THE TRIAL LOAD ANALYSIS

OF

CLARK DAM,

BUTLER'S GORGE,

TASMANIA.

Thesis submitted by

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SYNOPSIS

This paper describes the application of the Trial Load Method to the analysis of Clark Dam, a unit of the Tarraleah Development built and operated by The Hydro Electric Commission of Tasmania. Samples of the actual calculation forms used are included and the methods of computation are explained. The final analysis of the dam as constructed is presented with specimen computations, and the remainder suitably condensed and tabulated.

CONTENTS

Frequent references are made throughout to the book "Trial Load Method of Analyzing Arch Dams", without which the paper is incomplete. This book is Bulletin 1 of Part V. Technical Investigations, of the Boulder Canyon Project Final Reports, published by the United States Bureau of Reclamation. It is hereafter referred to as "The Bulletin".

CHAPTER I - INTRODUCTION

1. Clark Dam. The Tarraleah Power Development is a base load unit in the Hydro Electric Commission's electricity supply network. The scheme utilises the headwaters of the Derwent River, regulated by two reservoirs in series to provide a continuous flow of 900 cusecs to Tarraleah Power Station. The first of these is Lake St. Clair (260 square mile-feet), the second the artificial lake (380 square mile-feet) created by the construction of Clark Dam in Butler's Gorge.

The dam is a variable thickness concrete arch with gravity abutments on both banks. The outflow from the reservoir is normally passed through a vertical shaft Francis turbine or regulator valves in a powerhouse immediately downstream from the dam. A 20,000 cusec capacity ski-jump spillway is located on the left bank adjacent to the gravity tangent block.

2. Previous Methods of Design. Previous to the development of the trial load method most curved masonry dams had been designed by one of four methods:

(i) The cylinder formula required circular arches of uniform thickness and neglected the effects of bending, rib-shortening and shear deflection.

(ii) Formulae were developed for elastic arches, Cain's being the best known. Many dams designed on this basis have overhanging upstream faces to obtain efficient arch action.

(iii) All gravity dams even though curved in plan were treated purely as gravity sections.

(iv) The "Arch and Crown Cantilever" method assumed that the arches carried portion of the water load, uniform from crown to abutments, and of such intensity as to produce equal radial deflections of arches and cantilever, at the crown only. This is the forerunner of the present trial load method developed by the U.S. Bureau of Reclamation.

3. The Trial Load Method. For the purposes of the analysis, the dam is replaced by two systems of elements; a system of vertical cantilevers and a system of horizontal arches. Both of these substituted structures occupy the entire volume of the dam. One structure carries the load supported by cantilever action, the other carries the load supported by arch action. Each arch and each cantilever may move independently of other arches and cantilevers, but at the conclusion of the analysis geometrical continuity must be restored at all points in the structure. That is the arch and cantilever structures must have identical linear and angular displacements throughout, so that the continuous structures of arches and cantilevers occupy the position of the loaded dam. Between the initial state of the unloaded dam and its final position, elements are

free to deflect as they may under the loads applied. However, being component parts of the dam, there are certain restrictions on the nature of their movements. For example, tangential movement of a cantilever takes place by shearing deformation instead of bending.

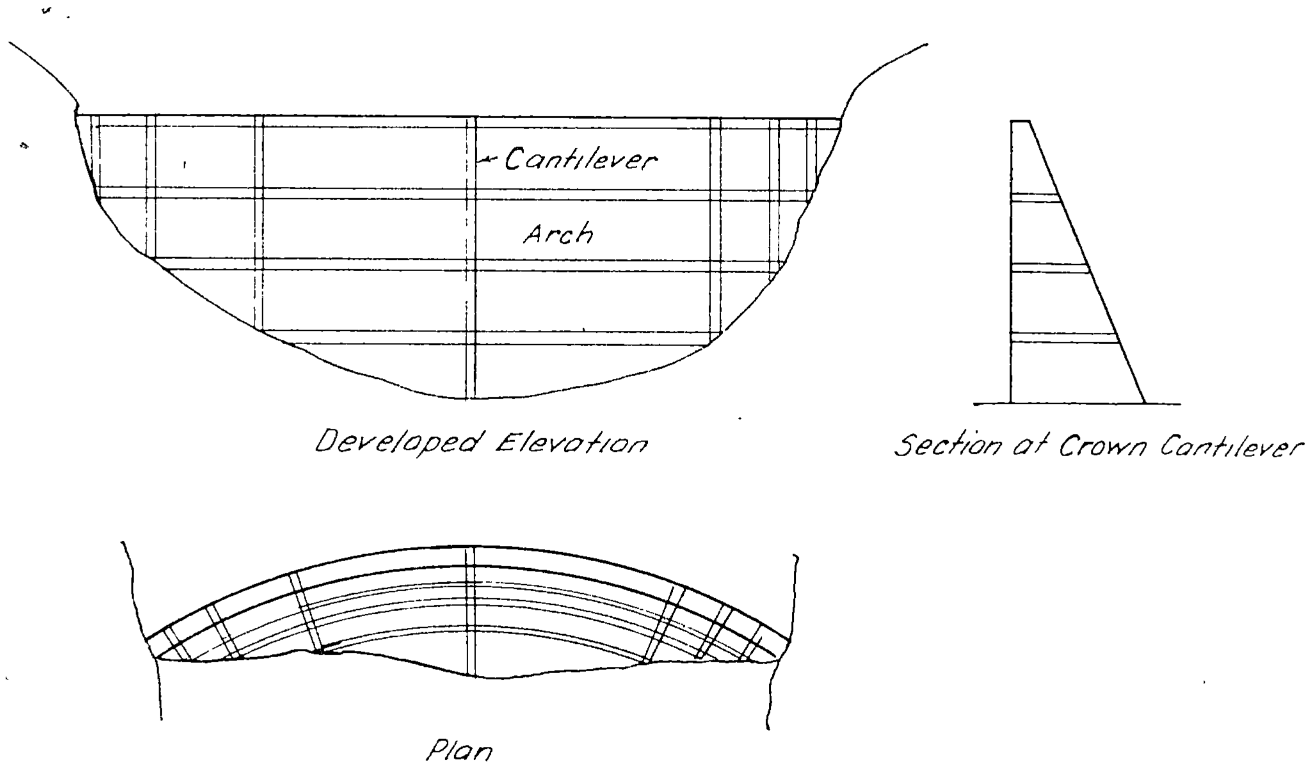


FIGURE 1 LOCATION OF ARCHES AND CANTILEVERS

Instead of investigating a great number of vertical and horizontal elements, only a few sample arches and cantilevers are analysed. The analysed structure then becomes a grid of intersecting sample arches and cantilevers, as shown in figure 1. Obviously, the greater the number of elements considered the more accurate the results of the analysis. The complexity of resulting operations, however, holds the practical limit to a moderate number of elements. With the simplified structure it is sufficient to secure coincidence of sample arches and cantilevers at their points of juncture.

The trial load method as now used by the Bureau assumes that the water load is divided between arch and cantilever elements; that the division may or may not be constant from abutment to abutment at each horizontal element; and that the true division of load is the one which causes equal arch and cantilever deflections at all points on all arches and cantilevers. Furthermore, the trial load method assumes that the distribution of load must be such as to cause equal arch and cantilever deflections in all directions; that is, in tangential and rotational directions as well as in radial directions.

Since the required agreement of arch and cantilever deflections can be obtained only by assuming different distributions of load and calculating resulting arch and cantilever movements until the specified criterion is fulfilled, the procedure is logically called the "trial load" method. The method has been gradually amplified until now the formulae consider the effects of radial sides of cantilever elements, twist action, tangential shear, Poisson's ratio effects and movements of foundation and abutment rock, as well as the more commonly considered effects of thrust, shear and flexure in concrete elements.

The analysis is carried out in steps, or adjustments as they are called. At present three adjustments are made; radial, tangential and twist. As the names imply, these serve to bring the arch and cantilever movements into agreement in radial and tangential directions and in vertical and tangential rotations. Needless to say, with a small number of elements, exact geometrical congruence of the two systems is an ideal which can only be approximated. When the adjustments are completed, the grid occupies very closely the position assumed by the loaded dam. It is then a relatively simple matter to compute stresses in the arch and cantilever elements.

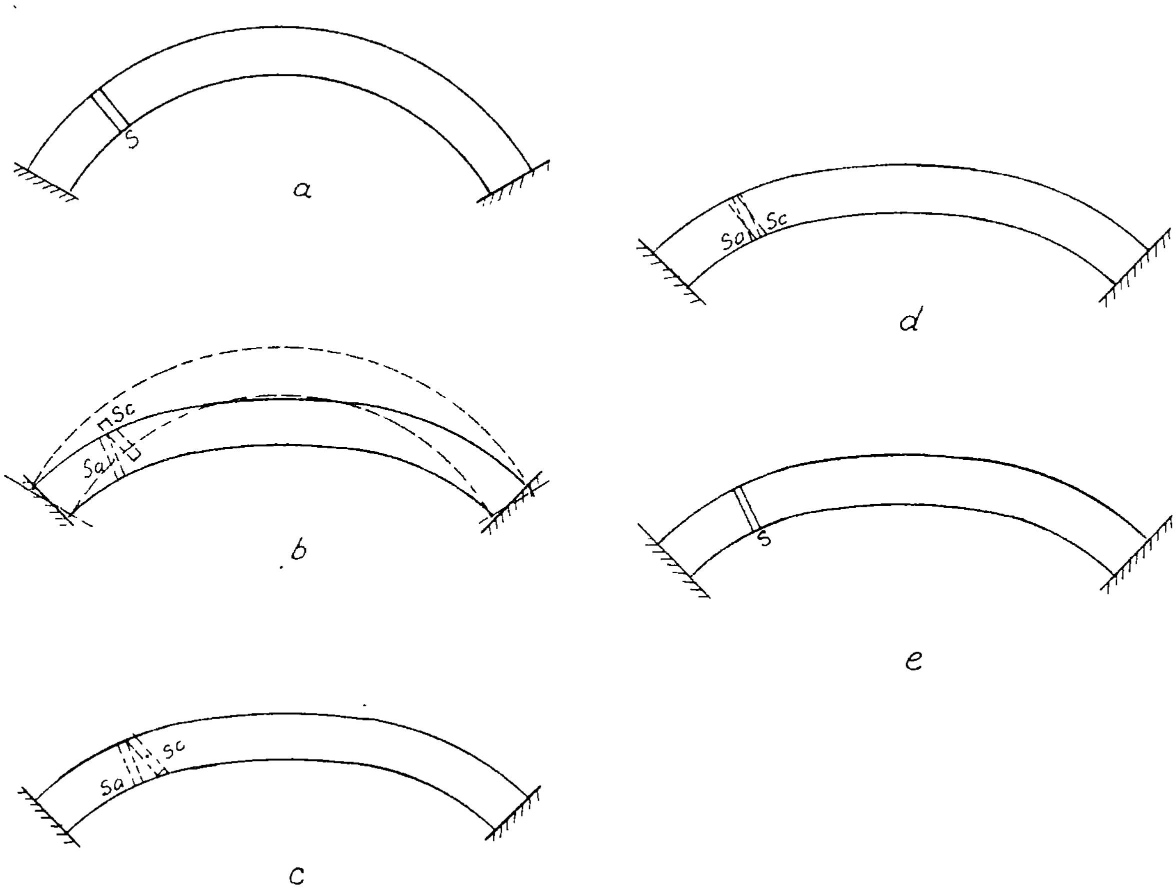


FIGURE 2 DEFORMATION OF A TYPICAL ARCH AND CANTILEVER

Figure 2 shows a picture of the nature and use of internal loads and the successive steps in a trial load analysis.

Figure 2a shows a plan of a typical arch which is intersected by a cantilever element at S, as indicated by the heavy lines. This is for the unloaded condition. The section which is a part of both the arch and the cantilever is hereafter referred to as the common section.

In Figure 2b external loads have been placed on the dam and the first trial load distribution made between arch and cantilever elements. Note that the cantilever has moved radially to S_c while the arch movement has a tangential and a rotational component, bringing the arch section to S_a. All displacements are shown to a highly exaggerated scale. In Figure 2c the radial adjustment has been completed. Although radial deflections are equal, it is evident that the arch and cantilever are out of alignment, both tangentially and in rotation. The first is corrected by the tangential adjustment in which a tangential load is applied to the cantilever, displacing it to the left, and an equal and opposite load applied to the arch. The loads are proportioned by trial to bring the centres of the two common sections together as in Figure 2d.

Next a twisting moment is applied to the cantilever and an opposing couple of equal magnitude applied as a bending moment on the arch. The magnitude of the couples is adjusted until the two common sections are in rotational agreement as in Figure 2e This is the twist adjustment. The trial load adjustment is now complete and Figure 2e represents the loaded condition of the dam with the common sections again congruent as they were in Figure 2a.

In practice, the latter adjustments throw the structure out of radial adjustment, so that a radial readjustment is necessary. In exceptional cases readjustments of tangential and twist loads are necessary. Roughly, this is an outline of the steps necessary in a trial load analysis. The many details and modifications required in an actual analysis are discussed later. In order to simplify the computations as much as possible, both internal and external loads are represented by systems of unit loads.

The use of the trial load method now enables the designing engineer to analyse load distributions, deflections, and stresses in curved concrete dams of all sizes and shapes, whether of the massive arched gravity type or the relatively thin, monolithic arch type.

(The material of this section has been selected directly from relevant sections of The Bulletin).

4. <u>The Design of Clark Dam</u>. The gorge profile and height of Clark Dam were found to compare closely with those of Gibson Dam, Montana, U.S.A. --- namely a wide trapezoidal profile of crest length approximately 1000 feet and total height about 200 feet. The dimensions of Gibson Dam were therefore used as a basis for Study A in which an arbitrary geometric profile, based on test borings, was used. When the adjustment was begun on the assumption that the cantilevers were uncracked, it was found that the full water load deflection of the cantilevers was less than the temperature deflection of the arches. Calculations showed that high tensions were developed in the cantilevers, necessitating a cracked cantilever analysis. Therefore with a view to gaining preliminary experience in effecting an adjustment using unit loads throughout, Study A was set aside temporarily and an entirely arbitrary set-up, involving much more flexible cantilevers and uniform thickness arches, was analysed. This provided the necessary experience in studying quickly the effects of various load

patterns for this type of foundation profile but, as was expected, high tensile stresses were developed at the upstream faces of the cantilevers. Attention was then redirected to Study A and three loading conditions examined --- full water load and temperature drop, full water load and temperature rise, and temperature rise with no water load.

As a result of this study it was decided to amend the previous design by stiffening the arches at and above mid-height. Sufficient progress had been made on excavation to indicate that an increased crown depth and crest length were required. An analysis (Study B) of the stresses due to full water load and temperature drop showed that the proposed crown section and downstream face radii were suitable. The only differences between this and the final design were the introduction of sloping faces on the gravity abutments and a decrease in height of the crown cantilever of about 25 feet. This latter was because the rock under the river-bed proved much sounder than had been expected.

It was intended that in the final study of the dam as built foundation deformation and the tangential adjustment should be included in the analysis. Unfortunately however, an acute staff shortage prevented the inclusion of these effects which would have vastly increased the amount of arithmetic in all phases of the analysis.

5. Scope of this Thesis. The thesis describes the method as simplified for use on Clark Dam, where the analysis was limited to the radial adjustment, with symmetrical arches on rigid abutments. This forms the basis of the method in which the succeeding refinements may be incorporated; a designer must master the simple radial adjustment before embarking upon more complex problems. An arch cannot be designed directly for a given load -- a profile must be assumed and analysed. Design therefore is a matter of selecting a suitable arch in the first place and intelligent application of the results of the analysis to the modification of the original section. This paper is primarily concerned with the methods of analysis.

CHAPTER II - PREPARATORY WORK

PART A - INTRODUCTORY.

Definition: The radial adjustment is the process of dividing up water and temperature loads between the arches and cantilevers so that the radial deflections of the two systems at corresponding points are equal.

6. Initial Loads. When a dam is built in vertical blocks to its full height before the contraction joints are grouted, the dead load is transferred to the foundation by column action. The vertical blocks are therefore treated as cantilevers fixed at the base by their own weight. Then, provided the portion of the water load is insufficient to cause cracking, the cantilever deflects as an elastic member, unaffected by dead weight. In an adjustment with uncracked cantilevers the

dead load is accordingly assigned to the cantilevers as an initial load which does not influence the division of the water load. Uplift pressure on the base of uncracked cantilevers must be included as an initial load which does affect the radial adjustment.

Atmospheric temperature changes cause temperature changes in the interior of a concrete dam. These cause no radial movements of the cantilevers alone, hence they are assigned to the arches as an initial load. For a temperature drop, the arches shrink downstream, working with the water load. For a rise they work against the water load.

7. Initial Relative Deflections.

(a) Uncracked cantilevers.

For uncracked cantilevers the deflections of the centre lines of the arches and cantilevers must be expressed relative to the unloaded position of the dam - that is, at the same temperature as existed throughout at the time the contraction joints were grouted. Although the centre lines of the arches and cantilevers are not straight lines it is convenient to regard them as such for the plotting of deflections. The cantilevers tilt downstream due to the uplift on the base. If the arches are below grouting temperature, they move downstream, so that the cantilevers take a definite portion of the water load before they overtake the arches. The remainder of the water load is then divided (by unit loads) to secure radial adjustment. For a temperature rise the arches must be pushed downstream before the cantilevers begin to share the water load. The uplift and temperature movements may be combined as one initial deflection of the arches relative to the cantilevers.

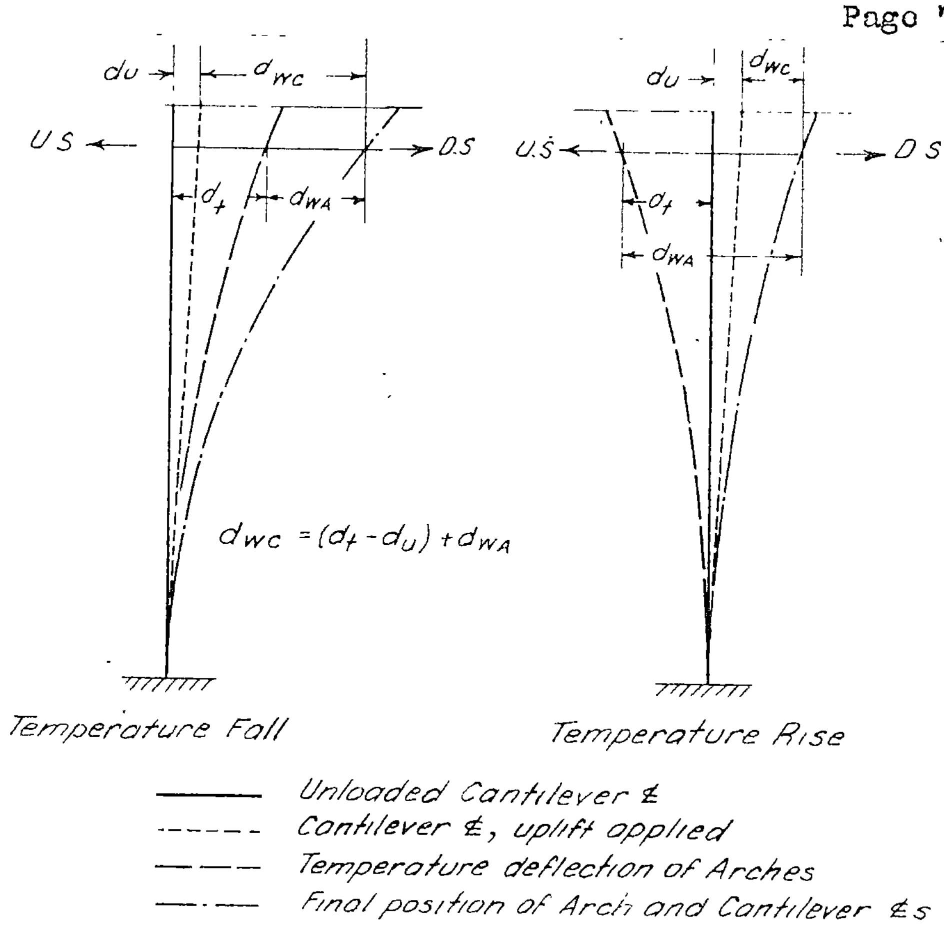


FIGURE 3 INITIAL RELATIVE DEFLECTIONS.

In Figure 3
d_t = arch temperature deflection)arith-
d_{wa} = arch water load deflection)metic
d_u = cantilever uplift deflection)values
d_{wc} = cantilever water load deflection)

Then $d_u + d_{wc} = d_t + d_{wa}$

$$d_{wc} = (d_t - d_u) + d_{wa}$$

where $(d_t - d_u)$ is the initial relative deflection.

(b) Cracked cantilevers.

If however, the tensile stress in the cantilevers exceeds the ultimate strength of the concrete a crack will form, and the water pressure must then work against the weight of the concrete above the crack. But although the concrete weight affects the radial movements of a cracked cantilever, it has already been allocated to the cantilever as an initial load, (by virtue of block construction). Hence the procedure for the radial adjustment consists of computing deflections due to concrete weight and subtracting them (algebraically) from the total cracked cantilever movement to find the deflections used in the trial load adjustments.

If d_{wc} = deflection of cantilever as computed on Form C7

d_d = dead load deflection of cantilever

d_t = temperature deflection of arches

d_{wa} = waterload deflection of arches

(arithmetic values)

then the cantilever deflects d_{wc}, but as dead load is not shared, the dead load deflection should be added arithmetically to d_{wc} to give the total downstream movement of the cracked cantilever.

for radial adjustment $d_{wc} + d_d = d_t + d_{wa}$

or $d_{wc} = (d_t - d_d) + d_{wa}$

where $d_t - d_d$ is the initial relative delection of arch to cantilever.

It is unusual for all the cantilevers in a dam to be cracked, those near the wings remaining in compression at the upstream face. The deflections of these uncracked cantilevers for dead load are combined with arch temperature movements to find the initial relative deflections. If this were not done there would be a sudden break in the graph of the relative deflections at the end of the central cracked portion. As the adjustment proceeds the dead load deflection is subtracted arithmetically from each trial load deflection for these uncracked cantilevers for comparison with the arch deflections. When the process is complete the dead load stresses are added to the live load stresses for these uncracked cantilevers.

If the cantilevers are cracked on the downstream face (when maximum temperature rise occurs on an empty reservoir) the initial deflection of the arches relative to the cantilevers is $(d_t + d_d)$, both components positive. The only other deflections are due to internal self-balancing radial loads.

8. Conditions of Loading. It is advisable to analyse the dam for at least these types of loading:-

(i) Maximum temperature fall occurring when the reservoir is full. This condition imposes the greatest load on the cantilever system causing high compression at the downstream face and possibly tension at the upstream face.

(ii) Maximum temperature rise and full water load occurring simultaneously. This might happen if the reservoir were to fill rapidly after a long period of heavy depletion during summer. The arches are more heavily stressed than in case (i).

(iii) Maximum temperature rise occurring when the reservoir is empty. Studies of the Butler's Gorge storage indicate that complete depletion is possible, and if this occurs during the summer, the arches are restrained in their upstream movement by the cantilevers only. Tension may then develop at the downstream face.

Silt, ice and earthquake loads have not been considered in the design of Clark Dam.

9. Selection of arches and cantilevers. For a reasonably symmetrical dam only half the structure need be analysed, in which case the left side (looking upstream) is considered. For preliminary studies four arches and four cantilevers are ample, although six to eight may be analysed in the final study. One cantilever is located at the crown and the others on arch abutments or at changes of slope of the abutment profile.

The amount of dissymmetry permissible before the analysis of unsymmetrical arches is undertaken is not clearly stated in Trial Load literature, but it would appear evident that much experience of the method is necessary before a decision could be made on any doubtful case.

PART B - CONSTANTS AND STRUCTURAL DATA

10. Structural Constants. The following values have been used in Study C:-

1. Weight of water = 62.5 lb./c.ft.
2. Weight of concrete, w = 160 lb./c.ft. The value of 150 was used in Studies A and B, before tests of the actual mix were made. The maximum value by actual test is 167 lb./c.ft.
3. Modulus of elasticity of concrete, $E = 4 \times 10^6$ lb./sq.in.
4. Poisson's ratio for concrete, $\mu = 0.2$. This value is mandatory, as it is incorporated in tables of arch and load constants in the Bulletin.
5. Shear modulus for concrete, $G = \frac{E}{2(1+\mu)}$. To account for non-linear distribution of shear stress the value of K, ratio of detrusion caused by actual shear distribution to the detrusion caused by the equivalent shear uniformly distributed, is assumed to be 1.25. $\frac{K}{G}$ then equals $\frac{3}{E}$. This also is included in the Bulletin tables.
6. Coefficient of thermal expansion of concrete, $c = 5.5 \times 10^{-6}$ per degree Fahrenheit.
7. Thermal diffusivity of concrete, h = 0.25 sq.ft./hr.

