Numeracy within reform-based learning environments: A synthesis of five dimensions of practice

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Abstract

Numeracy has become as essential as literacy for any individual who wishes to participate fully in democratic society. Alongside a growing awareness of the importance of developing school students' numeracy capabilities has been a curriculum reform movement that emphasises values-based, authentic experiences and transdisciplinary learning. This reform has translated into a focus on developing lasting conceptual understandings of a coherent set of the key ideas and skills that students need to become critical and productive members of society.

The aim of this thesis is to further understanding of numeracy from both a theoretical and a practical perspective in the context of schools undergoing curriculum reform. The study is concerned with the enactment of curriculum in the classroom, in which the roles and experiences of teachers and students are equally important. It considers the question, How are teachers positioning, and how are students experiencing, numeracy in reform-based learning environments?

A synthesised view of numeracy, underpinned by social constructivist theory, is presented in this thesis. It acknowledges the complexity involved in numeracy and argues that multiple aspects, beyond mathematical skill, are necessary for the development of competent and effective numeracy practice. A focus of the study is the development of a conceptual framework for numeracy incorporating five dimensions of practice: *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity*. These dimensions provide the lens through which the beliefs and practices of the teachers who participated in the study are considered and the learning exhibited by individual student participants is examined.

A qualitative collective case study was conducted through four phases of inquiry. Phase 1 involved an interview with the five participant teachers prior to their commencement of a unit of work that they had planned to achieve numeracy learning outcomes and that was informed by the Tasmanian Essential Learnings

curriculum, the local curriculum framework at the time of the study. Phase 2 involved classroom observations of the units of work and incorporated researcher observations, teacher records, and student outcomes. During Phase 3, interviews were conducted with six students in each case study school. The final phase of the study was a reflective interview with each of the participant teachers. The results of the research are presented by case with data for teachers and their respective students reported together.

Outcomes associated with the teachers and the students are presented in relation to the numeracy framework developed, conceptualised through five dimensions of practice. Within this broad view of numeracy, the diverse possibilities available both for teaching and for student learning are discussed, along with implications for curriculum design and professional learning. The thesis emphasises the potential for this newly developed multi-dimensional view to guide the numeracy education of students, thereby fulfilling the democratic goals of preparing students for their roles as future citizens.

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Chapter One

Introduction

1.1 Numeracy: A golden anniversary

Fifty years ago, in 1959 in the United Kingdom, The Crowther Report introduced the term *numeracy* as being the "…mirror image of literacy … [and recognised] the need in the modern world to think quantitatively" (quoted in Cockcroft, 1982, p. 11). Crowther's notion of numeracy was a sophisticated one, encompassing an understanding of scientific method, thinking quantitatively, and avoiding statistical fallacies. Although the Crowther Report implied a broad concept of numeracy that incorporated higher level thinking processes, since that time numeracy has developed across the globe in many different forms, reflected by many different terms and definitions. In the United Kingdom itself, the term numeracy moved away from the sophisticated conceptualisation of Crowther when, in a British government report on mathematics education (Cockcroft, 1982), numeracy was identified as an "at homeness" with numbers and an ability to use mathematics in everyday life as well as an ability to understand and interpret information presented in mathematical ways.

Australia essentially inherited the term numeracy from the United Kingdom. Although sharing a predominantly functional view of numeracy that emphasises the value of individuals having the mathematical skills necessary to cope with their everyday life experiences, Australia has moved away from the mostly number-based conception of numeracy that educators in the United Kingdom have adopted (Doig, 2001). Numeracy, in Australia, is perhaps more aligned with the conceptions of "quantitative literacy" as proposed in the United States (e.g., Steen, 2001) and "mathematical literacy" as defined by the Organisation for Economic Co-operation and Development (OECD, 2006) in Europe. Within Australian education systems, across both public and private sectors, the most widely adopted definition is that articulated by the Australian Association of Mathematics Teachers (AAMT):

> To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life.
>
> In school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of:
>
> - Underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic);
> - Mathematical thinking and strategies;
> - General thinking skills; and
> - Grounded appreciation of context. (AAMT, 1998, p. 2)

Although there are variations in how numeracy is conceptualised, it has forged an identity of its own having been previously overshadowed in education practice by literacy definitions and interventions (Luke et al., 2003). How numeracy is conceptualised affects not only the school mathematics curriculum but also the practice of teachers and ultimately the learning outcomes of students. Current conceptualisations of numeracy are considered in more depth in Chapter 2, both in relation to the context of this study and more broadly.

1.2 Numeracy and its relationship to mathematics

Although the terms numeracy and mathematics often appear to be used interchangeably (Groves, Mousley, & Forgasz, 2006), there are a number of views regarding the relationship between the two. The debate largely revolves around whether numeracy is a subset of mathematics, more than mathematics, or if they are synonymous with each other. Zevenbergen (2005) represented the interaction between school mathematics and numeracy as a "dynamic model" of two overlapping circles (p. 5). She argued that in primary school the overlap between numeracy and mathematics is almost complete and as students move through to senior secondary school the distinctions between the two become more evident as increasingly complex and abstract mathematical content is introduced. In some cases the term numeracy is used to replace the term mathematics with the aim of inferring a more democratic and accessible mathematics curriculum available to all students. The AAMT (1997) described mathematics and numeracy as being "clearly interrelated" but not synonymous with each other.

> All numeracy is underpinned by some mathematics; hence school mathematics has an important role in the development of young people's numeracy. The implemented mathematics curriculum (i.e. what happens in schools) has a responsibility for introducing and developing mathematics, which is able to underpin numeracy. However this 'underpinning of numeracy' is not all that school mathematics is about. Learning mathematics in school is also about learning in the discipline – its structure, beauty and importance in our culture. Further, while knowledge of mathematics is necessary for numeracy, having that knowledge is not in itself sufficient to ensure that learners become numerate. (p. 11-12)

However the relationship between mathematics and numeracy is viewed, there is little argument that numeracy is regarded as having the potential to provide a connection between mathematics and the real world if students are provided with opportunities to apply and use their mathematical knowledge and skills (Kemp & Hogan, 2000; Morony & Brinkworth, 2003; Scott, 1999; Willis, 1998). In this respect, numeracy is also considered to have cross-curricular relevance in that an appropriate level of numeracy underpins learning and progress in other areas (AAMT, 1997; Steen, 2001). A cross-curricular view of numeracy has teaching implications, in that it implies that numeracy learning not only is the responsibility of the mathematics teacher but also requires a commitment and contribution by all teachers (Berlin, 2003; Frykholm, 2002; Hughes-Hallett, 2003; Price, 1997). In the primary school context, where the teacher usually has responsibility for teaching all or most subject areas, the implication is that the teacher needs to recognise opportunities to embed numeracy when appropriate across a range of areas. In secondary schooling this perspective necessarily involves teachers from discipline areas other than mathematics identifying and making explicit use of mathematics to support effective learning.

A broad and synthesised view of numeracy is presented in this thesis. It acknowledges the complexity involved in numeracy and argues that multiple aspects, beyond mathematical skill, are necessary for the development of competent and effective numeracy practice. It also emphasises the place of numeracy across the curriculum, and aligns with the work of Steen (1997, 2001) who argued that numeracy is not the same as mathematics, but rather "an equal and supporting partner in helping students learn to cope with the quantitative demands of modern society." (Steen, 2001, p. 115). A conceptual framework for numeracy, describing five dimensions of practice, is developed in Chapter 3. The five dimensions of practice: *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity* developed in this thesis provide the lens through which the beliefs and practices of the teachers who participated in the study are considered and the learning exhibited by individual student participants is examined.

1.3 The context of the study: Reform-based learning environments

What competencies do citizens need in order to participate actively within democratic societies? This is a question that has driven educators to examine school curricula and address these needs with regard to the students of the 21st century. The *Melbourne Declaration on Educational Goals for Young Australians* (Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA], 2008a), which sets the direction for Australian schooling and followed the 1999 Adelaide Declaration (MCEETYA, 1999), places education as the driving force in equipping young people with the "knowledge, understanding, skills and values to take advantage of opportunity and to face the challenges of this era with confidence" (p. 4).

Within the broader context of education, three reform agendas, that have been influential over the past decade, are important for this thesis: first, the curriculum reforms that have encouraged values-based, transdisciplinary learning; second, the mathematics education reforms that have placed importance on the connections among, and applications of, the domains within mathematics; and third, the middle years' reforms that have argued that adolescent learners require programs that develop knowledge, skills, and habits of mind that are relevant to their concerns. These three reform agendas are discussed in this section. They influence the enactment of curriculum, and more specifically the teaching and learning of numeracy, within the classroom learning environment, which is the focus of this study.

1.3.1 Curriculum reform agendas

Many Australian states have been reconceptualising curriculum in terms of a coherent set of the most important ideas and skills, sometimes referred to as "big ideas" or "key ideas," that will enable students to become productive and critical members of society. The curricula are values-based and call for the study of these "big ideas" across the traditional discipline boundaries. The development of students' capacities for thinking and building understandings is placed at the centre. Internationally, reconceptualisation of curricula is also occurring in response to changing views of knowledge and learning and consideration of the goals of education both for individual students and within the broader context of society. A curriculum of the future (Young, 1999) makes assumptions about key values and purposes held by society, about knowledge, and about learning. Curriculum, now, needs to provide "opportunities for participation in learning communities and strengthening the links between participation in school-based learning communities and in other contexts for learning… and must provide access to 'specialist knowledge communities'" (Young, 1999, p. 476).

As with the broader reforms, recent curriculum reform in Tasmania has also been guided by a consideration of the knowledge, skills, and attributes required of students living in today's world. The curriculum context at the time of the study was a reform-based, values-focused curriculum that encouraged transdisciplinary activities. Transdisciplinary activities are concerned with relationships between, across, and beyond the disciplines. They are free from the constraints of discipline-boundaries, rather focusing upon issues, contexts, and values deemed important for students (Drake, 1993). The underlying assumption is that the curriculum can enable meaningful learning for students. The reform-based curriculum in Tasmania at the time of the study was centred around five Essential Learnings: Thinking, Communicating, Personal Futures, Social Responsibility, and World Futures (Department *of* Education, Tasmania [D*o*ET], 2002). The

Essential Learnings framework placed thinking skills and strategies at the centre of the curriculum and encouraged the connection of knowledge and concepts across the curriculum. It emphasised the importance of being numerate rather than purely knowing and doing mathematics. An ability to understand and apply mathematical concepts is valued alongside the development of students' abilities to problem solve, reason, communicate, and reflect upon their learning. Curriculum construction and the local reform environment are expounded in Chapter 2.

1.3.2 Mathematics education reforms

Over the past 20 years, reforms have also been occurring within mathematics curricula (National Council of Teachers of Mathematics [NCTM], 2000). These reforms have come about with a change in focus from mathematics content to how students can best learn mathematics (Van de Walle, 2004). They place importance on the connections among, and applications of, the domains within mathematics. Advocates of reform urged a move away from traditional teaching approaches that emphasise telling and practice of procedures (Olson & Barrett, 2004), to approaches that support a constructivist view of learning (Van de Walle, 2004), in helping students construct personally meaningful conceptions of mathematical topics (Fraivillig, Murphy, & Fuson, 1999). The United States NCTM Standards document (1989) that encapsulated much of the reform ideas, advocated that students should value mathematics, be confident in their ability to do mathematics, become mathematical problem solvers, learn to communicate mathematically, and learn to reason mathematically.

The mathematics education reforms are also consistent with the wider curriculum reforms that promote the importance of the development of students' capacities for thinking and understanding. The reforms have seen a significant increase in curriculum development and examination of curricula and teaching practices that impact positively upon student learning within this context. In order to meet the reform goals of student learning that advocate mathematical thinking, problem solving, and connecting and communicating ideas there has been a shift away from text books and materials that emphasise skills and procedures toward tasks and resources that focus on conceptual understanding. Conceptual understanding refers to the development and deepening of connections among mathematical facts, procedures, and ideas (Hiebert & Carpenter, 1992).

The mathematics education reforms reported in the United States have largely been mirrored within the Australian context. A problem solving approach was launched in the 1980s and around the same time the AAMT focused its activities on developing communication and collaboration between mathematics teachers and educators from across the country (Ellerton & Clements, 2007). This resulted in the development of resources for teachers such as The Mathematics Curriculum and Teaching Program (MCTP) (Lovitt & Clarke, 1988; 1989). This national professional development program saw a focus on the reform goals of mathematical thinking, group work, problem solving, and context in order to support conceptual understanding. The program culminated in 114 lessons or investigations (Lovitt & Clarke, 1992) to encourage teachers to adopt student-centred approaches to teaching and learning. The program was a pre-cursor to the kinds of issues that teachers and researchers in Australia have been grappling with, in relation to numeracy, over the past ten years and these are discussed in the Literature Review (Chapter 2).

1.3.3 Middle years' reforms

Within the context of curriculum reform in Tasmania this study situates the investigation of the phenomenon of numeracy positioning within the middle years of schooling. The middle years is a time for fostering curiosity, problem solving, and critical thinking among adolescents; and for developing skills and habits of mind, in addition to knowledge, that is relevant to their concerns (Jackson & Davis, 2000). It is also a time when adolescent learners begin to choose pathways to go on to further mathematics study. For students the "middle school years" are marked by the transition from the final years of primary school to the early years of secondary school. In Australia this involves, for the most part, moving through Grade 5 to Grade 8, in some states incorporating Grade 9. Students moving through the middle years of schooling are typically between the ages of 10 and 15.

Some school sites cater for all compulsory years of schooling including primary and secondary. Other schools offer a primary or secondary education only.

Educators are interested in this particular stage of schooling because of the "unique developmental and educational needs of young adolescent learners" (Barber, 1999). Reform agendas have been driven by those who believe students in these years require a stronger association between the particular developmental traits of adolescent learners and the way schooling is structured to meet these traits. Such reforms have led to the development of specific middle years' programs and in some cases dedicated middle school environments (Beane & Brodhagen, 2001). In the report *Beyond the Middle*, Luke et, al. (2003) write, "middle years education has become a clear motivating force for reform and for the framing and focusing of teachers' and students' work in schools and classrooms" (p. 12).

In this study, two of the schools were primary schools teaching grades from Kindergarten to Grade 6. These two schools did not incorporate specific middle years' programs, but did include young adolescent learners aged 10 – 12 years. The other school in this study catered for all the school grades from Kindergarten to Grade 12 and had a dedicated middle school for Grades 7 and 8, with a program specifically designed for these students based on the perceived needs of those learners.

In considering the implementation of values-based curricula in Australia, the learning needs of students and how to best prepare students for life in democratic society and in a global economy is not insignificant. This thesis argues that students' numeracy capabilities bring a crucial element to this agenda and the middle years are arguably an important time in addressing the essential needs of numeracy.

1.4 Aim and objectives of the research

Innovative and reform curricula are filtered through teachers' beliefs and practices (Wilson & Lloyd, 2000). Although researchers are aware of the broader contexts and policy-driven environments that influence curriculum construction, it is the curriculum that is enacted in the classroom that is of interest in this study. Teachers add a pedagogical dimension to curriculum to create daily learning experiences for their students. It is that knowledge that equips teachers to "lift the curriculum away from texts and materials to give it an independent existence" (Doyle, 1992, p. 499). The role of the student in curriculum is also acknowledged. Students determine their own levels of engagement and interest in classroom activity and therefore exert some control over their learning and knowledge construction. Snyder, Acker-Hocevar, and Snyder (1994) suggest that "curriculum enactment" appropriately describes the process of implementation and educational experience that teachers and students jointly undertake as they negotiate and determine what the curriculum will be like in each classroom. With respect to the teaching of mathematics, teachers' knowledge, beliefs, and practices play a significant role in the learning of their students (Hill, Rowan, & Ball, 2005).

The aim of the study is to uncover the dynamics of numeracy positioning in the context of schools undergoing curriculum reform. It is a study investigating how classroom teachers are positioning numeracy in an emerging values-based curriculum setting and how their numeracy pedagogies affect students' experiences of numeracy in the classroom. The study is designed to investigate this central phenomenon where it is actually occurring and to understand what this reform means to those participating in it. Two objectives underpin the research:

- First, a theoretical objective, to deepen understanding of the construct of numeracy in the context of current reform agendas, through the development of a conceptual framework for numeracy that aligns with and extends current research about numeracy and its capacity to equip students for their current and future lives as democratic citizens.

- Second, a practical objective, to contribute to an understanding of the complex nature of the teaching and learning of numeracy. The study is concerned with the enactment of curriculum in the classroom, in which the roles and experiences of teachers and students are equally important.

The following research questions are posed:

1. How are teachers positioning numeracy in reform-based learning environments according to five dimensions of practice?
2. How are students experiencing numeracy in these reform-based learning environments according to five dimensions of practice?
3. How does a five-dimensional framework for numeracy, developed to align with a transdisciplinary curriculum context, contribute to an understanding of numeracy teaching and learning?

Research questions 1 and 2 are generated from a review of the literature (Chapter 2) and refined based upon the conceptual framework developed for the study (Chapter 3). An additional question (question 3) results from the development of the conceptual framework.

Given the aim of the study, in exploring the beliefs and practices of teachers in relation to the learning of students in Tasmania's curriculum context, the methodological approach taken is a qualitative naturalistic inquiry incorporating a collective case study. The inquiry follows four classroom case studies (where one case includes two teachers) in different educational settings within the middle years. The data collection period for the study spanned a full year and involved eight teacher interviews, 55 classroom observations totalling 75 hours and 15 minutes, 24 student interviews, photographs, and the collection of teacher records and student work samples. The large and varied amount of data required a highly organised approach to data management across the four cases and this was assisted in this research by the use of a motif that became intrinsically connected to each case beyond the initial purpose of data organisation. The motif (Figure 1.1), that appears throughout the thesis, is used to identify the four classroom case studies. Details about the use of the motif are described in Section 4.8.5 of the Methodology chapter.

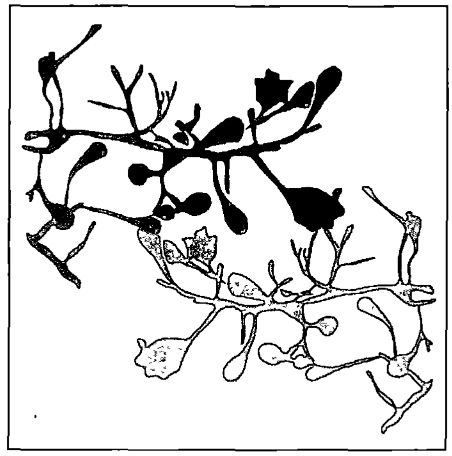


Figure 1.1. The motif.

1.5 Thesis overview

This chapter has provided an introduction to the major components of the thesis, numeracy and the reform agendas that form the context of the study. The principle aim and objectives of the research have also been described. Chapter 2 contains a review of the literature in the fields of curriculum and numeracy, drawing on the work of related fields to support the aim and objectives of the research.

Chapter 3 expands upon the review of the literature by explaining the comprehensive view of numeracy adopted in the study and by presenting the development of a conceptual framework for numeracy, incorporating five dimensions of practice. The framework, developed by the author, is derived from a thorough review of the literature. It not only articulates clearly the conceptualisation of numeracy for the thesis and guides the data analysis of all phases of the study, but also provides a means with which to interpret numeracy in reform-based learning environments in a way that has not been previously considered.

In Chapter 4, the research perspective and methodology are presented with specific discussion as to why these theoretical choices suit the objectives of the study. The design of the research, which is a qualitative collective case study conducted through four phases of inquiry, is introduced. The life of the project is also detailed in Chapter 4 and includes the methods of inquiry, procedures employed, role of the researcher and the participants, ethical considerations, data analysis procedures, and an evaluation of the study and its trustworthiness.

Chapter 5 presents the background to the results by discussing the distinctive characteristics of each of the schools that provided the setting for the case study research. The results of the study are then presented over four chapters, Chapters 6 to 9. The presentation of the results brings together the teachers and the students in each of the four case studies.

Finally, in Chapter 10, the findings of the study in relation to the research literature, along with the observed outcomes for the research questions, are discussed. The chapter discusses implications of the study for both curriculum design and professional learning and considers not only limitations of the study but also recommendations for further research. The conclusion brings together the motivation of the research and highlights its significance.

Chapter Two

Literature Review

2.1 Introduction

This chapter considers numeracy, the central phenomenon being studied, within the broader context of curriculum reform. The review of the literature spans three main areas. A discussion of curriculum begins the review by considering its role as a social construct and consequent influence on classroom teaching and learning. The reshaping of Australia's curricula and the motivation for the recent reforms is considered and the specific curriculum context of this study, Tasmania's Essential Learnings, is described. The second section of the review examines how numeracy has been defined and conceptualised both nationally and internationally. Some of the key factors influencing student learning of numeracy are considered and ways that numeracy is interpreted in practice are also discussed. Finally, a synthesised view of numeracy is proposed that aligns with a cross-curricular view of numeracy and also its place within the local curriculum context, a values-focused transdisciplinary curriculum. The literature review closes with the aim of the current study and the objectives and research questions proposed for the study.

2.2 Curriculum: A social construct

Definitions and characteristics of curriculum are many, varied, and often at odds. Views of curriculum have shifted over time and have included emphases on curriculum as subject matter, as a plan or intention, as an experience, and more recently as an outcome (Wiles, 2005). Schwab (1969) described four "commonplaces" of schooling – the teacher, students, subject matter, and milieu – and definitions of curriculum tend to place more or less importance on one or more of Schwab's four aspects. How ever the curriculum is defined, be it what curriculum writers describe, what the teacher teaches, or what the student learns, it takes place in a broader context of the social, political, and economic structures of society. "The curriculum does not stand apart from society – it is firmly embedded in it" (Brady & Kennedy, 2003, p. 3).

Construction of curriculum is being increasingly influenced by those beyond the education community. Stakeholders, including government, business, parents, community groups, and students, all declare views about the purposes and outcomes of curriculum. Governments themselves view the school curriculum as an instrument of social and economic development (Lee, 2001). In Australia, this is evidenced in the *Adelaide Declaration on National Goals for Schooling in the Twenty First Century*:

> Australia's future depends upon each citizen having the necessary knowledge, understanding, skills and values for a productive and rewarding life in an educated, just and open society. High quality schooling is essential to achieving this vision. This statement on the national goals for schooling provides broad directions to guide schools and education authorities in securing these outcomes for students.
> (MCEETYA, 1999, para. 1)

With respect to corporate and business interests, a 2002 report for the Commonwealth Department of Education Science and Training (DEST) (2002) identified key employability attributes and skills deemed necessary for young people in addition to any job specific or relevant technical skills. These attributes and skills were identified as: communication, team work, problem-solving, initiative and enterprise, planning and organising, self-management, learning skills, and technology skills (DEST, 2002, p. 7). The Australian Council of State School Organisations (ACSSO), a national body representing parents and school communities, declared its commitment to the principles of access, equality, equity of outcomes, excellence, and participatory democracy in the provision of a public education system (ACSSO, 2006). Students themselves also have their own personal, social, academic, and vocational aspirations.

This thesis acknowledges the highly complex and variable nature of curriculum in its role as a social construct, changing in response to and often as a reaction to societal changes, and representing each generation's view of what is deemed as worth knowing. A curriculum of the future (Young, 1999) makes assumptions about key values and purposes held by society, about knowledge, and about learning. The increased level of curriculum change in the later half of the twentieth century has mirrored an acceleration of societal change. Grundy (2005) describes the many facets of these contextual factors in relation to the notion of position and opposition. The curriculum is positioned within a local context and a series of broader contexts including the global world in which Australia participates. Curriculum is enacted at the classroom level to support the local school and community context, at the same time as negotiating the broader social contexts of stakeholder interests and aiming to serve the needs of students living in an increasingly complex and global world environment. Where "globalisation appears to be breaking down national barriers related to cultures, social values and economies ... school curriculums remain firmly attached to the needs of individual nation states" (Kennedy, 2005, p. 1).

2.2.1 Curriculum in the classroom

> Curriculum [is] a particular form of specification about the practices of teaching and not … a package of materials of a syllabus or ground to be covered … curriculum is a means of studying the problems and effects of implementing any defined line of teaching … Curriculum research and development is based on the study of classrooms. It thus rests on the work of teachers. (Stenhouse, 1975, p. 142)

Although being aware of the broader contexts and policy-driven environments that influence curriculum construction, it is the curriculum that is enacted in the classroom that is of interest in this research. Teachers are often at the end of a chain of curriculum development where they have had minimal input, making daily decisions regarding the implementation of system-approved frameworks. It is this "process of translating system documents into teaching and learning for students … that lies at the heart of teaching" (Brady & Kennedy, 2003, p. 24) and therefore at the heart of this thesis.

Teachers add a pedagogical dimension to curriculum to create daily learning experiences for their students (Doyle, 1992). Shulman (1987) describes such knowledge as "pedagogical content knowledge" and situates it within a broader model of teacher education with seven categories of important teacher knowledge: content knowledge; general pedagogical knowledge; curriculum knowledge; pedagogical content knowledge; knowledge of learners and their characteristics; knowledge of educational contexts; and knowledge of educational ends, purposes, and values. Shulman's model embeds curriculum as an important aspect of teaching and this thesis maintains the connectedness between the two, thus grounding the research in the classroom. Ben-Peretz (1990) encouraged teachers to see themselves as "informed and creative interpreters" of curriculum being able to reflect upon and reconstruct the curriculum to meet their learning objectives (p. xv).

Although Stenhouse (1975) placed emphasis on the teacher, the role of the student in curriculum is also central to this thesis. Students negotiate and "experience" the curriculum; they determine their own level of engagement and interest in classroom activity and therefore exert some control over their own learning and knowledge construction. "In the end it is teachers and students who exercise the ultimate control over the curriculum" (Brady & Kennedy, 2003, p. 25). Outcomes and content may be given in curriculum documents, yet teachers develop learning experiences for their students in the contexts of their schools and their classrooms, and students negotiate with teachers the level to which they engage in and undertake these learning experiences. Snyder et al. (1994) similarly suggest that "curriculum enactment" appropriately describes the process of implementation and educational experience that teachers and students jointly undertake as they negotiate and determine what the curriculum will be like in each classroom. It is in classrooms that curriculum becomes a social practice (Wexler, 1990) and is something to be understood (Pinar, Reynolds, Slattery, & Taubman, 1995). This thesis aims to deepen understanding of the construct of numeracy through the lens of the teacher positioning curriculum in the classroom where students themselves experience the curriculum.

2.2.2 Curriculum reform

Change is an inherent part of society and therefore of the construction of schooling and curriculum. Particularly in the last five decades change has been frequent and significant with respect to the aims and objectives of schooling, its content, assessment processes, teaching strategies, funding, and provisions (Marsh, 2004). Planned change is described by Fullan (1991) as being multi-dimensional – related to changes in goals, skills, beliefs, and behaviour – and ultimately concerned with change in practice. It may be focused on classroom or school level initiatives or on broader reforms and reconstructions of part or whole educational systems (Poppleton, 2000). This study is situated within the systemic reform of a state education system, with particular philosophical and pedagogical underpinnings. It is concerned with the impact of the reform at the classroom level with respect to the goals of the research: the positioning of numeracy within the reform-based environment. Reforms are usually directed at teachers and students in classrooms (Glatthorn & Jailall, 2000).

The central role that teachers play in the reform process is well recognised, not only in their roles as individuals but also as members of a team working collegially to introduce reform into the classroom (Fullan & Miles, 1992; Hargreaves, 1994; Little, 1999). In this sense it is collaboration that promotes professional growth and acknowledges individual capacity, organisational capacity, and the relationships between them (Knapp, 1997). Borko, Wolf, Simone, and Uchiyama (2003) advanced the work of Newmann, King, and Youngs (2000) and highlighted the complex nature of curriculum reform and the requirement for the coordination of all parts of the educational system to enable any likelihood of improved student learning. They identified six dimensions of school capacity: principal leadership; professional community; program coherence; technical resources; knowledge, skills, and dispositions of individual teachers; and learning opportunities for teachers (Borko et al., 2003).

Kennedy (1995) alleges that although curriculum reform may be about changes in content and organisation of curriculum, the limitations placed by social, economic, and political contexts are significant. Hargreaves (1994) highlights the embedded nature of curriculum reform within societal changes when considering the transition to a postmodern, post-industrial society in the late 1900s and into the twenty-first century: "at the heart of the transition is the globalisation of economic activity, political relations, information, communications and technology" (p. 47). The reform agendas in advanced industrialist societies have been driven by the increased economic competitiveness brought about by globalisation. The focus then for schooling has become the development of the skills and knowledge that can add value to the economy. This is predominantly discussed in terms of innovation, creativity, and entrepreneurship. Porter (1999) argued that these economic purposes are now dominating the political agenda to such an extent that other important functions of education, such as the social and the cultural, are being diminished. In this context, curriculum reform has risen to prominence in many countries. In the United States of America, and in the West in general, reforms are concerned with an interest in national standards and a core set of knowledge and skills for all students. In the United Kingdom, a National Curriculum was introduced in 1988, but more recent agendas focus on literacy and numeracy standards. In the non-industrialised regions of the Asia Pacific a very different set of issues is driving reform agendas. Access to schooling, health education, and vocational education are highly important in these contexts (Kennedy, 2003).

Within these broader contexts of society, re-conceptualisation of curricula is occurring in response to changing views of knowledge and learning and the consideration of the goals of education. Core curriculum, higher-order thinking skills, diversity and inclusive education, lifelong learning, multi-literacies, integrated curriculum, middle schooling, formative assessment, information and communication technologies, and citizenship education are some of the current priorities impacting upon the construction of curriculum. Both curriculum construction and numeracy itself are situated not only within the broader contexts of globalisation and political agendas, but also within the many re-constructions of teaching and learning that continue to occur.

2.2.3 Australia's curriculum landscape

In Australia, the curriculum landscape in many ways mirrors changes occurring internationally with respect to the goals of a democratic nation positioning itself within a global marketplace. As in other industrialised countries, the over-riding goal is to equip students with the competencies that will enable them to live successfully as adults in this century, to contribute to the social needs of democratic society, and to gain the necessary attributes and skills to make a positive contribution to the nation's role in a global economy.

Within this broader setting, much of Australia's curriculum landscape is contradictory. Since the inception of public education in Australia in the 1870s education has been a state-controlled area of government. This has resulted in key differences between the states in the conception and implementation of curriculum. In 1963 the responsibility for the funding of education in Australia became a federal responsibility and thus resulted in the tension between the "nation-building aspirations of the Commonwealth government on the one hand, and the constitutional responsibility of the states for education (and thus for curriculum) on the other" (Reid, 2005, p. 39). Numerous attempts to implement a national curriculum have been unsuccessful although the development of the "national" Statements and Profiles (Australian Education Council [AEC], 1991) did result in a description of curriculum across eight Key Learning Areas (KLAs) and the states and territories have adopted these in varying degrees (Reid, 2005). These eight KLAs are English, Mathematics, Science, Studies of Society and Environment, Health and Physical Education, Languages other than English, Technology, and the Arts. At the time of the study a national curriculum was very much on the Commonwealth's agenda and the Federal Government was increasingly utilising funding and policy to influence curriculum across the country. Initiatives included national literacy and numeracy benchmarks and testing (e.g., Curriculum Corporation, 2000; MCEETYA, 2008b), establishing national curriculum bodies to develop policies of national interest (e.g., Australian Curriculum, Assessment and Reporting Authority [ACARA], 2009; National Curriculum Board [NCB], 2008), and attaching the provision of education funding to specific reporting requirements (DEST, 2005).

Wilson (2004) describes the dichotomy that represents Australia's curriculum development as involving two contrasting views of knowledge: "cultural transmission" and "higher-order capacities." "Cultural transmission" places value on specified areas of knowledge and skills that are clearly defined whereas "higher-order capacities" prioritises the development of higher-order thinking and generic skills such as communication, problem-solving, and decision-making over particular knowledge. Many Australian states have been re-conceptualising curricula in terms of the second view. They are focusing on a coherent set of the most important ideas and skills, sometimes referred to as "big ideas" or "key ideas," which will enable students to become productive and critical members of society. The curricula are values-based and call for the study of these "big ideas" across the traditional discipline boundaries. The development of students' capacities for thinking and building understandings is placed at the centre.

Table 2.1 provides a summary of the organisation of curricula in all the Australian states and territories at the time of this study. In the table, the term "framework" implies a more integrated approach to teaching and learning, whereas the term "syllabus" indicates emphasis on individual subjects (Harris, 2005). Whether focusing on individual subjects or a more integrated approach, all the states and territories in some way relate their curriculum organisation to the eight KLAs.

Table 2.1

Australian curriculum organisation (excerpts taken from Harris, 2005, p. 55 and www.curriculum.edu.au)

State/Territory	Formal curriculum	Curriculum organiser and emphasis	Essential or Core Learnings (where declared as central to the framework).
New South Wales	Curriculum Framework K-10	Syllabuses - KLA emphasis although syllabus differentiation within KLAs	N/A
Victoria	Victorian Essential Learning Standards P-12 (VELS)	Framework - VELS endorses a discipline emphasis alongside integrated approach	Discipline-based learning Interdisciplinary learning Physical, personal and social learning
Queensland	Curriculum Framework K-12 (New Basics)	Framework - KLA emphasis within four overarching core learnings	Life pathways and social futures Multiliteracies and communication media Active citizenship Environments and technologies
Western Australia	Curriculum Framework K-12	Framework - 8 Learning Areas (congruent with KLAs) With 5 underpinning core shared values	Pursuit of knowledge Self acceptance Respect for others Social and civic responsibility Environmental responsibility
South Australia	Curriculum Standards and Accountability Framework (R-12)	Framework - 5 Essential Learnings supported by Learning Areas (includes KLAs and age group levels)	Futures Identity Interdependence Thinking Communication
Tasmania	Essential Learning Framework	Framework - 5 Essential Learnings supported by KLAs	Thinking Communicating Personal futures Social responsibility World futures
Australian Capital Territory	Curriculum Framework P-12	Statements - KLA emphasis	N/A
Northern Territory	Curriculum Framework (based on EsseNTial Learnings)	Framework – 4 Essential Learnings supported by 8 Learning Areas (congruent with KLAs)	Inner learner Creative learner Collaborative learner Constructive learner

Juxtaposed with this curriculum reform at the state or territory level is concern at the Commonwealth level for high level quantitative, scientific, and technological literacy and for productive innovation to occur if Australia is to continue to grow as a knowledge-based economy and society.

> Special emphasis is needed now on improving scientific and mathematical education and technological capability. (Committee for the Review of Teaching and Teacher Education [CRTTE], 2003b)

> A fundamental question being asked as part of this process is how schools can best develop, in all students, a capacity to innovate, to be creative and to take great control of their lives... (CRTTE, 2003a)

There is much debate amongst educators about the place of the disciplines in these new curricular frameworks. A values-focused curriculum is aimed at enabling legitimate connections between disciplines as well as engendering understanding through engagement. Likewise, people with singularly well-developed expertise are necessary to push the boundaries of innovation in Australian society. Are these two goals at odds? The needs for both numerate citizens *and* innovative and creative scientists, mathematicians, and technologists, should not be mutually exclusive (Skalicky, 2006; Watson, Beswick, Brown, & Callingham, 2007). Curriculum, now, needs to provide "opportunities for participation in learning communities and strengthening the links between participation in school-based learning communities and in other contexts for learning... and must provide access to 'specialist knowledge communities'" (Young, 1999, p. 476). Hanlon (2004), Executive Director Curriculum Standards and Support, D*o*ET, at the time of the study, acknowledged that there is a place for the disciplines within the broader goals of thinking and understanding that the Tasmanian curriculum reform was seeking to engender.

> From a strong base of values and purpose determined through broad community consultation we are focusing on higher order thinking and understanding, and building a conceptual framework for curriculum that respects and encompasses discipline/subject knowledge and the forms of inquiry at the heart of the disciplines. (Hanlon, 2004, p. 55)

2.2.4 Essential Learnings: A curriculum for Tasmania

Tasmania is one of several Australian states that have been implementing curriculum reform founded on a values-based philosophy (e.g. Education Queensland, 2000; South Australia Curriculum Standards and Accountability

[SACSA], 2001). The setting for this study is the reform-based curriculum in Tasmania. The reform was initiated in 2000 in response to the D*o*ET's *Learning Together* initiative (D*o*ET, 2000). This initiative was established to complement a broader political initiative, Tasmania Together, a strategy of community consultation intended to help Tasmanians create a shared environmental, social, and economic vision for the next 20 years. *Learning Together* recognised the need for a curriculum that addresses the knowledge, skills, and confidence required of students in the context of the twenty-first century. It presented a plan for systemic reform of Tasmania's education system. A values and purposes statement was released and formed the basis of the development of Tasmania's Essential Learnings curriculum formally released with Essential Learnings Framework 1 (D*o*ET, 2002).

This initial curriculum construction period involved a consultative process with schools, teachers, child-care workers, business people, community members, and students all involved in its conceptualisation, whilst the implementation of the curriculum, planned to occur over a five year period from 2004 – 2009, involved a more traditional authority innovation decision-making approach (Watt, 2006). The Tasmanian curriculum from 2002 was centred around five curriculum organisers, termed Essential Learnings, with 18 key elements structured as shown in Figure 2.1. "The five curriculum organisers, Thinking, Communicating, Personal Futures, Social Responsibility and World Futures, provide a framework that can be held in the mind, a focus for teaching and learning and a means of selecting content that is significant" (D*o*ET, 2002, p. 11). Each of the 18 key elements had specified learning outcomes for students spread across five standards to cater for the needs of students from Kindergarten to Grade 10 and to guide planning, instruction, and assessment within the curriculum (D*o*ET, 2003).

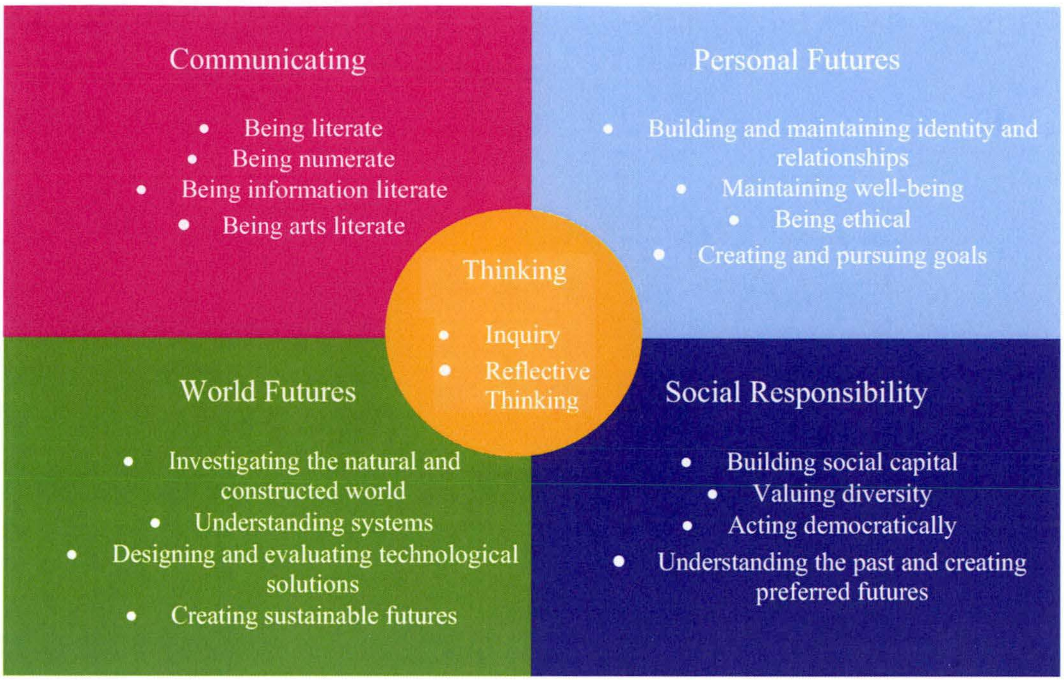


Figure 2.1. The Essential Learnings Framework.
(D*o*ET, 2002, p. 7)

It is within the Tasmanian Essential Learnings reform curriculum setting that the major focus of this thesis, numeracy, is explored. How are teachers positioning numeracy in a values-focused curriculum and how do students experience numeracy in these classrooms? As Australia grapples with the re-conceptualisation of curriculum, it is crucial that the place of numeracy is considered. "Critical and active engagement with democracy in the curriculum and wider society is the key" to preparing students for participation in a global world (Vidovich, 2005, p. 113). This thesis argues that numeracy has an important part to play in supporting this critical engagement with democracy.

2.3 Numeracy: An essential capability

Numeracy has become an essential capability for any individual who wishes to participate fully in a democratic society, to apply not only knowledge and skills, but also critical reasoning capabilities, in learning and in everyday life. Citizens are making judgments and decisions on a daily basis with regard to the situations they encounter, be these situations personal, work-related, or on a broader societal level. There are many examples in daily life and in the media where an ability to analyse information critically is required to make informed judgments. Politicians, media, and industry are talking regularly about many issues that impact on people's lives, such as interest rates, environmental issues, health policy, and national security. At an individual level people are dealing daily with work-related challenges and issues such as personal finance, health, shopping choices, and time management.

2.3.1 Conceptualising numeracy

The concepts and skills required to meet the numeracy demands of everyday life are defined and examined under various names, including "quantitative literacy" (Steen, 2001), "mathematical literacy" (OECD, 2003), "statistical literacy" (Watson, 2006), "critical numeracy" (Gal, 2002; Johnston, 1994; Lake, 2002), "critical mathematical literacy" (Frankenstein, 1998), "mathemacy" (Skovsmose, 2004), and "numeracy" (AAMT, 1997). Discussion amongst academics, mathematicians, and industry leaders, concerning numeracy, revolves around its increasing relevance for today's citizens. This conversation has extended to those involved in education at the school level and explores how to bring about learning that acknowledges the quantitative challenges of life in the twenty-first century. "In the twenty-first century, literacy and numeracy will become inseparable qualities of an educated person" (Steen, 2001, p. 9).

In 1959, The Crowther Report introduced the notion of numeracy as being the "...mirror image of literacy ... [and recognised] the need in the modern world to think quantitatively" (Quoted in Cockcroft, 1982, p. 11). Although the Crowther Report implied a broad concept of numeracy that incorporated higher level thinking processes in the same way as literacy does, since that time numeracy has developed across the globe in many different forms, reflected by many different terms and definitions. Willis (1990) defined being numerate as being able "to function effectively mathematically in one's everyday life, at home and at work" (p. vii). The perception that numeracy involved "using some mathematics to achieve some purpose in a particular context" (AAMT, 1997, p. 13) is representative of the intent of most attempts to describe numeracy during the 1980s and 1990s.

Although recognition of the importance of context is a feature of most definitions of numeracy, Doig (2001) notes that in the United Kingdom numeracy is a concept very much aligned with number. This sense that numeracy requires a lower level of mathematics than that implied by Crowther appeared in a British government report on mathematics education (Cockcroft, 1982). The Cockcroft Report identified two aspects of numeracy: an "at homeness" or being comfortable with numbers, and an ability to understand and interpret information presented in mathematical ways. Recent reforms in the United Kingdom have been initiated by a national literacy and numeracy strategy; however, the emphasis on number remains a focus for numeracy reforms with the other strands of mathematics, higher order thinking, and understanding of mathematical concepts all neglected (Fullan & Earl, 2002).

In Australia, the term "numeracy" is used to describe the quantitative capabilities of both students and adults. A view of numeracy has developed that more closely matches the term "mathematical literacy" used in Europe, and the term "quantitative literacy" as used in the United States of America. A view, beyond the ability to handle number concepts in context, is reflected by the AAMT definition:

> To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life.
> In school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of:
> - Underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic);
> - Mathematical thinking and strategies;
> - General thinking skills; and
> - Grounded appreciation of context. (AAMT, 1998, p. 2)

The OECD Programme for International Student Assessment (PISA), a three-yearly survey of the knowledge and skills of 15-year-olds in the principal industrialised countries, defines "mathematical literacy" as:

> An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that

> individual's life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 72)

The term "quantitative literacy" was used as early as 1974 when Professor Jerrold Zacharias of Massachusetts Institute of Technology identified "the varieties of competence that a citizen must possess in order to handle the matters and arguments that affect him and his country and his world" (p. 9). Public discourse surrounding quantitative literacy has grown significantly since that time, principally as a result of the writings and edited works of Lynn Arthur Steen, professor of mathematics at St. Olaf College in Minnesota, as documented in the publications, *Why Numbers Count* (Steen, 1997), *Mathematics and Democracy* (Steen, 2001), and *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges* (Madison & Steen, 2003). A comprehensive definition of "quantitative literacy" is detailed in Steen's case statement (2001), in which he describes ten elements of quantitative literacy.

> *Confidence with Mathematics*. Being comfortable with quantitative ideas and at ease in applying quantitative methods. Individuals who are quantitatively confident routinely use mental estimates to quantify, interpret, and check other information. Confidence is the opposite of "math anxiety"; it makes numeracy as natural as ordinary language.
> *Cultural Appreciation*. Understanding the nature and history of mathematics, its role in scientific inquiry and technological progress, and its importance for comprehending issues in the public realm.
> *Interpreting Data*. Reasoning with data, reading graphs, drawing inferences, and recognizing sources of error. This perspective differs from traditional mathematics in that data (rather than formulas or relationships) are at the center.
> *Logical Thinking*. Analyzing evidence, reasoning carefully, understanding arguments, questioning assumptions, detecting fallacies, and evaluating risks. Individuals with such habits of inquiry accept little at face value; they constantly look beneath the surface, demanding appropriate information to get at the essence of issues.
> *Making Decisions*. Using mathematics to make decisions and solve problems in everyday life. For individuals who have acquired this habit, mathematics is not something done only in mathematics class but a powerful tool for living, as useful and ingrained as reading and speaking.
> *Mathematics in Context*. Using mathematical tools in specific settings where the context provides meaning. Notation, problem-solving strategies, and performance standards all depend on the specific context.
> *Number Sense*. Having accurate intuition about the meaning of numbers, confidence in estimation, and common sense in employing numbers as a measure of things.
> *Practical Skills*. Knowing how to solve quantitative problems that a person is likely to encounter at home or at work. Individuals who possess these

> skills are adept at using elementary mathematics in a wide variety of common situations.
> *Prerequisite Knowledge.* Having the ability to use a wide range of algebraic, geometric, and statistical tools that are required in many fields of postsecondary education.
> *Symbol Sense.* Being comfortable using algebraic symbols and at ease in reading and interpreting them, and exhibiting good sense about the syntax and grammar of mathematical symbols. (p. 8)

Such a broad view serves to move quantitative literacy away from being positioned as a subset of literacy, as reflected in the International Adult Literacy Survey (IALS) (OECD, 2000), where it was only concerned with the understanding and ability to manipulate numbers within text. This broader view is also evident in the redesigned 2003 International Adult Literacy and Lifeskills Survey in which numeracy "applies to the knowledge and skills required to manage mathematical demands of diverse situations" (National Center for Education Statistics, 2005, p. 1). It is seen as an important link between mathematics and the real world, and is concerned, not only with quantity and number, but also with other strands of mathematics: shape, patterns and relationships, chance and data, and change.

