected to have had a significant effect on the effectiveness and viability of learning undertaken at such a time, although there has been little written on this aspect.

Tennant (1993) and Squires (1993) both pointed to weaknesses in lifespan development theories, which could tend to favour conventional, limited points of view, and often suffered from social, historical or gender bias. In addition they criticised such theories for their lack of adequate concepts for fundamental terms such as 'stage' or 'span'.

Davenport (1993) discussed several major criticisms of Knowles' theory of andragogy. Rather than a theory, it was seen as more accurately described as an educational ideology. Knowles' basic assumptions were criticised for containing both descriptive and prescriptive statements. Research based on the theory does not support many of the assumptions made.

### 2.32 Adult life span theory - conclusions

In conclusion, despite the undoubted insights consideration of lifespan development theory offered, caution was needed in drawing conclusions based on these theories, for the possible social, historical and gender bias, or conceptual weaknesses involved in such theories. In particular, the validity and possible gender bias of Levinson's theory, based on a research sample of 40 males only, could be questioned.

## 2.4 Everyday intelligence

### 2.41 Everyday intelligence in operation

Research studies by people such as Scribner (1984), Nunes-Carraher, Carraher and Schliemann (1985), Lave, Murtaugh and de la Rocha (1984) comprehensively documented the high level of competence displayed by people in managing problems in their everyday lives. Their reports described complex strategies and virtually error free performances by people in these everyday situations.

Several writers advanced a theoretical base to explain such competence. Biggs and Collis (1991) and Campbell, Collis and Watson (1995) proposed that it is derived intuitively, from the ikonic mode, which they saw developed in such people to a highly sophisticated level, and interacting with the logical structures of the concrete symbolic or formal modes. Similarly, Nunes, Schliemann and Carraher (1993) cited the idea of a pragmatic schema. This was first proposed by Cheng and Holyoac (1985), and they defined it as an abstract knowledge structure derived from ordinary life experiences. Similarly, Kolb (1993) described the process "... whereby knowledge is created through the transformation of experience" (pp. 138-155). Hiebert and Lefevre (1987) related such expertise to the linking of conceptual and procedural knowledge--the **context gives meaning** to the procedure. This was the underlying notion in all the writers.

Writers such as Resnick (1987), Biggs and Collis (1991), Bishop (1993), and Nunes et al (1993) attempted to distinguish the characteristics of everyday learning which made it so effective in comparison to school learning. If superficially similar tasks in the two settings, work and school, were compared the following differences emerged. Work related tasks were often shared; success often depended on successful meshing of a group's performances. However, in school the focus was on individual performance. Outside of school, both the setting itself and the availability of cognitive aids and tools helped to shape the task and its resultant solution. However, in school, unaided thought was prized most highly. Most importantly, outside school, tasks involved objects and situations that made sense to the people involved and were often used directly by them in the solution. School tasks, however, often became an exercise in symbol manipulation according to prescribed rules. Skill in a work context usually meant the development of highly specific competencies, yet one of the main aims of schooling was to impart skills and principles that are general and transferable (Resnick, 1987). In addition, a task outside school was usually directed towards some broader goal, not simply an aim in itself, as in the school situation. The situation in which the task was performed outside school **gave it meaning.** Many systematic inconsistencies which cropped up in school-based tasks occurred because students had lost the meaning of the problem. In general, practical thinking was more concerned with modelling processes and preservation of meaning within a specific context. (Nunes et al, 1993).

Nunes, Schliemann and Carraher (1993) cited Cole and Scribner (1974), Luria (1976), and Scribner (1975), on one major constraint found in the logicomathematical reasoning of unschooled adults. Their research indicated that such people treated premises as statements about reality. This appeared to indicate that street mathematics was restricted to reality and could not be easily transferred or generalised. However Dias (1988) cited by Nunes et al (1993), investigated further, and found this conclusion did not apply generally, but depended upon the structure of the interaction. Campbell, Collis and Watson (1995) noted in their study of visual processing by school students during mathematical problem solving, that students needed to transcend the limiting and inflexible nature of specific concrete images if they were to proceed into the use of visualising in a more abstract sense.

### 2.42 Everyday intelligence - conclusions

There was widespread recognition of the expertise displayed by people in everyday situations. However, such expertise was seen as strongly **contextually based**, and often lacked transference. "Their mathematics is a tool for solving problems in meaningful, environmentally familiar situations and is not seen as a culturally developed system of symbols, operations and relationships which can be applied generally" (Collis & Romberg, 1991, p. 94). Most writers recognised a strong visual element in the strategies developed, but appeared to differ in their interpretation of the logical abstract component present.

These ideas were particularly relevant for the purposes of this study. A significant number of the adult learners of mathematics taking part were chosen for their wide work experience. It was hoped to investigate whether any strongly visually based or context-based strategies for dealing with work-related mathematics were transferable to their present learning situation.

## 2.5 Cognitive theory and the learning of mathematics

### 2.551 Maths learning in the early concrete-operational to formal operational stages

Collis (1975) described clearly the development of mathematical learning as students progress through four cognitive stages from 7 years to 17 years approximately. His description is reproduced in some detail because of the relevance of the stages in mathematics learning to the participants in the study. The majority of mature age students in the research sample finished their schooling at the Grade 10 level, around 15 years.

In the early concrete-operational stage (7-9 years), children work with concretely based phenomena. Knowing is related directly to physical elements and operations.

In the middle concrete-operational stage (9-12 years), the child still has a concrete base for both operations and elements, but is able to draw some qualitative correspondences and perceive some consistencies.

In the late concrete-operational or concrete-generalisation stage (13-15 years), children come close to formal operations, but are more truly seen as working with concrete-generalisations, where they perceive consistency from a few specific instances. For instance, even an operation on variables is considered to be unique, with each variable representing a unique number i.e. there is no true understanding of the concept of 'lack of closure'.

In the formal-operational stage (16 years), students can conceive and manipulate abstract variables, construct abstract hypotheses and develop transformation rules unrelated to concrete reality. At this stage students work on the operation themselves and do not need to relate the elements or operation to a concrete reality. They appreciate the concept of acceptance of lack of closure and have the ability to handle multiple interacting systems (Collis, 1975, pp. 1-30).

Sfard and Linchevski (1994) described progress through the latter two cognitive stages from a different conceptual point of view. They described mathematics, including algebra , as a multi-level structure with the same ideas or concepts viewed differently from different positions in the structure. The same mathematical concepts could be interpreted as processes i.e. structurally, or at other times as objects, i.e. operationally. They proposed that the crucial points in the development of mathematics were at the junctions where there was a transition from one level to another. Competence in algebra was marked by flexibility of perspective, moving to and from operational and structural modes of thinking.

Sweller (1991), Silver (1987), and Schoenfeld (1987) citing Bundy (1975), used similar concepts in their descriptions of the competence of expert mathematicians. Such expertise they saw as derived from extensive experience with examples over an extended period of time, leading to the acquisition of fully automated rules and a comprehensive knowledge base. Derived from this was a range of higher order strategies or schema for the classification of problems.

Similar processes were described by Hoyer (1989) for age related cognitive growth generally. He used such processes to explain the characteristic differences between novices and experts e.g. in chess.

### 2.52 Learning of mathematics - conclusions

Essentially, competence in mathematics was seen by most writers as arising from a flexible and adaptable perspective which allowed for versatility of interpretation of mathematical material. Such competence appeared to be closely tied to growth of and ease of access to a vast knowledge base.

The possession of such a knowledge base, arising from life and work experiences, was seen to account for some of the differences between an adult learner of mathematics, and a school-age student dealing with the same material. If the adult learner had extensive experience of practical mathematics in everyday life or workplace, then ideally this constituted a wide knowledge base of examples and strategies, with the added advantage of rich contextual meaning attached to such experiences. However, the degree to which transfer of such context-based knowledge was possible had not been clearly established.

## 2.6 Visualising ability in mathematics

### 2.61 Type differences in mathematics abilities

The Russian psychologist, Kruteskii (1976) reported on detailed studies of the components of mathematical abilities in gifted young Russian students. He proposed the presence of "types" based on the "relative role of the verbal-logical and visual-pictorial components of a pupil's mental activity" (p. 314). He analysed the visual-pictorial component to be composed of two distinguishable elements: (a) the reliance on visual images to solve problems and the visualisation of even relatively abstract mathematical relationships, and (b) the ability to visualise spatial geometric concepts. He found a very high intercorrelation between these two elements. From these results, he proposed three different mathematical casts of mind. These were the analytic type (verbal-logical approach), the geometric type (visual-pictorial approach), and the harmonic type (both verbal-logical and visual-pictorial approaches). He concluded that the first two types tended to be limited to certain provinces of mathematics.

Kruteskii described the geometric type: "These pupils feel the need to interpret visually an expression of an abstract mathematical relationship ... figurativeness replaces logic for them ...They persist in trying to operate with visual schemes ... even when a problem is easily solved by reasoning" (1976, p.322). These students had a very high development of spatial concepts: "... the pupils were at pains to emphasise that they were not solving the problem, that they saw what was asked for" (p. 322).

Elements of these comments related strongly to descriptions of strategies used in everyday mathematics. (Scribner, 1984, Lave et al, 1984) In addition, anecdotal evidence of relatively unschooled adults returning to learning mathematics instanced similar comments.

Kruteskii noted that the dependency on a visual context can be a hindrance to the development of true generalising abilities, but strongly qualified this assertion. He described a graphic scheme as an abstract and generalised expression of mathematical relationships: "... the image is in a certain sense the 'bearer' of the sense and content of an abstract concept" (p. 326). He commented, however, that students of less ability are more likely to be bound by visual images to a concrete level, and hindered in moving to a more abstract generalisation. This correlated with the limited transfer of mathematical strategies acknowledged in much workplace mathematics (Nunes, Schliemann & Carraher, 1993), and the results found by Campbell, Collis and Watson (1995) for the use of visualising strategies in solving mathematics problems by school students.

### 2.62 Visualising ability - conclusions

Kruteskii's work was based on comprehensive and detailed testing of Russian pupils, with the Russian propensity for interview type investigations, contrasting with Western preference for psychometric type research. Both offer valuable but different insights into the nature of mathematical abilities. Although the translators, Kilpatrick and Wirzup (1976) expressed reservations about the acknowledged cultural bias and differing standards of objectivity in Soviet research, they felt that these qualifications did not detract from the immense value of Kruteskii's work .

## 2.7 Cognitive obstacles to the understanding of mathematics

### 2.71 Obstacles in the light of cognitive theory

Most cognitive obstacles to the learning of mathematics discussed in the literature were associated with a lack of '**learning with meaning**', the essential tenet of constructivism. Hiebert and Lefevre (1987) described several factors which they saw inhibiting the construction of relationships for rich conceptual knowledge in mathematics. They noted that an actual lack of knowledge may have lead to an incomplete or incorrect conceptual base.

Writers noted the difficulties that may have been experienced in constructing the relationship between items of information. Sweller (1991) explained the problems students experienced in attempting to transfer learned material from one context to another, as a lack of automated rules. He proposed that such automated rules free cognitive capacity to investigate the unfamiliar.

Again, Hiebert and Lefevre (1987) identified as a cognitive obstacle, the tendency for new knowledge to be compartmentalised, aiding in the natural resistance to conceptual change. Similarly, Herscovics (1989) described the acquiring of new knowledge as a process of assimilation and accommodation in the Piagetian sense. The existing knowledge itself became a cognitive obstacle which had to be overcome to accommodate the new knowledge.

Several writers, Schoenfeld (1987), Gerdes (1988), and Herscovics (1989), identified the over formal presentation of material without an intuitive foundation of existing knowledge, as presenting a cognitive obstacle to the acquiring of the new knowledge.

When these and related obstacles were examined in the context of learning algebra, considerable insight was gained into the difficulties beginning students often find in the study of algebra.

**2.72 Cognitive obstacles to the understanding of algebra**

There have been detailed studies into students' interpretations of and difficulties with algebraic letters and algebraic notation. Collis (1975), Kuchemann (1981), Booth (1984), MacGregor (1991), and Quinlan (1992), all documented research into the understanding of elementary algebra by various segments of the population of school-age students. Coady and Pegg (1993) documented similar research with a group of first year tertiary students. From these studies and related research, student difficulties with algebra can be broadly classified into the following six groups:

a) Difficulties arise from the meanings associated with letter use.

b) Difficulties are experienced with algebraic notation and conventions.

c) Difficulties are created by natural language use.

d) Difficulties arise from inadequate knowledge of underlying arithmetic structures.

d) Difficulties arise from rote or mechanistic learning of surface features.

e) Difficulties arise due to systematic flaws or "bugs" of the students' own invention.

f) Difficulties are created by the `name-process' dilemma.

Booth (1984) summarised the possible meanings students attached to letters:

1 Letters are regarded as shorthand for objects.

2 Different letters represent different numbers.

3 Letters represent whole numbers only.

4 Letters are seen as part of a pattern or code.

5 In abstract examples, the meaning of letters is ignored.

In her research, Booth (1984) found that the level of letter interpretation was not necessarily related to the degree of success in dealing with algebraic problems. Kuchemann (1981) commented that the difficulty of attaining a true understanding of the concept of variable could be partially explained by the fact that many items involving variables can be solved at a lower level of interpretation .

Kuchemann (1984, based on Collis 1975) described six successively more abstract interpretations of algebraic letters:

1 The letter is evaluated, or assigned a numerical value.

2 The letter is not used, nor given any meaning.

3 The letter is used as an object.

4 The letter is regarded as a specific unknown.

5 The letter is used as a generalised number.

6 The letter is used as a true variable, and seen as representing a range of unspecified values.

The reduction in meaning in the first three categories particularly may make some algebraic questions accessible, but any real understanding of elementary algebra required the ability to use letters at least as specific unknowns (Kuchemann, 1984). Collis (1975) further suggested that development of understanding in algebra may correspond to a progression in the ways in which letters are interpreted. The research by Coady and Pegg (1993) supported this notion. They found that the responses to several of the more demanding CSMS algebra items could be categorised according to the level of interpretation of algebraic letters. The Piagetian view proposed that a consistently made error to a given problem reflected a way of viewing the phenomena involved in a way which was consistent with the student's cognitive structure (Booth, 1984).

Kuchemann (1981) detailed a hierarchical ordering of items in the algebra test used in the research programme, Concepts in Secondary Mathematics and Science [CSMS]. These items were assigned to one of four levels, depending on the degree of abstractness and complexity of structure involved. The levels were approximately linked with the neo-Piagetian stages developed by Collis. Level 1 items required a response below the late concrete stage, Level 2 items required a response at the late concrete stage. Level 3 items were related to the early-formal stage, and Level 4 items required a response at the late formal stage. Several of these items were utilised in the algebra test for the present research, and this categorization into levels aided in the interpretation of results.

Difficulties in algebra arising from an inadequate understanding of algebraic notation and function were manifested in several ways. Students saw conjoined letters as representing the sum of those letters. There was confusion with powers and the use of brackets. Some of these problems appeared to stem from the absence of an operational model in arithmetic, making generalisation to the algebraic expression difficult (Booth, 1984; MacGregor, 1991).

MacGregor and Stacey (1993) and Herscovics (1989) highlighted difficulties in algebra arising from the translations from natural language to algebraic expressions. MacGregor and Stacey suggested that the most common cause for such incorrect translation from words to an equation was "an inappropriate attempt to represent cognitive models of compared unequal quantities ( p. 230)". This manifested itself particularly in `reversal' errors, which commonly occurred when students were required to construct an equation relating two variables, and there was an incorrect association of the numeral with the larger variable (MacGregor, 1991).

One persistent aspect of this difficulty was recorded in the literature as the `Student-Professor problem'. The problem is as follows:

**Write an equation that represents the statement:**

**"At this university there are six times as many students as professors", using S for the number of students and P for the number of professors.**

Clement and co-workers have researched this problem extensively (1982, 1981, 1979 cited in Herscovics, 1989). They found that natural language impeded the translation into algebra in both a syntactic and a semantic sense.

Explanations of the reversal error have been in terms of the misinterpretation of algebraic letters, syntactic errors, misleading influence of mental pictures, the selection of the wrong cognitive frame, conflict between a cognitive model of a relation and its syntactic structure algebraically; and, most recently, in terms of the conflict between the intuitive procedures of natural language processing, and the formal structures and symbols of mathematics (MacGregor, 1991). The 'Students-Professor problem' has been investigated with a very wide range participants, including university academics and teachers. The prevalence of the reversal error was almost universal. An adapted form of this problem was used in the present algebra test.

Sfard and Linchevski (1994), Hiebert and Lefevre (1987), and Herscovics (1989) all discussed the phenomena of rote learning. This very common feature of beginners' learning of algebra involved knowledge of procedures that are tightly linked to surface features. Then manipulations required no more than the application of mechanical algorithms; algebraic symbols have no link with conceptual content. They noted that such mechanistic rules and algorithms can take the students well beyond their actual level of conceptual understanding.

Silver (1987), Hiebert and Lefevre (1987), Schoenfeld (1987) and Maurer (1987) all discussed the 'invention' aspect of learning algebra: a consistent interpretation of subject matter that is actually wrong, resulting in 'bugs', systematic flaws in otherwise correct procedures. They often arose from an incorrect generalisation of a procedure which was correct for only limited applications. The writers noted that these generalisations were based on surface properties rather than on the underlying meaning. Wenger (1987) and Sfard and Linchevski (1994) saw a major portion of this difficulty stemming from limitations of textbooks examples and arrangements. In addition, Davis (1986) argued that school insistence on 'conventional procedures' lead to an erroneous concept of 'rightness' or 'wrongness' that depended on orthodoxy, and was only incidentally related to useful or meaningful relationships.

Resnick (1983) presented a slightly different aspect of such flawed understanding. Her view was that learners constructed understanding, and looked for regularity even when their information was incomplete, giving rise to `naive theories', which were very difficult to displace. She felt that students needed to be involved in making sense of procedures and formulae, or surface learning would result, tightly linked to the original situation.

Sfard and Linchevski (1994), Herscovics (1989), Kieran (1989), and Booth (1984) all cited various studies which showed that students have great difficulty with operational-structural or name-process duality. Where a concept is interpreted as a process, it is used structurally e.g. in the equation $5x + 2 = 9$, the expression $5x + 2$ represents a process. However, if a concept is interpreted as a product, it is used operationally e.g. in the equation $3x - 7 = 5x + 2$, the expression $5x + 2$ represents a product. Collis described the acceptance of such algebraic expressions, as answers to problems, as the ability to understand the notion of closure. Student's prior knowledge of arithmetic can present obstacles to their acceptance of closure (Collis, 1975).

Difficulties also arose from an incorrect notion of the `=' sign. The equality sign was seen as a signal to `do' something, and not as a symbol of equivalence. MacGregor (1991) also described its inappropriate use to mean 'the answer is', separating the operation from the answer. This idea was reinforced by the use of the `=' key in calculators as a `run' command (Sfard & Linchevski, 1994). In the context of some problems it was incorrectly translated to mean `is associated with', or `for every' (MacGregor, 1991). All these misconceptions made correct use and interpretation of algebraic notation difficult, especially for the beginning student of algebra. Kieran (1989) quoted the large body of research which showed that beginning students often have enormous problems in seeing the `surface' structure of an algebraic expression containing combinations of operations and variables.

The underlying cause of most of the cognitive difficulties with algebra discussed above could be linked to a failure to relate symbols and procedures in a meaningful way. Biggs and Collis (1991) allied this to the nature of school learning of mathematics, which they described as "... often decontextualised, abstract, impersonal and almost by definition, not within the student's direct experience" (p. 69).

### 2.73 Cognitive difficulties in understanding algebra - conclusions

The overall cause of student difficulties with algebraic concepts documented above appeared to be the failure to link symbols and procedures with appropriate meaning.

Although expressing their ideas differently, all writers supported this underlying theme. This was seen as the key issue in understanding adults' learning of mathematics. The research undertaken attempted to discover if their life and work experience gave them a rich knowledge base, which was then accessed through a high level of development in the ikonic and concrete symbolic modes, enabling a meaningful linkage of symbols and procedures.

Since most of the documented research into students' difficulty with mathematics has involved school-age students, the conclusions drawn about the nature of these cognitive difficulties cannot be applied to adult learners of mathematics without qualification.

## 2.8 Summary of literature review

In examining the nature of cognitive development, the constructivist perspective was seen as particularly useful by many writers. This view was usefully developed by Biggs and Collis (1982) into a neo-Piagetian theory of the structure of observed outcomes of learning, SOLO. The large body of research on the competence of individuals in everyday life was usefully examined by several writers in the light of SOLO theory. However the 'visualising' ability displayed in such contexts, needed to be carefully defined and examined for the degree of its true 'ikonic' component and logical-abstract component.

Adult cognitive development beyond the Piagetian formal stage was generally recognised to occur, but research indicated that it was substantially environmentally determined. In addition, recent research on the misleading nature of results from single task analysis, especially if the task is very complex, qualifies the conclusions drawn about adult cognitive developmental stages from such research.