11. Dimensions. From the specified crown section and the radii of the arcs of upstream and downstream faces the dimensions of the sample arches and cantilevers may be computed. The upstream face is cylindrical and the downstream face is specified by circular arcs at 25' intervals. The dam is constructed so that any section normal to the upstream face cuts the downstream face in a series of straight lines. All angles and arcs are measured in the upstream face, irrespective of whether the arches are of uniform or variable thickness. The position of a cantilever is defined by its angular distance from the crown. The half angle of the arches must be taken to the nearest whole degree to utilise the tables in the Bulletin.

Although the parapet is designed to withstand water right to the top maximum water level for the trial load analyses has been taken at crest level because of the generous assumption for floods and the inherent factor of safety of an arch. Normal full supply level is 2.5 feet below the crest.

Any holes in the dam - penstocks and galleries - are ignored in the analysis and stresses calculated as though they did not exist. The amount of reinforcing required round these openings is based on these stresses.

(Figure 4, "Dimensions of Clark Dam" has been cancelled and incorporated in Figure 21, Chapter IV).

12. Temperature Range in Arches. For the first study the relation between concrete thickness and mean temperature variation was taken from figure 54 in The Bulletin. This curve was derived for the arid areas of western U.S.A., and hence gives too high a range for Tasmanian conditions. For Study B a similar curve for Butler's Gorge was compiled from short term temperature records, extended by correlation with readings at Tarraleah, twelve miles distant. Records now available for a longer period have been utilised in the final Study. The method of deriving the thickness-temperature relationship is set out in a paper "Flow of Heat in Dams" by R.E. Glover, Nov.-Dec.1934 A.C.I. Journal.

Figure 5 shows the data for Clark Dam.

It is assumed that a temperature change is uniform throughout an arch and has the value corresponding to the mean thickness.

An appreciable quantity of heat is developed in a mass concrete structure by the exothermic actions involved in the hydration of the cement. The rate at which this internal heat is dissipated by conduction to the surface depends upon a combination of conductivity, specific heat, and specific gravity, called thermal diffusivity. If this is low a long time (perhaps years) will elapse before the dam has cooled to mean annual temperature, when the contraction joints may be grouted and the storing of water begun.

As the diffusivity of the concrete in Clark Dam is unusually low, a "modified" cement with a heat of hydration less than that of normal Portland cement has been used and a system of cooling pipes has been embedded in the concrete,

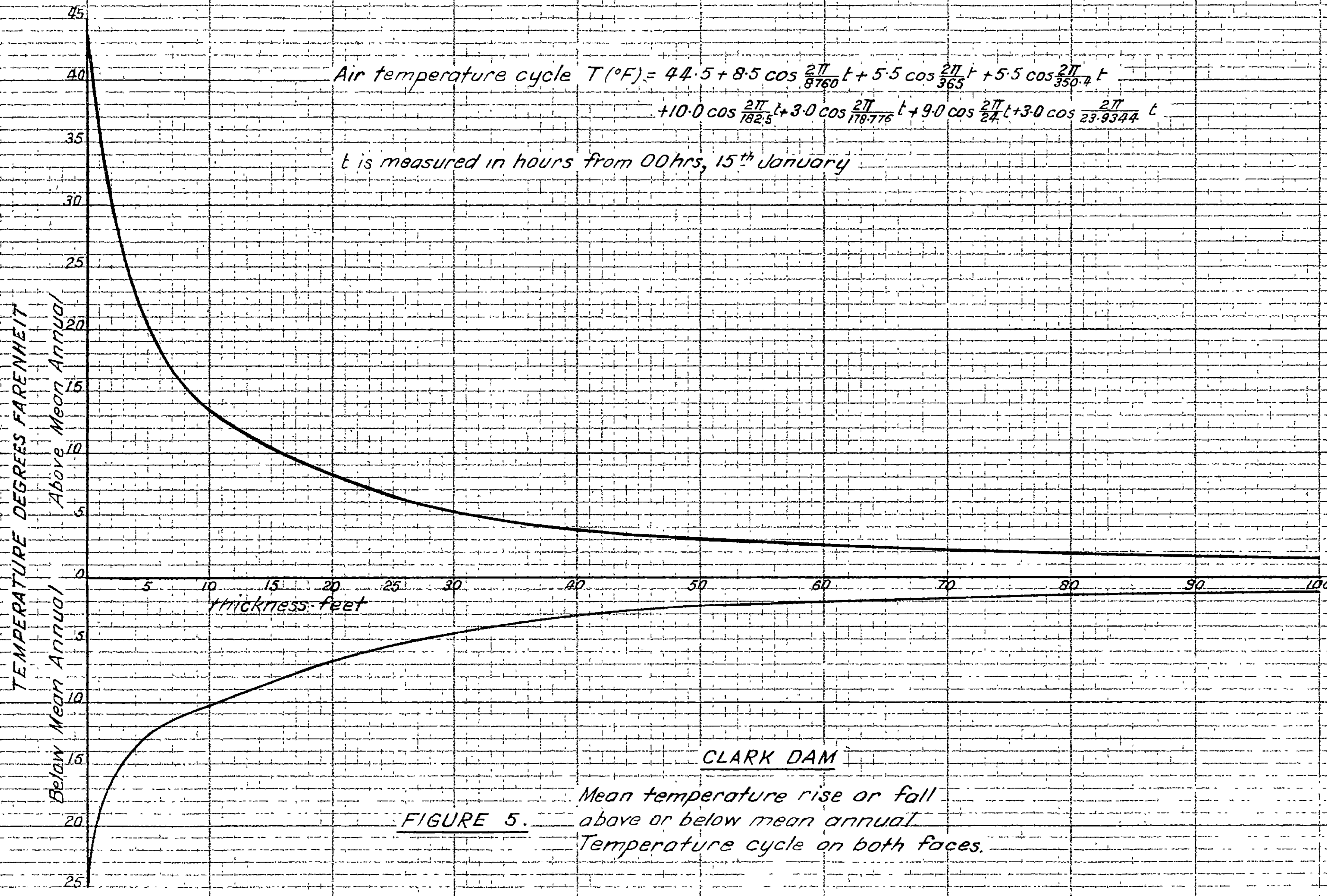


FIGURE 5. CLARK DAM

Mean temperature rise or fall above or below mean annual. Temperature cycle on both faces.

even though the dam is comparatively thin. Water from the Derwent is circulated through these pipes to accelerate the reduction of the internal temperature to mean annual. In some dams refrigerated water has been used to "sub-cool" the concrete below mean annual, thereby reducing the maximum temperature drops in the arches. The detailed study of temperature control in mass concrete is a highly specialised task, involving much advanced mathematics. A recent article of interest to engineers is "Cracking and Temperature Control of Mass Concrete" by Clarence Rawhouser in the Feb. 1945 Journal of the American Concrete Institute.

PART C - ANALYSIS OF CANTILEVERS

13. Description of Cantilever.

A cantilever is an element of the dam contained between the upstream and downstream faces and two vertical planes which pass through the centre of curvature of the upstream face (i.e. radial planes). It has a width at the upstream face of one foot. For simplicity the upstream and downstream edges of a horizontal section are taken as straight lines normal to the radial centre line. See Figure 6.

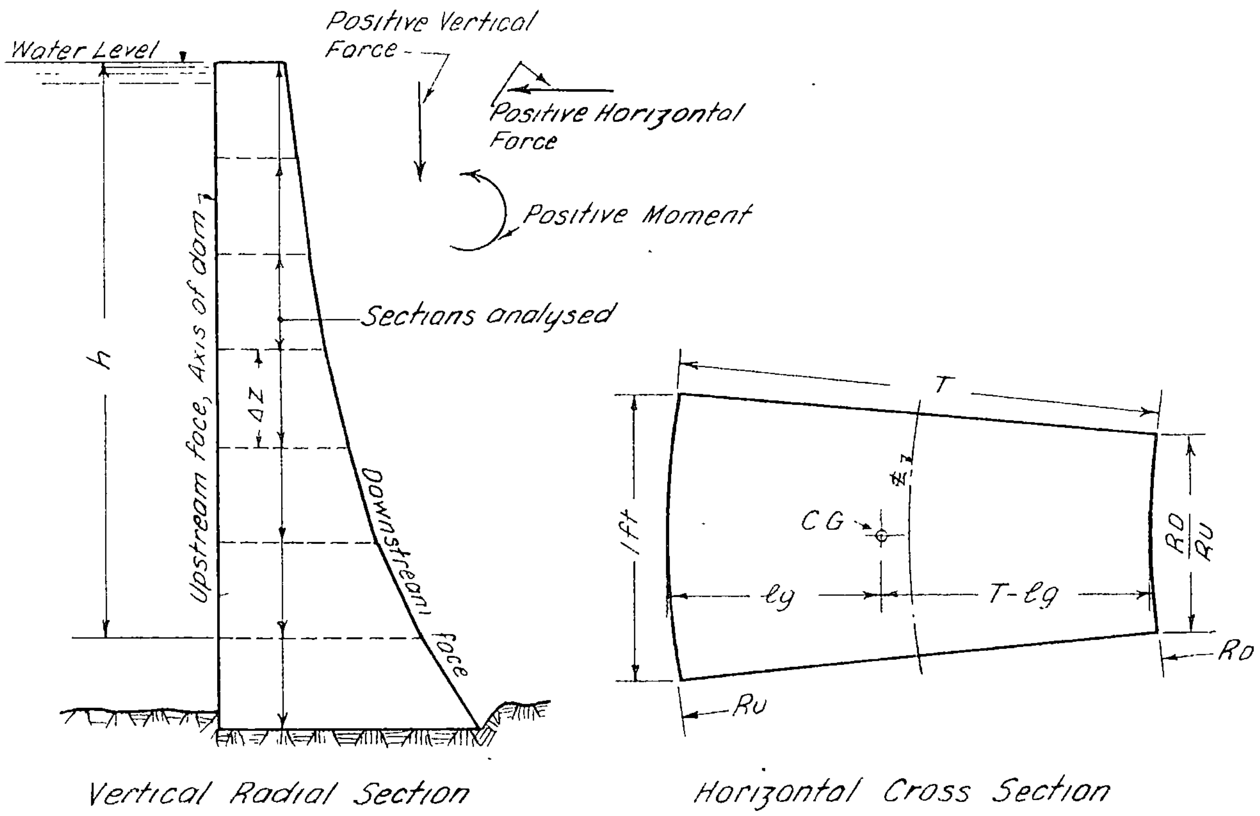


FIGURE 6 CANTILEVER

Cantilever Notation

All foot-pound units.

T = radial thickness

T_c = crown thickness

R_u = radius of upstream face

R_D = distance to any point on D.S. face from the centre of curvature of the U.S. face.

a = radius of curvature of D.S. face, at elevation considered.

x = distance between centres of curvature of U.S. and D.S. faces.

ϕ = angle from crown to cantilever, degrees.

ϕ_D = angle downstream edge makes with vertical.

α, β See Fig. 7

lg = distance from U.S. face to centre of gravity (C.G.).

$T-lg$ = distance from D.S. face to C.G.

A = area of horizontal cross-section.

I = moment of inertia of horizontal cross-section, about a circumferential line through C.G.

h = depth below reservoir surface.

P = water pressure

ΣW = weight of concrete above a section, positive downwards.

M = bending moment in vertical plane. Positive moment causes compression at the U.S. face.

M_W = moment of ΣW about C.G. of section.

V = radial shear, positive directed upstream.

ΔZ = increment of height between sections for which the properties have been evaluated.

Δr = radial deflection of centre line of cantilever, positive upstream.

U = uplift on a horizontal section, positive downwards.

M_U = moment of U about C.G. of section

For symbols in cracked cantilever analysis, see p.27.

14. Cantilever and Arch Thicknesses. Form C/A1

When the cantilevers and arches have been selected as previously described, their dimensions must be computed before any further properties can be evaluated. In a uniform thickness circular arch all radial sections are the same, but in a variable thickness arch, the radial thickness depends upon the angular distance from the crown. Since the calculation for arch quarter-points is similar to that for cantilevers it is included here.

Data:- R_U, T_c at the elevation considered, a and ϕ. In Fig. 7 A is the centre of curvature of the upstream face, F the c. of c. of the D.S. face.

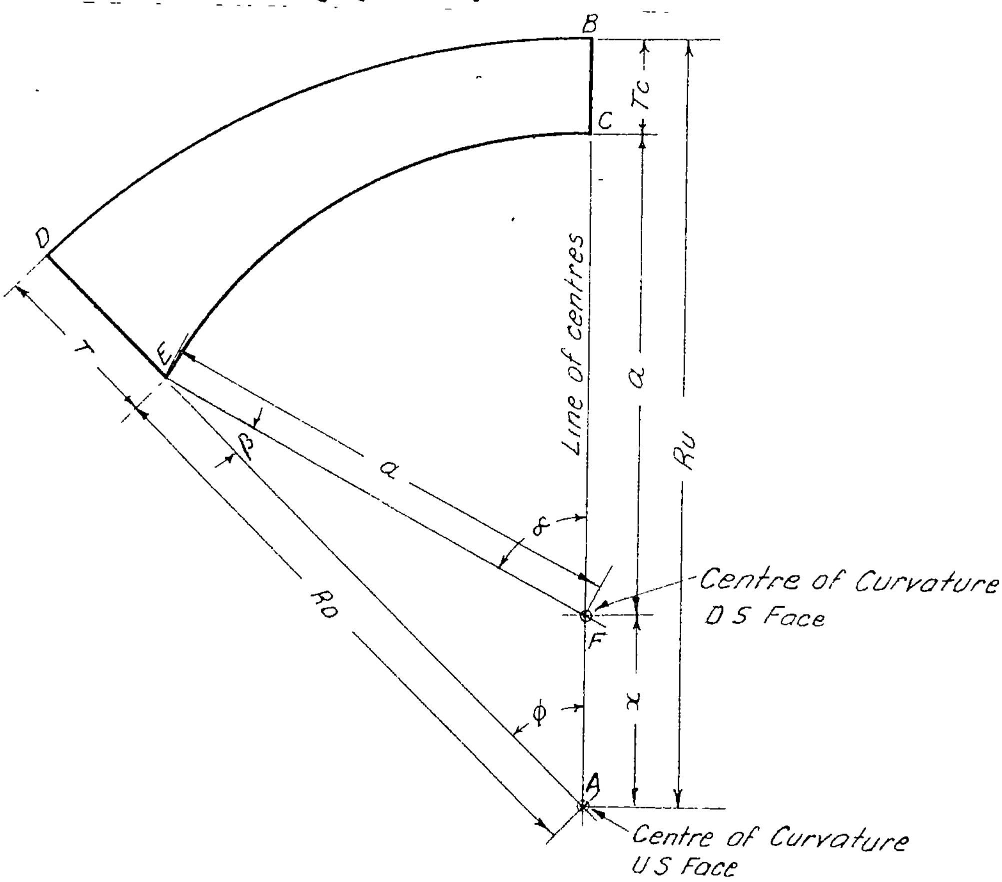


FIGURE 7. ARCH AND CANTILEVER THICKNESSES

$FA = x = R_u - (T_c + a)$

In $\triangle EFA$, $\sin\beta = \frac{x}{a}\sin\phi$

$\therefore \beta = \arcsin\left(\frac{x}{a}\sin\phi\right)$ and the external angle $\alpha = \phi + \beta$

Again in $\triangle EFA$, $EA = R_D = a\,\frac{\sin(180 - \alpha)}{\sin\phi}$

$$= a\,\frac{\sin(\beta + \phi)}{\sin\phi}$$

$$DE = T = R_u - R_D$$

(In U.S.B.R. Fig. 36, p.116, R_D is the true D.S. face radius, despite their definition of R_D).

Because T is a small fraction of R, one significant figure is lost in the subtraction, and hence sufficient figures should be used to evaluate T correct to three decimal places.

A set of 8-place natural sines, tabulated for every second, issued by the U.S. Coast and Geodetic Survey, were used in computing the thicknesses.

The values of T are computed on Form C/A 1, which has been drawn up for use with a calculating machine. As $\frac{x}{a}$ is the same for all points on an arch its value is noted on the sheet and sin β obtained for each case by multiplying $\frac{x}{a}$ by the requisite value of sinϕ. For a cantilever sinϕ is constant, while both x and a vary with the elevation.

For speed in computation the columns are completed one at a time, as this entails similar operations or entries into tables being made in groups. In this way any mistake or pronounced deviation from a series is more readily detected. The values of T computed here are transferred to Form C.2 in the case of cantilevers, or A.2 for arches, for the determination of further properties.

As in all trial load work each sheet should be worked independently by two persons for checking purposes. A discrepancy of one or two in the final figure here will cause increasingly large divergences as the work proceeds, thus hampering the checking in the later stages. It may be noted at the outset that much time is saved by having all forms of which more than one or two are required, duplicated or printed in readiness for use as the analysis proceeds.

Form C/A1 for cantilever B is given on the next page. This cantilever is used as the example in all the specimen computation forms included.

Form C/A.1

Sheet ....

THE HYDRO-ELECTRIC COMMISSION, TASMANIA

CLARK DAM

Study ....C........

Computed ....[illegible]....

Date ....

Checked .....DR.........

Date ....

Radial Thicknesses of ....CANTILEVER.... No ..B......

S.L. or Point	a	x	ϕ	$\sin\phi$	$\frac{x}{a}\sin\phi$	β	$\alpha = \beta + \phi$	$\sin\alpha$	$R_D = a\frac{\sin\alpha}{\sin\phi}$	$T = R_U - R_D$, $R_U = 405$
2345	—								388.000	17.000
2320	377.5	7.5	15°	.258,819,0	.005,142,099	0°17'41"	15°17'41"	.263,784,2	384.742	20.258
2295	360.0	20.0	15°	"	.014,378,83	0°49'26"	15°49'26"	.272,681,4	379.282	25.718
2270	337.5	37.0	15°	"	.028,374,23	1°37'33"	16°37'33"	.286,120,4	373.101	31.899
2245	310.0	58.0	15°	"	.048,424,20	2°46'32"	17°46'32"	.305,289,1	365.659	39.341
2220	277.5	82.5	15°	"	.076,946,19	4°24'47"	19°24'47"	.332,376,0	356.366	48.634
2195	240.0	110.0	15°	"	.118,625,4	6°48'46"	21°48'46"	.371,574,9	344.557	60.443
*[2170	197.5	140.0	15°	"	.183,466,6	10°34'18"	25°34'18"	.431,639,7	329.376	75.624]
2165									325.377	79.623
*[2145	150.0	172.0	15°	"	.296,779,1	17°15'51"	32°15'51"	.533,823,6	309.380	95.620]

* Sections not analysed

15. Calculation of Properties. Form C.2.

In order that deflections due to loads may be calculated the area, location of centre of gravity and moment of inertia of a number of horizontal sections are required. Throughout the work on Clark Dam horizontal sections at 25' intervals have been analysed. A, lg and I are used in the computation of dead weight, uplift forces and stresses in addition to deflections.

The properties are evaluated on Form C.2, which is the Bureau Form, Fig.24, modified to suit this dam by deletion of all references to a battered upstream face.

The column $\frac{1}{I}$ was soon found to be of little value (because $\frac{1}{EI}$ has to be computed only once) and in place of it a column $\frac{1}{A}$ was added. This is frequently used in cracked cantilever work (where V varies with each trial load).

Experience has shown that the curves for $\frac{A}{T}$, $\frac{lg}{T}$ and $\frac{I}{T^3}$ in the Bureau Fig. 16 are inadequately graduated for this work. Hence these quantities were computed from formulae 51, 52, 53 for the requisite values of $\frac{R_D}{R_U}$.

More properties are required if the cantilevers crack, but as a preliminary trial is necessary to determine whether the cantilevers do so these are not computed unless cracking is indicated. They are described in the section on cracked cantilever work.

Form C2.

THE HYDRO ELECTRIC COMMISSION TASMANIA

CLARK DAM

STUDY C

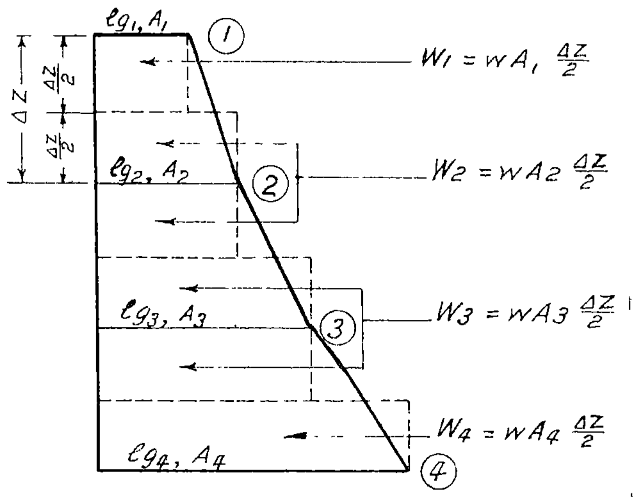
PROPERTIES OF CANTILEVER B

Sheet ____
Computed [illegible]
Date ____
Checked DR
Date ____

S.L.	T	R_U	R_D	$\frac{R_D}{R_U}$	$\frac{A}{T}$	A	$\frac{1}{A}$	$3\frac{lg}{T}$	lg	$T - lg$	$\frac{I}{T^3}$	I
2345	17·000	405·000	388·000	·958,02	·979,01	16·643	·060,085	1·489,3	8·439	8·561	–	–
2320	20·258	"	384·742	·949,98	·974,99	19·751	·050,630	1·487,2	10·043	10·215	·081,231	675·32
2295	25·718	"	379·282	·936,50	·968,25	24·901	·040,159	1·483,6	12·718	13·000	·080,659	1,372·0
2270	31·899	"	373·101	·921,24	·960,62	30·643	·032,634	1·479,5	15·732	16·167	·080,007	2,596·9
2245	39·341	"	365·659	·902,86	·951,43	37·430	·026,717	1·474,5	19·336	20·005	·079,217	4,823·4
2220	48·634	"	356·366	·879,92	·939,96	45·714	·021,875	1·468,1	23·800	24·834	·078,223	8,998·2
2195	60·443	"	344·557	·850,76	·925,38	55·933	·017,879	1·459,7	29·410	31·033	·076,948	16,992
2165	79·623	"	325·377	·803,40	·901,70	71·796	·013,928	1·445,5	38·365	41·258	·074,844	37,781

16. Dead Load Weights and Moments. Form C.3.

Forces and moments due to dead loads are computed on Form C3, instead of by the graphical method suggested in the Bureau Bulletin. The cantilever is treated as a series of blocks, rectangular in side elevation, whose cross-section is taken as that at the middle of the block, and which extends $\frac{\Delta Z}{2}$ above and below the horizontal sections for which the properties have been determined. The quantities required at all sections are the total weight above, ΣW and the moment M_W of ΣW about the centroid.



W = weight of half-block

$= w.A.\frac{\Delta Z}{2}$

e.g., at section 3 in Figure 8

$\Sigma W = W_1 + 2W_2 + W_3$

FIGURE 8 CONCRETE WEIGHT

$$\text{and } M_W = W_1(lg_3 - lg_1) + 2W_2(lg_3 - lg_2) + W_3(lg_3 - lg_3)$$
$$= W_1 lg_3 + 2W_2 lg_3 + W_3 lg_3 - W_1 lg_1 - 2W_2 lg_2 - W_3 lg_3$$
$$= (\Sigma W) \times lg_3 - \Sigma(W \times lg).$$

The weight W of each half block is recorded twice, above and below the line corresponding to the elevation of its mid-section, to facilitate the summations for ΣW and $\Sigma(W \times lg)$. By the sign convention used for cantilevers the dead load moment is positive.

If any of the cantilevers in the dam are cracked, W and M_W for all cantilevers are transferred to the sheets for uplift forces, C.5, for combination with U and M_U. If none of the cantilevers are cracked dead load deflections are not required, ΣW and M_W being used for stresses only.

Form C.3

Sheet ....

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

CLARK DAM

Study ...C...