More specifically, the field of adult numeracy has contributed to the concept of critical numeracy when considering the relationship between mathematics and numeracy. Johnston (1994) argues:

> Numeracy is a critical awareness which builds bridges between mathematics and the real world, with all its diversity. ... In this sense ... there is no particular 'level' of mathematics associated with [numeracy]: it is as important for an engineer to be numerate as it is for a primary school child, a parent, a car driver or a gardener. The different contexts will require different mathematics to be activated and engaged in (p. 34).

Such a stance, acknowledging the social and cultural aspects brought to bear on the use of mathematics in context, is important when considering numeracy, not only for adults but also for students who need to develop these numeracy capabilities.

More recent terms, "mathemacy" (Skovsmose, 2004) and "critical mathematical literacy" (Frankenstein, 1998), are informed by the work of critical theorists such as Foucault (1989) and Freire (1970). Such approaches move beyond the recognition of social and cultural contexts to an awareness of the role of politics and power within those contexts, and the ability of critical numeracy capabilities to transform. Mathematics is seen as a tool to interpret and challenge inequities in society. Frankenstein (1990) argues that critical mathematical literacy involves "the ability to ask basic statistical questions in order to deepen one's appreciation of political issues ... critical understanding of numerical data thus prompts individuals to question taken for granted assumptions" (p. 336).

Although closely related, the use of the term "critical" here is distinguished from higher order or metacognitive thinking capacities, included in many conceptions of numeracy as "critical thinking." Although critical thinking may at times include thoughtful consideration of political and social inequities presented in mathematical "texts," it is about the cognitive processes brought to bear on problems or investigations. Processes such as analysing, evaluating, and creating, as described by Anderson and Krathowl (2001), exemplify those that are used in critical thinking. Critical mathematical literacy, or critical literacy, however, refers more specifically to the attitude with which such knowledge or situations are approached. It is about looking for underlying explanations and questioning whose interests are being served. It is argued that a person can exhibit a high level of quantitative or mathematical literacy without necessarily acting for social justice (Gutstein, 2005). Critical thinking is a necessary component of critical numeracy, whereas critical numeracy is not a prerequisite for critical thinking.

Media messages are presented from a diverse range of sources, such as journalists, political commentators, and advertisers, all of whom represent particular points of view and potential bias. The need for critical evaluation of quantitative messages is an important element of both literacy and numeracy. It is expressed in Steen's (2001) work within the element *Logical Thinking.*

Watson (2006), in her research into school students' understanding of statistical concepts, also expresses the importance of recognising the social context in which mathematics is used and demonstrating higher level thinking skills to interpret information, question claims, and exercise judgment. Statistical thinking is an important element of descriptions of numeracy and quantitative literacy (Gal, 1995; Steen, 2001). Watson and Callingham (2003) describe a six-level developmental model of statistical literacy requiring mathematical understanding, appreciation of context, and critical thinking capabilities. At the highest level students demonstrate quantitative reasoning capabilities that enable them to synthesise complex mathematical skills, such as proportional thinking and inference, together with "appreciation of subtleties of language and context" (Watson, 2006, p. 267) in considering and responding to tasks.

2.3.2 Factors influencing numeracy learning

Just as changing views of knowledge, learning, and broader goals of education impact upon re-conceptualisation of curricula and the conceptualisation of numeracy, so also do they impact upon the daily learning experiences and outcomes of students. As argued in Section 2.2.1 it is the enactment of curriculum within the classroom learning environment that is of interest in this study and more particularly the enactment of the teaching and learning of numeracy within the context of curriculum reform. In this section, some key factors that pose particular pedagogical challenges for teachers and influence students' numeracy learning are considered. In acknowledging the large research area that this entails, four areas are briefly discussed. They are those that have featured within mathematics education research in the past fifteen years and that intersect broadly with the aims and objectives of the study – curriculum, teaching, assessment, and learning environments. It is acknowledged that these are not discrete areas and overlap considerably, but they are addressed separately in order to emphasise key points of interest.

2.3.2.1 Curriculum influencing numeracy learning

The broader influence of curriculum on student learning has been discussed in Section 2.2. How numeracy is conceptualised within curricula also influences the way that teachers interpret and position it within the classroom. The ways that numeracy has been interpreted in practice are discussed in more detail in Section 2.3.3. This section will consider the influence of curriculum on student learning within the field of mathematics education.

In the field of mathematics education, there has been an explosion of curriculum materials in the last fifteen years influenced by the momentum of the mathematics education reforms of the 1980s (Van de Walle, 2004). These reforms had their beginnings in the 1960s when, in response to space exploration, countries across the world, including the United Kingdom, the United States, the Netherlands, and Australia, independently began reviewing what should be taught in school mathematics curricula (e.g. Blane, Maurer, & Stephens, 1984; Cockroft, 1982; De Corte & Verschaffel, 1986). These reforms saw a change of focus from mathematics content to how students can best learn mathematics. The NCTM Standards document in the United States (1989) encapsulated much of the reform ideas, urging for a move away from an emphasis on the practice of procedures (Olson & Barrett, 2004), to approaches that support a constructivist view of learning (Van de Walle, 2004). Some reform advocates also saw new curricula as having the potential to influence teachers and their instruction to move from a focus on basic skills to more conceptually-based problems that required reasoning, problem solving, and communicating (Senk & Thompson, 2003).

Studies have argued that students taught using standards-based curricula, as compared with students taught using conventional mathematics curricula, generally evidence greater conceptual understanding and higher levels of problem-solving (Briars, 1999; Thompson & Senk, 2001). Furthermore, students have also been found to perform equally well on traditional mathematical tests that assess predominantly skills and procedures (e.g., Thompson & Senk, 2001).

Wilkins (2000) examined the Trends in International Mathematics and Science Study [TIMSS] study – a study involving the collection of educational achievement data at Year 4 and Year 8 from over 60 countries, to provide information about trends in mathematics and science achievement over time. – from the perspective of five components of quantitative literacy: mathematical content knowledge, mathematical reasoning capability, recognition of societal impact and utility of mathematics, understanding of nature and historical development of mathematics, and disposition towards mathematics. Although Wilkins acknowledged the rich data source that the TIMMS study offered, he cast a timely warning:

> Future research needs to focus on creating measures that more precisely capture the essence of each component of quantitative literacy... In addition, the effects of different curricula and instructional methods on the development of quantitative literacy should be examined. (Wilkins, 2000, p. 416)

As argued earlier in this chapter, the influence of curriculum on student learning cannot be understood without considering its interpretation by teachers and enactment within the classroom learning environment.

2.3.2.2 Teaching influencing numeracy learning

In keeping with the view of curriculum argued in Section 2.2, teaching in this thesis is about the interactions that occur between teachers and students with the aim of facilitating student understanding of planned for learning goals. This view aligns with that of Cohen, Raudenbush, and Ball (2003) who argue that "instruction consists of interactions among teachers and students around content" (p. 122) and also with Lampert (2001) who described the practice of teaching as involving "a teacher doing something with students around something to be learned" (p. 1).

Teaching is not deemed to be synonymous with teachers themselves, as there are broader factors beyond teachers' backgrounds, knowledge, beliefs, skills, and dispositions that can impact upon the learning interactions that occur within the classroom. Some of these factors might include the curriculum itself, professional learning and collaborative opportunities provided, and available resources. These factors are not dissimilar to those identified by Borko et al. (2003) as being dimensions that impact on school's capacities to implement reform agendas, and listed earlier in Section 2.2.2.

In any research study concerned with the complex interaction between teaching and learning it is important therefore to acknowledge and consider these broader contextual factors. It is however the beliefs and practices of teachers themselves that still remain at the fore of this study. The educational beliefs that teachers hold regarding teaching and learning are an important feature of the successful implementation of reforms and innovation and have a significant effect on classroom practices (Fullan, 1993; Richardson, 1996). Beliefs also form a filter for teachers' instructional and curricular decisions and actions and can therefore promote or inhibit change (Prawat, 1992). It is also acknowledged that classroom practice can encourage teachers to review and revise their beliefs (Guskey, 2002).

In relation to conceptions of learning, emphasis on conceptual understanding underpins the reform movement and is widely associated with effective teaching of numeracy (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997; Clarke, 2005; Clarke et al., 2002; Grouws & Cebulla, 2000). The success of particular Asian countries in the TIMSS studies conducted in 1995, 1998, and 2003 sparked an interest in describing commonalities between classrooms of particularly effective countries (Clarke & Clarke, 2002). In response to recommendations to redefine numeracy for teaching practice, much of the research in the last decade focused on teachers' everyday practices (Groves, Mousley, & Forgasz, 2006), with a common theme involving the description of characteristics of effective teachers of numeracy (Askew et al., 1997; Clarke et al., 2002). Across these studies there is general consensus which suggests that there are characteristics that are common to effective teachers of numeracy (Groves et al., 2006). These teachers:

- have high expectations of their students;
- focus on children's mathematical learning, rather than on providing pleasant classroom experiences;
- provide a challenging curriculum;
- use higher-order questioning;
- make connections both within mathematics and between mathematics in different contexts; and
- use highly interactive teaching involving students in class discussion.

Many of the characteristics of effective teaching of numeracy relate to what Askew and his colleagues termed "connectionist" teachers (Askew et al., 1997). The studies all determined "effectiveness" based on measures of aspects of students' mathematical growth and the findings were consistent with the recommendations made by the mathematics reform movement. It is noted that in Askew et al.'s (1997) *Effective teachers of numeracy* study, numeracy was defined as "the ability to process, communicate and interpret numerical information in a variety of contexts" (p. 7). The student assessment instruments focused largely on number, with a mix of both out of context and within context questions.

In Tasmania, at the time of this study, teachers had access to guidelines, practices and resources for teaching numeracy through the Department *of* Education's Learning, Teaching and Assessment Guide (LTAG) web-site (http://www.LTAG.edu.au). One of the resources on this site, "Principles for improving numeracy in schools" (McIntosh, 2002) advocated the type of teaching that reflects reform-oriented teaching practices. McIntosh described these as incorporating conceptual understanding, promoting connections, encouraging communication, consolidating concepts, coordinating across classes, creating a community of learners, and enacting a coherent curriculum. The principles are consistent with the broader reform movement in mathematics. With regard to communication, McIntosh emphasised that teachers must value the reasons for answers as well as the correctness of answers and emphasised the need for children to explain their answers. The emphasis on communication is reflected in the literature, with research recommending that students be given the opportunity not only to explain their answers, but also to question, justify, and articulate their thinking (e.g., Askew et al., 1997; Clarke et al., 2002; Luke et al., 2003; Van de Walle, 2004; Watson & De Geest, 2005).

2.3.2.3 Assessment influencing numeracy learning

A specific challenge for teachers of numeracy is the bringing of current ideas about assessment in line with ideas about the teaching and learning of numeracy. The conceptualisation of assessment and its role and function in the classroom has attracted increasing attention from the international mathematics education community over the past twenty years (Niss, 1993a; 1993b). Much has been written concerning assessment that is educative, formative, an integrated part of teaching and learning, and aimed at improving student performance (Black, Harrison, Lee, Marshall, & Wiliam, 2003; Blythe, 1998; Newmann, King, & Secada, 1996; Wiggins, 1998; Wiske, 1998). Despite this shift, the move towards new forms of assessment, in particular standards or performance-based assessment of student levels of understanding, is inconsistent. Support for reform and performance assessment at classroom level is occurring at the same time as an increasing use of standardized and external testing (Dwyer, 1998). Wiggins (2003) discusses this difficulty in relation to quantitative literacy and the need for tasks to reflect the actual practice of real-world problems in order for them to be realistic and provide evidence of students' abilities to use mathematics in varying and complicated situations. Making the boundaries between school and the world less distinct by bringing authentic contexts into the classroom still needs to be considered in terms of the effect these contexts might have on student learning (Anderson, Reder, & Simon, 1996). Franklin (2002) noted difficulties in moving to authentic assessment as including clarity for parents, teacher preference for traditional methods, and the time-consuming nature of the assessments themselves.

Assessment items examining mathematical literacy, numeracy, or quantitative literacy have moved toward a reflection of a changing view of assessment by embedding key mathematical skills to be tested in a context or story that provides relevance to the skills to be tested. Such tasks require students to draw on multiple knowledge constructions and sometimes both in-school and out-of-school experiences. Students may privilege some constructions over others, negotiate between areas of knowledge construction and, at the highest level, integrate multiple constructions to fit the assessment task (Kastberg & D'Ambrosio, 2004). In real life such integration of knowledge is critical to successful problem solving experiences but item developers still tend to use a context to engage students, not for it to be considered realistically (Kastberg & D'Ambrosio, 2004) and not for the assessment of multiple objectives (Skalicky, 2004).

There are many examples of units of work in which mathematics has been approached from a real-world setting or from another discipline, such as science or literature (Billings & Lakatos, 2003; Leonard & Campbell, 2004; O'Donnell, 2001; Ronau & Karp, 2001). In these examples, such conclusions as students "better understood the problem of litter through collecting and analysing actual data" (Ronau & Karp, 2001, p. 31) are not supported by any discussion of the form of assessment upon which the conclusions were based. Despite connections being made, a commensurate shift in assessment practices that acknowledges the multiple elements within the units of work is yet to be realised. Educators interested in the bringing together of disciplines for the purpose of teaching concepts in a contextual framework are grappling with this issue. "How will mathematical and scientific understanding be assessed, particularly when the concepts may be embedded in rich and complicated contexts?" (Frykholm, 2002).

2.3.2.4 Learning environments influencing numeracy learning

As is known from research on the effectiveness of schools (Hopkins, 2005; Lee & Williams, 2006), learning environments play a major role in affecting student learning and outcomes. The culture of the classroom learning environment is dependent on the shared meanings and social norms that both teachers and students bring to the classroom and how they interact within it (Nickson, 1992). As a result of an increasing interest in the social and contextual nature of learning (e.g., Cobb, 1986; Lave, 1988) teachers have been encouraged to ensure that teaching and learning within mathematics education is based upon shared activity, where ideas and strategies are investigated and discussed. Nickson (1992) argues that a move toward the valuing of more open, participative, and questioning learning environments has resulted in an increasing diversity of classroom cultures.

Within the field of mathematics education there have been few studies that have connected the curriculum as enacted in the classroom learning environment with student learning. One study that has, the 1999 TIMMS video study (National Center for Education Statistics [NCES], 2003) investigated a random sample of 100 eighth grade mathematics classes from across six countries (Australia, the Czech Republic, Hong Kong, Japan, Switzerland, and the Netherlands) that had performed significantly higher the United States on the 1995 TIMSS mathematics achievement test (Stigler & Hiebert, 2004). Overall the 1999 video study revealed that the higher-achieving countries implemented a greater percentage of tasks that encouraged students to make connections between mathematical concepts, and not reduce these tasks to procedural exercises (Stigler & Hiebert, 2004).

The student-centred approach to the learning of mathematics has emerged from the mathematics education reforms and broader reforms of learning and teaching.

It places a focus on teachers developing learning environments that are conducive to student inquiry rather than having mathematical concepts and procedures directly taught to them (Blythe, 1998; Rafferty, 1999; Scott, 2001). Schoenfeld (1992) argued that becoming a good thinker in any domain of learning is about acquiring the habits and dispositions of sense-making as much as the skills, strategies, and knowledge. To provide students with these opportunities requires that mathematics education is more of a socialisation process than an instructional one (Resnick, 1988).

It is acknowledged that it is difficult to separate the learning environment from the teacher. Within a broader view of teachers and students together creating the culture of the learning environment, it is acknowledged that teachers play a significant role in creating an environment for learning in order for students to relate to both the subject matter and to each other (Ball & Bass, 2000; Cobb, 2000; Lampert, 2001).

2.3.3 Numeracy in practice: Examples

The definitions and descriptions of numeracy, and associated terms highlighted in Section 2.3.1, reveal the complex nature of numeracy and the differing emphases placed on its conceptualisation, from a focus on basic mathematical skills to a consideration of the interplay among numerous complex factors that contribute to a person's capacity to interpret and think critically about quantitative information in a variety of contexts. These conceptualisations, together with the complex interplay of factors that combine to influence student learning of numeracy as discussed in Section 2.3.2, strongly influence how numeracy is interpreted in practice both from a research perspective and within the classroom learning environment. In this section, three ways that numeracy has been interpreted in practice are considered by examining some of the projects and learning activities being undertaken in schools to develop students' numeracy capabilities.

2.3.3.1 Numeracy as computation and mathematical skills

Research projects that interpret numeracy as being about computation and the development of mathematical skills focus on the learning of mathematics within the mathematics classroom. There is generally an emphasis on improving students' basic mathematical skills and there has been a predominance of work in the area of number, particularly in the United Kingdom where a number-based conception of numeracy has been adopted (Doig, 2001). As Brown (2000) noted,

> our UK politicians are more concerned with the traditional 'basic skills' characterised by what they believe the public wants, i.e. knowing addition and subtraction number-bonds and multiplication tables, together with facility in both mental arithmetic and traditional written procedures. (p. 1)

The *National Numeracy Project*, established in the United Kingdom in 1999 in response to the TIMSS video study (NCES, 2003), was concerned with the improvement of mathematics, including, number, measurement, and data handling, in primary schools. Later space and shape were added. It focused upon improving "practical teaching strategies," by using "detailed plans, practical guidance" and "offering out-of-school courses and in-school support for professional development" (Department for Education and Employment, 1999). A *Numeracy Task Force* was set up as a research project for five years and produced the *National Numeracy Strategy* and training programs for teachers (Askew, 1999; Price, 1998). The *Leverhulme Numeracy Research Project* (Brown & Askew, 2000) looked at underachievement in numeracy, incorporated the *National Numeracy Strategy*, and focused on the mathematics in the classroom (Brown, 2000; Brown & Askew, 2000).

In Australia and New Zealand, the projects *Count Me In* and *Count Me In Too* were school-based professional development projects that focused on students' mathematical strategies and have an emphasis on number (Gould, 2000; Bobis & Gould, 1999; Mulligan, Bobis, & Francis, 1999). A further program *Count Me Into Measurement*, focused on measurement (Outhred, 2001). The *Early Numeracy Research Project* (ENRP) in Victoria also focused on teaching and learning mathematics in the classroom (Clarke, 2000; Clarke, Sullivan, Cheeseman, & Clarke, 2000), as did the *First Steps in Mathematics* project from Western Australia (Department of Education and Training of Western Australia, 2004). The ENRP developed a framework for key aspects of early numeracy learning and ways to assess the mathematical profiles of students. The project considered the development of mathematical skills in the strands of number, measurement, and space using growth points as guides. The growth points were all described in terms of mathematical achievement with the impetus for the project being the desire to improve mathematics learning (Clarke, 2000).

2.3.3.2 Numeracy as mathematics in context

Griffin (1995), in considering occurrences of the mismatch between mathematical skill and numerate behaviour, suggested a lack of understanding of context or failure to transfer existing mathematical understanding between contexts. This difficulty in achieving transfer of mathematical knowledge across context is well recognised (Anderson, Reder, & Simon, 1996; Hughes-Hallett, 2001).

This perspective of numeracy places emphasis on the effective use of mathematics in everyday life by incorporating real-life contexts into the teaching and learning of mathematics within the mathematics learning area. Students may experience real-world mathematical problems in text books, on worksheets, or within investigations that require the application of mathematics to a context outside the classroom, such as planning a holiday or evaluating shopping choices.

In most middle school contexts, problem solving is confined to traditional word problems where students are not required to access knowledge or consider experiences related to their everyday life (Lowrie, 2005). Some studies, however, have demonstrated improved mathematics learning when skills and concepts are experienced in a real-world context that has meaning for the students (Bonotto & Basso, 2001; de Corte, Verschaffel, & Greer, 2000; Ronau & Karp, 2001; Steen, 1999). Creating a culture for learning mathematics differently that allows for reflective thinking and reasoning poses challenges and teacher influence is not underestimated. Teachers require the mathematical knowledge, understandings, and connections themselves in order to facilitate effective student learning (Ma, 1999). Furthermore, Sullivan, Zevenbergen, and Mousley (2003) argue that before teachers use context, there are many complex factors they need to consider, including the mathematical suitability of the context, the interest or relevance of the context to the students, the potential motivational impact, and the possibility of adverse effects or tendency to exclude some students.

For projects with this perspective the aim is to support the co-development of mathematical understanding. In Australia, for example, the *Victorian Early Years Program* (Hammond & Beesey, 1999) used mathematics that is seen as relevant to the children and related to their experience outside of school. The *Junior Secondary Numeracy Project* in South Australia (Kuss, 2000) developed a series of assessment items related to the interests of students outside of school. The *SAUCER* Project in Western Australia (Northcote & McIntosh, 1999) looked at mathematics used in adult life in order to incorporate appropriate mathematics in the classroom. Both the *Improving Numeracy for Indigenous Students in Secondary Schools* (INISSS) *Project* (Callingham & Griffin, 2001) and the *Middle Years Numeracy Research* Project (Siemon, 2000; Siemon & Griffin, 2000) incorporated many real-life contexts, such as a street party, CD sales, medicine doses, travel times, and soccer tournaments, into the teaching and assessment tasks used to evaluate students' numeracy capabilities.

The *Scaffolding Numeracy in the Middle Years* project (Court, 2005), involving teachers from both Victoria and Tasmania, was designed to investigate the efficacy of an assessment-guided approach to improving student numeracy outcomes in Years 4 to 8. In particular, it was aimed at developing a learning and assessment framework to support multiplicative thinking using context-based assessment tasks. As with the *Middle Years Numeracy Research* Project these tasks involved everyday contexts such as adventure camps and birthdays.

2.3.3.3 Numeracy across the curriculum

> Numeracy is not the same as mathematics, nor is it an alternative to mathematics. Rather it is an equal and supporting partner in helping students learn to cope with the quantitative demands of modern society. Whereas mathematics is a well-established discipline, numeracy is necessarily interdisciplinary … numeracy must permeate the curriculum. When it does … it will enhance students' understanding of all subjects and their capacity to lead informed lives. (Steen, 2001, p. 115)

In an education system designed to equip students for their lives outside of school, both as young people and future adults, a foundational aim becomes the promotion of student learning across the curriculum. Chapman, Kemp, and Kissane (1990) maintained that "if students have not learned how to use mathematics across the curriculum unless explicitly asked to do so, it is unlikely that they will use mathematics beyond the mathematics classroom" (p. 117).

Much comment has been made concerning the need for numeracy to be the responsibility not only of the mathematics teacher but also of the teachers of other disciplines as well, if students are going to have the opportunity to see numeracy as relevant to all aspects of their learning (Berlin, 2003; Frykholm, 2002; Price, 1997).

> Only by making quantitative ideas as pervasive in the curriculum as they are in life will numeracy become, like literacy, part of the fabric of liberal education. (Steen, 2000, p. 30)

> Instructors in middle school, high school and college need to join forces to deepen students' understanding of basic mathematics and to provide opportunities for students to become comfortable analyzing quantitative arguments in context... The development of quantitative literacy is the responsibility of individuals throughout the education system. (Hughes-Hallett, 2003, p. 92)

Within the projects that advocate a cross-curricular view of numeracy, primary school teachers encourage students to use mathematics in all areas of their learning. In secondary schooling this perspective necessarily involves teachers from other learning areas and disciplines other than mathematics, and requires them to identify and make explicit the use of mathematics to support effective learning.

In Australia, the *Integrated Curriculum Project* (Goos, 2001) was aimed at providing pre-service teachers with a cross-curricular approach to the teaching of numeracy. Hogan, in the *Numeracy across the curriculum* project in Western Australia (Hogan, Murcia, & van Wyke, 2004), advocated a blend of mathematical, strategic, and contextual knowledge required to take on three roles of being numerate: the fluent operator, the learner, and the critic. Thornton and Hogan (2004) also reported on a project in the Australian Capital Territory (ACT) aimed at a school-wide focus on numeracy across the curriculum, including professional development activities and classroom observation and reflection. In Tasmania, the *MARBLE* project, *Providing the Mathematical Foundation for an Innovative Australia within Reform-Based Learning Environments*, claims that the development of quantitative literacy for all students is as equally important as the higher foundational mathematics required for those students that go on to study higher level mathematics and sciences (Watson, Beswick, Brown, & Callingham, 2007). The investigators in the MARBLE project provided professional learning and support for teachers of mathematics with a cross-curricular focus.

An educational framework that embeds numeracy as one of the key cornerstones is also one focused on the development of lasting conceptual understandings. The test of this understanding, with regard to quantitative literacy, as for any other literacy, is whether a student is able to apply the appropriate skills in many different contexts. This transferability of knowledge and skills is a key aspect of numeracy and necessitates it being the responsibility of the whole school community, not just the mathematics teacher. The *National Numeracy Review Report* [NNR], (Department of Education, Employment and Workplace Relations [DEEWR], 2008) identified numeracy as requiring an across-the-school commitment.

As time passes and curriculum frameworks change, numeracy across the curriculum is vital in every context. Despite its many conceptions, numeracy today not only has clearly emerged as an integral element of the teaching and learning of mathematics but also has also been identified as being an important capability that should be developed within other curriculum areas (NCB, 2008).

> Numeracy includes capacities that enhance the lives of individuals by enabling them to interact with the world in quantitative terms, communicate mathematically, and analyse and interpret everyday information that is represented mathematically. It incorporates aspects such as number sense, measurement, estimating quantities, bearings, map reading, networks, properties of shapes, and personal finance and budgeting. Numeracy also includes the mathematics used by professionals such as economists, psychologists, architects and engineers, the mathematics that is useful in learning disciplines such as geography, chemistry, physics and electronics, and the everyday vocational mathematics used in fields such as building, sports, health and catering. It involves aspects of accurate measurement, ratio, rates, percentages, using and manipulating formulas, the mathematics of finance, modelling and representing relationships especially graphically, and representing and interpreting sophisticated data. (NCB, 2008, p. 4)

Numeracy research has been positioned primarily in the mathematics classroom. Indeed, educational frameworks themselves, position numeracy within the mathematics classroom. Tasmania's Essential Learnings provides a unique opportunity to consider the positioning of numeracy as a cross-curricular construct in a curriculum framework that itself promotes transdisciplinary education.

2.4 Numeracy: Towards a synthesised view

Green (2002), in considering the role of literacy in the English classroom and the wider curriculum, acknowledges the different discourses of language, meaning, and power that play a role in the development of literacy. He advocates the synthesis of these dimensions in forming a three-dimensional model of literacy where "the most worthwhile robust understanding of literacy is one that brings together the 'operational,' 'cultural,' and 'critical' dimensions of literate practice and learning" (Green, 2002, p. 27). Although Green acknowledges the political nature of literacy as a social practice, he calls for a balance between all the important dimensions of literacy with the aim being to support students in meaning-making in context (Durrant & Green, 2000).

It is equally important for mathematics educators to acknowledge the different dimensions that are necessary for the development of competent and effective numeracy practice. Such a balance informs this research. The critical and cultural aspects are not ignored, but neither are they preferenced over the mathematical language, skills, and functions required for students to make sense of, and critically evaluate, the contexts in which the mathematics is embedded. This study acknowledges the important contribution each element brings to a comprehensive definition of numeracy. Numeracy is about making meaning of mathematics, at whatever level of mathematical skill. In Australia, the term numeracy has become accepted as no longer being inferior to mathematics (AAMT, 1997; Johnston & Tout, 1995). It is about understanding and using mathematics, in all of its representations: for making sense of the world, for considering critically information presented, and for being critical of mathematics itself.

Numeracy as a practice, or repertoire of practices, positions both the learner and the teacher in the process of learning where classroom environments need to be cultures of "sense-making" (Mathematical Sciences Education Board & National Research Council [NRC], 1990, p. 32). One such approach to numeracy within the Australian education context is that described in the Queensland School Curriculum Council's Numeracy Position Paper (QSCC, 1999). This paper provided a framework of four organisers to describe numeracy, based upon a similar model for literacy (Freebody & Luke, 2003). Freebody and Luke (2003), in a similar way to Green (2002), advocate a synthesis of psychological, socio-cultural, and critical theory in their "four resources" model of literacy.

The QSCC framework (Appendix A) describes the resources needed for numeracy in terms of four areas of practice:

- Foundational – mathematical concepts and skills associated with the strands of mathematics; number, measurement, chance and data, space, and pattern and algebra.
- Linking – strategic processes and skills to enable appropriate use of mathematical knowledge.
- Pragmatic – personal and contextual application of mathematical skills and strategies.
- Critical – analytical and critical aspects of the use of mathematics and judgement and evaluation of representations and information.

In Tasmania the established definition of numeracy weaves together the five strands of the mathematics curriculum as outlined in the *Mathematics Guidelines K – 8* (Department of Education and the Arts Tasmania [DEAT], 1993).

> To be numerate is to have and be able to use appropriate mathematical knowledge, understanding, skills, intuition, and experience whenever they are needed in everyday life. Numeracy is more than just being able to manipulate numbers. The content of numeracy is derived from five strands of the mathematics curriculum – space, number, measurement, chance and data, and (pattern and) algebra – as described in the National Statement and Profile. (*Numerate Students, Numerate Adults* (DEAT, 1995, p. 6))

The move to the values-focused Essential Learnings as outlined in Section 2.2.4, although moving away from a discipline-based focus, still acknowledges the role of the five strands of mathematics for the development of numeracy. Reference to Being numerate is listed as a key element of one of the curriculum organisers, Communicating. The first part of the description associated with Being numerate links mathematical concepts and skills to "everyday problems" and the "demands of everyday life." It upholds that:

> Being numerate involves having those concepts and skills of mathematics that are required to meet the demands of everyday life. It includes having the capacity to select and use them appropriately in real settings. (D*o*ET, 2002, p. 21)

Although school-based mathematics is a foundation learning area for numeracy it is not an exclusive one. Developing the desired skills and competencies for being numerate becomes a responsibility across the curriculum, with the second part of the Being numerate description moving to a cross-curricular focus:

> Being truly numerate requires the knowledge and disposition to think and act mathematically and the confidence and intuition to apply particular mathematical principles to everyday problems. An opportunity to understand and use pattern, order and classification through first-hand experiences builds vital conceptual foundations for thinking and representation beyond the concrete and the immediate. These understandings are fundamental to being numerate. Access to high levels of abstract symbolic operation opens new ways of thinking and future academic and vocational pathways. (D*o*ET, 2002, p. 21)

The third part of the Being numerate description acknowledges the important critical dimension of numeracy that enables students to make informed decisions:

> Being numerate not only includes numeracy skills and understandings, but it also involves the critical and life-related aspects of being able to interpret information thoughtfully and accurately when it is represented in numerical and graphical form. This aspect of numeracy is akin to critical literacy – being able to recognise that information can be constructed to influence the reader or viewer. Developing the critical skills to analyse quantitative and spatial information when it is presented in various forms – for example graphs, tables, spreadsheets, charts and comparative models – enables young people to make more informed decisions, personally in everyday life, as consumers and as citizens. (D*o*ET, 2002, p. 21)

In moving to a values-based curriculum, a comprehensive view of numeracy as expounded by Steen (2001) and the need for numeracy to be taught across the curriculum is embodied. Numeracy entails both thinking and communicating; it plays a role in the other Essential Learning outcomes, and it is a basic concern for schools and for the community. Therefore, numeracy not only belongs in the key element Being numerate, but also must be developed in connection with all of the Essential Learning key elements as listed in Figure 2.1. In addition to being a vehicle for developing critical and reflective thinking, numeracy has the ability to enhance students' understandings of their personal futures, world futures, and social responsibilities.

There is a much consistency among the conceptions of numeracy as expressed by AAMT (1997), Steen (2001), QSCC (1999), and the Tasmanian Being numerate key element (D*o*ET, 2002). All four articulate a comprehensive multi-faceted view of numeracy and emphasise its relevance to all areas of the curriculum. These four perspectives have been used as the foundation for this thesis.

2.5 Chapter summary and research questions

This chapter has outlined the literature relevant to the aim of the study: to uncover the dynamics of numeracy positioning in the context of schools undergoing curriculum reform. The chapter has reviewed the literature in order to (i) define what is meant by the terms "curriculum" and "numeracy" in the context of this study, and (ii) examine some of the complex factors that influence numeracy learning as well as the ways that numeracy has been interpreted in practice. The research reveals not only the complex nature of numeracy and the differing emphases placed on its conceptualisation but also the interplay among numerous complex factors that contribute to students' capacity to interpret and think critically about quantitative information in a variety of contexts.

Two objectives underpin the research:

- First, a theoretical objective, to deepen understanding of the construct of numeracy in the context of current reform agendas, through the development of a conceptual framework for numeracy that aligns with and extends current research about numeracy and its capacity to equip students for their current and future lives as democratic citizens.

- Second, a practical objective, to contribute to an understanding of the complex nature of the teaching and learning of numeracy. The study is concerned with the enactment of curriculum in the classroom, in which the roles and experiences of teachers and students are equally important.

Based on the foregoing literature review of the field of curriculum, numeracy, and related areas, two (of three) research questions are posed for the study:

1. How are teachers positioning numeracy in reform-based learning environments?
2. How are students experiencing numeracy in these reform-based learning environments?

The next chapter draws on and extends the synthesised view of numeracy proposed in Section 2.4 through the development of the conceptual framework for the study. The framework presented in the next chapter results in the above research questions being refined and an additional research question being posed.

Chapter Three

Conceptual Framework

3.1 Introduction

In acknowledging the transdisciplinary curriculum context in which the study is situated, an important step is the conceptualisation of numeracy within this context. This chapter expands on the literature review by explaining the comprehensive view of numeracy adopted in this study and by describing the development of a framework for numeracy with which to examine classroom practice in depth. The framework for numeracy incorporating five dimensions of practice is presented and its development articulated with reference to appropriate literature. It is underpinned by social constructivist theory and based upon what is understood about learning, contemporary thinking about numeracy, and how numeracy is translated and described in transdisciplinary curriculum documentation. The framework for numeracy is critical to this study in that it provides a lens through which to examine the positioning of numeracy by teachers and the learning of numeracy by students within reform-based classroom learning environments.

After presenting the development of the framework, the chapter moves on to consider each of the five dimensions in depth. In doing so, a framework for numeracy is presented that can be applied to the positioning of numeracy in a reform-based curriculum environment in a meaningful way, both in terms of teacher beliefs and practices and also in terms of student learning. The framework developed in this chapter is used for analysis of all phases of the research (as detailed in Chapter 4).

3.2 Numeracy defined in this study

In considering the relationship between curriculum reform and the teaching and learning of numeracy, this thesis argues for an awareness of both the individual and the social in the construction of knowledge, for a balance between the building of abstract knowledge and knowledge connected to real-world settings, and for learning and instruction to provide students with opportunities to develop and demonstrate knowledge and cognitive processes across a range of dimensions.

The view of numeracy developed in this chapter and adopted in the study grows out of the synthesised view expounded in Section 2.4 of the Literature Review. It is underpinned by social constructivist theory, in which multiple aspects of knowledge construction are recognised and valued. As with the transdisciplinary curriculum context within which this study is situated, social constructivism expounds that mathematics and other areas of knowledge are "richly and organically connected" (Ernest, 1998, p. 263). The social constructivist perspective assigns "a prominent role to both the social and the individual in the development of meaning" (Prawat, 1996). Shepard (2001) expounds the principles of social constructivism as drawing from contemporary cognitive, constructivist, and socio-cultural theories. Although valuing the sense-making and active process of mental construction that individuals undergo to construct their own knowledge, the importance of the social and cultural interactions is not neglected. "School learning should be authentic and connected to the world outside of school not only to make learning more interesting and motivating for students but also to develop the ability to use knowledge in real-world settings" (Shepard, 2000, p. 7).

Ernest (1998) in *Social constructivism as a philosophy of mathematics* incorporates a "critical" perspective into his social constructivist philosophy. A critical approach is emphasised by researchers such as Skovsmose (2004) and Frankenstein (1998) and is outlined in Section 2.3.1 of the Literature Review. Ernest (1998, 2001) asserts that all students have the capacity to use mathematics to be "empowered" as individuals and as citizens. Numeracy, in this sense, has the potential to support a climate of critical questioning.

> Social constructivism regards mathematics as value laden and sees mathematics as embedded in society with social responsibilities, just as every other institution, human activity, or discursive practice is. ... education is a thoroughly value laden and moral activity, since it concerns the welfare and treatment of young persons. If, as in many education contexts, social justice values are adopted, then additional responsibility accrues to mathematics and its related institutions to ensure that its role in educating the young is a responsible and socially just one. (Ernest, 2001, p. 272)

As argued in Section 2.4 of the Literature Review, this thesis proposes a synthesised view of numeracy that acknowledges the different dimensions that are necessary for the development of competent and effective numeracy practice. The critical and cultural aspects are not ignored, but neither are they preferenced over the mathematical language, skills, functions, and disposition required for students to make sense of, and evaluate critically, the contexts in which the mathematics is embedded.

It is acknowledged that many areas within educational research more widely have not only influenced the theorisation of social constructivism but also informed the increasingly recognised place that numeracy has taken within mathematics education in the last ten years. Four of these areas are briefly discussed - constructivist theories of learning, conceptual understanding, metacognition, and the situated learning movement.

Constructivist theories of learning emphasize the ways in which learners construct knowledge for themselves into an integrated and holistic understanding. This ability to act upon knowledge brings together the connection between thinking and understanding. If understanding is "the ability to think and act flexibly with what one knows" (Perkins, 1998), then a performance view of understanding is advocated. Resnick and Klopfer (1989) also proposed this view within the "Thinking Curriculum" where content and skill are viewed as connected and students are encouraged to build their own connections and constructions about knowledge.

The conceptualisation of learning has shifted from a focus on remembering of facts and knowledge to a focus on seeking to understand and bring a critical awareness to learning. The relationship between content and skill is an age-old concern and many distinctions have been drawn including Anderson's (1983) declarative and procedural knowledge, Scheffler's (1965) "knowing that" and "knowing how to," and Piaget's (1978) "conceptual understanding" and "successful action." Glaser (1984) described thinking in relation to knowledge as being accessible and usable knowledge.

> The task is to produce a changed environment for learning – an environment in which there is a new relationship between students and their subject matter, in which knowledge and skill become objects of interrogation, inquiry, and extrapolation. As individuals acquire knowledge, they also should be empowered to think and reason. (Glaser, 1984, p. 103)

Within mathematics education research, a distinction is often made between conceptual knowledge, concerned with the relationships between individual facts and propositions, and procedural knowledge, the language and symbols of mathematics that go together to form algorithms or rules (Hiebert & Lefevre, 1986). Conceptual knowledge must be learned meaningfully to enable the relationships between units of knowledge to be recognised, but procedural knowledge may or may not be learned with meaning, as exemplified by rote learning. Hiebert and Lefevre proposed that "procedures that are learned with meaning are procedures that are linked to conceptual knowledge" (p. 8) and it is this connection that is significant in the building of mathematical knowledge and understanding.

Developments in fields such as meta-cognition (Metcalfe & Shimamura, 1994) have brought an emphasis to critical, reflective, and higher order thinking. This has resulted in more complex models of knowledge and cognition. Anderson and Krathwohl (2001) revised Bloom's (1956) original learning taxonomy into a two dimensional model. It incorporates both a knowledge dimension, the kind of knowledge learned – factual, conceptual, procedural, and meta-cognitive – and a cognitive process dimension, the processes used to learn – remembering, understanding, applying, analysing, evaluating, and creating.

Alongside research in the field of cognition has been a growing interest in the situated nature of learning. The situated learning movement (Lave, 1988; Lave & Wenger, 1991) has brought a needed examination of the relationship between in-school and out-of-school learning but warnings by Anderson, Reder, and Simon (1996) are important in terms of considering the complex nature of transfer. "The real goal should be to get students motivated and engaged in cognitive processes that will transfer… Often real-world problems involve a great deal of busy work and offer little opportunity to learn the target competences" (p. 9). Authentic, real-world investigations that are valued within reform environments, and specifically in the local curriculum context within which this study is situated, need to be planned thoughtfully and designed explicitly to target the key outcomes specified by the curriculum.

As summarised in Section 2.4 of the Literature Review, the work of the AAMT (1998) and Steen (2001), together with consideration of the values-based curriculum context of QSCC (1999) and the local curriculum definition of "Being numerate" (D*o*ET, 2002), inform this study as key sources. All of these four conceptions of numeracy articulate a comprehensive multi-faceted view of numeracy and emphasise its relevance to all areas of the curriculum. Essential to all these conceptions of numeracy is the view that mathematics is a vital tool in today's society, a tool that should be accessible to all members of society. They acknowledge the complexity involved in numeracy and the many aspects, beyond mathematical skill, that contribute to a high level of numerate behaviour. In this way they also align well with the social constructivist approach taken in this study.

From the comprehensive, balanced perspective of numeracy described in this thesis five dimensions of numeracy have been determined. The five dimensions extend across foundational mathematical concepts and skills, strategic thinking, disposition, recognition of context, and critical practice. These five dimensions are entitled *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity*. They are drawn from the research in the field of numeracy and underpinned by a social constructivist approach to knowledge construction.

Table 3.1 summarises the five dimensions of numeracy chosen in this study and identifies where each of the five dimensions, drawn from the research, falls within the numeracy definitions of the AAMT (1998) and Steen (2001) and where they are reflected within numeracy curriculum (QSCC, 1999; D*o*ET, 2002). Each of the five dimensions is developed further and discussed in detail in Sections 3.3 to 3.7.

Table 3.1

Five dimensions of numeracy

Dimensions of numeracy	Description	Links to numeracy definitions and curriculum documents
Mathematics	The foundational understanding and use of the language, concepts, and skills of mathematics, as they relate to number, measurement, space, data and chance, and pattern and algebra.	Underpinning mathematical concepts & skills from across the discipline (AAMT, 1998) Number sense, interpreting data, prerequisite knowledge, symbol sense (Steen, 2001) Foundational practice: uses the knowledge and skills of mathematics (QSCC, 1999) Concepts and skills of mathematics (D*o*ET, 2002)
Reasoning	The use of (i) mathematical thinking strategies to question, identify, represent, explain, and justify mathematical approaches relevant to a given context, and (ii) general thinking strategies to support the problem solving process, from lower level cognitive processes, such as recall and application, to higher level critical thinking processes involved in evaluation, judgment, decision making, and creativity.	Mathematical thinking and strategies, and general thinking skills (AAMT, 1998) Logical thinking, making decisions, interpreting data, practical skills (Steen, 2001) Linking practice: uses strategic processes and strategies (QSCC, 1999) Thinking and acting mathematically (D*o*ET, 2002)
Attitude	The confidence and disposition to choose and use mathematical understandings whenever required. Willingness to take risks and persevere in approaching new mathematics and new contexts.	Disposition to use mathematics (AAMT, 1998) Confidence with mathematics (Steen, 2001) Pragmatic practice: disposition to use mathematics, including affective components (QSCC, 1999) Disposition to think and act mathematically and the confidence and intuition to apply mathematical principles (D*o*ET, 2002)
Context	The ability to select and apply the appropriate mathematical tools for sense-making in a given context and understanding how the context impacts on the mathematics. Contexts related to school and everyday life, public and social issues, and an awareness of mathematics connected to history and culture.	Grounded appreciation of context (AAMT, 1998) Cultural appreciation, mathematics in context, practical skills (Steen, 2001) Pragmatic practice: choosing and using mathematics in context (QSCC, 1999) Selecting and using mathematics appropriately in real settings (D*o*ET, 2002)
Equity	Awareness that mathematics can be used inappropriately, can be represented to promote bias, and can therefore promote inequities in society. The ability to question assumptions and use mathematics in an analytical and critical manner to make decisions and resolve problems and investigations.	A fundamental component of learning … and critique across all areas of the curriculum (AAMT, 1998) Logical thinking, making decisions (Steen, 2001) Critical practice: knowing what mathematics is and how it can be used (QSCC, 1999) Critical and life-related aspects of being able to interpret information thoughtfully and accurately, and make informed decisions (D*o*ET, 2002)

As this study is concerned with the enactment of curriculum in the classroom, the roles and experiences of both teachers and students are equally important. The research is not relying only on judgments of what teachers say or what they do in the classroom to evaluate the students' learning of numeracy. It also considers the pedagogical decisions made and implemented by teachers and how these align with student learning: are students obtaining the outcomes that teachers plan for and implement in their classrooms? The conceptual framework for numeracy, incorporating five dimensions of practice, provides the lens through which the beliefs and practices of the teachers who participated in this study are considered. The framework also enables the research to examine closely the learning exhibited by individual student participants in the study.

For each of the five dimensions of numeracy categories are developed and described that reflect the main aspects of each dimension. In considering these key aspects and the nature of each dimension, this thesis draws upon the main ideas and frameworks described by research within each area. The categories and ideas encapsulated within four of the dimensions – *Mathematics*, *Reasoning*, *Context*, and *Equity* – are assumed to have a hierarchical nature, in the sense that they are presumed to be ordered in terms of increasing complexity. As Anderson and Krathwohl (2001) argued, in revising Bloom's (1956) original taxonomy of educational objectives, "much greater importance [is placed] on teacher usage [and in this thesis, relevance to the classroom learning environment] than on developing a strict hierarchy" (p. 309). Although the hierarchical nature of learning is a contested area (Goldin, 2008), it has been extensively used within mathematics education research and has strongly influenced development of progressions of learning within curricula, teaching, and assessment. The use of the term hierarchy in this thesis is intended to reflect a progression of ideas and concepts related to learning within each of the dimensions. The hierarchies used in this thesis are aligned with frameworks of other theorists (e.g. Anderson & Krathwohl, 2001; Clarke, Cheeseman, McDonough, & Clarke, 2003; Mooney, 2002), and these are examined in detail in Sections 3.3, 3.4, 3.6, and 3.7. Based upon research, the dimension of *Attitude* is assumed to not be hierarchical and is examined in Section 3.5.