Conclusions drawn from lifespan development theories also needed to be treated with caution, in view of the limitations now recognised in much of the foundation research and related conceptual difficulties, although such theories gave very useful insights into many aspects of adult learning.

The massive body of research on student difficulties with mathematics, and with algebra in particular, identified an underlying weakness in connecting procedural and conceptual knowledge. All aspects of the research reiterated the same basic tenet: the need for learning to be **meaningful.** The great bulk of this research has involved school age students. The validity of applying the results of such research to adult learners of mathematics cannot be assumed, but there is very little research recorded on this issue. The universality of many of the particular difficulties, across a wide range of age of students gave some credibility to extending insights into the causes of such difficulties with mathematics, to adult learners.

# CHAPTER 3

# RESEARCH METHODOLOGY

## 3.1 Introduction

A detailed description of the participants in the study is given. This is followed by a description of the test instruments, their administration, and comments on the validity of the tests for this particular research.

## 3.2 Subjects

A case study was undertaken of eight students chosen as a purposive sample from the student population of engineering and science students at the North West Regional College of TAFE [NWRCOT], Burnie campus. Engineering students were enrolled in the Associate Diploma courses for mechanical, civil or electrical engineering. The science students were enrolled in either the Associate Diploma course for laboratory technology, or the Advanced Certificate for laboratory technology. In 1995, there was a total of 162 students enrolled full-time or part-time in these courses, all of which involved two years full-time study or four to five years part-time study. Each course included several mathematics and calculus subjects. All mathematics and calculus classes for engineering and science students at NWRCOT Burnie campus were conducted by the same teacher, the researcher, in this case.

Unless they had a satisfactory pass in an appropriate Year 11 mathematics subject, engineering and science students completed Mathematics I, a bridging subject which aimed to provide foundations in basic calculator use, algebra, trigonometry, graphing, exponentials, logarithms and geometry. Although this subject had no prerequisites, a competency to the traditional Level 2 in Year 10 mathematics was implicitly assumed in the depth and speed with which the material was covered.

TAFE engineering and science students are drawn from the whole north west coast, a predominantly rural area of relatively high unemployment. The table below shows unemployment percentages from the 1991 census for the north west coast compared to Tasmania, and to Australia as a whole (Australian Bureau of Statistics [ASB], personal communications, November, 1995).

| LOCATION | AGE GROUP | PERCENTAGE UNEMPLOYED |
|---|---|---|
| North west coast | 20 - 24 years | 24.1% |
| Tasmania | | 20.9% |
| Australia | | 17.4% |
| North west coast | 25 - 34 years | 15.6% |
| Tasmania | | 13.4% |
| Australia | | 11.5% |
| North west coast | 35-44 years | 10.2% |
| Tasmania | | 9.1% |
| Australia | | 8.3% |

**Table 3.1 - Percentage unemployment in 1991 by age group and area.**

**(1991 census, ASB)**

The need for qualifications and retraining has become increasingly important in maintaining an edge in the employment market. There are several major employers on the north west coast. The food industry includes companies such as Lactos, Vecon, Edgells and U.M.T. Major industrial employers include Tioxide, Australian Paper; and engineering firms such as Dale Elphinstone.

Engineering students at NWRCOT tend to be predominantly male, and, although there were female engineering students in 1995, none were in the mathematics or calculus classes for that year. The laboratory technology courses attracted basically equal numbers of male and female students.

The students chosen represented two of the groups making up the population of engineering and science students at the NWRCOT. The first larger group consisted of mature age (twenty to over forty years old) students, who had undertaken engineering studies to secure promotion or to improve their employment prospects. Many of these students had a trade background. The majority had finished their formal schooling at Grade 10 or equivalent, and most had had wide experience in the workforce. Formal learning of mathematics also finished at Grade 10 for this group, apart from the practical mathematics involved in their apprenticeships in fitting and turning, carpentry or the electrical trades. This practical use of mathematics usually continued in their workplace or trade. This group contained only male students in 1995 and four were included in this case study. This group was of particular interest in the research, as experience had shown that students with this type of background appeared to have a great facility for learning mathematics, including algebra, and the main aim of the study was to try and discover how such aptitude arose.

The second group consisted of students in the twenty to forty or over age bracket without a trade background or any other formal training since leaving school. Such students may have worked in relatively unskilled positions, been unemployed, or been involved in home-making and family responsibilities. This group contained both male and female students. Four were included in the study, two female and two male. Again this group showed reasonable facility in mastering mathematics, and it was hoped to investigate their learning strategies.

Although the groups were too small for statistical analysis with any wide generalisability, it was hoped that an in-depth case study of the background, attitudes to mathematics, and mathematical learning processes of the two groups would provide some insights into the effects of maturity and different work experiences on cognitive styles in learning mathematics, and particularly in the understanding of algebraic concepts.

Apart from these two groups which were of interest to this research, the classes also included a few much younger students who had come into TAFE directly from a senior secondary college, with virtually no work experience. In addition, there was a range of other students, both young and middle-aged, usually with fairly inadequate mathematics background. The extent and type of work experience varied greatly from student to student.

The research was scheduled for Semester 2, 1995. During that semester, two mathematics subjects were being taught to engineering and science students;

Mathematics I to engineering students from all streams and laboratory technology students, and Applied Calculus to mechanical and civil engineering students who had completed Mathematics I in the first semester. After consent had been obtained from the director of the NWRCOT, students from both mathematics classes were informed in detail of the proposed research by letter, and written consent obtained for their possible participation. Most of the students completed the background questionnaire, and, on the basis of the information given, the eight representative students were chosen. The eight participants consisted of three laboratory technology students, two female and one male; one male civil engineering student; three male mechanical engineering students; and one male electrical engineering student.

In the administration of all the questionnaires and tests, care was taken to preserve a balance between the needs of the research, and the students' own pressures of time, study and work commitments. It was decided it was not in the best interests of the students to use a large amount of class time for the tests and interviews. For the research, the participants were mainly chosen from the full-time population as several of the part-time students were required to make up at work the time spent at TAFE studies. All participants were given great flexibility in the time and place for completion of the tests and questionnaires.

Anonymity and confidentiality were guaranteed for all observations and results. The students were assured that the research results would have no bearing at all on the final results in their own subjects. In the written accounts of the research, the use of pseudonyms preserved anonymity.

## 3.3 Instruments

### 3.31 Background questionnaire

Initially, students from each class were asked to complete a confidential questionnaire (Appendix A) which investigated educational background and work experience. Only personal information considered to be relevant to the present investigation was sought. Particular emphasis was given to past experience in mathematics learning, and **feelings** about mathematics, and algebra in particular. Details of past work experience were sought, and any **work-related** use of mathematics. Such background knowledge of the students' past mathematical experiences, and work experiences was considered essential in investigating any relationship between mathematical competency and work-related mathematical strategies.

A follow-up interview on the background questionnaire was conducted for the eight main participants. Relevant material on the questionnaire was clarified and expanded where necessary, and an interview protocol (Appendix B) used as a guide, to ensure that similar questioning occurred for all subjects. The interviews were recorded on audio tape and subsequently transcribed. In addition, brief notes were made at the time of the interview.

The first group of four male students ranged in age from 24 years to 31 years. Three had completed formal schooling to Grade 10 at north-west high schools, the fourth had begun Year 12 in a Victorian high school, but had not studied mathematics beyond Grade 10. All four had served apprenticeships, two in the electrical trade, one fitting and welding and one fitting and turning. All had undertaken various trade-related courses. Joseph, an electrical engineering student, listed further studies in "refrigeration, electronics, P.L.C.'s and industrial control". Sam, a mechanical engineering student, listed studies in "hydraulics, pneumatics, welding, plastic welding and computing". All four had been employed in a variety of trade-related jobs. Mark described his work experience: "Ten years as a textile fitter with Tascot Templeton, two years as a fitter and turner with Australian Weaving Mills, and one year as a maintenance fitter/welder with Vecon". Sam listed his work experience: "Fitting and machining with Savage River Mines for eight years, then work on the Burnie Wharf, with Skilled Engineering, then with Vecon".

The four students chosen as representative of the second group consisted of two females and two males, aged from 31 years to 36 years. Arthur had finished schooling after Grade 9, Christine and Maurice had finished after Grade 10 and Rose had begun but not completed a Year 11 in a Western Australian High School. None of the four had studied mathematics beyond Grade 10, neither had any of this group undertaken any form of study or training since leaving school. Their work experience however was quite varied. Maurice, from a family of thirteen, began his working life as a labourer for two years. He then worked in a timber mill pallet factory for three years. Finally, through a 'mate', he had worked in a laboratory for the past eleven years. Christine had worked as a pharmacy assistant for seven years, then as a part-time office worker. Rose had worked in a supermarket delicatessen, eventually as manager, for twelve years. Arthur described his varied existence as a contractor, "barking logs, picking apples, racking timber, whatever".

### 3.32 ACER Operations test

It was considered desirable to have a measure of the students' basic mathematical competencies, as well as investigating in detail their visualising abilities, and facilities with algebraic concepts.

In order to obtain a measure of the level of reasoning ability in mathematical operations, the ACER Mathematics Profile Series - Operations Test was chosen as suitable for several reasons. Its purpose is to assess students' ability to handle familiar operations in the real number field (Cornish & Wines, 1978a). The sixty items were in three subsets of twenty, each dealing with different elements: a) small numbers, b) large numbers and c) pronumerals. Students were required to operate on equations involving each of these elements. Sample items for each of the three elements are displayed in Figure 3-1. The complete test is presented in Appendix C. The inclusion of pronumerals as an extension of familiarity with the real number field was considered as particularly appropriate to the overall purposes of the study. The items wre based on those used by Collis in his research into levels of mathematical development which he linked to Piagetian theory and stages of development of concrete and formal operations (Collis, 1975).

53 $s \div t = rs \div \triangle$

A $rt$ C $t$

B $\frac{rs}{t}$ D $\frac{t}{r}$

54 $p + q = (p + r) + (q + \triangle)$

A $s$ C $-r$

B $r$ D $p + q - r$

**Figure 3-1: Selected items from ACER Mathematics Profile Series - Operations Test**

All tests in the ACER Mathematics Profile Series use the MAPS scale, a measurement scale based on the Rasch approach. This is based on a probabilistic measurement model developed by G. Rasch, the main advantage of which is the provision of objectivity of measurement (Cornish & Wines, 1978a). Items in all areas of the ACER tests are calibrated onto the MAPS scale which then proposes to measure mathematical development across all content areas. Because the MAPS scale was originally developed using similar items to those utilised by Collis in his research on neo-Piagetian levels in mathematical development, levels can be superimposed on the MAPS scale which indicate student progress through the identified stages of operational thinking (Cornish & Wines, 1978a). This whole approach was of particular relevance to this study, since a broader interpretation of student performance in terms of operational levels, independent of student age, provided a framework for understanding individual student's difficulties with specific algebraic concepts. Although the tests were written and validated with school-age students, the mathematical content of the items related reasonably well in terms of language and approach with the corresponding course content of the Mathematics I unit common to all participating TAFE students.

The ACER Mathematics Series Operations Test (Appendix C) was administered to the participants. No time limit was set on the test and students were asked to complete the test at a time to suit themselves. The procedures for administration explained in the handbook accompanying the test were adhered to, and student confidentiality assured. The students were asked to complete item numbers 11-20, and 31-60 (See Appendix C). The Operations Test Teacher's Handbook identified the average difficulty of these items to be such that students operating at the early formal operational (concrete generalisation) stage would have 50% mastery over these items as a whole (Cornish & Wines, 1978a). Collis and Biggs (1991) identified the transition from the concrete generalisation stage to the formal operational stage as of crucial importance for a student's ability to comprehend and apply more abstract algebraic concepts.

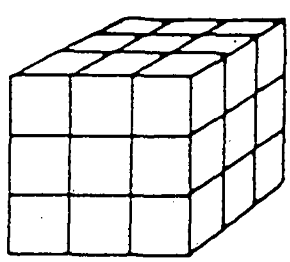
### 3.33 Tests of visualising abilities

Two instruments were chosen to measure visualisation abilities, one of the ACER Mathematics Profile Series Space Test Units I to IV, and the Betts Questionnaire on Mental Imagery [QMI]. The Space Tests are part of the ACER Mathematics Profile Series, and items from the tests with their possible scores were calibrated on to the MAPS scale. Theoretically, it was then possible to compare students MAPS abilities for the Operations Test and the Space Test.

Student results for the ACER Operations test were analysed according to the instructions in the test handbook, and from the resulting levels of mastery, a choice was made of the ACER Profile Series Space Test Unit IV (Appendix D) in order to investigate the students spatial and visualising ability in a more logical and abstract sense. No time limit was set on this test and students were left free to complete it in class or in their own time.

The Space Test Unit IV is a thirty-two item, group administered test, with multiple choice answers. It is designed to measure four content areas of spatial ability: visualisation, spatial orientation, form-perception and attention to detail. Taken together, an assessment of these abilities appeared to give a reliable measure of visualisation of images possessing some degree of symbolism and abstraction. Sample items measuring each of the four content areas are presented in Figure 3-2. It was hoped that this test would relate well to the TAFE students' possible work-related visualisations, associated with such practical quantities as volumes or areas.

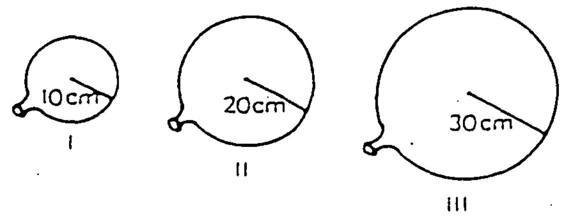
23 This pile of small cubes was placed on a desk and its exposed faces were spray painted, without the cube being disturbed.



How many cubes did not get any paint on them?

A one  C eight
B two  D nine

31 This diagram shows three stages of a balloon being blown up.




Compared with stage I, the volume of air needed in the balloon would be increased by what factors in stages II and III?

A 2 and 3  C 8 and 27
B 4 and 9  D 20 and 30

**Figure 3-2: Items from ACER Mathematics Profile Series Space Tests - Unit IV**

The test was administered to the students according to the procedure outlined in the ACER Mathematics Profile Series Space Test Teachers' Handbook (Cornish & Wines, 1978b). No time limit was set on this test and students were left free to complete it in class or in their own time.

The Betts Questionnaire on Mental Imagery (Appendix E) measures vividness of visual imagery directly. The shortened form developed by Sheehan (1967) was used. His research demonstrated that this instrument gave a reliable measure of the ability to visualise in a rage of sensory modalities. Subjects were asked to rank on a seven point scale the clearness and vividness with which they could generate a series of specified visual images, for example 'the sun as it is sinking below the horizon'. A rating of 1 corresponded to: 'Perfectly clear and as vivid as the actual experience', and a rating of 7 corresponded to: 'No image present at all, you only "know" that you are thinking of the object'. Sample items from the questionnaire are presented in Figure 3-3. This questionnaire had the potential to give a reliable measure of the ability to form vivid visual images, corresponding to ikonic mode functioning in Biggs and Collis' theory (1991). The test was distributed to all the participants, and they were requested to complete it at their convenience.

Think of 'feeling' or touching each of the following, considering carefully the image which comes to your mind's touch, and classify the images suggested by each of the following questions as indicated by the degrees of clearness and vividness specified on the Rating Scale.

| *Item* | *Rating* |
|---|---|
| 11. Sand | ( ) |
| 12. Linen | ( ) |
| 13. Fur | ( ) |
| 14. The prick of a pin | ( ) |
| 15. The warmth of a tepid bath | ( ) |

**Figure 3 -3: Items from Betts QMI.**

A descriptive correlation of results on the ACER Space Test IV and the Betts QMI was expected to provide a reasonably reliable measure of the visualising capacity of participants.

### 3.34 Algebra test

To investigate students' meanings and uses of letters in algebra, items for an algebra test were based on the Algebra Project New Test 1990, devised by Quinlan (1992) (Appendix E). This test itself is based on the extensive body of research into students meaning for variables (Collis, 1975; Harper, 1979; Rosnick & Clement, 1980; Kuchemann, 1980; Booth, 1983; quoted in Quinlan & Collis, 1990, Pp 435 - 441). Additional items were based on the twelve-item test developed by MacGregor (1991) to investigate difficulties experienced by students in using simple algebraic notation and in formulating logical relations.

In the Mathematics I class, the Algebra Test (Appendix D) was given as part of a general revision of algebraic processes. Participating students in the Applied Calculus class were given the Algebra Test individually and asked to complete it in their own time. No time limit was set for the test. The tests were marked, and the errors carefully analysed to discover any consistently made error, as these can reflect a way of viewing a problem or dealing with its solution consistent with the student's cognitive structure (Booth, 1984, p.7).

The participants were then interviewed individually and asked to explain their thinking on the questions for which they had given incomplete or incorrect answers. Initially, their original answers were not shown to them. This served two purposes. Where the wrong answer had been due simply to carelessness or haste, the answer and related reasoning given in the interview situation clearly indicated the student's mastery of the particular concept being tested. Where the student's incorrect response was repeated, the explanation given allowed some insight into the faulty cognitive processes giving rise to the erroneous result. These interviews were particularly useful in investigating the students' thought processes as they attempted the different problems. In particular, attempts, successful or otherwise, to use visual strategies or real-world referents to overcome cognitive obstacles were noted. These interviews were audio-taped and later transcribed for more careful scrutiny. Notes and observations on the participants' learning processes, and any relevant comments or conversations from classroom interactions, were kept.

## 3.4 Limitations of the study

The selection of a case study as a research method involves limitations. There is the almost inevitable factor of observer bias, as discussed in the introduction. In the present study, where the researcher was actually the students' teacher, great care was exercised to preserve objectivity, so that observations were not influenced by the researcher's preconceived ideas of an individual student's ability and attitudes. As already discussed, the strong rapport and trust already established between the researcher and students greatly facilitated and enhanced the students' openness in the interviews.

The great advantage of the more qualitative approach of a case study is its ability to provide a more comprehensive perspective on the situation, and to detect more subtle nuances and insights into the situation. In this case, it gave a more comprehensive and insightful picture of the cognitive processes involved in an adult learning mathematics. Great care was taken to make the choice of students as 'representative' as possible. However, the definition of 'representativeness' was ultimately based on subjective judgement. It was nevertheless possible to decide on several objective criteria. These were age, last year at school, work experience, and the presence or absence of trade training. The small size of the total engineering and science student population, and the even smaller number of those involved in mathematics classes placed some limitations on the choice of such 'representative' students.

Finally, it was possible that factors such as age, maturity or overall intelligence may have been far more significant in determining the degree of understanding of algebraic processes, than work experience or visualising ability. Some monitoring of this would have been possible with an investigation of the past academic performance and IQ levels of the students involved. Apart from the questionable validity of such results, such an investigation was considered too intrusive, particularly as the students involved in the research had a student-teacher relationship with the researcher. The students own report of their past success or otherwise in mathematics at school was taken as a guide. However, this performance may not have reflected their true potential. Self report on such matters as education are known to be inaccurate to some degree. In addition, the background questionnaire revealed that many of the students' schooling was disrupted or impeded for reasons often beyond their control.

The retention rates for students beyond Grade 10 for some of the north-west schools attended by the participants, in the appropriate years were investigated. For the relevant years between 1974 and 1980, the percentages of Grade 10 students proceeding on to Year 11 ranged from 22% to 28%. In 1985, the average retention rate in Year 11 for north west high schools was still only 29%. This compares with an average retention rate of 61.3% for the north west high schools in 1995 (Department of Education and the Arts, Resource Planning Services Section, Hobart, personal communications, November, 1995; January, 1996). Such low retention rates in previous decades may indicate that many of the students leaving school after Grade 10 in those years had above average academic ability. This could have introduced a considerable intelligence bias in mature-age students currently returning to study. The magnitude of this factor, however, wuld have been very difficult to establish.

The assumption that students who have completed mathematics studies to Grade 10 have similar mathematical backgrounds could be criticised as an oversimplification. The standard of mathematics teaching can vary from school to school, and, even within a school, and there are major differences in the mathematics covered at different levels. To a large extent this disparity was offset in the present study, since the all the participating students had completed or were completing the preparatory subject Mathematics I, and so had experienced the same exposure to basic algebra, trigonometry and geometry.

The disadvantages in the use of interviews as a data collection technique are acknowledged. The TAFE students' time was very limited. In addition, it was difficult to ensure replication of interview context for each interviewee. Also, since the students were from the researcher's mathematics classes, their answers may have tended to reflect what they assumed she wanted to hear. However, it was felt that the advantage gained from the trust existing between the researcher and the students was of much greater significance. The direct transcripts of the interviews provided objective records of the interactions. These transcripts gave invaluable insights into the participants' own perceptions of their progress in mathematics and their candid feelings about mathematics, past and present. They helped to clarify some of the cognitive blocks in their treatment of algebraic concepts.