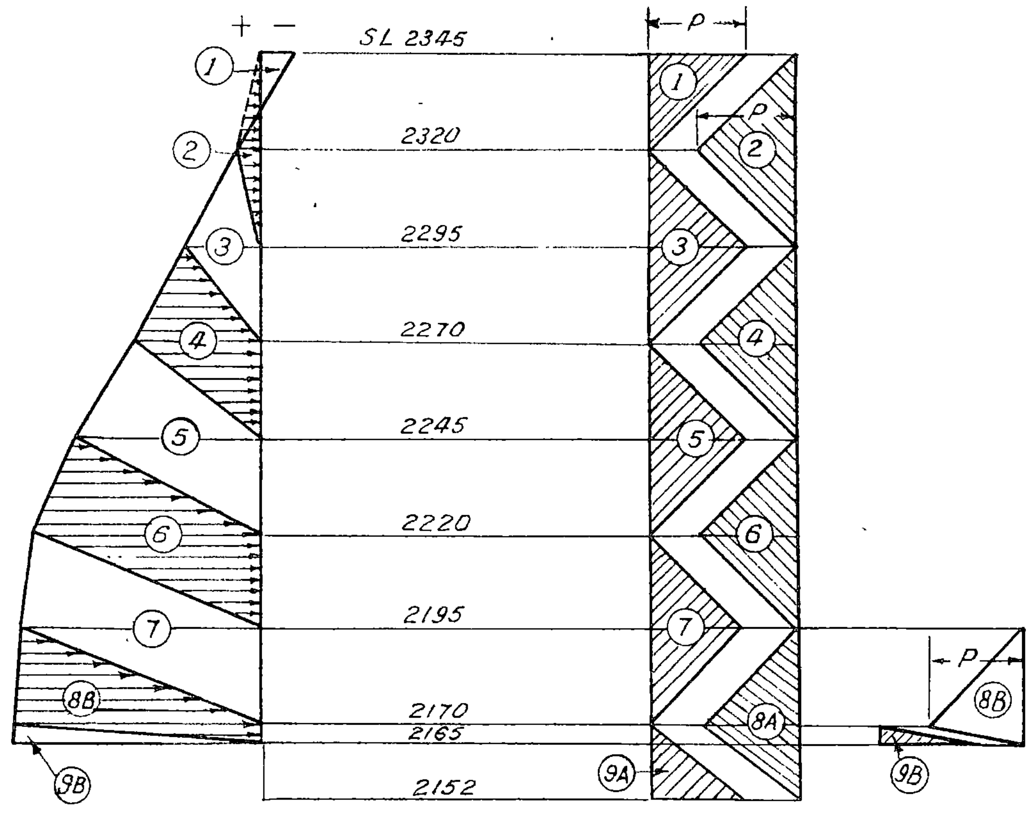
Computed ...T.W.L...
Date ....
Checked ...R...
Date ....

Dead Load Weights and Moments, Cantilever ...B...

S.L.	Height	Area	Weight of Half-Block	Total Weight above section	lg	(i)	(ii)	Moment about C.G.
	$\frac{\Delta Z}{2}$	A	$W = wA\Delta Z/2$ $w = 160$	$\sum W$		$(\sum W) \times lg$	$\sum(W \times lg)$	$M_W = (i) - (ii)$
2345		16·643		0	8·439	0	0	0
	12·5		33,286					
2320		19·751	39,502	72,788	10·043	731,010	677,620	53,390
	12·5		39,502					
2295		24·901	49,802	162,092	12·718	2,061,490	1,707,720	353,770
	12·5		49,802					
2270		30·643	61,286	273,180	15·732	4,297,670	3,305,250	992,420
	12·5		61,286					
2245		37·430	74,860	409,326	19·336	7,914,730	5,716,900	2,197,800
	12·5		74,860					
2220		45·714	91,428	575,614	23·800	13,699,600	9,340,380	4,359,200
	12·5		91,428					
2195		55·933	111,866	778,908	29·410	22,907,700	14,806,300	8,101,400
	15·0		134,239					
2165		71·796	172,310	1,085,457	38·365	41,643,600	25,365,000	16,279,000

17. Unit Radial Loads.

To facilitate finding the effect on deflection of a change in the loading on a cantilever, a system of unit loads has been devised by the U.S.B.R. This unit load is triangular on side elevation, varying linearly from 1000 lb./sq.ft. at the section considered to zero at the sections immediately above and below. Fig. 9 shows how a typical load pattern is built up of multiples of the unit loads. Although the Bureau suggests that unit loads be taken at the elevation of sample arches only, this work has shown that loads at every horizontal section analysed (when there are more of these than sample arches) give a more flexible load system for the radial adjustment.



Typical Load Pattern
Cantilever B

Unit Loads, Clark Dam
Study C.

FIGURE 9 UNIT RADIAL LOADS

The shear V due to radial load acting on a vertical face is the area of the load diagram above the section under consideration, and the moment M of the load about this section is the product of V and the distance of the centroid of this portion of the load, above the section. A table of V and M for all unit loads is given (p. 68) for reference in computing deflections and stresses.

The sign convention is that a radial load acting downstream is positive and causes a negative shear and moment. Care is necessary where a cantilever does not sit on an arch abutment as the height of the lowest step differs from those above it. Σ (V x P) and Σ (M x P) for evaluation of stresses are easily summed on the computing machine without recording individual values.

18. Uplift Forces and Moments. Form C.5. The subject of uplift (pore pressure) in concrete is a controversial one. In the absence of any more recent authoritative statements the suggestions advanced in The Bulletin for inclusion of uplift in the analysis of arch dams have been adopted. If the tension in the concrete is below 50 lb./sq.in. it is assumed that the concrete does not crack and that pore pressure plays an insignificant part in the load distribution. Uplift on the base of all cantilevers, cracked or uncracked, is included in the analysis. If the tension is greater than 50 lb./sq.in. the concrete is assumed to crack to the point of zero stress and uplift pressure then exerts a wedging effect on the cantilever.

Schoklitsch (Hydraulic Structures p.512) says in reference to uplift pressure on foundations: "This upward pressure constitutes an effective uplift only in those portions of the foundation joint in which it exceeds the pressure due to the weight of the dam. In the remaining parts of the joint this upward pressure merely takes the place, partially or completely, of the foundation reaction, since it is immaterial as far as the stability (against overturning only) of the dam is concerned, whether the reaction along parts of the base is due to the foundation itself or to water under pressure." This is illustrated in Figure 10.

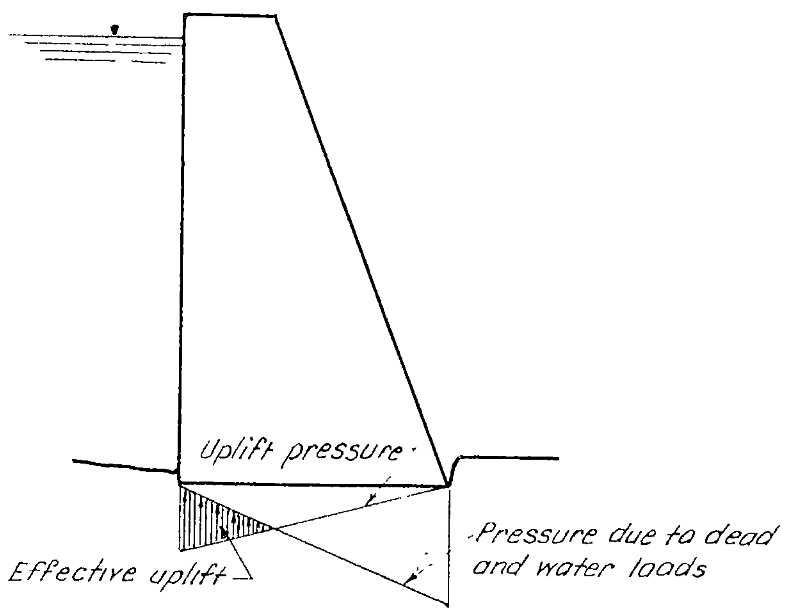


FIGURE 10 UPLIFT ON BASE

The accurate determination of uplift by this method is a matter of trial and error since an assumed uplift will shift the resultant downstream, thereby increasing the effective uplift slightly. This process would have to be repeated until agreement was reached and obviously would be a very tedious process for a complete trial load on all cantilevers. For this reason the accurate inclusion of uplift effects in trial load analysis is not usually feasible.

For Clark Dam uplift pressure was taken as varying from full water pressure at the upstream face to zero at the downstream face applied to half the area, on the bases of all cantilevers and other horizontal sections cracked by excessive tension.

In view of the assumptions involved a simple method of calculating uplift forces and moments was employed, thus:-

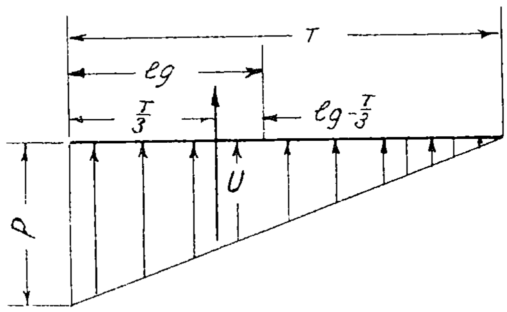


FIGURE 11 ASSUMED UPLIFT

Referring to Figure 11,

Uplift U = average pressure x area

$= \frac{P}{2} \quad x \quad \frac{A}{2}$

$= \frac{PA}{4}$, positive downwards

acting at $\frac{T}{3}$ from the upstream face.

The moment of U about the C.G. of the total section is

$M_u = U\ (lg - \frac{T}{3})$.

M_u has the same sign as U.

This method neglects the radial sides of the cantilever. U and M_u are computed on Form C.5 for the base of all cantilevers and for other horizontal sections where cracking is considered probable (found by experience or trial). ΣW and M_W are transferred from Form C3 for all horizontal sections of all cantilevers. Where uplift occurs U and M_u are combined with ΣW and M_W to find the total vertical force and moment respectively. The tabulation of ΣW, M_W and $\frac{1}{EI}$ on this sheet reduces the number of forms to which reference must be made when Forms C.6 and 7 are being compiled.

Form C.5

THE HYDRO-ELECTRIC COMMISSION, TASMANIA

Sheet ....

CLARK DAM

Computed ....

Date ....

Study ....C.........

Checked ....DR.........

Date ....

Uplift Forces and Moments, Cantilever....B....

S.L.	Area A	Pressure $\frac{P}{4}$	Uplift $U = \frac{PA}{4}$	$lg - \frac{T}{3}$	M_u $U(lg - \frac{T}{3})$	ΣW	$\Sigma W + U$	M_w	$M_w + M_u$	Uncracked $\frac{1}{EI}$
			−		−	+	+	+	+	10^{-14}
2345						0		0		—
2320						72,790		53,390		257.08
2295						162,090		353,770		126.54
2270						273,180		992,420		66.853
2245	37.430	1,562.5	58,484	6.222	363,890	409,330	350,850	2,197,800	1,833,900	35.994
2220	45.714	1,953.1	89,284	7.589	677,580	575,610	486,330	4,359,200	3,681,600	19.294
2195	55.933	2,343.8	131,100	9.262	1,214,200	778,910	647,810	8,101,400	6,887,200	10.217
2165	71.796	2,812.5	201,930	11.824	2,387,600	1,085,460	883,530	16,279,000	13,891,000	4.595,2

19. Deflections of Uncracked Cantilever, Form C.4.

The radial deflection of the centre line of a radial sided cantilever is computed by the ordinary mechanics formul - the deflection is the double integral of the curvature of the centre line due to bending plus the integral of the shear deformation from the base upwards. In this analysis the integration is replaced by summation. This computation for an uncracked cantilever is made on Form C.4 for unit, dead and uplift loads.

The moment at each elevation is taken from the unit load tables, Form C.3, or C.5 as required. $\frac{M}{EI}$ is the curvature of the centre line at each section. In Figure 12 the change of slope of the centre line at section 2 relative to that at section 1 is (average of end curvatures) x ΔZ. In the summation $\Sigma \frac{M \Delta Z}{EI}$ it is found expedient to express the change of slope as (lower end curvature x $\frac{\Delta Z}{2}$ + upper end curvature x $\frac{\Delta Z}{2}$). The summation of deflection is treated similarly and in this way all calculation is performed on the machine and sub-totals at each elevation recorded on the form.

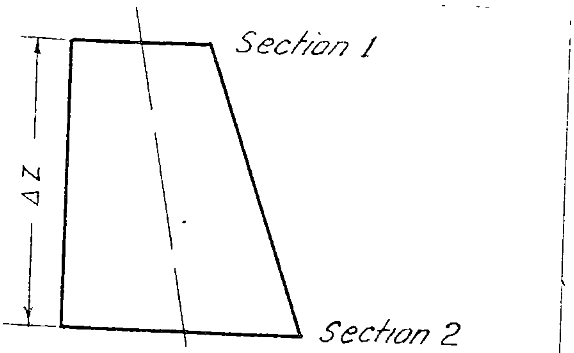


FIGURE 12 CANTILEVER DEFLECTION

V is found in the unit load tables and $\frac{1}{A}$ on Form C.2. The detrusion of the top of a block relative to the bottom is (bottom shear stress x $\frac{K}{G}\frac{\Delta Z}{2}$ top + shear stress x $\frac{K}{G}\frac{\Delta Z}{2}$), and the total shear detrusion is the summation of this.

For unyielding foundations the slope and the bending and shear deflections of the base are obviously zero. The larger the number of steps the more closely does the summation approach the true integral. The total movement is the sum of the bending and shear deflections, and for positive unit loads the (downstream) deflection is negative. Dead load deflection is positive. It is not really necessary to insert all the + and - signs on these forms as they are all of the same nature on one sheet, and the results are tabulated elsewhere for use in the radial adjustment.

A sample deflection computation is appended.

Form C.4

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

CLARK DAM

Study C

Radial Deflection of Cantilever B for Load 4

Sheet ____
Computed ____
Date ____
Checked ____
Date ____

$\frac{I}{E} = 1{\cdot}736{,}1 \times 10^{-9}$

$\frac{K}{G} = 5{\cdot}208{,}3 \times 10^{-9}$

S.L.	M	$\frac{1}{EI}$	$\frac{M}{EI}$	$\frac{\Delta Z}{2}$	$\sum \frac{M}{EI} \Delta Z$	$\sum\left(\sum \frac{M}{EI} \Delta Z\right)\Delta Z$	V	$\frac{V}{A}$	$\frac{K}{G} \times \frac{\Delta Z}{2}$	$\sum \frac{V}{A} \frac{K}{G} \Delta Z$	Total Deflection
	10^3	10^{-14}	10^{-7}		10^{-6}	10^{-3}	10^3	10^2	10^{-8}	10^{-3}	– ve
2345	0	—	0		20·472	2·548,6	0	0		·302,51	·002,851,1
				12·5					6·510,4		
2320	0	—	0		20·472	2·036,8	0	0		·302,51	·002,339,3
				12·5					"		
2295	0	—	0		20·472	1·525,0	0	0		·302,51	·001,827,5
				12·5					"		
2270	104·17	66·853	·696,41		19·601	1·024,1	12·5	4·079,2		·275,96	·001,300,1
				12·5					"		
2245	625	35·994	2·249,6		15·919	·580.06	25	6·679,2		·205,92	·000,785,98
				12·5					"		
2220	1250	19·294	2·411,8		10·092	·254,93	25	5·468,8		·126,83	·000,381,76
				12·5					"		
2195	1875	10·217	1·915,7		4·682,8	·070,242	25	4·469,8		·062,123	·000,132,36
				15·0					7·812,4		
2165	2625	4·595,2	1·206,2		0	0	25	3·482,0		0	0

20. Forces and Properties, Cracked Cantilever, Form C6.

If the tension at the face of the cantilever exceeds 50 lb./sq. in. it is assumed that the cantilever is cracked to the point of zero stress, the position of which is fixed by the criterion that the resultant of the forces above the section considered is equal to the total stress on the uncracked area. Stresses are assumed to vary as a straight line from zero at the end of the crack to a maximum at the uncracked face. (See sections 76 and 77 of the Bulletin).

Additional symbols required in this phase of the analysis are:-

- e = eccentricity of the resultant force, measured from the c.g. of the total horizontal section, positive when upstream of the c.g.
- F = function for determining the depth of the crack, $\left(\frac{R_D}{R_O}\right)$.
- R_O = distance to end of crack from the centre of curvature of the upstream face.
- T_1 = radial thickness of uncracked portion of section.
- A_1 = area of uncracked portion of section.
- I_1 = moment of inertia of uncracked portion of section about the circumferential line through its c.g.
- M_1 = resultant moment about c.g. of uncracked portion.
- σ_{ZD} = vertical stress at uncracked downstream face.
- σ_{ZU} = vertical stress at uncracked upstream face.

Form C.6 is used to compute the forces and properties for each trial load in the radial adjustment. V and M are calculated from the unit load tables, with due attention to the direction of the loads.

(a) Cracked Upstream Face.

To the water load moment M_W or $M_W + M_u$ is added to find the total moment on the section. If it is known that cracking does not occur, M_W is added; if cracking does occur, $M_W + M_u$ is added; if the position is doubtful both calculations are carried out until it is determined whether the section is cracked or not. The eccentricity of the force $\sum W + U$ is computed only for those sections likely to crack. For the others $\sum W$ falls either within the middle third or sufficiently close to it to cause less than the permitted tension. $\frac{R_D}{R_U}$ is copied from Form C.2.

The function $F = \frac{R_U - lg + e}{R_D}$ is computed next. lg is positive, and e is negative if downstream of the c.g. Experience soon showed that a small difference in the value of $\frac{R_D}{R_O}$, as determined by F, causes large discrepancies in subsequent operations, and makes checking more difficult. As the graph in the Bulletin is inadequately graduated a table of F to six places for values of $\frac{R_D}{R_o}$ from 0.80 to 1.00 in steps of 0.01 was prepared. From this $\frac{R_D}{R_o}$ may be found to the same number of places as R_D/R_U.

Division of R_D/R_U by R_D/R_o yields R_o/R_u. The radial thickness T_1 of the uncracked portion equals $R_o - R_D$.

The reduced area $A_1 = \frac{1}{2} T_1 \ (R_o/R_U + R_D/R_U)$

$$\text{i.e. } \frac{A_1}{T_1} = \frac{1}{2} \ (1+R_D/R_o) \times R_o/R_U$$

As the order of accuracy of cracked cantilever analysis is of necessity less than that for uncracked cantilevers it is sufficient to determine $I_1/(T_1)^3$ from a graph of the values of this function. The value of $I_1/(T_1)^3$ for the requisite value of R_D/R_o is multiplied by R_o/R_U before being recorded on Form C.6. There is no need to write I for the sound sections on this sheet as 1/EI is shown on Form C.5.

The resultant moment M_1 (Formula 71, Bulletin) is computed in two steps. Multiply the value of $\frac{M}{(\Sigma W+U)T_1}$ (for the requisite value of R_D/R_o) by the corresponding T_1 and write the product in the column $\frac{M}{(\Sigma W+U)}$. This is then multiplied by $\Sigma W+U$. In the case of the uncracked sections transfer $M+M_W$ across to the column M_1. Example: Cantilever B, Trial 11, Study C.1.

The shear V is unaltered by the cracking of horizontal sections.

(b) Cracked Downstream Face. In the case of Clark Dam this happens only when maximum temperature rise in the concrete occurs at a time of reservoir depletion. The computations are much the same as for the cracked upstream face, but are simpler in two respects. Firstly as there is no water load there is no uplift, and secondly the width of the upstream end of the uncracked zone remains constant at one foot. Example: Cantilever B, Trial 8, Study C.3.

The alterations in this form (compared with that in the Bulletin) have been made to avoid three-factor multiplications, which cannot be performed in one set-up on the particular calculating machines used on this work. This Form C.6 provides an excellent example of the advisability of completing a column at a time, since reference is made to so many other quantities in the course of its compilation.

Form C.6

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

Sheet

Computed *[illegible initials]*

Date

Checked *DR*

Date

Trial Load No *11*

$\frac{1}{E} = 1{\cdot}736{,}1 \times 10^{-9}$

$\frac{K}{G} = 5{\cdot}208{,}3 \times 10^{-9}$

C L A R K   D A M

Study *C1*

Properties and Forces. - Cracked Cantilever *B*

		Loads and Eccentricities				Properties at Horizontal Sections									Resultant Moment.	
	Cracked U.S. Face		V	M	$\Sigma M+M_U$	$\frac{\Sigma M+M_U}{\Sigma W+U}$	$\frac{R_D}{R_U}$	$\frac{R_U l_g+e}{R_D}$	$\frac{R_D}{R_O}$	$\frac{R_D}{R_U} \div \frac{R_D}{R_O}$	$\left(\frac{R_O}{R_U}-\frac{R_D}{R_U}\right)R_U$	$\frac{A_1}{T_1}$		$\frac{I_1}{T_1^3}$		$\frac{M_1}{(\Sigma W+U)}$	
	Cracked D.S. Face		V	M	$\Sigma M+M_U$	$\frac{\Sigma M+M_U}{\Sigma W+U}$	$\frac{R_U}{R_{axis}}=1$	$\frac{l_g-e}{R_U}$	$\frac{R_O}{R_U}$	$\frac{R_O}{R_U}$	$\left(1-\frac{R_O}{R_U}\right)R_O$	$\frac{A_1}{T_1}$		$\frac{I_1}{T_1^3}$		$\frac{M_1}{(\Sigma W+U)T_1}$	
	S.L.	Load	Horiz Force		Total Moment	e		F		$R_O \cdot R_U$	T_1		A_1		I_1		M_1
			10^3	10^3	10^3												10^3
1	2345	−780	0	0	0												0
2	2320	+360	+5.250	+125.00	+178.39												+178.39
3	2295	+2125	−25.812	−40.106	+313.66												+313.66
4	2270	+3400	−94.875	−1,482.30	−489.88												−489.88
5	2245	+4550	−194.25	−5,036.5	−2,838.7												−2,838.7
6	2220	+6000	−326.12	−11,465.7	−7,106.5												−7,106.5
7	2195	+7375	−493.31	−21,637	−14,750	−22.769	·850,76	1.024,0	·933,59	·911,28	24.511	·881,03	21.595	·073,376	1,080.5	−4.130,1	−2,675.5
8B	[2170	+8700]															
9B	2165	+9000	−738.50	−39,994	−26,103	−29.544	·803,40	1.036,0	·904,05	·888,67	34.534	·846,03	29.217	·070,463	2,902.0	−5.849,0	−5,167.8

Cracked

Form C.6

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

Sheet

Computed CAW

Date

Checked DR

Date

Trial Load No 8

CLARK DAM

Study C3

$\overline{E} =$

$\frac{K}{G} =$

Properties and Forces. - Cracked Cantilever B

	Loads and Eccentricities				Properties at Horizontal Sections										Resultant Moment.	
Cracked U.S. Face		V	M	$\Sigma M+M_U$	$\frac{\Sigma M+M_U}{\Sigma W+U}$	$\frac{R_D}{R_U}$	$\frac{R_U l_g+e}{R_D}$	$\frac{R_D}{R_O}$	$\frac{R_D}{R_U}\cdot\frac{R_D}{R_O}$	$\left(\frac{R_O}{R_U}-\frac{R_D}{R_U}\right)R_U$	$\frac{A_1}{T_1}$		$\frac{I_1}{T_1^3}$		$\frac{M_1}{(\Sigma W+U)}$	
Cracked D.S. Face		V	M	$\Sigma M+M_U$	$\frac{\Sigma M+M_U}{\Sigma W+U}$	$\frac{R_U}{R_{axis}}=1$	$\frac{l_g-e}{R_U}$	$\frac{R_O}{R_U}$	$\frac{R_O}{R_U}$	$\left(1-\frac{R_O}{R_U}\right)R_U$	$\frac{A_1}{T_1}$		$\frac{I_1}{T_1^3}$		$\frac{M_1}{(\Sigma W+U)T_1}$	
S.L.	Load	Horiz Force		Total Moment	e		F		$\frac{R_O}{R_U}$	T_1		A_1		I_1		M_1
		10^3	10^3	10^3												10^3
1 2345	-530	0	0	0												0
2 2320	-490	+12·75	+161·46	+214·85												+214·85
3 2295	-440	+24·375	+628·13	+981·90												+981·90
4 2270	-510	+36·250	+1,382·3	+2,374·7												+2374·7
5 2245	-580	+49·875	+2,455·2	+4,653·0	+11·367	1	·019,677	·940,36		24·154	·970,18	23·434	·080,81	1,138·8	+3·983,2	+1,630·4
6 2220	-680	+65·625	+3,893·7	+8,252·9	+14·338	1	·023,363	·929,05		28·735	·964,52	27·715	·080,33	1,905·9	+4·728,1	+2,721·5
7 2195	-750	+83·500	+5,754·2	+13,856	+17·789	1	·028,694	·912,60		35·397	·956,30	33·850	·079,63	3,531·6	+5·806,5	+4,522·7
8B 2170	-850															
9B 2165	-900	+107·88	+8,614·8	+24,894	+22·933	1	·038,104	·883,32		47·255	·941,66	44·498	·078,38	8,270·8	+7·706,3	+8,365·2

21. Deflections of Cracked Cantilever. Form C.7.

The computation of the movements of a cracked cantilever, made on Form C.7, is similar to that for an uncracked cantilever. The method assumes that the concrete in the cracked zone plays no part in the elastic movement of the cantilever. Due to the reaction of the top arch $\frac{M_1}{EI_1}$ and $\frac{V}{A_1}$ frequently change sign near the top of the cantilevers. This must be noted in the summations.