3.3 Mathematics dimension

The learning of mathematics is acknowledged as being developmental in nature with many existing frameworks describing in detail sequences of learning for specific content areas of mathematics. The two strands of mathematics that are described in this section are graphing and data analysis, and measurement. The context of the units of work planned and implemented by the teachers in each the four case studies presented in this thesis have determined these strands. It is not the intention of the thesis to discuss all of the strands of mathematics.

3.3.1 Mathematics dimension – as it pertains to graphing and data analysis

Curriculum documentation, guiding the teaching of mathematics throughout the middle years of schooling, evidences the value placed on students developing the ability not only to collect, organise, and represent data, but also to be able to analyse and interpret data and graphs in order to answer questions, make inferences, and evaluate arguments (AEC, 1991; D*o*ET, 2002; NCTM, 1989). Van de Walle (2004), in considering the teaching of middle school mathematics, reiterates the goal of teaching in this area being for students to "learn how a graph conveys information" and in particular develop the skills of "analysis and communication" (p. 392). Research in the field of statistical literacy also emphasises the ability of students to evaluate data and graphs critically in a variety of contexts and to be able to use data and graphs to inform decision making (Gal, 2004; Watson, 2006).

Other theoretical models of statistical thinking and reasoning that have informed the categorisation of the *Mathematics* dimension as it pertains to graphing and data analysis include Friel, Curcio, and Bright (2001), Mooney (2002), Pfannkuch and Wild (2004), and Shaugnessy (2007). All of these models recognise a shift from students focusing purely on the data within a graph, whether it be single or multiple aspects of the data, toward capabilities that involve being able to consider the data as a whole and consider relationships among data, before moving to being able to evaluate and consider implications from the data based upon knowledge or experience beyond the graph itself.

The key curriculum "big ideas" of data analysis and graphing, together with theoretical models of the learning of these concepts, have been combined to inform the development of the categories of student learning. The current hierarchical models discussed are easily adapted for this study. For the purpose of evaluating student learning about graphs, both constructed and interpreted by students, this dimension is divided into four hierarchical categories: Reading and describing graphs, Making meaning from graphs, Analysing and interpreting graphs, and Evaluating and thinking beyond the graph. These categories are described in Table 3.2.

Table 3.2

Mathematics dimension – as it pertains to graphing

Category	Description
Reading and describing graphs	Uses language of graphs Identifies components of graphs Is aware of some different types of graphs Describes individual aspects of data
Making meaning from graphs	Makes comparisons between two data points Understands purpose of graphs Is aware of the importance of accuracy of data Chooses type of graph appropriate to purpose
Analysing and interpreting graphs	Compares multiple aspects of the data in the graph Identifies trends and integrates ideas Considers data as a whole Identifies inconsistencies Considers relationships among variables (observed) Considers variation
Evaluating and thinking beyond the graph	Makes inferences or predictions (explains/speculates) Considers alternatives Evaluates data based upon other knowledge/experience

3.3.2 Mathematics dimension – as it pertains to measurement

Measurement is one of the major strands of mathematics and is a central component of the primary and secondary school curriculum both in Australia and around the world (AEC, 1991; Department for Education and Employment [DfEE], 1999; D*o*ET, 2002; NCTM, 2000). "The fundamental idea which underlies measurement is the comparison of one thing with another according to some specified feature" (AEC, 1991, p. 136). Measurement is, however, more complex than is often assumed and has connections to many other mathematical topics, including number, place value, proportional reasoning, fractions, geometry, algebra, and data (Van de Walle, 2004).

The "big ideas" of measurement, as informed by *Mathematics – A curriculum profile for Australian schools,* are choosing units, measuring, estimating, time, and using relationships (AEC, 1994). In addition, research, particularly in the domains of length, area, and volume, highlights important concepts and skills that underpin a conceptual understanding of measurement. These include conservation, attribute identification, the use of formal and informal units, comparison of measures, choosing appropriate measuring tools (Clarke et al., 2003; Lehrer, 2003; Outhred, Mitchelmore, McPhail, & Gould, 2003), and also measuring from a fixed point and unit reiteration (Lehrer, 2003).

These concepts form the basis of a number of theoretical frameworks related to the learning of measurement, for example, Clarke et al. (2003), Outhred et al. (2003), and van den Heuvel-Panhuizen and Buys (2005). The frameworks articulate developmental levels or "growth points" that students go through as they gain deeper understanding of the concepts and processes involved in measurement. The core of the *Count Me Into Measurement* program (Outhred et al., 2003) for example, describes three key stages students must progress through:

- identification of the attribute (direct/comparison/partitioning/ conservation);
- informal measurement (counting units/relating number of units to quantity, comparison of measurements); and
- unit structure (replicating a single unit/relating size of units to number required) (p. 85)

The *Early Numeracy Research Project* (ENRP) (Clarke et al., 2002) identifies five generic growth point descriptors for the learning of measurement.

1. The child shows awareness of the attribute and its descriptive language.
2. The child compares, orders, and matches objects by the attribute.
3. The child uses uniform units appropriately, assigning number and unit to the measure.

4. The child chooses and uses formal units for estimating and measuring, with accuracy.
5. The child can solve a range of problems involving key concepts and skills.

Although these frameworks are situated within the early years of schooling, it is recognised that students are perceived to pass through these stages for each of the attributes of measurement, not simultaneously, but rather recognising the increasing complexity of concepts with the increasing number of dimensions involved (Van de Walle, 2004). These stages of understanding would therefore be of equal relevance to middle school students and their developing understanding of measurement concepts, in particular as they move to considering area and volume.

The key curriculum "big ideas" of measurement, together with theoretical models of the learning of measurement, have been combined to inform the development of the categories of student learning for this thesis. The hierarchy of development in the area of measurement as described by Clarke et al. (2002) and Outhred et al. (2003), as well as the ideas of Lehrer (2003), have been translated into the same underpinning format as used in Table 3.2 in order to ensure consistency within the framework developed for this study. For the purpose of evaluating student learning in relation to the area of measurement, this dimension is divided into four hierarchical categories: Reading and describing measurement, Making meaning from measurement, Analysing and interpreting measurement, and Evaluating and thinking beyond measurement. These categories are described in Table 3.3.

Table 3.3

Mathematics dimension – as it pertains to measurement

Category	Description
Reading/describing measurements	Uses language of measurements Identifies measurement attributes Is aware of some different types of measurement Describes individual aspects of measurement
Making meaning from measurement	Makes comparisons between two aspects Understands purpose of measurement Is aware of the importance of accuracy of measurement Chooses measurement tool/unit appropriate to purpose
Analysing and interpreting measurements	Compares multiple aspects of measurements in task Integrates ideas Considers measurement data as a whole Identifies inconsistencies Considers relationships among attributes
Thinking beyond the measurement	Makes inferences or predictions Transfers knowledge to new contexts Considers alternative ways of measuring Evaluates measurement data based upon other knowledge/experience

3.4 Reasoning dimension

The role that thinking plays in the development of numeracy is well recognised both from the perspective of reforms in mathematics education (NCTM, 1989, 2000; Van de Walle, 2004) and from the perspective of wider curriculum reforms (Blythe, 1998; Wiggins & McTighe, 1999). The Tasmanian curriculum context (D*o*ET, 2002) has perhaps as its most recognised feature, the situating of Thinking at the centre of the curriculum, acknowledging it as "a prerequisite to fulfilling a role as an active and concerned citizen and to following personal pursuits successfully" (p. 13). Ritchhart (2002) challenges all teachers to develop classrooms as cultures of thinking, arguing that reasoning is critical in enabling students to build the intellectual capacities required of them in the twenty-first century.

For the purpose of this thesis, the term "reasoning" encompasses all the terms that refer to the reasoning skills students bring to their learning in order to develop conceptual understanding (Wiggins & McTighe, 1998). Common terms used include "thinking mathematically" and developing the "strategic processes and skills" needed to choose and use mathematics in a variety of contexts (Hogan, 2000; QSCC, 1999). Van de Walle (2004) describes a "problem solving" approach that similarly focuses on the value of supporting students to develop the strategies and processes of questioning, identifying, representing, explaining, justifying, and generalising for approaching mathematical tasks and solving problems.

The categories used to analyse and describe student learning in this dimension, based upon interview data, were informed by the work of Anderson and Krathwohl (2001), who described processes of thinking from lower level cognitive processes, such as recall and application, to higher level critical thinking processes involved in evaluation, judgment, decision making, and creativity. Anderson and Krathwohl's taxonomy was a revision of Bloom's original taxonomy of educational objectives (Bloom, 1956). Their motivation in doing this included a desire to broaden its application to all teachers involved in the planning, teaching, and assessment of curriculum, and to make a purposeful shift in distinguishing the dimension of knowledge from the cognitive processes that support learning, and to more clearly define the place of content in the learning process.

Although Anderson and Krathwohl (2001) use the term "Understand" for the second cognitive process category, this thesis retains Bloom's original term "Comprehend" for this category, as the term "understand" in this thesis refers more broadly to the impact that all the levels of reasoning may have on developing student understanding (Wiggins & McTighe, 1998). For the purpose of considering students' learning, the *Reasoning* dimension is divided into six categories: Remember, Comprehend, Apply, Analyse, Evaluate, and Create. These categories are described in Table 3.4.

Table 3.4

Reasoning dimension (Anderson & Krathwohl, 2001)

Category	Description
Remember	Observation and recall of information, knowledge of major ideas For example: identify, recognise, recall
Comprehend	Constructs meaning from oral, written, and graphic information For example: exemplify, classify, summarise, compare, explain
Apply	Carries out or uses a skill, concept, or procedure in a given situation For example: executive, implement
Analyse	Understands overall structure and how components relate to one another For example: differentiate, organise, attribute, distinguish
Evaluate	Compares and discriminates between ideas, makes judgements based on reasoned argument For example: critique, make decisions, judgements, justify
Create	Puts elements together to form a coherent whole or to create new ideas For example: hypothesise, generate, produce, generalise

3.5 Attitude dimension

Wilkins (2000) identified that positive attitudes toward mathematics were vital to numeracy. Although definitions of numeracy do not necessarily emphasise the role of affect in the development of numeracy, they do acknowledge its place in noting the importance of students developing a "disposition" towards (AAMT, 1998; D*o*ET, 2002), a "capacity … to engage" (OECD, 2006), or "confidence" (Steen, 2001) in their interaction with mathematics and its application. Curriculum developers also recognise that "an important aim of mathematics education is to develop in students positive attitudes towards mathematics and their own involvement in it, and an appreciation of the nature of mathematical activity" (AEC, 1991, p. 31).

The term attitude is used to describe an evaluative response to a psychological object (Ajzen & Fishbein, 1980) and therefore individuals' attitudes to mathematics refers to their evaluation of mathematics. Attitude to mathematics has been identified as a multi-dimensional construct (Ma & Kishor, 1997) and there are a number of studies, predominantly based upon self-report scales, that have considered a variety of affective factors in the learning of mathematics, (Beswick, Watson, & Brown, 2006; Galbraith & Haines, 2000; Tapia & Marsh, 2004).

Beswick et al. (2006) considered eight aspects that contributed to a positive attitude toward the learning of mathematics from a synthesis of the literature and in the context of a middle school mathematics classroom. These include: confidence or anxiety (Ernest, 1988); like or dislike; engagement or avoidance; high or low self efficacy; and beliefs that mathematics is important or not important, useful or useless, easy or difficult (Ma & Kishor, 1997), and interesting or not interesting (McLeod, 1992). Cretchley (2008) evaluated four research studies on affective factors in mathematics learning and summarised the key areas of interest as being:

- Self-concept factors: mathematics talent, confidence, self-efficacy, anxiety
- Other motivational factors: interest, enjoyment, intellectual stimulation, reward for effort, valuing mathematics, diligence.

(p. 152)

For the purpose of this study, the work of Beswick et al. (2006) and Cretchley (2008) informed the use of six categories for the *Attitude* dimension: Confidence, Interest, Enjoyment, Intellectual stimulation, Diligence, and the Valuing of mathematics. The emphasis is on those categories that could possibly be observed in discussions with students about their learning. The categories are not hierarchical, but it is assumed that the combining of these aspects of attitude contributes to greater potential success in the classroom. The categories are described in Table 3.5.

Table 3.5

Attitude dimension

Category	Description
Confidence/self-efficacy	Is confident in own ability to do and learn mathematics.
Interest	Is disposed towards engagement in mathematics due to inherent nature of the task, often accompanied by positive emotions.
Enjoyment	Has an expressed liking for mathematics, accompanied by feelings of excitement and/or sense of fun from doing mathematics.
Intellectual stimulation	Gains a sense of satisfaction when doing mathematics, enjoys the challenge of doing and thinking through problems.
Diligence	Perseveres and re-works problems, attempts tasks, checks work, planning.
Valuing of mathematics	Appreciates the importance and relevance of mathematics to school and to life.

3.6 Context dimension

Researchers have long recognised that the context of mathematics use plays a part in determining student success or otherwise in solving mathematical problems (Carraher, Carraher, & Schliemann, 1985; Lave, 1988; Willis, 1990). The perception that numeracy involves "using some mathematics to achieve some purpose in a particular context" (AAMT, 1997, p. 13) is representative of the intent of most attempts to describe numeracy during the 1980s and 1990s and recognition of the importance of context has been a feature of most definitions of numeracy since that time. From the perspective of such a situated view, numeracy is regarded as having the capacity to "bridge the gap" between mathematics and the real world through the use of context (Johnstone, 1994; Willis, 1998).

Fosnot and Dolk (2001) advocate the use of "context" problems in the mathematics classroom rather than the more traditional and contrived story problems, with context problems being more closely related to students' lives and thereby "designed to anticipate and to develop children's mathematical modelling of the real world" (p. 24). The importance of context has also been emphasised by Kemp and Hogan (2000) who argue that students need opportunities to use mathematics regularly to solve problems in a variety of real-life situations, in particular, contexts that are relevant to them so that they can see the purpose and usefulness of the mathematics.

Griffin (1995), in considering occurrences of mismatch between mathematical skill and numerate behaviour, suggested that a lack of understanding of context or a failure to transfer existing mathematical understanding between contexts was a factor. This difficulty in achieving transfer of mathematical knowledge across context is well recognised (Anderson, Reder, & Simon, 1996; Hughes-Hallett, 2001; Kemp & Hogan, 2000). With respect to student learning it implies that it is not the sole responsibility of mathematics teachers to develop students' numeracy, but rather *all* teachers contribute to their students' developing numeracy (DEAT, 1995; Morony & Brinkworth, 2003). Numeracy, like literacy, is increasingly being regarded as a cross-curricular construct. In the Tasmanian curriculum context, the key element "Being numerate" was situated with the Communicating Essential Learning informing learning across the curriculum (D*o*ET, 2003).

In the middle school classroom, context may take on an additional meaning in that hands-on activities are often required in order to support student learning of mathematics. In this study, therefore, context includes both real-life application of mathematics and experiences that have a practical nature related to a meaningful context.

In recognising the fundamental importance of context in developing numeracy, juxtaposed with the challenges in providing meaningful contexts that not only engage and provide relevance for students, but also support the learning of important mathematical concepts and the capacity of students to transfer their learning to new contexts, four categories of student learning are developed and described. These categories are presented in Table 3.6. The *Context* dimension is somewhat different from the other dimensions because students do not have to demonstrate the first category, Personal experience of context, before the second category – they can move past it in their sophistication. Also personal experience might be unreasonable or quite reasonable in a particular context. A decision was made therefore that having personal experience of a context was not a prerequisite for demonstrating learning in this dimension. In constructing the dimension, however, it was determined that demonstrating an understanding of context and its relationship to mathematics then becomes more sophisticated in nature and this is reflected in the last three categories.

Table 3.6

Context dimension

Category	Description
Personal experience of context	Focuses on personal experience or personal opinion to make sense of graphing/measurement and to discuss context Makes some connections with that which the graph/measurement might represent but not informed by the mathematics together with the context
Context integrated with mathematics as presented	Integrates context, as presented, with the graph/measurement to make sense of the mathematics Relies wholly on context as presented, and integrates this context with the mathematics Selects appropriate mathematics for context
Context integrated with mathematics, from both prior knowledge and as presented	Integrates both prior knowledge of context, and context as presented, with the mathematics to inform sense-making Looks at graph/measurement and understands what it represents in the context, from informed knowledge of context
Relational understanding of the mathematics and the context and can transfer to new contexts	Demonstrates an understanding of both the distinguishing and relational features of the mathematics and the context in the specific task, and can transfer that knowledge to new contexts Makes decisions and draws inferences based on the context and the mathematics

3.7 Equity dimension

In situating the *Equity* dimension alongside the other four dimensions, this thesis argues that in a twenty-first century globalised world the boundaries of numeracy should be pushed so that all students can not only think mathematically and use mathematics in their lives, but also are "empowered" as individuals and as citizens (Ernest, 2001; Skovsmose, 2004).

In mathematics education, the area of equity is a very broad research field and much has been written about student groups who experience disadvantage in their access to equality of mathematics teaching and learning, or who evidence lower mathematical learning outcomes than other groups, based upon characteristics such as gender, ethnicity, social class, and culture (e.g, Bishop, 1988; Leder, 1992; Secada, 1992; Zevenbergen, 2001). The use of the term "equity" in this thesis refers to that aspect of numeracy that acknowledges that mathematics should be accessible to all members of society. All students must be supported in the learning environment to develop the knowledge and competencies that will enable them to become competent and critical citizens, able to question assumptions and to use mathematics in an analytical and critical manner to make decisions, resolve problems, and challenge inequities in society.

This dimension of numeracy is strongly supported by the research of Skovsmose (2004), Frankenstein (1998), and Gutstein (2003) in the field of critical mathematical literacy. They all emphasise the capacity for mathematics to equip students with the capabilities and tools to interpret and challenge inequities in society. Jablonka (2003) indicated that any conception of mathematical literacy inherently promotes a particular social practice, whether or not it is made explicit. This was exemplified through discussion of the potential for mathematics to be used to develop human capital and cultural identity, to promote social change and environmental awareness, and to evaluate mathematics itself. Ernest (2001) contends that students' mathematics education should encourage

> critically understanding the uses of mathematics in society: to identify, interpret, evaluate and critique the mathematics embedded in social, commercial and political systems and claims, from advertisements to governments and interest-group pronouncements; ... (p. 285)

Furthermore, Ernest (2001) notes that building this awareness is done through connecting with students' own interests and experiences, and through developing a culture of questioning, discussion, and decision-making in the classroom. This thesis argues that the dimension of numeracy entitled *Equity* begins in the classroom with the manner in which students are encouraged to work together, question, take risks in their approaches to tasks, consider alternative strategies, and make and share solutions and decisions. Using mathematics in a critical manner to make decisions, resolve problems, and undertake complex investigations engenders the capacities that can then be applied in social, cultural, and political contexts within and beyond the classroom environment. As researchers have only recently been discussing equity in this way, an early attempt at bringing together the fundamental ideas of equity within a hierarchical model is deemed important.

In acknowledging the importance of developing student learning in relation to the *Equity* dimension, both within the classroom learning environment and also as part of each student's engagement with the curriculum, five categories of student learning are developed and described. These are presented in Table 3.7. The first category describes the students' personal social engagement with their own work and with their peers within the classroom learning context. This category is supported by the work of Boaler (2008) who coined the term "relational equity" to emphasise the ways students are able to be supported in classrooms to act equitably in the way they treat and respect each other. The remaining four categories – informed by the research of Ernest (2001), Frankenstein (1998), and Skovsmose (2004) – are hierarchical in nature and describe the development of students' capacities to engage with and use mathematics to be critical about social, political, economic, and cultural contexts within which learning is embedded.

Table 3.7

Equity dimension

Category	Description
Personal social engagement	Questions and makes decisions based on own and others' mathematical strategies in the social context of the learning environment Awareness of the ideas of others Appreciates that mathematics can be useful Appreciates that other people's views and strategies might be useful Collaborates, listens and shares ideas with others *Pre-requisite disposition for the development of equity*
Awareness of issues	Awareness and questioning of issues as a result of exploring the mathematics in social, cultural, economic, or political contexts
Considering viewpoints	Considers a variety of perspectives Expresses opinions about own and others' viewpoints or circumstances Communicates viewpoints in written, oral, and visual forms and listens and engages in discussion with peers
Relating mathematical information to social and political consequences	Makes critical interpretation of mathematical information, by considering social, ethical, and/or political consequences Uses mathematics effectively to support viewpoints and arguments
Challenging inequity	Uses mathematics to operate powerfully, by challenging inequities in society, and by considering or taking action

3.8 Chapter summary

This chapter has presented a conceptual framework for numeracy, within a transdisciplinary curriculum, based upon five dimensions of practice. The five dimensions recognise the complex nature of numeracy and align with the underpinning ideas of a social constructivist view of knowledge construction. Descriptions of each dimension are drawn from the literature relevant to each area. In the case of the *Mathematics* dimension, only the mathematical areas of measurement and data and graphing are described. It is the context of the units of work planned and implemented by the teachers in this study that determined these strands. It is acknowledged that within numeracy education research, the dimensions of *Mathematics, Reasoning*, and *Context* are well considered by others (e.g. AAMT, 1997; Hogan, 2000; NCTM, 2000; Van de Walle, 2004; Willis, 1998). The dimension of *Attitude*, although widely acknowledged as contributing to effective numeracy practice, does not form a strong part of previous research studies in this area (e.g. Kemp & Hogan, 2000). The fifth dimension, *Equity*, is less well considered outside of the work of critical mathematical theorists and researchers interested in this area and who tend to have it as their primary focus (e.g., Frankenstein, 1998; Gutstein, 2003; Skovsmose, 2004).

The framework developed and presented in this chapter is informed by a social constructivist view of learning, contemporary thinking about numeracy, and how numeracy is translated and described in curriculum documentation. It is critical to this study in that it provides a lens through which to examine the positioning of numeracy by teachers and the learning experiences of students within reform-based classroom learning environments. Four of the five dimensions are being interpreted as hierarchical in nature, whereas the dimension of *Attitude* is made up of six categories that together are deemed to contribute to greater potential success in the classroom. This framework for numeracy, incorporating five dimensions of practice, underpins all aspects of the analysis of the research.

Based on the conceptual framework developed in this chapter, the research questions for the study are refined as:

1. How are teachers positioning numeracy in reform-based learning environments according to five dimensions of practice?
2. How are students experiencing numeracy in reform-based learning environments according to five dimensions of practice?
3. How does a five-dimensional framework for numeracy, developed to align with a transdisciplinary curriculum context, contribute to an understanding of numeracy teaching and learning?

The research perspective taken, the methodology employed, and the practicalities of exploring the three research questions posed for the study through the methodology form the focus of Chapter 4. As a qualitative naturalistic inquiry, the conceptual framework for numeracy developed in this chapter was not shared with the teachers or the students who participated in the study, but was used at the point of data analysis when the researcher exited the field.

Chapter Four

Methodology

4.1 Introduction

The aim of the thesis is to explore the positioning of numeracy in a reform-based curriculum. The researcher worked alongside five middle years' classroom teachers who were implementing units of work within the local reform environment, the Tasmanian Essential Learnings framework. In this chapter the research perspective and methodology are examined with specific discussion as to why these theoretical choices suit the objectives of the study. The design of the research, the methods of inquiry, and the procedures employed are detailed and address the role of the researcher and the participants, as well as ethical considerations. Finally, the processes for analysing the data are presented together with an evaluation of the study design and its trustworthiness.

4.2 Research perspective

The thesis is founded upon philosophical and theoretical principles that were developed to address the purpose of the research. These principles are discussed in this section and it is these principles that inform the research framework and design of the study, and drive the research methodologies described later in this chapter.

4.2.1 Qualitative research

Qualitative research is conducted in natural settings, building a complex holistic picture of phenomena with multiple dimensions (Creswell, 1998). It enhances understanding of these phenomena with "as little disruption to the natural setting as possible" (Merriam, 1998, p. 5). The aim of the study influenced the decision to employ a qualitative approach to the research, the thesis recognising that social phenomena are inevitably complex, and it is this complexity that yields richness in the data.

The predominant philosophical assumption of qualitative research is "that reality is constructed by individuals interacting with their social worlds" (Merriam, 1998, p. 6). Eisner (1991) asserts "if qualitative inquiry in education is about anything, it is about trying to understand what teachers and children do in the settings in which they work" (p. 11). In this sense, the research sought to increase understanding of how teachers are positioning numeracy in a values-focused curriculum environment and how this positioning impacts on the learning experience of students. "The goal of research then is to rely as much as possible on the participants' views of the situation being studied" (Creswell, 2003, p. 8)

Merriam (1998) discusses five main characteristics of qualitative research:

- It is concerned with understanding the phenomenon of interest from the participants' perspectives.
- The researcher is the primary instrument for data collection and analysis.
- It usually involves fieldwork, where activity is observed in its natural setting.
- It employs an "inductive research strategy" (p. 7), the research driven by a purpose of building theory to explain the phenomenon.
- It focuses on "process, meaning, and understanding" and therefore results in a "richly descriptive" product. (p. 8)

The methodological objective of the thesis was to ensure that these characteristics were consistently applied throughout the study.

Mathematics education researchers also consider as a primary goal that of understanding what is studied with well-framed studies being able to satisfy "both fundamental or theoretic aims and practical aims simultaneously" (Hiebert, 1998, p. 141). Systemic research, as opposed to experimental, investigates phenomena in their natural settings and this style of research is predominant in mathematics education because it "prioritizes authenticity" (Wiliam, 1998, p. 7). Furthermore, it ensures that a variety of data can be collected to provide a range of perspectives through which the phenomenon can be interrogated: "no one kind of data will provide complete solutions to the complex problems in mathematics education" (Hiebert, 1998, p. 151). Denzin and Lincoln (1998) highlight this characteristic suggesting that "qualitative research is multi-method in focus, involving an interpretive, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret phenomena in terms of, the meanings people bring to them" (p. 3).

4.2.2 Social constructivism

The embedding of a social constructivist approach to the teaching and learning of numeracy is expounded in Chapter 3, particularly as it informed the development of a broad and synthesised conceptualisation of numeracy across five dimensions of practice: *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity*. In ensuring consistency in applying the same theoretical principles throughout the conceptualisation, design, and implementation of the study, a social constructivist approach has also informed the methodological principles underpinning the research.

The adoption of a qualitative approach to research provides an opportunity to "get closer to the actor's perspective through detailed interviewing and observation" (Denzin & Lincoln, 1998, p. 10). Qualitative researchers acknowledge the "socially constructed nature of reality, the intimate relationship between the researcher and what is studied, and the situational constraints that shape inquiry" (Denzin & Lincoln, p. 8). The adoption of a constructivist perspective in this study recognises the establishment of a respectful and interactive researcher-respondent relationship (Guba & Lincoln, 1989; Manning, 1997).

Merriam and Caffarella (1999) suggest that a constructivist stance "maintains that learning is a process of constructing meaning; it is how people make sense of their experience" (p. 261). Beyond this assumption, however, constructivists hold varying beliefs in regard to the nature of reality, the role of experience, what knowledge is of interest, and whether the process of meaning making is an individual or social process (Merriam & Caffarella). A social constructivist approach "takes the primary reality to be persons in conversation" (Ernest, 2001, 275), and this informs the current study design with key data collection sources being interview and classroom observation.

4.2.3 Research-Practice link

A natural assumption of the research, leading from the qualitative, social constructivist perspective taken in the thesis, is a valuing of the link between research and practice. In shaping the thesis, the researcher wanted to undertake research that was of relevance to teachers, educators, and students. The implementation of values-based curricula in Australian schools creates a plethora of issues for those involved. The impact of such curricula on the numeracy pedagogies of teachers and the numeracy learning of students is of interest. It is important, therefore, that the research is designed to link research and practice. Burkhardt and Schoenfeld (2003) argue for a strong link between "research-based insights and improved practice" (p. 3) in education research and state that such work attends to both theory and fundamental problems of practice.

Schoenfeld (2000) points out that research in mathematics education is very different from research in mathematics itself. He contends that what counts is not the "trappings of science, such as the 'experimental method,' but the use of careful reasoning and standards of evidence, employing a wide variety of methods appropriate for the tasks at hand" (p. 18). In developing the research framework and in designing the research process, the ideals proposed by Schoenfeld are considered.

Although the research remains primarily about understanding the central phenomenon of numeracy positioning, it is important to ensure that such understandings are shared with those who have vested interests in the phenomenon. The thesis is also a celebration of teachers' work, acknowledging their central role in the change process of education reform. Ensuring that the research is not only disciplined and rigorous but also accessible to teachers is valued within the mathematics education community (Sowder, 2002). The findings of the research, therefore, have begun to be shared beyond the thesis by the presentation of elements of the research in journals and at conferences, both academic and professional.

In considering the perspective taken here, evaluating the adequacy of the research in terms of its *relevance* and *communicability* is deemed important. These two criteria are included in Section 4.11 of this chapter, where the overall trustworthiness of the research is considered in terms of Guba and Lincoln's criteria of *credibility*, *dependability*, and *transferability* (1989).

4.3 Research framework

The research framework of the thesis is informed by the two objectives that underpin the research (Section 2.5). The framework and the assumptions on which it is based are outlined in this section.

4.3.1 Theory building

Theory refers to going beyond the descriptive, the "what" and the "how" of the research, to the "why," with theory enabling explanation for the characteristics of constructs being explored (Miles & Huberman, 1994; Whetten, 1989). Therefore, both description and explanation are important in the thesis. The phenomenon of numeracy positioning is considered in terms of:

1) What factors or constructs contribute to our understanding of the phenomenon?
2) How are these factors related?
3) Why do these relationships exist? and
4) In what context do these relationships exist? What are the limitations?

(Whetten, 1989)

"Theory construction in social research is always undertaken against a background of more general underlying assumptions" (Layder, 1993, p. 15). The underlying assumptions of the thesis are described in the research perspective (Section 4.2) and conceptualisation of the research framework is further informed by acknowledging the context within which the research is conducted. The phenomenon of numeracy positioning cannot be considered outside the broader context of the curriculum. To assist in recognising context, Layder's research map (1993) is used as a means of portraying the integration of layers of experience over time.

4.3.2 Layder's conceptual map

Layder (1993) developed a conceptual map "to help in the planning and ongoing formulation of field research which has theory generation as a primary aim" (p. 73). It reflects four different dimensions, or layers, of social reality, with a fifth dimension, history, applying at each level (see Figure 4.1). Although, in reality, the layers represented in the map have no clear boundaries, by representing the distinctive natures of the characteristics of each layer, the framework assists in examining how these layers combine to influence social activity.

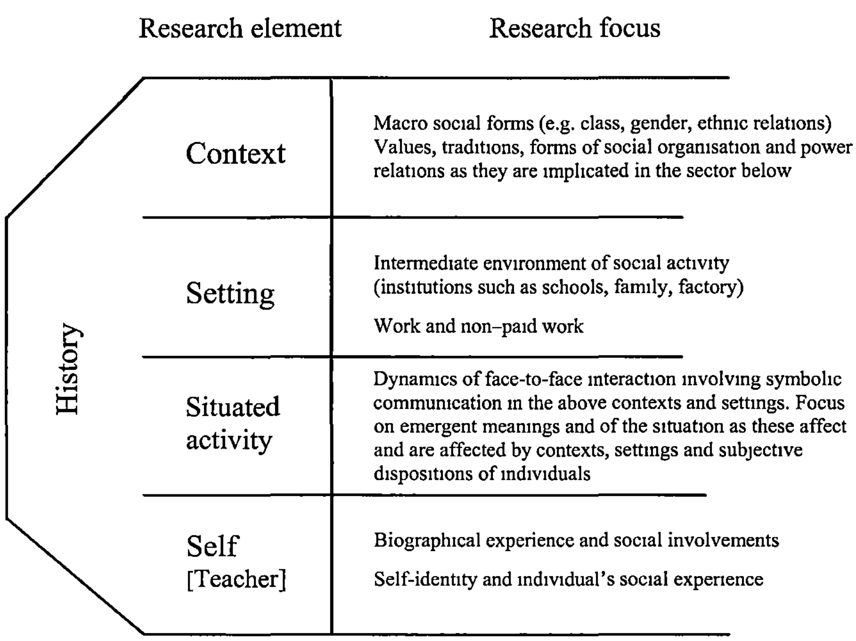


Figure 4.1. Layder's Research Map. (Layder, 1993, p.8)

In this study, the focus of investigation of the central phenomenon of numeracy positioning is on the teacher/s and the situated activity of the implementation of the unit of work in the classroom and how this affects the experiences and outcomes of the students. Awareness that this activity does not occur in isolation from the school, the D*o*ET and the Essential Learnings curriculum context, nor from the wider context of education in Australia is also important. Teachers themselves are juggling their obligations not only to their students as learners, but also to their immediate school employer, and the wider educational institution to which they are contracted (Newman, 1995). In this thesis there is a need to understand activity in the context of mandated curriculum reform. Macro-processes, although occurring outside the classroom environment, feed into the activity occurring within the classroom. The implementation of a reform-based curriculum tends to emphasise these outside influences. For example, in this study teachers were assessing student learning against a backdrop of conflicting state and federal requirements with respect to reporting: "micro-phenomena have to be understood in relation to the influence of the institutions that provide their wider social context" (Layder, 1993, p. 102). Layder's map caters for these issues as it incorporates history and power into all elements of the map.

Layder's map serves to focus the research in a way that reflects the philosophical and theoretical stance taken. It informs the construction of the research framework (Table 4.1) that places boundaries on the study investigating how classroom teachers are positioning numeracy in an emerging values-based curriculum setting and how their numeracy pedagogies affect students' experiences of numeracy in the classroom. The framework informs the study design and methodology and how they may best answer the research questions in a qualitative interpretive study.

Table 4.1

Research framework

Conceptual: level of positioning	Research Focus in this thesis	Questions	Design Element of the Study	Resulting instruments for data collection
Context	State Education Department	How has the state positioned numeracy in the Essential Learnings curriculum?	Background Chapter. DoET context	Essential Learnings (ELs) curriculum documents Establishing relationship with DoET numeracy coordinators Attendance at state professional learning days in relation to the ELs
Setting	School	How are schools positioning numeracy as they design, implement, and report on the Essential Learnings?	Background Chapter: School context Introduction to case studies.	School documents Informal interview with principals, recorded and transcribed
Situated activity	Teachers	How are teachers positioning numeracy in their classrooms as they plan, teach, and assess within the Essential Learnings curriculum?	Initial teacher interview Classroom observation of unit of work Teacher planning and assessment records Final teacher interview Classroom observation of unit of work	Initial teacher interview transcript Field notes Photographic records Researcher reflective transcripts Teacher planning and assessment documents Final teacher interview transcript Field notes Photographic records Research reflective transcripts
	Students	How do students experience numeracy in these classrooms?	Classroom observation of unit of work Student outcomes Student interviews: beliefs and understandings	Field notes Photographic records Research reflective transcripts Student work samples Student interview transcripts
Self	Teachers	What are the histories and perceptions of teachers as they position numeracy in the Essential Learnings?	Teacher background Initial teacher interview Final teacher interview	Teacher background Initial teacher interview transcript Research reflective transcripts Final teacher interview transcript

4.4 Case study methodology

As Yin (2003) observes, case study is suited to research where phenomenon are inseparable from their context. Yin talks about case study as a "comprehensive research strategy" (p. 14) encompassing design, data collection techniques, and approaches to data analysis. "A case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2003, p. 13). Case study can therefore be differentiated from other research designs by what Cronbach (1975) calls "interpretation in context" (p. 123).

The central phenomenon of numeracy positioning is explored through the activities and experiences of middle years' teachers and their students. An examination of the complexities of the phenomenon necessitated that the researcher immerse herself in that aspect of the lives of teachers where they were enacting the phenomenon in the reform environment: their classrooms. By focusing on a single phenomenon, the researcher's aim was to "uncover the interaction of significant factors characteristic of the phenomenon" (Merriam, 1998, p. 29).

In this study, perhaps more relevant than Yin's emphasis on case study as a process, is the description of case study as identifying boundaries to that which is being studied (Creswell, 2003; Miles & Huberman, 1994; Stake, 1995). Whether emphasising the study as a system, or a phenomenon studied in a bounded context, writers agree that case study research aims to capture the complexity and the uniqueness of that "bounded system" to study the case holistically and naturalistically, to uncover the meanings that participants give to their actions, and to "… look for the detail of interaction within its contexts" (Stake, 1995, p. xi). Merriam (1998) also concludes that the boundaries and limits of the "object of study" are its defining characteristic. These boundaries and limits are identified and described fully in Section 4.5, The Study Design, and enable the defining of the unit of analysis. They also guide the data collection and data analysis process.

It has also been suggested that case study research could make a significant contribution to educational practice by developing and testing theory as opposed to simply describing reality (Eisenhardt, 2002; Hammersley, 1995; Stenhouse, 1985). Stenhouse makes the additional point that "case study research should be of benefit and interest to those people who are studied... [and]... should be directed towards improving the capacity of those studied to do their job..." (Stenhouse, 1985, p. 269). In undertaking this study, the thesis not only contributes to education research literature but also informs the education community and teaching profession in a way that it is hoped will assist in their practice.

A case study approach was taken here for two reasons. First, it provided a means of examining the relationship between numeracy and the Essential Learnings in living schools and classrooms and with those making the learning and teaching decisions that affect the numeracy experiences and outcomes of students: teachers. A key question is 'How are teachers positioning numeracy in reform-based learning environments?' The focus of investigation of the central phenomenon was on the teacher/s and the situated activity of the implementation of the unit of work in the classroom and how this affected the experiences and outcomes of the students.

Second, it provides an holistic, in depth method for investigation that draws on multiple perspectives and sources of information. "The greatest advantage of a case study is that it permits a researcher to reveal the way a multiplicity of factors have interacted to produce the unique character of the entity that is the subject of the research" (Thomas, 2003, p. 35).

Gillham (2000) suggests that a case can be "an individual; it can be a group – such as a family or a class ... it can be an institution – such as a school ... it can be a large scale community – a town, an industry" (p. 1). In this study the phenomenon of numeracy positioning is explored through the examination of four cases, in which each teacher, or pair of teachers, was undertaking a unit of work within the reform environment. The form of case study is in this sense collective with each case study instrumental in learning about the central phenomenon

(Stake, 1995). In adopting the case study approach, the study seeks to arrive at an understanding of each individual case. The research is intrinsically interested in the beliefs and teaching practices of each of the case study teachers. Its primary purpose, however, remains the gaining of a deeper understanding of the central phenomenon.

> The risk of error in assuming that the results in other cases will be identical to the results in the present case can be reduced if the investigator studies more than one entity in order to identify likenesses and differences between entities and thereby recognize how much confidence can be placed in conclusions drawn from the first case studied. (Thomas, 2003, p. 35)

As the overall intent of the research is to go beyond the descriptive to the interpretive (Merriam, 1998), the collective case study design described in the next section is seen as enabling added persuasion to the interpretations made.

4.5 The study design

The research design for this thesis was informed by both the research framework as described in Section 4.3, together with the case study methodology as discussed in Section 4.4. It is that part of the research where the researcher defines the boundaries of the case and establishes the unit of analysis to be studied. In this thesis, the unit of analysis incorporates four phases of inquiry within a background of school setting and educational system context. The study design is represented in Figure 4.2.

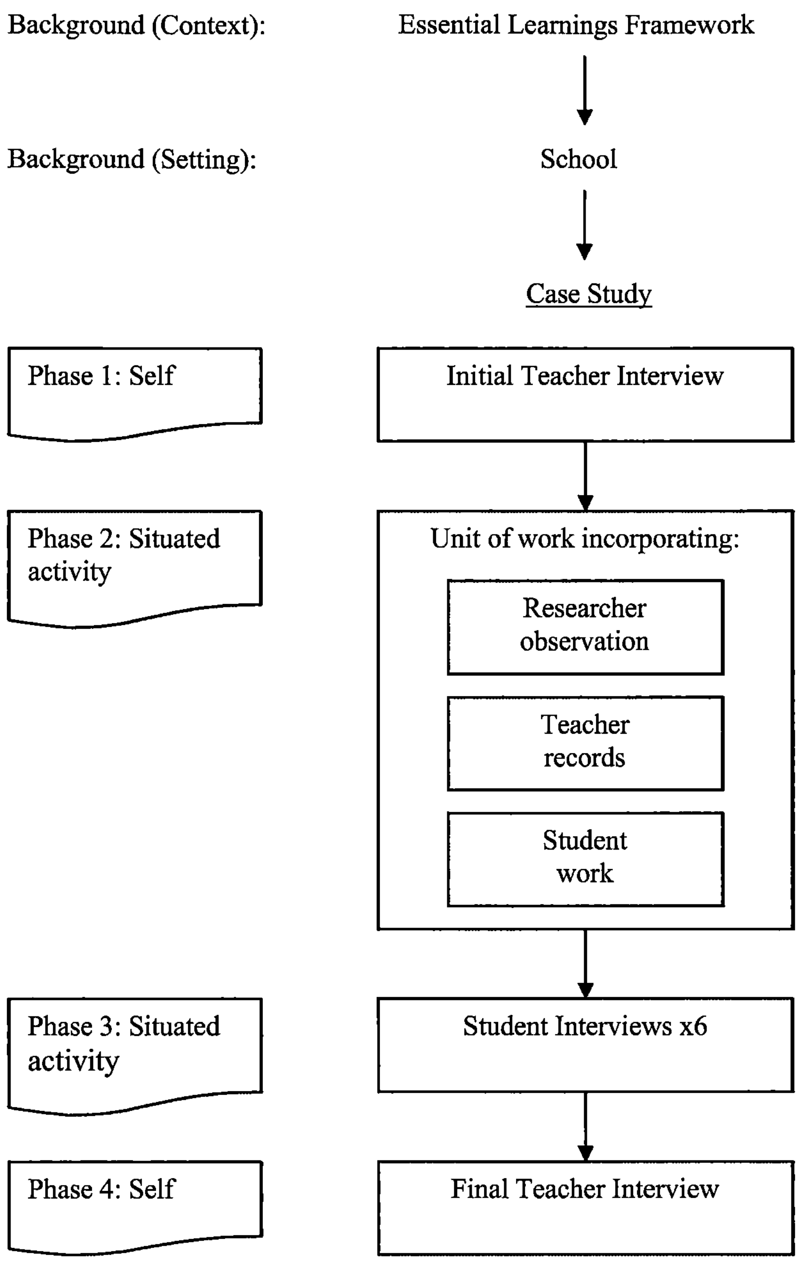


Figure 4.2. Study design: Four phases of inquiry.

The design reflects all four layers of inquiry as described by Layder (1993; 1998): context, setting, situated activity, and self, and where these conceptual levels are situated with respect to the elements of the research. The levels of context and setting form an important background to the case study itself and acknowledge that the case, although 'bounded,' does not exist in isolation from these influences. The predominant focus of the study is at the level of self and situated activity, the positioning of numeracy by teachers in their classrooms within the Essential Learnings and how this positioning affects the experiences and learning of students. The four phases described here were designed to capture the phenomenon of numeracy positioning comprehensively and together combine to form the case study unit of analysis. The two background elements and the four case study phases of inquiry together ensure that all design elements, as described in the research framework (Table 4.1), would be comprehensively explored.

Phase 1 involved an interview with all the participant teachers prior to their commencement of a unit of work that they had planned to achieve numeracy learning outcomes and that was informed by the Essential Learnings. The specific nature of this interview is described in Section 4.9.1.

Phase 2, the most time intensive phase of the research, involved classroom observation of the unit of work. This period of observation was different in each case and incorporated researcher observation, photographs, teacher records, and student work. Details of this phase are described in Section 4.9.2.

Phase 3, student interviews with six students in each case study school, occurred toward or after the completion of Phase 2 and are described in Section 4.9.3.

The final phase of the study was a reflective interview with all the participant teachers and this phase is described in Section 4.9.4.

The design was repeated four times in three Tasmanian schools. The details of how the design specifically pertains to the case study participants are summarised in Section 4.6.3.

4.6 Case study participants: Sampling

As this study sought to investigate the phenomenon of numeracy positioning in depth, non-probability sampling techniques (Merriam, 1998), appropriate to the qualitative nature of the study, were employed. As with most qualitative case studies, two levels of sampling were necessary in this study: first, selection *of* the case described in Section 4.6.1 and second, by selection *within* the case described in Section 4.6.2.

4.6.1 Case selection

The study phenomenon is that experienced by classroom teachers and their students, within the broader setting of their schools. It is defined by the unit of work that each case study teacher planned and implemented to ensure appropriate boundaries were set for the research. Selection of both schools and teachers was purposeful (Creswell, 2005). A purposeful sampling approach is one in which "researchers intentionally select individuals and sites to learn or understand the central phenomenon" (p. 204), in this case numeracy positioning. It "involves the conscious selection ... of certain subjects or elements to include in the study" (Burns & Groves, 1995, p. 243).

4.6.1.1 Selection of schools

Principals of seven schools: six state government and one non-government, were approached to gauge interest in the project. These schools included five primary (Grades K – 6), one secondary (Grades 7 – 10), and one K – 12 school. In the case of the six state government schools, they were schools that were identified in consultation with the State-wide Coordinator for Numeracy as being supportive of the Essential Learnings and having an interest in numeracy. The non-government or independent school was also recommended as meeting these criteria. Therefore, in terms of the criteria of implementation of the reform-based Essential Learnings curriculum across Grades 5 – 8, and with support for effective programs for numeracy, the cases were in this sense typical (Creswell, 2005). More particularly, they were schools in which teachers who had been recommended by the State-wide Coordinator for Numeracy were teaching (reputation sampling). Of the schools approached, all seven principals supported the project and provided the researcher with permission to contact the respective teachers.

At the time of the study, the state was divided into 27 geographical clusters that provided an organisational framework and support to approximately 70,000 children in 217 Tasmanian government schools. The Association of Independent

Schools of Tasmania (AIST) provided support for 12,000 students in 33 non-government schools across the state. Three of the 27 geographical clusters are represented in the study, and include semi-rural, urban, and semi-urban areas representing differing socio-economic backgrounds and school structures. Both government and non-government schools were included as examples of mandated and voluntary implementation of reform. This maximum variation sampling (Creswell, 2005) of obtaining schools that provided as much variation as possible within a three school sample was sought to "present multiple perspectives of individuals [and] in order to represent the complexity of our world" (p. 204). The demographic details of the three case study schools that went on to participate in the study are described in the School Background chapter (Chapter 5).