It was recognized that there was an inherent lack of validity in using tests and material specifically designed for school-age students, but suitable instruments for adults were not readily available. The 'school' nature and wording of some of the problems may have been off-putting for these mature-age students, and reduced motivation in completing the task. After taking the nature of the students' mathematical experiences into account, however, the tests used were selected as the most appropriate available for both the students and the needs of the research.

# CHAPTER 4

# RESULTS AND DISCUSSION

## 4.1 Results and discussion - background questionnaire

A summary of the participating students' ages, number of years of secondary schooling, highest year, level and grade of mathematics achieved, and results from all tests used in this research, is given in Appendix G

Although students' background details varied greatly, several common factors emerged for the eight participants. The majority of students grew up in rural areas of Tasmania, most on the north-west coast, although one student, Joseph, spent his school years in a lower socio-economic area near a large Victorian city. As he described:

> "...the area I was from was fairly rough ... a lot of people have gone to gaol from the area ... it was a full-on Commission area."

As has been discussed in Chapter 3, continuing school beyond Grade 10 was relatively uncommon for students of north west high schools in the seventies and eighties. This circumstance is supported by the anecdotal evidence supplied by the TAFE students who were high school students during these decades. Mark, a Grade 10 student in 1979, commented:

"I didn't know people stayed on, that's fair dinkum. I just thought people left at Grade 10; that that was the done thing."

Christine, a Grade 10 student in 1974, made the observations,

"I think with the girls it just wasn't pushed; this is 20 years ago. You just thought--leave school, get a job in a supermarket or a milk bar and have kids--and that would be it."

Economic factors were also a significant deterrent to further education. Maurice, who was in Grade 10 in 1979, explained:

"See I'm sort of the third last of 13. Austudy and that sort of thing wasn't about then, so ... when you go to higher education you've got to have the quids."

Mark commented:

"... I suppose I do know a few people that have (gone on) sort of thing, but they've mainly come from richer type families.... It certainly didn't happen in East Devonport very often."

Students memories and feelings about mathematics at school were, with a couple of exceptions, negative or vague or both. Comments varied:

"...hated it ... didn't understand it",

"...didn't like it ... struggled through",

"...difficult ... did a lot of work to get through", "Have no recollections of likes of dislikes of maths in high school ... did poorly."

A couple of students felt positive about their mathematics experiences. Maurice commented:

"(It was) my best subject ... it was okay, I found it all right to do."

James memories were also positive:

"I enjoyed it ... worked hard to achieve results."

One interesting factor emerged which appeared to have had a significant influence on mathematical progress and even the decision to continue or to stop schooling after Grade 10. One half of the participants had experienced some major disruption to their schooling during their high school years. The perception appeared to be that this was where the major block to any further progress in mathematics really occurred.

Maurice:

"Well I was very sick in Grade 9, I missed 55 days in the last two terms of school and failed the maths.... They couldn't do much about it."

Rose:

"I hated school full stop. Probably because we moved around so much ... 30 schools in all, ...Year 10 in Holland..... I had trouble understanding the maths, that was more to do with the language ... So there was no point any more, I hated it."

Joseph:

"I went to England in 1980, Year 9. We were sort of everywhere and I found it really hard to do the homework. That triggered the whole thing; that was the end of me then. They put me in a vegie maths after that."

Mark:

"...There was about 10 (schools), the last three were three different high schools. When I was younger my parents passed away and I went from cousin to cousin.... Dad died when I was in Grade 10.... I didn't like maths and had trouble concentrating. I think it had a lot to do with my private, personal life; I couldn't settle down. And I think I found maths the hardest to grab hold of."

Students memories about algebra were less specific. For most it appeared to have been a confusing irrelevant subject, if remembered at all. Comments on their feelings about algebra included the following:

"Confusing."

"I never really had a good grasp, I think."

"... that I didn't need it, and what would I use it for, and that it did not make sense."

" I did not like it."

"I couldn't do it so I didn't like it."

"Can't remember."

"None (algebra) that I can recall."

As already enumerated in Chapter 3, students taking part in the research had wide and varied work backgrounds, especially those students with a trade background. When questioned about mathematics 'on the job', some students initially did not relate their daily use of measurements, weights, volumes, and areas to `school mathematics'. Sam, with a fitting and turning background, stated:

"...small calculations ... measurements ... I don't class that as maths."

When questioned further, Sam reiterated:

"You're estimating ... you're always checking fit, ....using a few formulas ... but that's not real maths."

Interviewer:

"What is real maths?"

Sam:

"Algebra!"

Other students gave quite detailed lists of mathematics applied in the workplace. James enumerated:

"Maths used to find current carrying capacities of cables and all other electrical calculations, basic Ohm's Law, power calculations, motor design change, HP, RPM, electronics."

Several students gave instances of the workplace expertise, in context based mathematics, reported by Scribner (1986) and other workers in this field. Joseph, from an electrical background, described the older expert `sparkies' on the job:

"Some guys have a natural flair and understanding of electrical, and with skills they have acquired e.g. good techniques--can get around problems (e.g. maths) . Some guys are really good at sight ... perfect at visualising.... Conduits is a hard art, but they'll bend it and do it quick, do a better job than anyone else."

Again, Rose described the skills of judgement developed working in a delicatessen:

"It becomes very automatic; you know approximately how much you've got to pull out of the cabinet and throw on the scales ... Sliced meat--you know there's 10 slices in 250 grams.... You look at it, you just approximate when you're looking at it."

The final part of the background interview was concerned with the students' own accounts of their current success in mathematics. They were virtually unanimous in attributing this success to three factors: firstly, the strong conviction that mathematics is a very important and necessary foundation in the courses they are undertaking; secondly, the fact that for most of these students, the return to study involved considerable financial sacrifice; and thirdly, the tremendous commitment in time and effort these students have been prepared to give to achieve their success . These views were expressed variously.

Interviewer:

"Why do you think you've been successful in maths this time compared to your school experience?"

Christine:

"I know that that's what I need to be good at ... to be what I want. I can see the relevance in it"

Mark:

"Well, I think it's because of having to this time. I left a full-time job to come here and do it, so I don't really want to fail."

Rose:

"... I want to make it work.... I put a lot of hours in. I've been brain dead for 12 years. Working in the supermarket is terrible for that, it's really dead end stuff."

James:

"I think I'm older and more willing to learn.... I'm paying for it you see ... you put in the effort."

Sam:

"At school you didn't really care what you were doing. You've come back for yourself this time. It's six months off work. You could earn a heck of a lot in that time ... I sit down at home and do hours and hours of work."

Maurice:

"It will help me greatly in my chemistry studies."

This commitment, combined with the wide-ranging background experiences of these students, appeared to facilitate consistently successful performances on most mathematical tasks. These results are discussed in the next three sections.

## 4.2 Results and discussions of the QMI, Operations Test and Space Test

The Betts Questionnaire on Mental Imagery proposes to measure direct visual imaging. A score of 35 represented perfectly clear and vivid images for every item, while a score of 245 indicated no images formed at all for any item. The results for the two groups are illustrated below in Table 1. Group 1, the "trade group", scored an average of 88.8, and Group 2, the "non-trade group" scored an average of 86. Although scores varied considerably within the groups, the final averages were surprisingly close Although this appeared to indicate that students from the "trade group" had a slightly greater ability overall to visualise, the groups were too small, and the variability too great ($\sigma$ = 224.7), for this to be of any significance. The results could equally validly be interpreted as indicating no real difference between the two groups in the ability to form direct visual images.

| NAME | Age | QMI Score | NAME | Age | QMI Score |
|---|---|---|---|---|---|
| **Group 1** | | | **Group 2** | | |
| Joseph | 29 | 116 | Arthur | 36 | 126 |
| Sam | 24 | 64 | Maurice | 31 | 86 |
| James | 31 | 72 | Christine | 36 | 63 |
| Mark | 32 | 103 | Rose | 32 | 69 |
| **Average** | **29** | **88.8** | **Average** | **33.8** | **86** |

**Table 1 - Betts QMI results**

The ACER Mathematics Profile Series - Operations Test (Appendix C) proposes to assess students' abilities to handle familiar operations in the real number field (Cornish & Wines, 1978a). For the purposes of this research, items 11 - 20 and 31 - 60 (see Appendix C), were chosen to be completed by the participants. The results of the test indicated an average MAPS ability of 65.4 for the eight students. The broad interpretation of this related the level of challenge offered by items at this point, to the Piagetian operational stages. A MAPS ability of 65 indicated a level where students were likely to show almost complete mastery in concrete operational tasks, and considerable facility with formal operational tasks i.e. with theoretical or closed systems, the student could operate on all combinations of the factors involved (Cornish & Wines, 1978a). The results indicated that the participants could operate at this level.

Results for the Operations test in terms of MAPS ability for the two groups in the test are displayed in Table 2 below. Group 1, "trade background", averaged 66.5 and Group 2, "non-trade", averaged 64.25 . The difference of 2.25 appeared to indicate that Group 1 had a slightly greater mastery in formal operational tasks than Group 2. Comparisons between students however, are recommended only if their MAPS abilities differ by more than the sum of the scaled errors associated with those MAPS abilities (Cornish & Wines, 1978a). For the eight results, the average scaled error was 2.5. This implied that the difference in results between the two groups may be explained in terms of the errors associated with the students' test scores, and could not validly be interpreted as a difference in ability between the two groups. The two groups were considered to have comparable mathematical operational ability

| NAME | Age | Operations Test | NAME | Age | Operations Test |
|---|---|---|---|---|---|
| Group 1 | | MAPS Ability | Group 2 | | MAPS Ability |
| Joseph | 29 | 62 | Arthur | 36 | 62 |
| Sam | 24 | 64 | Maurice | 31 | 66 |
| James | 31 | 72 | Christine | 36 | 66 |
| Mark | 32 | 68 | Rose | 32 | 63 |
| **Average** | **29** | **66.5** | **Average** | **33.75** | **64.25** |

**Table 2 - Results of Operations Test**

The ACER Mathematics Profile Series [MPS] - Space Tests were designed to monitor students' spatial development, both in traditional spatial topics in mathematics and in the perceptual aspects underlying these topics. Based on the results from the Operations Test, in which the students recorded an average MAPS ability of 65.4, Space Test Unit IV was chosen. The ACER MPS Space Test Teachers' Handbook specified this test as having a mean item difficulty of 57.7 on the MAPS scale (Cornish & Wines, 1978b). Theoretically, this placed the Space Test items well below the average MAPS ability of 65.4 recorded for the participants in the Operations Test. This implied that they should have been capable of completing up to 90% of the items on the Space Test (Cornish & Wines, 1978b). However, the students in Group 1, "trade background," only managed an average of 59.5, while Group 2, "non-trade background" showed even less ability in this area, with an average of 56.5 on Space Test IV. The results for the two groups are displayed below in Table 3. There was a difference of three between the two groups. Again, this difference was only marginally outside the difference which could be accounted for, in terms of the scaled errors associated with the students' test scores. For the Space Test, the students' average scaled error was 1.9 (Cornish & Wines, 1978b).

If considerable experience in working with three and two dimensional entities gave an advantage in visualisation and form-perception, then Group 1, students with trade experience, would have been expected to perform significantly better on the Space Test. The small margin of difference between the two groups did not support this hypothesis.

Correlation between the results from the Operations Test and the Space Test was very significant ($r = 0.965$ $p < .001$). Clearly both tests appeared to measure the same fundamental cognitive mathematical ability, rather than differentiating between spatial and visualising abilities, and basic mathematical competence.

| NAME Group 1 | Age | Space Test MAPS Ability | NAME Group 2 | Age | Space Test MAPS Ability |
|---|---|---|---|---|---|
| Joseph | 29 | 54 | Arthur | 36 | 55 |
| Sam | 24 | 56 | Maurice | 31 | 59 |
| James | 31 | 66 | Christine | 36 | 57 |
| Mark | 32 | 62 | Rose | 32 | 55 |
| **Average** | **29** | **59.5** | **Average** | **33.75** | **56.6** |

**Table 3 - Results of Space Test**

The differences **between** the overall MAPS ability scores on the Operations Test and the Space Test appeared much more significant. There was a difference of approximately 7 between the average results for the two tests. It was difficult to account for such a significant difference, if a true mapping between the abilities displayed on the two tests existed.

The explanation of the higher overall MAPS abilities displayed by both groups in the Operations Test may have been in terms of all participants' greater facility with real number and algebraic relationships. Several factors could account for this competence. All the groups had studied or were studying Mathematics I, which placed far greater emphasis on algebra, than geometry or related spatial topics. In addition, for students of both engineering and science, the majority of other subjects studied involved algebra in the form of the evaluation and transformation of formulae. In the Space Test, the unfamiliar language in some items, and items on school-related topics, may have been an additional inhibiting factor.

When the results from the Betts Questionnaire on Mental Imagery and the Space Test were compared, there was no significant correlation ($r = 0.232$, ns). This appeared to indicate that, for this particular group, the ability to form direct visual images, and the ability to perceive and manipulate spatial concepts were two independent facilities. Again, this would have been based on the assumption that the Space Test gave a true measure of spatial abilities. As discussed above, this did not appear to be the case. The low value of r in fact was probably more related to a lack of correlation between <u>mathematical operational ability</u> and visualising ability.

## 4.3 Results and discussion - Algebra test

Results for each question are summarised, and compared with results from previous research. Where appropriate, the question is discussed in detail, and some interpretation given for the results.

**QUESTION 1**

| | | | | |
|---|---|---|---|---|
| **1.** | **If y=3, find the value of** | **(i)** | **2y** | ................. |
| | | **(ii)** | **2y + 5** | ................. |
| | | **(iii)** | **2(y + 5)** | ................. |
| | | **(iv)** | **2y + y** | ................ |
| | | **(v)** | **3y - y** | .................. |
| | | **(vi)** | **2(5y)** | .................. |

This item was adapted from item 3 in Quinlan's Algebra New Test 1990 (1992). The question involves simple substitution, and does not require any real understanding of the concept of variable. The students' grasp of standard conventions of algebraic notation for multiplication, use of brackets and order of operations are measured.

All eight TAFE students had no difficulty with this question, with almost 100% correct answers. The couple of errors made were identified by the students involved as careless slips. In Quinlan's research with 517 Australian students from Years 7 to 12, he recorded relatively high success rates for the similar items, ranging from 86% to 77%

correct answers (Quinlan, 1992, pp. 109-110). Lower scores were associated with items involving brackets, but these items presented no difficulty for the TAFE students. The high success generally for all students can be attributed to dealing with a familiar arithmetic system.

**QUESTION 2**

**2. 't is equal to the sum of 4 and s'. Write this information in mathematical symbols.**

Question 2 was adapted from item 9 on MacGregor's 12-item test for investigating Year 9 students' difficulties with elementary algebra (1991, pp. 66-67). This question investigated students' abilities to form a simple algebraic statement involving the 'sum of' relationship between variables. The research by MacGregor discovered that over a third of the students tested expressed 'the sum of two variables' as two adjacent variables. For these students expressions such as **xy** represented the sum of **x** and **y**. In the Assessment of Performance Unit, 1985, involving British students of the same age and year level, only 48% were successful on the same item (MacGregor, 1991).

Apart from one student, Rose, who had joined the Mathematics I class seven weeks into the semester, the TAFE students all gave the correct answer, and appeared to have no difficulty with the technical meaning of 'sum', nor with expressing an answer with a 'lack of closure' (Collis, 1975). Rose however, gave her answer as **t = 4s.**

When questioned about her understanding of the item, Rose's answers display the classic confusion recorded in the literature, between 'times' and 'sum' when expressed in algebraic notation.

Interviewer:

"Question 2, 't is equal to the sum of 4 and s'. How did you understand that?"

Rose:

"Just exactly what it says".

Interviewer:

"So how would you write it?"

Rose:

"t is equal to 4 **times** s".

She then appeared to realise the contradiction between what she had described and what was really asked for, and quickly corrected herself:

"4 plus s."

Interviewer:

"Are you sure of that now?"

Rose:

"Yes, because you've got **and.**"

QUESTION 3

3. **'The number z is nine times the number x.' Write this information in mathematical symbols.**

Question 3 corresponds to item 10 on MacGregor's test. The ability to write an algebraic expression involving the relation 'times', is being tested. Care was taken to ensure that factors that might have caused the 'reversal' error were not contained in the item (MacGregor, 1991).

Every student in the TAFE group answered this question correctly, displaying a clear understanding of the concept 'times', and a good grasp of algebraic notation. Surprisingly, in the light of the results discussed below, no student made the 'reversal' error, but all appeared to have a clear sense of the meaning of the sentence, and be able to express it correctly in mathematical symbols.

Students in MacGregor's research scored a very low rate of success for this item. Only just over 56% of the students gave the correct answer. Over one third gave answers containing the reversal error in some form, in spite of the care taken to eliminate known causes of this error from the form of the item (MacGregor, 1991, pp. 92-95).

QUESTION 4

4 **A number p is six more than another number q. If q is 21, what is p?**

Question 4 corresponded to item 1 of MacGregor's 12-item test. This question was designed to investigate students' understanding of the written mathematical relation 'more than'. Previous research had shown that some students were not sure if 'more than' referred to a sum or a product, particularly if there were no real-world referents (MacGregor, 1991). The syntax of the item was structured deliberately; this item is described as a 'consistent' language problem. "The syntactic structure is such that the unknown is the grammatical subject of the sentence, and the relational term is consistent with the arithmetic operation to be used" (MacGregor, 1991, p.67, quoting Lewis & Mayer, 1987).

MacGregor's research quoted an 83% correct answer response for this item. Misinterpretation of the syntax of the problem accounted for 8% of the errors. The TAFE students also found this item relatively easy, with only one careless mistake by Mark. When presented with the same item in an interview he solved it correctly, without hesitation.

## QUESTION 5

**5 The number x is five times the number y. If x is 30, what is y?**

Question 5 corresponded to item 2 on the 12-item test devised by MacGregor The question is similar in format to question 4. It is a short statement describing a simple relationship between two variables, involving only small whole numbers. However, his item is an 'inconsistent' language problem: "the unknown is the grammatical object and the necessary operation of division is not consistent with the relational term `times'" (MacGregor, 1991, p. 68, quoting Lewis & Mayer, 1987).

The percentage of correct responses for this item in MacGregor's research was only 60%. A high proportion of the total answers, 31%, revealed students' difficulties with the syntactic structure of the statement. When compared to the previous question, there were 23% more incorrect answers. This increase was associated with the change from consistent to inconsistent structure. However, the TAFE students again demonstrated their competence with such algebraic items. All answered correctly except Christine, who did make a syntax error. Her thoughts on the problem were quite revealing.

Christine:

> "... If x is 30, you divide by 5. Divide by 5 till you get the number ... Six, is that right?"

Interviewer:

> "It's right. Why do you think you wrote that (150) in the test?"

Christine:

> "I've just timesed it by 5, it's just a mistake."

Interviewer:

"Do you think there's anything about the question that would lead to that sort of mistake?"

Christine:

"Probably because the 30 is first."

Christine recognised that the syntactic structure of the question had misled her.

## QUESTION 6

6 **In an engineering class, there are 12 more men than women. There are 14 men. How many women are there?**

Question 6 corresponded to item 3 from MacGregor's test. It was paraphrased to make it slightly more relevant to the TAFE students! Both the students involved in MacGregor's research and the TAFE students found this type of question easier than the corresponding item involving variables (Question 4) . Ninety two per cent of the school students gave a correct response, although syntax errors still accounted for the majority of mistakes. The TAFE group had a 100% correct response. Although this is also an inconsistent language problem, the concrete or real-world situation appeared to guide the TAFE students in choice of an appropriate operation.

**QUESTION 7, 8 and 9**

Questions 7 , 8 and 9 were similar to items 5, 7 and 8 in the twelve-item test. These three items were designed to test students' ability to understand simple equations and the basic conventions of algebraic notation. (MacGregor, 1991, p. 68).

**QUESTION 7**

7 **If $a + 7 = b$, which is the greater number, a or b?**

There was a correct response of 90% from the school students, with reversal errors forming the majority of incorrect responses. The TAFE students also found this question uncomplicated; all gave the correct response. These results support Wollman's (1983) research quoted by MacGregor (1991, p. 86). He found that although students had difficulty constructing equations, they were more able to interpret them.

**QUESTION 8**

8 **If $5y = p$, which is the bigger number, y or p?**

As expected, reversal errors accounted for most of the incorrect responses to this item, 13% of the total responses, in MacGregors's research. Only 77% of these students gave the correct response, indicating that students have more difficulty with the conventions of writing algebraic multiplication than simple sums, as in Question 7. However, there was 100% correct response from the TAFE students, indicating again the relatively clear understanding these students appear to have of basic algebraic conventions and procedures.