The vertical stress σ_Z at the face is the sum of the direct and bending stresses. $\frac{\sigma_Z}{(\Sigma W + U)/A_1}$ (Formulae 72 and 79 in the Bulletin) is read from a graph, and then multiplied by $(\Sigma W+U)/A_1$ to obtain the vertical stress. The computation of the stress parallel to the downstream face is treated in the next section. Example: Cantilever B, Trial 11, Study C.1.

Form C.7

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

CLARK DAM

Study C1

Trial Load No 11

$\frac{1}{E} = 1{\cdot}736{,}1 \times 10^{-9}$

$\frac{K}{G} = 5{\cdot}208{,}3 \times 10^{-9}$

Deflection and Stresses. - Cracked Cantilever B

Sheet

Computed [illegible]

Date

Checked DR.

Date

	Bending Deflection				Shear Deflection			Total	Stresses at Uncracked Face lb/sq.in.			
S.L.	$\frac{M_1}{EI_1}$	$\frac{\Delta Z}{2}$	$\sum\left(\frac{M_1}{EI_1}\right)\Delta Z$	$\sum\left(\sum\frac{M_1}{EI_1}\Delta Z\right)\Delta Z$	$\frac{V}{A_1}$	$\frac{K}{G}\frac{\Delta Z}{2}$	$\sum\left(\frac{V}{A_1}\frac{K}{G}\right)\Delta Z$	Δr	σ_z $(\Sigma W+U)/A_1$	Vert	$Sec^2\phi$	Parallel to Face
	10^{-8}		10^{-6}	10^{-3}	10^{2}	10^{-8}	10^{-3}					
2345	0		-211·22	-32·351	0		-7·354,7	-·039,71		0	1·044	0
		12·5				6·510,4						
2320	+45·861		-216·95	-26·999	+2·658		-7·372,0	-·034,37		+7	1·048	+7
		12·5				"						
2295	+39·691		-227·64	-21·441	-10·366		-7·321,8	-·028,76		+24	1·061	+25
		12·5				"						
2270	-32·750		-228·51	-15·379	-30·962		-7·052,8	-·022,43		+83	1·088	+90
		12·5				"						
2245	-102·18		-211·64	-10·237	-51·898		-6·513,3	-·016,75		+159	1·138	+181
		12·5				"						
2220	-137·11		-181·73	-5·320,3	-71·339		-5·711,0	-·011,03		+224	1·222	+274
		12·5				"						
2195	-429·89		-110·86	-1·662,9	-228·44		-3·759,3	-·005,422	+·014,048	+421	1·403	+591
		15·0				7·812,4						
2165	-309·16		0	0	-252·76		0	0	+·014,135	+427	1·394	+595

uncracked

cracked

22. Slope of the Downstream Face. It is shown in The Bulletin that the stress σ'_{ZD} parallel to the face of the cantilever is equal to the vertical stress σ_{ZD} multiplied by $\sec^2\phi'_D$, where ϕ'_D is the inclination to the vertical of the line of maximum slope in the cantilever face at the point under consideration.

For the crown cantilever, and all cantilevers in a uniform thickness arch dam, $\phi'_D = \phi_D$, the inclination of the face in a vertical radial section through the centre of curvature of the upstream face. $\sec^2\phi_D$ equals $1+\tan^2\phi_D$.

Where the cantilever is located away from the crown in a variable thickness dam the line of greatest slope in the face is in the vertical radial plane through the centre of curvature of the downstream face at that elevation. In Figure 13 OABC is a tetrahedron at the D.S. face. ABC is the downstream face of the dam. AOD is the vertical radial plane through the c. of c. of the D.S. face. AOC is the vertical radial plane through the c. of c. of the U.S. face. AD is perpendicular to BC. β_D is the angle between the tangent to the intrados (BC) and a line (OB) normal to the radial arch section OC. β_D has been evaluated on Form C/A.1.

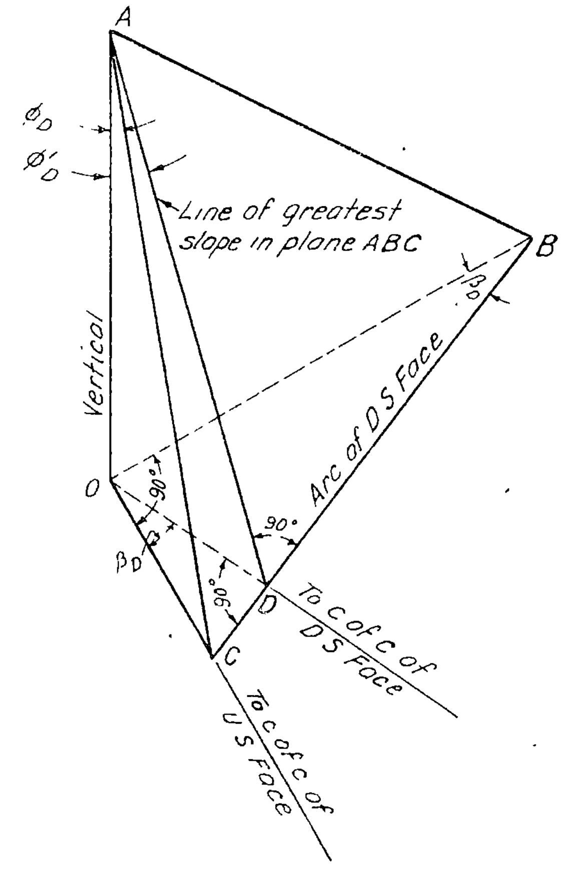


FIGURE 13 COMPUTATION OF ϕ'_D

Then $\tan \phi_D' = \frac{OD}{OA} = \frac{OD}{OC} \cdot \frac{OC}{OA} = \cos\beta_D \cdot \tan\phi_D$

and $\sec^2\phi_D' = 1 + \tan^2\phi_D'$.

At breaks of slope in the downstream face take the value of ϕ_D for the greater slope. $\tan\phi = \frac{\Delta T}{\Delta Z}$.

Example: Cantilever B. at 2245, T = 39.341 & at 2220 T=48.634

$\therefore \tan\phi_D = \frac{9.293}{25} = \cdot 371{,}72$. From Form C/A1,

β_D at SL2245 is $2^{\circ}46'30''$ (to nearest 10"),

so $\cos\beta_D = \cdot 998{,}83$

$\therefore \tan\phi_D' = \cdot 371{,}29$

$\sec^2\phi_D' = 1.137{,}9$.

PART D - ANALYSIS OF ARCHES

23. Theory of Arch Analysis. The prime requisites are:-

(1) The moment, thrust and shear at the crown, quarter-points and abutment for water and temperature loads for calculating deflections and stresses.

(2) Deflections of the arch centre line at the crown and quarter points (called "arch points") due to these loads for use in the radial adjustment.

The full trial load method provides for external radial, and internal tangential and twist loads, all of which cause radial, tangential and twist movements of the arch. This discussion is limited to radial loads and deflections.

The method is:- The arch is treated as two curved cantilevers, with crown forces applied to each half arch to replace the effect of the other. When the load is applied the crown deflections (angular, radial and tangential) for the two halves of the arch are equated. From the three simultaneous equations thus derived the crown forces are determined and used to compute the forces at the other arch points. From these, in turn, the arch deflections may be calculated.

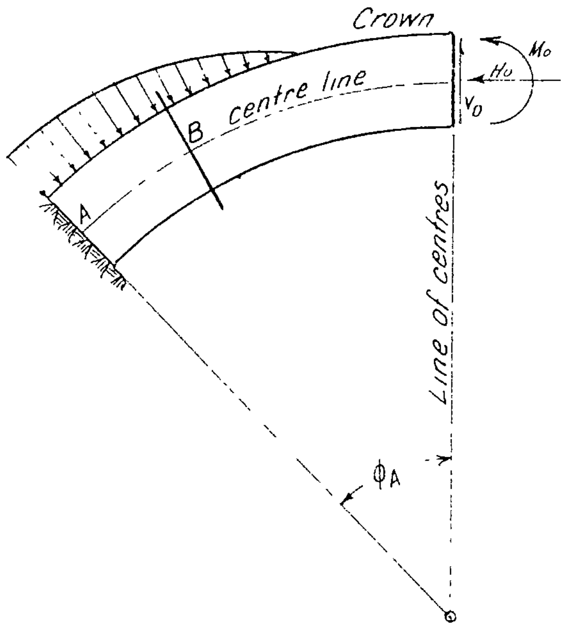


FIGURE 14   HALF ARCH

Referring to Figure 14, the deflection of the arch centre line at B is composed of two parts - (i) the deflection of the loaded cantilever AB due to load on it, between B and the abutment; (ii) the deflection of the unloaded cantilever AB due to the moment, thrust and shear at B which are computed from the crown forces and the effects of any load between B and the crown. Deflection (i) is called a Load Constant or D-term; its magnitude depends on the dimensions of the arch and the shape of the load pattern. Deflection (ii) is equal to (deflection at B due to unit force at B) x (force at B) . "Deflection at B due to unit force at B" is termed an Arch Constant, and is dependent only upon the dimensions of the arch. Hence the one set of Arch Constants suffices for all load patterns.

24. Arch Notation

All foot-pound units.

r = radius to centre line of arch.
r_v = radius to centre line of voussoir.
T_v = average radial thickness of voussoir (assumed equal to mean of end thicknesses).

r_p = $r_{0, 1,2,3,A}$ = radius to centre line of variable thickness arch at crown, $\frac{1}{4}$-point, $\frac{1}{2}$-point, $\frac{3}{4}$-point abutment respectively.

e = eccentricity of arch centre line at quarter points to centre line of voussoirs.

A = area of vertical cross-section of arch.
I = moment of inertia of vertical cross-section of arch.
P = magnitude of radial load at abutment, normally 1000 lb./sq.ft. for the unit loads.

t = temperature rise or fall above that at which the contraction joints were grouted, degrees Fahrenheit.

M_O = moment at crown. Positive M_O causes compression at the upstream face.

H_O = thrust at crown. Positive H_O causes compression.

V_O = shear at crown. Positive V_O causes positive moments in the arch to the left.

M_L, H_L, V_L - Moment, thrust and shear due to applied load on the left part of the arch, considered as a curved cantilever. A positive unit load causes positive M_L, H_L, V_L.

M,H,V - moment, thrust and shear at arch points due to crown forces and applied load. Same sign convention as crown forces. Subscript A refers to abutment forces.

Δr - radial deflection of centre line of arch, positive upstream.

x, y - coordinates. See pp.40,44.

ϕ arch angle. The Bulletin uses ϕ for every arch angle, changing the meaning of the subscripts every few pages, especially in the derivation of arch equations. In the radial adjustment ϕ has the following meanings:-

Form C/A.1. ϕ is the angle from the crown to the point considered, degrees.

Forms A.2, A.4, A.6. ϕ_1, ϕ_2, ϕ_3, ϕ_A - angles from the crown to the $\frac{1}{4}$-point, $\frac{1}{2}$-point, $\frac{3}{4}$-point, and abutment respectively.

Form A3. ϕ_1, ϕ_2, ϕ_3, ϕ_A - magnitude of angles from crown to $\frac{1}{4}$-point, $\frac{1}{2}$-point, $\frac{3}{4}$-point, and abutment respectively.

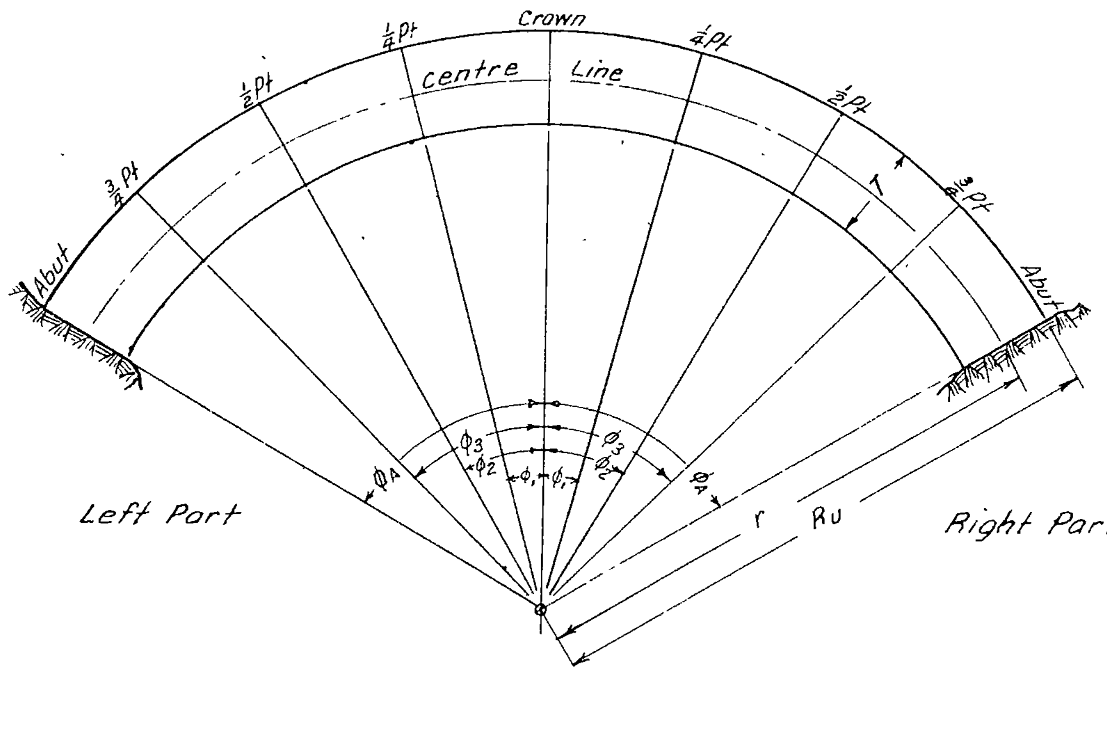


FIGURE 15 UNIFORM THICKNESS CIRCULAR ARCH

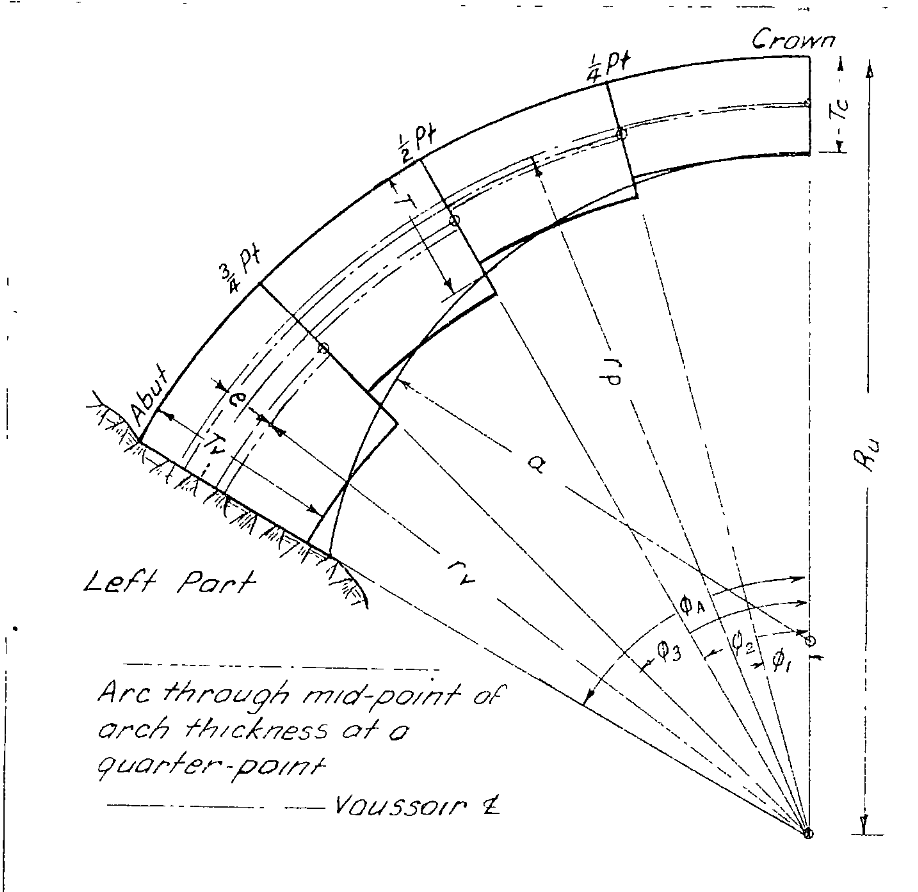


FIGURE 16 - VARIABLE THICKNESS ARCH

Arch Constants

Constant	Movement	at point due to	Unit force	at the point
A_1	Angular		Moment	
B_1	Angular or Tangential		Thrust Moment	
C_1	Angular or Radial		Shear Moment	
B_2	Radial or Tangential		Thrust Shear	
C_2	Radial		Shear	
B_3	Tangential		Thrust	

Load Constants

D_1 is the angular movement) at a point
D_2 is the radial deflection) due to loads between the
D_3 is the tangential deflection) point and the abutment.

A_1' etc. - "primo" values of A_1 etc. - i.e. the tabular values of the constants.

$\Delta A_1'$ etc. the difference between tabular values of A_1 etc. for the angles to the ends of the voussoir from the point considered.

25. Arch Thicknesses. Form C/Al. The radial thickness of a variable thickness arch is required at the crown, ¼-point, ½-point, ¾-point, and abutment. The method employed is set out in Section 14.

The figures for Arch V., the sample arch for this description, are given on the next page.

Form C/A.1

Sheet ....

THE HYDRO-ELECTRIC COMMISSION, TASMANIA

CLARK DAM

Computed ....Jw....

Date ....

Study ....C....

Checked ....DL....

Date ....

Radial Thicknesses of ....ARCH.... No ....V....

S.L. or Point	a	x	ϕ	$\sin\phi$	$\frac{x}{a}\sin\phi$	β	$\alpha = \beta + \phi$	$\sin\alpha$	$R_D = a\frac{\sin\alpha}{\sin\phi}$	$T = R_U - R_D$, $R_U = 405$
Cr.										37.000
¼ Pt	310.0	58.0	11°00'	·190,809,0	·035,699,75	2°02'45"	13°02'45"	·225,730,4	366.735	38.265
½ Pt	310.0	58.0	22°00'	·374,606,6	·070,087,68	4°01'08"	26°01'08"	·438,667,4	363.013	41.987
¾ Pt	310.0	58.0	33°00'	·544,639,0	·101,900,2	5°50'55"	38°50'55"	·627,264,8	357.029	47.971
Abut.	310.0	58.0	44°00'	·694,658,4	·129,968,3	7°28'04"	51°28'04"	·782,257,9	349.092	55.908
			$\frac{x}{a}$ =	·187,096,77						

UNIFORM THICKNESS CIRCULAR ARCHES

26. Arch Constants and Trigonometric Functions, Form A4.

The arch constants are composed of two factors - a "multiplier" dependent on the thickness, centre line radius and modulus of elasticity of the arch, and a "trigonometric part" dependent only on the arch angle. The multipliers (e.g. $\frac{12\,r}{ET^3}$) are computed and recorded for ready reference.

The list of multipliers required, the same as for the variable thickness arch, Form A.2, is shown in Figure 40 of The Bulletin. It is expedient to calculate also at this stage the multipliers required for the load constants. The trigonometric parts (A_1 etc.) for the angle from the abutment to the arch point considered (ϕ_1 for the $\frac{3}{4}$-point, ϕ_2 for the $\frac{1}{2}$-point, etc.) are tabulated in Table 10 of The Bulletin. Some of these consist of two parts, the first due to bending, and the second to shear deflection.

e.g. For Arch No. I, $\phi_A = 60^\circ$

$$B_3 \text{ (at crown)} = \frac{12r^3}{ET^3} B'_{31} + \frac{r}{ET} B'_{32}$$

$$= (2\cdot218{,}333\times10^{-4}) \times \cdot055{,}251{,}87 + (3\cdot819{,}444 \times 10^{-8}) \times 1\cdot661{,}382$$

$$= 1\cdot232{,}016 \times 10^{-5}$$

In this simplified analysis C_2 at the crown is not required because the shear at the crown of a symmetrical arch is zero. The crown values only of A_1, B_1 and B_3 are required for solution of crown forces, as these arch constants are for angular and tangential movements. The arch constants are now tabulated on Form A4, and the remainder of the sheet filled in.

For the "ϕ-Points" x and y are the coordinates of the point with respect to the crown and are used in the computation of the moment, thrust and shear at the arch points. See Fig. 17a.

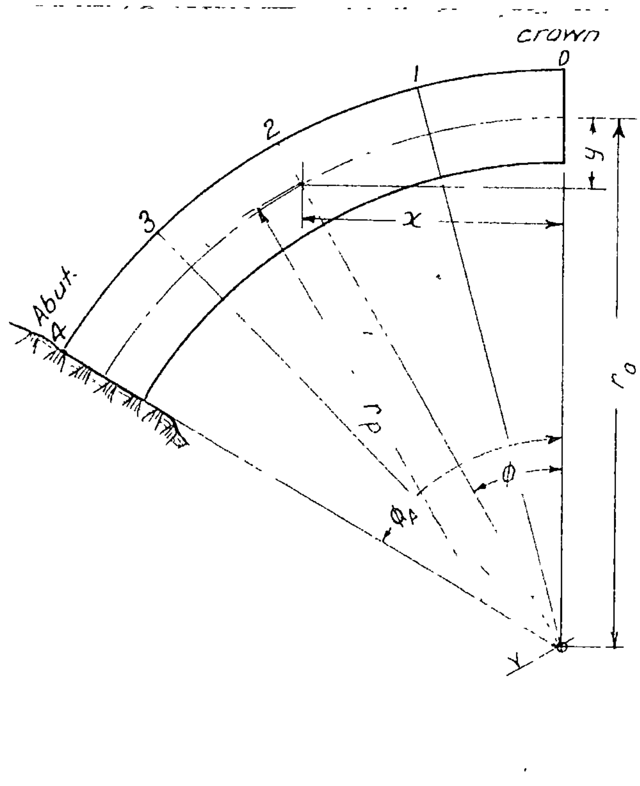


FIGURE 17a - ϕ-POINT CO-ORDINATES

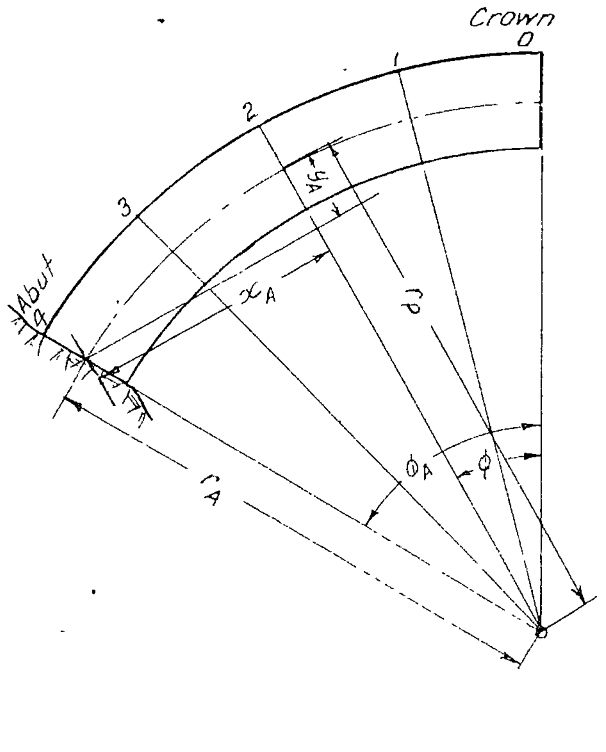


FIGURE 17b - (ϕ_A-ϕ)-POINT CO-ORDINATES

For a uniform thickness arch, $r_o = r_p = r$, and hence

$$x = r \sin\phi, \quad y = r \operatorname{vers}\phi.$$

For the "$(\phi_A - \phi)$-Points" x_A and y_A are the coordinates of the arch points with respect to the abutment. See Figure 17b. These are used in determining the temperature D-terms. In a uniform thickness arch these two sets of coordinates are complementary so that the one calculation serves both.

27. Load Constants. Similarly the load constants equal "multiplier" x "trigonometric part", the latter dependent upon three factors - the load pattern, the arch angle ϕ_A and the position of the point considered. These "prime" D-terms have been evaluated by the U.S.B.R. for five basic unit loads and are tabulated in The Bulletin, Tables 17-21. The unit radial loads are:-

No.1 - a uniform load of 1000 lb./sq.ft.

Nos. 2-5 tapered loads, varying linearly from 1000 lb./sq.ft. at the abutment to zero at the $\frac{3}{4}$-point (No.2), $\frac{1}{2}$-point (No.3), $\frac{1}{4}$-point (No.4) and crown (No.5), all applied to the upstream face. See Fig. 18. Any load pattern which varies linearly between the quarter points may be built up with positive and negative multiples of these unit loads. D_1 and D_3 for each load are required at the crown only (for solution of the crown forces). D_2, the radial deflection, is computed for all quarter points. Note that the crown values of D_1' and D_3', and the values of D_2' at all arch points are listed in the one line for ϕ_A in Table 17. For a temperature load $D_1 = 0$, $D_2 = -cty_A$ and $D_3 = ctx_A$.