It is also acknowledged that school selection involved an element of convenience sampling (Creswell, 2005). As the research was being conducted by a sole PhD student investigator, and involved regular repeated visits to the research sites, practical and financial constraints were not insignificant. The three case study schools were all located within the south of the state to enable access by the researcher on a daily basis when required. Walford (2001) suggests that although convenience should not be the sole reason for selecting a sample "it is understandable that academics and research students should include convenience in their consideration of which sites to approach to try and gain access" (p. 14).

4.6.1.2 Selection of teachers – Phases 1 to 4

Upon gaining principal permission, eight teachers, from six schools, were invited to participate in Phase 1 of the research. These teachers were purposefully selected based upon criteria established for the case study and also reputation sampling. Reputation sampling (Cohen, Manion, & Morrison, 2000) applied as the researcher used the recommendations of experts in the field, state government numeracy coordinators, in selecting an initial group of schools and teachers to approach. Due to the in-depth nature of the study, a greater number of teachers were initially approached with the aim of having three or four teachers participate in the case study incorporating all phases.

Of these eight teachers, five agreed to take part in the Phase 1 interviews and a further discussion about the research study. After the Phase 1 interviews, all five teachers, two working together, wanted to proceed with the remainder of the case study phases. Shortly after, one of these teachers was promoted out of the classroom to an administrative position and a second moved overseas. The two teachers were replaced with first, a volunteer who was from the same school as the teacher who moved overseas, and second, a teacher from a different school, who also was recommended by the State-wide Coordinator for Numeracy and whose school met the case study school selection criteria. These changes occurred prior to the commencement of Phase 2 of the research and therefore did not hinder the research in anyway. The final five case study teachers were:

- Alice, from Snowgum Primary School, teacher of a Grade 5/6 class, working individually.
- Ophelia and Samantha, from Stringybark Primary School, teachers of separate Grade 6 classes, involved in collaborative planning but teaching separately.
- Ange and Jen, from Tanglefoot School, home tutor teachers for two Grade 8 classes, implementing a combined unit of work together.

Table 4.2 sets out data on the professional background of the five final case study teachers: Alice, Ophelia, Samantha, Ange, and Jen. Pseudonyms are used for each case study teacher and school for reporting purposes and to maintain participant anonymity.

The boundaries of the case as described in the Study Design (Section 4.5) were clearly identified with each teacher as being one unit of work planned and implemented during the 2005 school year. It was important that the units of work were ones that the case study teachers would have planned and run with their classes whether they had decided to participate in the research or not. For this reason the time length for each case was determined by each individual case and ranged from four to twelve weeks.

Table 4.2

Teachers professional backgrounds

School	Teacher (pseudonym)	Qualifications	Teaching Experience (yrs) (inc. 2005)	Grade levels taught	Grade level currently teaching	Subject areas taught	Subject areas currently teaching	Time ELs in school (yrs) (inc. 2005)	Time using ELs in planning (yrs) (inc. 2005)	Time using ELs in assessment(yrs) (inc. 2005)
Snowgum Primary School	Alice	Dip. Ed., B.Ed. (incomplete)	30	Prim. 3-6	5/6	Primary	Primary	3	3	1
Stringybark Primary School	Samantha	B.Ed.	25	Prim. 3-6 (all)	6	Primary	Primary	5	3	1
	Ophelia	B.Fine Arts, Dip. Ed.	10	K – 12 1 yr UK	6	Primary Art, IT, English, SOSE	Primary	5	2	1
Tanglefoot School	Ange	B. Human Movement, M.Ed. (Sports psychology)	7	Sec. 7 -10 Prim. 5, 6 (Science)	7, 8	Mathematics, Science, Health and Physical Education, English	Mathematics, Science	3	3	3
	Jen	B.A. (Psychology), B.Teach.	3	Sec. 7 - 10	8, 10	English, History, SOSE, Mathematics	English, SOSE, History	3	3	3

4.6.2 Selection within the case

Once the case has been defined and selected, decisions remain as to whom to interview, what to observe, and which documents to collect for analysis. In this research, these within-case sampling decisions included selection of students and decisions with respect to observations, photography, and document collection. These theoretical sampling decisions (Merriam, 1998) were guided by the research questions and the researcher's own ethic of ensuring as thorough an understanding as possible would be gleaned about the study phenomena. The selection of students involved different sampling techniques for Phases 2 and 3 of the project. These are described in Sections 4.6.2.1 and 4.6.2.2 respectively. Further description concerning these specific methods of inquiry is detailed in Section 4.9.

4.6.2.1 Selection of students – Phase 2

The student participants were selected with respect to their automatic association with the purposefully selected case study schools and teachers. The use of convenience sampling techniques was therefore of relevance in this study. Associated with the case study teachers, 112 students from Grades 5 – 8 participated in Phase 2 of the study. The students shared their thoughts and their work and allowed photographs to be taken of both their work and of themselves undertaking activities in the classroom environment. The sample of students included 78 primary students in Grades 5 or 6 (aged 10 to 12), and 34 secondary students in Grade 8 (aged 13 to 14). The distribution of students across each case is presented in Table 4.3.

Table 4.3

Students participating in the research: Distribution across each case

School	Teacher	Grade	Number of Students (Phase 2)*
Snowgum	Alice	5/6	23
Stringybark	Samantha	6	27
Stringybark	Ophelia	6	28
Tanglefoot	Ange & Jen	8	34

* During Phase 3, six students from each of the four classes participated in a student interview.

4.6.2.2 Selection of students – Phase 3

From the four classes that participated in the study, 24 students were selected to take part in an individual semi-structured interview: six students from each case study as described in Table 4.3. These students were selected to represent a spread of academic ability in each class. In addition, the teachers of those students selected were asked to recommend students who would be able and willing to discuss their work and their thoughts with the researcher. All the students recommended by their teachers were then asked by their teacher if they would be happy to participate in a one-on-one interview with the researcher. Parental consent was gained from the parents of these students.

4.6.3 Design summary as it applies to participants

A summary of the case study participants: schools, teachers, and students is represented in Figure 4.3. It brings together the overall design of the study with the sampling decisions described in this section. The three schools involved in the research were all located in southern Tasmania and were operating under the new Essential Learnings framework. The two state government schools, Stringybark and Snowgum, came directly under the control of the D*o*ET, whereas Tanglefoot School, an independent girls' school, was implementing the Essential Learnings in a way determined solely by that school body. Within the three schools, five case study teachers participated in the research: one teacher, Alice, and her Grade 5/6 class at Snowgum Primary School; two teachers, Samantha and Ophelia, and their respective Grade 6 classes operating independently at Stringybark Primary School; and two teachers, Ange and Jen, and their respective Grade 8 classes operating together at Tanglefoot School.

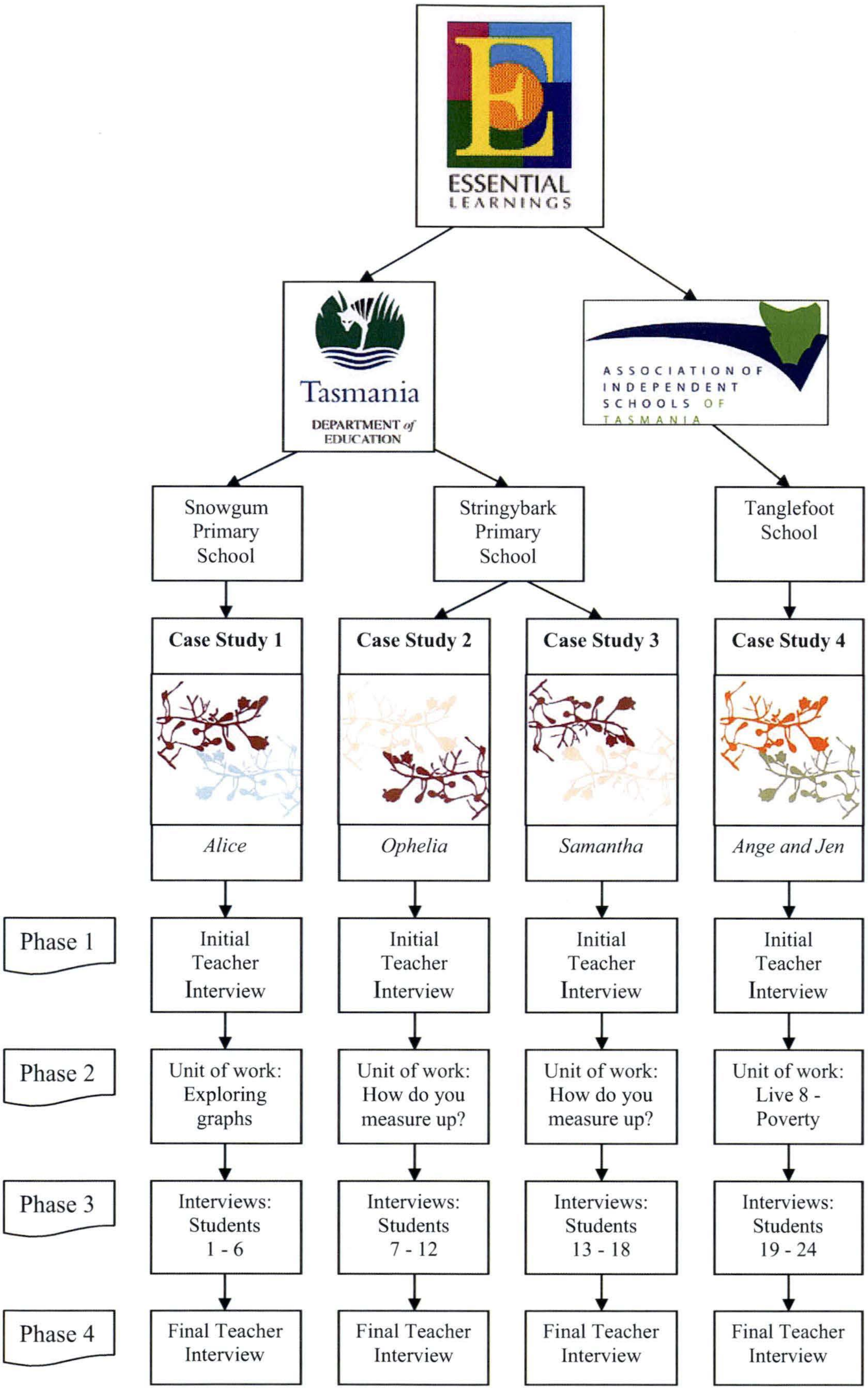


Figure 4.3. Collective case study: The participants.

4.7 The researcher's role

"The researcher is the research instrument in qualitative research projects" (Janesick, 2004, p. 103). Two major roles were required of the researcher in terms of data collection, that of interviewer and of observer. As interviewer, the objectives of the researcher were to communicate the purpose of the research effectively and to ask questions. The Phase 1 interviews were quite formal as they occurred at the beginning of the data collection period. Phase 3 and 4 interviews were more relaxed in view of relationships that had been established whilst the researcher was in the field.

The in-depth nature of being in the field for significant lengths of time in this study required the researcher to consider and define the role to be taken clearly. In this regard, Wolcott (1994) suggests that "somewhere between 'going out to places' and 'coming back with information,' every fieldworker has to achieve some workable balance between participating and observing" (p. 95). Woods (1986) raises an interesting question in regard to observations: Why participate? For Woods, the answer is based on the belief that "the central idea of participation is to penetrate the experiences of others within a group or institution" (p. 33). The understanding of the experiences of the case study teachers and their students is central to this thesis.

The discussion associated with the role of participation or non-participation is something that tends to be portrayed as a dichotomy, a decision that must be made. Gillham (2000) however, suggests that each form of observation, participant or non-participant, represents two ends of a continuum: rather than choosing one end or the other the researcher moves between roles. In this study, the researcher was free to move between roles depending on the situation or the data needing to be collected. There were times, for example, when the researcher participated in classroom activities when a student asked for help with an activity or when the teacher asked for the researcher's assistance with the supervision of a group of students undertaking an activity, particularly during small group work. The researcher remained aware of engaging in activities appropriate to the situation. This may have involved talking to or questioning students about their work, enabling teachers to share their thoughts, or attending meetings that the case study teachers participated in during the study. The purpose of doing this was to be able to observe closely the phenomenon of study within each of the four case study classroom learning environments. At no stage, however, was the researcher involved in collaborating with teachers to influence planning, teaching, or assessment of students.

The way Spradley (1980) describes the movement from outsider to insider during field work is most relevant. In all four case studies, the researcher commenced the time in the classroom by sitting towards the back of the room, observing and taking notes. It was not long before students became comfortable with the researcher's presence and the researcher moved from being an "outsider" to becoming an "insider." This change occurred within only a few visits and was very much initiated by the teachers and their students as they felt comfortable to engage in conversation with the researcher. The researcher was responsive to the context and participated at some times and not at others. This was critical to developing a trusting and respectful teacher/researcher relationship and was achieved by ensuring effective communication was established and maintained throughout the data collection period.

The length of time in the field for each case was also a factor in terms of how far the "insider" or participant role went. With the two longer case studies, both at Stringybark Primary School, the researcher was, by the end of the time in the field, welcomed by the larger staff in the staffroom and happily greeted by students upon arrival. Therefore, not only did the researcher's role alternate *within* each case study, it also varied *between* each case study.

The nature of the relationships with each case study teacher changed as the research process progressed. The effectiveness of these relationships was vitally important because if the researcher/participant relationships are positive, "participants will be more willing to share everything, warts and all, with the researcher" (Janesick, 1998, p. 40). The researcher put a high priority on forming and maintaining relationships with the participant teachers and students. Relationships were maintained by a genuine interest in the work of the teachers and their students, effective communication whilst in the field, the sharing of digital photographs taken by the researcher immediately after return from each field visit with the respective teachers (many of which were displayed in classrooms), and ongoing email communication whilst away from the research sites.

As a participant observer it would be naïve to assume that the presence of the researcher in the classroom with pen, notebook, and digital camera in hand would not have an influence. The teacher, students, parents, and other staff who were present would at times interact with the researcher and it should be noted that this is one of the main criticisms levelled against observation as a method of data collection (Gillham, 2000). While in the field, Goetz and LeCompte (1984), when discussing how the researcher should act, suggest that some may enter the field with an "assumption of ignorance or naiveté ... others simply suspend preconceived notions and even existing knowledge of the field under study" (p. 9). The researcher undertook the role professionally and respectfully at all times and as far as possible suspended preconceived notions that would hinder the data collection process. The researcher, however, clearly brought herself to the field, as a personality and as a researcher with a theoretical position. This must be acknowledged but is not believed to have detracted from the study. On the contrary, it is believed that these aspects brought their own positive elements to the field as it supported the teachers and students to undertake their daily roles as they usually would without putting on pretence or feeling compelled to operate in any particular way.

4.8 Data collection: Methods and techniques

Having developed the conceptual research framework and formulated research questions, the next step in focusing and bounding the data is deciding on the methods of inquiry that best suit the purposes of the research. In a case study such as this, attention cannot be paid to every phenomenon. Decisions as to which people, activities, relationships, and physical contexts deserved attention were initially made through reference to the research frameworks and questions as discussed in Section 4.3.2. Within this framework, further decisions needed to be made. Which teachers and students would make the best participants? Which documents would provide the best source of data? Therefore, within-case sampling decisions were more purposive than random.

The thesis incorporates two major primary sources of data – interviews with teacher and student participants, and direct observation of classroom activities. The collection of documents, both teacher and student, and the taking of photographs formed secondary data sources to strengthen the analysis and representation of the study results. The inquiry methods of interview, observation, document review, and photographic records are considered in this section.

4.8.1 Interview

Research interviews are predominantly defined in terms of the degree of structure that is applied to the interview process and may be viewed along a continuum from the very structured to the very open-ended. "For the most part, however, interviewing in qualitative investigations is more open-ended and less structured" (Merriam, 1998, p. 74). For the purposes of this study, the semi-structured interview was adopted (Fontana & Frey, 1998). In this type of interview the questions are "more flexibly worded, or the interview is a mix of more and less structured questions" (Merriam, 1998, p. 74). The interview is, for the most part, guided by a list of questions or issues to be explored. "This format allows the researcher to respond to the situation at hand, to the emerging worldview of the respondent, and to new ideas on the topic" (Merriam, 1998, p. 74). The interviews in the study were not rigidly structured although each of the teachers and students were asked the same basic questions. Additional questions were used as the need arose to follow a particular theme or to elicit greater detail in regard to a specific topic.

Qualitative case study methodology suggests the use of interview since it is through conversations with a range of informants, or participants, that "multiple realities" or view of actors on the case are accessed (Stake, 1995, p. 64) and knowledge is constructed (Kvale, 1996); "the one-to-one personal relationship that an interview provides is usually more effective in eliciting respondents' sincere participation in a research project" (Thomas, 2003, p. 66).

Interview was the method of data collection for Phases 1, 3, and 4 in this study and was employed to "gain access to teachers' and students' impressions, beliefs, assumptions, and justifications of observed events" (Wiliam, 1998, p. 23). All interviews explored the issues central to this thesis: reform-based curriculum and numeracy. The interviews were all audio-taped and transcribed by the researcher.

4.8.1.1 Initial teacher interview – Phase 1

The purpose of the Phase 1 teacher interviews was to gain insight into the teachers' beliefs and practices regarding the Essential Learnings, and about numeracy and its place in the curriculum. The interview also provided opportunity for the investigator to explain the project fully and answer any questions the teachers had as well as provide an opportunity for teachers to share experiences. A copy of the semi-structured interview schedule that guided the Phase 1 interviews is provided in Appendix B.

4.8.1.2 Student interviews – Phase 3

Six students from each case study were selected to participate in an individual interview (Section 4.6.2.2). These interviews were scheduled toward the end of or on completion of the classroom-based unit of work and served to "supplement, clarify, or validate the data" gained from the classroom observations and student work samples (Wiliam, 1998, p. 23). The purpose of the student interviews was to gain an overview of the students' numeracy learning. More particularly, the researcher was interested in the students' thoughts on the unit of work, their learning, and their particular work samples that they brought to the interview. Student interview schedules are included in Appendix C.

4.8.1.3 Final teacher interview – Phase 4

Final teacher interviews were undertaken to gain feedback from teachers on their perceptions of the outcomes of the unit of work specifically in relation to student learning and to the place of numeracy in the Essential Learnings. During this interview the teachers were asked to reflect on their experiences during the unit of work. They were also encouraged to use specific examples where appropriate. The final teacher interview schedules are included in Appendix D.

4.8.2 Observation

The direct and naturalistic nature of observation has been acknowledged by Adler and Adler (1998) who state that qualitative observation:

> occurs in the natural context of the occurrence, among the actors who would normally be participating in the interaction, and follows the natural stream of everyday life. As such, it enjoys the advantage of drawing the observer into the phenomenological complexity of the world, where connections, correlations, and causes can be witnessed as and how they unfold (p. 81).

Conducting observations can be overwhelming during the initial period in the field due to the amount of activity to be observed and more importantly filtered, interpreted, and recorded (Taylor & Bogdan, 1998). To overcome this, a pre-determined protocol for the recording of field notes was thoroughly planned and developed prior to entering the field. The approach to observation taken by the researcher was informed by Merriam (1998), Thomas (2003), and Wolcott (1995). The protocol is included here in Figure 4.4 and was physically glued to the inside cover of every observational notebook, for the researcher to refer to and reflect upon continually.

Field Notes

Record keeping: to include descriptive and reflective notes
Wolcott (1994), Thomas (2003), Merriam (1998)

- Descriptive: setting, people, activities, conversations, subtle factors, own behaviour.
    - What people actually doing and saying, concrete
    - Reconstruction of dialogue
    - Portraits of subjects
    - Description of physical setting
    - Accounts of events
    - Depiction of activities
    - Observer's behaviour
- Reflective: include reflections, interpretations, ideas, concerns, questions, feelings, methodology, theory, beginning analysis.
    - Speculation, feelings, problems, ideas, hunches, impressions, prejudices
    - Plans for future, questions to clarify
    - Reflections on analysis
    - Reflections on method
    - Ethical dilemmas and conflicts
    - Observer's frame of mind
    - Points of clarification

Start notes with:
DATE
START TIME
FINISH TIME
LENGTH OF ACTIVITY
WHO
WHERE
NO. OF VISIT

Descriptive notes on RHS page (leave margin for analysis pts) any reflections within observation in [ ]; afterwards any additions/reflections on LHS page.

Points to consider:
- Outsider to insider
- Negotiate access to documents, photography
- Observations and field notes involve asking questions; answers to be discovered
- Levels of participation; continuum
- Factual, accurate, and thorough

Figure 4.4. Field notes protocol designed for the study.

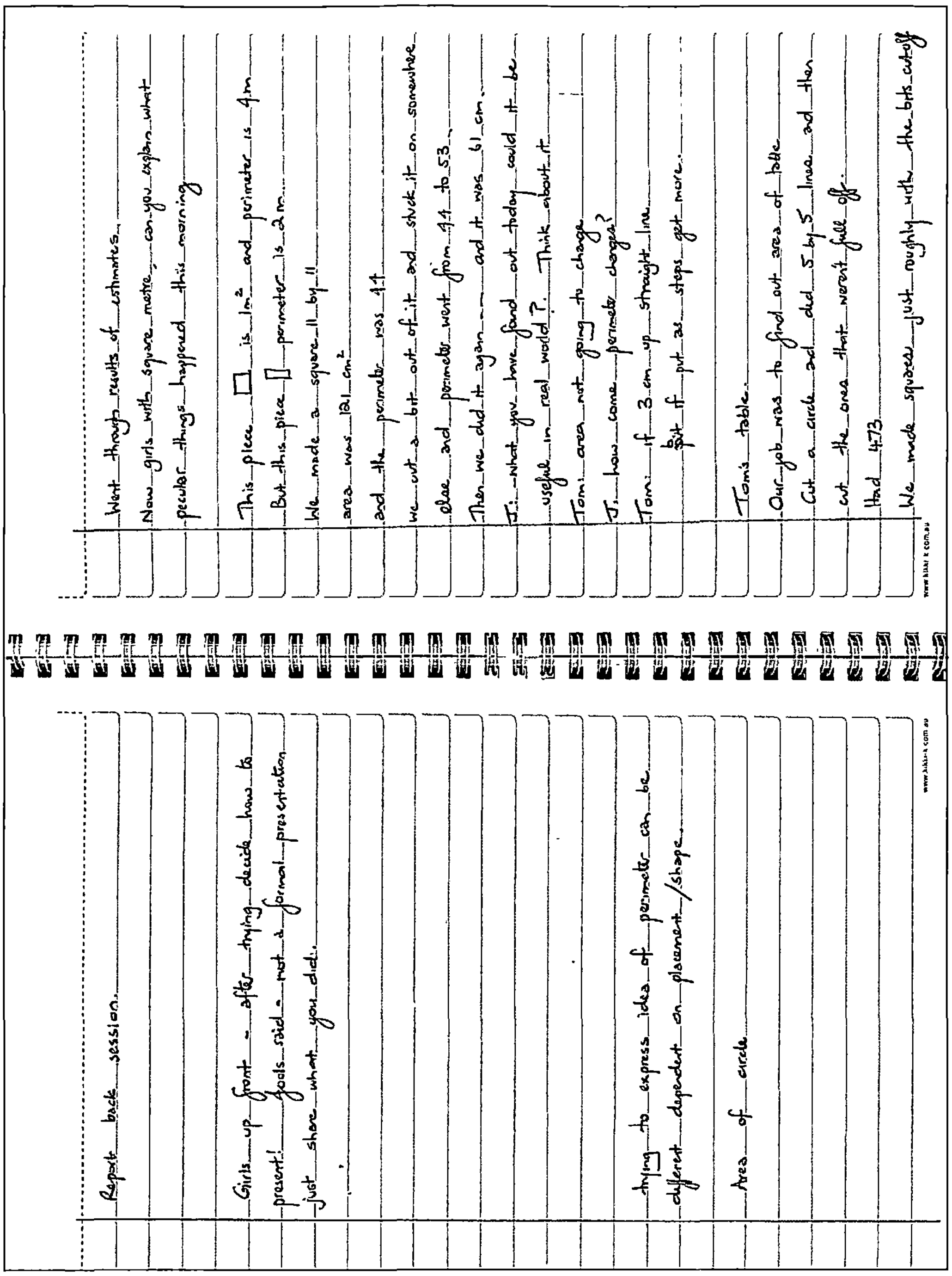
The descriptive field notes were hand-written in A4 notebooks, each case study identified by the use of the notebook motif, as described in Section 4.8.5. The researcher found the recording of observations by hand a relatively straightforward task due to much previous experience with note-taking.

The researcher was never without her notebooks and both teachers and students quickly became used to this note-taking role. Notes were not taken when engaging in informal conversation with teachers or students, out of both respect and genuine interest in these conversations. The researcher moved from participant in the classroom to observer and the note-taking role adjusted accordingly. "The role of participant observer is more appropriate for contemporary mathematics classrooms" (Wiliam, 1998, p. 21). It involves becoming part of the life of the school, asking questions, and being involved with the students. By being an active participant in the classroom the researcher was able to ask spontaneous questions as they arose and Thomas (2003) refers to this role as enabling the researcher to be perceived as a member of the community. On the other hand it must be acknowledged that taking on this role does suggest "that a possible compromise in the quality and quantity of the field notes is necessary because the researcher would not be able to simply sit, observe, and record" (Wiliam, 1998, p. 27). This compromise is in some ways mitigated by the inclusion of reflective notes.

Reflective notes were made in two ways: first, the researcher audio-taped her own thoughts upon exiting the field. These notes included observational thoughts that may have been initially overlooked, further questions to explore, analytical issues for consideration, and any reflections as listed in the field note protocol (Figure 4.4). These reflective notes were transcribed by the researcher in between site visits. Second, the researcher re-read the hand-written field notes after each site visit and recorded further comments on the left-hand page when deemed relevant. These notes were usually when a clarification was needed or a summary provided.

In addition to the observation techniques describe above, the aim of the research guided the observations. The research questions, "How are teachers positioning numeracy in reform-based learning environments?" and "How are students experiencing numeracy in reform-based learning environments?" guided the classroom observations. "For the researcher entering the mathematics classroom, the purpose of observation is to develop an understanding of the ways in which a mathematics culture is being constructed and reproduced within the context of that classroom or school" (Wiliam, 1998, p. 21). It is noted, that as a qualitative naturalistic inquiry, the conceptual framework for numeracy developed in Chapter 3 was not shared with the teachers or the students who participated in the study, but was used at the point of data analysis when the researcher exited the field and is discussed in Section 4.10.

In focusing the observations, specific attention was given to the numeracy connected to the unit of work. The units of work were already scheduled into the year's program for each particular case and involved work that the students would normally undertake on a day-to-day basis in the classroom. The important elements of the classroom observation were the teachers' planning and implementation of the unit of work, and the teachers and students conversations about the work and with each other during each classroom visit. If the researcher herself formed part of any conversation with students or teachers this was also recorded. Observations included general observation notes, written records of conversations, photographing of a specific activity [indicated in the observation notes by number], and the collecting of student work samples. A scanned copy of one page from an observation record is included in Figure 4.5 to exemplify the observation note-taking process. The elements of documents and photography are discussed in the following sub-sections.


Report back session.

Girls up front - after trying decide how to
present! Fools said - not a formal presentation
just show what you did.

trying to express idea of perimeter can be
different dependent on placement/shape.

Area of circle

Went through results of estimates.
Now girls with square metre, can you explain what
peculiar things happened this morning

This piece ☐ is 1m² and perimeter is 4m
But this piece ▯ perimeter is 2m.
We made a square 11 by 11
area was 121 cm²
and the perimeter was 44
we cut a bit out of it and stuck it on somewhere
else and perimeter went from 44 to 53,
Then we did it again — and it was 61 cm.
T: what you have found out today could it be
useful in real world? Think about it
Tom: area not going to change
T: how come perimeter changes?
Tom: if 3 cm up straight line
bit if put as steps get more.

Tom's table.
Our job was to find out area of table
Cut a circle and did 5 by 5 lines and then
cut the area that weren't full off.
Had 473
We made squares just roughly with the bits cut off

Figure 4.5. Sample observation note record.

4.8.3 Documents

The data and collection processes discussed to date are materials produced by the researcher: the field notes written, as well as the interviews conducted and transcribed. This section discusses data written by the participants themselves and includes teachers' records, student work samples, and more general school-related documents.

In determining documents to collect and in maintaining the focused nature of each case study, the criteria proposed by Merriam (1998) were used to judge the value of each data source: whether the document contained information relevant to the research questions, and whether it could be easily yet systematically acquired. All document collection was negotiated with each respective case study teacher and was done with her full knowledge and permission. Similarly, student work samples were photocopied with consent from both the teacher and the student.

School documents, such as school newsletters and school assessment booklets, were collected at each school site where they provided insight into the broader setting of each case. Lastly, policy and other documents were collected to provide a comprehensive overview of the Essential Learnings and its implementation in Tasmania. These were predominantly documents from the D*o*ET, but also included some newspaper articles and national reports.

4.8.4 Photographs

Researcher-produced photographs (Bogdan & Biklen, 1992, p. 142) were used to support participant observation. "In this capacity it is most often used as a means of remembering and studying detail that might be overlooked if a photographic image were not available for reflection." (p. 143). The researcher took photographs of classrooms, students working, and student work samples to complement other methods of data collection.

Although it has been argued that "a camera emphasizes the researcher's role as an outsider" (Bogdan & Biklen, 1992, p. 142) it did not serve to distance the researcher from the participants in this study, perhaps even the contrary. Teachers and students in classrooms today are very familiar with digital photography and it is incorporated in most classrooms by teachers on occasion. The researcher emailed all photographs back to each teacher after exiting the field on each occasion, and would often return again to the field to find photographs of students' working on display. Both the teachers and the students appreciated this effort made by the researcher and saw it as affirming the value the researcher placed on the work being done and supporting the work of the teacher. Teachers often have insufficient time to photograph student work and appreciated this extra element to their classrooms.

4.8.5 The motif

The importance of data management was not underestimated (Miles & Huberman, 1994). This was assisted in this research by the use of stationery purchased for the purpose of recording field notes and keeping administrative records in relation to each case. The notebooks' 'branches' design (Kikki-k) became intrinsically connected to each case beyond the initial purpose of data organisation. This motif (Figure 4.6) is used to identify the three case study schools, the data of which are presented in four results chapters. The names assigned to each case study school are derived through a connection with the colours of Tasmanian native tree species:

- Snowgum Primary School – The colours pale blue and brown in this thesis representing the *Eucalyptus coccifera*, common name: Snowgum.
- Stringybark Primary School – The colours beige and brown in this thesis representing the *Eucalyptus obliqua*, common name: Stringybark.
- Tanglefoot School – The colours olive and orange in this thesis representing the *Nothofagus gunnii*, common name: Tanglefoot.

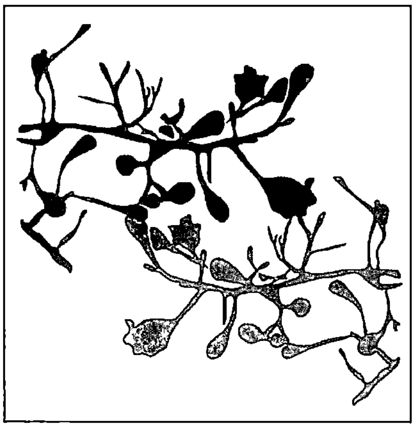


Figure 4.6. The motif.

The use of the motif not only assisted a planned data collection process but continued throughout data analysis with transcripts, student and teacher records, field notes, and analysis notes and displays, all kept within the structure of the four case studies and their respective motifs.

4.9 Procedures

Acknowledging that "access and entry are sensitive components in qualitative research" (Janesick, 1998, p. 40), the process of gaining access to each research site was similar to that described by Walford (2001). Walford suggests that the issue of access can be viewed as an "incremental continuum, where the researcher is gradually able to move from the initial permission to enter the building to a series of trusting relationships with some teachers and students" (p. 34).

Ethical approval for the research was gained from the *Southern Tasmania Social Sciences Human Research Ethics Committee* at the University of Tasmania in 2004 (Ethics reference approval number H7988). The committee adheres to the guidelines outlined in the *National Statement on Ethical Conduct in Research Involving Humans* (National Health and Medical Research Council, 1999). The research also had permission and approval from the D*o*ET, and satisfied department criteria for *Conducting Research in Tasmanian Government Schools* . Ethics approvals from the University of Tasmania and the D*o*ET are found in Appendices E and F respectively.

All teacher and student data were collected in 2005 to coincide with a full school year in Tasmania. Principal support for the research and the Phase 1 teacher interviews were completed in the first school term of 2005. It is noted that an interview was completed with each of the three school principals in order to gain school background information and insight into each school setting. Phase 2 of the study, the most time intensive phase, was conducted in the second and third school terms. This phase involved regular classroom observations and the collection of teacher and student records. Phase 3 student interviews were scheduled where possible at the completion of the classroom-based units of work. The Phase 4 final teacher interviews all took place in the third school term when all other data collection had finished. Appendix G outlines the data collection timeline for the research, including ethical requirements for each phase. The following sections detail the procedures for each phase of the study.

4.9.1 Phase 1 – Initial teacher interviews

School principals were informed about the project with a letter of invitation and accompanying project information sheet. These letters were followed with a phone call from the researcher. All the principals contacted expressed their support of the research and agreed to pass on the letters of invitation to the selected teachers. They also gave permission for the teachers to be contacted by the researcher. The selection of the five final case study teachers was described in detail in Section 4.6.1.2.

Letters of invitation to the case study teachers were accompanied by a project information sheet. Letters included an overview of the purposes and aims of the research, the procedures involved in the study, appropriate contacts, and statements of the treatment of confidentiality and withdrawal. A consent form was included containing a statement of informed consent. Before the interview the consent forms were explained and signed.

The Phase 1 teacher interviews were scheduled at a time to suit each respective teacher. At the primary grades (Grade 5 and 6) one classroom teacher is usually responsible for the implementation of the total program and therefore involvement in the project was based upon one case study teacher. At the grade 7 and 8 level, more than one teacher may be working together to implement a unit of work. Therefore in this setting the project enabled the inclusion of more than one teacher for a particular case study to reflect the actual classroom situation. Teachers Alice, Samantha, and Ophelia were interviewed individually, and in the case of Tanglefoot, Ange and Jen, were interviewed together. Interviews were approximately 30 – 40 minutes in length. The interviews took place at the respective school site at a time chosen as convenient by the respective teachers. The day before the interview, the researcher emailed an outline of the interview to the teachers to enable them to consider the issues and bring any supporting documentation with them to the interview if they wished. These sessions were audiotaped with permission.

The decision to continue through Phases 2 – 4 of the project was made after the Phase 1 Initial interviews. This provided both the investigator and the teacher with the option of not proceeding with the main part of the project for whatever reason. This option was clearly stated in the consent form. None of the teachers interviewed chose to withdraw at that point but two teachers had to withdraw from the project prior to the implementation of Phase 2 for personal or professional reasons. These teachers were replaced as described in Section 4.6.1.2.

4.9.2 Phase 2 – The unit of work

Classroom observation of the unit of work for each case study was negotiated at the initial teacher interview as this was the most time intensive phase of the research and required the biggest commitment by the case study teachers. It was a unit of work already scheduled into the year's program and ranging in time from four weeks to twelve weeks depending on the case. The researcher entered each case site during pre-determined times as advised by the case study teachers as being those times when the unit of work was being taught.

During each field visit the researcher took comprehensive field notes, participated in conversations with teachers and students, and took photographs when relevant and sometimes even requested by the students. The researcher made her field notes available to the teachers if requested and emailed all photographs back to the classroom teacher after each field visit. This ensured transparency and communication during the research. A summary of the classroom observation phase for each case is as follows:

- Snowgum – Alice: 7 classroom observations over four weeks, total 9 hours and 30 minutes, predominantly 90 minutes each, with one at 45 minutes [48 pages in total].

- Stringybark – Ophelia: 15 classroom observations over ten weeks, total 18 hours and 10 minutes, predominantly 80 minutes each, with three 50 minutes or below [60 pages in total].
- Stringybark – Samantha: 18 classroom observations over eleven weeks, total 33 hours and 35 minutes, predominantly 110 to 120 minutes each, with one at 30 minutes [126 pages in total].
- Tanglefoot – Ange and Jen: 15 classroom observations over six weeks, total 15 hours, predominantly 40 to 50 minutes each, with one at 95 minutes and three at 100 minutes [59 pages in total].

Teacher records relevant to the unit of work were discussed and photocopied. Photocopying usually occurred after the periods of classroom observation and with the permission of the school.

Student data were relevant when they related to specific activities and conversations concerned with the interaction of numeracy and the key elements of the unit of work. Student work samples were photocopied when possible, predominantly at the end of the unit of work, so as to cause the least disruption to classroom activities and enable the completion of work samples. As the data were work that the students were doing as part of their daily work in the classroom with their class teacher, parent/guardian consent was not applicable. Students could not withdraw from the unit of work itself but parents/guardians could choose not to have their children's work used by the researcher. It was necessary to give parents/guardians an informed option, via a withdrawal of participation form, not to have the students' work samples and specific observations/photographs used by the researcher. No parent elected to withdraw their child's work from the research. Students' privacy was protected by having the observations coded and re-identifiable. A large number of photographs were taken and student and teacher records and work samples collected during the study. Those pertinent to exemplifying the results are included throughout Chapters 6 to 9.

4.9.3 Phase 3 – Student interviews

The students selected for interview were interviewed individually in a separate room in their respective schools. Selection of students was described in Section 4.6.2. The interviews were approximately 30 to 40 minutes in length and were audiotaped. Students brought their major work samples with them to which to refer at anytime during the interview, particularly when discussing aspects of the work completed during the period of the research. Participation in these interviews was voluntary and parental permission was obtained.

For students to participate in the interviews, parental consent was an ethical requirement. This involved a second letter being sent to the parents of the selected students with the same information sheet as was contained with the original letter, but this time requesting informed consent for their child to be interviewed. The selected students were also asked if they would be happy to participate in the interview by their classroom teacher, not the researcher, so they did not feel obliged. At the beginning of the interview the researcher also reiterated with each student that the interview was voluntary and they could ask questions or stop at any time.

4.9.4 Phase 4 – Final teacher interviews

The case study teachers were invited to participate in a final reflective interview with the researcher. These sessions occurred after completion of the unit of work and after collection of all the teacher and student records. The interviews were organised at a time to suit each teacher and took place in a separate room at her school. The sessions were recorded and lasted approximately 40 to 60 minutes. Details of this phase of the project were included in the original letter of invitation and also incorporated in the consent form signed by the teachers prior to the commencement of the research.

Just as gaining and maintaining access to research sites is an important aspect of conducting research, leaving the field is also important. Stake (1995) suggests that "it is often unclear when the final visit is…" (p. 60). In each instance, several visits were made to the research sites after the formal observation period had ended, to complete collection of all relevant documents, to thank the teachers and students personally, and to present certificates of appreciation. In most cases, the Phase 4 final teacher interview coincided with the last site visit.

4.10 Data Analysis and reporting

In qualitative research the process of data analysis begins with methodological and analytic decisions made before entering the field and ends when the report of the research is complete. In this study, data collection, analysis, and reporting were "interactive" (Merriam, 1998, p. 152) and involved specific processes of reflection and organisation of the data for interpretation and reporting.

Whilst collecting data the researcher undertook rudimentary data analysis utilising suggestions provided by Bogdan and Biklen (1992). These included: making decisions that narrowed the study, developing analytic questions, writing observer's comments after each field visit alongside field notes, recording researcher reflective notes after each field visit for consideration between field visits, and continuing to explore the literature whilst in the field. These strategies served to ensure the data collection process remained focused and considered in line with the research questions and the design of the study. It also ensured that upon completion of all of the phases of data collection the researcher was ready to begin an intensive phase of qualitative data analysis. This intensive phase of data analysis and the use of the conceptual framework for numeracy (Chapter 3) was undertaken by researcher after exiting the field.

Data analysis was guided by Miles and Huberman (1984) and Creswell (2003). Miles and Huberman (1984) detail three primary modes of qualitative analysis: data reduction, data display, as well as the drawing and verification of conclusions. Creswell (2003) discusses six steps in the analysis process: organising and preparing the data, obtaining a general sense of the data, coding, description of themes, representation of the data, and finally interpretation or making meaning of the data. Although structured differently both Miles and Huberman and Creswell cover the same elements of qualitative analysis within their models. They elaborate on the importance of going beyond description in data analysis to *thinking* about the data in order to formulate theory. This aspect of theory building as it relates to this study was described in Section 4.3.1 and it is demonstrated here how it affected the analysis process practically. The following Sections, 4.10.1 to 4.10.5, describe the process of data analysis as informed by these authors. In Section 4.10.1 the process of data preparation is described. Sections 4.10.2 to 4.10.4 detail the reduction, representation, and interpretation of data as they applied to the four phases of the study. The analysis of the Phase 1 and Phase 4 teacher interviews are discussed together in Section 4.10.2 as the process of data analysis for all of the teacher interviews was similar. The structure of the presentation of the results is outlined in Section 4.10.5.

4.10.1 Data preparation

The data collection process across the three case sites and the classrooms of the four case study teachers necessitated a highly organised approach to managing the data and preparing it for analysis and reporting. The teacher and student interviews were transcribed, photographs were downloaded and computer-filed, and all teacher and student records were filed in folders according to case, together with field notes that were hand-written in the case study notebooks. Researcher reflective notes were also transcribed and filed according to case.

Organisation and transcription of data also enabled the researcher to continue the process of becoming more familiar with the data, to gain "a general sense of the information and to reflect on its overall meaning" (Creswell, 2003, p. 191). The digital audio recordings of the eight Phase 1 and Phase 4 teacher interviews 24 Phase 3 student interviews, three principal interviews, and 53 reflective field note recordings were fully transcribed including interviewer questions and interviewee responses. Attention to detail included recording pauses, additional questions, emphases by the interviewee on words or phrases, and any body language, such as hand movements. This was a lengthy process due to the number of hours recorded, approximately 40 hours of audio taped interviews in total, but it was important in the process of becoming familiar with the data before beginning the coding process. Full transcripts are included in Appendix H (Teacher and

Principal interviews), Appendix I (Student interviews), and Appendix J (Reflective transcripts).

4.10.2 Teacher interviews – Phases 1 and 4

Given the vast quantity of data, the qualitative data analysis package, NVivo Version 7 (QSR International Pty Ltd, 1999-2006), was used to support the process of data reduction for the teacher interviews. Having used NVivo to separate all of the teacher comments related to the broader curriculum and those related more specifically to numeracy, the coding process was undertaken in a more traditional manner, by hand. In relation to curriculum, key themes were identified through cluster analysis (Miles & Huberman, 1994). The results were synthesised to provide a description of each teacher's conceptualisation of curriculum, the results of which are presented at the beginning of each case study Results chapter.

In relation to numeracy, the teacher interviews were coded according to the conceptual framework for numeracy developed in Chapter 3. The teacher responses were grouped according to the five dimensions of numeracy, *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity*, proposed in the thesis. The grouped responses were then summarised to provide a description of each teacher's beliefs according the five dimensions of numeracy.

In addition to the coding as described, select teacher quotes from the Phase 1 initial teacher interviews and the Phase 4 final teacher interviews are also used at the beginning and end of each case study Results chapter to capture the teachers' voices. Appendix K contains a scanned copy of the analysis of the teacher interviews for the case of Alice, to exemplify the coding process.

4.10.3 The unit of work – Phase 2

The qualitative nature of the research provided the opportunity to use very rich descriptions of the enactment of numeracy in the classroom. A holistic approach was taken to the analysis of the classroom observations. This was achieved through "a careful reading (indeed probably several readings) of the corpus of data, in order to become thoroughly familiar with it" (Hammersley & Atkinson, 1995, p. 210). Scanned copies of the observation notebooks for each of the four case studies is provided in Appendix L.

In terms of reducing and interpreting these data, writing was an important stage in the process of data analysis and presentation of the results. Miles and Huberman (1994) suggest that by "deciding what to leave in, what to highlight, what to report first and last, what to interconnect, and what main ideas are important, analytic choices are being made constantly" (p.8).

Using the framework of the five dimensions of numeracy, selected elements of practice in the classroom were used to exemplify each dimension. Photographs and work samples were sourced to support and illustrate examples. Illustrations with examples are presented in the Results chapters and dates are reported in British format as Day/Month/Year.

4.10.4 Student interviews – Phase 3

After transcription the student interviews were analysed according to the conceptual framework developed in Chapter 3 of the thesis. Student responses were coded and grouped according to the five dimensions of numeracy – *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity*.

After initial coding, a second level of analysis was undertaken with the student interviews. As a focus of the research was the investigation of student learning in each of the case studies, the initial coding of the student interviews according to the five dimensions of numeracy was not sufficient. Further reduction and interpretation of the data needed to consider the constituent aspects of each dimension of numeracy as identified in Chapter 3 (refer Sections 3.3 to 3.7). The student responses, already coded according to each of the five dimensions of numeracy, were categorised with respect to each of the aspects of student learning within these dimensions (Appendix M). In each Results chapter, student learning data are presented in tables in which a tick represents the fact that the student was able to evidence that category of learning. Individual student responses are used throughout the presentation of the Student learning sections of each Results chapter to support and illustrate the findings.

In relation to the *Reasoning* dimension, six categories are described in the Conceptual Framework in Chapter 3 (refer Table 2.5). Five of these six categories of Anderson and Krathwohl (2001) were used for the second level analysis of the student interviews. The fifth process in the hierarchy, Apply, in identifying that element of reasoning that recognises a student's capacity to execute or implement their learning, was not gleaned from the student interviews in which students discussed the learning they had undertaken throughout the unit of work. This category of *Reasoning* did, however, inform Phase 2 of the study, the classroom observations of the units of work.

4.10.5 Presentation of results

The results of the research are presented by case with teachers and their respective students reported together. It is the relationship between the teachers' conceptualisations and enactment of numeracy in the classroom, and the students' learning experiences that is important, and which enables the research to consider the positioning of numeracy in reform-based learning environments.