QUESTION 9

9 **If $x = 3$ and $y = 6$, what is the value of $xy$?**

The TAFE students demonstrated sound competence in the use of algebraic notation for expressing a product, with 100% correct responses. This contrasted sharply with the school students in MacGregor's research. Only 61% gave a correct response to this item, and one third of the students tested gave an answer which interpreted **'xy'** as **'x+y'** (MacGregor, 1991, p. 68).

QUESTION 10

**10 x and y are numbers. x is seven more than y. Write an equation showing the relation between x and y.**

This question corresponded to item 11 on MacGregor's test, and investigated students' ability to form simple algebraic sentences involving the relations 'more than' and 'times'. The question was expressed as plainly as possible, and designed to avoid the factors assumed to cause reversal errors. In spite of this, students obviously had considerable difficulty. Only 28% of the school students in the MacGregor's research gave the correct answer. Reversal errors accounted 46% of the responses, e.g. $x + 7 = y$. In some responses, there was additional confusion about the relation 'more than' e.g. **7x=y.** This was also one item that challenged the TAFE students; three gave incorrect responses, and two of these were reversal errors.

Since the accepted causes for reversal errors had been factored out of the question, MacGregor proposed that psycholinguistic factors may be significant. She suggested that the semantic and syntactic structures underlying comprehension maybe independent and even in conflict (MacGregor, 1991, pp. 11-12).

Further difficulties encountered by students in tackling this question appeared to be confusion about the concepts of sum and product, and the algebraic notation for these. Poorer readers may have had difficulty with the words and concepts of `equation' and 'relation' (MacGregor, 1991, p. 101).

In interviews with the TAFE students, confusion in trying to express the relationship described in the problem was obvious. For example, Arthur had given as his answer **'x=y -7'**, a typical reversal.

Interviewer:

> "What did you understand by Question 10, do you remember tackling it before?"

Arthur:

> " Yes, x is seven more than y."

Interviewer:

> "Did you mean to write what your wrote?"

(Arthur's original answer was shown to him.)

Arthur:

> "Well it seems logical."

Interviewer:

"That $x = y - 7$?"

Arthur:

"Well, if $x$ equals seven more than y, then y must equal 7 less than x."

Arthur's reasoning was correct, but he did not see the mismatch between what he understood and expressed in words, and what he had written in his equation. Such a discrepancy would appear to support MacGregor's (1991) notion of conflict between semantic and syntactic structures within a person's cognition.

QUESTION 11

**11. The Niger River in Africa is $y$ metres long. The Rhine in Europe is $z$ metres long. The Niger is three times as long as the Rhine. Write an equation which shows how $y$ is related to $z$.**

Item 12 on the twelve-item test was used for Question 11 in the present test. It was hoped that the real-world referent of the relative lengths of the two rivers would reduce the confusion which appeared to occur with mental images of groups of counted objects side-by-side e.g. in the `students-professor' problem discussed previously. This was a language consistent problem, so it was not expected to produce syntax errors.

There was a dramatic difference between performances of the two groups of students for this question. The Year 9 students in MacGregor's study found this question very difficult. Only one third gave a correct response. In spite of the consistency of the syntactic structure of the problem, 44% of all responses were reversals of some form. General confusion or misunderstanding of concepts such as ratio, proportion, equations, sums/products, powers, 'times'/'times more' and algebraic variables generally accounted for the wide range of errors made by the Year 9 students in this item (MacGregor, 1991).

All the TAFE students answered correctly except Rose. These students appeared much more able not only to interpret the statement correctly, but also to express their understanding with the correct logical structure in a mathematical expression. This was in contrast to Question 7, which had a purely abstract content and was answered incorrectly by three of these students. Although the number of students involved in this research was small, the results appeared to indicate that for these mature-age students, a real-world referent aided in the construction of meaning for a mathematical statement, and in expressing it correctly in algebraic notation.

Rose's comments on this question in contrast, mirror the helplessness many students feel when faced with this type of problem.

Rose:

> "I didn't like those (types of question) at all ... I was lost with those. When you put it all into words, describing it in words, and then you've got to put it back into equations, I just don't see it."

QUESTION 12

**12 In a football match, one team scored $t$ points and the other scored $r$ points. How many points altogether were scored in the match?**

Question 12 was Item 5 in Quinlan's New Algebra Test 1990, which he had adapted from a similar question in Booth's (1983) Strategies and Errors in Secondary Mathematics [SESM] research questions. The item was designed to investigate students' understanding of variable, and their ability to handle lack of closure in an answer, expressed in algebraic letters whose values were unknown. It also measured their ability to express a sum in correct algebraic notation (Quinlan, 1992, p.104).

Quinlan's results for 517 Victorian students from Years 7 to 12 showed 63% correct answers for this item, with 12% of the responses representing the sum as **pr**. In Booth's (1984) SESM research with 50 students from 13 to 15 years, only 36% answered correctly, and 29% gave conjoined answers.

TAFE students had no difficulty with this item. Again, overall, the TAFE students demonstrated good mastery of algebraic notation and interpretation..

QUESTION 13

13. a) **Given c + d = 10,**

CIRCLE ALL THE POSSIBLE MEANINGS FOR c:

(i) 3

(ii) 10

(iii) 12

(iv) 7.4

(v) **The number of tools in a box.**

(vi) **An object such as a condenser.**

(vii) **An object such as a battery.**

(viii) **None of the above.**

(ix) **More than one of the above (if so, indicate which ones).**

(x) **Don't know.**

b) **If $c + d = 10$, what happens to d as c increases?**

c) **If $c + d = 10$, and c is always less than d, what values may c have?**

This question was amodified form of Item 6 in Quinlan's Algebra Test (1992). Part (a) investigated the level of meaning students ascribe to algebraic variables. This area has been extensively researched and documented (Collis, 1975; Booth, 1984; Kuchemann, 1981, 1983; MacGregor, 1991; MacGregor & Stacey, 1993; Quinlan, 1992). Parts (a) and (b) were also aimed at directing students' attention to the possibilities of zero, fractional and negative values for a variable, leading into part (c) (Quinlan, 1992, pp. 105-106).

Question 13 (c) demanded a true understanding of variables as generalised numbers. To consider the relationship between the two variables, the students needed this clear concept of a variable. "Letters are used as variables when a second (or higher ) order relationship is established between them" (Kuchemann 1981, p. 111).

For Question 13 (a) parts (i) to (iv), where numerical values are assigned to c, both Quinlan's Year 9 students, and the TAFE students in the present research, accepted the proposed values in the following order of popularity: '3', '7.4', '10' and '12'. Only 23% of Year 9 students, and 6 of the 11 TAFE students considered '12' a possible choice.

The results from this item and from the related problem set in Question 13 (b) appear to confirm students' difficulties in moving from a concept of a variable as a specific unknown, or as a set of simple whole numbers, to a true appreciation of a variable as a generalised number (Kuchemann, 1981, p. 109).

Several specific features of this particular question, especially for parts (a) and (c), and its context in the 1995 Algebra test, may have further inclined the TAFE students to a limited view of the possible numerical values for c.

The superficial semantic link between a **c+d = 10**, and the implication that 'a part cannot be greater than the whole' may have led to a hasty rejection of 12. This exclusion would have been unconsciously encouraged by the placement of this particular item after twelve items all of which deliberately involved only small whole numbers. TAFE students especially appear to be sensitive to context, and appear to limit themselves to what they perceive as the author's domain for given problems; i.e. relatively simple numbers for `school-based' algebraic exercises, or realistic estimates of range for practical values of quantities such as resistance, mass, or concentrations. Some of these perceptions, plus a confusion between negative values of a variable, and a variable preceded by a negative sign, emerge from their comments about Question 13 (a), part (iii) , **`12' as a possible meaning for c in `c + d = 10'.**

Arthur:

> "When I saw this, I thought - well, that's just plus that number. I didn't realise at the time that it could be plus minus d , which could make it any number."

Interviewer to Rose:

> "Why did you feel that part (iii) `12' could not be a possible answer?"

Rose:

> "I suppose it would work if d was minus ... It doesn't actually say minus there ... It didn't occur to me at the time."

Sam:

> "... If it was going to be a negative number it should have been written with a negative next to it in brackets."

On further questioning, Sam, an able student, conceded that he was usually happy to assign positive and negative values to any variable, but he added:

> "But at that point I didn't!"

Question 13 (a) parts (v), (vi) and (vii) investigated the students' perceptions of a algebraic letter as an object or as a shorthand for an object. Only 44% of the Year 9 students in Quinlan's (1992) research accepted `**number** of apples in the box' (part (v) alternative in Quinlan's test) as correct, however, 72% correctly rejected option (vi) and (vii), which were both objects. The TAFE students appeared confused on these items. Three able students rejected (v) `The number of tools in a box', as an option, and two less able students accepted both (vi) and (vii) as viable options for `**c**'. The competence these students showed in other parts of the test possibly demonstrated "that the elusiveness of the concept (of variable) may be in part because so many items involving variables can be solved <u>at a lower level of interpretation</u>" (Kuchemann, 1981, p. 110). In hindsight, it was realised that the generic nature of the word `tools', the alternative chosen for `apples' in part (v), may have further confused the TAFE students' practical understanding of this item.

Students' comments when interviewed revealed a possible additional factor. TAFE students generally are mature-age, with considerable work experience. They are used to dealing with real situations, with work-related mathematics in meaningful context. Even with mathematical tasks, their approach is not superficial or surface; they tend to want to make sense of what they are doing, and they assume logical consistency in learning material. If it is not there, these students will tend to ignore supereficial rules and generalisations, and try to make logical sense of the particular situation. Arthur's comments on Question 13 (a) illustrate this, and point to the logical inconsistencies in the question.

Arthur:

> "I wasn't real sure about that one because it wasn't stating the number of tools in the box ...The fact is, you've got one tool box with tools in it. So, it leads you to believe that there was more than one number there, and the amount of tools in the box wasn't stated ... I assumed that the amount of tools in the box was irrelevant up to a point, because that was equal to 'c', and it had plus 'd' which could have been plus minus 'd'. But the fact is, if you're talking about the tool box as well as the tools, then you've got two numbers there."

Joseph's comments show a slightly better grasp of the idea of a variable as a generalised number.

Joseph (in answer to the number of possible choices):

> "I just said it would be the number of tools in the box, and stuff like that, because you could divide the tools up into certain parts and make 10. The object, such as a condenser, did I circle that one?... I did. I thought, I suppose, it wasn't a number, it was an object. There was only one. I suppose I didn't think of that. I was probably thinking you could have a number of condensers, but it doesn't really say that ..."

James also objected to the practical ambiguity of the question.

James:

> "It says the number of tools - it could be one, so it could represent an object such as a condenser ... it's not logical'.

Mark's comment was to the point.

Interviewer:

"With question 13, parts (v) to (viii), do any of those apply to 'c'?"

Mark:

" It's too far removed from what that (the equation c+d=10) is."

Question 13 (b) looked again at the relationship between two variables, and theoretically should have presented greater difficulty than part (a). The TAFE students gave 100% correct responses. Even if for some, the concept of variable was anchored in the concrete, they were well able to interpret an equation. The Victorian Year 9 students also had an 85% success rate with this question (Quinlan, 1992, p. 108).

Question 13 (c) required the letters to be seen as generalised numbers for a correct response. In the British study, only 11 % interpreted it correctly, with 39% listing one value only (Kuchemann, 1991, P109) Quinlan felt that the inclusion of part (a) alerted students to the possibility of more than one value for `c' and attributed the greater success of the Victorian Year 9 students to this. Sixteen per cent gave correct answers (Quinlan, 1992, p. 109). In the research on this question with first year tertiary mathematics students by Coady and Pegg (1993) 36% gave a correct response.

TAFE students responses showed an understanding that there was a constraint on 'c', and that 'c' could take a range of values. This "ability to account for possibilities and the consequent limitations suggests the presence of higher order functioning" (Coady & Pegg, 1993, p. 194). However, the students' answers again indicated that the meanings they give for algebraic letters could be context bound. All of the TAFE students listed four or five possible whole number answers for 'c', with one indicating the inclusion of negative whole numbers. Two possible factors could give some explanation for these results. As mentioned above, this question occurs after twelve relatively simple problems involving only small whole numbers in the present test. This may have influenced the students to assume that answers were restricted to this domain. In addition, the interpretations of 'c' in part (a), as tools in a box or a condenser, or a battery, may have unconsciously led the students to assume that they were still dealing with simple numbers of objects. Student comments in interviews appear to support these views.

Interviewer:

"Have a look at number 13 (c). What values may 'c' have?

Christine:

"... Must be 1 to 4."

Interviewer:

"No other values?"

Christine:

"Negatives, no, it's not negatives."

Interviewer:

"Wouldn't it be?"

Christine:

"(Pause) ...Could be, yes, could be. One to 4 or negative ... It's always less than d, so it could be anything. So, it could go on and on and on ..."

Sam:

"I was just keeping it positive, yes, and whole numbers ... I suppose if I'd thought about it , it could be anything less than 5 ..."

QUESTION 14

14 **At a certain college, there are six times as many students as there are lecturers. This fact is represented by the equation**

$$S = 6L$$

CIRCLE YOUR CHOICES IN THE FOLLOWING QUESTION.

(a) **In this equation, what does the letter L stand for?**

(i) **Lecturers**

(ii) **Lecturer**

(iii) **Number of lecturers**

(iv) **Students**

(v) **Student**

(vi) **Number of students**

(vii) **None of the above**

(viii) **More than one of the above (if so, indicate which ones)**

(ix) **Don't know.**

(b) **In this equation, what does the letter S stand for?**

(i) **Lecturers**

(ii) **Lecturer**

(iii) **Number of lecturers**

**(iv) Students**

**(v) Student**

**(vi) Number of students**

**(vii) to (ix) Repeated the options given for part (a).**

This question repeats item 7 from Quinlan's Algebra Test 1990, This item in turn was a revised version of a the 'professors-and-students' problem posed in multiple choice format by Rosnick (1981, cited in Quinlan, 1992). Responses to this problem have been characterised by an almost universal tendency to make the reversal error i.e. any incorrect association of the numeral in the equation with the larger variable.

This problem has been the subject of much research, most recently by MacGregor (1991). From her examination of nine reports on this problem, she identified five causes of students' difficulties in writing simple equations:

a) the misinterpretation of algebraic letters as abbreviated words;

b) the attempt to translate literally from natural language to an algebraic equation;

c) the misleading influence of mental pictures;

d) the selection of the wrong cognitive frame;

e) conflict between the student's cognitive model of a relation between variables and the syntactic structure of its algebraic representation, and

f) the powerful misleading effect of a mental picture of two groups of objects (e.g. a large group of students and a small group of professors) (MacGregor, 1991).

Her own investigations led her to infer however that none of these factors accounted for students writing reversed equations when these factors had been minimised. She concluded that the intuitive procedures of natural language processing obstructed the comprehension and use of a formal language such as the symbol system of mathematics ( MacGregor, 1991, p. 111).

Various reports indicate that even university students, academics or teachers have had a generally low rate of success of around 60% on this problem. Reports on secondary students results showed around 50% success. The most common and persistent source of errors was the reversal error (Quinlan, 1992). As previous items on the present test, Questions 2, 3, 10 and 11, had already demonstrated, the group of TAFE students appeared on the whole to avoid the reversal error, although some confusion in this respect emerged in interviews.

Maurice (chose (v) 'Student' for 'S'):

> "...With the students, 6S,...obviously something single..."

Interviewer:

> "You felt that that looked like one student?"

Maurice:

> "Each individual one. I thought for every ... (pause, as Mark appeared to be trying to reconcile the algebraic formulation with the language sense). Yes, probably should just have been six times as many students as there are lecturers. Maybe it's a lot to do with the way you read it too."

Christine's reasoning displayed a similar conflict between her interpretation of the equation and her semantic understanding of the verbal statement.

Christine:

> "Well, we've got S = 6L ... there are six times as many students as there are lecturers. So your obvious instincts would be S equals student and L equals lecturer. But that's not it, because S equals 6L. So, if you've got 6 students, you've one lecturer ..."

In Quinlan's study, 39% of students in Years 8 to 12 chose 'Number of students' as the correct meaning for S. A lower percentage, 25% of the TAFE students gave the correct response for part (a). This is particularly interesting in the light of the much greater success they have enjoyed on most of the other items. Twenty five per cent of the TAFE students, compared to 17 % of the school students, chose a combination of persons and numbers for their answers.

Quinlan's version of the problem used in the present test, was particularly useful in determining the extent to which students regarded algebraic letters as representing just people or numbers of people. It was found to be a very appropriate item for the present test, as this dual view of variables appeared to be prevalent among the TAFE group of students. A major reason for this is their common usage of letters to represent varying quantities in formulae, a major component of most of their engineering and science subjects. In an equation such as **'v = u + at'**, the statement 'v represents velocity' is universally understood to mean that the symbol **`v'** represents the numerical values that the velocity can take within the context of the particular situation. In responses to question 13, the more able students on the whole appreciated the distinction between a letter used as a noun or as a number, in this mathematical context. However, six of the eight students said that `S' stood for `students' or `student' or both, although most displayed no difficulty in interpreting the equation using `S' and `L' in the correct numerical sense. This dual but logical understanding of variables is further supported by the low incidence of the reversal error. This is an area which could be investigated further. Some of the students' comments illustrate this dual understanding quite well.

Joseph:

> "I think I said the lecturers obviously were L and S was students. I started to think about the English equivalents and all the rest. What does the letter stand for? I thought six times <u>so many</u> lecturers. Have I circled lecturers?"

James (who included lecturer, lecturers and number of lecturers in his meaning for L):

> "...depends how you're going to word your English....I mean, R stands for resistance...V for voltage.".

**QUESTION 15**

**15 Consider the following statement:**

**"3a + 2b could represent the total number of people seated in a restaurant, some at 3 large tables (the same number at each) and some at 2 smaller tables (the same number at each)".**

**CIRCLE one of the following.**

**I agree/disagree with this statement. Give your reasons.**

This question was adapted from item 8 in Quinlan's Algebra Test 1990. The item further investigated students concept of variable as either object or number and, the degree of tolerance for lack of closure in an answer.

Joseph was representative of several TAFE students wavering between the need for what they perceived as an appropriate answer for a reality-based question, and their appreciation of an unclosed mathematical statement, e.g. **3a+2b** as an appropriate answer for an algebraic problem. Joseph had crossed out his first correct answer which contained the comment:

> "I agree with this statement, if it is desired to represent this sum, (if) this sum is appropriate."

He then wrote:

> "I disagree - need the number of a and b (substitution), (3a+2b is) not a proper representation."

Arthur simply wrote:

> "The question doesn't make sense."

When interviewed, he repeated this sentiment.

The majority of the TAFE students answered this question correctly, and appeared to find no problem with the lack of closure in the answer.

**QUESTION 16**

**16 (a) Add 4 on to n + 5**

**(b) Add 4 on to 3n**

**(c) Multiply n + 5 by 4**

This question was taken from item 9 in Quinlan's Algebra Test 1990 based on Kuchemann (1980), and investigated students' understanding of variables as specific unknowns, and operations on variables. Part (c) required the student's understanding of the structural complexity of **4 (n + 5).**

Kuchemann (1981) recorded low rates of success for all three items, especially part (c) in the CSMS research. Part (a) was correctly answered by 68% of the students, part (b) by 36%, and part (c) by 17%. Similar results were recorded by Booth (1984). Students appeared to have difficulty in writing generalised expressions to represent the arithmetic operation referred to, particularly when brackets are involved.

As had been found for all items which require operating and writing generalised algebraic expressions, the TAFE students performed extremely well. For parts (a) and (b), there were 100% correct responses. For part (c) two gave incorrect responses, but these appeared to be more the result of carelessness than lack of understanding of the expression or of the significance of brackets.

Christine originally gave as her answer: ' **n + 5 x 4'**. When given the same question later, without reference to her first answer, she wrote **4 (n + 5)**.

Interviewer:

"Explain your reasoning."

Christine:

"Multiply n + 5 by 4."

Interviewer:

"Why did you put the brackets there?"

Christine:

"Because you've got to multiply n + 5, so you've got to put brackets round them ... both of those together, add them together, and then multiply by 4."

When shown her original answer, she exclaimed,

"See what tests do to you!"

QUESTION 17

17. **This question is about $t + t$ and $t + 4$.**

   (a) **Which is larger, $t + t$ or $t + 4$? WHY?**

   (b) **When is $t + t$ larger ?**

   (c) **When is $t + 4$ larger?**

   (d) **When are they equal?**

This question was item 10 on Quinlan's Algebra Test 1990, adapted from Harper (1979). A concept of variable in its truest sense was necessary for a solution. Students needed to appreciate that the relative size of the two expressions was dependent on the value of '**t**'. School students generally found these very difficult items, with Kuchemann (1992) recording only a 6% success for a similar question. A similar question used by Coady and Pegg (1993) resulted in only 21% correct responses from the first year tertiary stsudents.