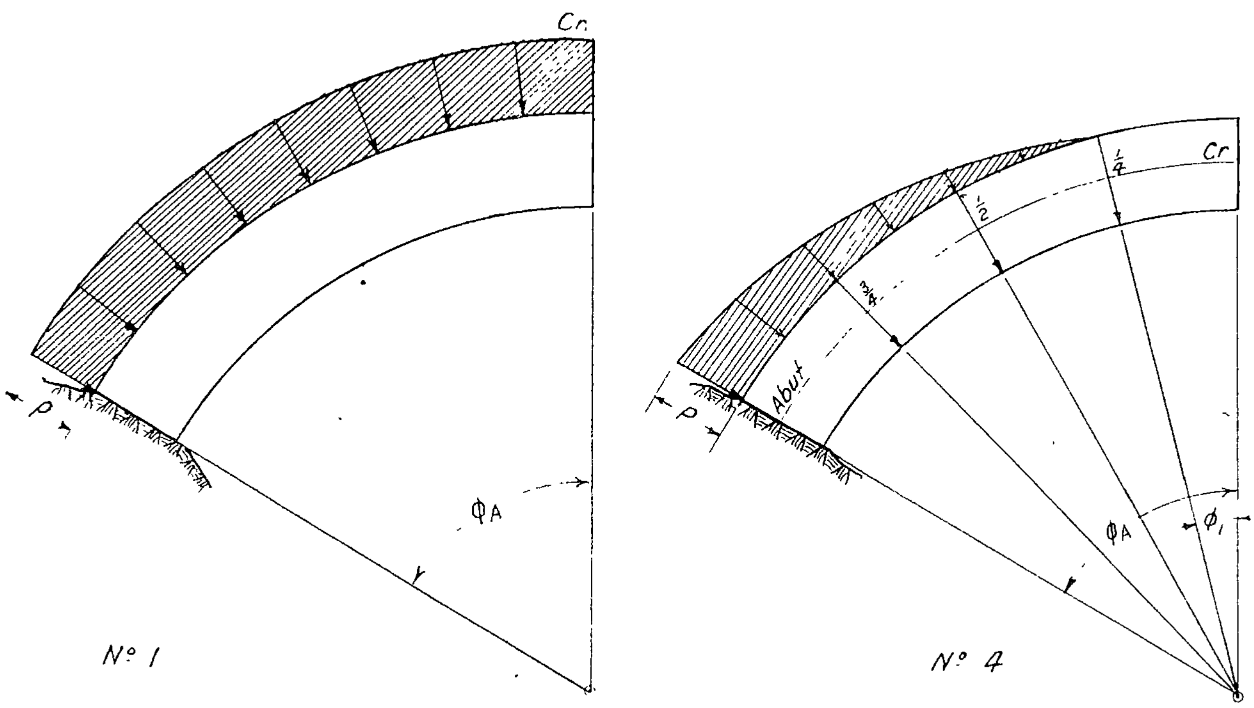


FIGURE 18 UNIT RADIAL LOADS

Tabulate D_1 and D_3 for all loads on the one arch on one page for reference in computing crown forces, and transfer D_2 to Form A.6 for each load.

All arch and load constants are assumed positive as calculated.

e.g. For Arch No.I, $\phi_4 = 60^\circ$, Radial Load No.4,

$$D_1 \text{ at crown} = \frac{12r^2R_u}{ET^3} \times D_1'$$
$$= 2{\cdot}268,750 \times 10^{-4} \times 19{\cdot}775,85$$
$$= 4{\cdot}486,646 \times 10^{-3}$$

$$D_2 \text{ at } \tfrac{1}{2} \text{ pt.} = \frac{12r^3Ru}{ET^3} \times D_{21}' \quad \frac{rRu}{ET} . D'_{22}$$
$$= (8{\cdot}984,250 \times 10^{-2}) \times 4{\cdot}592,150$$
$$+ (1{\cdot}546,875 \times 10^{-5}) \times 212{\cdot}458,1$$
$$= {\cdot}415,856,7.$$

The procedure from this stage onwards in arch analysis is the same for both uniform and variable thickness arches.

VARIABLE THICKNESS ARCHES

28. Eccentricities and Multipliers. Form A.2.

Both sides of the arch are divided into four voussoirs, each subtending $\frac{1}{4}\phi_A$ at the centre of curvature of the upstream face; then each voussoir is treated as a section of a uniform thickness circular arch whose width is equal to the average thickness of the voussoir. Therefore each voussoir has its own set of multipliers and its centre line is eccentric with respect to the true centre line at the arch points. These quantities are computed on Form A.2. which is self-explanatory. Example, page 42.

Form A.2.

THE HYDRO ELECTRIC COMMISSION TASMANIA

CLARK DAM

STUDY C

ECCENTRICITIES AND MULTIPLIERS FOR ARCH No V

Sheet ____
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Date ____
Checked DR.
Date ____

POINT	ϕ	VOUSSOIR	AT POINT		FOR VOUSSOIR		VOUSSOIR TO POINT			
			T	r_p	T_v	r_v	e_0	e_1	e_2	e_3
0 (Cr)	0°		37·000	386·500,0						
		0 - 1			37·632	386·184,0	0·316,0			
1 ($\frac{1}{4}$)	11°		38·265	385·867,5						
		1 - 2			40·126	384·937,0	1·563,0	0·930,5		
2 ($\frac{1}{2}$)	22°		41·987	384·006,5						
		2 - 3			44·979	382·510,5	3·989,5	3·357,0	1·496,0	
3 ($\frac{3}{4}$)	33°		47·971	381·014,5						
		3 - 4			51·940	379·030,0	7·470,0	6·837,5	4·976,5	1·984,5
4 (Abut)	44°		55·908	377·046,0						

$E = 5{\cdot}76 \times 10^{8}$ lb/sq. ft.

$\frac{1}{E} = 1{\cdot}736,111 \times 10^{-9}$

$R_U = 405{\cdot}000$ ft.

$r_p = R_U - \frac{T}{2}$. $r_v = R_U - \frac{T_v}{2}$ $I_v = \frac{T_v^3}{12}$

$e_0, e_1, e_2, e_3 = r_p - R_v = e$ for transfer of constants

VOUSSOIR	$\frac{1}{ET_v}$	$\frac{r_v}{ET_v}$	$\frac{r_v^2}{ET_v}$	$\frac{r_v}{EI_v}$	$\frac{r_v^2}{EI_v}$	$\frac{r_v^3}{EI_v}$	$\frac{r_v R_U}{ET_v}$	$\frac{r_v^2 R_U}{EI_v}$	$\frac{r_v^3 R_U}{EI_v}$
		10^{-8}		10^{-11}	10^{-8}	10^{-6}	10^{-6}	10^{-6}	10^{-3}
0-1		1·781,617		15·096,67	5·830,092	22·514,88	7·215,549	23·611,87	9·118,527
1-2		1·665,487		12·412,83	4·778,158	18·392,90	6·745,222	19·351,54	7·449,124
2-3		1·476,424		8·757,351	3·349,779	12·813,26	5·979,517	13·566,60	5·189,368
3-4		1·266,920		5·635,423	2·135,994	8·096,058	5·131,026	8·650,776	3·278,903

29. Arch Constants and Trigonometric Functions, Forms A.3 & A.4

The deflection of the arch centre line at any point is the sum of the contributions of the voussoirs 4-3, 3-2 -- outwards from the abutment. The application of arch constants for a uniform thickness arch to the analysis of a variable thickness arch requires the determination of the effects of movements of an assumed uniform thickness voussoir on all arch points between its outer end and the crown. The contribution of a voussoir to a constant at an arch point is the product of the multiplier for that voussoir and the increment of the uniform thickness values for the angles from the point considered to the limits of the voussoir.

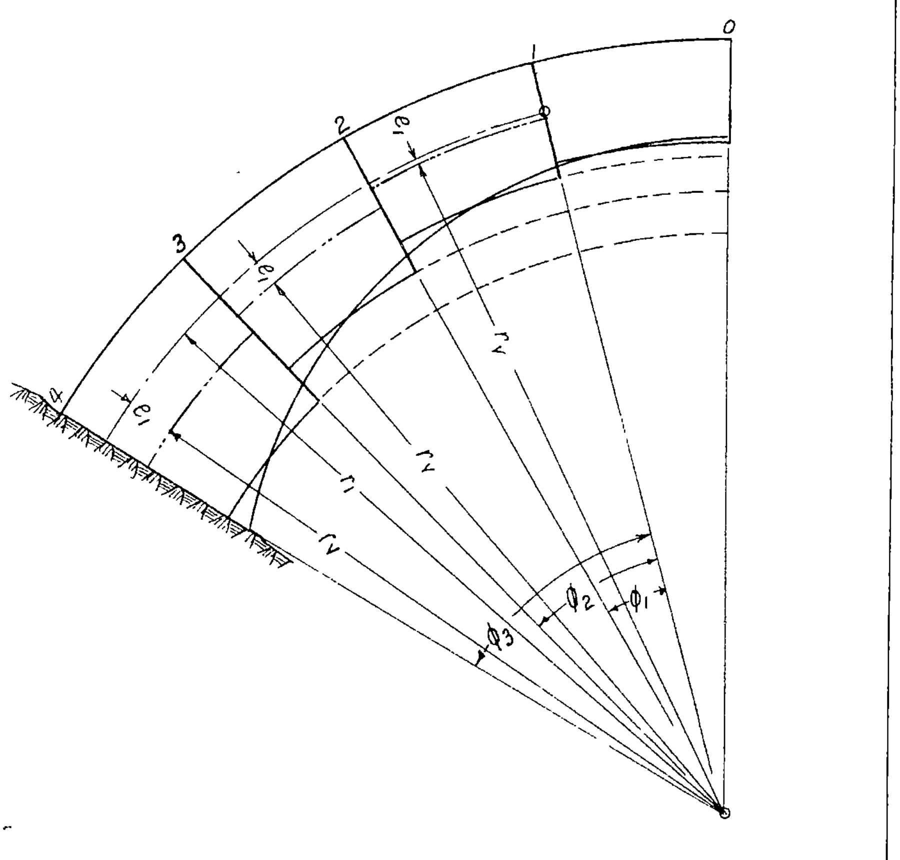

FIGURE 19 ARCH CONSTANTS FOR $\frac{1}{4}$-POINT

For example the angular deflection of the quarter point due to unit moment there = (i) contribution of voussoir 4-3 (ii) contribution of voussoir 3-2 (iii) contribution of voussoir 2-1. Item (i) = (deflection at the crown of a uniform thickness arch of length ϕ_3 and of thickness T_v for voussoir 4-3) - (deflection at the crown of a uniform arch of the same thickness, of length ϕ_2) = multiplier for voussoir 4-3 x (A_1' for ϕ_3 - A_1' for ϕ_2. The other items are found in a similar manner. See Figure 19.

The abrupt changes in r and T at the quarter points cause the appearance of constants of integration in the expressions for the arch constants. The physical interpretation of these is that a unit thrust applied at the true centre line of the arch at a quarter point is eccentric to the centre line of the voussoirs, thereby causing a moment. The extra terms involving the eccentricities appear then only in the Arch Constants B_1, B_2 and B_3 -movements due to unit thrusts.

A full list is given in paragraph 104 of The Bulletin. The methods of setting out and computing the Arch Constants are shown in the sample set of Form A.3 included. When computing terms involving eccentricities, use the values under "e", for the crown, "e_1" for $\frac{1}{4}$-point, etc., for the corresponding voussoirs.

Form A.4 may now be completed. The arch constants are transferred from Form A.3 and the trigonometric functions copied from Table 22. The coordinates of the arch points with respect to the crown are $x = r_p \sin\phi$, $y = r_o - r_p \cos\phi$, where ϕ is the angle from the crown to the point. The coordinates x_A, y_A with respect to the abutment are $x_A = r_A \sin(\phi_A - \phi)$,

$$y_A = r_p - r_A \cos(\phi_A - \phi).$$

Note that in The Bulletin the two sets of formulae for x_A, y_A are apparently inconsistent, because ϕ_A refers to a different angle in the two cases. The line of headings "($\phi_A - \phi$) Point" is not used for the variable thickness arch. Example: Form A.4 for Arch V.

Form A.3.

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

C L A R K D A M.

Study C

Sheet

Computed L.A.W.

Date

Checked D.R.

Date

Arch No. V ϕ_A 44° Functions A_1, B_1

	Pt.	ϕ	A_1'	$\Delta A_1'$	$\frac{r}{EI}\Delta A_1'$	$\frac{r}{EI}\Delta A_1' e$	$\Sigma\frac{r}{EI}\Delta A_1' e$ (B_1)	$\Sigma\frac{r}{EI}\Delta A_1' e^2$ (B_3)	$\Sigma\frac{r}{EI}\Delta A_1'$ $\boxed{A_1}$
CROWN	0	0	0		10^{-11}	10^{-12}	10^{-10}	10^{-10}	10^{-11}
				·191,986,2	2·898,352	9·158,792	1·943,013	9·324,313	8·044,658
	1	11°	·191,986,2						
				·191,986,2	2·383,092	37·247,73			
	2	22°	·383,972,4						
				·191,986,3	1·681,291	67·075,10			
	3	33°	·575,958,7						
				·191,986,2	1·081,923	80·819,65			
	4	44°	·767,944,9						

	1								
	2								

	$\frac{1}{4}$	ϕ	B_1'	$\Delta B_1'$	$\frac{r^2}{EI}\Delta B_1'$	$\Sigma\frac{r^2}{EI}\Delta B_1'$ (a)	$\Sigma\frac{r}{EI}\Delta A_1' e$ (b)	$\boxed{B_1}$ (a)+(b)	$2\Sigma\frac{r^2}{EI}\Delta B_1' e$ (B_3)
CROWN	0	0°	0		10^{-11}	10^{-9}	10^{-9}	10^{-9}	10^{-8}
				·001,177,222	6·863,313	2·091,711	·194,301	2·286,012	2·052,662
	1	11°	·001,177,222						
				·008,188,620	39·126,52				
	2	22°	·009,365,842						
				·021,953,78	73·540,31				
	3	33°	·031,319,62						
				·041,966,88	89·641,00				
	4	44°	·073,286,50						

Form A.3.

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Arch No. V ϕ_A 44°   Functions B_3, C_2

Prime values in red. Δ values in black

	Pt.	ϕ	B'_{31}	B'_{32}	$\Sigma \frac{r^3}{EI} \Delta B'_{31}$ (a)	$\Sigma \frac{r}{ET} \Delta B'_{32}$ (b)	$\Sigma \frac{r}{EI} \Delta A'_1 e^2$ (c)	$2\Sigma \frac{r'}{EI} \Delta B'_1 e$ (d)	a+b+c+d $\boxed{B_3}$
CROWN	0	0	0	0	10^{-7}	10^{-7}	10^{-7}	10^{-7}	10^{-7}
			·000,012,984,18	·196,669,1	1·174,933	·156,077,3	·009,324,3	·205,266,2	1·545,601
	1	11°	·000,012,984,18	·196,669,1					
			·000,397,074,7	·223,946,6					
	2	22°	·000,410,058,9	·420,615,7					
			·002,636,215	·274,528,9					
	3	33°	·003,046,274	·695,144,6					
			·009,401,996	·341,049,7					
	4	44°	·012,448,27	1·036,194,3					
			C'_{21}	C'_{22}	$\Delta C'_{21}$	$\Delta C'_{22}$	$\Sigma \frac{r^3}{EI} \Delta C'_{21}$	$\Sigma \frac{r}{ET} \Delta C'_{22}$	$\boxed{C_2}$
1/4	1	0	0	0			10^{-8}	10^{-8}	10^{-8}
					·002,341,461	·571,275,7	58·195,94	2·379,743	60·575,68
	2	11°	·002,341,461	·571,275,7					
					·015,980,17	·543,998,4			
	3	22°	·018,321,63	1·115,274,1					
					·041,271,33	·493,415,9			
	4	33°	·059,592,96	1·608,690,0					
1/2	2	0							
					·002,341,461	·571,275,7	15·937,81	1·532,648	17·470,46
	3	11°							
					·015,980,17	·543,998,4			
	4	22°							

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Arch No. V ϕ_A 44° Functions C_1

	Pt.	ϕ	C_1'	$\Delta C_1'$	$\frac{r^2}{EI}\Delta C_1'$		$\sum \frac{r^2}{EI}\Delta C_1' e$ (B₁)		$\sum \frac{r^2}{EI}\Delta C_1'$ $\boxed{C_1}$
CROWN	0	0	0		10^{-10}		10^{-10}		10^{-10}
				·018,372,82	10·711,52		352·735,9		91·864,37
	1	11°	·018,372,82						
				·054,443,33	26·013,88				
	2	22°	·072,816,15						
				·088,513,25	29·649,98				
	3	33°	·161,329,4						
				·119,330,8	25·488,99				
	4	44°	·280,660,2						
1/4	1	0							
				·018,372,82	8·778,824		198·663,7		45·922,51
	2	11°							
				·054,443,33	18·237,31				
	3	22°							
				·088,513,25	18·906,38				
	4	33°							
1/2	2	0							
				·018,372,82	6·154,489		67·079,13		17·783,55
	3	11°							
				·054,443,33	11·629,06				
	4	22°							

Form A.3.

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

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Arch No. V ϕ_A 44° Functions B2

	Pt.	ϕ	B'_{21}	B'_{22}	$\Sigma \frac{r^3}{EI} \Delta B'_{21}$ a	$\Sigma \frac{r}{ET} \Delta B'_{22}$ b	$\Sigma \frac{r}{EI} \Delta G'_e$ c		a+b+c $\boxed{B2}$
CROWN	0	0	0	0	10^{-9}	10^{-9}	10^{-9}		10^{-9}
			·000,168,780,2	·036,408,07	395·739,4	7·042,574	35·273,59		438·055,6
	1	11°	·000,168,780,2	·036,408,07					
			·002,482,316	·103,922,0					
	2	22°	·002,651,096	·140,330,1					
			·010,362,49	·156,301,6					
	3	33°	·013,013,59	·296,631,7					
			·026,371,48	·185,918,6					
	4	44°	·039,385,07	·482,550,3					
1/4	1	0							
			·000,168,780,2	·036,408,07	118·806,2	4·120,917	19·866,37		142·793,5
	2	11°							
			·002,482,316	·103,922,0					
	3	22°							
			·010,362,49	·156,301,6					
	4	33°							
1/2	2	0							
			·000,168,780,2	·036,408,07	22·259,60	1·854,146	6·707,913		30·821,66
	3	11°							
			·002,482,316	·103,922,0					
	4	22°							

Form A.4. THE HYDRO ELECTRIC COMMISSION TASMANIA

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STUDY C

FUNCTIONS OF ARCH No V

Arch Term	$\frac{3}{4}$ Point	$\frac{1}{2}$ Point	$\frac{1}{4}$ Point	Crown	
A_1				$8.044,658 \times 10^{-11}$	A_1
B_1				$2.286,012 \times 10^{-9}$	B_1
C_1	$3.924,423 \times 10^{-10}$	$17.783,55 \times 10^{-10}$	$45.922,51 \times 10^{-10}$	$91.864,37 \times 10^{-10}$	C_1
B_2	$2.606,517 \times 10^{-9}$	$30.821,66 \times 10^{-9}$	$142.793,5 \times 10^{-9}$	$438.055,6 \times 10^{-9}$	B_2
C_2	$2.619,421 \times 10^{-8}$	$17.470,46 \times 10^{-8}$	$60.575,68 \times 10^{-8}$	—	C_2
B_3				$1.545,601 \times 10^{-7}$	B_3

ϕ Point	Abut. $\phi A = 44^\circ$	$\frac{3}{4}$ Point, $\phi_3 = 33^\circ$	$\frac{1}{2}$ Point, $\phi_2 = 22^\circ$	$\frac{1}{4}$ Point, $\phi_1 = 11^\circ$	Crown
ϕ	·767,944,9	·575,958,7	·383,972,4	·191,986,2	0
$\sin\phi$	·694,658,4	·544,639,0	·374,606,4	·190,809,0	0
$\cos\phi$	·719,339,8	·838,670,6	·927,183,9	·981,627,2	1
Vers ϕ	·280,660,2	·161,329,4	·072,816,15	·018,372,82	0
$\phi - \sin\phi$	·073,286,50	·031,319,62	·009,365,842	·001,177,222	0
$x = r_p \sin\phi$	261·918	207·515	143·851	73·627	0
$y = r_o - r_p \cos\phi$	115·276	66·954	30·455	7·722	0

~~$(\phi_A - \phi)$ Point~~	~~Crown~~	~~$\frac{1}{4}$ Point~~	~~$\frac{1}{2}$ Point~~	~~$\frac{3}{4}$ Point~~	~~Abutment~~
$x_A = r_A \sin(\phi_A - \phi)$	0	71·944	141·244	205·354	261·918
$y_A = r_p - r_A \cos(\phi_A - \phi)$	0	10·896	34·416	69·650	115·276

30. Load Constants, Form A.3. Load constants are procured in much the same manner as arch constants. Paragraph 106 of The Bulletin gives the expressions for D_1, D_2 and D_3 for radial loads. In the case of the uniform load the computation is similar to that for the arch constants except that the prime values are selected from the one line of the Table for ϕ_A. Prime values D_2' are drawn from Table 17, and D_1' and D_3' from Table 18. As for the uniform thickness arch D_1 and D_3 are required at the crown only.

The case of the triangular loads is more complex, because for points under the load, the load between the point and the abutment is trapezoidal, and hence prime values cannot be selected directly from the tables. The essential point to remember is that the D-terms are deflections at a point due to loads between that point and the abutment. The method employed to determine the prime values to use is best explained by an example.

Consider the derivation of the D-terms at the crown for Load No.5.

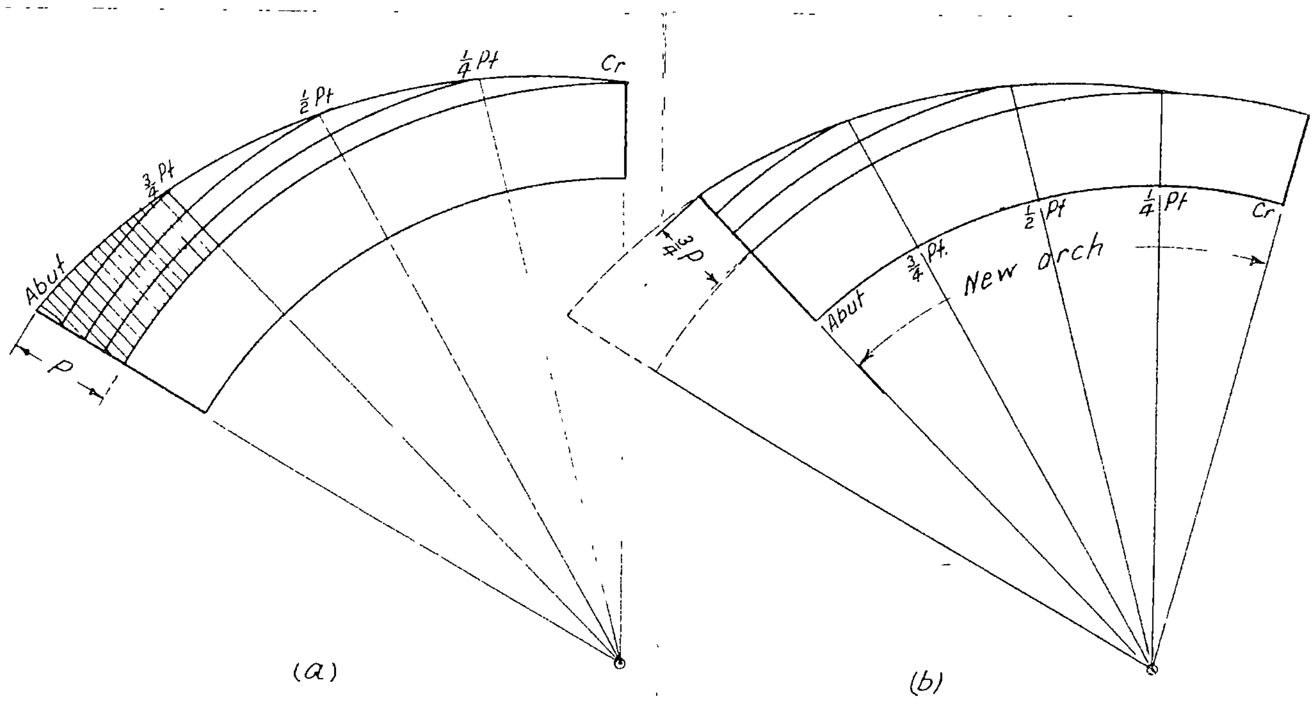

FIGURE 20 D-TERMS AT CROWN FOR N° 5 LOAD

From Figure 20a it may be seen that the contribution of voussoir 4-3 to D at the crown is:

multiplier x (D' for Load No.5, crown, for ϕ_A)

- multiplier x (D' at crown for load between $\frac{3}{4}$-point & crown)

Now by shifting the arch round to the right under the load (Figure 20b), until the new arch abutment is under the $\frac{3}{4}$-point of the load, it is seen that D' at the crown of the original arch (for the unshaded portion of the load) equals D' at the $\frac{1}{4}$-point of the new arch of angle ϕ_A for a No.4 load of intensity $\frac{3}{4}$P. Hence the contribution of voussoir 4-3 to D at the crown is:

multiplier x (D' at crown for No.5 - $\frac{3}{4}$ D' at $\frac{1}{4}$-pt. for No.4).