The problematic nature of writing up qualitative research has been highlighted by Woods (1986) who suggests that "the point where rich data, careful analysis and lofty ideas meet the iron discipline of writing is one of the greatest problem areas of qualitative research" (p. 188). Another difficulty associated with writing accounts of fieldwork is recognised by Edwards (2001) who states that:

> one of the challenges of telling the tale elicited from the field is to provide an account of what is going on with sufficient coherence to retain a reader's interest but also sensitive to the complexities and multiple perspectives revealed in the study (p. 133).

The focus of the Results chapters is on the beliefs and practices of each teacher and the experiences and learning of her respective students according to the five dimensions of numeracy. The results are richly illustrated in order to enhance the likelihood of illumination of the conceptualisation and learning of numeracy.

Wherever possible, the words of the teachers and students were used because this provided a means to convey their unique perspectives.

4.11 Trustworthiness: Addressing adequacy and limitations of the study

Given that the underlying paradigm of the research is that of social constructivism, it is appropriate that its adequacy and limitations be addressed in terms relevant to this paradigm. Lincoln and Guba (2000) discuss the importance of trustworthiness and authenticity as criteria for judging the outcomes of a constructivist inquiry. They developed the terms *credibility*, *dependability*, and *transferability* (Guba & Lincoln, 1989) and these are examined in order in this section. Two additional criteria, *relevance* and *communicability* (Malara & Zan, 2002), are reported at the end of this section because of their necessity in supporting the value placed upon the link between research and practice in this thesis. Together these five criteria enable a comprehensive evaluation of the research both in the qualitative education research community in general and in the mathematics education research community more specifically. The criteria discussed here overlap with Schoenfeld's (2002) three criteria for conceptualising mathematics education research: trustworthiness, generality (or scope), and importance.

4.11.1 Credibility

Creswell and Miller (2000) note that despite the varying approaches to judging qualitative research, there is a general consensus that researchers need to demonstrate that their studies are credible, a term used to replace the term "internal validity." The extent to which this study overcomes limitations by satisfying seven strategies espoused by Creswell (2003) is examined here. These strategies overlap with the six criteria proposed by Guba and Lincoln (1989) for judging the outcomes of a constructivist inquiry and by Merriam (1998) who also described six strategies to enhance the internal validity of a qualitative case study.

4.11.1.1 Triangulation

Four case studies were conducted across three schools over a twelve month period. This meant that the phenomenon was observed and considered at separate sites and on many different occasions. Interviews with principals, teachers, and students enabled the perspectives of different participants and activities within each case to be considered. Data were also collected in different ways including observation, interview, documents, and photography. This allowed for the verification and comparison of data. The use of these four methodological approaches to the same phenomenon also served to increase validity.

Interviews were taped and transcribed and observations were hand-written, and in all cases reflections were recorded after each observation. These different methods of recording data were particularly helpful when the researcher returned to the data for the intensive phase of analysis and so served to increase the validity of the interpretations. The study was also participative and collaborative, increasing the validity of interpretations and conclusions.

4.11.1.2 Rich, thick description

Thick description was obtained in two ways: first in the keeping of field notes that were comprehensive and supported by reflective notes after exiting the field on each occasion; and second, in presenting the thesis the researcher has sought to provide as much detail as is necessary to convey the findings and to provide the reader with a sense of the setting of each case, thereby reflecting the experiences of the teachers and their students.

4.11.1.3 Clarifying researcher bias

This criterion is achieved in the thesis by establishing a clear theoretical framework within which the research was conducted. The researcher has sought to be open and honest throughout the thesis.

4.11.1.4 Present negative information

"Because real life is composed of different perspectives that do not always coalesce, discussing contrary information adds to the credibility of an account for a reader" (Creswell, 2003, p. 196). Detailed analysis and triangulation of the data enabled a comprehensive presentation of results from multiple perspectives. This also enabled the researcher to discuss the reality of classroom practice and student learning, as they related to the conceptual framework for numeracy that the thesis explores.

4.11.1.5 Prolonged time in the field

Guba and Lincoln (1989) describe the importance of prolonged engagement in terms of building a rapport and getting behind "fronts" that participants may present. Each school was visited prior to the commencement of data collection to conduct the initial teacher interviews, to answer questions, and to establish a relationship with the participants. Considerable time was spent in the classroom of each teacher spread over a number of weeks, ranging from four weeks to twelve weeks. There were many opportunities for informal discussions with both the participating teachers and others in each school, all of which contributed to the researcher's understanding of the context in which the teachers were operating.

After the completion of field observations the researcher maintained occasional site visits until the end of the 2005 school year to complete all methods of data collection, in particular, collection of teacher and student work samples. Telephone and email contact with the five case study teachers was maintained throughout the duration of the study.

4.11.1.6 Peer debriefing

During the period of the research a number of papers and workshops were presented, prompting dialogue with researchers and practitioners. This enabled the researcher to obtain the feedback and opinions of others with "alternative theoretical viewpoints" (Stake, 1995, p. 113). Ongoing discussions with supervisors, research groups, and peers also ensured that the researcher's construction of the thesis was thoughtfully considered, sometimes challenged and ultimately, strengthened.

4.11.1.7 External audit

Creswell (2003) describes a final strategy that is perhaps the least frequently implemented but which is, nevertheless, valuable. It involves a person not familiar with the researcher or the project, as a peer debriefer would be, who is able to evaluate the research either throughout the study or at its conclusion. In this case, as a PhD student, this criteria is address by the submission of the thesis for evaluation and review by external examiners.

4.11.2 Dependability

The term “dependability” is used in place of “reliability” and refers to “the extent to which the research findings can be replicated” (Merriam, 1998, p. 205). In education it is acknowledged that the study of phenomena that are multifaceted, constantly changing, highly contextual, and provided from many perspectives, means that the traditional use of reliability is not relevant in qualitative research in education. In light of this, the strategy of triangulation, in addition to the length of time spent in the field, are deemed to have enhanced the dependability of the study’s results.

Dependability is also supported by the provision of an “audit trail” (Merriam, 1998, p. 207). A clear provision of the means by which the data were collected and analysed, and how decisions were made throughout the research process further serves to authenticate the research.

4.11.3 Transferability

Generalisability or external validity is associated with “the ability to generalise findings from a specific setting and small group to a broad range of settings and people” (Neuman, 2003, p. 187). In this study the term “transferability” (Lincoln & Guba, 1985) has been applied instead of “generalisability” or “external validity.” “In qualitative research, a single case or small non-random sample is selected precisely because the researcher wishes to understand the particular in depth, not to find out what is generally true of the many” (Merriam, 1998, p. 208).

In this study the sample size of five teachers and 112 students across four classrooms may seem small, particularly given that 24 of the students were selected for the Phase 4 student interviews. It is possible, however, for the qualitative researcher to provide enough rich description to enable the reader interested in making some form of transfer or to determine whether transfer may be achievable (Lincoln & Guba, 1985). Comprehensive, diverse, and in-depth data collection methods have enabled rich, thick, detailed descriptions to be provided so that anyone interested in transferability has a solid framework for comparison. Also, the collective case study design provides diversity amongst each case in which the phenomenon of numeracy positioning is investigated. This provides the reader with a greater range of situations upon which to draw when considering the issue of transferability.

4.11.4 Relevance

In this thesis, the term "relevance" is that espoused by Sierpinska (1993): "something is pragmatically relevant in the domain of mathematics if it has some positive impact on the practice of teaching; it is cognitively relevant if it broadens and deepens our understanding of the teaching and learning phenomena" (p. 38). In valuing the importance of gaining an understanding of the phenomena of numeracy positioning from teachers and students themselves, this study seeks not only to contribute to the mathematics education research community but also to have relevance to educators and teachers themselves. The study was designed in a way that was deemed most able to serve these goals through the articulation of both a theoretical and a practical objective (Section 2.5).

4.11.5 Communicability

"To be appreciated and have any feedback, research must be communicated" (Malara & Zan, 2002, p. 567). This thesis is submitted for rigorous review of its clarity, organisation, and synthesis in presenting its findings. In addition the researcher has communicated results of the research during the completion of the PhD to both academic and practitioner audiences. Articles have been published in peer-refereed journals and in teacher journals, and work has been presented at both research conferences and teacher conferences. These publications are referenced throughout the thesis where appropriate.

The researcher also sought to strengthen the criteria of relevance and communicability by meeting the seven criteria for credibility as discussed in Section 4.11.1.

4.12 Chapter summary

This chapter has explicated the design of the collective case study and how this design serves the broader aim of the study: to consider the phenomenon of numeracy positioning in a reform-based curriculum. The chapter has presented:

- the research perspective and framework from which the case study methodology was developed;
- the research design;
- a detailed account of the sampling techniques employed in the selection of participants and the role of the researcher in the study;
- an examination of the methods of observation, interview, documents, and photography in the generation of the data;
- the process through which the data were analysed; and
- a discussion of the issues associated with trustworthiness.

Before reporting the four cases of the thesis in Chapters 6, 7, 8, and 9, the three schools that participated in the study, Snowgum, Stringybark, and Tanglefoot are briefly described. The purpose of the following chapter is to provide insight into each school setting in terms of the implementation of a reform curriculum and the position of numeracy. The conceptual level of the research framework (Table 4.1) being considered in Chapter 5 is that of Setting.

Chapter Five

Background: School Setting

5.1 Introduction

In the previous chapter, the research design and methodology for a collective case study of numeracy positioning was explained as the central phenomenon of this study. The focus of the research is the teachers and the situated activity of the implementation of a unit of work in their classrooms and how this affects the numeracy experiences and outcomes of students. The research was theoretically framed recognising the influence of the settings and context within which the central phenomenon is being experienced (Layder, 1993). Four case studies are reported in this thesis and these occurred in three different school settings within the Tasmanian education system. Chapters 6, 7, 8 and 9 present these case studies.

The four Results chapters are presented from data collected and analysed acknowledging the wider setting in which they are situated. These theoretical assumptions were discussed in Chapter 4 in which the research framework for the study, underpinned by Layder's conceptual research map, was introduced (Table 4.1). The research framework was structured around Layder's four layers of social reality: context, setting, situated activity, and self (Layder, 1993). This chapter is about school setting, particularly the question: *How are schools positioning numeracy as they design, implement, and report on the Essential Learnings?* The question forms background to the presentation of each case study and is discussed from the descriptive analysis of school documents and from brief interviews with the school principals.

In this chapter the three schools: Snowgum, Stringybark, and Tanglefoot are described. A general overview of each school structure is also included. The purpose of the chapter is to provide insight into each school setting in terms of the implementation of a reform curriculum and the position of numeracy. The local curriculum was introduced in Chapter 2 (Section 2.2.4). When school websites and policy documents are quoted to describe the context of a particular case study school, formal referencing has not been used to ensure anonymity of the participants.

5.2 Snowgum Primary School

Snowgum Primary School is an urban government primary school established in the 1930s. At the time of this study Snowgum catered for approximately 310 students of predominantly low socio-economic background. The school had 11 classes of students from Grades Prep to 6 and two Kindergarten classes on a separate campus. These classes were staffed by 17 teachers and, together with the principal, three assistant principals, 14 teacher aides, five support staff, and two administration staff, formed the total school staff.

Due to the low socio-economic status of the school, social issues played a major role in the day-to-day running of the school and impacted upon curriculum and student learning.

> Teachers have to work pretty hard. They deal with a lot of the social issues that other schools would not have to deal with. It is time consuming and takes their mind off Essential Learnings if you like and you have to know that when this movement is on or this change process is on from a Department level that teachers' minds have to be on the curriculum, hence they need lots of support and understanding. (Principal, Snowgum Primary School, 16/11/05)

The research occurred in one Grade 5/6 classroom, with the teacher, Alice, teaching 23 students, 10 Grade 5 students and 13 Grade 6 students. Snowgum Primary School had a high proportion of part-time teachers and Alice spent four days a week with her class. On the other day she undertook a role as the Essential

Learnings coordinator for the whole school, guiding the process of implementation for Snowgum Primary School, with professional learning and support from the D*o*ET. In this case study the researcher was invited into Alice's classroom for seven numeracy sessions during a four week period. The researcher was not a participant in the wider school setting and had minimal opportunity to observe the whole school environment.

A focus for the school, alongside the systemic curriculum reform, was working towards the goal of "trying to become a full service school with a child care centre, a morning program, and after school child care" (Principal, Snowgum Primary School, 16/11/05). This included substantial building work and redevelopment of the school grounds, in the order of $720,000.

5.2.1 Snowgum and curriculum

> We don't have any worries about that [curriculum]... the issue for us is the time frames and the amount of change in that time frame. (Principal, Snowgum Primary School, 16/11/05)

As a state government school, Snowgum's curriculum is constructed in accordance with the D*o*ET's policies and guidelines. Snowgum began the process of planning together using the new Essential Learnings curriculum (D*o*ET, 2002; 2003) mid-way through 2003 and was supported by documentation and professional learning provided by the D*o*ET. Implementation in the classrooms commenced in 2004. As with all state government schools, and in view of the ten year plan for implementation, the four key element outcomes that formed an initial focus for teaching and learning at Snowgum Primary School were Being literate, Being numerate, Maintaining well-being, and Inquiry. Assessment against these key element outcomes did not commence until 2005 when state-wide reporting against the Essential Learnings standards was introduced for the first time.

> I think the curriculum in a primary school there is not a lot of change. What we have done, we have got our teachers together; they are planning together. That is not easy when you are used to doing your own thing. You've got your own directions and you need to compromise, like sharing a classroom. (Principal, Snowgum Primary School, 16/11/05)

As reflected by the principal, perhaps the most significant change for these primary school classroom teachers was the aspect of planning together. Regular collaborative planning times were introduced and supported by the school to enable teachers to plan units of work relevant to their grades and informed by the Essential Learnings.

5.2.2 Numeracy at Snowgum

At Snowgum Primary School, as in all state government schools, a strong emphasis was placed on students' literacy and numeracy outcomes. This was further reflected by the efforts of the D*o*ET to continually improve documentation for the focus key element outcomes: Being literate, Being numerate, Maintaining well-being, and Inquiry. In July 2005 the D*o*ET released updated Being numerate support materials (D*o*ET, 2005a). This thirty page document described in detail the mathematical experiences of a "typical" learner at each of the five standards and across the five strands of mathematics: number, space, pattern, measurement, and data handling, in addition to their capacities to think, act, and communicate mathematically. In September 2005 further support materials were released providing the teaching emphases relevant to each standard (D*o*ET, 2005b).

Snowgum Primary School staff participated in professional learning in the area of numeracy and as a staff they spent time considering assessment of the key element outcome, Being numerate. In October 2005 the researcher was invited to participate in a moderation day attended by all schools in the Hobart cluster, including Snowgum. The purpose of this day was for teachers to share and discuss student work samples. Work samples were specifically evaluated in terms of the Essential Learnings standards for the key element outcomes: Being literate, Being numerate, and Inquiry. Overall teachers found the additional support materials provided by the D*o*ET for Being numerate to be particularly helpful in assessing student work.

At the time of undertaking data collection at Snowgum Primary School, teachers were in the midst of reporting. They were reporting for the first time against the new curriculum standards using a new information technology support system specifically designed for state-wide reporting. At the same time, the Federal government introduced "plain English" reporting requirements nationwide, requiring the use of A, B, C, D and E to grade students. These requirements were mandatory and linked to funding for schools (DEST, 2005).

Although the researcher did not participate in the wider school setting, there was a general feeling of stress concerning both the new Student Assessment and Reporting Information System (SARIS) and the federal government requirements. This was possibly felt more in this case study as it took place in the last term of the school year when reporting was at the fore. The principal also confirmed this stress but noted positively that it was necessary to experience this time of frustration during the early stages of reform in order to move forward.

During 2005, The Australian Education Union (AEU) undertook a survey of Tasmanian teachers asking them whether they felt ready to report and assess students under the Essential Learnings. The first ballot, in May 2005, returned an overwhelming 'No' from teachers across the state with respect to their feelings of readiness to report (Principal, 16/11/05). Snowgum Primary School as a school reflected this overall negative response, with teachers not confident to report against the four introduced key element outcomes.

> We voted against reporting on Essential Learnings, not because we haven't put lots and lots of time into it, because we put a day a week of a teacher for three years ... but I have got a lot of part-time teachers, seven of my classes of the eleven are shared ... teachers get themselves very stressed when they are put under pressure. So when we voted not to go along those lines, they automatically said we would do 'Maintaining well-being' because we had done a good job of that. So a third of it will be done and the other two will be done, numeracy and literacy, based upon the old model ... [next year] we will do a full report on everything. (Principal, Snowgum Primary School, 16/11/05)

5.3 Stringybark Primary School

Stringybark Primary School is a government primary school located in a semi-rural setting, 30 km from the central business district of the state's capital. The school is over 50 years old and at the time of the study, catered for approximately 670 children from Kindergarten to Grade 6. The school aimed "to provide high-quality learning opportunities for all students so they can maximise their potential in supportive environments which will enable them to become effective members of the broader community" (Stringybark website). The school staff was comprised of two co-principals, two assistant principals, 21 classroom teachers, a physical education specialist teacher and a music specialist teacher, 13 support and administration staff, and 10 teacher aides.

This research involved two Grade 6 classes, incorporating 55 of the total of 68 Grade 6 students. The other 13 Grade 6 students were in a combined 5/6 class, not participating in the study. The Grade 6 classes were situated in different buildings on the school site, and staffed by the two case study teachers, Ophelia and Samantha. These two teachers participated in monthly collaborative planning meetings together with the other Grade 5/6 teacher. The researcher was invited to participate in these meetings during the twelve week research period. The researcher was also invited to whole school staff meetings during this time.

5.3.1 Stringybark and curriculum

> We believe that the child is central to all that we do. Our primary goal therefore, is to continually improve our teaching and learning processes to maximise the educational opportunities for the children in our care. The use of the Essential Learning Framework (ELF) is pivotal in achieving our aims. (Stringybark website)

Like Snowgum, Stringybark's school curriculum was based upon the D*o*ET's policies and guidelines (D*o*ET, 2002; 2003). The school was one of ten in the south of the state involved in the three year curriculum consultation process that took place from 2001 to 2004. This involved a small group of schools undertaking extensive professional learning and providing consultative feedback during the initial construction of the Essential Learnings, before it was implemented state-wide in 2005. Stringybark had, therefore, been planning and working together as a staff with the Essential Learnings since 2002. Teachers had begun to consider their teaching practice in light of the reforms and, in particular, in terms of teaching for understanding (Blythe, 1998), as this was one of the key philosophies underpinning the re-construction of curriculum. As well as whole staff development in this area, teachers met once every month, in grade specific collaborative planning teams, to develop units of work that would then be the basis for their own classroom teaching.

The researcher was welcomed into the school for a full term and invited to participate in the school community. Strong leadership was evidenced at Stringybark, together with a positive and vibrant environment where, in general, staff were optimistic towards the curriculum reform.

5.3.2 Numeracy at Stringybark

The school saw numeracy as important for all students, alongside literacy and social skills. This valuing of numeracy was evidenced in general staff discussions, sharing of information with regards to numeracy, time dedicated to professional learning, and the allocation of one of the case study teachers, Samantha, to the role of numeracy coordinator for the school. This role incorporated time off class during which Samantha was able to undertake professional learning with a specific numeracy focus, to support other staff in developing the numeracy of their students, and to keep abreast of current trends and research in relation to numeracy and the curriculum. Perhaps the positive view taken by the principals and staff in welcoming the researcher into the school and classrooms for a full twelve week term was another indication of the school's support for numeracy.

During the case study period, Stringybark Primary School participated in National Numeracy Week, a federal government initiative in which the importance of numeracy in education is recognised and celebrated across the country. At Stringybark, numeracy week was a time where the whole school celebrated the work of both teachers and students in the area of numeracy. Displays of students' work were thoughtfully constructed in the classrooms and school foyer for the wider school community, including parents, to enjoy.

With 2005 being the first year for reporting to parents, against a maximum of four of the Essential Learnings key element outcomes, there was a major focus on assessment at Stringybark Primary School during the case study period. Teachers had begun considering their assessment practices within the new curriculum framework in 2003 although reporting was not introduced until 2005. They were using assessment to guide planning, teaching and learning, and to inform reporting.

As with Snowgum Primary School, teachers at Stringybark returned an overwhelming "no" vote when the AEU ballot was held in May 2005, asking them whether they felt "ready to report." For Stringybark, however, this result changed significantly when the ballot was re-held in September. A majority of staff then felt confident in their abilities to assess and report against the four Essential Learnings key element outcomes: Being literate, Being numerate, Maintaining well-being, and Inquiry.

The researcher attended staff meetings at Stringybark throughout the research period and four of these meetings were dedicated to issues of assessment. Staff shared practices and ideas for assessing student work and for keeping assessment records. They discussed the D*o*ET supporting documentation and D*o*ET staff came to the school to introduce these documents (D*o*ET, 2005a; D*o*ET, 2005b). Teachers brought student work samples to staff meetings to share and consider in relation to the Essential Learnings standards. Towards the end of term, the staff also spent time learning the D*o*ET's new Student Assessment and Reporting Information System (SARIS) and developing a new Stringybark Primary School student report format.

5.4 Tanglefoot School

> The unique environment that is [Tanglefoot] seeks to challenge each student to develop her potential, to be socially aware, constructively critical and questioning. ... The school is committed to assisting all students develop to their fullest [sic] extent in order that they can confidently fulfil an effective role in the wider society they will occupy. (Tanglefoot website)

Tanglefoot School is an independent girls' school situated within one of the city's surrounding residential suburbs. It is a member of The Association for Independent Schools of Tasmania (AIST) that is comprised of 12,000 students across 33 non-government schools. This accounted for 14.6% of total school enrolments in Tasmania (AIST, 2004). Tanglefoot was established in 1935 and at the time of the study had a total enrolment of 365 students from Kindergarten to Year 12. The school was staffed by a principal, deputy principal/director of studies, middle school and junior school co-ordinators, a further 34 teaching staff, and 26 administrative and support staff.

The setting for this study was Tanglefoot's middle school, which consisted of two Grade 7 classes with a total of 39 students and two Grade 8 classes with a total of 34 girls. The two Grade 8 teachers, Ange and Jen, together with the 34 Grade 8 students were the participants in this study. The middle school was established in 2003 in response to research about meeting the particular needs of students moving from the primary area to the high school and encompassed students ranging from 12 to 14 years of age.

> You are a person who is living in a world that is changing fast. You need to develop many skills to live, learn, and work efficiently, flexibly and happily in it...the Middle School is simply timetabling the school day so that those things that you want can happen. It is also a way to ensure that you get the chance to understand and come to grips with the things that you are concerned about. And, it is also a way of organising each day so that the skills that the professionals (the teachers) think you need in a fast changing world can be learned. (Tanglefoot Middle School Handbook, 2005)

For these reasons each middle school class had a key teacher responsible for teaching one or two of the core subjects of English, Mathematics, Studies of Society and Environment, and Science and being responsible for the pastoral care of the girls. In addition, specialist teachers for Art, Drama, Health and Physical Education, Information Technology, French, Japanese, and Music were involved in the middle school to provide for the overall educational needs of the students and to provide a bridge to senior school.

> You have one teacher who is a tutor, who is Jen, and goes across English and SOSE and then you've got Ange who is a tutor and goes through Maths and Science and because they work together down here and often do their planning together it really works and they can make their work interdisciplinary. (Middle School Coordinator, Tanglefoot School, 21/08/05)

5.4.1 Tanglefoot and curriculum

Although part of the wider Tasmanian educational community, as an independent school, Tanglefoot did not fall directly under control of the state government education system. Tanglefoot School considered the Essential Learnings (D*o*ET, 2002) and its values and purposes, and as a school community constructed a curriculum in the middle school that incorporated the Essential Learnings key element outcomes, but in a manner unique to Tanglefoot.

> In a sense we have taken it all on board. It is just that our presentation of it is different as you have seen from the assessment booklet. There has been a lot of work to put it in our own words… Of course there will be changes down the track and we don't know what we are going to come up with anyway, what is going to be imposed upon us. At the moment all our aims are focused in on the Essential Learnings, on the outcomes of the Essential Learnings and we find that that suits the interdisciplinary way of teaching really, really well. (Middle School Coordinator, Tanglefoot School, 21/08/05)

The Essential Learnings was included in Tanglefoot's Middle School Handbook together with references to Bloom's taxonomy (1956), Blythe's teaching for understanding (1998), and Gardner's multiple intelligences (1983). These were presented as the underpinning theories that informed a curriculum incorporating three main aspects: traditional subject disciplines, interdisciplinary units of work, and six week mini-courses providing options in academic, life skills, and recreational areas of student interest.

> We are embedding the Essential Learnings in our curriculum… in our own way… In our own way refers to the way that the middle school is incorporating the values and outcomes of the Essential Learnings within the framework of the traditional disciplines" (Tanglefoot Assessment Booklet, 2005).

Whereas the D*o*ET was introducing the key element outcomes of the Essential Learnings in a staggered manner over five years, Tanglefoot's middle school incorporated all of the 18 key element outcomes throughout a subject, discipline-based framework and supported this with interdisciplinary, inquiry-based learning. The case study at Tanglefoot was situated at Grade 8 level, where students were taught by numerous teachers who at times worked together as a team to support this inquiry learning.

5.4.2 Numeracy at Tanglefoot

At Tanglefoot School, the discipline of mathematics was a core subject for students in the middle school. It involved the explicit teaching of the five strands of mathematics: number, space, measurement, chance and data, and algebra, in addition to thinking, acting, and communicating mathematically. Each Grade 8 class had four 50-minute mathematics lessons timetabled each week. Students' numeracy capabilities were encouraged through the discipline of mathematics. At times, students were also required to draw upon the knowledge and skills they developed in the Mathematics classes for use in other subject areas and in their interdisciplinary units of work.

The introduction of a dedicated middle school at Tanglefoot, coinciding with the move to Essential Learnings in the wider state education system, prompted the school to take a fresh look at assessment and reporting practices. As a result, a Middle School Assessment Booklet was developed for students, parents, and teachers as a clear and comprehensive document of assessment criteria. Twice yearly, samples of student work were presented in a portfolio together with a report. Although the values and outcomes of the five Essential Learnings were viewed by teachers as intrinsic to every subject discipline, they were situated and reported within specific areas of the middle school curriculum as described in Table 5.1.

Table 5.1

Tanglefoot's situating of the Essential Learnings for the purposes of reporting (excerpts from Tanglefoot Assessment Booklet, 2005)

Essential Learnings	Key Element Outcomes	Where assessment is focused for this key element outcome
THINKING	Inquiry	assessed by teachers collaboratively through the Inter-Disciplinary Negotiated Curriculum Report
	Reflective Thinking	assessed within the disciplines of English, Drama and Art and through the Inter-Disciplinary Negotiated Curriculum Report
COMMUNICATING	Being Literate	assessed by teachers collaboratively in the report called Learning to Learn. Being Literate is also assessed within the disciplines of English, French and Japanese
	Being Numerate	assessed through Mathematics
	Being Information Literate	assessed by teachers collaboratively through the Inter-Disciplinary Negotiated Curriculum Report
	Being Arts Literate	assessed within the disciplines of Drama, Art and Music
PERSONAL FUTURES	Building and Maintaining identity and Relationships	assessed within SOSE and HPE
	Maintaining Wellbeing	assessed by teachers collaboratively through Learning to Learn and within HPE
	Being Ethical	assessed by teachers collaboratively through Learning to Learn
	Creating and Pursuing Goals	assessed by teachers collaboratively through Learning to Learn and through the Inter-Disciplinary Negotiated Curriculum
SOCIAL RESPONSIBILITY	Building Social Capital	assessed by teachers collaboratively through Learning to Learn
	Valuing Diversity	assessed within SOSE, French and Japanese.
	Acting Democratically	assessed by teachers collaboratively through the Inter-Disciplinary Negotiated Curriculum Report
	Understanding the Past and Creating Preferred Futures	assessed within SOSE
WORLD FUTURES	Investigating the Natural and Constructed World	assessed within Science
	Understanding Systems	assessed within SOSE
	Designing and Evaluating Technological Systems	assessed by teachers collaboratively through the Inter-Disciplinary Negotiated Curriculum Report
	Creating Sustainable Futures	Assessed within Science and SOSE

The Tanglefoot School case study is distinctive from the other case studies in that:

- Tanglefoot constructed curriculum around the Essential Learnings in a unique manner, not as directed by the state government education department;
- Students at Tanglefoot were taught by numerous teachers. The other case studies took place in primary school settings where students had one classroom teacher;
- Tanglefoot described their curriculum and commenced assessment in a discipline-based framework supported by interdisciplinary inquiry with all 18 of the Essential Learnings key element outcomes.

5.5 Chapter summary

This chapter has presented the distinctive characteristics of each of the three schools that provided the setting for the case study research: Snowgum Primary School, Stringybark Primary School, and Tanglefoot School. All three schools had embraced the move to the reform curriculum, but at a pace and manner that suited each school context. In the two government schools this saw a focus on the planning, teaching, and assessment of four of the 18 key element outcomes of the Essential Learnings, whereas Tanglefoot School had moved to incorporate all 18 key elements into their middle school curriculum and assessment framework.

In all three schools the teaching and learning of numeracy and its relationship to other curriculum areas was at the fore of much of the schools' planning. At Snowgum and Stringyback there was an emphasis on professional learning for all teachers based upon the D*o*ET guidelines and supporting materials. At Tanglefoot School the development of students' numeracy was encouraged both within the discipline of mathematics and more particularly by providing students with opportunities to engage with interdisciplinary units of work.

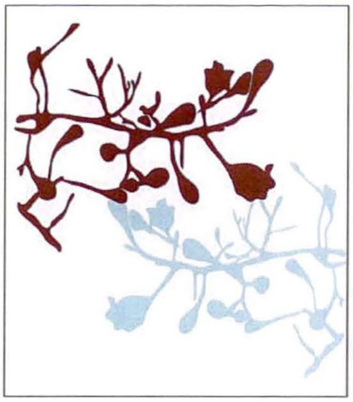
Given that this study occurred during the first year of mandated reporting against new curriculum standards for Being numerate in the government sector, for Snowgum and Stringybark Primary Schools, issues around the assessment and reporting of numeracy were considerable.

The thesis now moves to a presentation and discussion of the results, with each case considered separately across Chapters 6, 7, 8 and 9. The presentation of the results brings together the teachers and the students and considers all the data types collected.

As described in Chapter 4 the primary aim of the collective case study is to contribute to an understanding and develop theory with regard to the positioning of numeracy in a values-focused curriculum. The research questions which focus the case study flowed both from this original aim, a review of the literature, and from the research framework outlined in Chapter 4 (Table 4.1). The conceptual levels being considered in Chapters 6 to 9 are that of situated activity (teachers and students in relation to the unit of work) and self (teacher). Accordingly, the results address the first two research questions posed for the study:

4. How are teachers positioning numeracy in reform-based learning environments according to five dimensions of practice?
5. How are students experiencing numeracy in reform-based learning environments according to five dimensions of practice?

The first of the four Results chapters, Chapter 6, presents the findings of the research for the teacher Alice, of Snowgum Primary School, and for her students.



Chapter Six

Results:
Snowgum Primary School – Alice

Now my work is based on observation about students' understanding ... I find out what the kids know and I find out what they don't know and that's what I work on. ... Now they play around with numbers and they talk about numbers and the conversations in my classroom are really exciting. I give them lots of opportunities for checking reasonableness of answers... I'm now teaching explicitly the language, language of mathematics and of thinking. I was doing that in Thinking but I am much more conscious of how that connects now and my planning reflects key concepts and ideas, not just "Well today we'd better do some addition stuff" and thinking about what are the big ideas that I want these kids to know. (Alice, 28/4/05)

6.1 Introduction

The setting for this case study, Snowgum Primary School, is described in Section 5.2. As a state government school, Snowgum Primary School constructed its curriculum in accordance with the D*o*ET's policies and guidelines. The school had commenced implementation of the new Essential Learnings framework (D*o*ET, 2002; D*o*ET, 2003) in 2003 with planning, teaching, and assessment initially focused on the key element outcomes Being literate, Being numerate, Maintaining well-being, and Inquiry. Within this context, the Grade 5/6 teacher, Alice, implemented a three-week numeracy unit on graphing. This learning took place during three of the four weekly numeracy sessions that were part of Alice's usual teaching program.

Alice had been teaching for thirty years and was one of three Grade 5/6 teachers at Snowgum Primary School. Alice had spent all of her teaching life in Tasmanian schools predominantly working in upper primary grades. She had been at Snowgum Primary School for seven years and had assumed coordination roles in 2003 for both the Grade 5/6 team and also for the Essential Learnings. This involved Alice working off class for one day a week to coordinate and guide professional learning for her colleagues and to initiate and facilitate planning teams throughout the school. Alice had been using the underpinning ideas of the Essential Learnings for two years to inform her teaching.

Data analysed and reported in this chapter are two teacher interviews, field observations taken during seven classroom visits with Alice and her 23 Grade 5/6 students, and six individual student interviews, together with documents and photographs collected across the four phases of the research. It is noted that the case of Alice with respect to her beliefs about numeracy according to the five dimensions of practice conceptualised in Chapter 3 has been reported previously (Skalicky, 2008).

6.2 Curriculum

This section presents Alice's beliefs about curriculum and summarises the unit of work that Alice was teaching during this study.

6.2.1 Alice constructing curriculum

Alice had found the shift to transdisciplinary curriculum to be a natural one, particularly in relation to supporting students in their understanding of concepts and incorporating strategies that supported students with their thinking. Alice's expressed views of curriculum accentuated a social view of knowledge construction, together with an emphasis on the development of higher-order capacities.

The classroom learning environment was important to Alice, as she worked hard to establish a "community of learners" in which teacher and students could "learn … share …laugh …and …cry together" (Alice, 28/4/05). The opportunity to work continually on this aspect of her teaching was a feature of Alice's practice. She viewed the challenge of creating a supportive learning environment for her students as the foundation upon which learning occurred. Alice provided opportunities for her students to work in groups, to share their work with each other, and to take risks. She spoke of wanting "to find ways to discuss and celebrate our work, because we don't do enough of that," highlighting her underlying belief that meaningful social engagement was crucial for learning.

Alice viewed herself as a learner and appreciated the shift to staff working together in collaborative planning teams, an important element of the implementation process of the curriculum reform. Alice described the processes of planning, teaching, assessment, and reflection as aspects of her teaching that she was now more conscious of undertaking because of the sharing among staff.

Two underpinning notions of the Essential Learnings curriculum reforms, conceptual understanding and higher-order thinking, were informing Alice's teaching.

> I want to plan using the Essential Learnings framework, but I want to really think about what it is that I want these kids to come out understanding and plan for that specifically, and if those strategies that I am using don't do that then they are not good strategies and they have to go. They might be fun and they might be great and I've used them for years but if they're not developing the understandings that I am after then I've had to let them go. (Alice, 28/4/05)

She was confident in her ability to plan for understanding and often spoke of "putting that new lens over" when describing how she developed a unit of work, or "learning sequence," in which the understanding of key concepts guided planning, teaching, and assessment. For Alice, these learning sequences were "no longer an add on" in her classroom but, rather, formed the focus, and became "what we do, not what we do as well as" (Alice, 28/4/05). Discipline knowledge and important skills learning were situated as supporting the learning of important ideas and understanding of concepts. Alice gave an example of spending time with her students on the information literacy skills of collecting, collating, and summarising information to ensure that students were equipped to undertake more open-ended inquiry tasks that formed part of the learning sequences she planned.

For Alice, a capacity for deep understanding was closely connected with the development of students' abilities to "think." Alice felt that explicitly teaching thinking strategies and related language was important for students as it enabled them to approach tasks in a purposeful and meaningful way. Alice's own professional learning in the area of Thinking was contributing to her expressed desire and ability to embed Thinking in all areas of her teaching.

Within this context, it was in the area of numeracy that Alice experienced the greatest challenge. To position mathematics learning within the changed curriculum construction that was occurring in Alice's classroom, in which key understandings or concepts were developed across a range of subjects, was the area that Alice found most difficult.

> I was doing some great things out there with learning sequences and working with teachers on Thinking and building that into my practice. I was way up there, but my mathematics was still floating out here somewhere. [*Alice physically demonstrates with her hand outstretched representing the mathematics removed from her other work.*] I couldn't get it to fit. I couldn't drag it in because I just didn't know what to do. ... I was teaching the stuff, or some stuff, but I wasn't teaching them for understanding ... I wasn't making the connections. (Alice, 28/4/05)

Alice considered herself to be a naturally reflective person and appreciated the reflection that had been brought to bear on her teaching as a result of the curriculum reforms being introduced. Alice herself was aware of the tensions that existed between her strong beliefs about teaching and learning and the actual enactment of curriculum in the numeracy classroom.

6.2.2 The unit of work: Exploring graphs

During the case study, Alice taught a focused numeracy unit on graphing with three clearly identified understanding goals:

- Students will understand that there are a range of ways that information can be presented.
- Students will understand that some forms are more useful as a means of organising information.
- Students will understand that we use graphs, charts and tables to represent numerical aspects of the world.

(Alice's documentation, 11/10/05)

Alice identified the need for this learning following a unit of work, "What makes Australia special?" planned and taught by a pre-service teacher using the Teaching for Understanding Framework (Blythe, 1998). The unit was designed to provide students with the opportunity to learn about the unique features of the Australian continent. Alice "quickly discovered that [the pre-service teacher] was asking [the students] to record statistical information and ... they were just downloading information from the net, and when [Alice] questioned them more closely about what the information was telling them, or why they were including it in what they were doing, they really didn't have any understanding ..." (Alice, 8/12/05).

Alice became concerned about the students' difficulties in interpreting the information they were incorporating into their work. She was aware that they had had many past experiences making graphs, particularly bar graphs, "[but] they had never really done a lot of work on what it is telling [them] and how [they] can best use the information" (Alice, 8/12/05). Alice wanted her students to gain the understanding that there are a variety of ways that information can be represented and to have the knowledge and skills to make and implement those choices from an informed perspective.

6.3 Numeracy

This section presents the results for this case within three main sections: Alice's beliefs about numeracy, the enactment of numeracy in the classroom with Alice and her students, and the student learning outcomes of six individual students.

In 2004, Alice had participated in a DoET targeted professional learning program for teachers of middle years' students, Grades 5 – 8. The program, *Being numerate in the middle years*, occurred over six days and brought together 48 teachers from across Tasmania. It explored the role of numeracy, planning and structuring numeracy learning, thinking and working mathematically, mental computation, and the important middle years' concepts of proportional reasoning; average; and fractions, decimals, and percentages (Watson, Beswick, Caney, & Skalicky, 2005). Alice experienced a major shift in her beliefs and practices concerning numeracy teaching and learning as a result of participating in this program. Following this, Alice worked toward changing her classroom environment and her numeracy teaching.

Before her participation in the *Being numerate in the middle years* professional learning program, Alice described her previous numeracy practice as dominated by four aspects: preparing her students for high school; teaching algorithms; teaching mathematics concepts such as percentages, decimals, and fractions separately so as not to confuse them; and giving regular automatic response tests to practise number facts.

Alice's initial interview took place in April, 2005, approximately six months after her involvement in the program. Classroom observation of her teaching practice occurred in October and November of 2005. Alice's beliefs and practices concerning numeracy and its place in the curriculum were explored through the teacher interviews, documentation, and observational data.

6.3.1 Alice and her beliefs about numeracy

Alice was experiencing a significant shift in her view of numeracy. Alongside this, her overall teaching pedagogy had changed to one aligned with the philosophy that informed the Essential Learnings, promoting transdisciplinary learning constructed around developing student understanding of key concepts and ideas.

Alice was working on her numeracy pedagogy and bringing those aspects of her teaching that came naturally to her in other curriculum areas, particularly literacy, into her numeracy teaching. Since the introduction of the Essential Learnings, Alice was aware of and described some significant changes that were occurring in her practice. She was equipping her students with the language of both mathematics and thinking to support their participation in the learning environment. Alice was encouraging students to use and share their own strategies for problem solving and to develop understanding of mathematical concepts. She was building her teaching based upon "observation of students' understanding." Alice had begun to challenge her own ideas of mathematics teaching and was working with students to help them see the connections among concepts (for example, fractions, decimals, and percentages) rather than teaching them separately. Although Alice still found numeracy to be an area that did not naturally sit in her integrated learning sequences, she had made a significant shift in her focused numeracy time.

Alice was exploring ways that she could naturally connect mathematics with her integrated learning sequences but found this challenging. She was, therefore, focusing on a dedicated numeracy block and the learning experiences she was providing for her students during this time.

> I was teaching the stuff, or some stuff, but I wasn't teaching them for understanding ... my planning [now] reflects key concepts and ideas, not just "well today we'd better do some addition" but thinking about what are the big ideas that I want these kids to know. (Alice, 28/4/05)

The following subsections detail how Alice's conversation about her teaching could be described according to the five dimensions of numeracy developed in Chapter 3.

6.3.1.1 Mathematics

Alice expressed a sense that numeracy was more than the mathematical skills upon which she had previously focused. By sharing the challenges she faced in moving towards authentic numeracy pedagogy, Alice revealed some uncertainties in her own mind about the distinctions between mathematics and numeracy, and also whether such distinctions were important. At the same time, Alice described numeracy as "that whole notion of using mathematics and transference of that kind of knowledge." When Alice was describing the structure of her numeracy time she referred to focused time at the beginning of each numeracy block for the development of number sense. She also discussed the inclusion of "explicit teaching time" in which the strands of mathematics (including number, measurement, chance and data, pattern and algebra, and space) were explicitly taught, with her overall objective being to plan "for understanding of key mathematical concepts and ideas."

In sharing her discontent with her perception that her mathematics teaching was disconnected from her other work, Alice evidenced a belief that mathematics has a role to play in understanding ideas and concepts planned for in integrated units in other areas of the curriculum. She wanted to do this in a "positive way [that was] not a contrived way."

Alice sought to equip her students with the "language of mathematics" so that they could share effectively their strategies and solutions. The emphasis she placed on student understanding of important mathematics concepts was evident in Alice's conversations. For example, she discussed the concept of "doubling," and wanting to support her students in making connections among mathematics concepts, by considering the meaning and application of doubling in terms not only of number, but also in considering measurement and pattern relationships. For Alice, mathematical skills remained important: "I see these skills that you have to teach – data and how do we read and how do we collect and all those sorts of things… so many skills that we have to logically work through."

6.3.1.2 Reasoning

The development of mathematical thinking and reasoning played a very important role in Alice's classroom. She was very interested in seeing how the language of thinking applied to mathematics, with students often asked to "justify" their position or choice of a particular strategy or to "elaborate" on their thinking. Students in Alice's classroom were given the freedom to select and apply problem solving strategies. This had driven the shift in the culture of the classroom with students "beginning to see themselves as problem-solvers."

Alice had participated and led many professional learning sessions in the field of Thinking, as a result of the curriculum reforms. She was very interested in the application of this within the numeracy classroom. Alice worked with her students to use tools to support their thinking within mathematical settings. For example, cooperative learning strategies, such as jigsaw techniques (Aronson & Patnoe, 1997) and think-pair-share activities (Kagan, 1989) were specifically taught to provide students with tools to share and describe their own thinking. Time was allocated at the end of every numeracy session for the whole class to share, and to reflect upon and articulate their learning. Alice was clear about the outcomes that this shift in her teaching practice brought to student learning. "They are actually making connections and seeing a range of possibilities … they are making those connections for themselves. ... They are moving away from that memorisation … that rote learning being their only strategy."

6.3.1.3 Attitude

In sharing the changes in her numeracy pedagogy, Alice identified a distinct connection between these changes and the changing attitude of her students toward their mathematics learning. Alice had transformed not only the structure of her numeracy time and the classroom learning environment but also found that her own attitude to mathematics teaching had undergone a shift in which numeracy was highly valued and seen as an important component of the curriculum. She was no longer accepting disruptions during this time: "I had to change what was going on and let the kids see that this is really important to me and make it really important."

Alice wanted to build a "community of learners" in which students shared and reflected upon their learning and she described these times as "exciting" for both her and her students. Her efforts to "create a learning environment … so that the kids will take risks" were resulting in the students "taking chances in the classroom." Alice believed that there was "real potential for these kids to be self-motivated, confident, articulate users of mathematics."

> Students are now free to manipulate numbers. They were frightened my kids, of numbers because I was going to ask them something really hard and they never put their hands up. Now they play around with numbers and they talk about numbers and the conversations in my classroom are really exciting.

The increasing development of a positive attitude toward mathematics, for Alice and for her students, was very entrenched in the changed culture of the classroom and the embedding of thinking within mathematical learning – an impact of the curriculum context.

6.3.1.4 Context

In describing numeracy, Alice felt that "everything [had] to be in context to be meaningful." She was "trying to make [mathematics] meaningful and connected with the real-world." Her initial focus was on the transfer of number calculations to real-world situations. Alice also gave an example of using cooking to further develop students' understanding of the concept of doubling, and discussed the relevance of doubling in terms of measurement and the use of recipes.

She still felt that "things [were] a bit contrived" and, although she valued the transference of students' mathematical knowledge to new contexts, she expressed a desire to work on the implementation of this area in meaningful ways. Alice described her biggest challenge as incorporating numeracy into her planning of transdisciplinary units of work informed by the Essential Learnings: "dragging mathematics out there into my learning sequences in a more positive way that is not a contrived way."

6.3.1.5 Equity

Alice was focusing on the development of understanding of mathematical concepts and on building a classroom community that would enable students to begin to explore mathematical strategies. The students were, at times, sharing their mathematical strategies, and questioning each other about these strategies. Personal social engagement was being encouraged through Alice's provision of opportunities for students to collaborate on tasks, and to listen and share ideas with each other.

Alice was a highly self-reflective teacher who was aware of the importance of numeracy for all of her students. She was beginning to support in her students the dispositions that if developed further would have the potential to enable them to go on to consider mathematics and its implications in social, economic, and political contexts.

6.3.2 Numeracy as enacted in the graphing unit of work

Between morning tea and lunch, Alice's students spent a focused one and a half hours on their numeracy learning. During this time, every day, students engaged with a variety of concepts, with resources, with each other, and with their teacher, to develop their numeracy capabilities. During the seven classroom visits undertaken in this case, detailed observations were recorded and later analysed, enabling a picture to be created of Alice and her students' numeracy classroom according to the five dimensions of numeracy as proposed in this thesis.