The responses from the TAFE students had some interesting features. Five of the students demonstrated a clear understanding of the fact that the relative size of the two expressions depended on the value of '**t**'. They showed a well-developed concept of variable as a generalised number. This demonstrated the higher order functioning in the interpretation of algebraic variables discussed by Coady and Pegg (1993). However, several students made it clear, either in the expression of their answers, or in the interview situation, that they had restricted the domain of '**t**' to integral values. As in previous questions, it can be hypothesised that to these students, the context of the problem, among other problems dealing with simple whole numbers, implied this restricted domain. Students comments support these ideas.

Maurice:

"Really, you could put 'any number greater than 4' (for part (b)) and 't is less than 4' (for part (c)). I was only just thinking of whole numbers, I can remember that".

Interviewer to Mark:

"In question 17 (b), you said that `**t + t**' would be larger when '**t=5**' or more ... would 't' need to be as big as 5?"

Mark:

"Well, if it was 4, it would be the same, wouldn't it?...(Pause) Oh, right, ... above 4, yes!"

Interviewer:

"So `greater than 4'. So, initially you were thinking in terms of integers?"

Mark:

"I was thinking of whole numbers, yes."

Two students found the question very confusing, in spite of dealing competently with similar questions, 13 (b) and 18. For them, the presence of '**t**' in both expressions appeared to cause the difficulty.

Rose successfully reasoned that:

"... 't' could be any thing."

But she commented further:

"I thought that (Question 17) was horrific!"

She seemed disinclined to consider it further.

Joseph attempted to explain his difficulties with the question. He had written,

"Unsure of whole area - confusing!. **'t + t`....'t + 4'** - not same end product, can't be changed, unless add things, etc."

Faced with the structural complexity of comparing the two expressions, he appeared to revert to a notion of a variable as representing a specific unknown. His answer to part (a) read in part:

"None (i.e. neither is larger), because **'t + t'** has value of 2t, **'t + 4'** is greater number."

(This reflects an apparent confusions with the surface values of 2 and 4).

"Yet, they represent different info (so I'm confused)."

He appeared to have difficulty reconciling his understanding that **'t'** as a variable could take any value, when to him, the question appeared to be asking for specific values.

QUESTION 18

18. **This question concerns the length of the two poles shown in the sketch.**

**b metres** **a metres**

(a) CIRCLE YOUR CHOICE OF ANSWER.

(i) **The red pole is longer than the green pole.**

(ii) **The green pole is longer than the red pole.**

(iii) **They are equal in length.**

(iv) **Any of the above is possible.**

**Give reasons for your answer.**

(b) **When would the green pole be longer than the red pole?**

(c) **When would the red pole be longer than the green pole?**

(d) **When are they equal in length?**

This question was devised by Harper (1979), and used by Quinlan in his Algebra Test 1990. It was designed "to find out the degree to which students allowed variation in a geometrical setting, by using the variable notion to organise perception" (Quinlan, 1992, p. 125). In his research, Harper recorded performance improvement with year levels, from 8.3% correct in Year 1 to 83.3% in Year 6 (equivalent to an Australian Year 12). Overall, Harper recorded 33% of the students gave a correct algebraic interpretation, while Quinlan's (1992) results were 36% correct answers.

The results for the TAFE students were dramatically different, with all but one student giving a correct algebraic interpretation. These students clearly transcended any ordering suggested by the visual appearance of the figures, and did not attempt to measure the lines to obtain numerical substitutes for the letters. This success appears to further support the notion that these mature students have considerable skill in understanding and dealing with abstract concepts, where the context provides ground-to-figure relationship in the sense discussed by Marton (1988). In this item the real-life context does not appear to restrict the students' ability to reason abstractly, but appears to enable them to maintain consistency in their applications.

**QUESTION 19**

**19. For an industrial site visit, 3 buses take f students each and 4 cars take g students each.**

**(a) CIRCLE the ONE that best says what the value of 3f tells us:**

**(i) 3 buses x f students**

**(ii) How many students took buses.**

**(iii) That there is the same number of students on each bus.**

**(iv) Three buses, f students.**

**(v) The number of buses which take the students.**

**(b) Give the total number of students taken by these buses and cars.**

**(c) One car leaves early with g students. How many students remain?**

This question was adapted from item 14 in Quinlan's Algebra Test 1990. He created the item to "obtain information on students' ability to interpret the meanings of letters when they referred to a real-life context and to carry out operations on numerical variables without knowing their values" (Quinlan, 1992, p. 134). His results for 1990 indicated the three most popular choices were option (i) 41.4%, (ii) 20% , while 19% chose option (iv).

The TAFE students results for question 19 (a) were quite atypical. Three of the most able students chose (i) '3 buses x f students'. Only two made the most favoured choice, (ii) `How many students took buses', and three chose (iv) `three buses, f students'. In light of excellent performances on other more difficult items by the students making the choice of (i), the validity of the question in actually measuring the students' ability to interpret the meaning of letters in a real-life context is questionable. All of the students in question were engineering students, who in many related subjects make use of algebraic letters to represent varying quantities such as current, velocity, magnetic flux or power, for use in formulae. The format of question 19 (a) part (i) '3 buses x f students', would be far more familiar and meaningful to these students in an engineering context, in spite of the "fine technicality" mentioned by Quinlan: "..to be correct it would have had to be written as "3 buses x f students per bus".(1992, p. 134). The naive wording of (b) `How many students took buses', was probably a greater distracter to these mature-age students.

In Quinlan's Algebra Test 1990, student results for question 19 parts (b) and (c) were 52% and 41% correct respectively. The TAFE students performed more successfully on part (b), with 63% correct, but only 38% correct for part (c). Those who were wrong in both parts were generally the weaker students who gave incorrect algebraic expressions.

No students gave simple numerical answers or omitted the question altogether. The lower rate of success on this item by the TAFE students may have been due to the poor structure of the question, as discussed above.

QUESTION 20

**20 Decide whether the following statements are TRUE always, never or sometimes.**

**Tick the correct answer.**

**If you tick "true only when...", write when it is true.**

**All the letters stand for whole numbers or zero. (0,1,2,3,4,..)**

| | | |
|---|---|---|
| (i) | $a + b + c = a + x + c$ | **True always** |
| | | **Never true.** |
| | | **True only when** |
| (ii) | $2a + 3b + 7 = 5a + 7$ | **(options repeated as for (i))** |
| **(iii)** | $2a = a + 2$ | **(options repeated as for (i))** |
| **(iv)** | $a + 2b + 2c = a + 2b + 4c$ | **(options repeated as for (i))** |

Question 20 was adapted from Item 15 in Quinlan's Algebra Test 1990, based on the related question in Collis (1975). The items were "considered to provide valid measures of the students' ability to work with the concept of numerical variables.... They proved most suitable for providing data for investigating variations in the levels of difficulty across different algebraic tasks" (Quinlan, 1992, p.137). Each part required the student to work with an algebraic letter as representing a true variable, i.e. to see it as representing a range of unspecified values. The items progressed in structural complexity from (i) to (iv).

Research generally had shown that all students found items of this type particularly difficult. For part (i) 'a + b + c = a + x + c', Collis in 1974 found only 25% success (Collis, 1975, quoted in Quinlan & Collis, 1990, p. 435). Again, Kuchemann (1981) found only 25% of students identified the correct response. For the corresponding item in the SEMS study, Booth (1984) found only a 23% correct response. In Quinlan's (1992) research, 40% of students gave a correct response, and in Coady and Pegg's research, there was a 50% correct response. The TAFE students results were significantly better; 63% of this group answered this item correctly.

For parts of the item which have been researched, results indicate a similar trend of decreasing success for parts (iii), (ii) and (iv) (Collis, 1975). Quinlan reported in his study, successful results of 36% for part (iii), 20 % for part (ii) and only 4% for part (iv). The results for the TAFE students show a similar trend, but at a greater level of success for each part of the item; 50% of this group answered part (iii) correctly, 25% part (ii) and again a much lower percentage, 25% gave a correct response for part (iv). Because of the small size of the TAFE group, not too much reliance can be given to these specific percentages, but considered in total, they do appear to indicate a greater degree of success for this group.

The overall low success for these particular questions is partially explained by the fact that to successfully answer them, students need a true appreciation of a variable as a generalised number. In addition, parts (ii), (iii) and (iv) require the recognition of a systematic relationship between the sets of possible values. This level of abstraction and structural complexity appear to be beyond all but the most able students.

The greater success of the TAFE students on every part of the question reflects again their apparent greater facility with the notion of variables as generalised numbers, and with interpreting relationships between variables. These mature-age students have had considerable experience both in the workplace, and in their present studies, of working with variables in an applied mathematics context, with formulae, where the letters stand for the values of such measured quantities as power, work, current and acceleration. This exposure to real-world referents for variables enables these students to interpret relationships between variables in a more logical and meaningful way. This relates to the figure-ground relationship developed by Marton (1988). It is an area needing further investigation.

Although all the TAFE students in the research were generally successful in the use and manipulation of algebraic expressions, for several of the less able students among the group, the real-world context appeared to limit their ability to generalise their concept of variable. Although such students seemed to be limited to a notion of a variable as standing for a specific object, their actual concept of variable appeared to have the more subtle dual nature discussed, above in the analysis of question 14. For instance, for such students, the interpretation of V as voltage does not prevent them from interpreting a formula involving V in its true mathematical sense. The statement 'V represents voltage' includes the notion that in an equation involving V, the possible numerical values of the voltage within the context of the situation are understood. In a formula involving V (voltage in volts) and I (current in amperes), to say that V = I is a practical nonsense. They may have the same numerical value, but could not conceptually said to be equal. An additional complication is the common use of **'a, b and c'** to represent fixed parameters in generalised equations such as the general quadratic equation:

$$\mathbf{'ax^2 + bx + c'}.$$

These conflicting interpretations are exemplified in comments made by some of the students, although they had difficulty in expressing their ideas.

Interviewer:

> "Number 20, part (ii) when is the statement '2a + 3b + 7 = 5a + 7' true? You felt that it could never be true?"

Arthur:

> "Well, the way I looked at this one, you couldn't add these, because they were different ... They could stand for numbers but they could have a different number."

Interviewer:

> "Could they stand for more than one number?"

Arthur:

> "Yes,...or a multiple of..., or a <u>different formula</u>, or whatever."

Interviewer:

> "... so when you first see letters, what do you see them standing for?"

Arthur:

> "Something different, or the same letter would have been used."

Similarly, in the interview with Maurice, the interviewer pursued Maurice's ideas about 'a' and 'b'.

Interviewer:

"...so, 'a' only stands for one number?"

Maurice:

"Well, the way I understand it, when you write 'a' and 'b', the only way that (2a+3b+7=5a+7) would be true is if 'a' and `b' equalled the same number, and I didn't think they would..."

Interviewer:

"They actually only stand for a single number?"

Maurice:

"It doesn't have to be a single number,...could be any number."

James explains this idea more effectively.

James:

"I guess if they're (variables) in simultaneous equations, or you're plotting points for x and y, they can be the same. Normally I assume letters don't stand for the same number."

The results discussed above show TAFE student performances on virtually all the algebra questions were notably better than those of school students recorded in the research. These mature-age students appeared to be able to handle algebraic concepts and notation in a consistently logical manner, and usually avoided the common types of errors associated with limited and superficial notions of algebraic coventions. The relation of this mathematical competency to their work and life experiences is discussed in the next chapter. The important implications of the facilitation of learning by such a real-world knowledge base, for the teaching and learning of mathematics to students of all ages, is also explored.

# CHAPTER 5

# GENERAL CONCLUSIONS AND IMPLICATIONS OF THE RESEARCH

## 5.1 Introduction

The results from all the components of the research point to several conclusions about the common characteristics displayed by the group of TAFE students who took part in the research, as adult learners returning to the study of mathematics. The possible implications of these shared features, both for adult learners generally, and the for the teaching of mathematics at any age or level are discussed below.

The background material aided in determining the factors identified by the students as contributing to their present successful study of mathematics. Other significant factors in their past and present mathematical learning are discussed.

Although results from the Betts QMI test and the Space test appeared inconclusive in differentiating spatial abilities and perceptions, the results from the Operations Test linked very usefully to the neo-Piagetian stages developed by Collis (1975). The implications of this for the students' successful handling of algebraic concepts are discussed.

The major discussion concerns the conclusions drawn from the results of the algebra test, and the interviews on items from the test. These conclusions have implications for the theory of cognitive development of adults, particularly in terms of the cognitive strategies employed in their understanding and facility with algebraic concepts. The results support the notion of a "bottom-up" facilitation of the learning of mathematics, with both ikonic and concrete-symbolic modes augmenting the higher order learning in the formal mode for these mature-age students. The two lower modes of functioning appear to offer a rich knowledge base grounded in real-world referents, which enable the student to develop consistent and logical approaches in dealing with algebraic concepts. The able student then moves confidently into the realm of abstract entities and systematic relationships.

The small size of the research group necessarily limits the degree to which the results of the research could be said to apply more generally. Research on a much wider scale into some of the proposed hypotheses would be needed to validate the conclusions drawn.

## 5.2 Results and discussion - background questionnaire

In the background investigation, the students identified several common factors to which they attributed their current success in their mathematics studies at TAFE. The students all evinced a very strong commitment to their study of the subject, in terms of time given and efforts made. The central importance of mathematics in the successful mastery of other subjects in their chosen field gave mathematics immediate relevance for them. Many had made considerable financial sacrifice to undertake their present studies, and were very determined to succeed. The literature supported these factors as crucial to successful learning. Biggs and Telfer (1987) described mature-age students as being characterised by deep approaches to learning. When this was combined with the achieving approach displayed by many of these students, optimal learning resulted.

Another significant factor emerged from the background material. Half of the students involved in the research had experienced some major interruptions or impediment to continuing their studies at school. A deep-seated need to be involved in more satisfying and stimulating work provided a further strong motive for success. These students appeared to have reached a 'critical' stage in their lives and had re-evaluated their career choices, as described in the andragogical literature. Such life situations are seen as prompting task-centred and intrinsically motivated learning (Knowles, 1990).

## 5.3 Results and discussion - Operations Test, Space Test and QMI

The ACER Maths Profile Series [MAPS] Operations Test proposed to give a measure of students' abilities to handle mathematical operations across the real number system, extending to simple operations on algebraic letters. The MAPS score from the test was linked to the neo-Piagetian levels developed by Collis (1975), ranging from early concrete-operational stage to late formal-operational stage, as discussed previously. All the TAFE students in the research group attained MAPS ability scores from the Operations Test of between 62 and 72. In the Teacher's Handbook for the Operations Test, MAPS ability scores of between 60 to 70 are linked to students operating in the late formal-operational stage. By this stage, it is proposed, rules and relationships are so familiar that they have become part of the student's reality. They can be operated upon to produce any logical transformations (Cornish & Wines, 1978a). Biggs and Collis describe the formal stage as characterised by purely abstract thought (1982). All of the students involved, however, completed their mathematics schooling, either in Grade 9 or Grade 10. From Collis' (1975) data approximate age norms have been calculated for expected levels of ability. This placed Year 10 students as most likely to operate at the concrete-generalisation stage. Even allowing for the approximate nature of such generalisations and their restriction to the observed outcomes in school-based tasks, the results in the Operations test appear to indicate a major advance for most of these students in the level of mathematical operations since their school mathematical experiences.

The ACER Maths Profile Series Space Test IV proposed to measure students' perceptions of spatial and geometrical concepts. The results of the test indicated that all students operated at a much lower level of mathematical operations on this test when compared with their performances on the Operations test. However, there was a very significant correlation with the Operations Test ($r = 0.965$ $p < .001$). Several reasons can be hypothesised for this combination of results. The very high correlation with the Operations Test results suggested that the Space Test primarily measured the same cognitive abilities as the Operations Test, rather than differentiating spatial and visualising abilities. The much lower MAPS ability recorded for every student may be due to the nature of the test itself. It is almost twenty years old, and syllabii, language and approaches to geometrical and spatial topics may have changed sufficiently for the test to be an inappropriate measure of spatial abilities.

Students' ability to form direct and rich mental images was measured by Betts Questionnaire on Mental Imagery [QMI]. The results from the QMI revealed no real differences between the "trade" and "non-trade" groups, in the ability to form direct visual images . No correlation ($r = 0.232$, ns) was found for the results from the Betts QMI, and the ACER Maths Profile Series, Space Test IV. This probably reflected the fact, noted above, that the Space Test appeared to be more a measure of mathematical operational abilities, than spatial abilities.

A major aim of this research was to attempt to discover the reasons for the high level of mathematical competence displayed by these mature age students, and reflected in their performances on the Operations Test and the algebra test. As discussed in Chapter 3, maturity and life experience may have contributed to the higher degree of metacognitive activity displayed by these students. In addition, their work experiences may have been a significant factor; this is discussed in the next section. Finally, in the majority of the subjects which form their current courses, these students have broad experience of applied mathematics, and this may have significantly contributed to the development of their mathematical expertise. This idea is also explored further in the next section. However, attempts to identify differentiated levels of visualising skills resulting from work experiences, by means of the QMI and Space Test were not successful, for the reasons outlined above.

## 5.4 Results and discussion - Algebra test

Overall, the TAFE students performed significantly better than any group of students mentioned in the various studies quoted, on almost all items in the algebra test. However, the composition of the various research groups varied, from a group of 517 students from Year 7 to Year 12 in Quinlan's (1990) study, to 50 Year 9 students in

Booth's (1984) study, and 278 first year tertiary students in Coady and Pegg's (1993) study. Comparing results across these groups is not entirely valid, in terms of age, mathematical background and experience. However, the comparisons were useful in identifying cognitive obstacles common to all age and year groups, and then investigating the TAFE students' approaches to and success in overcoming such obstacles. In addition, as already mentioned, the TAFE students' background details revealed that these students had varying success in their high school mathematics, with the majority studying mathematics at least to Grade 10 Level II in the old system. In terms of school mathematics experience, this put them on a par with a large number of the students involved in the other research projects.

From the results of the Algebra test, two main components of the TAFE students' algebraic skills could be identified. All the students displayed considerable ability in dealing with the expression, interpretation and manipulation of simple algebraic expressions and notation. However, the level of interpretation of the meaning of algebraic letters divided the students into two groups. The more able group displayed no difficulty in moving from real-world referents to more abstract expressions where successful interpretation depended on the ability to use variables as true generalised numbers, and facility in establishing second-order relationships between the same or different variables. However, the less able group found difficulty in differentiating variables in the true mathematical sense, from the use of variables in the formulae of applied mathematics.

Insights into these two related components of the TAFE students algebraic skills--the high level of operational skill with algebraic expressions, and the possible dual notion of the meaning of a variable--can be explored from the theoretical perspective of the neo-Piagetian cognitive levels developed by Collis and Biggs (1982) and described in detail previously in this paper. This perspective corresponds well with a constructivist approach to learning, which also offers a framework for understanding the particular skills developed by these students.

The five broad stages of development described by Collis and Biggs (1982) have increasing levels of abstraction, with a unique mode of functioning associated with each stage. These modes are viewed as developing throughout life, and in interaction with subsequent modes. Multimodal learning is possible, with one mode facilitating and supplementing adjacent modes (Biggs & Collis, 1991; Collis & Romberg, 1991). As discussed in Chapter 2, Collis and Biggs (1991) define *bottom-up facilitation of higher order learning,* as learning in which lower levels are invoked to facilitate learning in a higher level mode. In a discussion of students' cognition tasks in assessment situations, Collis and Romberg (1991) developed an analysis of possible courses of action taken by students when faced with a mathematical problem. Their analysis of possible solution routes include movement between the ikonic mode, the basis for intuitive thinking, and the concrete symbolic mode, the basis for thinking in elementary mathematical systems. This is an example of bottom-up facilitation of learning, where the ikonic mode provides richness of experience and flexibility to the conventional logic of the concrete symbolic mode.

A parallel process may be hypothesised to account for the TAFE students successful handling of algebraic expression and notation. Evidence from the answers given in the algebra test, from interview transcripts, and from the researcher's own experience of these students over many years, all point to a fundamental concern of these students when tackling any mathematical problem. These students demand **meaning,** they want to **understand** and **make sense** of any task given. This point was well put by Richard, a mature student with a carpentry background, in the Mathematics I class. As he explained,

> "... At school you just did an exercise and that was it. You didn't do maths anywhere else.... When it means something, that's the difference.... Even in formulas, I don't understand (them), if I can't understand exactly what the symbol is (standing for). If you don't know what they stand for, you're lost.... So this is why just working on a formula, (if) you know what it means, I reckon that makes a difference."

Such an approach can be linked to the concrete or reality-based nature of both the ikonic and the concrete-symbolic modes of functioning. Both these modes of functioning clearly maintain links with real-world experience. In contrast, functioning in the formal mode, discussed in detail previously, is concerned with truly abstract elements and systems.