Similarly, for voussoir 3-2:

multiplier x ($\frac{3}{4}$D' at $\frac{1}{4}$-pt., No.4 - $\frac{1}{2}$D' at $\frac{1}{2}$-pt.,No.3); and so on. All D' terms are taken from the line for ϕ_A.

Likewise for a point under the load, for Load No.5, D at $\frac{1}{2}$-pt. = multiplier for voussoir 4-3 x (D' at $\frac{1}{2}$-pt., No.5 - $\frac{3}{4}$D' at $\frac{3}{4}$-pt., No.4) + multiplier for voussoir 3-2 x($\frac{3}{4}$D' at $\frac{3}{4}$-pt., No.4).

A complete list is given in Table 9 in The Bulletin.

Temperature load D-terms for a load of +1°F. are computed as for the uniform circular thickness arch, using the coordinates x_A, y_A.

Tabulate D_1 and D_3 on one page, and transfer D_2 to the relevant Form A.6.

Example: D_1, D_2 and D_3 for Loads 1 and 4 on Arch V.

Form A.3.

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Arch No. V ϕ_A 44° Functions Load (1), D_1 & D_3

	Pt.	ϕ	D_1'	$\Delta D_1'$	$\frac{r^2R_v}{EI}\Delta D_1'$		$\sum\frac{r^2R_v}{EI}\Delta D_1'e$ (D_2)		$\frac{\sum r^2R_v}{EI}\Delta D_1'$ $\boxed{D_1}$
CROWN	0		0		10^{-4}		10^{-3}		10^{-4}
				1·177,222	·277,964,2		4·156,640		8·471,431
	1		1·177,222						
				8·188,620	1·584,624				
	2		9·365,842						
				21·953,78	2·978,382				
	3		31·319,62						
				41·966,88	3·630,461				
	4		73·286,50						
	1								
	2								

Y_Y			D_{31}'	D_{32}'	a $\sum\frac{r^3R_v}{EI}\Delta D_{31}'$	b $\sum\frac{rR_v}{EI}\Delta D_{32}'$	c $\sum\frac{r^2R_v}{EI}\Delta D_1'e$		a+b+c $\boxed{D_3}$
CROWN	0		0	0	10^{-2}	10^{-2}	10^{-2}		10^{-2}
			·012,984,1	5·860,143	4·758,478	·191,811,8	·415,664,0		5·365,954
	1		·012,984,1	5·860,143					
			·397,074,8	40·148,95					
	2		·410,058,9	46·009,09					
			2·636,215	104·496,4					
	3		3·046,274	150·505,5					
			9·401,996	191·030,5					
	4		12·448,27	341·536,0					

Form A.3. THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

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Arch No. V ϕ_A 44° Functions Load (1), D_2

PT.		ϕ	D'_{21}	D'_{22}	$\Delta D'_{21}$	$\Delta D'_{22}$	a $\sum \frac{r^3 R_v}{EI} \Delta D'_{21}$	b $\sum \frac{r R_v}{ET} \Delta D'_{22}$	a+b $\boxed{D_2}$
CROWN	0		0	0					
					·168,780,2	54·780,89	·160,274,4	·004,493,601	·164,768,0
	1		·168,780,2	54·780,89					
					2·482,316	158·365,3			
	2		2·651,096	213·146,2					
					10·362,49	244·814,9			
	3		13·013,59	457·961,1					
					26·371,48	305·249,4			
	4		39·385,07	763·210,5					

1/4	1								
					·168,780,2	54·780,89	·048,116,51	·002,572,609	·050,689,12
	2								
					2·482,316	158·365,3			
	3								
					10·362,49	244·814,9			
	4								
1/2	2								
					·168,780,2	54·780,89	·009,015,136	·001,140,140	·010,155,28
	3								
					2·482,316	158·365,3			
	4								
3/4	3								
					·168,780,2	54·780,89	·000,553,413,9	·000,281,082,2	·000,834,496,1
	4								

Form A.3.

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Arch No. V ϕ_A 44° Functions Load 4, D_1 & D_3

	Pt.	ϕ	D_1'	$\Delta D_1'$	$\frac{r^2 R_v}{EI} \Delta D_1'$		$\sum \frac{r^2 R_v}{EI} \Delta D_1' e$ (D3)		$\sum \frac{r^2 R_v}{EI} \Delta D_1'$ D_1
CROWN	0		0		10^{-6}		10^{-4}		10^{-5}
				0	0		4·900,190		7·637,125
	1		0						
				·098,162,17	1·899,589				
	2		⅓ No2, ½ pt ·098,162,17						
				1·466,655	19·897,52				
	3		⅔ No3, ½ pt 1·564,817						
				6·308,583	54·574,14				
	4		No. 4, Cr. 7·873,400						
	1								
	2								

¼		D_{31}'	D_{32}'	a $\sum \frac{r^2 R_v}{EI} \Delta D_{31}'$	b $\sum \frac{r R_v}{EI} \Delta D_{32}'$	c $\sum \frac{r^2 R_v}{EI} \Delta D_1' e$		a+b+c D_3
CROWN	0	0	0	10^{-3}	10^{-3}	10^{-3}		10^{-3}
		0	0	5·777,829	·473,002,8	·490,019,0		6·740,851
	4	0	0					
½ CROWN		·005,848,397	1·927,760					
	2	⅓ No2, ½ pt. ·005,848,397	1·927,760					
		·187,739,1	19·156,95					
	3	⅔ No3, ½ pt ·193,587,5	21·084,71					
		1·451,710	67·325,78					
	4	No 4, Cr. 1·645,297	88·410,49					
¾	3							

Form A.3.

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Arch No. V ϕ_A 44° Functions Load (4) D_2

	Pt.	ϕ	D'_{21}	D'_{22}	$\Delta D'_{21}$	$\Delta D'_{22}$	$\sum \frac{r^3 R_v}{EI} \Delta D'_{21}$	$\sum \frac{r R_v}{ET} \Delta D'_{22}$	$\boxed{D_2}$
	0		0	0					
					0	0	·017,109,40	·000,741,515,2	·017,850,92
	1		0	0					
					·033,234,16	5·818,250			
CROWN	2		⅓ No 2, ½ pt ·033,234,16	5·818,250					
					·714,269,8	38·085,41			
	3		⅔ No 3, ¼ pt ·747,504,0	43·903,66					
					4·012,082	92·483,94			
	4		No 4, Cr. 4·759,586	136·387,6					

1/4	1		0	0					
					·015,009,25	6·079,187	·012,358,81	·000,818,144,6	·013,176,95
	2		⅓ No 2, ¾ pt ·015,009,25	6·079,187					
					·457,117,6	41·040,99			
	3		⅔ No 3, ½ pt ·472,126,9	47·120,18					
					3·011,634	103·631,1			
	4		No 4, ¼ pt. 3·483,761	150·751,3					
1/2	2		0	0					
					·071,269,33	24·339,48	·004,581,777	·000,626,979,5	·005,208,756
	3		⅔ No 3, ¾ pt ·071,269,33	24·339,48					
					1·284,556	93·829,42			
	4		No 4, ½ pt. 1·355,825	118·168,9					
3/4	3		0	0					
					·127,529,4	42·599,78	·000,418,156,5	·000,218,580,6	·000,636,737,1
	4		No 4, ¾ pt. ·127,529,4	42·599,78					

UNIFORM AND VARIABLE THICKNESS ARCHES

31. Solution of Crown Forces, Form A.5. For radial adjustment only, with symmetrical arches and rigid abutments, the solution of the crown forces is very rapid. The arch and load constants already computed are the total constants under these circumstances, and are thus substituted directly in the two simplified simultaneous equations for crown movements:-

$$A_1M_o + B_1H_o = D_1$$
$$B_1M_o + B_3H_o = D_3$$

By virtue of symmetry $V_o = 0$. Solution of these two equation gives $M_o = \frac{1}{K}(D_1B_3 - B_1D_3)$ and

$$H_o = \frac{1}{K}(A_1D_3 - B_1D_1), \text{ where } K = A_1B_3 - B_1^2.$$

The computations are recorded on Form A.5. Note that K is the same for all loads, as it involves arch constants only. H_o is always positive for radial loads, but M_o may be positive or negative. Because significant figures are lost in the subtractions performed here in determining K, M_o and H_o, any small mistakes in the constants will cause greatly magnified errors in M_o and H_o.

Example: Solution of crown forces for Loads 1, 4 and Temp., Arch V.

32. Forces and Radial Deflections, Form A.6.

The forces and radial deflections caused by the unit and temperature loads are computed with the aid of Form A.6.

The forces M_o and H_o, transferred from Form A.5, are written in the row "crown" under the headings "M" and "$H_o \cos\phi$" respectively. M_L, H_L, and V_L are calculated by the formulae on page 137 of The Bulletin, and are positive as computed. For the variable thickness arches r in these formulae is $r_p = R_u - \frac{1}{2}T$, for the arch point considered, recorded on Form A.2. ϕ is the angle from the crown to the quarter-points. Trigonometric functions and coordinates are taken from Form A.4. For the temperature load ML, H_L, and V_L are zero. The term $\frac{PR_u}{\phi}$ which occurs in all the triangular load formulae may be recorded in the line under the table of forces for ready reference.

D_2 for positive radial loads is positive, so that $-D_2$ is negative. In the case of a positive temperature load $-D_2$ is positive. C_1, B_2 and C_2, on Form A.4, are multiplied by the corresponding force on this form, the sign of each component being determined by the sign of the force. Several significant figures may be lost in the deflection summation.

Example: Form A.6 for Loads 1, 4 and Temperature, Arch V.

Form A.5 Sheet..........

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Solution of Crown Forces, Arch No..V....

Load No...1...

$K = A_1B_3 - B_1^2 = 7\cdot207,981 \times 10^{-18}$

$M_o = \frac{1}{K}(D_1B_3 - B_1D_3) = +1,147,085$

$H_o = \frac{1}{K}(A_1D_3 - B_1D_1) = +330,210\cdot0$

$V_o = 0$

Load No...4...

$K = A_1B_3 - B_1^2 = 7\cdot207,981 \times 10^{-18}$

$M_o = \frac{1}{K}(D_1B_3 - B_1D_3) = -500,239\cdot7$

$H_o = \frac{1}{K}(A_1D_3 - B_1D_1) = +51,011\cdot90$

$V_o = 0$

Load No. Temperature, +1°F

$K = A_1B_3 - B_1^2 = 7\cdot207,981 \times 10^{-18}$

$M_o = \frac{1}{K}(D_1B_3 - B_1D_3) = -456,870\cdot3$

$H_o = \frac{1}{K}(A_1D_3 - B_1D_1) = +16,077\cdot63$

$V_o = 0$

Form A.6.

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C L A R K   D A M.

Study C

Arch No V   Load No 1

Sheet ____
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Checked DR
Date ____

	MOMENTS M			THRUSTS. H			SHEARS V		
POINT	M_L	$H_o y$ +	$= M_o + H_o y - M_L$	H_L	$H_o \cos \phi$ +	$= H_o \cos \phi + H_L$	V_L	$H_o \sin \phi$ +	$= H_o \sin \phi - V_L$
CROWN	0	0	+1,147,085	0	+330,210·0	+330,210·0	0	0	0
$\frac{1}{4}$ ϕ_1	2,871,237	2,549,882	+ 825,730	7,440·992	+ 324,143·1	+331,584·1	77,277·64	63,007·04	− 14,270·60
$\frac{1}{2}$ ϕ_2	11,324,559	10,056,546	− 120,928	29,490·54	+ 306,165·4	+335,655·9	151,715·67	123,698·85	− 28,016·82
$\frac{3}{4}$ ϕ_3	24,894,882	22,108,880	−1,638,917	65,338·41	+276,937·4	+342,275·8	220,578·80	179,845·24	− 40,733·56
ABUT ϕ_4	42,857,839	38,065,288	−3,645,466	113,667·4	+237,533·2	+351,200·6	281,336·65	229,383·15	− 51,953·50

+ Use ϕ - Point Functions on Form A.4. $PR_v = 405{,}000$

RADIAL DEFLECTIONS

Temperature load D - Terms.
D1 = 0   D2 = − cty*   D3 = ctx.*

*Use (ϕ A − ϕ) - Point Functions on Form A.4.

	$\frac{3}{4}$ Point	$\frac{1}{2}$ Point	$\frac{1}{4}$ Point	Crown
$-D_2$ =	− ·000,834,496,1	− ·010,155,28	− ·050,689,12	− ·164,768,0
$C_1 M$ =	− ·000,643,180,4	− ·000,215,05	+ ·003,791,96	+ ·010,537,6
$B_2 H$ =	+ ·000,892,147,7	+ ·010,345,47	+ ·047,348,05	+ ·144,650,3
$C_2 V$ =	− ·001,066,983,4	− ·004,894,67	− ·008,644,51	0
TOTAL Δr =	− ·001,652,512	− ·004,919,53	− ·008,193,62	− ·009,580,1

Form A.6.

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

C L A R K D A M.

Study C

Arch No V Load No 4

Sheet
Computed MW
Date
Checked DR
Date

	MOMENTS M			THRUSTS. H			SHEARS V		
POINT	M_L	$H_o y$ +	$= M_o + H_o y - M_L$	H_L	$H_o \cos\phi$ +	$= H_o \cos\phi + H_L$	V_L	$H_o \sin\phi$ +	$= H_o \sin\phi - V_L$
CROWN	0	0	-500,239·7	0	+51,011·90	+51,011·90	0	0	0
$\frac{1}{4}$ ϕ_1	0	393,913·9	-106,325·8	0	+50,074·67	+50,074·67	0	+9,733·530	+9,733·530
$\frac{1}{2}$ ϕ_2	317,878·1	1,553,567·4	+735,449·6	827·793,6	+47,297·41	+48,125·20	12,919·32	+19,109·39	+6,190·07
$\frac{3}{4}$ ϕ_3	2,509,297	3,415,451	+405,914	6,585·830	+42,782·18	+49,368·01	51,202·53	+27,783·07	-23,419·46
ABUT ϕ_4	8,303,756	5,880,448	-2,923,548	22,023·19	+36,694·89	+58,718·08	113,442·87	+35,435·84	-78,007·03

+ Use ø - Point Functions on Form A.4. $\frac{PR_u}{\phi_3} = 703,175·4$

RADIAL DEFLECTIONS

Temperature load D - Terms.
D1 = 0 D2 = - cty* D3 = ctx.*

*Use (ø A - ø) - Point Functions on Form A.4.

	$\frac{3}{4}$ Point	$\frac{1}{2}$ Point	$\frac{1}{4}$ Point	Crown.
$-D_2$ =	-·000,636,737,1	-·005,208,756	-·013,176,95	-·017,850,92
$C_1 M$ =	+·000,159,297,8	+·001,307,890	-·000,488,27	-·004,595,42
$B_2 H$ =	+·000,128,678,6	+·001,483,299	+·007,150,34	+·022,346,05
$C_2 V$ =	-·000,613,454,3	+·001,081,434	+·005,896,15	0
TOTAL Δr =	-·000,962,215,0	-·001,336,133	-·000,618,73	-·000,100,29

Form A.6.

THE HYDRO-ELECTRIC COMMISSION, TASMANIA.

G L A R K D A M.

Study C

Arch No V Load No Temperature, +1°F.

Sheet
Computed mw
Date
Checked DR
Date

	MOMENTS M			THRUSTS H			SHEARS V		
POINT	M_L	$H_o y$ +	$= M_o + H_o y - M_L$	H_L	$H_o \cos \phi$ +	$= H_o \cos\phi + H_L$	V_L	$H_o \sin\phi$ +	$= H_o \sin\phi - V_L$
CROWN	0	0	−456,870·3	0	→	+16,077·63	0	→	0
$\frac{1}{4}$ ϕ_1	0	124,151·5	− 332,718·8	0		+ 15,782·24	0		+3,067·757
$\frac{1}{2}$ ϕ_2	0	489,644·2	+ 32,773·9	0		+ 14,906·92	0		+6,022·786
$\frac{3}{4}$ ϕ_3	0	1,076,461·6	+ 619,591·3	0		+ 13,483·84	0		+8,756·504
ABUT ϕ_4	0	1,853,365	+1,396,495	0		+ 11,565·28	0		+11,168·46

+ Use ∅ - Point Functions on Form A.4.

RADIAL DEFLECTIONS

Temperature load D - Terms.
D1 = 0 D2 = - cty* D3 = ctx.*

*Use (∅ A - ∅) - Point Functions on Form A.4.

	$\frac{3}{4}$ Point	$\frac{1}{2}$ Point	$\frac{1}{4}$ Point	Crown.
$-D_2$ =	+·000,059,928	+·000,189,288	+·000,383,075	+·000,634,018
$C_1 M$ =	+·000,243,154	+·000,058,284	−·001,527,928	−·004,197,010
$B_2 H$ =	+·000,035,146	+·000,459,456	+·002,253,601	+·007,042,896
$C_2 V$ =	+·000,229,370	+·001,052,208	+·001,858,315	0
TOTAL Δr =	+·000,567,598	+·001,759,236	+·002,967,063	+·003,479,904

CHAPTER III - THE RADIAL ADJUSTMENT

33. Introduction. When computations of arch and cantilever data have been completed the results are tabulated for ready reference in the radial adjustment. All data and graph sheets should be prepared before the adjustment is started so that no delays occur during the actual process. A list of the information required is given in the subsequent sections.

34. Cantilever Data. Tabulate (1) unit load shears and moments, (2) unit, dead, and uplift-on-base load deflections, (3) ϕ_D. Forms C.2 and C.5 are used frequently during radial adjustments. By graphical interpolation determine the dead load deflections of the cantilevers situated at the arch quarter points for combination with temperature deflections in cracked cantilever studies.

35. Arch Data. Tabulate (1) crown and abutment forces for all unit and temperature loads, (2) crown and abutment cross-sectional area and section modulus, (3) temperature deflections for maximum temperature rise and fall above and below mean annual (see Figure 5), (4) arch unit load and initial relative deflections (see section 6).

36. Load and Deflection Sheets. Prepare on a separate sheet of squared paper for each arch a load and deflection diagram. Draw the arch centre line as a straight line, and mark off on it the quarter points and the cantilevers which intersect the arch. Negative (downstream) deflections are plotted below the line at a suitable scale. Above the arch centre line plot the total water load at that elevation. Similarly draw up a sheet for each cantilever, except that in this case it is better to draw separate load and deflection diagrams. The type of form required is indicated in the condensed result sheets, Figures 23-28.

37. Procedure for the Radial Adjustment. This phase of the work is simple in comparison with the lengthy arithmetic of the preparatory work as it consists almost entirely of "trial and error" adjustment of loads and deflections, calling for little more than patience and perseverance.

The best method of beginning the adjustment, for all the loading conditions mentioned in section 8, is to secure agreement of the crown cantilever and arch crowns by using only No.1 unit loads on the arches. Then apply this loading to the other arch points and cantilevers, and plot the deflections. Now by trial and error alter the load distribution and plot each successive load and deflection curve until reasonable agreement is reached. Careful study of the general trend of the unit and total load deflection curves will benefit this phase of the work. The loads on arch and cantilever at conjugate points sum to the external load (i.e. the water load or zero) at that elevation.

To build up a load pattern on an arch it is necessary to apply No.1 load first, then 5, 4, 3 and 2 in that sequence. For this reason it would appear advisable to have numbered the triangular loads in the reverse order. Although The Bulletin states that for the lower arches loads 1, 3 and 5 are usually sufficient, loads 2 and 4 were found essential in most of the Studies on Clark Dam.

In cracked cantilever analyses difficulty arises in estimating the load required to produce a given deflection, or the change in deflection caused by an alteration of the load, because when the concrete is assumed to crack under tension, deflection is no longer directly proportional to the applied load. In addition, as the total load increases, there are large "jumps" in the deflection as successive sections crack. However, "Experience teaches".

The load distribution determined by the radial adjustment is assumed to be the true division of load for the purpose of computing stresses.

38. Stresses. Once the forces acting on the component parts of the dam have been determined (by any method of analysis), the stresses may be computed by the usual methods. The Bulletin treats the subject very fully, developing many formulae for the general case of a variable thickness arch dam with sloping upstream face subject to radial, tangential, twist and vertical loads.

As the forces determined by the simple radial adjustment would be modified by the other adjustments elaborate stress calculations are not justified. Stresses computed for Clark Dam are:- arch bending and direct stresses normal to radial planes at both faces, vertical cantilever stress at the upstream face, inclined cantilever stress at the downstream face and average radial shear on vertical and horizontal sections.

These are tabulated on the result sheets in Chapter IV The moments, thrusts and shears at the abutments of the upper arches were used in checking the stability of the gravity tangent blocks on both banks.

The requisite formulae in The Bulletin are equations 232, 244 (arch), 238 and 243 (cantilever).

CHAPTER IV - ANALYSIS OF THE FINAL DESIGN

39. Loading Conditions.

(a) Study C1. Full water load (to S.L.2345) and maximum temperature drop below mean annual. The cantilevers in the central half of the dam were cracked at the upstream face. Arch temperature and cantilever dead load deflections were combined as an initial relative deflection. Hence the dead load deflection of cantilever D was added algebraically to the external load movement for comparison with the position of the arch centre-line.

(b) Study C2. Full water load and maximum temperature rise. As there was no cracking of the cantilevers, unit loads were utilised throughout the analysis.

(c) Study C3. No water load and maximum temperature rise. Temperature and dead load deflections were combined as initial relative deflections. Cantilevers A, B & C were cracked at the downstream face.

(d) Study C4. An investigation into the effect on stresses of the procedure used in the grouting of the radial contraction joints.

40. Dimensions of Arches & Cantilevers.

Work on Study C was withheld until most of the excavation had been completed, so that it is an examination of the dam as constructed. Where the arch lengths in the right and left parts of the dam differ they have been averaged for the analysis. A deep pocket in the left bank near the gravity abutment has been backfilled with concrete and treated as part of the foundation rock.

The overhang of the roadway slab and parapet on the downstream face is supported on a continuous curved fillet, thereby stiffening the crest considerably. The thickness of this arch has been taken as 18 feet - a compromise between the thickness at S.L.2345 of 20 feet and at S.L.2335 of 17 feet. For the computation of cantilever weights a crest thickness of 17 feet has been assumed.

Although it had been intended to analyse six cantilevers and seven or eight arches in the final study, limited time prevented the analysis of more than four of each. The position and main dimensions of these are shown in Fig.21.

Additional arch dimensions are:-

Arch	T_c	T_A	$T_{average}$
I	18.000	18.000	18.0
III	25.000	34.802	28.5
V	37.000	55.908	43.7
VII	55.000	75.791	61.9

Individual unit loads below S.L.2195 for Cantilevers A and B were necessitated by the irregular "steps" at their bases of 18 and 30 feet respectively. Although there is a change of slope of the downstream face five feet from the base of B the assumption of a straight line variation of thickness from the base to S.L. 2195 introduces no appreciable error.

FIG. 21

CLARK DAM

PLAN, PROFILE AND SECTIONS

41. Arch Loads to 1/8th-Points. In Studies A & B difficulty was encountered in securing radial adjustment of the crest arch with the cantilever tops because of the extreme flexibility of the arch and the inadequacy of the five unit loads to represent the loading curve. Accordingly for Study C the deflections due to four additional loads, extending from a maximum at the abutment to zero at the $\frac{7}{8}$-pt. (No. 2a) $\frac{5}{8}$-pt. (No. 3a), $\frac{3}{8}$-pt. (No. 4a) and $\frac{1}{8}$-pt. (No. 5a), were computed. They are tabulated with the other arch deflections.

Prime D-terms were computed from the formulae of sections (Bulletin) 94 and 95 by the method shown in section 204. A check on the correctness of the computations was that some prime values could be selected from Table 17, e.g. D_2' at the 1/4-pt. of a 60° arch for a triangular load extending $22\frac{1}{2}°$ (Load 3a) is the same as D_2' at the crown of a 45° arch for a No. 3 load.