6.3.2.1 Mathematics

Although the overriding aim of the unit of work was the development of student understanding of graphing, Alice continued to incorporate time during each lesson for building students' capacities and confidence with number. Mental computation formed a focus at the beginning of each numeracy class, to develop this area further, and to address gaps in students' number understanding. This was an area Alice had begun to develop after her participation in the professional learning program, *Being Numerate in the Middle Years*, as she felt it was foundational for students' mathematics learning. Alice did this using number boards and games such as "Today's number is ...," "Follow Me," and "What's my number?" (McIntosh, deNardi, & Swan, 1994). During the data collection period Alice focused on place value, operations, decimals, and doubling. Number boards were provided to students as jigsaw puzzles for them to complete (Figure 6.1). As students worked together to create the puzzle they used counting, pattern, and place value understanding to construct the completed number board.

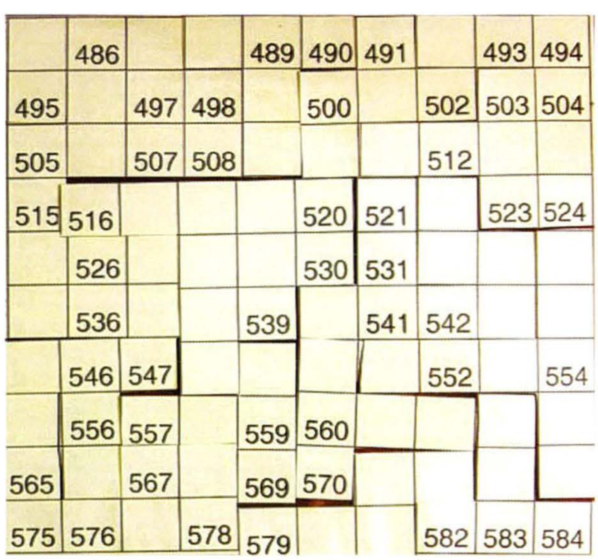


Figure 6.1. Completed number chart puzzle.

The graphing work commenced with Alice finding out what the students knew about different types of graphs. Typical responses included:

> Pie graphs are round graphs cut up like pies. Bar graphs are graphs with bars.
> Some graphs are more difficult to read than others.
> I know how to understand and make bar graphs.
> A graph shows the number of things you are presenting.
> Pie graphs show how big things are next to each other.
> (Classroom observations, supporting documentation, 11/10/05)

Student responses demonstrated that their prior knowledge and understanding were related to reading and drawing bar graphs and pie charts. Due to students' limited understanding of graphing, in particular the purpose of different types of graphs, Alice chose to focus on line graphs and the characteristics of different types of graphs, specifically that "the same information [can be represented] in many different ways [and] some ways may be more appropriate than others" (D*o*ET, 1993, p.10). Alice also reflected that in looking at her students' mind maps about graphs they knew "a lot about bar graphs and pie charts but had gaps in other types of graphs," so she looked for graphs that could extend their knowledge when designing tasks for each lesson. (Classroom observations, 14/10/05). Alice's planning for the focused unit (Figure 6.2) was developed from this early work on gathering students' current understandings of graphing.

'Host' Essential Learning: Communication and Being Numerate

Understanding Goals	Performance of Understanding	Ongoing Assessment and Feedback
• Students will understand that - there are a range of ways that info can be presented. (graphs and charts) - some forms are more useful as a means of organising our info. - we use graphs, charts & tables to represent numerical aspects of the world We can represent the same information in many different ways Some ways may be more appropriate than others (Act BK1 C&D P10)	• Mindmap - 'Graphs' KWL - graphs - links made • Pie Graph - 'What can you determine from this pre-graph' • Venn diagram - comparison of Line & Bar Graphs • A visit to Sizzlers - literacy - tell the story of the graph → own graph/story → • Interpretation of 2 shaded bar graph 'What does this represent?) • Data given (tallest buildings in Aust) - design a graph based on one aspect of the info. →	• Review Mindmaps at end of L.S. - add new understandings. • What have you learned?

Figure 6.2. Alice's planning for the graphing unit of work.

Establishing the language of graphs, and the knowledge required to name graphs and the components of graphs occurred early on in the unit. As the unit progressed, Alice provided students with opportunities to interpret graphs, make their own graphs, and to compare and contrast different types of graphs. This was done through a series of different graphing tasks, with each task building on the previous one, in terms of developing student understanding and moving students from reading and describing graphs, to being able to analyse, interpret and create graphs.

A task completed by students early in the unit of work required them to write a story that explained a graph of "someone's visit to Sizzlers Restaurant with the number of portions on their plate at a certain time of day" (Classroom observation, 24/10/05) and then to graph and interpret their own visit to Sizzlers. Alice explicitly emphasised to her students that their "first job was to interpret the graph" (Classroom observation, 24/10/05). Figure 6.3 provides an example of a student interpretation of the graph provided (incomplete). Figure 6.4 presents one student's own graph created and then interpreted.

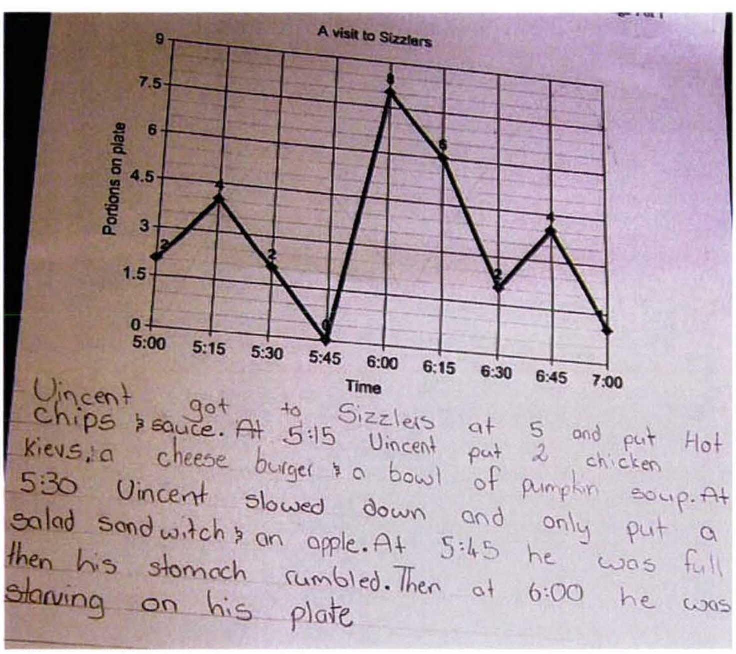


Figure 6.3. Student story to explain the graph presented.

MY MEAL AT SIZZLERS

I entered Sizzlers at 5:00pm and sat down with my family and talked for a while. Then I went and got myself some potato salad, fried rice, a type of fish called Blue Grenadier, carrots, broccoli and some pork at 5:15. I returned to the table and ate slowly until I'd cleared my plate at 5:30. I then went back and refilled my plate which I ate quickly because there was only a small amount of each food. I finished my plate a bit after 5:45. Then I filled my plate to the brim with fried rice which I finished at 6:15. I then went and served my self some ice-cream and jelly witch I ate slowly until 6:30. Then I went and got four slices of different cake which I finished a bit after 6:45. Then I just chattered with my family until leaving at 7:00.

Figure 6.4. Student's own graph (in red) and his story explaining the graph.

Figures 6.5 and 6.6 are examples of the variety of graphs produced by students later in the unit of work, when given a table of data about the construction of the 30 tallest buildings in Australia that included their name, location, height, number of storeys, and date of completion. The student completing the graph in Figure 6.5 chose to compare the buildings by their names using a bar graph, whereas the student completing the graph in Figure 6.6 chose to explore whether or not a relationship existed between the heights of the buildings and their year of construction, using a scatter plot.

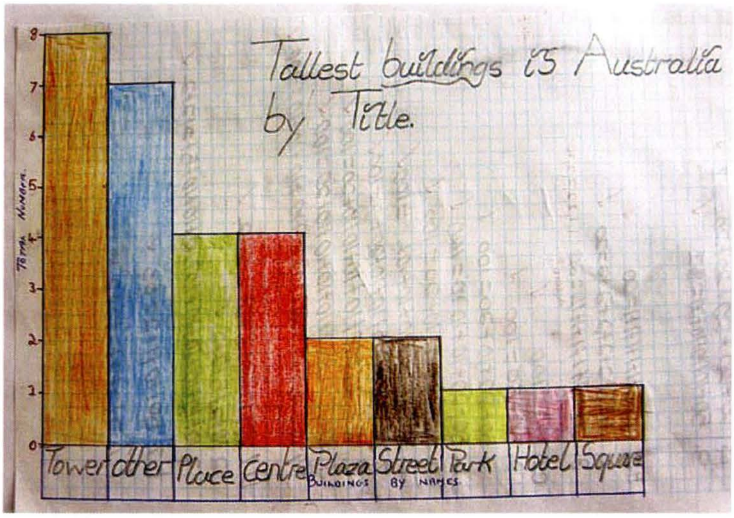


Figure 6.5. Student bar graph comparing the tallest buildings in Australia by their name.

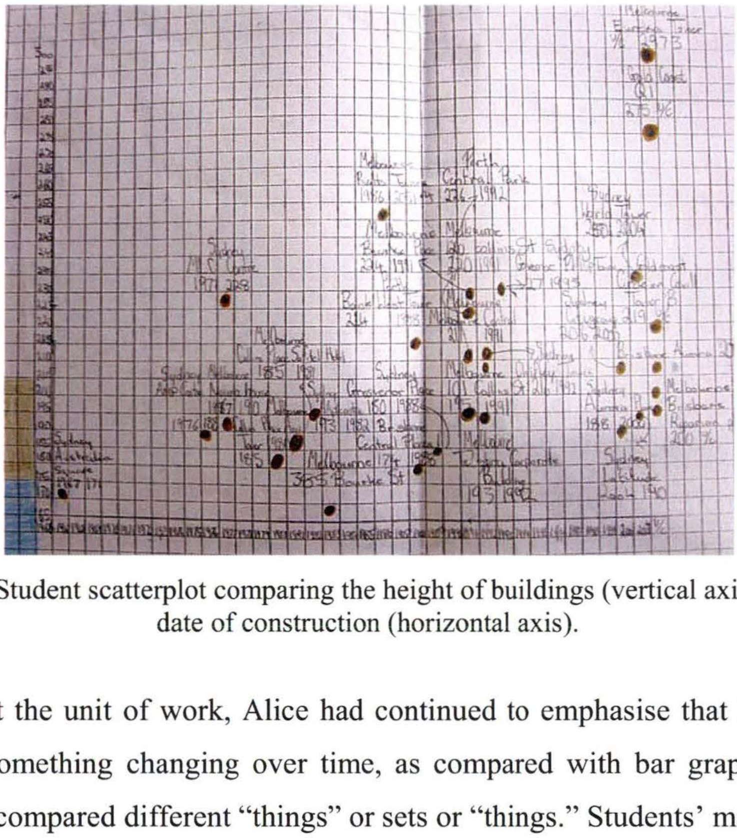


Figure 6.6. Student scatterplot comparing the height of buildings (vertical axis) with their date of construction (horizontal axis).

Throughout the unit of work, Alice had continued to emphasise that line graphs represent something changing over time, as compared with bar graphs and pie charts that compared different "things" or sets or "things." Students' mathematical learning of these distinctions was evidenced when working in pairs to consider the common and distinctive features of line and bar graphs at the end of the unit of work (Figure 6.7).

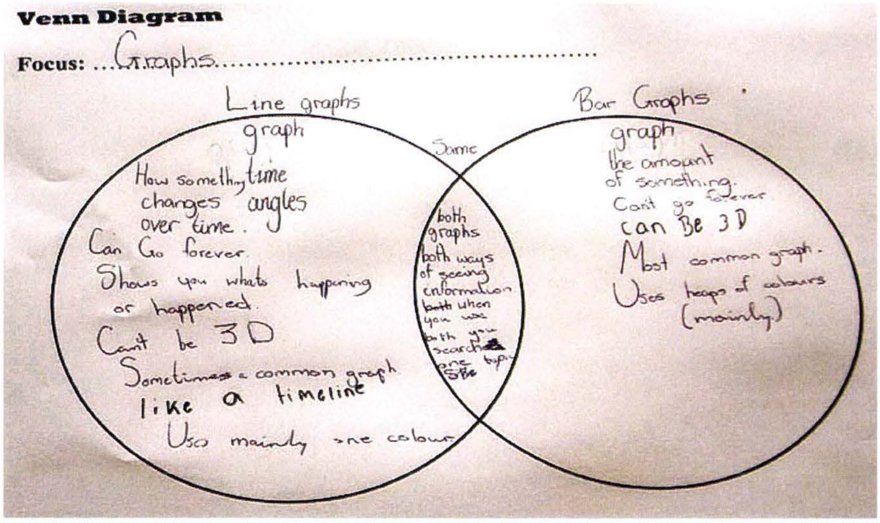


Figure 6.7. Student's comparison of the common and distinctive features of line graphs and bar graphs.

6.3.2.2 Reasoning

Reasoning played a major role in Alice's classroom. Although Alice would ask the occasional knowledge question, such as "What is a pie graph?" (11/10/05) or "What word do we use?" (26/10/05), she continually questioned students to extend their thinking. Alice encouraged her students to exemplify or explain specific elements of their work, for example,

> So what might he have been doing in that time?
> What does the change in the angle of the graph tell us?
> Give me an example? (Classroom observation, 26/10/05), and
> What is this telling me? (Classroom observation, 31/10/05)

Alice also asked her students what they did and how they had gone about it, with questions that encouraged them to articulate their thinking and their mathematical strategies.

> How do you know that? (Classroom observation, 11/10/05)
> Tell me some of the strategies you used, what did you do?
> How would that help us? (Classroom observation, 26/10/05)

Some of the questions Alice posed required students to think beyond their current ideas, particularly when sharing together as group. Questions that stretched students to analyse and critique their own and others work and thinking included:

> Anything we have missed? (Classroom observation, 26/10/05)
> What is another way we represent statistics and data?
> Are you happy with that [explanation]? (Classroom observation, 31/10/05)

When one student wrote her Sizzlers Restaurant story and had the person spilling all the food and then refilling their plate, the student himself identified a discrepancy between the story and the graph, because of Alice's question, "Why was that a problem?" (Classroom observation, 24/10/05). The student realised the graph "should have gone down to zero" to represent the ideas in the story accurately.

Students participated in their mathematics learning by explaining strategies, by questioning each other, and by actively engaging in discussion and sharing their findings and the way they approached tasks. As the unit progressed the activities moved from identifying types of graphs and their components to tasks in which students were required to interpret graphs and to come up with their own examples to show comprehension. They also had opportunities to apply their knowledge by considering the purpose of different types of graphs and their distinguishing attributes.

Before choosing or implementing a graphing activity, Alice found out what the students already knew and used this, together with any misconceptions that she was aware of, to her inform her planning. A variety of tools was used to uncover students' thinking such as mind maps, think-pair-share activities, and reflective journal writing. At times students used their own strategies to record their thinking. Figure 6.8 shows the work of two girls who planned their story first, by making notes of the times and number of portions on the plate when interpreting the graph in the Sizzlers Restaurant task (Classroom observation, 24/10/05).

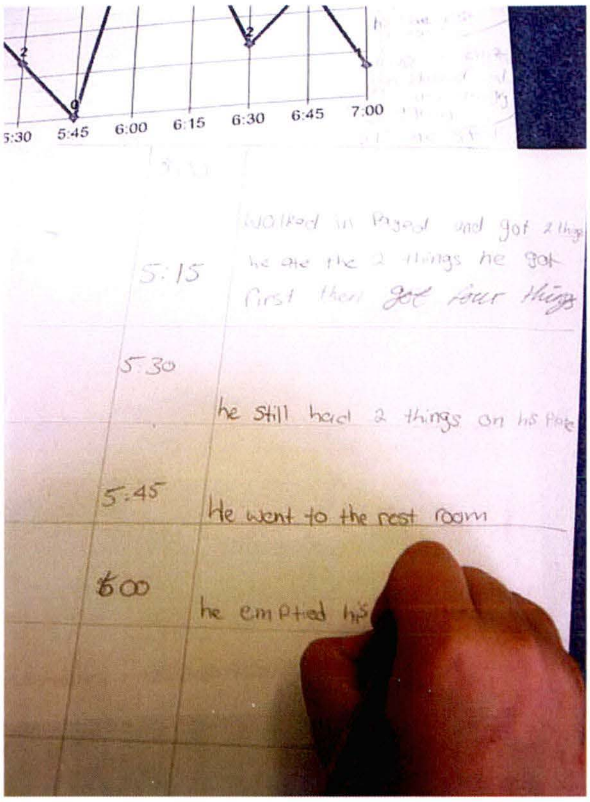


Figure 6.8. Student recording data from graph prior to writing interpretative story.

During a task in which students worked in pairs to write an interpretation of a graph, either a bar graph comparing data on levels of bullying at school or a line graph of changing petrol prices throughout the year (Classroom observation, 14/10/05), Alice noticed that "students tended to start with general statements, for example, 'Grade 6 is bullied more than Grade 12s' or 'the price of petrol increased,' and then as [students were] questioned they started to include statistics" and be more specific in their interpretations (Classroom observation, 14/10/05).

Students were constantly sharing ideas in small groups or as a whole class, and constructing collaborative and shared understanding. After working in pairs to create a Venn diagram about the similar and distinguishing features of line graphs and bar charts, two pairs then shared their work to determine features that they agreed on, before sharing these outcomes with the class as a whole to create a class Venn diagram (Figure 6.9).

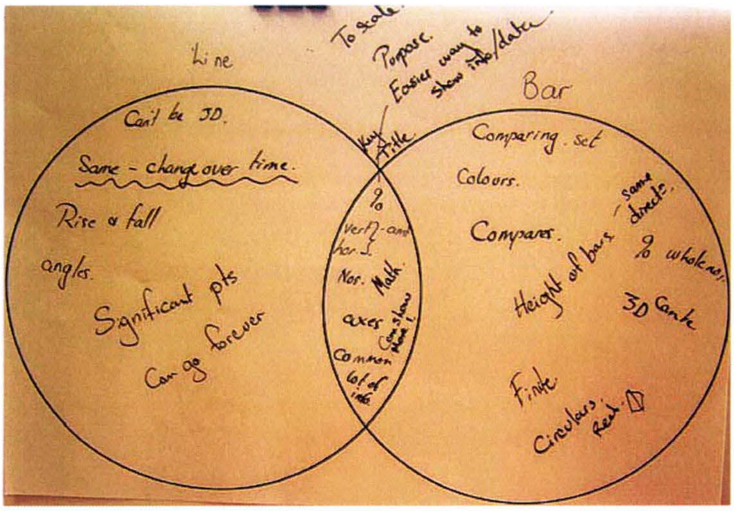


Figure 6.9. Whole class Venn diagram of features of line graphs and bar charts (26/10/05).

Alice encouraged her students to make decisions, with most tasks involving some element of choice so that the tasks were accessible at many levels. Students stopping and re-evaluating, and at times even changing their work, was supported and seen as a normal part of the learning process.

Alice, herself, was always considering where to take the learning next, by evaluating and reflecting upon students' understanding. She did this by listening to student discussions, by questioning students, and by assessing their recorded work, all of which revealed student reasoning in Alice's classroom. Alice's commitment to continue to work on graphing and statistical reasoning with her students was evidenced at the end of the unit of work, when she reflected in reference to curriculum documentation, that "representations of data [had] been covered, but not the aspects relating to interpreting in terms of identifying median, mode and spread" (Classroom observation, 1/11/05).

She turned to the researcher and posed the question, "What do I do now… ?" (Classroom observation, 1/11/05)

6.3.2.3 Attitude

Alice's numeracy classroom was a supportive one in which students were provided with opportunities to work on their own (Figure 6.10a), as well as with a partner or in groups (Figure 6.10b). Alice acknowledged students' prior learning and sort to find out their current understanding. She constantly explained to them what was required and why she had chosen a particular task. Students were provided with many opportunities to explore their own ideas and there were times when students were clearly excited by some of their learning opportunities.

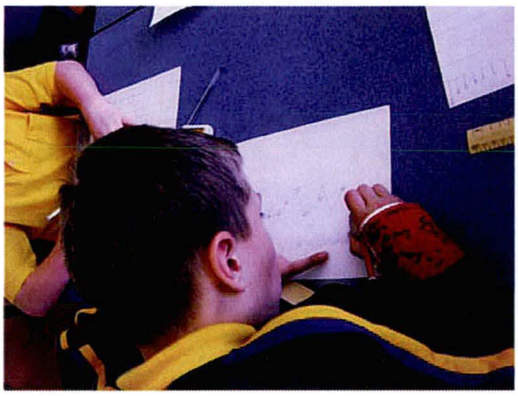


Figure 6.10a. Working individually.

Figure 6.10b. Working in groups.

Discussion with each other about their ideas was valued and students could change their ideas and strategies as their understanding developed, by listening to other students, by considering the questions Alice was asking them, and by recognising their own errors and misconceptions. Developing a supportive learning environment was a conscious part of Alice's pedagogy, as she wanted to develop students' confidence in numeracy. She encouraged perseverance by enabling students to re-consider and re-work their ideas based upon new knowledge or listening to others.

Many of Alice's classroom strategies were aimed at having students feel comfortable so they would be prepared to take risks and see themselves as capable learners of mathematics, as this was something that Alice had previously shared as an issue for her students. Whole class sharing was common and formed a key part of building a secure environment for students to feel comfortable and confident about their own numeracy learning (Figure 6.11). Alice's students were discussing, sharing, and at times taking risks with their learning during the unit of work as a result of the classroom culture Alice was developing.

Figure 6.11. Whole class sharing.

6.3.2.4 Context

Throughout the graphing unit, many different contexts were both presented to students and chosen by students as they developed their mathematics understanding of graphing. Although understanding of the contexts themselves did not form part of Alice's overall objective, they played an important role in helping students present or interpret the data and graphs involved.

In most cases the contexts in which the tables and graphs were embedded were contexts with which students were relatively familiar, such as eating at restaurants, construction of buildings, and school bullying. Students were also able to choose their own contexts in a few of the graphing tasks, and they tended to choose contexts relevant or familiar to themselves, such as students in school, pets, sports, and death rates in Tasmania. When Alice's students had a choice to interpret a graph on bullying or one involving oil prices, the majority of students chose the one to which they could relate, that of school bullying. Those students who went on to model a graph of their own creation, on either the bar graph or the line graph, chose familiar contexts, such as school sports as represented in the example in Figure 6.12.

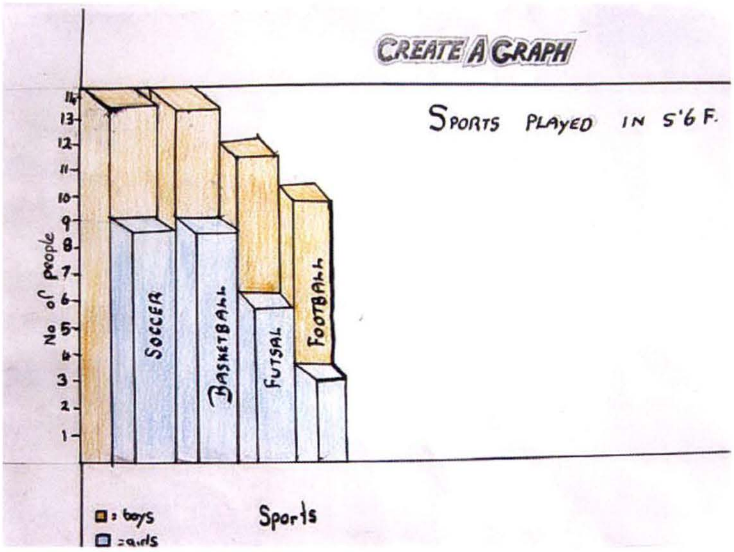


Figure 6.12. Student created graph comparing sports played by boys and girls in Grade 5/6.

In another graphing activity, students were presented with a bar graph, with no detail, and they had to construct meaning by completing the appropriate components that would make the graph meaningful. Some students became very focused on choosing a meaningful context when completing this task. As with the earlier activity, student choice of context represented something with which they were personally familiar or in which they were interested, such as pets (Figure 6.13).

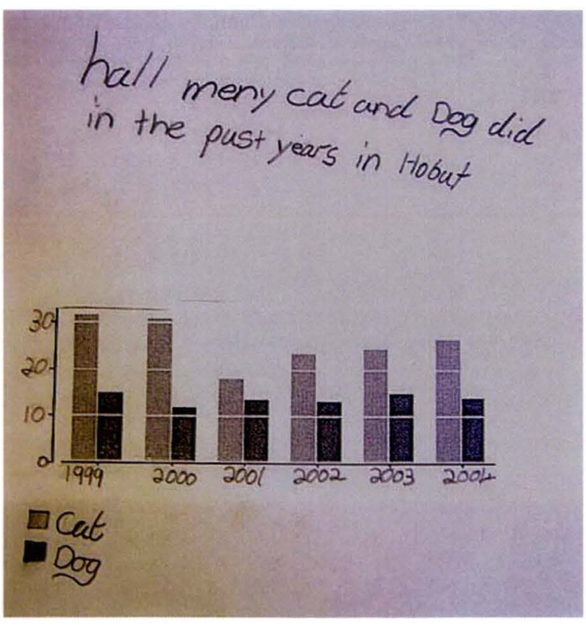


Figure 6.13. Student example of providing meaning to a blank graph.

6.3.2.5 Equity

In Alice's classroom, the differing mathematical ideas of students were valued and all students were encouraged to contribute and share these ideas. They were also questioned to extend their understanding and encouraged to question each other; this questioning was modelled by the teacher. Students were provided with many opportunities to work together, to share ideas, and to consider a variety of viewpoints (refer Figure 6.11). Students also had opportunities to make their own choices, with tasks having a degree of flexibility, such as choosing which aspect of the data to graph, or choosing a context. This enabled Alice's students to access tasks at their own level and also to extend themselves by working with others.

The culture of the classroom was being established that would enable students to develop the capacities that could eventually be applied in social and political contexts. There were contexts embedded within some of the tasks in this unit of work that did have social implications, such as bullying, health, and nutrition, but Alice did not choose to take the learning beyond developing students' capabilities to create, understand, and interpret graphs. At this stage, Alice was focusing on the important mathematical foundational learning, together with developing students' reasoning capabilities and a sense of confidence in the numeracy classroom.

6.3.3 Student learning

The six students interviewed in this phase of the study were asked to describe and discuss the graphs they had created or interpreted during the unit of work on graphing. The interviews were analysed with respect to the five dimensions of numeracy. Chapter 3 introduced how each dimension was categorised and described. The following subsections detail how the students' learning was evidenced across the five dimensions of numeracy. The reporting of the results is intended to provide representative evidence of the range of responses for each category within the dimension. In doing this, at least one comment from each student is included for each of the five dimensions.

6.3.3.1 Mathematics

The numeracy unit of work in this case had very focused learning outcomes around the nature and purpose of graphing. Students undertook tasks involving drawing graphs, comparing types of graphs, and interpreting graphs. Within this context, the students demonstrated specific mathematics understandings in the area of data and graphing, across a range of levels. Table 6.1 summarises the observed categories of Mathematics for each student, as evidenced by the student interview.

Table 6.1
Student learning: Mathematics dimension (Snowgum Primary School)

Student / Category	S1	S2	S3	S4	S5	S6
Reading and describing graphs	✓	✓	✓	✓	✓	✓
Making meaning from graphs	✓	✓	✓	✓	✓	✓
Analysing and interpreting graphs			✓		✓	
Evaluating and making informal inferences			✓		✓	

All six students were able to *Read and describe graphs*, identifying components of their graphs when sharing their work.

> Like up here say, how many cats and dogs died, so you know the numbers in Hobart. So you know it is Hobart and saying which one is cats and which one is dogs. [S2]
>
> You could just look at this and see that in 1996 there were a bit more than thirty houses built and fifteen destroyed. [S4]
>
> That was a line graph and it went down. [S6]

Further, all of the students could also *Make meaning from graphs.* Student 1 evidenced some understanding of comparison when she stated that "less [sic] people were bullied in 1999," although she did not go on to indicate to what this was compared. The other five students evidenced this level of learning, by making clear mathematical statements about graphs, for example when Student 6 discussed a line graph representing both the time of day and the number of items of food a person had on their plate during a meal at a restaurant.

> I chose my Dad and then we had to say he arrived at 5 pm and put two things on his plate and then he added two more things and then he ate two things and then he ate it all up. Then he put eight things on his plate and then he ate two at 6:15, when he had six things on his plate and then at 6:30 he had two so ... [S6]

Only two students, Students 3 and 5, went on to *Analyse and interpret graphs.* In relation to different tasks, both students compared multiple aspects of their data and made interpretive statements.

> It was really hard, because I said he ate two things and it went back down to here, and then I thought, no, that's not right, because he got two more things. So I had to write more for his meal and with red, it shows what I would get and I did it straight up here ... [S3]

> [Alice] had printed out a sheet showing the top thirty tallest buildings in Australia and the ones that were under construction at the moment, and other people did how many tallest buildings in each [city] but I decided to do storeys of buildings ... and in Melbourne in every case came up on top with the most ... except Sydney maybe got it for more buildings because I think it has more little buildings. [S5]

The same two students could also *Evaluate and makes informal inference.* In the following example, Student 3 discussed a scatter plot she created to compare the heights of buildings with their construction completion date.

> There were a couple [of points] like this [separated], and then there would be more bunched out around here [date of completion data]... It was really weird, and then when I started getting into the heights, the heights were completely spread out ... they weren't near each other at all.
> Int: Okay, so the really tall buildings weren't actually built more recently?
> S3: No, they weren't near each other at all. [in terms of time S3 was expecting more recently constructed buildings to be taller] [S3]

When sharing work from the same task, Student 5 went beyond the information of the number of storeys for buildings in Melbourne and Sydney. Due to other information on the total number of buildings in these cities being missing from the table of data, Student 5 considered that, although Melbourne may have more storeys, it may still have fewer taller buildings than Sydney. This student inferred that variation could involve more buildings of fewer storeys, or taller buildings with more storeys.

> It's saying that Melbourne has 650 storeys and Sydney has 553, etc. but it doesn't actually say how many buildings, you have only got the number of storeys, so even though Melbourne has 654, it might only have like 15 majorly [sic] big buildings. ... If I did another graph and then compared it I could show number of buildings and number of storeys. [S5]

6.3.3.2 Reasoning

Observed student learning in this dimension of numeracy very closely matched that for the Mathematics dimension. Table 6.2 summarises the observed categories of Reasoning for each student, as evidenced by the student interview.

Table 6.2
Student learning: Reasoning dimension (Snowgum Primary School)

Category \ Student	S1	S2	S3	S4	S5	S6
Remember	✓	✓	✓	✓	✓	✓
Comprehend	✓	✓	✓	✓	✓	✓
Analyse			✓		✓	
Evaluate			✓		✓	
Create			✓			

All of the students were able to *Remember* and identify specific information related to their graphs. The following are two examples.

> Thirty cats died in 1999. [S2]

> Well line graphs just show how things change over time and some other things. [S4]

They could all also *Comprehend* and, due to the unit of work focusing on different types of graphs, students demonstrated this category of Reasoning when they were explaining their graphs. In the two examples included, Student 1 pointed to a section of a line graph in which the line fell at a steeper gradient than the previous section of that graph.

> He probably started eating faster. [S1]

Student 6 summarised a graph on types of cancers found in men, by focusing on the most prevalent cancer, lymphoma.

> More people get lymphoma … and then [it shows] that men get other … kinds of cancers as well. [S6]

Students 3 and 5 went on to demonstrate both their ability to *Analyse* and to *Evaluate.* Student 5, in analysing the benefits of putting information into graphs, emphasised the point he was making by using very large numbers that are perceived to be difficult to grasp when expressed in words.

> Well if you write it out in words like that, 365 million people go to work and 365 million zillion go to school, you are really not going to know how much that is, besides from being a big number. If you put it into a graph you can automatically see, without even reading the numbers that there are more people going to school than going to work or something like that. [S5]

Student 3 demonstrated her ability to *Evaluate* by providing a critique of her own work when she discussed the process she went through to graph data about the number of buildings in Australia. After initially trying a line graph to plot building completion dates, she recognised that it did not provide a meaningful picture of the data and therefore re-considered her decision.

> I went for when the buildings ended, when they were finished and tried it with a line graph, a big piece of paper for a line graph, but that didn't work because I went up and then I went down and it was telling me that I had no buildings in Australia, and I went 'I don't want it like this, I want to put that there and everything.' After I tried it [another way], it actually worked. [S3]

In the following example, Student 5 began by *Analysing* the differences between bar graphs and line graphs, including the purpose for which they are best suited, how their components can be distinguished and also key similarities between the two types of graphs. Into his analysis, he incorporated judgments about the effectiveness of the graphs, how "easy" they are for representing data, and also why the use of colour is recommended.

> For line graphs, I learnt that it was time and amount comparing, like it's an amount over a period of time, comparing one thing over it. It could be many things, but there would be many line graphs on one sheet, and on bar graphs, comparing a group of things, but it is usually [comparing] the same type of thing, The most important thing I learnt is that there is always one axis showing the number of something, like one hour, two hours, three hours … and the similarities, there is always an amount and a total of something, like at the top so both have axes, they both have numbers, they are both mathematical, they collect and show data, they use measurement, they're an easy way of showing something, they both have titles, they compare things, just in different ways. You should use colour because it helps make it more standoutish [sic], and it can be used vertical and horizontal and scales … oh yeah because you can enlarge something or shrink it and it will still show the same thing. [S5]

Only one of the six students evidenced the ability to *Create* new and original ideas when discussing their graphing work during the student interviews. Student 3, when preparing to write a story that would interpret a graph provided about a person's visit to Sizzler's Restaurant, explained how she generated a table with the time of day and summary comments about how much food was on the plate at each time of day, to help her plan her story.

> So I thought I might make a plan, and so I wrote the times and what he did, like, what did he eat, like did he eat fast or something like that and then I just wrote my items and what he was going to eat there ... when I do graphs again I am pretty sure I am going to plan it out before I do it. [S3]

The same student also created a scatter plot due to her interest in looking at the relationship between the height of buildings in Sydney and Melbourne and their dates of completion. This type of graph had not been taught previously and was not one of the types of graphs discussed by Alice during the unit of work.

6.3.3.3 Attitude

As students shared their work from the graphing unit, their attitudes toward their learning were revealed by both the comments made and by the manner in which they conveyed them. Table 6.3 summarises the observed categories of Attitude for each student, as evidenced by the student interview.

Table 6.3
Student learning: Attitude dimension (Snowgum Primary School)

Student / Category	S1	S2	S3	S4	S5	S6
Confidence/ self-efficacy	_*		✓	✓	✓	✓
Interest	_*	_*	✓		✓	✓
Enjoyment			✓			
Intellectual stimulation			✓		✓	
Diligence/Perseverance		✓	✓	✓	✓	
Appreciates value of mathematics	✓		✓	✓	✓	✓

* denotes negative attitude.

Of the six students, Student 3 and Student 5 evidenced a highly positive attitude to numeracy by demonstrating most, or in the case of Student 3 all, of the affective factors deemed to contribute to successful numeracy learning. In terms of *Confidence* four of the six students displayed a positive self-concept about their capabilities in the context of the unit of work. This confidence was both general and sometimes related to a specific aspect of the work. The following are two examples.

> I knew how to do some things that others didn't and I could just help them ... I could get mine done quickly because I knew more about it. [S4]

> I already know about the main types of graphs. [S5]

Student 1 shared a lack of confidence in relation to creating graphs.

> I am not the best at them. [S1]

In terms of *Interest*, three of the six students demonstrated an engagement with a particular task, for example when Student 3 discussed her learning about a new type of graph, the scatter plot, and how she took it home to show and explain it to her mother.

> I showed my Mum the scatter plot and she understood why it was called a scatter plot as soon as she saw it. [S3]

These students also discussed particular areas of graphing that they wanted to explore further. Student 5 for example wanted to pursue pie graphs.

> [I want to learn more] because I already know about the main types of graphs, like line, bar, pie and I still want to try and make a pie. [S5]

Students 1 and 2, when sharing their work during the interview, demonstrated a clear lack of interest, both by their comments and at times their manner. The following are two examples.

> Int: What would you like to learn more about?
> S1: Not much, but a few more graphs if there are any. [S1]

> I don't know, I just picked one. [S2]

Student 3 was excited when discussing her work. She shared the *Enjoyment* she experienced when working on different tasks and discovering new knowledge about graphs and graphing.

> I like both [tasks]. … I like using my own ideas. I like using what I know about, but with [the other task], I liked it because it's kind of easy to find information if I look it up. [S3]

> I didn't even know you could do that. That was good fun. [S3]

Two of the six students demonstrated the *Intellectual stimulation* they gained by working on the graphing tasks. This was predominantly evidenced by the depth of their responses, and the way in which they described working on tasks that may have been quite challenging. In the following examples, the challenge of tasks led the students to consider other things that they would like to pursue. In the first example, Student 3's comment also shows her *Interest* in exploring different types of graphs.

> I would like to make different graphs and see how they are different. Like with a bar graph and a line graph, I can show the difference between. I would like to do that with all the others: see what you have to do with them, and I could see how different they are and how much you have to work on each one like that. [S3]

> But it doesn't actually say how many buildings [when discussing number of storeys for each city in Australia in his graph], as you have only got the number of storeys so even though Melbourne has 654 [storeys] it might only have like 15 majorly [sic] big buildings.... If I did another graph and then compared it [I could show number of buildings and number of storeys] because I remember another girl in my class had done the number of buildings and we were going to compare those but we ran out of time. [S5]

In one instance the intellectual stimulation gained was associated with an explicit expression of how difficult the task was.

> That was very hard because you have got heaps of things in together. When ... the buildings are finished, they were just all over the place. It was so hard. [S3]

Four of the six students showed their *Diligence* by discussing times when they persevered on tasks and planned and checked their work. Student 3 did all of these and at many different times. She also related persevering on tasks to achieving a better final outcome. The following examples represent the perseverance of two other students.

> It took me a while to figure out what animal or thing to choose. [S2]

> I used to think that doing a pie graph was just cutting up a circle ... but now I understand that you have to, it's a lot more work than that and you have to find out the degrees and everything with angles, and I did a pie graph about two weeks ago ... and I did all the work even though I was working with someone else. [S4]

Five of the six students evidenced an *Appreciation of the value of mathematics* when sharing the usefulness of graphs in everyday life and work contexts. The following are examples.

> [It is helpful to use graphs to describe things] like when prices change, like in petrol ... and then in food. [S1]

> Graphs are used for many things. They are used for recording data, and finding the most used items, etc, etc. We use graphs every day ... If we just wrote everything down like in business and companies, if you just wrote all these numbers down, you would lose track of things way too easy [sic]. But if you put it all into one sheet, comparing things, you can easily see the difference. [S5]

> You could use [graphs] on jobs to show how many people have different jobs, just helpful to have them. [S6]

6.3.3.4 Context

As a unit of work with focused learning outcomes related to the nature and purpose of graphing, different contexts were both presented to students and chosen by students as they developed their mathematics understanding in this area. The contexts themselves played an important role in helping students represent and interpret the data and graphs involved. Table 6.4 summarises the observed categories of Context for each student, as evidenced by the student interview.

Table 6.4
Student learning: Context dimension (Snowgum Primary School)

Student / Category	S1	S2	S3	S4	S5	S6
Personal experience of context	✓	✓				✓
Context integrated with mathematics as presented	✓	✓	✓		✓	✓
Context integrated with mathematics, from both prior knowledge and as presented			✓		✓	
Relational understanding of the mathematics and the context and can transfer to new contexts						

For three of the six students, their *Personal experience of context* dominated their sense-making of the graph, irrespective of the embedded mathematics. In the following example, Student 1 began by considering possible reasons for there being less bullying amongst Grade 6 students in 1999, as compared with Grade 6 students in 2003. The student then got confused and was unable to relate his ideas to bullying levels changing over time, and instead focused on students growing up between 1999 and 2003.

> Int: Why do you think less people were bullied in 1999?
> S1: Well because there weren't probably as many [students] as 2003 … it would take a long time for people to grow up and people wouldn't reach Grade 6 within 4 years. [S1]

Student 2, when discussing a comparative bar graph, used his own experience playing soccer to discuss the reasons for there being more boys than girls playing soccer in the represented graph. Although in this case the student gave a reasonable explanation for the graphical representation, it was not informed by the mathematics.

> There are only 9 [girls playing soccer] and the boys are 14, because when I played soccer this year, it was just mostly all [boys] ...this year there were no girls [at soccer]. [S2]

In the following example, Student 6 was more interested in describing the sleeping habits of her father than focusing on the mathematics embedded with the context.

> He slept for half an hour [describing graph she had created for a person eating at a restaurant]... because he likes to have cat naps, [referring to her Dad] and just sits there with his arms on his stomach and goes to sleep. [S6]

At times the same three students, together with Student 3 and Student 5, were able to share their understanding of graphing and data by more clearly focusing on the *Context integrated with the mathematics as presented.* The following examples are representative of the types of responses in which students remained focused on the data and context presented within a task, but without thinking or discussing beyond these parameters.

> Reading it, it was like the bullying is getting bigger, like 2003 it's got more bullying. [S2]

> So [I] looked for how many people were dying here [Tasmania] and it came up with a big graph, and it showed how ... many older people are going to be here and everything and it went whoosh! 2009 it was whoosh! [indicating a steep increase] [S3]

> [The table showed] the top thirty biggest buildings in Australia and the ones that were under construction at the moment ... I did [a graph of the number of] storeys and Melbourne in every case came up on top with the most ... I think except Sydney maybe got it in more buildings ... [S5]

Students 3 and 5 demonstrated a capacity to integrate both the mathematics and the context at a higher level in order to make sense of the graph or data presented to them. There were times during their interviews when *Context was integrated with the mathematics, from both prior knowledge and as presented.* Two examples are provided. Student 3, when sharing the interpretive story she had written about a graph of someone's visit to Sizzlers Restaurant, was continually considering what would have been reasonable within the context of eating at a restaurant and the time it might take not only to eat, but also to put food on the plate.

> It was really hard, because I said he ate two things and it went back down to here, and then I thought, no, that's not right, because he got two more things. So I had to write more for his meal and with red, it shows what I would get and I did it straight up here which is wrong, because if I went straight up well that means I started with 9 … I walked in and when I sat down I had 9 on the plate! [referring to 9 being on the y axes, and therefore 9 items of food on the plate as soon as she walked in the door of Sizzlers Restaurant, without any time allowed to put the items on the plate.] [S3]

Student 5 discussed a task involving the interpretation of data about the construction of buildings in cities within Australia. The student initially focused on the number of storeys for buildings in Melbourne and Sydney as represented in the table. He then went beyond the context as presented to discuss his inability to make particular conclusions about how tall the buildings might be because the number of buildings in each city was not provided. The student also considered that even though Melbourne had more storeys, Sydney may have more buildings that are taller.

> It's saying that Melbourne has 654 storeys and Sydney has 553, etc. but it doesn't actually say how many buildings. You have only got the number of storeys. So even though Melbourne has 654, it might only have like 15 majorly [sic] big buildings. [S5]

None of the students interviewed evidenced *Relational understanding of the context and the mathematics* with an ability to *transfer to new contexts*. This is likely to have been a question of opportunity provided by the unit of work.

It is noted that from the interview data, Student 4 did not provide clear evidence to enable a code for this dimension. When discussing her work, Student 4's responses were quite limited and there was minimal connection with context. In one instance, when being questioned about her selection of scale on a graph that she drew, Student 4 included in her justification for changing her scale, a reference to the context that she had chosen, which was houses destroyed in Tasmania, as well as thinking about what might be realistic.

> I was going to do it [scale] in groups of five, but each year there would be more than just ten houses destroyed in Tasmania, so I did it about that [scaled in tens, up to a maximum of 30] …but I still don't think it is very realistic. [S4]

The response demonstrates a definite attempt to integrate the context with the mathematics, but she remained concerned with the number and did not explain why it may not be realistic.

6.3.3.5 Equity

Table 6.5 summarises the observed categories of Equity for each student, as evidenced by the student interviews. In this case, there were a number of contexts that had social implications embedded within the tasks provided to the students. These contexts included bullying, health, and nutrition. When the students were sharing their work during the interviews, the social issues or consequences related to these contexts did not form part of their conversations. One student did, however, demonstrate his engagement with and appreciation of the mathematical strategies of others in the classroom.

Table 6.5
Student learning: Equity dimension (Stringybark Primary School – Alice)

Student / Category	S1	S2	S3	S4	S5	S6
Personal social engagement					✓	
Awareness of issues						
Considering viewpoints						
Relating mathematical information with social and political consequences						
Challenging inequity						

Student 5 provided emerging evidence in the student interview of his *Personal social engagement* when working on graphing tasks. When discussing his bar graph of the number of total storeys in buildings across major cities in Australia, Student 5 evidenced his awareness that, although his graph presented accurate information, by including further information he may be able to learn more about the relationship between the number of storeys and the number of buildings in each city. He considered this because of discussions with another student and the graph she had chosen to draw showing the number of buildings in each city.

> But it doesn't actually say how many buildings, as you have only got storeys so even though Melbourne has 654 [storeys] it might only have like 15 majorly [sic] big buildings ... If I did another graph and then compared it [I could show the number of buildings and the number of storeys] because I remember another girl in my class had done the number of buildings and we were going to compare those but we ran out of time. [S5]

During the student interview, students also shared some of the connections they saw between the numeracy unit of work on graphing and the integrated unit of work on Australia they had been completing prior to the graphing unit of work. It was the unit of work on Australia that had motivated Alice's planning to develop further the students' capabilities in the area of graphing. Student 5 and Student 6, for example, when discussing the relevance of their learning about graphs to the integrated unit, mentioned collecting statistical data such as crime rates and under-age pregnancy. They did not, however, evidence a disposition to question these statistics or interpret them critically in relation to any social or political implications or inequities.