While several of the items in the algebra test were deliberately given a real-world context, many of the items were given in totally abstract form, with no link to reality. In particular, inconsistent language problems were posed in abstract form. These items usually produced a high percentage of reversal errors among school students.. Kuchemann allocated the items used in the CSMS algebra test to four levels. He tentatively linked these levels with the neo-Piagetian stages developed by Collis, with Level 1 algebra items requiring a response below the late concrete level, and items at the highest level, four, requiring a response at the late-formal stage (Kuchemann, 1981). Several of the Level 4 items were used in the present algebra test, in Questions 16 (c), 17, and 20. Questions 3, 4, 5, 10 and 13 (b) and (c) in the present test appeared to offer the same level of abstract difficulty.

The success of the majority of the TAFE students with most of these questions suggests that these students operate at the formal operational level with these types of mathematical tasks. This conclusion is qualified by the fact that, apart from the first year tertiary students in Coady and Pegg's sample, all test results and assignment of operational levels have applied to school students only. However, because the TAFE students began their present mathematics studies with a similar background of secondary school mathematics, and were set virtually identical items, it was felt reasonable conclusions could be drawn about their level of mathematical operation based on these results.

It is hypothesised that the TAFE students use a dual bottom-up facilitation of higher order learning, with the ikonic mode augmenting the concrete symbolic and formal modes with flexibility and richness of background . Functioning in the formal mode is further augmented by invoking the concrete symbolic mode, which for these students involved a wealth of experience of second order symbols, with direct referents in the experienced world.

Unlike their school-age counterparts, these mature-age students, particularly those with a trade background, have had an extensive and varied experience of mathematics in the workplace. The comprehensive literature on the mathematical expertise displayed by minimally schooled adults in the workplace documents their use of intuitive processes and imagery in unconventional mathematics (Nunes, Schliemann & Carraher, 1993; Scribner, 1986; Lave, Murtaugh & de la Rocha, 1984). Such processes represent ikonic mode functioning of a highly developed level and complexity. Students comments in interviews appeared to give evidence of the use of such ikonic mode strategies in approaching and solving mathematical problems in the workplace, as in Joseph's description of expert electricians:

> "... I reckon they always had a natural feel. They might know a few things of maths, or they'd remembered the particular angle or measurement. Put it this way, <u>they'd work out a system that they know how to get things done</u>."

Joseph described his own use of such strategies in working out mathematics problems :

> "I just see a picture if I can. I just kind of picture things that I find in the trade or that I've come across, and use these things to help".

Maurice illustrated the difficulty of communicating the insights arising from the ikonic mode:

> " ... I **know** what I'm thinking. I **know how** to do it. I'm able to get the answer, but expressing it on paper is a totally different thing."

James described an 'Aha' experience:

> "I couldn't do transpositions ... I was doing an electrical contractors course, and we got these huge big formulas to transpose ... I just asked a friend next to me how to do it, and just watched him, and, then all of a sudden, I could do anything, no matter how difficult ... I just suddenly got it .... I can just do any hard formula whatsoever."

Biggs and Collis (1991) describe the major task in primary and secondary schooling as mastery of the symbol systems of written language and signs, including, "writing, mathematical symbol systems, maps, musical notation, and other symbolic devices" (p. 63). Learning in school then, centres on the concrete-symbolic mode. This can lead to a disembodied and depersonalised approach to much of school learning. "When students perceive school learning as unrelated to their personal goals, ... they tend to adopt a 'surface approach' to learning" (Biggs & Collis, 1991, p. 69, quoting Biggs, 1987; Marton & Saljo, 1975). Surface learning in this sense is characterised by rote learning, the memorisation of unrelated facts, and procedural learning which is tightly linked to surface features. Such learning in the concrete symbolic mode only, without the support and enrichment of the ikonic mode, may lead to an impoverished and superficial understanding of the learning material. Many of the common mistakes made by school students on items from the algebra test appear to stem from such a unimodal approach.

Some explanation of the TAFE students' success with the same items can be explained in terms of their rich and varied experience in mathematics with a real-world base. This came first from their work experience, where the numbers and formulae used referred to real quantities, and the relationships between these quantities were not just mathematical equations, but physical realities they had experienced. In addition, these students were studying a number of other subjects involving considerable use of formulae to express relationships between physical quantities such as velocity, current, or magnetic flux. As a result, these students have had extensive experience in using their mathematics in meaningful situations. This rich context-based experience appears to have assisted these students to work consistently and logically with mathematical entities such as algebraic letters and expressions in more abstract contexts.

For the less able students, the real-world referent may inhibit their ability to understand and manipulate variables in more abstract and structurally complex situations, as the results of the algebra test showed. This limited transference has been noted and discussed in the literature concerning everyday mathematical competence (Carraher, 1985; Collis & Romberg, 1991). As discussed in Chapter 2, use of the ikonic mode can be associated with inflexibility in mathematical thinking, if specific concrete images are not transcended by more abstract image schema (Campbell, Collis & Watson, 1995, citing Bishop, 1989).

Students taking part in the research appeared to relate their use of algebraic letters in mathematical situations, to the use of letters to represent varying physical quantities such as temperature and mass in physical formulae. In this context, as already discussed, 'm represents mass' is quite an acceptable statement. The students knew that where 'm' occurs in an equation, the numerical value of the mass in kilograms or some other unit, was implied, and the two uses of the letters were neither confusing nor misleading in this practical context. The more able students appeared to be able to use their background experience to facilitate a well-developed understanding and use of algebraic variables with such concrete referents, but they were then able to successfully transcend this concrete base, and interpret and extend their concept of variable in a purely abstract sense.

Biggs and Collis (1992) describe thinking in the formal mode as both incorporating and transcending specific circumstances. They suggest that this mode begins to appear, in some individuals, from around 14 years of age. Formal responses "transcend the tangible, ... (deal with) theoretical constructs, physical laws, and unobservables" (p.72).

The transition from concrete-symbolic to formal mode is described by Sfard and Linchevski in terms of the ability move flexibly from the perspective of viewing mathematical concepts as objects or operationally, to viewing the same concepts as processes or structurally. Sweller (1991), Silver (1987), and Schoenfeld (1987) expressed the same ideas in describing expertise in mathematics. Such expertise they see as <u>derived from extensive experience with examples over an extended period of time, leading to the acquisition of fully automated rules and a comprehensive</u>

knowledge base. Hoyer (1989) in fact described age related cognitive growth in terms of these processes. Such references appear to support the notion of extensive experience in work-related and applied mathematics leading to highly developed ikonic and concrete-symbolic modes of functioning which then enrich and facilitate learning in both the concrete-symbolic and formal modes. This was demonstrated in the current study, where the TAFE students were uniformly more successful than their school age counterparts in the other studies, on the abstract items, Questions 3, 4, 5, 10,13 (b) and (c), and Questions 16 (c), 17, and 20 in the algebra test. Mick, an articulate mature-age student in the Applied Calculus class, expressed his ideas about his own mathematical competency:

> "If you are taught a method, you can really only do it the way of the method until you realise that there is a concept behind it. If you know the concepts, you can apply your own form of how you do maths and algebra to the problems ... Leaps of understanding--due more to experience, trade experience.... When I look at a problem, I try to relate it to the real world."

Much of the writing about mathematical learning from the constructivist perspective centres on the fundamental concept of 'meaningful learning'. The viewpoint of Hiebert and Lefevre summarised earlier in the paper, stated "Procedures that are linked with meaning are linked to conceptual knowledge", where procedures referred to symbol manipulations or strategies, and conceptual knowledge referred to "knowledge that is rich in relationships" (1987, p. 8). Mick's comments seem to exemplify these ideas in a very real sense, and typify the reality-based mathematical mastery demonstrated by the TAFE students.

## 5.5 Implications and further research

These conclusions have implications both for mature-age and school-age students. Full advantage needs to be taken of the potential of the rich and varied background of mature-age students, for enhancing and facilitating their learning of mathematics. For such students who may be tackling the more formal mathematics involved in engineering and science courses, preliminary mathematics textbooks would facilitate the learning of mathematics if real-world referents were used much more extensively in exercises and examples throughout the text, not, as is often the case, just as extension exercises in applied mathematical concepts towards the end of each chapter.

The results of this research would suggest if the mathematics teacher of such students can relate the mathematics they are doing to the students' past experiences, and place their mathematics learning in a meaningful context, the learning taking place will be greatly enhanced.

When school students' difficulties with the more abstract algebraic expressions and use of variables are considered, the results of this research would indicate that their understanding could be facilitated by much more extensive use of real-world referents for the mathematics they are learning, even in the initial stages. Such an approach has nothing to do with the `fruit salad' approach of teaching the meaning for algebraic letters with 'a' standing for apples, and 'b' standing for bananas. The real-world referents chosen could be familiar **varying** quantities, such as speed, distance or mass. Students usually already have reasonably consistent and logical notions about the way in which such quantities vary. If these can be carefully linked to and expressed in algebraic notation, the students' understanding of such notation can be greatly facilitated by this connection to a familiar and consistent framework. These ideas are supported by Resnick (1992). She points out that "School instruction tends to aggravate the tendency to separate the formal notation of mathematics from its <u>external referents</u>." She goes on to suggest that even at the primary school level, "instruction that develops a fundamental attitude towards arithmetic as grounded in meaningful relationships is needed" (pp. 393, 394).

The most fundamental conclusion drawn from the research concerns the theoretical implication of the cognitive strategies employed by mature-age students in their learning of mathematics. The conclusions suggest that these adult learners make use of the lower order ikonic mode to facilitate learning in the concrete symbolic mode; and of both the ikonic and concrete symbolic modes, to facilitate learning in the formal mode. These are areas deserving a great deal more research. The greater the understanding of the cognitive mechanisms employed by adult learners, particularly with respect to mathematics, the more effectively teaching strategies can be developed to capitalise on the potential for learning, if these natural strategies are utilised. Further research into the differences between mature-age and school-age students in cognitive mechanisms utilised in the learning of mathematics, would provide insights of benefit to both groups.

While the aims to find an association between known tests of visual and spatial ability, and link this with background trade experience, were not productive with the small sample in this study, the analysis of the algebra test results, and the subsequent discussion on test items, did support the developmental models of multi-modal functioning. In particular, the research appeared to demonstrate the use by the group of work-based and real-life referents to facilitate and enrich competence with algebraic notations and manipulations.

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## APPENDIX A

## ADULT LEARNERS OF MATHEMATICS

## BACKGROUND QUESTIONNAIRE
(Confidential)

Name:.........................................................................................................

Address.......................................................................................................

Year of Birth............................

Maths Background:
Last High School/Secondary College attended

Highest Grade achieved at school

Last year at school 19...........

Highest level of Maths achieved ( e.g. Grade 10 Level II, Grade 11 Maths Applied).

What did you feel about Maths in High School/College?

How did you go?

How much algebra did you study?

What were your feelings about algebra at the time?

[redacted]

Have you undertaken any further studies since leaving school?

Has any of your further studies involved maths in particular?

Could you describe any work experience or jobs you have held since leaving school .

Could you describe any maths you have used "on the job". (dealing with accounts, measuring volume, weight, use of charts and maps, etc.)

In your present study of mathematics, what parts of your past work experience have helped you most?

Would you like to add any further comments?

**APPENDIX B**

INTERVIEW PROTOCOL

Preliminary

A Welcome P (participant) and thank for making time available.

B Briefly explain procedure, and use of tape to ensure accurate record. Assure P. of confidentiality and anonymity in any published results.

C Explain purpose of interview:
"I am interested in all of the factors that have contributed to you as an adult learner, especially in relation to your learning of mathematics."

INTERVIEW

1) "Could you tell me about your primary schooling?"

PROBE 1: Where? How big?
PROBE 2: Feelings about primary school?
PROBE 3: Feelings about mathematics at primary school?

2) "Could you tell me about your secondary schooling?"

PROBE 1: Where? How big?
PROBE 2: Feelings about secondary school years?

3) "How did you go in maths in secondary school?"

3 PROBE 1: Final level and grade?
3 PROBE 2: Were classes streamed?
3 PROBE 3: Feelings about streaming?
3 PROBE 4: Feelings about maths generally at secondary school?
3 PROBE 5: Feelings about maths teachers?
3 PROBE 6: Feelings of your friends about mathetics?
3 PROBE 7: What parts of mathematics seemed the most difficult to you?
3 PROBE 8: What things stopped you from doing really well in maths?
3 PROBE 9: What happened in your maths lessons ?
3 PROBE 10: What do you think about the value of the maths you learned at school?

4) "Could you tell me about the algebra you studied at secondary school?"

4 PROBE 1: Feelings about algebra?
4 PROBE 2: Understanding?
4 PROBE 3: How do you remember it being taught?
4 PROBE 4: Did anything in particular help you to understand algebra?
4 PROBE 5: Why did you think you learnt algebra?

5) "How did you feel about leaving school?"

5 PROBE 1: How old?
5 PROBE 2: Any intention to go on with schooling?
5 PROBE 3: Did many of your friends go on?
5 PROBE 4: Parents' feelings about your leaving school?
5 PROBE 5: Would you do things differently now, looking back?

6) "Could you tell me about your career since you left school?"
PROBE 1: Types of jobs?
PROBE 2: Job satisfaction?
PROBE 3:

7) "What sort of maths did you use on the job?"

7 PROBE 1: Measuring, weighing, estimating, dealing with money, surveying, scaling, timing. etc.?
7 PROBE 2: How did job experience increase maths abilities?
7 PROBE 3: What sort of maths was most needed on the job?

8) "Have you done any other study of mathematics since leaving school?"

PROBE 1: Maths-related subjects?
PROBE 2: Hobbies involving maths?
PROBE 3:

9) "Why have you come back to study?"

9 PROBE 1: Better job? Retrenchment? Financial reasons? Family commitments? Pressure from family/partner? Boredom with present job? Feel you can do much better?

10) "In your study of mathematics now, what do you think has helped you most from your past experiences?"

10 PROBE 1: Meaning to maths?
10 PROBE 2: Lots of real mathematics on the job?
10 PROBE 3: See more clearly what you want, want to study i.e. maturity

APPENDIX C

ACER MATHEMATICS PROFILE SERIES

# OPERATIONS TEST

## Test Booklet

**Directions**

This test booklet contains a total of 60 questions covering a range of mathematical operations met in primary and secondary schools. Students are not expected to do all 60 questions at the one time – your teacher will tell you which ones you are to do.

Each question is a mathematical sentence in which one of the terms has been replaced by △. A question is followed by four alternative answers, labelled **A**, **B**, **C**, and **D**. You should choose the alternative to replace the △ and make the sentence true.

The following practice questions will show you how to answer the questions in the test.

Wait until you are told how to answer the questions before going on.

**Practice Questions**

**P1** $6 + 10 = 6 + \triangle$

**A** $-16$ **C** $10$

**B** $-10$ **D** $16$

**P2** $5 \times 17 = 5 \times \triangle$

**A** $\frac{17}{5}$ **C** $17$

**B** $5$ **D** $85$

**Please do not make any marks in this booklet.**

**BACKGROUND INFORMATION FOR TEACHERS**

Test materials in the *ACER Mathematics Profile Series* are designed so that teachers may monitor students' mathematical development throughout primary and secondary schooling. This is achieved by converting raw scores on any of the tests to a common scale called the MAPS scale. Conversion tables for this test are incorporated in the *ACER Mathematics Profile Series —Operations Test Teachers Handbook*. The handbook discusses interpretative procedures concerning the likely mastery of all items on the MAPS scale. These interpretations enable teachers to identify a range of suitable learning experiences, in relation to each student.

To assist in the calculation and use of mastery levels the *answer sheet* has a specially designed student record section and the score key displays a *mastery profile 'cursor'*.

In general, different groups of items in the *Operations* test will be selected for administration, depending on the particular class. The handbook recommends various 30- and 40-item tests and suggests their appropriate year levels. The suggested testing times are

- about 30 minutes for a 30-item test
- about 40 minutes for a 40-item test

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The Australian Council for Educational Research Limited
Frederick Street Hawthorn Victoria 3122

1 $3 + 4 = 4 + \triangle$

A 7 C 3

B 5 D $-7$

2 $7 \times 8 = 8 \times \triangle$

A 56 C $\frac{7}{8}$

B 7 D $\frac{1}{7}$

3 $15 \div 3 = \triangle \div 3$

A 15 C 3

B 5 D $\frac{5}{3}$

4 $6 \times 1 = \triangle$

A 61 C 6

B 7 D 1

5 $(3 \times 2) \times 5 = \triangle \times (2 \times 5)$

A 15 C 3

B 6 D $\frac{1}{3}$

6 $5 - 2 = \triangle - 2$

A 5 C 1

B 3 D $-5$

7 $5 + 0 = \triangle$

A 6 C 0

B 5 D $-5$

8 $(5 + 4) + 6 = \triangle + (4 + 6)$

A 15 C 6

B 9 D 5

9 $9 + 1 = \triangle$

A 91 C 9

B 10 D $-8$

10 $8 \times 0 = \triangle$

A 80 C 1

B 8 D 0

11 $(8 - 4) + 4 = \triangle$

A 12 C 4

B 8 D 0

12 $(12 \div 2) \times 2 = \triangle$

A 12 C 3

B 6 D $\frac{1}{12}$

13 $15 \div 5 = 30 \div \triangle$

A 15 C 3

B 10 D 2

4 $4 + 5 = (4 + 6) + (5 + \triangle)$

A 6 C $-1$

B 4 D $-6$

15 $7 - 4 = \triangle - 7$

A 11 C 4

B 10 D 3

16 $8 \div 4 = \triangle \div 8$

A 32 C 4

B 16 D 2

17 $(7 \times 2) - (3 \times 2) = (7 + \triangle) \times 2$

A 4 C $-3$

B 3 D $-4$

18 $(24 \div 6) \div 2 = \triangle \div (6 \div 2)$

A 24 C 6

B 12 D 4

19 $(12 - 6) - 4 = \triangle - (6 - 4)$

A 12 C 4

B 8 D 2

20 $(40 \div 8) \times 4 = (40 \times 4) \div (\triangle \times 4)$

A 32 C 4

B 8 D 2

21 $123 + 456 = 456 + \triangle$

A 579 C $-123$

B 123 D $-333$

22 $44 \times 125 = 125 \times \triangle$

A 5500 C 44

B 125 D $\frac{125}{44}$

23 $864 \div 432 = \triangle \div 432$

A 864 C 2

B 222 D $\frac{1}{2}$

24 $984 \times 1 = \triangle$

A 9841 C 984

B 985 D 1

25 $(23 \times 24) \times 25 = \triangle \times (24 \times 25)$

A $23 \times 25$ C $\frac{23}{25}$

B 23 D $\frac{1}{23}$

26 $654 - 543 = \triangle - 543$

A 6003 C 111

B 654 D $-432$

27 $876 + 0 = \triangle$

A 8760 C 876

B 877 D 0

28 $(89 + 67) + 33 = \triangle + (67 + 33)$

A 122 C $-89$

B 89 D $-156$

29 $578 + 1 = \triangle$

A 579 C 577

B 578 D 1

30 $769 \times 0 = \triangle$

A 7690 C 769

B 770 D 0

31 $(987 - 321) + 321 = \triangle$

A 987 C 345

B 666 D 321

32 $(625 \div 25) \times 25 = \triangle$

A 625 C 25

B 125 D 1

33 $240 \div 15 = 480 \div \triangle$

A 30 C $\frac{15}{2}$

B 16 D $\frac{2}{15}$

34 $123 + 456 = (123 + 789) + (456 + \triangle)$

A 789 C $-123$

B 123 D $-789$

35 $654 - 543 = \triangle - 654$

A 1197 C 543

B 765 D 111

36 $468 \div 234 = \triangle \div 468$

A 936 C 117

B 234 D $\frac{1}{234}$

37 $(72 \times 25) - (60 \times 25) = (72 + \triangle) \times 25$

A 60 C $-12$

B 12 D $-60$

38 $(900 \div 30) \div 10 = \triangle \div (30 \div 10)$

A 900 C 30

B 90 D 9

39 $(89 - 56) - 21 = \triangle - (56 - 21)$

A 110 C 68

B 89 D 47

40 $(72 \div 36) \times 9 = (72 \times 9) \div (\triangle \times 9)$

A 324 C 4

B 36 D 2

41 $p + q = q + \triangle$

A $r$ C $-p$

B $p$ D $p - 2q$

42 $r \times s = s \times \triangle$

A $t$ C $\frac{r}{s}$

B $rs$ D $r$

43 $h \div j = \triangle \div j$

A $hj^2$ C $\frac{h}{j}$

B $hj$ D $h$

44 $y \times 1 = \triangle$

A $y + 1$ C $1$

B $y$ D $\frac{1}{y}$

45 $(u \times v) \times w = \triangle \times (v \times w)$

A $u$ C $uv$

B $uw$ D $\frac{u}{w}$

46 $m - y = \triangle - y$

A $m + 2y$ C $m$

B $m + y$ D $m - y$

47 $g + 0 = \triangle$

A $h$ C $0$

B $g$ D $-g$

48 $(h + j) + k = \triangle + (j + k)$

A $h$ C $h + k$

B $k$ D $h - k$

49 $v + 1 = \triangle$

A $w$ C $v - 1$

B $v$ D $v + 1$

50 $w \times 0 = \triangle$

A $w$ C $0$

B $1$ D $w \div 0$

51 $(j - k) + k = \triangle$

A $j - 2k$
C $j - k$
B $j - k^2$
D $j$

52 $(m \div g) \times g = \triangle$

A $mg$
C $m$
B $\frac{m}{g^2}$
D $\frac{m}{2g}$

53 $s \div t = rs \div \triangle$

A $rt$
C $t$
B $\frac{rs}{t}$
D $\frac{t}{r}$

54 $p + q = (p + r) + (q + \triangle)$

A $s$
C $-r$
B $r$
D $p + q - r$

55 $f - e = \triangle - f$

A $g$
C $e$
B $e + f$
D $2f - e$

56 $k \div p = \triangle \div k$

A $kp$
C $p$
B $\frac{p}{k}$
D $\frac{k^2}{p}$

57 $mp - np = (m + \triangle) \times p$

A $n$
C $-n$
B $m - n$
D $p - n$

58 $(p \div q) \div r = \triangle \div (q \div r)$

A $p$
C $pr$
B $\frac{p}{r}$
D $\frac{p}{r^2}$

59 $(u - v) - t = \triangle - (v - t)$

A $u$
C $u - 2t$
B $u - t$
D $u - v$

60 $(y \div z) \times w = (y \times w) \div (\triangle \times w)$

A $z$
C $zw$
B $\frac{z}{w}$
D $\frac{y}{z}$

ACER MATHEMATICS PROFILE SERIES

# OPERATIONS TEST

## Answer and Record Sheet

Name ..........................................................................

| MASTERY LEVELS | I | II | III | IV | |
|---|---|---|---|---|---|
| 90% | | | | | − 10 |
| 75% | | | | | − 5 |
| 50% | | | | | *MAPS ability* |
| 25% | | | | | + 5 |
| 10% | | | | | + 10 |

*Raw Score* ....................