The work was an experiment, as the calculation of the prime D-terms involves much arithmetic, and satisfactory results had been obtained in the previous studies by assuming two loading curves, one for the arches and one for the cantilevers, which were approximately equivalent. Experience with the loads in Study C showed that they were not as valuable as had been hoped.

Appendix to Section 41.

To demonstrate the method of computation, D_2' at 1/2-pt for Load 5a on Arch I will be derived.

It will be seen from Figure 22 that the load between the 1/2-pt. and the abutment is trapezoidal. For purposes of calculation this is divided into two portions - a uniform and a triangular load. The notation used here agrees as far as possible with that used in the Tables. Unfortunately the notations used in the formulae and the Tables of the Bulletin are not consistent.

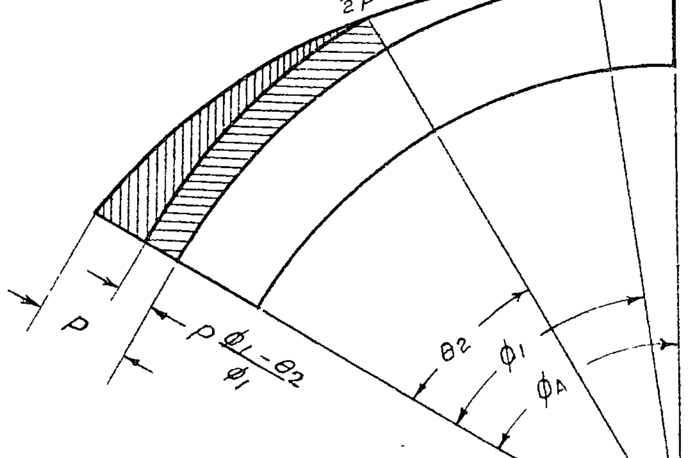


FIGURE 22 - D-TERMS AT $\frac{1}{2}$ PT FOR N° 5a LOAD

ϕ_A the angle from crown to abutment.

ϕ_1 the angle from the abutment to the beginning of the whole triangular load.

θ_2 the angle from the abutment to the point considered (here the 1/2-point). It is also the angle from the abutment to the apex of the triangular portion of the trapezoidal load, i.e. it is the "ϕ_1" for that load for the purpose of entering the tables of integrals.

ϕ an angle used in the case of points not under the triangular load. $\phi = \phi_1$ (or θ)$+\phi_0$, but in the case of uniform load and the apex of the triangular load $\phi = \phi_1$ and $\phi_0 = 0$. This substitution is made to reduce the number of integrals tabulated. Refer to Sections 199 and 200 of The Bulletin.

In this example, $\phi_A = 60^\circ, \phi_1 = 52^\circ 30', \theta_2 = 30^\circ, \phi = 30^\circ \& \phi_0 = 0^\circ$.

(a) Uniform Portion of Load.

The ordinate of the load is $\dfrac{(\phi_1 - \theta_2)}{\phi_1}$ $P = 3/7\ P$.

From Bulletin Equation 138,

$$D_2' \text{ 1st term} = 3/7\ P \int_0^{A\phi_1} \sin\phi_1 \operatorname{vers}\phi_1\, d\phi_1 \text{(Bulletin notation)}$$

$$= 3/7\ P \int_0^{\theta_2} \sin\theta_2 \operatorname{vers}\theta_2\, d\theta_2$$

This integral is in Table 14 under the heading $\int \operatorname{vers}\phi_1 \sin\phi\, d\phi_1$ in the column headed "$\cos\phi_0$", in the line θ_2(ϕ_1 in the Table) $= 30^\circ$.

Multiply the tabulated value by $\cos\phi_0 (= 1)$. Then

$$D_2' \text{ 1st term} = \frac{3}{7} \times 1000 \times .008{,}974{,}596{,}216$$
$$= 3.846{,}255{,}521$$

In a similar manner D_2' 2nd term is derived from Bulletin Equation 139,

$$D_2' \text{ 2nd term} = D_2' \text{ 1st term} + 3(3/7P)(\text{Table 14, Col.6, for } \theta_2),$$
$$= 3.846{,}255{,}521 + 160.714{,}286.$$

(b) Triangular Portion of Load.

All terms in the formulae for the triangular load include the factor (load ordinate / load angle). For the small triangular load the load ordinate is $P.\ \theta_2/\phi_1$ and the load angle is θ_2. Hence the factor is P/ϕ_1, the same as for the total load.

From Equation 146,

$$D_2' \text{ 1st term} = (P/\phi_1).\ (\text{Table 14, Column 3, for } \theta_2)$$

$$= \frac{1000}{.916,297,857} \times .001,257,122,089$$

$$= 1.371,958,91$$

From Equation 147,

$$D_2' \text{ 2nd term} = D_2' \text{ 1st term} + 3(P/\phi_1)(\text{Table 14, Col. 4, for } \theta_2)$$

$$= 1.371,957 + 71.027,978$$

D_2' for the whole load is the sum of the values for the component loads. Thus $D_{21}' = 5.218,213$

$$D_{22}' = 236.960,5$$

42. Arch and Cantilever Tabulations. These results of preparatory computations are tabulated on the succeeding pages:-

Cantilevers: Table 1 - Unit Load Shears and Moments.

Table 2 - Unit Load Deflections,

Dead Load Deflections,

Deflections due to uplift on the base of uncracked cantilevers.

Arches: Table 3 - Unit and Temperature Load Moment, Thrust and Shear at Crown and Abutment.

Table 4 - Unit and Temperature Load Deflections,

Dead Load Deflections of dam at arch points.

The methods of calculating these quantities have been explained in Chapter II with the aid of a full set of completed computation forms. Chapter II also includes a list of the structural constants (section 10) and a diagram of the cantilever unit loads (Figure 9, section 17).

S.L.	UNIT LOADS										
	1	2	3	4	5	6	7	8A	8B	9A	9B
2345	V = 0 M = 0										
2320	V = 12.5 M =208.34	V = 12.5 M =104.17									
2295	V = 12.5 M = 520.84	25 625	V = 12.5 M =104.17			For unit loads of peak intensity 1,000 lbs/sq. ft. V and M as tabulated are in units of 1,000. Positive radial loads cause negative V and M					
2270	V = 12.5 M = 833.34	25 1250	25 625	V = 12.5 M = 104.17							
2245 Base of D	V = 12.5 M =1,145.8	25 1875	25 1250	25 625	V = 12.5 M = 104.17						
2220	V = 12.5 M =1,458.3	25 2500	25 1875	25 1250	25 625	V = 12.5 M = 104.17					
2195 Base of C	V = 12.5 M =1,770.8	25 3125	25 2500	25 1875	25 1250	25 625	V = 12.5 M = 104.17				
2170	V = 12.5 M = 2,083.3	25 3750	25 3125	25 2500	25 1875	25 1250	25 625	V = 12.5 M = 104.17			
2165 Base of B	V = 12.5 M = 2,145.8	25 3875	25 3250	25 2625	25 2000	25 1375	25 .750		V = 15 M = 175		V = 2.5 M = 4.167
2152 Base of A	V = 12.5 M = 2,308.3	25 4200	25 3575	25 2950	25 2325	25 1700	25 1075	21.5 437.16		V = 9 M = 54	

CANTILEVER UNIT LOADS - SHEARS AND MOMENTS

CLARK DAM - STUDY C.

TABLE 1.

S.L.	UNIT LOADS									DEAD LOAD	UPLIFT ON BASE
	1	2	3	4	5	6	7	8	9		
					CANTILEVER A.						
2345	.006,774	.010,51	.006,711	.003,981	.002,160	.001,035	.000,401,6	.000,099,73	.000,010,20	.014,72	.000,197,5
2295	.003,158	.005,368	.003,936	.002,588	.001,505	.000,760,7	.000,311,2	.000,082,07	.000,009,039	.008,726	.000,143,8
2245	.001,040	.001,857	.001,521	.001,186	.000,834,1	.000,488,0	.000,220,9	.000,064,41	.000,007,883	.003,777	.000,090,16
2195	.000,189,0	.000,348,8	.000,305,2	.000,261,5	.000,217,9	.000,174,2	.000,116,9	.000,046,75	.000,006,726	.000,793,3	.000,036,49
					CANTILEVER B.						
2345	.005,442	.008,266	.005,065	.002,851	.001,440	.000,613,8	.000,189,1	.000,036,22	.000,003,194	.011,82	.000,271,5
2295	.002,385	.004,022	.002,888	.001,828	.000,997,4	.000,453,5	.000,148,7	.000,030,19	.000,003,051	.006,674	.000,189,2
2245	.000,700,6	.001,247	.001,017	.000,786,0	.000,539,4	.000,293,2	.000,108,2	.000,024,16	.000,002,907	.002,574	.000,107,0
2195	.000,093,96	.000,174,0	.000,153,2	.000,132,4	.000,111,5	.000,090,71	.000,054,81	.000,018,13	.000,002,763	.000,354,6	.000,024,68
					CANTILEVER C.						
2345	.003,347	.004,820	.002,650	.001,290	.000,523,6	.000,150,3	.000,021,84			.006,945	.000,168,6
2295	.001,246	.002,063	.001,407	.000,801,6	.000,361,2	.000,114,0	.000,018,26			.003,394	.000,107,3
2245	.000,252,4	.000,449,1	.000,365,7	.000,282,2	.000,183,7	.000,077,63	.000,014,68			.000,862,5	.000,045,98
					CANTILEVER D.						
2345	.001,552	.001,967	.000,790,5	.000,218,5	.000,030,25					.002,331	.000,098,46
2295	.000,368,9	.000,586,4	.000,344,9	.000,129,4	.000,021,90					.000,688,9	.000,042,20
	1	2	3	4	5	6	7	8	9	DEAD	UPLIFT

NOTE: All deflections except those due to dead load are negative.

CANTILEVER DEFLECTIONS

CLARK DAM - STUDY C.

TABLE 2.

	Load	CROWN		ABUTMENT		
		Mo	Ho	Ma	Ha	Va
ARCH 1.	1	+ 171,000	+ 402,500	- 323,200	+ 403,800	- 2,160
	2	- 133,200	+ 3,610	-1,244,000	+ 6,420	- 49,580
	3	- 752,300	+ 24,230	-3,183,000	+ 30,370	- 82,640
	4	-1,604,000	+ 67,620	-4,203,000	+ 74,180	- 92,480
	5	-1,819,000	+ 129,700	-3,889,000	+ 134,900	- 81,070
	T	- 24,080	+ 350	+ 45,510	+ 180	+ 300
	2a	- 19,750	+ 500	- 378,900	+ 1,400	- 26,040
	3a	- 380,900	+ 11,170	-2,259,000	+ 15,920	- 68,830
	4a	-1,192,000	+ 43,140	-3,662,000	+ 49,870	- 90,520
	5a	-1,859,000	+ 96,870	-4,198,000	+ 102,800	- 89,030
ARCH 111.	1	+ 332,500	+ 387,400	- 983,500	+ 395,700	- 14,960
	2	- 88,030	+ 3,050	-1,187,000	+ 5,930	- 48,380
	3	- 509,900	+ 20,760	-3,243,000	+ 28,070	- 82,720
	4	-1,143,000	+ 60,340	-4,506,000	+ 69,780	- 95,330
	5	-1,317,000	+ 120,000	-4,330,000	+ 129,300	- 86,290
	T	- 87,710	+ 1,750	+ 240,600	+ 930	+ 1,490
ARCH V.	1	+1,147,000	+ 330,200	-3,645,000	+ 351,200	- 51,950
	2	- 48,830	+ 2,830	- 658,600	+ 4,520	- 36,790
	3	- 245,200	+ 17,830	-1,915,000	+ 22,700	- 64,420
	4	- 500,200	+ 51,010	-2,924,000	+ 58,720	- 78,010
	5	- 448,500	+ 101,600	-3,308,000	+ 111,700	- 77,430
	T	- 456,900	+ 16,080	+1,396,000	+ 11,570	+ 11,170
ARCH V11.	1	+2,366,000	+ 173,100	-6,607,000	+ 202,200	- 112,400
	2	- 17,720	+ 2,250	- 288,900	+ 3,040	- 24,500
	3	- 42,300	+ 11,520	- 981,200	+ 14,390	- 45,390
	4	+ 18,160	+ 29,820	-1,862,000	+ 35,740	- 61,500
	5	+ 329,500	+ 56,130	-2,804,000	+ 66,170	- 73,110
	T	-1,441,000	+ 96,410	+3,957,000	+ 84,330	+ 46,740

ARCH UNIT LOAD FORCES.

CLARK DAM - STUDY C.

Table 3.

ARCH DEFLECTIONS - CLARK DAM - STUDY C
TABLE 4

Point	Maximum Temperature Rise	Maximum Temperature Fall	Cantilever Dead Load	UNIT LOADS 1	UNIT LOADS 5	UNIT LOADS 4	UNIT LOADS 3	UNIT LOADS 2
	+9.4°F.	- 7.5°F.		ARCH I				
Crown	+.038,54	-.030,75	+.014,72	-.029,12	+.052,63	+.055,65	+.031,36	+.006,494
1/4 Point	+.033,73	-.026,91	+.011,82	-.025,51	+.020,29	+.025,71	+.016,85	+.003,867
1/2 Point	+.021,32	-.017,01	+.006,500	-.016,11	-.031,83	-.028,39	-.013,26	-.001,984
3/4 Point	+.007,124	-.005,684	+.002,000	-.005,381	-.033,24	-.034,12	-.022,01	-.005,232
	+5.7°F.	-4.7°F.		ARCH III				
Crown	+.023,37	-.019,27	+.008,726	-.017,98	+.008,331	+.010,42	+.005,886	+.001,208
1/4 Point	+.019,97	-.016,47	+.006,900	-.015,43	+.000,913,8	+.003,564	+.002,721	+.000,650,0
1/2 Point	+.011,85	-.009,772	+.003,394	-.009,305	-.009,057	-.007,329	- .003,221	-.000,484,2
3/4 Point	+.003,694	-.003,046	+.000,750,0	-.003,068	-.007,089	-.006,891	-.004,262	-.000,990,9
	+3.6°F.	-2.8°F.		ARCH V.				
Crown	+.012,53	-.009,744	+.003,777	-.009,580	-.001,491	-.000,100,3	+.000,210,0	+.000,077,14
1/4 Point	+.010,68	-.008,308	+.003,000	-.008,194	-.001,906	-.000,618,7	-.000,051,09	+.000,024,28
1/2 Point	+.006,333	-.004,926	+.001,600	-.004,920	-.002,116	-.001,336	-.000,529,6	-.000,083,54
3/4 Point	+.002,043	-.001,589	+.000,450,0	-.001,653	-.001,176	-.000,962,2	-.000,562,3	-.000,139,3
	+2.3°F.	-1.8°F.		ARCH VII				
Crown	+.004,054	-.003,172	+.000,793,3	-.003,148	-.000,859,4	-.000,388,2	-.000,122,4	-.000,017,86
1/4 Point	+.003,537	-.002,768	+.000,640,0	-.002,756	-.000,813,8	-.000,390,4	-.000,130,2	-.000,020,75
1/2 Point	+.002,261	-.001,769	+.000,370,0	-.001,785	-.000,631,6	-.000,359,4	-.000,142,3	-.000,026,96
3/4 Point	+.000,864,3	-.006,676,	+.000,120,0	-.000,705,3	-.000,316,4	-.000,217,5	-.000,117,4	-.000,030,85

Additional Loads for Arch I
(See Section 39)

POINT	5a	4a	3a	2a
Crown	+.059,54	+.045,48	+.017,22	+.001,042
1/4 Point	+.025,25	+.022,83	+.009,615	+.000,648,2
1/2 Point	-.031,76	-.021,49	-.006,285	-.000,251,1
3/4 Point	-.035,24	-.029,49	-.013,13	-.000,847,6

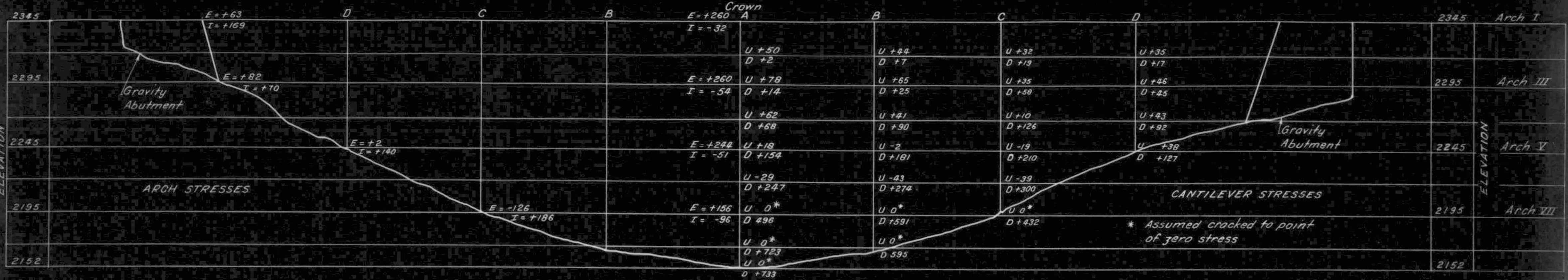
43. Results of Studies C1, C2 and C3.

The stresses, load patterns and deflection diagrams for Study C1 are shown on Figures 23 and 24, Study C2 on Figures 25 and 26, and Study C3 on Figures 27 and 28.

Studies C1 and C2 reveal that the majority of the water load is carried to the foundations by gravity action. Under both loading conditions the maximum arch stresses are less than the maximum cantilever stresses. The results of Study C3 show that undesirable tensions will occur at the downstream face of the cantilevers, should the reservoir be completely empty for a long period in summer. Under dead load alone tensile stresses exist near the base at the downstream face of the cantilevers in the central portion of the dam.

Maximum stresses (all in lb./sq.in) are:-

Study C1	Cantilever bending and direct stress	733
	Cantilever shear stress	217 (300 permissible)
	Arch bending and direct stress	260 (compression)
		126 (tension)
	Arch shear stress	22
Study C2	Cantilever bending and direct stress	335
	Cantilever shear stress	70
	Arch bending and direct stress	294 (compression)
		No tension
	Arch shear stress	16
Study C3	Cantilever bending and direct stress	367
	Cantilever shear stress	19
	Arch bending and direct stress	131 (compression)
		No tension
	Arch shear stress	6



PROFILE ON AXIS LOOKING UPSTREAM

(Developed)

0 50 100

Scale of Feet

LOADING CONDITIONS

STUDY C1

Reservoir water surface level 2345
Temperature Drop.

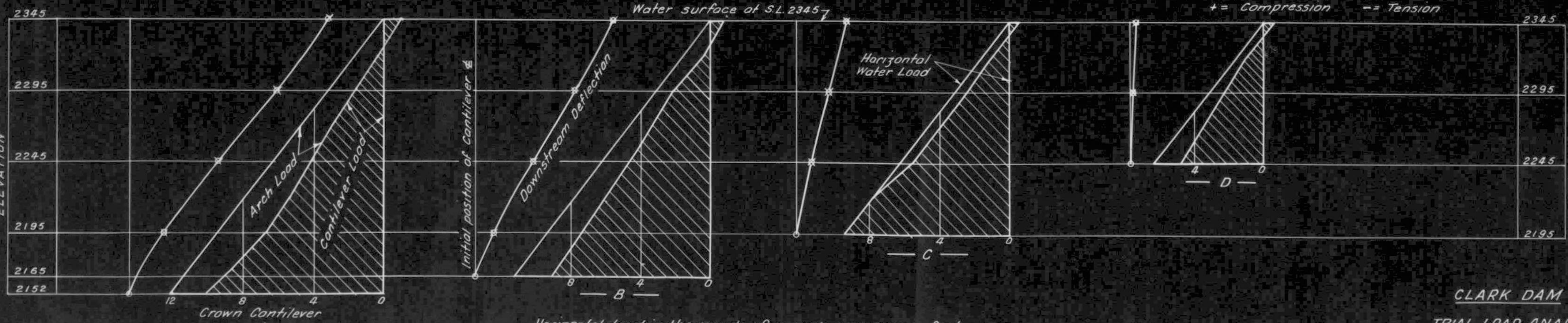
Maximum Radial Shear
Cantilever A — -217 lb/sq. in.
Arch VII — -22 lb/sq. in.

Cantilever stresses parallel to cantilever edges
U = Stress at upstream edge of cantilever
D = Stress at downstream edge of cantilever

Arch stresses are acting in horizontal directions parallel to arch axis
E = Stress at extrados of arch
I = Stress at intrados of arch.
All stresses in lbs per square inch
+ = Compression — = Tension

+.05 0 -.05

Radial Deflection in Feet



Horizontal Load in thousands of pounds per square foot.

○ Cantilever Deflection
× Arch Deflection

LOAD DISTRIBUTION AND RADIAL ADJUSTMENT

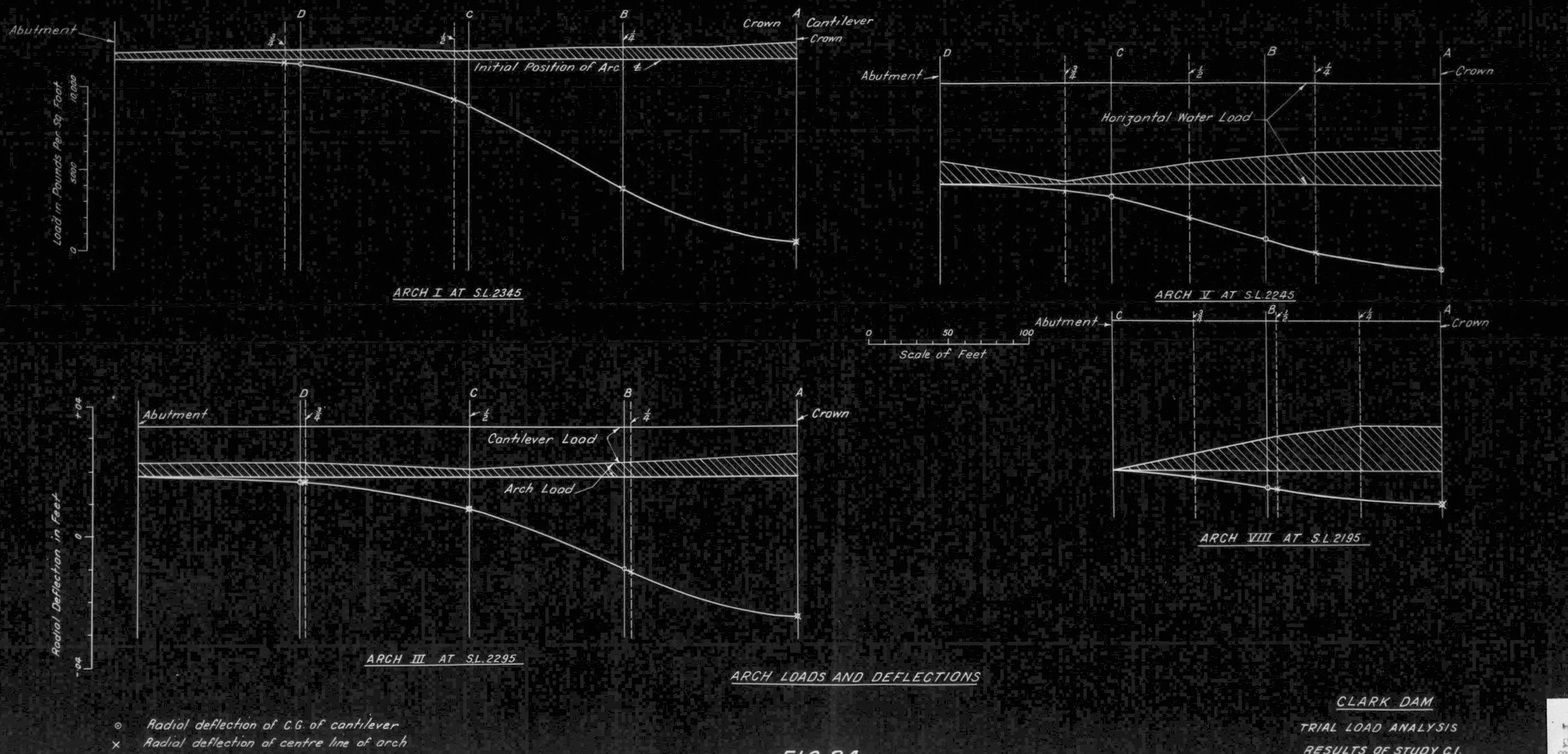
CLARK DAM

TRIAL LOAD ANALYSIS

RESULTS OF STUDY C1

Sheet 1 of 2.