> S5: We were doing New South Wales and we had to put like where a nature park reserve was and all of that, from a facts file.
> Int: Was there much data in that work that you had to put into tables and graphs when you were doing that?
> S5: Not really with tables and graphs, but while we were doing that [Alice] wanted us to go and find statistical stuff about our state, like the crime rate ... And we found stacks of graphs on that and it is in a flip folder we gave to [Alice].
> Int: So that is why she thought she would take it a bit further maybe. ... Do you think putting those statistics into that unit of work helped you understand your State a bit more?
> S5: Yeah. It wasn't the most crime-rated or death-rated place in the country but it was still probably as much as anywhere else. ... [graphs] would have helped a lot because you can compare against other States and learn about stuff. [S5]

> Int: Did you use graphs much in the work to do with your Australia project?
> S6: With my group, we did Tasmania and we did use some graphs ...
> Int: Can you think what sorts of things you put or used your graphs for?
> S6: How many babies are born with under-age women. ... that was a line graph and it went down.
> Int: So how many, were many born [to women under the age of 18]?
> S6: Yeah, about I think it had eighteen, around about eighteen to twenty sixteen year olds had kids. Yeah that was the lowest it got to and that.
> Int: Did that surprise you?
> S6: Not really. [S6]

6.4 Chapter summary

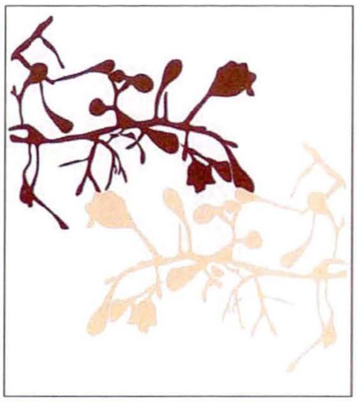
This chapter has shared the beliefs and practices of Alice and the learning experiences and outcomes of her students. Alice was an experienced Grade 5/6 teacher for whom the curriculum reforms being implemented in Tasmanian schools intersected with a major shift that she was experiencing in her beliefs about the teaching and learning of numeracy. She had participated in a numeracy professional learning program for middle years' teachers earlier in the same year that the research study was conducted. Alice was at a critical and complex stage of moving between her old ideas and where she wanted to be. She was working toward changing her numeracy teaching and the numeracy learning environment in which her students participated.

The dimensions of Mathematics, Reasoning, and Attitude featured in this case as Alice planned and implemented a focused unit of work on graphing. In working to incorporate her broader views of teaching and learning into her numeracy classroom, Alice was using a diverse range of thinking tools and strategies to support her students to share and reflect upon their learning and to become problem solvers. She was also very conscious of building a supportive classroom environment in which her students could take risks and develop increasing confidence of their own knowledge and skills as users of mathematics.

> *In terms of what the students learnt they can actually read graphs in a whole range of styles and they can actually represent information now in a whole range of ways so I think that has been quite successful because it is that one step on from what I would normally do. Normally I would just say "here is a bar graph" or "make a bar graph about that" without really thinking is this the best way to do this. Now the kids have those great ideas about whether it is the best or most appropriate way to represent the data. ... I think they would approach it differently now because they would see a graph and think "Is that important for what I want to know." ... Now they will be able to interpret that information and make better choices about whether it enhances their work or not."... The other thing that is happening is that the numeracy in my classroom is much more based on all those ideas, that thinking, reflection, sharing, all of those things. ...*
>
> *I guess I saw it as a skills-based unit and that's the big dilemma we have had with the Essential Learnings up until now, where teachers*

say "Where are the skills?" I guess I have been able to represent that ... if I identify a need and that it is a skill that you have to have then you can take the same approach as with a bigger learning sequence, as opposed to "let's just teach them about graphs," you can take a broader focus. (Alice, 8/12/05)

The presentation of the results continues in Chapter 7 with the findings of the research produced for the teacher Ophelia of Stringybark Primary School and for her students.



Chapter Seven

Results:
Stringybark Primary School – Ophelia

> *I see mathematics as the skills ... the process, actually teaching a process. Whereas I see numeracy as developing confidence with maths and developing problem-solving skills and being able to use numeracy in the real-world. Being able to apply knowledge to different situations as well so a student can take the knowledge that they have of one particular process and use that knowledge and apply it to something else. ... You have to teach the processes but it's also really important to develop the numeracy, the confidence and the ability to transfer that knowledge.* (Ophelia, 24/5/05)

7.1 Introduction

The setting for this case study, Stringybark Primary School, is described in Section 5.3. As a state government school, Stringybark constructed its curriculum in accordance with the D*o*ET's policies and guidelines. The staff had been planning and working collaboratively, using a whole of school approach, to implement the Tasmanian curriculum reforms since 2001.

Ophelia, a Grade 6 teacher at Stringybark Primary School, worked closely with a team of four Grade 6 teachers to plan transdisciplinary units of work informed by the Essential Learnings framework (D*o*ET, 2002). In this case, the unit of work "How do you measure up?" was planned to develop student understandings of measurement-related concepts and the processes of inquiry and reflective thinking. Ophelia implemented the unit of work over a two-month period during her usual one-hour-and-twenty-minute numeracy block.

Ophelia was in her tenth year of teaching during which she had spent most of her time as a specialist art teacher in both primary and high schools. She had commenced at Stringybark Primary School in 2004 and was very motivated to take on the challenge of becoming a capable and effective Grade 6 classroom teacher. Ophelia had been using the Essential Learnings (D*o*ET, 2002) to underpin her planning and teaching for eighteen months.

Data analysed and reported in this chapter are two teacher interviews with Ophelia, field observations taken during fifteen classroom visits with Ophelia and her 28 Grade 5/6 students, six individual student interviews, together with documents and photographs collected across the four phases of the research.

7.2 Curriculum

This section presents Ophelia's beliefs about curriculum and summarises the unit of work that Ophelia was teaching during this study.

7.2.1 Ophelia constructing curriculum

For Ophelia, the shift from being a specialist art teacher to becoming an upper-primary classroom teacher, responsible for all areas of learning, occurred alongside the implementation of the Tasmanian curriculum reforms. Ophelia found that the structure and nature of the reforms provided her with not only a philosophy of curriculum from which to shape her practice, but also a supportive and collaborative setting in which to share and develop her own ideas about classroom teaching and learning.

Ophelia emphasised the importance of identifying "what [she wanted] the children to learn and what [she wanted] the children to understand" so that her planning, and the activities and tasks she designed, were purposefully developed to support the learning of thoughtfully constructed understanding goals. She also felt that this enabled her to focus on "the really important things" – the concepts, processes, and strategies that were relevant to the "generative topic" or "guiding question" being explored.

The disciplines were brought together as they related to a particular unit of work, to encourage the connection of ideas and to support the understanding goals. Ophelia gave an example of a previous collaboratively planned unit of work, "Are you looking after yourself?" that had drawn on aspects of literacy, numeracy, art, science, and technology to investigate the over-riding concepts of health and well-being. Strategies that supported students to develop their "thinking" and "problem-solving skills" were also viewed as central and embedded in all units of work. Ophelia saw the development of these higher-order capacities as a key aspect of the Tasmanian curriculum reform and it was apparent that they also informed her own construction of curriculum.

> I think that is what the Essentials [Learnings] is all about. It's all about logical thinking, making decisions, that sense of your subject, that power, making connections across things and looking at the bigger picture. (Ophelia, 24/5/05)

Whilst discussing her beliefs and practices, Ophelia used many verbs and phrases that evidenced her role as a reflective practitioner. She took every opportunity to consider the implications of her teaching for student learning. She was very positive about the opportunity that the curriculum reform had provided her in relation to reflecting upon and changing her practice.

> [I] think very carefully about the tasks and activities [I] get the children to do and keep linking them back to what it is that [I] want them to understand. So [I] keep analysing, "Does this activity actually help the children gain the understanding and the knowledge that I want them to about a particular idea, or a particular question and also, could the children do this task without having, without understanding the ideas behind it?" ... I think that [the Essential Learnings] has provided me with a lot of self-reflection about what I teach and why I teach it. (Ophelia, 24/5/05)

7.2.2 The unit of work: How do you measure up?

The Grade 5/6 collaborative planning team at Stringybark Primary School planned a minimum of one "generative topic" each term. These units of work focused upon a guiding question or idea for the students to investigate through tasks and activities designed to support the understanding goals. The units were informed by the Essential Learnings, with key element outcomes identified that were relevant to the generative topic and that would guide the development of understanding goals.

In this case, the unit of work "How do you measure up?" had been planned and implemented alongside another unit of work, "Me: An Author," exploring different writing genres and resulting in the students publishing their own books. The teachers had originally intended the two units of work to be more connected, with the understandings of measurement influencing the planning and publishing of the books, but they "found that the link wasn't naturally there and agreed that there was no point ... forcing a connection that just didn't seem to be right so they ended up running as two distinctive units of work" (Ophelia, 14/12/05). Natural connections made by the students between these two units of work were explored by the researcher during the Phase 3 student interviews.

In planning "How do you measure up?" the teachers wanted to develop students' capacities for inquiry and reflective thinking within the context of their mathematics learning. Two understanding goals were included for the important measurement-related concepts identified from the Being numerate curriculum documents as being relevant for the upper-primary grades. In addition three goals related to the Thinking Essential Learning were included. The five understanding goals were:

- Objects and events have attributes that can be measured and there are standard units that we use to describe and communicate measures of attributes.
- We use our knowledge and understanding of measurement to answer questions about our world.
- Students will understand how to pose and define a problem, clarify the issues involved and select and monitor the most effective process to use.
- Students will be able to collect and record information, with an understanding of accuracy and reliable results.
- Students will understand that reflective thinking is a deliberate process ... and that it is used to develop and refine ideas and beliefs and to explore different and new perceptions.

(Grade 5/6 planning team documentation, 21/6/05)

The initial unit plan was revisited by the collaborative planning team as the unit of work progressed, and in response to the shared needs of the students. The final version of the unit plan was completed on 1st August 2005, with the major change being the inclusion of a further task related to the second understanding goal, and involving the relevance of measurement to different occupations. A culminating performance was also designed although Ophelia did not implement this with her class due to time constraints.

7.3 Numeracy

This section presents the results for this case within three main sections: Ophelia's beliefs about numeracy, the enactment of numeracy in the classroom with Ophelia and her students, and the student learning outcomes of six individual students.

7.3.1 Ophelia and her beliefs about numeracy

Ophelia, being relatively new to the teaching of mathematics, was very open to learning new ideas and was shaping a philosophy of mathematics teaching very much informed by the Essential Learnings' ideas on the teaching of mathematics, in which the importance of being numerate was emphasised rather than purely knowing and doing mathematics. Ophelia felt that numeracy was more than mathematics and reflected upon whether she was putting this into practice in her classroom. She described numeracy as much more than mathematics as she discussed her aim of supporting her students to "develop confidence," "problem-solving skills," and the ability to "use" and "apply" mathematics. In discussing the varied aspects of learning that she saw as contributing to numeracy, Ophelia described them as "all essential parts of a person being numerate" and "all areas that are sort of addressed within [the] classroom." The following subsections detail how Ophelia's conversations about her teaching could be described according to the five dimensions of numeracy developed in the Conceptual Framework (Chapter 3).

7.3.1.1 Mathematics

Ophelia described mathematics as the important "knowledge," "skills" and "processes" that students needed to understand and be able to use in order to be numerate. She had a much broader conception of numeracy, beyond these skills and processes: "You have to teach the processes, but it's also really important to develop the numeracy, the confidence, and the ability to transfer that knowledge to other processes" (Ophelia, 24/5/05).

In sharing her beliefs about teaching processes, Ophelia also emphasised the importance of conceptual understanding over procedural understanding: "They need the understanding of why it is that you do a process … rather than learning it off by heart." Her own developing understanding of mathematics and the connections among concepts was evident as she gave examples of students being taught processes when they were "ready," not just because it was scheduled into the curriculum. "There's no point in teaching them to do a long multiplication process when they're not comfortable with place value." Ophelia explained her approach in not teaching skills to the class as a whole, but rather "teach[ing] it individually when [she could] see that somebody has got that understanding and they're almost there but they just need to know a process, then [she] would teach them that process."

Ophelia gave examples of open-ended mathematics questions she had recently begun incorporating into her numeracy time to focus on conceptual understanding, such as "the answer is one hundred, what could the question be?" and "here's the graph, explain what the story might have been to go with the graph."

7.3.1.2 Reasoning

Ophelia saw a particularly strong connection between the two Essential Learnings, Thinking and Communicating, of which Being numerate was one key element. Thinking and its role in the teaching of numeracy was evident in Ophelia's conversations. She described the majority of her numeracy teaching as involving an emphasis on "problem-solving" with the discussions that students had being an integral part of numeracy. The students were encouraged to explain "how they actually did it," and Ophelia "[found] out about a student's numerical understanding" by their explanations.

Ophelia described two main outcomes to her problem-solving approach: first, students were being supported to develop "a language … to explain their thinking step by step" and second, they were becoming aware that "there is more than one way to approach a problem." Both of these outcomes support the important role that reasoning played in Ophelia's numeracy pedagogy. Ophelia was starting to move towards the use of more open-ended questions in her numeracy teaching because of the possibilities that this provided for the students to "use and share different strategies," thereby giving student reasoning an important place in the numeracy classroom.

7.3.1.3 Attitude

Ophelia described numeracy as "developing confidence with maths." It was this confidence that she saw as enabling her students to participate in a problem-solving classroom environment. Ophelia wanted her students to feel comfortable sharing their ideas, strategies, and solutions, and was aware that the development of a positive attitude toward numeracy was important in helping them to do this.

As Ophelia shared her desire to encourage students to apply their understanding of mathematical ideas and processes to different problems on which they were working, it was evident that she aimed to support students in taking risks with their mathematical learning.

7.3.1.4 Context

In describing numeracy, Ophelia talked about the importance of students being able to "use numeracy in the real world" and to be able to "apply [mathematical] knowledge to new situations." Although no specific examples of context were given, Ophelia did refer to the "blending" of context and mathematics, when she described how she would teach students a specific mathematical process when they needed it, based on their need to apply mathematics skills and processes relevant to the questions they were answering.

7.3.1.5 Equity

With Ophelia focusing on developing her own view of numeracy teaching and sharing her focus on the development of problem-solving skills with her students, this dimension of numeracy was not well developed. Her problem-solving focus did, however, imply that she wanted her students to engage with their learning and with the learning of their peers.

7.3.2 Numeracy as enacted in the "How do you measure up?" unit of work

Between morning tea and lunch, Ophelia's students spent a dedicated one hour and twenty minutes on their numeracy learning. During this time Ophelia provided students with learning experiences related to the mathematical area of focus, in this case, measurement. She also continued to provide students with opportunities to develop their number sense throughout these lessons. During the fifteen classroom visits undertaken in this case, detailed observations were recorded and analysed, enabling a picture to be created of Ophelia and her students' numeracy classroom according to the five dimensions of numeracy as proposed in this thesis.

7.3.2.1 Mathematics

Ophelia's numeracy block consistently began with a mathematics game as a context for exposing students to number facts and times tables, and for assessing their understanding without testing them. This was followed by a group of twelve short calculation questions covering a wide range of mathematical learning areas and including the four operations: place value understanding, factors, patterns in the form of number series, and measurement conversions. Students could either calculate their answers mentally or use pencil and paper to support their thinking if they preferred. The following set of questions is an example.

1. 567 + 132
2. 181 – 63
3. Product of 90 and 8
4. Remainder when 51 is divided by 6
5. ¼ of 20
6. Average of 151 and 179
7. Is 60% less than 0.59 ?
8. 9300 ÷ 10
9. Next ordinal number after 107th
10. $4^2 + 6^2$
11. Total cost if I bought nine 45 cent stamps and twelve 50 cent stamps
12. Total cost if I bought 5 pens at 90 cents each and 3 books at $4.50 each

(Classroom observation, 1/8/05)

The majority of the numeracy lesson was spent on solving problems related to the measurement unit of work. Ophelia initially found out students' prior knowledge in the area of measurement by asking them what measurement is used for, what things can be measured, and with which units of measurement they were familiar (Figure 7.1). The unit of work covered the topics of time, length, area, volume, mass, and temperature and, over the term, students were provided with opportunities to develop their measurement language, their capacities to select and use appropriate measurement tools and measurement units, and also their capabilities in estimation and accurate measuring.

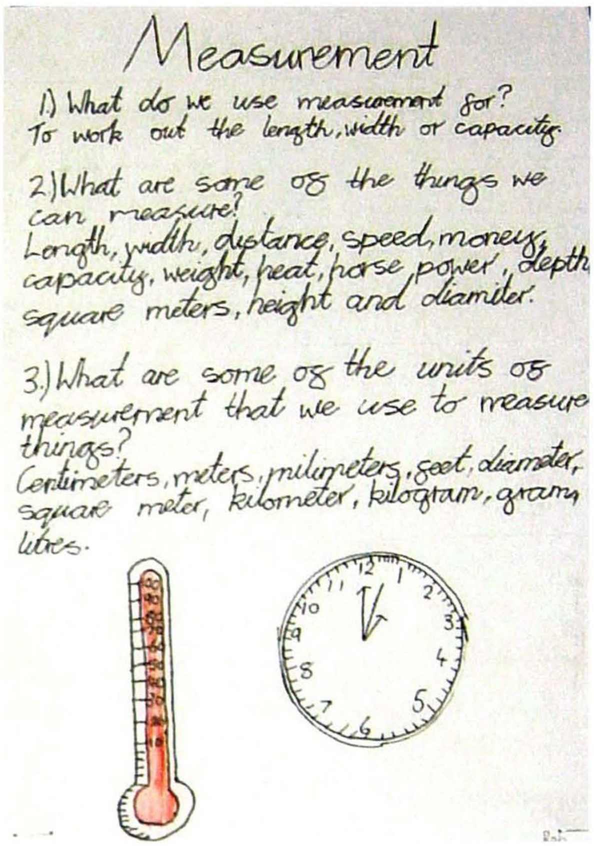


Figure 7.1. Example of student's prior knowledge.

Ophelia then introduced a number of questions and open-ended tasks each week that required the students to select and apply measurement processes and skills, to use resources, and to work together to solve the tasks. Although Ophelia would introduce tasks that covered a few different measurement topics, such as time, weight, and length, as the unit of work progressed Ophelia began to focus on specific topics to address those concepts that the students found more challenging, such as capacity and volume. The following questions represent examples of the tasks given to students during the first half of the case study. The questions selected demonstrate the variety of topics, a shift from one- and two-dimensional concepts to three-dimensional concepts, and also a move toward incorporating more open-ended questions.

> Draw a 1m long line on a piece of A4 paper.
> Find a collection of objects that together weigh 1kg. What were they?
> (Classroom observation, 27/6/05)
>
> Estimate the length/height of the following objects: door, person opposite, your foot, your desk, art area floor, the tote trays
> Write down your estimation and what you have based your information on.
> Measure these objects. Record their actual length/height. How close/far off were you? (Classroom observation, 11/7/05)
>
> What size containers would you need if you were sharing 1½ litres of lemonade among three people? (Classroom observation, 19/7/05)

Key measurement concepts on which Ophelia spent more time with her students as the unit progressed included the relationship between area and perimeter, conservation of area, and a conceptual understanding of volume and capacity.

> A school bus has a 2600 kg carrying capacity. How many people could it carry? (Classroom observation, 21/7/05)
>
> What does $1m^2$ look like? (Classroom observation, 21/7/05)
>
> Estimate the capacity of my mug. What unit of measure would you use? Measure its capacity. (Classroom observation, 15/8/05)
>
> A rectangle has an area of 36 cm^2. What might its perimeter be? (Classroom observation, 18/8/05)

At all times students were encouraged to draw or make models to represent the concepts with which they were working. This was of particular relevance as the learning focus shifted from area to volume, with students making nets and forming three-dimensional shapes to help them develop a conceptual understanding of volume (Figure 7.2). Ophelia had noticed that some of the students found calculation of volume, and drawing and understanding three-dimensional shapes such as a cube, very difficult (Classroom observation, 15/8/05). As a result she moved from using worksheets to tasks involving the students constructing three-dimensional shapes. Understanding and calculating volume was supported by the visualisation that came with constructing three-dimensional shapes.

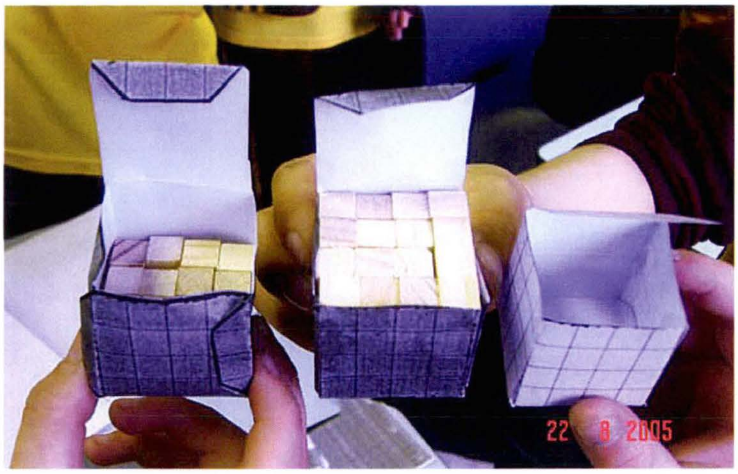


Figure 7.2. Constructing nets and considering the volume of shapes.

7.3.2.2 Reasoning

With key words from Bloom's taxonomy (Bloom, 1956) hanging from the ceiling in Ophelia's classroom, the place of thinking in developing capable learners was clearly valued by Ophelia. She encouraged her students to share their answers and to explain how they had worked out their solutions. Ophelia did this with the whole class at specific times during the numeracy lesson, both after the short-answer calculation questions, and sometimes at the completion of the open-ended tasks, or at the end of the lesson. Ophelia regularly asked her students how they worked things out and what they needed to know to answer the question or to complete the task. She valued the processes students went through to reach an answer and used questioning techniques to develop in her students the ability to explain.

> If you couldn't have done that, what would you do? (Classroom observation, 4/7/05)
> Can you explain why? (Classroom observation, 25/8/05)
> How accurate were you? (Classroom observation, 1/8/05)
> Tell me what you have done so far. (29/7/05)

It was this type of questioning that enabled students to consider other strategies for solving the same problem. This occurred with both the short answer questions that Ophelia posed, as well as the more open-ended questions. With the problem "Multiply 145 by 20" (29/7/05), three solutions were shared by students:

1. 5 x 20 = 100, 4 x 20 = 80 and add a 0 for the tens makes 800, and one more hundred is 900, then 100 x 20 = 2000, so the answer is 2900
2. I doubled 145 to 290 then added a 0 [because x 20] = 2900
3. Traditional method of long multiplication (Classroom observation, 29/7/05)

Figure 7.3 provides examples of the diversity of ways students represented 10 cm^2 when exploring relationships between area and perimeter.

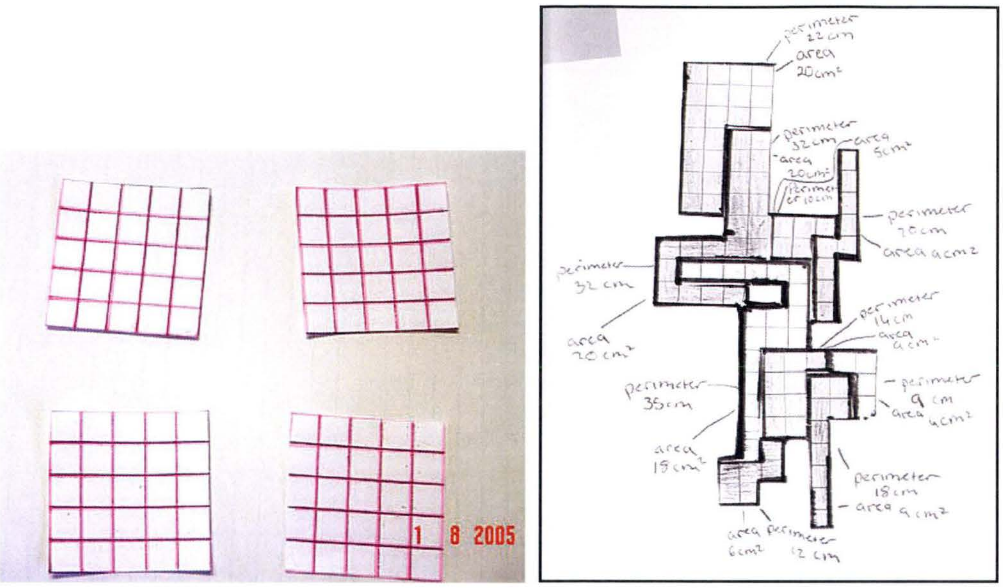
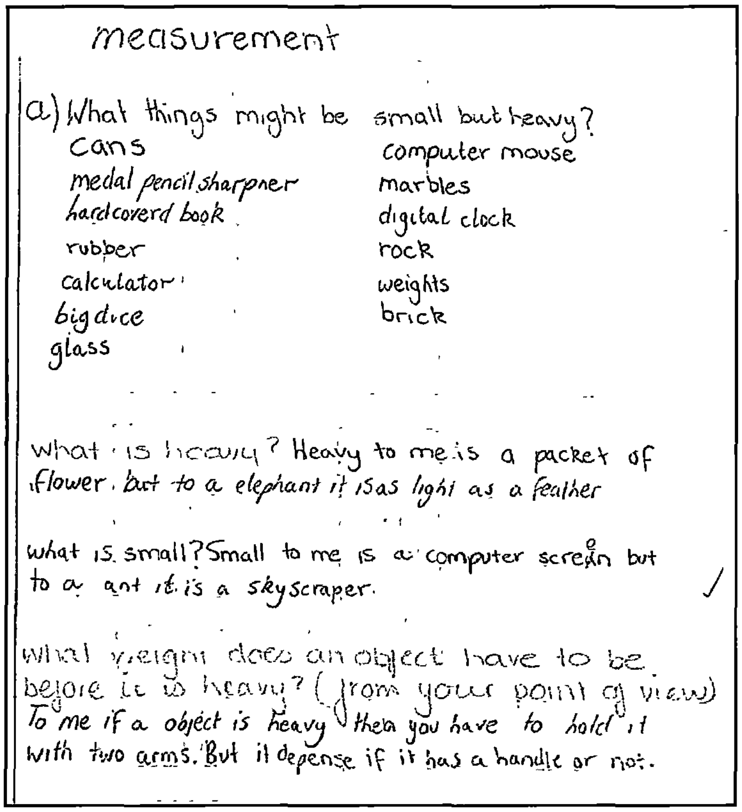


Figure 7.3. Representing 10 cm^2.

When students were working independently or in small groups Ophelia would spend time moving among groups and questioning them about their work. She challenged students' preconceptions with questions like "What might be your definition of small?" (Classroom observation, 27/6/05), and "Does it say it has to be straight?" (Classroom observation, 29/7/05). Students were also encouraged to record their thinking in their mathematics books as seen in the following example of a student considering objects that might be small, but heavy (Figure 7.4).


measurement

a) What things might be small but heavy?

cans
medal pencil sharpner
hardcoverd book
rubber
calculator
big dice
glass

computer mouse
marbles
digital clock
rock
weights
brick

what is heavy? Heavy to me is a packet of flower. but to a elephant it is as light as a feather

what is small? Small to me is a computer screen but to a ant it is a skyscraper.

what weight does an object have to be before it is heavy? (from your point of view)
To me if a object is heavy then you have to hold it with two arms. But it depense if it has a handle or not.

Figure 7.4. Student work sample, small but heavy?

Ophelia was becoming more comfortable with students sharing their ideas with each other while working together on tasks. Early on in the case study she had shared that this resulted in a noisier classroom environment (Classroom observation, 27/6/05). Although she still appreciated a balance of times when students were working quietly at their desks and times when they were moving around and working together, Ophelia placed a great deal of value on the learning opportunities for students, and the assessment opportunities provided to her, by giving her students opportunities to discuss their ideas with each other and to make explicit their reasoning.

At times, Ophelia discussed her numeracy teaching with the school's numeracy coordinator, Samantha. As Samantha and Ophelia were part of the same planning team, Ophelia would often share her ideas and consider ways to improve her teaching of numeracy. It was these discussions that led Ophelia to shift gradually to including more open-ended tasks in her numeracy classroom as she wanted to give her students opportunities to use their own strategies and share their thinking

(Classroom observation, 18/8/05). Ophelia was very excited about the positive effect this had on students in her classroom who had found traditional skills-based questions more difficult. Given the opportunity, these students began to explore tasks in a more open-ended way, had posed some excellent questions, and produced work to a high standard. She also shared that some of the traditionally brighter students still focused on getting to an answer when doing open-ended tasks (Classroom observation, 29/7/05). Ophelia thoughtfully used questions to extend students' approaches to more open-ended tasks.

> How might you get an accurate measurement? (Classroom observation, 27/6/05)
> How many beats does a normal adult heart beat? (Classroom observation, 18/7/05)
> Now you have a metre square what can you measure with it? (Classroom observation, 29/7/05)
> How did you know it wasn't 3157? (Classroom observation, 2/8/05)

Two students in Ophelia's classroom decided to take on the challenge of using understanding of area, in particular $1m^2$, to calculate the area of their table, that was a trapezium shape (Classroom observation, 29/7/05). Once they had cut a piece of paper to the size of the table, they began to rule it up into 1cm squares until they realised they could be more efficient by also using larger shapes of 10cm by 10cm, for 100 cm^2 and some 7cm by 7cm, for 49 cm^2 (Figure 7.5).

Figure 7.5. Finding the area of a trapezium-shaped table.

The emphasis on reasoning in Ophelia's classroom was well aligned with the understanding goals for Inquiry and Reflective thinking that were planned for by the Grade 5/6 planning team:

- Students will understand how to pose and define a problem, clarify the issues involved and select and monitor the most effective process to use.
- Students will be able to collect and record information, with an understanding of accuracy and reliable results.
- Students will understand that reflective thinking is a deliberate process ... and that it is used to develop and refine ideas and beliefs and to explore different and new perceptions.

(Grade 5/6 planning team documentation, 21/6/05)

7.3.2.3 Attitude

Ophelia created a supportive classroom environment by constantly reflecting on her own practice and how she might engage students in their numeracy learning. The students were very comfortable learning together and they were given opportunities to share their work, both formally with the whole class and informally with each other (Figure 7.6). Ophelia's students did not hesitate to ask questions, seek clarification, and share any difficulties they were having with their learning.

Figure 7.6. Students working mathematically and sharing ideas together.

Although students were encouraged to complete all of the required tasks by the end of each week, they were able to select which tasks to complete first, and how to go about it. Activities were designed to be hands-on, when possible, with resources and materials available to support students in their learning. Both the verbal and body language observed conveyed the positive attitudes of students in

Ophelia's classroom towards the tasks, the context, and the mathematics. Figures 7.7a and 7.7b are two examples of the ways in which students used resources to support their learning.

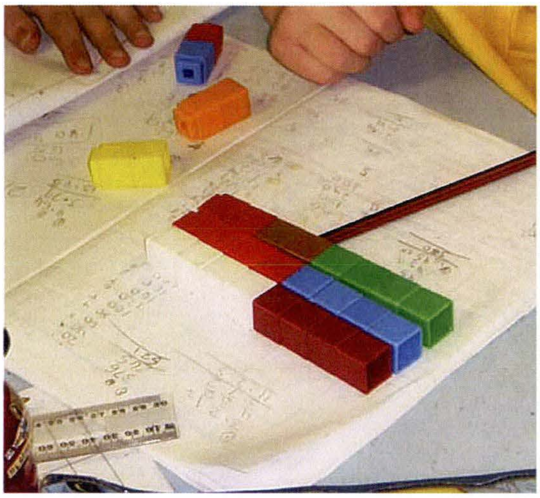


Figure 7.7a. Using centicubes to calculate volume. *Figure 7.7b.* Using materials to weigh 1kg.

Ophelia enjoyed creating displays in her classroom, and there was always a variety of student work around the room (Figure 7.8). During this unit of work, students' measurement learning was displayed alongside their learning in other areas. Ophelia was also clear about the learning goals of the unit and both displayed them in the classroom, as well as referring to them throughout the unit of work so that students were aware of the reasons for their learning (Figure 7.9). The students themselves were asked to choose samples of their numeracy work selectively at the end of the unit of work that would show that they had demonstrated the understanding goals for the unit (Classroom observation, 29/8/05). This explicit approach to setting the learning goals for the unit of work, and allowing students to demonstrate how they had achieved them, was a further consolidation of Ophelia's purposeful aim in developing positive attitudes toward numeracy in her classroom.

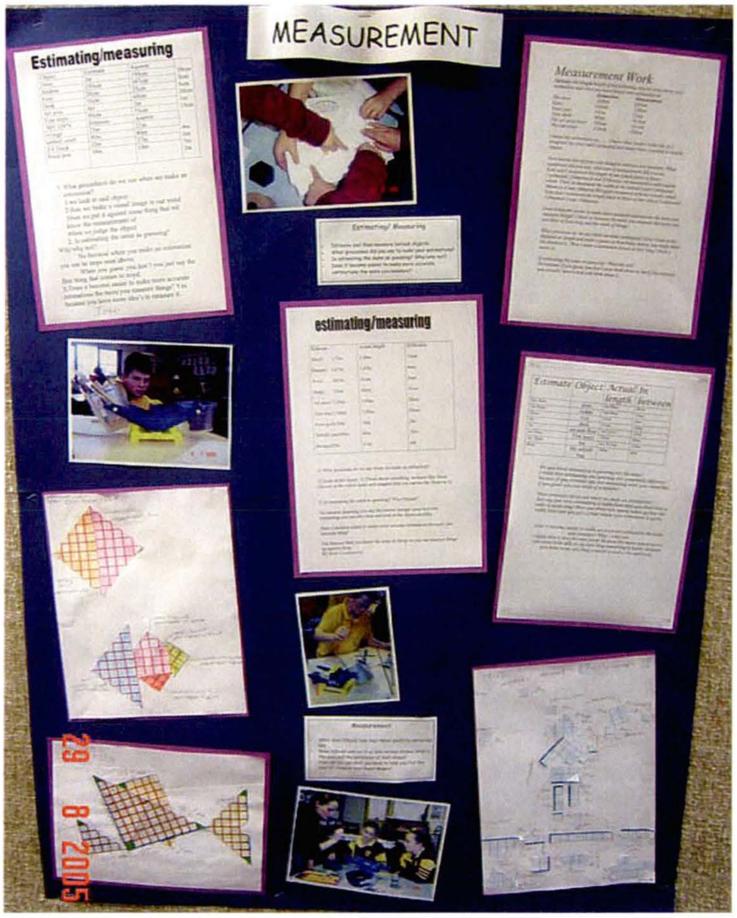


Figure 7.8. Display of students' measurement work.

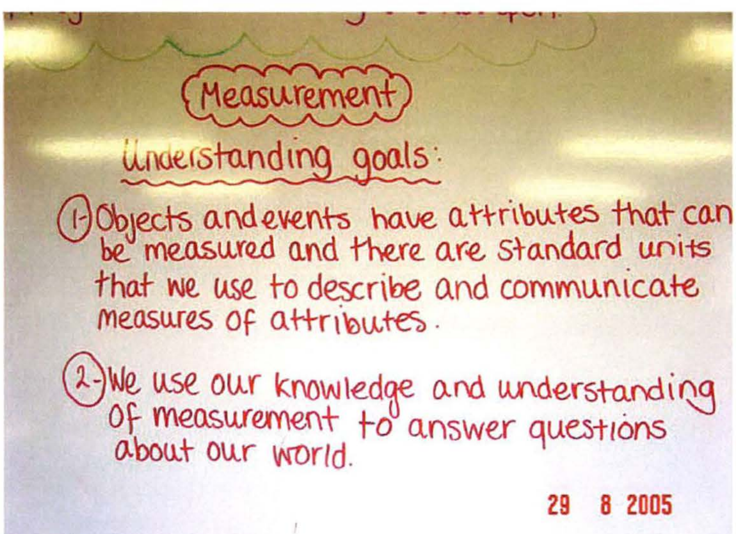


Figure 7.9. Understanding goals for the measurement unit of work displayed on the whiteboard.

Ophelia wanted her students to feel comfortable taking risks with their learning, and in discovering that open-ended tasks were engaging a wider range of students in their mathematics learning, she sought to continue to provide a variety of both structured and open-ended tasks so that all her students would both enjoy and experience success in the numeracy classroom.

7.3.2.4 Context

The unit of work "How do you measure up?" had two content focused understanding goals (Figure 7.9) that formed the basis of planning for student learning throughout the term. Inherent in these learning objectives is the practical and contextual nature of measurement. Ophelia, together with her colleagues, planned for students to develop the knowledge and skills that would enable them to measure accurately and to describe and communicate these measurements using formal measurement language and units. They also wanted their students to see the relevance of measurement in life, both at school and outside of school.

During this case, Ophelia provided both structured and open-ended tasks for her students to work with. There were many short calculation questions each week that required students to focus purely on the mathematics of conversions and identification of units and attributes. For these questions, context was not included. Some tasks incorporated some aspect of context in the question posed, such as calculating the cost of orange juice or the capacity of a paddling pool. The open-ended tasks, however, incorporated many varied contexts. Predominantly, the tasks were hands-on and contextually relevant to students, such as estimating and measuring objects, measuring a person, considering the capacity of a school bus, and finding objects that collectively weighed 1 kg. As shown in Figure 7.7b for example, students were given the challenge of finding objects in the classroom that would together have a mass of 1kg to support their developing understanding of mass (Classroom observation, 27/6/05). This practical task, using real-world objects, provided a meaningful context for students to consider what 1kg represented, as compared with a more traditional mathematics tasks that might have had students use other smaller weights to collectively weigh 1kg. It is noted that in this case the teacher used the words weigh and weight to indicate mass.

Throughout the case study Ophelia observed that some students who had found traditional skills-based questions in mathematics quite difficult, when presented with the more open-ended tasks that were practical and contextual in nature, did "great work" and took things in quite unexpected directions (Classroom observation, 29/7/05). For the task "A school bus has a 2600 kg carrying capacity. How many people [Grade 6] could it carry?" (Classroom observation, 27/705), two students, who had struggled with pure mathematical calculations involving large numbers and multiple steps, were very creative. They came to a very realistic and considered answer based on the contextual nature of the question. The solution incorporated addition, subtraction, division, averages, and rounding (Figure 7.10).

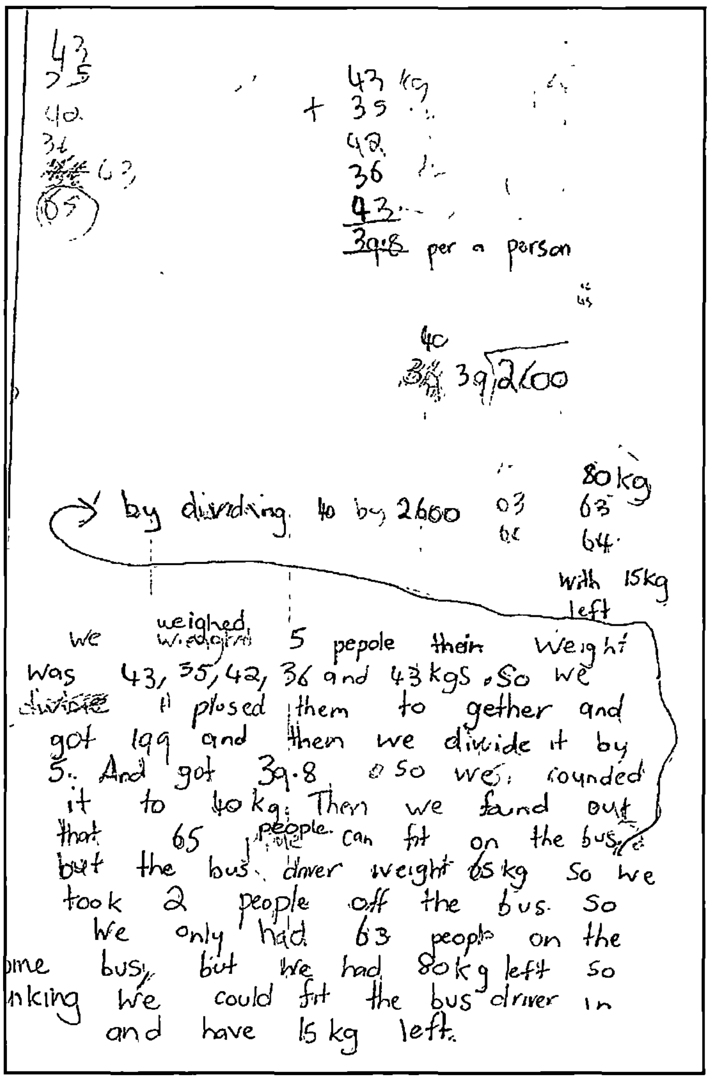


Figure 7.10. Student work sample for the bus problem.

7.3.2.5 Equity

In Ophelia's classroom, students were supported in their learning and encouraged to contribute and share their ideas. They were also questioned to extend their understanding. They were provided with opportunities to work both independently and together, so that they could see the different ways that tasks might be approached and solutions reached. Students also had opportunities to make their own choices, with tasks having a degree of flexibility, such as choosing which tasks to solve, and what materials and resources might be used to go about it. This enabled Ophelia's students to access tasks at their own level and also to extend themselves by working with others.

Ophelia shared the challenges of deciding how much information to give students when supporting their learning, so that they have enough "to get them going" but not too much so that they "discover things for themselves … and show the students [who are] at a higher level, compared to those at a lower level staying with the standard things to measure." (Classroom observation, 18/7/05). In working to give students more open-ended tasks Ophelia was providing the foundations within a supportive context that encouraged students to make decisions, to consider and question assumptions, and to consider other students' points of view in relation to mathematical problem solving.

Although the contexts embedded in tasks during the unit of work were many and varied they were not designed to be socially or politically loaded. The culture of the classroom, however, was being established that would enable students to develop the capacities that might eventually be applied in social and political contexts. At this stage Ophelia was focusing on the important mathematical foundational learning together with a pedagogical shift toward tasks that would support students' deeper understanding of mathematical concepts and, in particular, their reasoning capabilities.

7.3.3 Student learning

The six students interviewed in this phase of the study were asked to describe and discuss the tasks they completed during the measurement unit of work. The interviews were analysed with respect to the five dimensions of numeracy as developed in Chapter 3. The following subsections detail how the students' learning was evidenced across the five dimensions of numeracy. The reporting of the results is intended to provide representative evidence of the range of responses for each category within the dimension. In doing this, at least one comment from each student is included for each of the five dimensions.

7.3.3.1 Mathematics

The numeracy unit of work in this case had two planned and articulated mathematical learning outcomes relating to the learning of a variety of measurement attributes and the use of formal units of measure appropriate for the respective attribute. Students undertook many measurement tasks, both closed and open, to support student learning across a range of measurement attributes. Within this context, the students demonstrated specific mathematics understandings in the area of measurement, across a range of levels. Table 7.1 summarises the observed categories of Mathematics for each student, as evidenced by the student interviews.

Table 7.1
Student learning: Mathematics dimension (Stringybark Primary School - Ophelia)

Student / Category	S7	S8	S9	S10	S11	S12
Reading and describing measurement	✓	✓	✓	✓	✓	✓
Making meaning from measurement	✓	✓	✓	✓	✓	✓
Analysing and interpreting measurement	✓	✓			✓	✓
Evaluating and transferring measurement understandings to new or different contexts		✓				✓

All of the six students were able to *Read and describe measurement.* They did this using appropriate measurement language and units of measure and across a variety of tasks, involving the attributes of length, area, volume, weight, and capacity. The following are examples.

> Capacity, I didn't really use that term very often and now I use it a lot ... like a jug can hold so much capacity, have so much capacity, and say it has a capacity of 125 millilitres. [S11]

> You can go if it's five centimetres down and five centimetres across, then it's five times five. [S12]

All of the six students could also *Make meaning from measurement.* This was evidenced both in relation to specific aspects of learning about a particular measurement concept, such as conservation of length or area, and also in relation to appreciating the purpose of measurement, providing accurate measures, and units of measure more broadly. In the first two examples, students shared specific discoveries, such as one metre in length does not have to be a straight line, and the relationship between time on a clock and fifteen minutes representing a right angle. The third example demonstrates an understanding that units of measure provide accurate information and in the final example, Student 10 shares the value of measurement in describing many activities undertaken in real life.

> I found out that [a straight line] was a one metre line, but that [a curvy line] was also a one metre line, you'd never think that ... I used a piece of string and sticky tape. [S11]

> [A right angle] is ninety degrees, so you can see at three o'clock [ninety degrees] and there was a pattern.
> Int: What was the pattern.?
> S12: It goes five, it goes one one and two two , three three, four four, five five [referring to the seconds going up from 20,25,30,35,40,45 etc.], and it goes up by ones there [referring to the hours going up in ones] ... the question was at what times are the hour hand and the minute hand at right angles ... and that helped because I didn't know what right angles were [before]. [S12]

> [Units are included] so you know what you are measuring, instead of just 98 metres [for example]. I did that in centimetres not metres so it was more accurate. [S9]

> If you didn't have measurement you wouldn't know how much water and milk to bake a cake. You wouldn't be able to describe how [large pause] I just thought of something! Nearly everything you can do can do something with measurement...you can even use it to do something in the air... yeah oxygen and how clean the air is and the same with water. [S10]

Four of the six students demonstrated learning at the level of *Analysing and interpreting measurement*. These students were able to consider multiple aspects of the task they were undertaking and shared their interpretations and findings. Two examples are presented.

> Like here [estimations of] the height of the door, foot, desk, and then actual measurements of them ... some of them [estimations] were a long way off and some of them really close, one I was two centimetres off and one I was 27 [centimetres off] … cause if it is small you can base it on like a ruler, cause we use rulers everyday and we know how big they are. So if it is small, it is easier and if it something really far away you can't really tell how far it is. Kate and I measured the oval and … I estimated 250 metres and it was 279 metres. [S7]

> It was still a hundred centimetres squared but it was just in different shapes.
> Int: So the area was still a hundred centimetres squared, what about the outside, what was the perimeter of that?
> S11: It's one metre isn't it? Oh yeah, forty [cm].
> Int: So when you measured all these shapes, did you find anything out about the perimeter?
> S11: The forty, it changed, the forty went up because it was in different shapes. Even though the area stays the same the perimeter doesn't have to. [S11]

Of these four students, two went on to *Evaluate and transfer measurement understandings to new or different contexts*. Student 8 considered the implications of measuring and the use of area, size of pictures and text, and the overall use of space in making a book for a literacy unit of work that was being taught alongside the numeracy unit.