*Scaled Error* ..................

*Practice Examples:* P1 A B C D    P2 A B C D

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | A | B | C | D | 21 | A | B | C | D | 41 | A | B | C | D |
| 2 | A | B | C | D | 22 | A | B | C | D | 42 | A | B | C | D |
| 3 | A | B | C | D | 23 | A | B | C | D | 43 | A | B | C | D |
| 4 | A | B | C | D | 24 | A | B | C | D | 44 | A | B | C | D |
| 5 | A | B | C | D | 25 | A | B | C | D | 45 | A | B | C | D |
| 6 | A | B | C | D | 26 | A | B | C | D | 46 | A | B | C | D |
| 7 | A | B | C | D | 27 | A | B | C | D | 47 | A | B | C | D |
| 8 | A | B | C | D | 28 | A | B | C | D | 48 | A | B | C | D |
| 9 | A | B | C | D | 29 | A | B | C | D | 49 | A | B | C | D |
| 10 | A | B | C | D | 30 | A | B | C | D | 50 | A | B | C | D |
| 11 | A | B | C | D | 31 | A | B | C | D | 51 | A | B | C | D |
| 12 | A | B | C | D | 32 | A | B | C | D | 52 | A | B | C | D |
| 13 | A | B | C | D | 33 | A | B | C | D | 53 | A | B | C | D |
| 14 | A | B | C | D | 34 | A | B | C | D | 54 | A | B | C | D |
| 15 | A | B | C | D | 35 | A | B | C | D | 55 | A | B | C | D |
| 16 | A | B | C | D | 36 | A | B | C | D | 56 | A | B | C | D |
| 17 | A | B | C | D | 37 | A | B | C | D | 57 | A | B | C | D |
| 18 | A | B | C | D | 38 | A | B | C | D | 58 | A | B | C | D |
| 19 | A | B | C | D | 39 | A | B | C | D | 59 | A | B | C | D |
| 20 | A | B | C | D | 40 | A | B | C | D | 60 | A | B | C | D |

ACER MATHEMATICS PROFILE SERIES

# SPACE TEST
## Unit IV

Directions
This unit contains 32 questions which cover spatial aspects of mathematics. Although some questions may not be directly related to the work you have done in class, you should try to work out the correct answer.

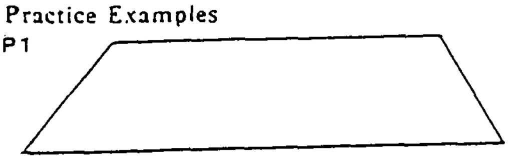
In each question the alternative answers are represented by the letters A, B, C, D, or E. Mark your chosen answer by circling the appropriate letter on the Answer and Record Sheet.

The following practice examples will show you how to answer the questions on the test.

Wait until you are told how to answer the questions before going on.

Practice Examples

P1



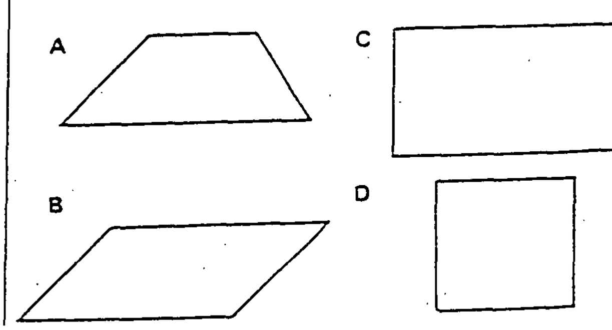
How many sides does the shape have?

A 4 C 2
B 3 D 1

Since 4 is the correct answer the corresponding letter A has been circled on the answer sheet.

P2

Which one of the following figures is a square?




**Please do not make any marks in this booklet**

BACKGROUND INFORMATION FOR TEACHERS

Test materials in the *ACER Mathematics Profile Series* are designed so that teachers may monitor students' mathematical development throughout primary and secondary schooling. This is achieved by converting raw scores on any of the tests to a common scale called the MAPS scale. Conversion tables for this test are incorporated in the *ACER Mathematics Profile Series – Space Test Teachers Handbook*. The Handbook discusses interpretive procedures concerning the likely mastery of all items on the MAPS scale. These interpretations enable teachers to identify a range of suitable learning experiences, in relation to each student.
To assist in the calculation and use of mastery levels the *Answer and Record Sheet* has a specially designed student record section and the score key displays a *mastery profile 'cursor'*.
The suggested testing time for a *Space Test Unit* is about 30 minutes.

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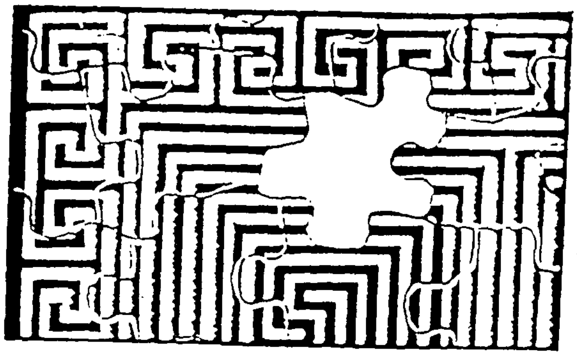
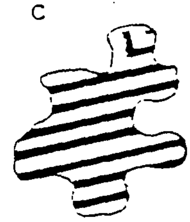
1 From this section of a jigsaw puzzle, one piece is missing:



Which one of the following pieces should it be?

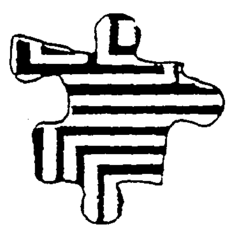
A

C



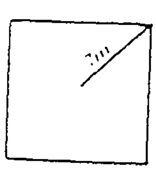
B

D

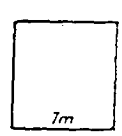


2 In each of these squares, one of the dimensions has been marked to represent 7 metres. Which one of the squares would have the greatest perimeter?

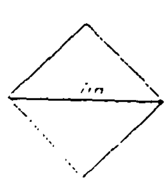
A



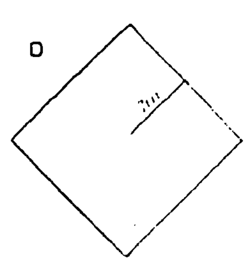

C



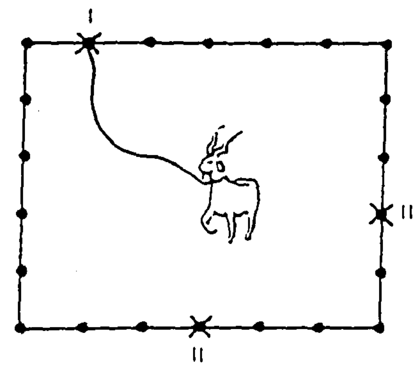

B




D



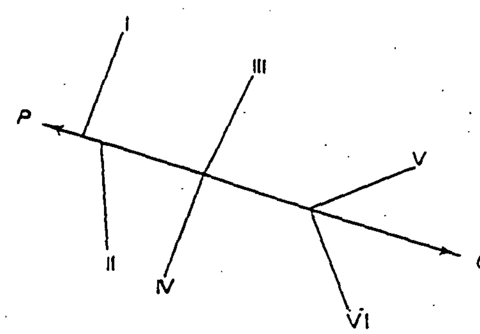

3 In this rectangular field 50 m by 60 m, there are fence posts at the corners and every 10 m along the sides. A goat is tethered with a rope 30 m long.




If the rope is attached to one of the posts I, II, or III, from which post can he graze on the greatest area?

A post I
B post II
C post III
D The area is the same from posts I, II, and III.

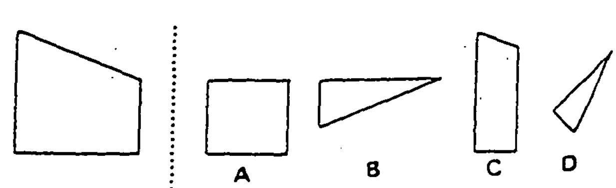
4 Which two lines in the diagram are perpendicular to the line $\overline{PQ}$?




| | |
|---|---|
| A I and IV | C III and IV |
| B II and VI | D V and VI |

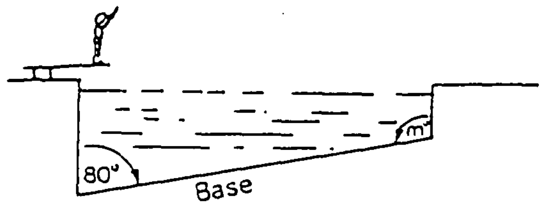
5 The shape to the left of the dotted line can be made by fitting together three of the four shapes on the right, without overlap.

Which shape is not needed?



A B C D

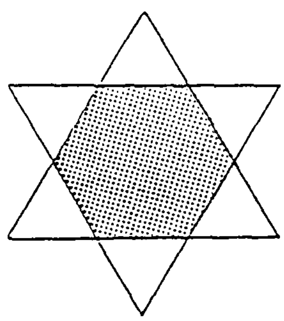
On the side where the boy is diving, the pool slants upwards at 80° with the vertical side.




What is the value of angle m?

A 80°
B 100°
C 120°
D It depends on the depth of the pool.

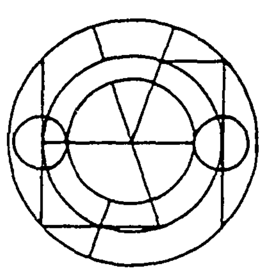
7 A six-pointed star can be made by overlapping two equilateral triangles as shown in the diagram.

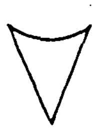


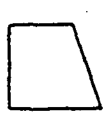
If the area of each of these triangles is 9 sq cm then the area of the star is 12 sq cm.
What is the area of the shaded hexagon?

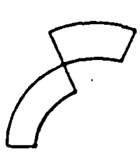
A 3 sq cm
B 6 sq cm
C 7 sq cm
D 9 sq cm

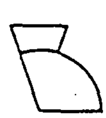
8 Which one of the following shapes appears in this pattern.



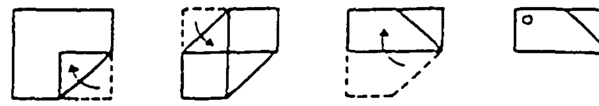
A


C


B


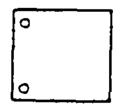
D


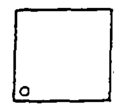
9 A square piece of paper was folded, as shown in Figures I, II, and III. Figure IV shows where a small hole was then punched through the paper.

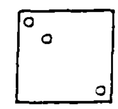


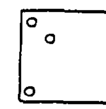
I II III IV

If the paper was completely unfolded, which one of the following shows how it would look?

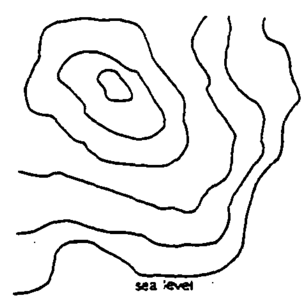

A


B


C


D

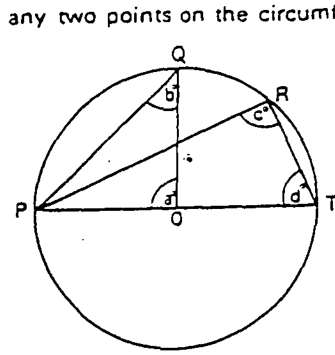
10 The map shows the contours of a hill near the coast. Contour lines are drawn joining points at each multiple of 10 metres above sea level.




What is the height of the hill above sea level?

A between 40 and 50 metres
B exactly 50 metres
C between 50 and 60 metres
D exactly 60 metres
E between 60 and 70 metres

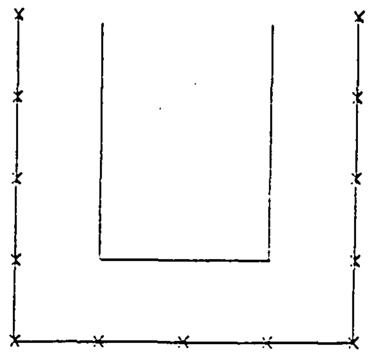
11 POT is the diameter of a circle, centre O. Q and R are any two points on the circumference.




Which one of the angles marked in the diagram must be 90°?

A a
B b
C c
D d

12 The x's form an 'open square of order 5', since the sides are equal in length and there are 5 x's spaced evenly along each side.



The inner formation is

A an open square of order 4.

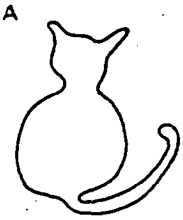
B an open square of order 3.

C an open square whose order can't be specified.

D not an open square.

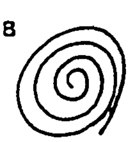
13 Which one of the following is a simple closed curve?

A

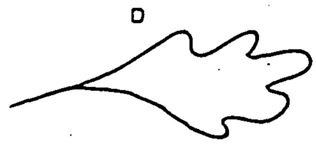


C

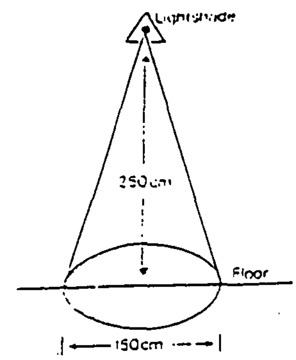
B



D



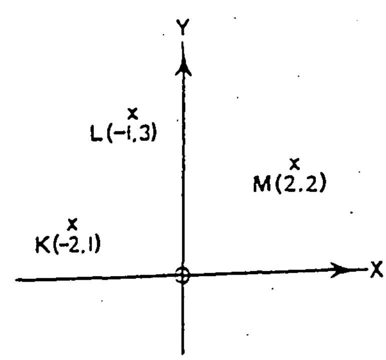
14 A light, with a conical shade, shines directly down onto the floor 250 cm below. Its beam illuminates a circular region of diameter 150 cm as shown in the diagram.




If the light is raised to 300 cm above the floor, the diameter of the circular region illuminated is

A 100 cm.
B 180 cm.
C 200 cm.
D 250 cm.
E 500 cm.

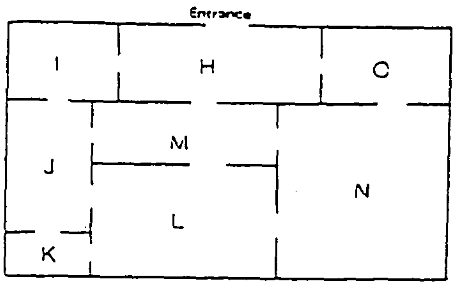
15 M is the image of L under a translation.




The image of K under the same translation will be at the point

A (−5, 2).
B (1, 0).
C (3, 4).
D (4, −3).

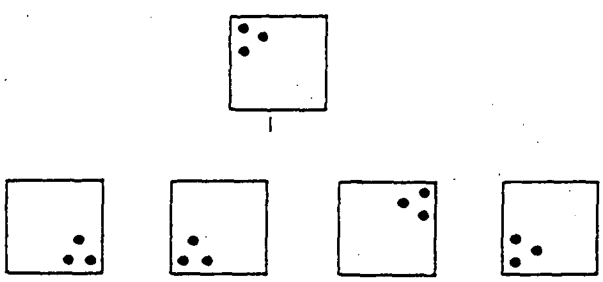
16 This diagram represents the plan of an art gallery with only one entrance. Arthur, a frequent visitor, wondered if it would be possible to walk through every internal doorway exactly once, starting in the entrance hall, H.




Arthur would find that

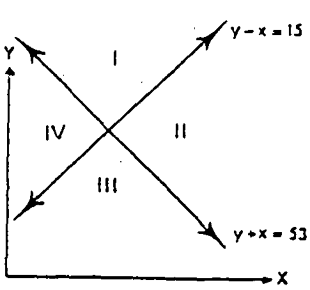
A it is possible and he finishes at the entrance hall.
B it is possible but he always finishes in room N.
C it is possible but he finishes in different rooms, depending on his route.
D it is impossible to do.

17 Which one of the following shows the same face as card I?




A B C D

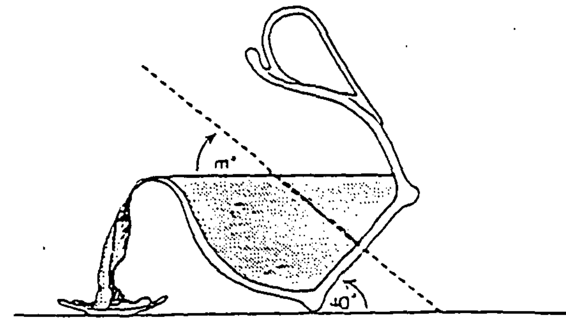
18 These lines meet at the point (19,34). Between them are four regions labelled I, II, III, and IV.




Which region contains ordered pairs (x,y) such that $y + x$ is greater than 53 and $y - x$ is less than 15?

A I
B II
C III
D IV

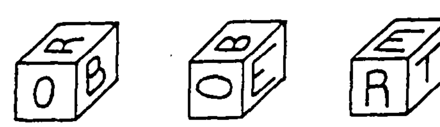
19 Sally is pouring some milk for her cat, with the base of the jug at 40° tilt from the table.




What is the angle m between the level of the milk and the axis of the jug (marked as a dotted line)?

A 140°
B 90°
C 50°
D 40°
E m° cannot be calculated because of the curved shape of the jug.

20 Robert wrote the letters of his name on the six faces of a cube. These show three different views of the cube.



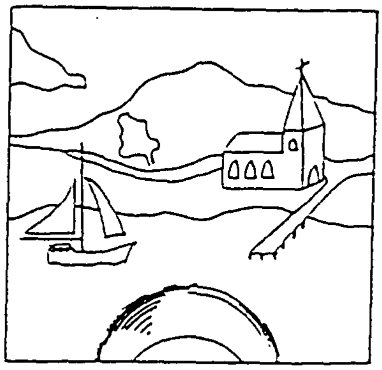

Which one of the following is the letter opposite the B on the cube?

A R
B O
C E
D T

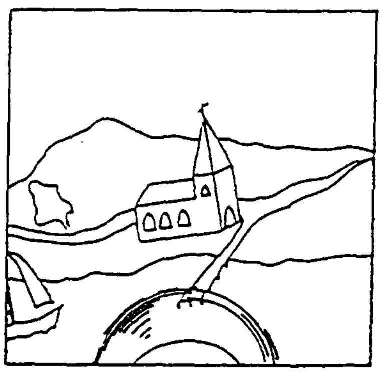
21 As a gramophone record is played, the locus of the point of the needle is best described as . .

A an arc of a circle.
B concentric circles.
C a spiral.
D a straight line.