FIG. 23



ARCH LOADS AND DEFLECTIONS

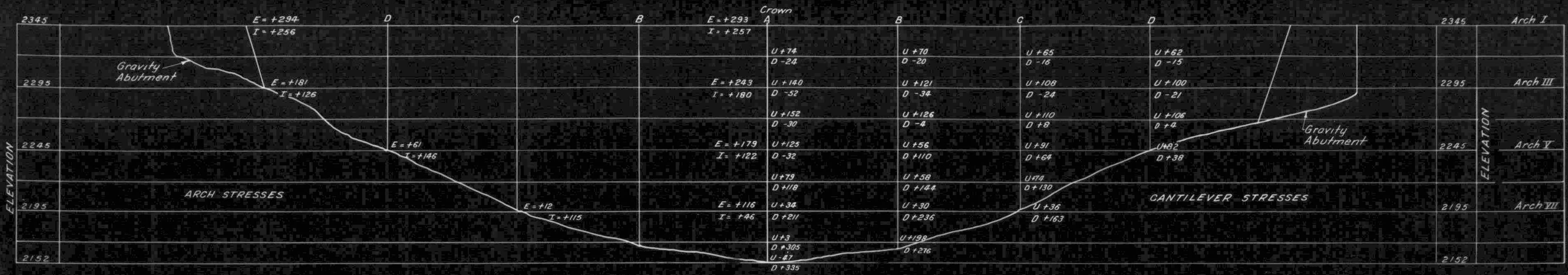
FIG. 24

CLARK DAM

TRIAL LOAD ANALYSIS

RESULTS OF STUDY C.I.

Sheet 2 of 2.



PROFILE ON AXIS LOOKING UPSTREAM

(Developed)

0 50 100

Scale of Feet

LOADING CONDITIONS

STUDY C 2

Reservoir water surface level 2345

Temperature Rise

Maximum Radial Shear

Cantilever A - -70 lb/sq. in.

Arch VII - -16 lb/sq. in.

Cantilever stresses parallel to cantilever edges

U = Stress at upstream edge of cantilever

D = Stress at downstream edge of cantilever

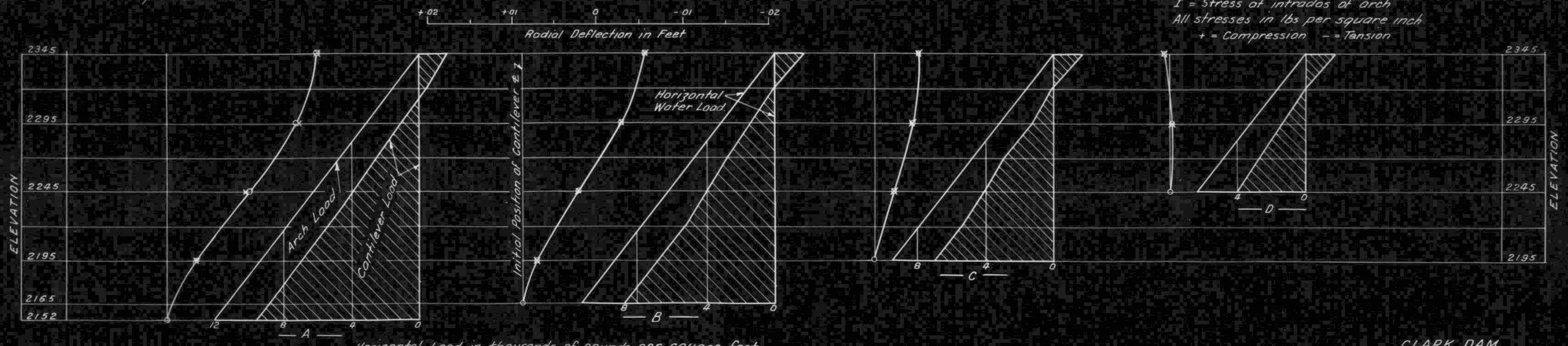
Arch stresses are acting in horizontal directions parallel to arch axis

E = Stress at extrados of arch

I = Stress at intrados of arch

All stresses in lbs per square inch

\+ = Compression - = Tension



Horizontal Load in thousands of pounds per square foot

∘ Cantilever Deflection

× Arch Deflection

LOAD DISTRIBUTION AND RADIAL ADJUSTMENT

FIG. 25

CLARK DAM

TRIAL LOAD ANALYSIS

RESULTS OF STUDY C 2

Sheet 1 of 2.

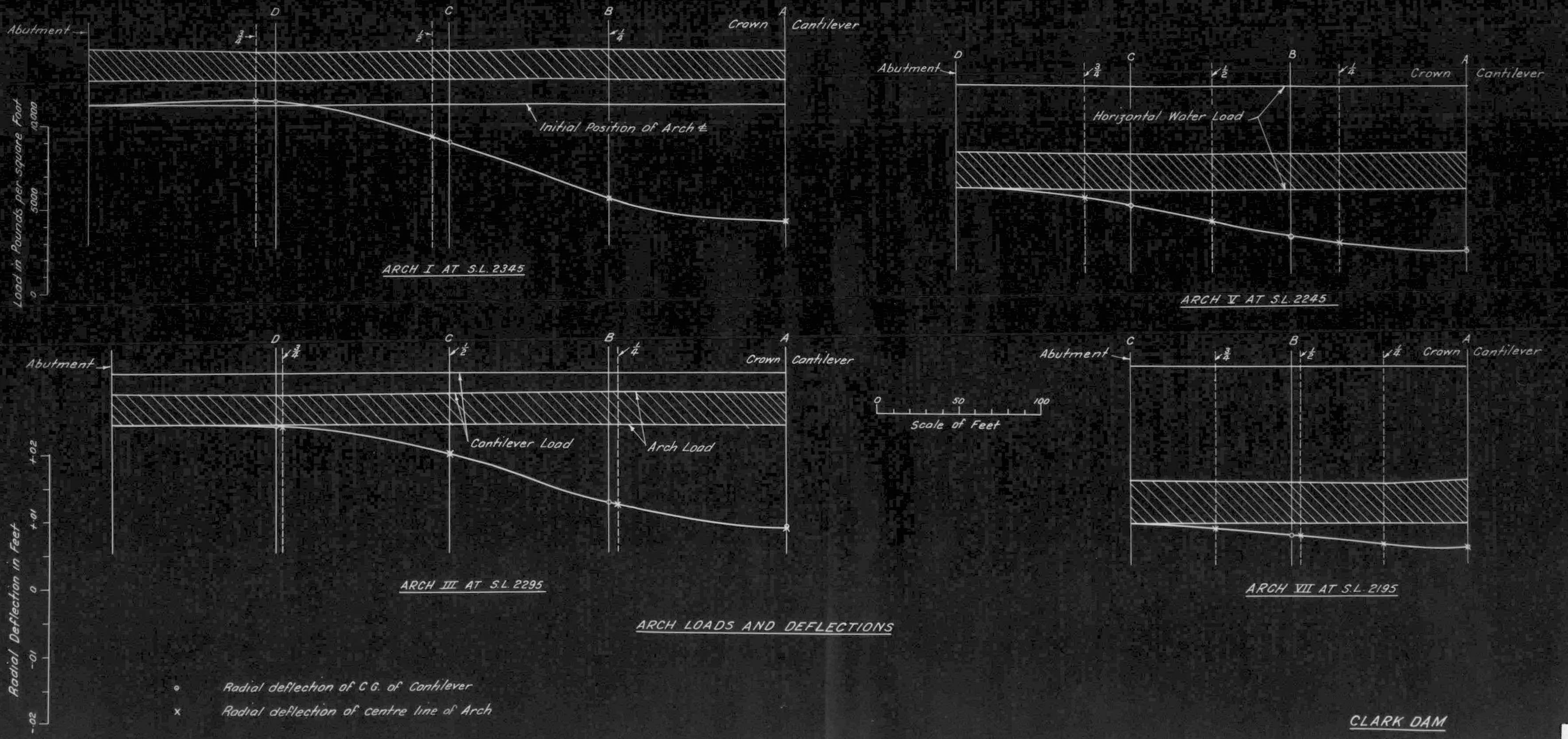


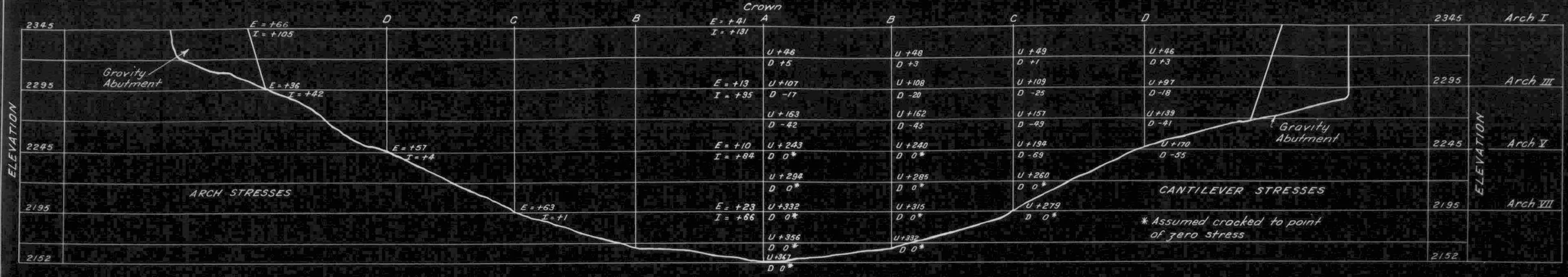
FIG. 26

CLARK DAM

TRIAL LOAD ANALYSIS

RESULTS OF STUDY C2

Sheet 2 of 2.



PROFILE ON AXIS LOOKING UPSTREAM

(Developed)

0 50 100

Scale of Feet

LOADING CONDITIONS

STUDY C3

Reservoir empty, Temperature-Rise

Maximum Radial Shear

Contilever A +19 lb/sq. in.

Arch I - -6 lb/sq. in.

Contilever stresses parallel to contilever edges.

U = Stress at upstream edge of contilever

D = Stress at downstream edge of contilever

Arch stresses are acting in horizontal directions parallel to arch axis

E = Stress at extrados of arch

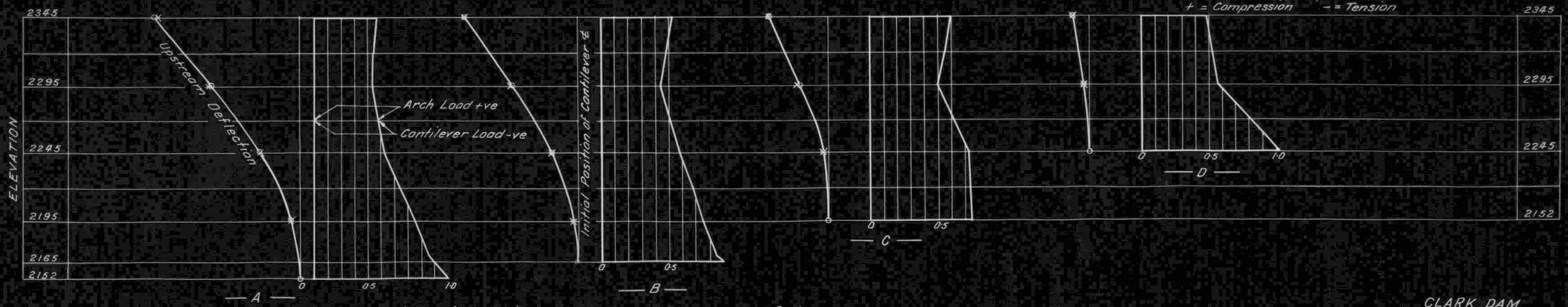
I = Stress at intrados of arch

All stresses in lbs per square inch

+ = Compression - = Tension

+.05 0 -.05

Radial Deflection in Feet



Horizontal Load in thousands of pounds per square foot

○ Contilever Deflection

× Arch Deflection

LOAD DISTRIBUTION AND RADIAL ADJUSTMENT

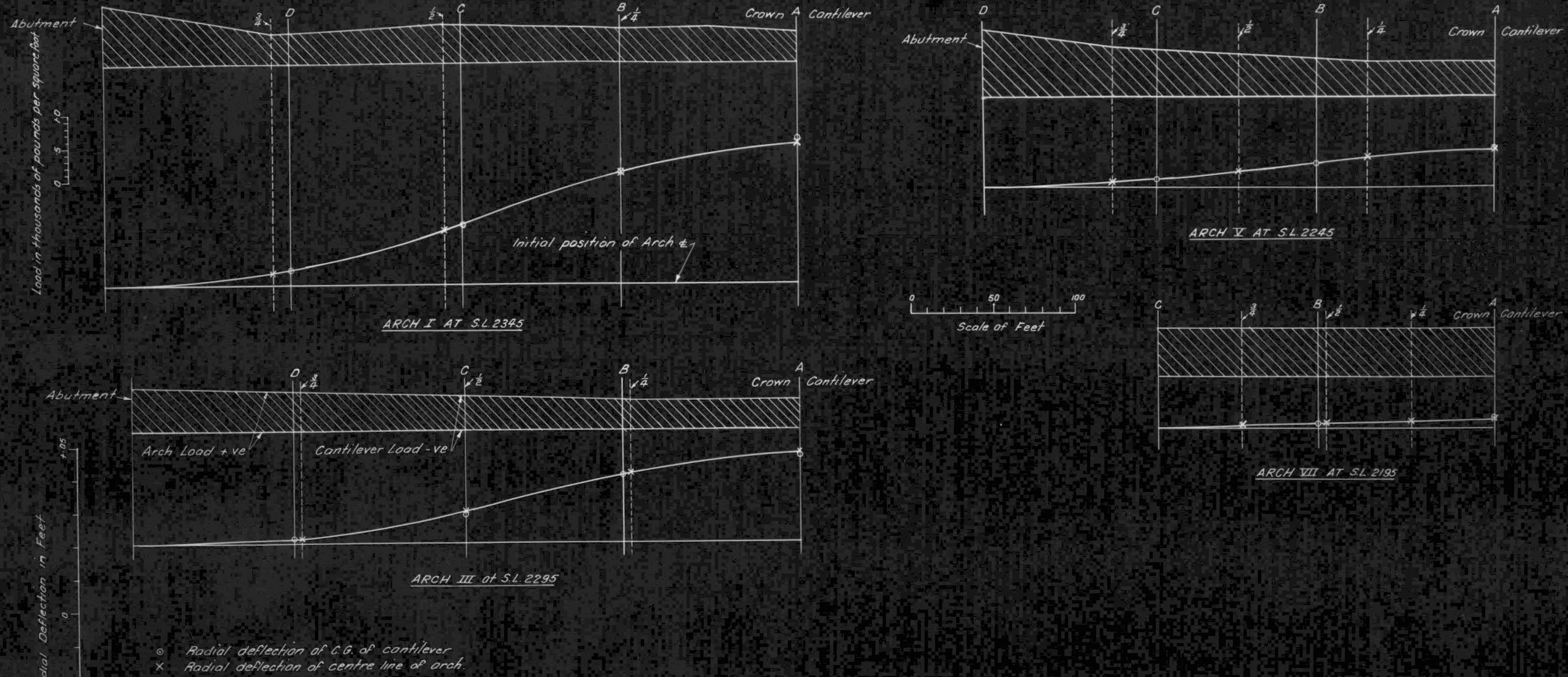
FIG. 27

CLARK DAM

TRIAL LOAD ANALYSIS

RESULTS OF STUDY C3

Sheet 1 of 2



ARCH LOADS AND DEFLECTIONS

CLARK DAM
TRIAL LOAD ANALYSIS
RESULTS OF STUDY C3
Sheet 2 of 2

FIG. 28

44. <u>Grouting of Contraction Joints. Study C4.</u>

Clark Dam was built as a series of vertical blocks, fifty feet long at the upstream face, with radial contraction joints between the blocks. When the dam has been built to its full height and the temperature of the mass concrete reduced to a predetermined level (mean annual) these joints will be filled with cement grout to ensure monolithic action of the structure as soon as the reservoir begins to fill. The radial adjustment as described in the previous chapters assumes that the only stresses in the dam at the conclusion of the grouting process are those due to dead load. This presupposes that the reservoir is empty at the time of grouting and that the grout has been injected in such a manner that when solidified it fills exactly the space which previously existed between adjacent blocks.

In practice, however, the grout must be forced in under pressure (50-100 lb./sq.in.) to ensure that it penetrates all parts of the joint and that as much as possible of the excess mixing water is forced into the pores of the concrete. This pressure exerted on the radial faces of the block has an unbalanced component which deflects the cantilevers upstream and induces tension at the downstream face. Although this upstream movement would greatly improve the stress conditions revealed by Studies C1 and C2 (particularly the former) it would greatly aggravate the cracking of the downstream face under loading C3. Therefore to offset this upstream deflection it is necessary to have a partial water load at the time of grouting. Accordingly Study C4 was undertaken to determine a procedure of joint grouting that would be both practical and of assistance in reducing undesirable stress conditions.

Since the grout pressure will vary in a somewhat indeterminate manner over the area inside the grout stops due to friction in the pipes and outlets, the static head of the column of grout, leakages and variations in joint widths a simplifying assumption must be made as to an average pressure in the joint. This pressure on the radial faces of a 50 ft. block is treated as an equivalent radial load on the unit cantilevers so that the unit radial loads may be used in assessing its effects.

After the grout has set it ceases to exert a fluid pressure on the blocks. It is desirable for stress conditions that the grout pressure should deflect the cantilevers upstream slightly more than the partial water load has deflected them downstream. Therefore if the partial water load were removed there would be no elastic movement of the cantilevers and they would have an initial upstream deflection (beyond the dead load position) due to the excess of grout load over water load. The amount of the initial deflection will be governed by the crown cantilever for three reasons:-

(1) The crown cantilever is the critical section for determining the safe water level before the joints are grouted.

(2) The water loads caused by this safe reservoir level decrease as the foundation rises towards the gravity abutments, i.e. the partial water load deflections decrease very rapidly towards the wings.

(3) Grout pressure must be the same in all joints to prevent lateral movements. Thus any arrangement of water and grout loads which restores the crown cantilever to its dead load position will deflect all the remaining cantilevers upstream.

These initial loads should be included in the cracked cantilever analyses but provided they are small they may be neglected without serious effect on stresses computed for full reservoir conditions.

Proposed Grouting Procedure.

The proposal under consideration at present is to grout the contraction joints to the full height in one operation maintaining an average pressure of about 75 lbs./sq. in. in the joints. The reservoir level is to be held at S.L.2265 during the grouting operations.

The maximum computed deflection due to grout pressure is 0.2" at the crest of the crown cantilever. At the conclusion of the grouting operations it is expected that the residual cantilever moments will be positive at all locations except hear the base of the crown cantilever where small negative moments remain.

It is possible that practical considerations may cause this proposal to be modified, either intentionally or unintentionally.

45. Time Required for Trial Load Analysis.

The time spent on Study C may be subdivided thus:-

(a) Preparatory Computations	-	10	man-weeks
(b) Study C1	-	5	" "
(c) Study C2	-	4	" "
(d) Study C3	-	5	" "
(e) Study C4	-	5	" "
(f) Tabulation of Results	-	2	" "

In the preparatory work every sheet was calculated independently by two men, but only the final trials were checked in the radial adjustments. Experience in the method of analysis and use of computing machines gained in the previous studies caused much of the work to be somewhat mechanical, thereby reducing very considerably the time consumed in those portions of the work. This is clearly demonstrated by the fact that Study C was completed in less than half the time spent on the first Study.

The calculating machines used were Monroe 10 x 10 x 21 High Speed Adding-Calculators, Model MA-7. This model has electric carriage-shift and dial clearances, and is equipped with automatic division. The computation forms were drawn up to suit the capabilities of these machines.

CHAPTER V - CONCLUSION

SUPPLEMENTAL EFFECTS

46. Foundation Deformation. The manner in which foundation deformation is incorporated in trial load analysis is, briefly, this:

(1) Using the formulae and curves of Chapter III of The Bulletin (based on the work of Boussinesq, Cerruti and Vogt) the movements of the arch abutments and cantilever foundations for unit forces and moments are determined.

(2) The effect of yielding abutments on arch elements is included in the solution of crown forces by means of additional D-terms and in the computation of deflections by the addition of movements at the arch points caused by the deformation of the abutments.

(3) Where a cantilever does not sit on an arch abutment, its foundation movement is computed directly from the radial shear, tangential thrust, twisting moment and moment in the vertical radial plane at the base of the cantilever due to cantilever loading alone. The rotation of the base in the vertical plane is included in the summation of the change of slope of the centre line (for radial deflection). The other movements are simply added to the cantilever elastic deformations.

(4) In the case of a cantilever which sits on an arch abutment, however, the cantilever foundation movement is composed of two parts --- the arch abutment movements (radial, tangential and twist in the horizontal plane) and rotation in a vertical radial plane. These two components are included in the cantilever movements as indicated in the preceding paragraph. The arch abutment movements are also composed of two parts --- those due to arch loading and those due to concentrated loads at the abutments coming down from the cantilever resting thereon. Thus the cantilever deflections are modified by the arch loading and the arch deflections are changed by an alteration in the cantilever loading. In this way the interaction of the arch and cantilever loads on the abutment is brought into the analysis.

Allowance for deformation of abutments increases the computed deflections. The general result seems to be somewhat lower arch and cantilever stresses along foundation and abutment locations, without material stress changes in central and upper portions of the structure.

47. Tangential and Twist Adjustments. As described in section 3 the tangential and twist adjustments are made after the first radial adjustment has been completed. The tangential and twist loads on arch and cantilever elements are internal self-balancing loads. Tangential shear effects are important in relatively long arches like Clark Dam. Their main results are a decrease in radial deflection at the crown, an increase in radial deflections near the abutment, a reduction in stress at the upstream face of cantilevers near the crown and a slight increase in arch stresses at the abutments. The general effect of considering twist action is a decrease in radial deflections at practically all locations, a decrease in maximum arch stress and an increase (positive) in cantilever stress at the upstream face. Twist action is important in most arch dams, particularly the high curved gravity type.

48. Extra Work Entailed in Supplemental Effects.

The inclusion of abutment deformation in the radial adjustment would cause a slight increase in the amount of preparatory computations and slow down the adjustment considerably. The addition of the tangential and twist adjustments would cause an almost ten-fold increase in the amount of preparatory work because each of the three types of load produces angular, radial and tangential deflections. The extra adjustments and readjustments would also consume a great amount of time. It would seem therefore that where a dam is comparable with one already existing a relatively small organisation cannot cope with these supplemental effects. As indicated above the general results of including foundation deformation, tangential shear and twisting moments all tend to improve the stress conditions computed from a simple radial adjustment - thus providing another justification for their omission from the analyses of Clark Dam.

CONCLUSION

49. Conclusion. In view of the extensive research into the many phases of arch dam design by the skilled engineers and mathematicians of the United States Bureau of Reclamation any criticism of the method as a whole, based on the limited experience of one dam only, is unwarranted. Some minor points have been raised in the relevant sections of the text. The trial load method has been developed slowly, new refinements of theory and procedure being added one by one. Engineering design is now progressing very rapidly, and it is possible that the trial load method will be supplanted. In the meantime, however, it is the best available tool for the analysis of curved masonry dams. Measurements of deflections and rotations taken on several dams, both model and prototype (notably Boulder, Gibson and Stevenson Creek) have shown excellent agreement with the calculated deflections obtained from the trial load analyses.

The feature in the simplified method as applied to Clark Dam which seems most open to criticism is the manner in which the effects of tension and cracking are incorporated in cantilever analysis. The strength of the cantilevers is very greatly reduced by the assumptions that the concrete cracks to the point of zero stress and then exerts no restraining influence whatever on the bending and shear deflections.

As provision is being made at Clark Dam for the measurement of radial deflections and the width of contraction joints, it is suggested that after the dam has been placed in service a comparison of the computed and actual behaviour of the dam should prove of value to the engineering profession. The action of uplift pressure in mass concrete and its effect on cantilever deflection are also matters which require further investigation.

This thesis, as stated at the beginning, has been confined almost entirely to an explanation and description of a simple radial adjustment of a symmetrical dam. Sufficient information has been given to indicate how the results of the analyses may be applied to design, including the improvement of stress conditions by carefully regulated grouting. An outline of the extension of the trial load method of analysis to cover the supplemental effects should suffice to show how unwise it would be to attempt a full analysis before the simple radial adjustment has been mastered.

50. Acknowledgements. My thanks are due to the Commissioner of The Hydro-Electric Commission of Tasmania for permission to use the numerical data and examples from the analyses of Clark Dam, for the supply of specimen computation forms, prints and photostats, and for the use of the duplicating machine.

The studies of Clark Dam have been performed by the methods developed by the United States Bureau of

Reclamation, as set forth in their Bulletin "Trial Load Method of Analyzing Arch Dams." The same notation has been used as far as possible, but new symbols have been introduced where necessary to avoid ambiguity. The majority of the computation forms used have been adapted from the general forms shown in The Bulletin.

Thanks must also be given to many friends for their helpful advice as to the general arrangement of the paper and assistance in the preparation and checking of drawings, numerical examples and tables.

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