22 These two sketches show the view from the cockpit immediately before and after an aeroplane made a manoeuvre.



Before

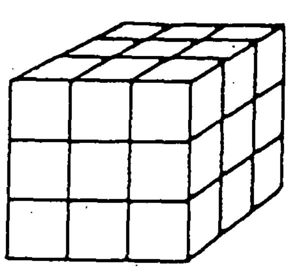


After

The aeroplane has

A turned right and dipped its right wing.
B turned left and dipped its left wing.
C turned right, climbed, and dipped its right wing.
D turned left, climbed, and dipped its left wing.

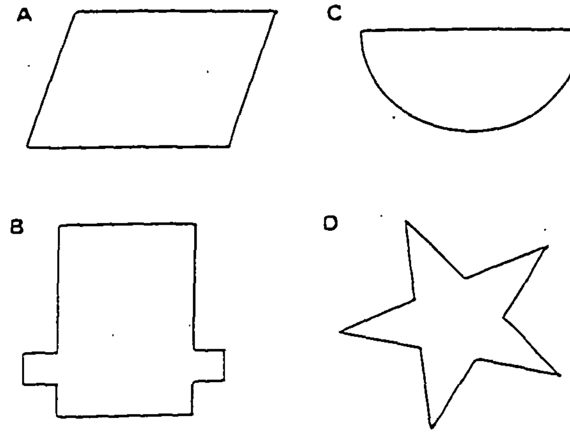
23 This pile of small cubes was placed on a desk and its exposed faces were spray painted, without the cube being disturbed.



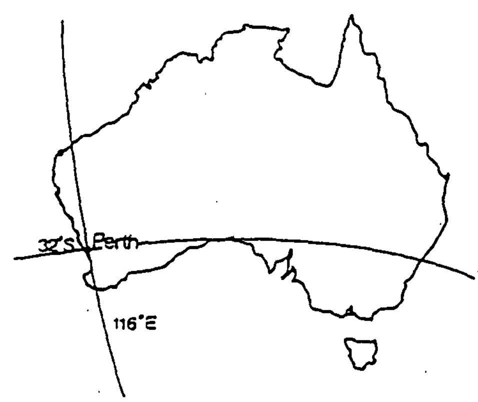
How many cubes did not get any paint on them?

A one
B two
C eight
D nine

24 Which one of the following shapes has no axes of symmetry?



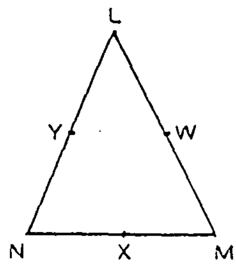

25 Perth has a latitude 32° South and a longitude 116° East. Bermuda is directly opposite Perth on the earth's spherical surface.




The latitude and longtitude of Bermuda must be

A 32°S 116°W.
B 32°N 116°W.
C 32°S 64°W.
D 32°N 64°W.

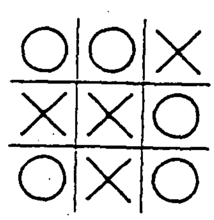
26 LMN is any triangle and W, X, and Y are midpoints of the sides, as shown in the diagram.
$L \curvearrowright Y = N$ means that N is the reflection of L in Y.




The location of $(M \curvearrowright X) \curvearrowright Y$ is

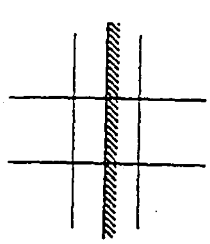
A inside the triangle LMN.
B outside the triangle LMN.
C one of the vertices L, M, or N.
D one of the midpoints W, X, or Y.
E an unmarked point of the perimeter of triangle LMN.

27 This game of noughts and crosses ended up in a draw.

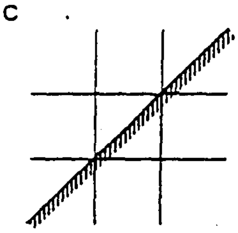


Using a mirror (the shaded lines show the reflecting surface), which one of the following positions would make it appear as though there were only 3 X's in a line?

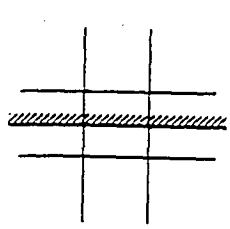
A



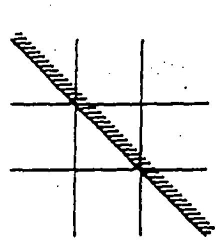
C



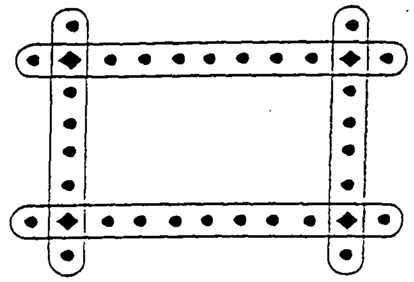
B



D



28 Four strips of a construction set form a rectangle as shown. All the screws are loose enough to vary the shape by pushing in or out on one of the corners.



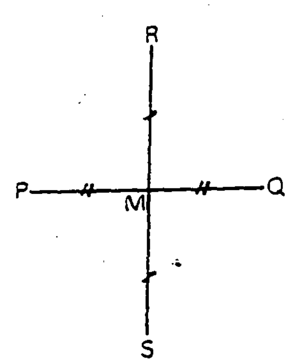
Which one of the following relationships remains or becomes true when the shape is varied?

A The diagonals are equal in length.
B The diagonals bisect each other.
C The diagonals are perpendicular to each other.
D The shape is symmetrical about both the diagonals.

29 How long is it between times when the two hands of a clock are together and when they are opposite?

A exactly six hours
B about 65 minutes
C exactly one hour
D about 33 minutes
E exactly 30 minutes

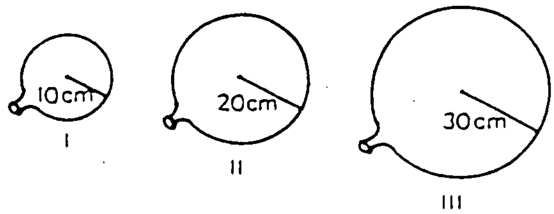
30 $\overline{PQ}$ and $\overline{RS}$ bisect each other at M, as shown in the diagram.




Which one of the following statements is always true?

A $\overline{RM}$ is perpendicular to $\overline{MQ}$.
B $\overline{PS}$ is parallel to $\overline{RQ}$.
C $\overline{PR} = \overline{RQ}$.
D All of the above are true.

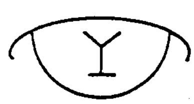
31 This diagram shows three stages of a balloon being blown up.




Compared with stage I, the volume of air needed in the balloon would be increased by what factors in stages II and III?

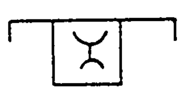
A 2 and 3
B 4 and 9
C 8 and 27
D 20 and 30

32 Which one of the following figures is not topologically equivalent to Figure I?

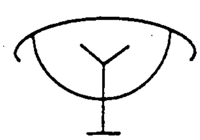


I

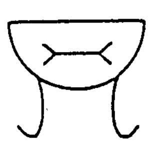
A



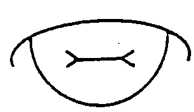
C



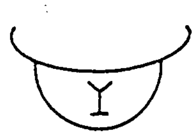
B



D



E



ACER MATHEMATICS PROFILE SERIES

## SPACE TEST

### Answer and Record Sheet

Name ................................................................

Class ..................................

Date of Testing .........../ ........../ ..........

Date of Birth ............../ ........../ ..........

Age ............year ............months

| MASTERY LEVELS | I | II | III | IV | |
|---|---|---|---|---|---|
| 90% | | | | | – 10 |
| 75% | | | | | – 5 |
| 50% | | | | | *MAPS ability* |
| 25% | | | | | + 5 |
| 10% | | | | | + 10 |

*Raw Score* ....................

*Scaled Error* ..................

*Practice Examples:* P1 A B C D E    P2 A B C D E

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | A | B | C | D | E | 17 | A | B | C | D | E |
| 2 | A | B | C | D | E | 18 | A | B | C | D | E |
| 3 | A | B | C | D | E | 19 | A | B | C | D | E |
| 4 | A | B | C | D | E | 20 | A | B | C | D | E |
| 5 | A | B | C | D | E | 21 | A | B | C | D | E |
| 6 | A | B | C | D | E | 22 | A | B | C | D | E |
| 7 | A | B | C | D | E | 23 | A | B | C | D | E |
| 8 | A | B | C | D | E | 24 | A | B | C | D | E |
| 9 | A | B | C | D | E | 25 | A | B | C | D | E |
| 10 | A | B | C | D | E | 26 | A | B | C | D | E |
| 11 | A | B | C | D | E | 27 | A | B | C | D | E |
| 12 | A | B | C | D | E | 28 | A | B | C | D | E |
| 13 | A | B | C | D | E | 29 | A | B | C | D | E |
| 14 | A | B | C | D | E | 30 | A | B | C | D | E |
| 15 | A | B | C | D | E | 31 | A | B | C | D | E |
| 16 | A | B | C | D | E | 32 | A | B | C | D | E |

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ACER 2M 4/88

## APPENDIX E

# THE BETTS QMI VIVIDNESS OF IMAGERY SCALE

*Instructions for doing test*

The aim of this test is to determine the vividness of your imagery. The items of the test will bring certain images to your mind. You are to rate the vividness of each image by reference to the accompanying rating scale, which is shown at the bottom of the page. For example, if your image is 'vague and dim' you give it a rating of 5. Record your answer in the brackets provided after each item. Just write the appropriate number after each item. Before you turn to the items on the next page, familiarize yourself with the different categories on the rating scale. Throughout the test, refer to the rating scale when judging the vividness of each image. A copy of the rating scale will be printed on each page. Please do not turn to the next page until you have completed the items on the page you are doing, and do not turn back to check on other items you have done. Complete each page before moving on to the next page. Try to do each item separately independent of how you may have done other items.

*Rating Scale*

The image aroused by an item of this test may be:

| | |
|---|---|
| Perfectly clear and as vivid as the actual experience | *Rating* 1 |
| Very clear and comparable in vividness to the actual experience | *Rating* 2 |
| Moderately clear and vivid | *Rating* 3 |
| Not clear or vivid, but recognizable | *Rating* 4 |
| Vague and dim | *Rating* 5 |
| So vague and dim as to be hardly discernible | *Rating* 6 |
| No image present at all, you only 'knowing' that you are thinking of the object | *Rating* 7. |

An example of an item on the test would be one which asked you to consider an image which comes to your mind's eye of a red apple. If your visual image was moderately clear and vivid you would check the rating scale and mark '3' in the brackets as follows:

| *Item* | *Rating* |
|---|---|
| 5. A red apple | (3) |

Think of some relative or friend whom you frequently see, considering carefully the picture that rises before your mind's eye. Classify the images suggested by each of the following questions as indicated by the degrees of clearness and vividness specified on the Rating Scale.

| *Item* | *Rating* |
|---|---|
| 1. The exact contour of face, head, shoulders and body | (   ) |
| 2. Characteristic poses of head, attitudes of body, etc. | (   ) |
| 3. The precise carriage, length of step, etc. in walking | (   ) |
| 4. The different colours worn in some familiar costume | (   ) |

Think of seeing the following, considering carefully the picture which comes before your mind's eye; and classify the image suggested by the following question as indicated by the degree of clearness and vividness specified on the Rating Scale.

| | |
|---|---|
| 5. The sun as it is sinking below the horizon | (   ) |

Think of each of the following sounds, considering carefully the image which comes to your mind's ear, and classify the images suggested by each of the following questions as indicated by the degrees of clearness and vividness specified on the Rating Scale.

| *Item* | *Rating* |
|---|---|
| 6. The whistle of a locomotive | (   ) |
| 7. The honk of an automobile | (   ) |
| 8. The mewing of a cat | (   ) |
| 9. The sound of escaping steam | (   ) |
| 10. The clapping of hands in applause | (   ) |

Think of 'feeling' or touching each of the following, considering carefully the image which comes to your mind's touch, and classify the images suggested by each of the following questions as indicated by the degrees of clearness and vividness specified on the Rating Scale.

| *Item* | *Rating* |
|---|---|
| 11. Sand | (   ) |
| 12. Linen | (   ) |
| 13. Fur | (   ) |
| 14. The prick of a pin | (   ) |
| 15. The warmth of a tepid bath | (   ) |

*Rating Scale*

The image aroused by an item of this test may be:

| | |
|---|---|
| Perfectly clear and as vivid as the actual experience | *Rating* 1 |
| Very clear and comparable in vividness to the actual experience | *Rating* 2 |
| Moderately clear and vivid | *Rating* 3 |
| Not clear or vivid, but recognizable | *Rating* 4 |
| Vague and dim | *Rating* 5 |
| So vague and dim as to be hardly discernible | *Rating* 6 |
| No image present at all, you only 'knowing' that you are thinking of the object | *Rating* 7 |

Think of performing each of the following acts, considering carefully the image which comes to your mind's arms, legs, lips, etc., and classify the images suggested as indicated by the degree of clearness and vividness specified on the Rating Scale.

| *Item* | *Rating* |
|---|---|
| 16. Running upstairs | (  ) |
| 17. Springing across a gutter | (  ) |
| 18. Drawing a circle on paper | (  ) |
| 19. Reaching up to a high shelf | (  ) |
| 20. Kicking something out of your way | (  ) |

Think of tasting each of the following considering carefully the image which comes to your mind's mouth, and classify the images suggested by each of the following questions as indicated by the degrees of clearness and vividness specified on the Rating Scale.

| *Item* | *Rating* |
|---|---|
| 21. Salt | ( ) |
| 22. Granulated (white) sugar | ( ) |
| 23. Oranges | ( ) |
| 24. Jelly | ( ) |
| 25. Your favourite soup | ( ) |

Think of smelling each of the following, considering carefully the image which comes to your mind's nose and classify the images suggested by each of the following questions as indicated by the degrees of clearness and vividness specified on the Rating Scale.

| *Item* | *Rating* |
|---|---|
| 26. An ill-ventilated room | ( ) |
| 27. Cooking cabbage | ( ) |
| 28. Roast beef | ( ) |
| 29. Fresh paint | ( ) |
| 30. New leather | ( ) |

Think of each of the following sensations, considering carefully the image which comes before your mind, and classify the images suggested as indicated by the degrees of clearness and vividness specified on the Rating Scale.

| *Item* | *Rating* |
|---|---|
| 31. Fatigue | ( ) |
| 32. Hunger | ( ) |
| 33. A sore throat | ( ) |
| 34. Drowsiness | ( ) |
| 35. Repletion as from a very full meal | ( ) |

## APPENDIX F

# ADULT LEARNING OF MATHEMATICS
# UNDERSTANDING VARIABLES

Name.......................................................................................................

**Directions:** There is no time limit for the completion of these questions.
Answer each question in the space provided.
Attempt as many questions as possible.

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1. If $y = 3$, find the value of

   (i) $2y$ ..............................

   (ii) $2y + 5$ ..............................

   (iii) $2(y + 5)$ ..............................

   (iv) $2y + y$ ..............................

   (v) $3y - y$ ..............................

   (vi) $2(5y)$ ..............................

2. 't is equal to the sum of 4 and s.' Write this information in mathematical symbols.

   ..........................................................................................................

3. 'The number z is nine times the number x.' Write this information in mathematical symbols.

   ..........................................................................................................

4. A number p is six more than another number q. If q is 21, what is p?......................

5. The number x is five times the number y . If x is 30, what is y?.............................

6. In an engineering class, there are 12 more men than women. There are 14 men. How many women are there?

   ..........................................................................................................

7. If $a + 7 = b$, which is the greater number, a or b?.............................................

8. If $5y = p$, which is the bigger number, y or p? ..............................................

9. If $x = 3$ and $y = 6$, what is the value of $xy$?....................................................

10. x and y are numbers. x is seven more than y. Write an equation showing the relation between x and y.

    ..........................................................................................................

11. The Niger River in Africa is $y$ metres long. The Rhine in Europe is $z$ metres long. The Niger is three times as long as the Rhine. Write an equation which shows how $y$ is related to $z$.

..........................................................................................................................

12. In a football match, one team scored $t$ points and the other scored $r$ points.

How many points all together were scored in the match?..........................................

13. (a) Given $c + d = 10$,

CIRCLE ALL THE POSSIBLE MEANINGS FOR $c$:

(i) 3

(ii) 10

(iii) 12

(iv) 7.4

(v) The number of tools in a box

(vi) An object such as a condenser

(vii) An object such as a battery

(viii) None of the above.

(ix) More than one of the above (if so, indicate which ones).

(x) Don't know.

(b) If $c + d = 10$, what happens to $d$ as $c$ increases?

..........................................................................................................

(c) If $c + d = 10$, and $c$ is always less than $d$, what values may $c$ have?

..........................................................................................................

14. At a certain college there are six times as many students as there are lecturers. This fact is represented by the equation

$$S = 6L$$

CIRCLE YOUR CHOICES IN THE FOLLOWING QUESTIONS.

(a) In this equation, what does the letter L stand for?

(i) Lecturers
(ii) Lecturer
(iii) Number of lecturers
(iv) Students
(v) Student
(vi) Number of students
(vii) None of the above
(viii) More than one of the above (if so, indicate which ones)
(ix) Don't know.

(b) In this equation, what does the letter S stand for?

(i) Lecturers
(ii) Lecturer
(iii) Number of lecturers
(iv) Students
(v) Student
(vi) Number of students
(vii) None of the above
(viii) More than one of the above (if so, indicate which ones)
(ix) Don't know.

15. Consider the following statement:

3a + 2b could represent the total number of people seated in a restaurant, some at 3 large tables (the same number at each) and some at 2 smaller tables ( the same number at each).

CIRCLE one of the following

I agree/disagree with this statement. Give your reasons.

..........................................................................................................................

..........................................................................................................................

16. (a) Add 4 onto n +5 ..............................................

(b) Add 4 onto 3n ..............................................

(c) Multiply n + 5 by 4 ..............................................

17. This question is about t + t and t + 4.

(a) Which is larger, t + t or t + 4? WHY?

......................................................................................................................

......................................................................................................................

......................................................................................................................

(b) When is t + t larger?

......................................................................................................................

(c) When is t + 4 larger?

......................................................................................................................

(d) When are they equal?

......................................................................................................................

18. This question concerns the length of the two poles shown in the sketch.


(a) CIRCLE YOUR CHOICE OF ANSWER

(i) The red pole is longer than the green pole.

(ii) The green pole is longer than the red pole.

(iii) They are equal in length.

(iv) Any of the above is possible.

Give reasons for your answer.

..........................................................................................................................

..........................................................................................................................

(b) When would the green pole be longer than the red pole?

..........................................................................................................................

(c) When would the red pole be longer than the green pole?

..........................................................................................................................

(d) When are they equal in length?

..........................................................................................................................

19. For an industrial site visit 3 buses take f students each and 4 cars take g students each.

(a) CIRCLE the ONE that best says what the value of 3f tells us:

(i) 3 buses x f students

(ii) How many students took buses.

(iii) That there is the same number of students on each bus.

(iv) Three buses, f students.

(v) The number of buses which take the students.

(b) Give the total number of students taken by these buses and cars.

.................................................................................................................................

(c) One car leaves early with g students. How many students remain?

.................................................................................................................................

20. Decide whether the following statements are TRUE always, never or sometimes.
Tick the correct answer.
If you tick "True only when....", write when it is true.
All the letters stand for whole numbers or zero. (0, 1, 2, 3, 4, ...)

(i) $a + b + c = a + x + c$

True always.
Never true.
True only when....................................

(ii) $2a + 3b + 7 = 5a + 7$

True always.
Never true.
True only when....................................

(iii) $2a = a + 2$

True always
Never true
True only when....................................

(iv) $a + 2b + 2c = a + 2b + 4c$

True always
Never true
True only when....................................

# APPENDIX G - TABLE OF RESULTS

| NAME | Age | Maths - Year/ Level/Grade | QMI Score | Operations Test Maps Ability | Space Test Maps Abilitiy | Algebra Test |
|---|---|---|---|---|---|---|
| Group 1 | | | | | | |
| Joseph | 29 | 10.2F | 116 | 62 | 54 | 59.4 |
| Sam | 24 | 10.2C | 64 | 64 | 56 | 75.9 |
| James | 31 | 10.3P | 72 | 72 | 66 | 72.9 |
| Mark | 32 | 10.2P | 103 | 68 | 62 | 81.9 |
| Group 2 | | | | | | |
| Arthur | 36 | 9.2F | 126 | 62 | 55 | 63.5 |
| Maurice | 31 | 10.2HP | 86 | 66 | 59 | 75 |
| Christine | 36 | 10.2P | 63 | 66 | 57 | 61 |
| Rose | 32 | 9.2P | 69 | 63 | 55 | 57 |
| | 31.375 | | 87.375 | 65.375 | 58 | 68.325 |