> Int: How did you use any of the things you learnt in measurement to create your book …?
> S8: [It] relates to how big the page is going to be and how much text and how much space you have for your pictures. …
> Int: Why do you think the page looks good that way?
> S8: Because you've got your simple text and you've got a lot of area around it so it's not too big in your face and you've got a nice simple picture to go with it. [S8]

Student 12 began describing a task involving finding how many people could fit on a 2600kg bus. The student not only used quite complex number calculations involving addition, division, averages, and rounding to come to a solution, but also went on to evaluate the implications of having a bus driver on the bus, and made decisions to cater for that scenario.

> Well I weighed five people, at 37, 45, 42, 36, and 43, so I plussed them all together and then divided them [by 5] and I got 199… and [I] got an average of 39.8 kilos. So I rounded it to 40 kilos and then found out when I divided 40 into 2600 and found out sixty five could fit on the bus, cause that was the answer when I divided. But then we had the bus driver, so we weighed [the teacher] and she weighed 65 kilos so we needed to take two people out of the bus [so the bus

> driver can fit on]. So we had 80 kilos. So we plussed the 65 in. So we had sixty four people in the bus with fifteen kilos left. [S12]

7.3.3.2 Reasoning

Observed student learning in this dimension of numeracy very closely matched that for the Mathematics dimension. Table 7.2 summarises the observed categories of Reasoning for each student, as evidenced by the student interview.

Table 7.2
Student learning: Reasoning dimension (Stringybark Primary School - Ophelia)

Category \ Student	S7	S8	S9	S10	S11	S12
Remember	✓	✓	✓	✓	✓	✓
Comprehend	✓	✓	✓	✓	✓	✓
Analyse	✓	✓			✓	✓
Evaluate	✓	✓		✓		✓
Create						

All of the students interviewed were able to *Remember* and recognise aspects of measurement in their work. Predominant responses involved identifying units of measure relevant to measurement attributes. The following are examples.

> There are one thousand [grams in a kilogram]. [S7]

> Yards, miles, kilograms, kilometres, centimetres and all that stuff. [S10]

> When you're talking about capacity, you don't talk about kilos; you don't say centimetres, you say litres. [S12]

All six students also demonstrated an ability to *Comprehend.* In this category, students typically explained the purpose of measurement, and an understanding of the value of units of measure as exemplified by Student 9 and Student 11.

> We use measurement to find out how tall or wide something is, or how heavy things are and what are the things we can measure and what are the units of measurement that we can use to measure things. [S9]

> [Without measurement] 8.5, that would just sound like eight and a half, you wouldn't know what it was. It sounds better in centimetres term, or metres or litres or whatever you are trying to measure. [S11]

The students also evidenced more specific comprehension related to measurement attributes or concepts of measurement, and examples are included in Section 7.3.3.1, in relation to the category *Making meaning of measurement.*

Four of the six students *Analysed* information related to the measurement unit. Student 7 distinguished between the learning that occurred for students when doing volume calculations on a worksheet, as compared with creating three-dimensional shapes to help develop a conceptual understanding of volume. In doing this, the student was making a connection between measurement and shape. In both examples the students were analysing in order to be able to evaluate, as they were beginning to make judgements toward the end of their comments.

> When we were doing the cubes, when we were doing our volume sheets some people thought that when we were making it three dimensional that was another shape behind it but when we actually got to make it you could tell it is actually not another shape behind it and so it was easier to actually make it and see it. [S7]

> Pages, how big the pages are and the left, you've got to have the widening not too far ... and gaps, spacing between letters .... Probably grade four up and because it's a chapter book you can't have like size 36 font ... you have to kind of measure the book and try out different styles, like fourteen might be too big , ten [font] might be too small so you need to try them out. [S12]

Although only two students demonstrated an ability to *Evaluate* within the Mathematics dimension (in which mathematical data as presented within measurement tasks were required to justify ideas, decisions, and inferences, within the Reasoning dimension) four of the six students evidenced this level of thinking.

> There is no point having a book for kinder [children] with little tiny writing ... it's better to have the book that way round [landscape] and to have bigger writing. [S7]

> The area and problem solving skills [are most important] like we did a lot of problem solving ... on the page it is simple, do this do this do this [but] with real objects they can be different sizes and a lot bigger than you or a lot smaller than you. ... well it's a lot harder to do. Just say you've got a house and it's that long and this big and how much water is going to fit inside it. It would be a lot harder than just doing [a problem] on an A4 piece of paper. [S8]

> It is better if you actually make the shapes and objects because when you just do that [worksheet] you are just counting and it is not very interesting, but when you write it down and use it to explore and measure and find out things but just say you've got a whole lots of shapes on the page all you have to do is measure them with a ruler, but if you teach intelligence, it makes it a lot more fun. [S10]

Reasoning at the highest level, *Create*, was not evidenced in the student interview data.

7.3.3.3 Attitude

As students discussed the different tasks they had undertaken during the measurement unit of work, their attitudes toward their learning were revealed by both the comments they made and also by the manner in which they conveyed them. Table 7.3 summarises the observed categories of Attitude for each student, as evidenced by the student interviews.

Table 7.3
Student learning: Attitude dimension (Stringybark Primary School - Ophelia)

Student / Category	S7	S8	S9	S10	S11	S12
Confidence/ self-efficacy	✓	✓	✓	✓	✓	
Interest	✓	✓		✓	✓	✓
Enjoyment	✓	✓		✓		
Intellectual stimulation	✓	✓		✓	✓	
Diligence/Perseverance	✓	✓				✓
Appreciates value of mathematics	✓	✓	✓	✓	✓	✓

Of the six students interviewed, three evidenced a highly positive attitude toward their numeracy learning, with Students 7 and 8 demonstrating all of the affective factors. Five of the six students displayed *Confidence* about their own capabilities in the learning of measurement. At times the confidence demonstrated was related to a specific aspect of the measurement work, such as in Student 10's remark about volume, but mainly students' confidence in this case revealed an overall confidence about their numeracy capabilities in the area of measurement.

> This year I really know how things are [with] measurement. [S9]
>
> I [hadn't done volume before] but I could work it out. [S10]
>
> I understand it really well and didn't find it that hard. [S11]

In terms of *Interest* five students discussed their interest in specific areas of the measurement unit, such as area, volume, capacity, and estimation, and as exemplified by Students 11 and 12. Student 8 shared his interest in shape as it related to tangrams.

> I'd really like to learn more about cubic centimetres and metre squares and all sorts of things. [S11]

> I'd like to learn how to estimate stuff better because I am not too good at it or capacity like the cubes, like how many cubes, one of them volume things. [S12]

> Well I like the tangrams, which is a square that is in lots of different shaped blocks and you can make lots of different shapes with it. Chinese used to have it and I used it to make a goose which has got triangles here. [S8]

Student *Enjoyment* went beyond students discussing areas of learning that they found interesting and incorporated students being clearly excited about their learning. Three of the six students displayed this sense of fun when sharing their work, and two examples are included.

> I'd like to make a maths book. [expressed with excitement] [S7]

> I really enjoyed the measurement [problem] where you had to figure out there was a school bus that could carry 2600 kilograms, then how many students? And we worked it out and eventually came up with the answer. Well no-one knows [what the answer could be] because anyone can weigh anything. [S10]

Four of the six students demonstrated the *Intellectual stimulation* they gained by exploring measurement concepts. This was evidenced by the sense of satisfaction they gained from thinking and talking through problems and the way they described working on tasks that may have been quite challenging.

> [With the measurement work] I'd go to my Nan's sometimes and we'd work on it together... [talk about] what terms are, like there were some tricky ones that we had to look in the dictionary and find out what they were. [S11]

For Student 8 her intellectual stimulation was maintained by being able to experiment and test her theories practically and for Student 7 the intellectual stimulation gained was associated with an explicit expression of how difficult the task was.

> Actually fill [the cup] with water and make sure it's accurate and explore it and find different ways to do it... it gives a picture of it and [you] actually get to do it and interact with the object instead of just playing with a piece of paper. [S8]

> We have done measurement work [before] but not as hard, like to different levels ... we had to do harder sums, longer sums, and different types, and we had to do area work with real objects instead of ones we had make [up]. [S7]

Three of the six students demonstrated their *Diligence* by sharing the value of persevering on measuring tasks as exemplified by Student 12.

> Yes the more I measured, I got better at it because at first you have no idea. You can guess but then like once you remember stuff you can actually see. [S12]

For Student 8, her perseverance was situated within the practical nature of the tasks in which she engaged. The same student evidenced intellectual stimulation by the same sorts of tasks, with a connection between the engagement she gained by exploring and modelling her ideas with the perseverance she was willing to put in.

> Like we did a lot of problem solving … on the page it is simple … [but] with real objects they can be different sizes and a lot bigger than you or a lot smaller than you. … well it's a lot harder to do. Just say you've got a house and it's that long and this big and how much water is going to fit inside it. It would be a lot harder than just doing [a problem] on an A4 piece of paper. [S8]

All of the students in this case evidenced an *Appreciation for the value of mathematics* when discussing the very practical nature of measurement and its relevance to every aspect of life. Two examples are given.

> What we use measurement for, we use it to find out how tall or wide something is or how heavy things are. And what we can measure and what are the units of measurement that we can use to measure things. [S9]

> I'd like to put it into practice, like every job has something to do with measurement. Someone on my table said that measurement 'oh measurement I won't need measurement for working at a salon' or something and I said 'yeah you have to use it for every job' and that's true, there's no job where you don't have to use measurement. [S11]

7.3.3.4 Context

As discussed in Section 7.3.2.4, the practical and contextual nature of measurement was a key aspect of the unit of work. The goals of the unit went beyond the development of mathematical knowledge and skills, to include the use of mathematics to be able to "answer questions about the world." The students demonstrated specific understandings of the role of context as it related to a diverse range of tasks across a range of levels. Table 7.4 summarises the observed categories of Context for each student, as evidenced by the student interviews.

Table 7.4
Student learning: Context dimension (Stringybark Primary School - Ophelia)

Category / Student	S7	S8	S9	S10	S11	S12
Personal experience of context			✓	✓	✓	
Context integrated with mathematics as presented	✓		✓	✓	✓	✓
Context integrated with mathematics, from both prior knowledge and as presented	✓				✓	✓
Relational understanding of the mathematics and the context and can transfer to new contexts		✓			✓	

When Students 9 and 10 were discussing their learning, *Personal experience of context* dominated their sense-making of measurement. Throughout their interviews, both of these students revealed a preference for discussing the practical tasks, such as measuring objects, and their comments were often about the process of measuring rather than the measurement attributes themselves. After sharing her work on estimating and measuring the heights and lengths of objects, Student 9 used an unrealistic context to explain why you might measure the width of a door.

> So if there was a big truck that had to go through the door, you'd have to measure it to see how big the truck was to see if it could actually go through the door. [S9]

Student 11 also demonstrated his personal experience of measurement when sharing his practical experiences involving measuring tasks with his father.

> Seeing as my Dad's a builder, we use measurement a lot and I helped him build the hand rail and we had to work out ... [S11]

Five of the six students evidenced an understanding of *Context integrated with the mathematics as presented.* The following are examples of when the students remained focused on the measurement data and the context as it related to the task, without being distracted or influenced by personal experience or opinion about the context.

> [The question was] A school bus has a 2600 kilogram carrying capacity, how many people could it carry? And we had to work out the average weight of

people, [so] we decided [it was] for our school, and we decided to measure it on Grade 3 because they are in the middle of our school, and then we worked out their average weight [44 kg] and divided it [2600 ÷ 44] and got 59 … it showed I knew how to work out averages and how to work out capacity. [S7]

> With the door I guessed 2 metres 20 and the actual was 1 metre 90. With Amy I guessed 1 metre 54 and she was 1 metre 44. My foot I guessed 25 centimetres and it was 23 centimetres. The desk I guessed 1 metre and it was 67 centimetres … [S10]

> This measurement thing with [another student] I actually had to put the ruler at his feet and measure him lying down and it was probably a better way of doing it that just drawing and saying what do you think? [S11]

> Let's say you got the ipod, well I've measured a pencil and it was sixteen centimetres so you can say is that a bit bigger and you go, oh yea that might be two centimetres bigger… because at first you have no idea, you can guess, but then like once you remember stuff you can actually see. [referring to his ability to estimate improving with the more things he estimated and then measured] [S12]

Three of the six students demonstrated a capacity to integrate both the mathematics and the context at a higher level when discussing the measurement work they had undertaken during the unit of work. There were times during their interviews when *Context was integrated with the mathematics, from both prior knowledge and as presented.* Three examples are included. In the first example, Student 7 considers the reasons for some estimations being more difficult to judge than others, based upon other known measurements that can be used as a point of reference.

> Like here [estimations of] the height of the door, foot, desk, and then actual measurements of them ... some of them [estimations] were a long way off and some of them really close. One I was two centimetres off and one I was 27 [centimetres off] … cause if it is small you can base it on like a ruler, cause we use rulers everyday and we know how big they are. So if it is small it is easier and if it something really far away you can't really tell how far it is. [S7]

In the second example, Student 11 continued his discussions about building a hand rail with his father with specific inclusion of some of the measurements that were involved.

> … we had to work out like it was about 100 centimetres, yeah it was about 2 metres, 52 centimetres and you had to work out exactly the measurement and all that sort of thing. I really enjoy building, doing that sort of thing, centimetres and metres. [S11]

In working on a task to find out how many people might fit on a 2600 kg bus, Student 12 explained the thinking processes he went through to calculate not only an answer but also an answer that would be contextually relevant based on his knowledge that in reality buses need to have a driver (p. 195).

Student 8 and Student 11 evidenced a *Relational understanding of the context and the mathematics and an ability to transfer this to new contexts*. In both cases the students shared some of the implications of their learning in the numeracy unit of work for the books they were making in a literacy unit of work that they were also undertaking. Student 8 discussed the relationship between the age of the target audience for a book and decisions regarding page size, amount of text, and spacing. Student 11 was also concerned with the borders and size of the text, including the setting out of a table of data to go in his book about football, and undertaking precise measurements to make the table.

> Int: How did you use any of the things you learnt in measurement to create your book …?
> S8: [It] relates to how big the page is going to be and how much text and how much space you have for your pictures. …
> Int.: Why do you think the page looks good that way?
> S8: Because you've got your simple text and you've got a lot of area around it, so it's not too big in your face and you've got a nice simple picture to go with it. [S8]
>
> I said the text it was 3.1 [border] and 18 size font … I had to decide all that at the start before I started writing my book … actually I worked out that, how they won the Brownlow [medal] in the table … I really decided 4.1 was where that might have to sit, like 4.1, 5.3, whatever it was. Int: So the distance between the actual width of each column in the table. [S11]

7.3.3.5 Equity

Table 7.5 summarises the observed categories of Equity for each student, as evidenced by the student interviews. This dimension of numeracy was all but absent from being observed in the student interviews. The students discussed their own learning about measurement concepts, measuring skills, and their experiences during the unit of work. Although there were many contexts used throughout the unit of work, the unit objectives were not looking at the place of context within a broader social or political perspective.

Table 7.5
Student learning: Equity dimension (Stringybark Primary School – Ophelia)

Category \ Student	S7	S8	S9	S10	S11	S12
Personal social engagement					✓	
Awareness of issues						
Considering viewpoints						
Relating mathematical information with social and political consequences						
Challenging inequity						

Student 11 did, however, provide emerging evidence in the student interview, of his *Personal social engagement* when working on measurement tasks. In the following example he shared his thoughts on the best way to represent measurements of greater than one metre. Although he did not clearly justify his position, he had made a decision about the appropriate representation of units based upon his own ideas and also upon the way his peers were recording their measurements. He referred to other students' choices but did not move on to a consideration of others' opinions nor why he had chosen to do it differently.

> So like 162, some kids just did 162 centimetres but I reckon metres probably sounds better, it sounds too plain 162 centimetres. Metres makes it more broken up, that's what I thought 1 metre and 62 centimetres. I also did that for how tall I was, that's what I did. [S11]

7.4 Chapter summary

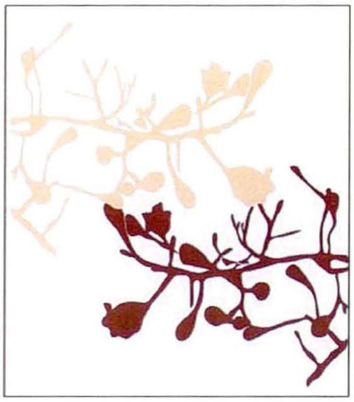
This chapter has shared the beliefs and practices of Ophelia and the learning experiences and outcomes of her students. For Ophelia, the implementation of the curriculum reforms was occurring at the same time as her move from being a specialist secondary art teacher to being an upper-primary teacher responsible for all areas of the curriculum. She was therefore relying on the reforms to shape her practice and found the collaborative and supportive school setting, to which she had moved, to be most helpful. In this case, Ophelia implemented a collaboratively planned unit of work, "How do you measure up?," with understanding goals that covered learning the use of appropriate measurement attributes and units of measure, the relevance of measurement to the real world, and the important processes of inquiry and capacities for reflective thinking.

The dimensions of Mathematics, Reasoning, and Attitude featured in this case. Ophelia had a strong sense of the reforms being concerned with making connections between ideas in order to understand key concepts and equipping students with both the knowledge and the thinking skills that would enable them to make informed decisions. In terms of the teaching and learning of numeracy, Ophelia was interested in supporting her students to go beyond their learning of the mathematics, to developing confidence, problem solving skills, and an ability to use and apply the mathematics in other areas.

> *I think the unit of work went really well in relation to my understanding goals. I think that the children really enjoyed it, they got a lot out of it, and they were really able to demonstrate their understanding of measurement. It became very broad and there wasn't enough time to cover everything that I would have liked to have done. ... I could see the progression in the kids' understanding of measurement. They enjoyed the actual practical process of measuring a range of objects. A lot of them started to pose their own questions about measurement and tried to find answers to it which was really good. ...*
>
> *I was surprised at the level of understanding that some of the children showed, particularly some children who had struggled in other areas of numeracy really grasped measurement a lot more than I thought they would have done. Probably because there was a fair degree of practical work, they actually got to use tape measures, they got to measure the weights of things, and use trundle wheels, and a lot of concrete materials which they really enjoyed. ... I think they were able to see the uses of measurement in real-world situations. They were able to see how measurement could be used outside of school, in the home, the workplace, um, and really got a connection to it knowing that it would be something that they would need to use for the rest of their lives. ...*
>
> *We have talked a lot about the different thinking skills that you can use and about justifying and explaining, questioning and we have talked about creative thinking and all that sort of thing so, for the whole year, the students have been asked to justify their answers and explain their thinking and explain how they did it, and a lot of questioning as well. Can they actually ask questions of themselves and of their own work and then try to find a way to answer those questions and those problems? So that has been a big part of it, and*

> *that really came through in the measurement work. ... I am really pleased with the work that I have done this year. I think my numeracy program has come a long way. I think there is a lot of stuff that I am really excited about trying next year. I really enjoyed the open-ended activities and really enjoyed getting the kids to explain processes of how they did it and will be looking at that a lot more next year.* (Ophelia, 14/12/05)

In Chapter 8, the presentation of the results continues with the findings of the research presented for the teacher Samantha, also of Stringybark Primary School, and for her students.



Chapter Eight

Results:
Stringybark Primary School – Samantha

> *I have enjoyed working in numeracy, particularly in the last two years, more than I have in my whole life. I believe that I am addressing a lot of areas within the Essential Learnings through my numeracy program. The kids are doing a lot of literacy work with recording and reporting. The thinking, inquiry and reflective thinking is obviously essential to numeracy. I have reached the point as a teacher now, where I can see where the numeracy can be drawn from whatever topic we do, and how that can work together ... that whole philosophy behind Essential Learnings and values and valuing children and developing confidence and belief in themselves and I have managed to do that, largely through the numeracy program.* (Samantha, 1/3/05)

8.1 Introduction

The setting for this case study, Stringybark Primary School, is described in Section 5.3. Stringybark was the setting for the case of Ophelia, reported in Chapter Six, and of Samantha, also a Grade 6 teacher at Stringybark Primary School. Samantha was a member of the same collaborative planning team as Ophelia, and they worked closely together to plan transdisciplinary units of work informed by the Essential Learnings (D*o*ET, 2002). The Grade 6 collaborative team planned the unit of work "How do you measure up?" with the aim of supporting student learning in the area of measurement related concepts and the processes of inquiry and reflective thinking. Samantha implemented the unit of work in her own classroom over a two-month period during her usual two-hour numeracy block.

Samantha had been teaching in the primary grades for twenty-five years. She expressed a particular joy in teaching upper primary students. Samantha had been at Stringybark Primary School for nine years and had become an integral member of staff, not only as a classroom teacher, but also as a senior member of staff and in her role as school numeracy coordinator. She supported and mentored all of her colleagues in the development of their numeracy teaching practice. She also undertook a leadership and mentoring role in the Grade 5/6 collaborative planning team. The Essential Learnings (D*o*ET, 2002) had played a major role in Samantha's teaching for the past three years

Data analysed and reported in this chapter are two teacher interviews with Samantha; field observations taken during eighteen classroom visits with Samantha and her 27 Grade 6 students; six individual student interviews, together with documents and photographs collected across the four phases of the research.

8.2 Curriculum

This section presents Samantha's beliefs about curriculum and summarises the unit of work that Samantha was teaching during this study.

8.2.1 Samantha constructing curriculum

Samantha revealed a strong sense of the Tasmanian curriculum reforms matching her own philosophy of teaching. She was passionate about the Essential Learnings and its role in shifting the focus of teaching away from "teaching kids heaps and heaps of stuff to teaching not as much but much much deeper" and in providing a framework to inform how this teaching could occur. When describing the new curriculum Samantha said she "absolutely love[d] it, and believe[d] in it one hundred per cent."

In addition to aligning her curriculum construction with the Essential Learnings and with respect to teaching for understanding, two distinctive themes arose that epitomised Samantha's construction of curriculum. First, Samantha established and maintained a vibrant and productive classroom culture in which engagement and conversation were natural components of the learning process. Second, Samantha held a long-term view of her students as future citizens and she constantly planned for, and provided her students with, opportunities to develop themselves as independent thinkers and decision-makers.

Samantha was a passionate teacher who, despite her years of experience and involvement in school leadership roles, found her greatest professional satisfaction in the classroom with her students. She was confident in her role as a teacher in the changing curriculum setting and found it "a very exciting environment to be in." This flowed into Samantha's classroom where she spent much time engaging students in discussion with herself and with each other. They were constantly encouraged to question, and to explain and justify their thinking.

> It's so sparky, once they are on a roll with it they get so excited about sharing back. They want to tell the rest of the class. They want to tell me what they have discovered and what they have found out. (Samantha, 1/3/05)

Samantha discussed the opportunities she provided her students to approach challenges "from any angle as long as they are able to justify … why they are doing that." In implementing integrated units of work in her classroom, Samantha explained that she allowed her students to approach the same question in different ways, to varying degrees, and even from different discipline-based perspectives. "I might have a group here doing an art activity in connection with what we are doing, a group here doing some science, and there's a group here doing some numeracy … but it's all exactly on the same topic." For Samantha, this autonomy provided to her students was a means both of supporting them to develop as independent learners and of providing her with "a good indication of just how deeply the kids [were] understanding."

In her planning, Samantha was very aware of ensuring that she aligned her understanding goals, tasks, and assessment criteria. She saw this as a very time-consuming but worthwhile process resulting from the reforms and from her own move to open-ended questioning and inquiry in her classroom. For Samantha, the roles of both teacher and student were shifting to a much more negotiated position in her classroom. She described her own role changing to become open to student ideas and to learning how to question students in meaningful ways. Students were learning "how to think and explain their thinking," and to be responsible for their own learning. Samantha believed in providing her students with the "maximum opportunity" to develop as capable learners who had roles to play not only in school but also in society.

8.2.2 The unit of work: How do you measure up?

Samantha and Ophelia were members of the same collaborative planning team at Stringybark Primary School. They worked together to plan units of work informed by the Essential Learnings. Section 7.2.2 describes the motivation for the unit of work "How do you measure up?" that was planned collaboratively but implemented by Samantha and Ophelia individually in their own classrooms with their own students.

Samantha's comment in relation to why the unit of work "How do you measure up?" became quite distinct and separated from the literacy unit of work, "Me; An Author," was that "the kids viewed them as two separate units, even though I did try to keep making those links, it ended up, I felt I was really forcing that issue, so we just left it" (Samantha, 14/12/05). Upon reflection, Samantha thought that if the literacy unit had been completed during first term and then used "as a jumping board into the measurement unit" then perhaps it may have enabled more meaningful connections for the students. Natural connections made by the students between these two units of work were explored by the researcher during the Phase 3 student interviews.

Samantha described the goals of the numeracy unit being to "get kids using measurement, understanding how formal units of measurement work, and how they apply to our real-world situation" (Samantha, 14/12/05). The five documented understanding goals, two measurement-related and three to support the development of inquiry and reflective thinking, were reported in Section 7.2.2. A culminating performance for the unit of work was developed by Samantha toward the end of the term and in response to the teachers' desire to provide the students with an opportunity to demonstrate their learning in a context of their own choosing. The culminating performance was completed by Samantha's students at the end of the unit of work.

> Thinking about all you have learned during our measurement unit, what are some of the BIG questions related to measurement that you may want to investigate?
> You will need to consider:
> - What makes a good question?
> - What are some big issues in our world that relate to measurement?
> - What would you like to find out more about?
>
> Pose the problem, ask the question.
> Conduct your investigation
> As you discover some possible or partial solutions to your question, make a note of new questions you need to ask as they arise.
>
> (Samantha's documentation, 1/8/05)

8.3 Numeracy

This section presents the results for this case within three main sections: Samantha's beliefs about numeracy, the enactment of numeracy in the classroom with Samantha and her students, and the student learning outcomes of six individual students.

8.3.1 Samantha and her beliefs about numeracy

"Numeracy and the rest!" Numeracy was obviously an area of the curriculum about which Samantha was passionate. She was very emotive when discussing her beliefs and practices and shared her concern for student learning and her desire to show the relevance of numeracy in all of their lives. Samantha referred to her own history of failing mathematics and how that had motivated her current "love" of it and commitment to make it engaging and relevant to students. Samantha clearly enjoyed her role as the school numeracy coordinator. She took every opportunity to engage in professional learning in the area, to extend her own mathematical knowledge and understanding of current research and practice in the field of numeracy, and to share these ideas with the other teachers in her school.

Samantha positioned numeracy alongside literacy and inquiry and reflective thinking as being the "core" areas of the curriculum that informed her planning across all areas, and in particular the integrated or "generative" units of work that were implemented regularly by Samantha and her colleagues in the Grade 5/6 collaborative team. The high priority of numeracy was evident as Samantha indicated that "there is numeracy involved in all the units" of work that she planned and implemented with her students. In many cases, Samantha planned numeracy units of work to run parallel with a generative unit. She did this to enable development of the numeracy learning required to support the broader unit to occur in parallel with the generative unit, and in a planned and focused manner.

Samantha wanted her students to be involved in much more than just "doing" mathematics. She wanted them to "explore" mathematics and this belief underpinned her practice. The following subsections detail how Samantha's conversation about her teaching could be described according to the five dimensions of numeracy as developed in Chapter 3.

8.3.1.1 Mathematics

Samantha was concerned with developing students' mathematical knowledge, "skills," and "processes" within the broader learning opportunities of all of her units of work. Although "check[ing students'] abilities in the four processes" at the beginning of a school year, Samantha talked about her main aim being to situate all of her "skills building maths lessons" in the context of a unit of work and relevant to the individual learning needs of students when solving problems.

Samantha was very comfortable and confident discussing many different areas of mathematics. She gave examples of the mathematical requirements of tasks involved in a recent unit of work the students had undertaken, "You are what you eat." Students were required to record data as part of keeping a diary, tabulate data when organising nutritional information, read and interpret these data, and understand number and units of measure when reading and working with information from pedometers. Samantha provided a copy of her unit planning with the numeracy learning outcomes she had designed related to this unit of work, and the tasks that supported student learning of the mathematical skills that students would need to be able to engage in the generative unit on nutrition.

Samantha also explained that she sought to identify the embedded mathematics in tasks or units of work. She then considered those students who may need additional teaching support to complete tasks using the particular mathematical skills and processes. She would then work with those students individually or in small groups to support their capacity to engage in the broader learning experience.

8.3.1.2 Reasoning

Samantha saw the major benefit of the Essential Learnings as being the overriding emphasis on Thinking. She described the major shift that happened each year with many students in her classroom as she embedded Thinking into the learning environment. Samantha found the most difficult shift for students with regard to this was in relation to their learning of mathematics as her classroom culture was very "verbal." Students were constantly questioned and required to explain and justify their thinking: "Are you going to do it this way? Why?"

The development of mathematical thinking and reasoning played a very important role in Samantha's classroom and matched her broader goal to create "independent thinkers." Students were required to "justify" their strategies and to "explain" what worked, what did not work, and why, when conducting mathematical investigations and solving problems. They were given the freedom to select and apply problem solving strategies. Samantha clearly had an inquiry focus in her numeracy teaching, with students undertaking in depth investigations and problem solving in which they were given time to work together, to discuss ideas, and to develop solutions. She talked about students not being expected to solve problems in one lesson, as she valued the capacity for reasoning that open-ended tasks allowed and the time students needed to explore the tasks in depth and from different perspectives. Samantha found out "just how deeply the kids [were] understanding" from their written and verbal explanations and from their sharing and justification of strategies.

8.3.1.3 Attitude.

Samantha herself had a very positive attitude toward numeracy. She declared that she "enjoyed working in numeracy … more than she [had] in her whole life." This resulted in her desire to make numeracy interesting and meaningful for her students. She shared that she did not always have this positive attitude, and it had really come from a curriculum change that matched her underlying philosophical beliefs about teaching and learning. Samantha shared a personal interest in the teaching of numeracy and also in supporting others to develop their own numeracy teaching. She worked with other teachers in their classrooms and assisted them with their planning. Samantha was very confident in her teaching of numeracy and could explain why she chose to teach the way she did.

Samantha shared some of her own personal experiences of mathematics learning and resultant negative attitudes as well as the value she placed on having her students engage with, not just "do" mathematics. In doing so, Samantha evidenced her belief that attitudes were strongly linked with student learning outcomes.

8.3.1.4 Context

Samantha's belief that mathematics was all about "life" and that numeracy was needed to "survive" in life, evidenced the important role that context played in her conception of numeracy. In sharing her practice, she spoke of her focus on open-ended problems in the teaching of numeracy. The open-ended problems that she used were based upon real-world examples.

Many of the problems Samantha used in her classroom were constructed from her own life and she encouraged her students to create their own problems for the class to work on. Her desire to create interesting and relevant mathematical problems for students to investigate shows that, for Samantha, numeracy was all about context and was not separable from it. Samantha also gave examples of broader contexts used in the units of work implemented each term and in which numeracy formed an element, for example, nutrition and book publishing.

8.3.1.5 Equity

Samantha's focus on developing students' capacities to question effectively was motivated by her desire to develop "independent thinkers." She focused on a long term vision of preparing her students for a "role in society" and talked about the importance of questioning in decision-making and not taking things for granted.

For Samantha, the extent to which students could question effectively directly impacted upon their development as independent thinkers. She found many ways to both question students herself and also to enable students to effectively question each other. Samantha was developing in her students the capacities that would enable them to go on to consider mathematics and its implications in social, economic, and political contexts.

8.3.2 Numeracy as enacted in the "How do you measure up?" unit of work

Samantha began each day with numeracy, sharing her interest and joy in the relevance of mathematics, for everyday life and learning, with her students. Samantha's numeracy classroom was an active one in which students worked together, with each other and with their teacher, to explore strategies and solutions. During this two-hour period each day, Samantha and her students engaged in open-ended problem solving, both in relation to the focus unit of work on measurement and also to other areas of mathematics learning. During the eighteen classroom visits undertaken in this case, detailed observations were recorded and later analysed, enabling a picture to be created of Samantha and her students' numeracy classroom according to the five dimensions of numeracy as proposed in this thesis.

8.3.2.1 Mathematics

The teaching and learning of mathematics in Samantha's classroom was situated within the context of ongoing problem solving. Samantha posed questions daily to her students based upon authentic experiences from her daily life and involving various mathematical concepts and ideas, as shown in the following examples,

> Tim went to Bridport on Friday night. It takes 4 1/2 hours from Hobart, without stopping. He left Hobart at 5:25 pm. What time might he have arrived? Explain

> your answer. He left Bridport at 2:15 pm. What time might he have got home? Explain your answer. (Classroom observation, 4/7/05)
>
> I went shopping. I had $130 to spend. I found a jumper; original price $168 with 50% off. I found a jacket, original price $120, 50% off. Which one did I buy? Why? (Classroom observation, 27/6/05)

Samantha also encouraged her students to contribute problems and questions themselves from their own daily experiences and that involved Being numerate.

> My netball team played on the weekend and we won. The amount [sic] of goals we scored has the factors 6, 1, 18, 2, 12, 3, 9, 4 and itself. We won by an even amount and scored an even amount [sic] and so did the other team. The other team's score has the factors 1, 2, 10, 5, 4 and itself. What did my team score? What did the other team score? How much did we win by? (Classroom observation, 8/8/05)
>
> On Friday our fridge stopped working at 7:30 pm and we got it fixed at 10:00 am on Monday morning. How many hours and minutes was our fridge not working for? (Classroom observation, 11/8/05)

During the measurement unit of work, Samantha continued to pose questions for her students to explore but chose to incorporate more questions that were related to the learning area of measurement. Samantha also introduced more tasks that could be explored in-depth and over numerous lessons. This was done with the aim of supporting her students to develop an understanding of the purpose of measurement, and of measurement attributes and units of measure, as well as to develop their practical measuring skills.

At the beginning of the unit of work, Samantha supported her students in developing the language of measurement by having them consider and compile a vocabulary of measurement terms and their relationships. The students shared these with each other and built on them throughout the unit of work. Samantha also asked her students what aspects of measurement they were interested in learning about and students were provided with opportunities throughout the term to explore these areas of interest. These areas included gaining a better understanding of specific measurement concepts such as height, area, angles, and energy; to measurement of people, gardens, and objects; and also how measurement relates to areas such as horses, pigeons, football, and the Great Wall of China.

Samantha's formative assessment of student learning by ongoing observation informed her selection of tasks. She recorded observations of student learning daily on sticky notes, and then reflected on them to consider further learning for the class and for each individual student. These observations specifically related student learning to mathematics. Example observations included:

> [Student] discovered connection between L x H x W.
> [Student] needed to measure capacity, not prepared to make an estimate.
> To estimate the height of the door, [student] went straight to the door, and used his own height as a benchmark. (Classroom observation, 25/7/08)

Although Samantha posed the same questions to all of her students and guided the overall experience, students could select tasks on which to focus and how they approached each task and for how long. As a result Samantha's classroom had many students or groups of students working on many different activities at any one time.

Throughout the unit of work, students were exposed to activities across the attributes of length, area, volume, capacity, weight, time, temperature, and distance. The following are examples.

> Using newspaper make a square metre. What could you measure with your square metre? (Classroom observation, 4/7/05)
>
> I put three objects on the scales and they weighed 3 kg. What were they? (Classroom observation, 4/7/05)
>
> I have a box which is 6cm long, 4cm wide and 2 cm high. How many chocolates can I fit in it if they are all perfect cubes with an edge length of 2 cm? (Classroom observation, 22/7/05)

Samantha's classroom was a hands-on and active one with students rarely sitting at their desks. They explored mathematical concepts through the use of real-world questions and were encouraged to use resources to explore their ideas. Figures 8.1a, 8.1b, 8.1c, and 8.1d show students working both inside and outside the classroom on measurement activities.

Figure 8.1a. Measuring with $1m^2$. *Figure 8.1b.* Stretching and measuring a lolly snake.

Figure 8.1c. Comparing water and milk. *Figure 8.1d.* Measuring a person.

Due to the open-ended nature of tasks in Samantha's classroom, there was a natural connection between mathematical strands and concepts. Not only were students exploring measurement concepts but they were also continually drawing on their number understanding, collecting and recording data to represent meaningful findings, considering and recording patterns among mathematical ideas, and often creating visual objects and shapes to help them understand the measurement relationships they were exploring.

A unique aspect of Samantha's classroom was the way she built upon and utilised the outcomes of open-ended tasks to develop new questions and new tasks for students to solve. She did this in a way that enabled deeper engagement with the mathematics embedded within the tasks and, for some students, extension to new higher level mathematical concepts. Three examples are presented here that exemplify this aspect of mathematical learning in Samantha's classroom. The chocolate box question, originally posed on the 22nd of July (p. 217) was extended to support a conceptual understanding of the formula for volume, by the further question:

> Can you find a connection between the chocolate box numbers?
> Try some boxes of different sizes. (Classroom observation, 25/7/05)

A task that involved students exploring the many possibilities of measuring a person was extended to consider the relationships among some of these measurements and the mathematical areas of ratio and proportion with the question:

> Our arm span is our height. What can you find out about this? (Classroom observation, 1/8/05)

Student work originally undertaken in early July, creating a square metre with newspaper and exploring the relationship between area and perimeter (p. 217), was extended for students to consider the practical application of this knowledge in the areas of design and use of space, with the question:

> We discovered that if we change the shape of a square metre it has the same area but the perimeter can change. How can I use this knowledge in real life? (Classroom observation, 1/8/05)

This work was taken even further with students specifically considering how to design a vegetable garden with 60 metres of fencing wire and then for some students, the challenge of exploring and discovering the area of other shapes, such as circles, and triangles with the following questions:

> I need to set out some garden beds in my vegie garden for carrots, strawberries, peas, spuds, and corn. I need paths between the beds. (Classroom observation, 22/8/05)

> How do I decide which garden plan to use? What are some of the things I need to consider when setting up my vegie garden?
> Would a triangle (P=60cm) give me a greater area than the rectangle or circle?
> We don't know the area of triangle so we have to do some finding out.
> How do I fence a circle? (Circumference = 60cm)
> (Classroom observation, 29/8/05)

During the last few weeks of the unit of work students were given the opportunity to evidence their learning in the area of measurement by choosing a "big" question to answer from their own area of interest. This culminating performance task is discussed in the Context dimension (Section 8.3.2.4).

8.3.2.2 Reasoning

Reasoning was central in Samantha's numeracy classroom and it was purposefully and explicitly taught to students. The unit of work that formed the focus of this case had three understanding goals specifically linked to the learning areas of Inquiry and Reflective Thinking.

> - Students will understand how to pose and define a problem, clarify the issues involved and select and monitor the most effective process to use.
> - Students will be able to collect and record information, with an understanding of accuracy and reliable results.
> - Students will understand that reflective thinking is a deliberate process … and that it is used to develop and refine ideas and beliefs and to explore different and new perceptions.
>
> (21/6/05, Grade 5/6 planning team)

Samantha focused on tasks that were open-ended, that could be solved using a number of strategies, and that required students to consider assumptions or ask questions in order to justify a solution. Figure 8.2 shows an example group of three questions posted on the whiteboard for students to solve. It includes two problems provided by the teacher and one from a student.

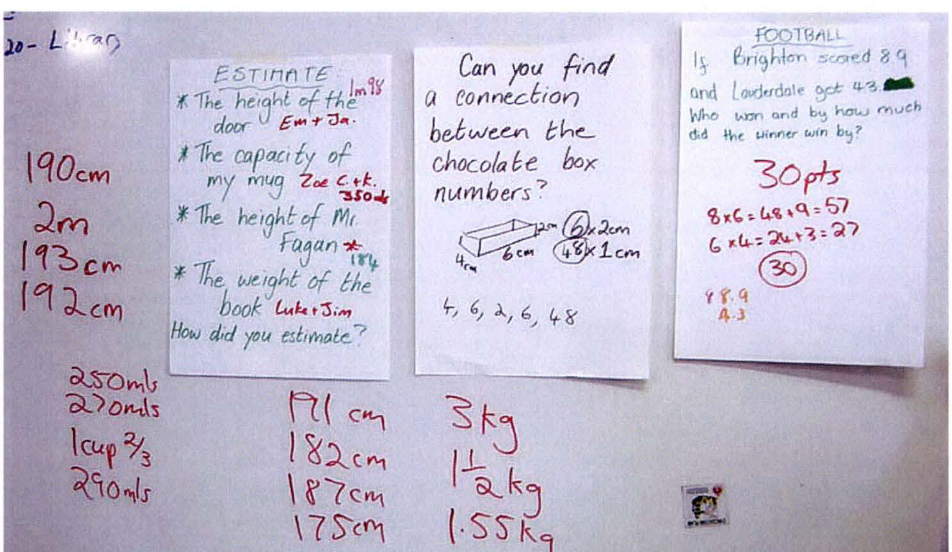
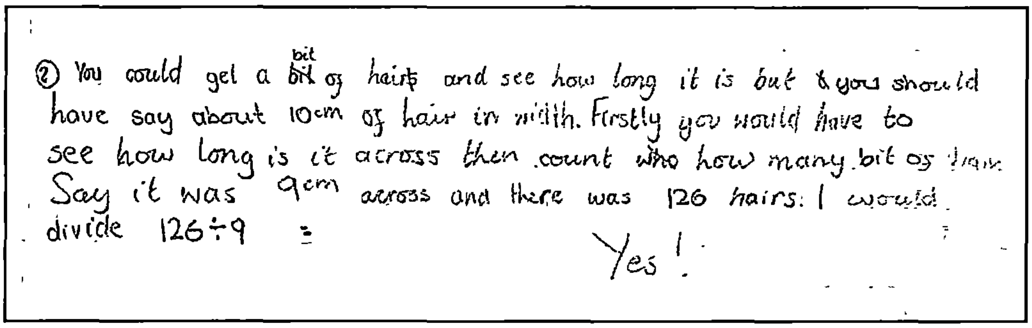


Figure 8.2. Sample of daily numeracy questions posed for students to consider.

Samantha's students were encouraged to record their thinking when solving problems, as shown in the example solution to the question, "How could we find the width of a hair?" presented in Figure 8.3 (Classroom observation, 22/7/05). It was always the mathematical thinking and understanding that was valued above an answer: "Be brave enough to make an estimate, it doesn't matter if you're not right" (Classroom observation, 25/7/05).


(2) You could get a bit of hair and see how long it is but & you should have say about 10cm of hair in width. Firstly you would have to see how long is it across then count who how many bit of hair. Say it was 9cm across and there was 126 hairs. I would divide 126÷9 =

Yes!

Figure 8.3. Student considering how to measure the width of one strand of hair.

Samantha posed questions and open-ended tasks that required of her students much more than recall of knowledge. In addition to using her own questions Samantha also obtained open-ended questions from other resources (e.g., Sullivan & Lilburn, 1997). In order to approach tasks, her students needed to make meaning of them, to consider assumptions, to "make decisions and justify them" (Classroom observation, 4/7/05). By establishing a classroom in which students shared their ideas with each other and with the whole class on a daily basis, students were also supported to critique each other's ideas and strategies, and to justify problem solving strategies and solutions. If students were not sure how to proceed or approach a problem, Samantha would not just point them in the right direction she would ask them what they might need to know, with questions like, "Do you have a question you would like to ask?" (Classroom observation, 4/7/05).

When questioning her students, Samantha asked them not only how they had gone about approaching a question or a task with questions like "Explain what you've done?" (Classroom observation, 11/7/05), but also why they had approached it in a particular way. In questioning her students, Samantha was constantly challenging them about their own and others' mathematical understanding and strategies.

> What strategy could you use to be fairly sure your values are right?
> How do you know? (28/7/05)
> I am wondering how you can find out? (15/8/05)
> How can we check to see who is right? (11/8/05)

She also challenged her students' ideas to push them to think further and continually relate what they were doing to the mathematical learning goals that underpinned the unit of work.

> Is speedo the actual word? What does a speedo measure? (20/6/05)
> What do you mean by eyes, and what would you have to measure? (15/7/05)
> How are you going to measure it? (18/7/05)

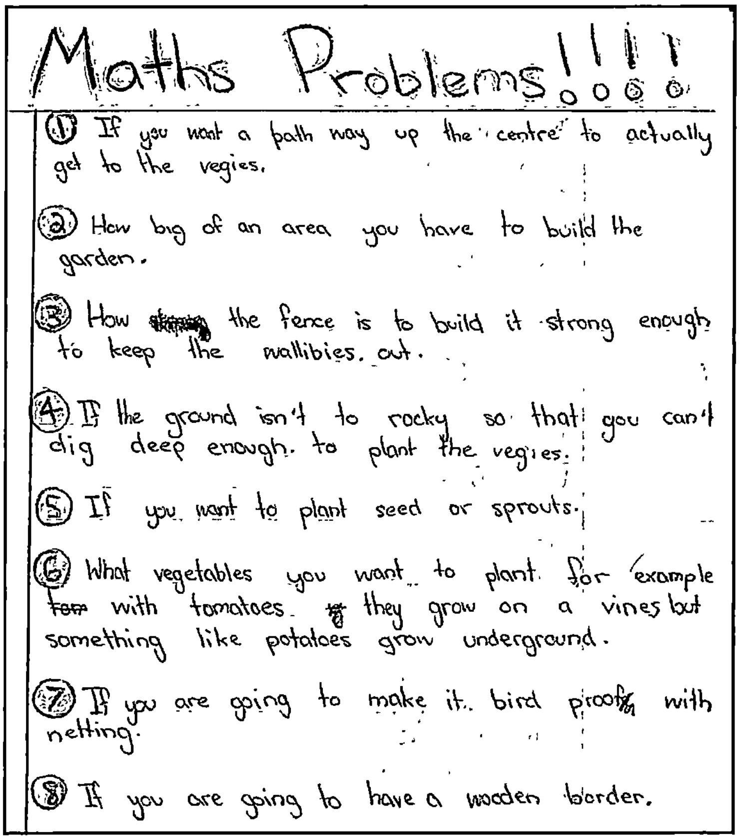
Numeracy was not viewed as a quick process but, rather, questions and tasks were able to be worked on over many days and even weeks. Samantha enabled this by both the questions she posed and the way in which she encouraged her students to persevere with tasks and with their own ideas. She posed questions that led to further questions or ideas for students to consider.

> What would you measure in feet? (20/6/05)
> Is there a word for how much space? (20/6/05)
> Is there another way I could express it? [168cm] (15/7/05)
> How many nines could we get [in a game of MULTO]? (18/8/05)

She was also explicit with her students about the value she placed on mathematical thinking in her numeracy classroom.

> Nice thinking (20/6/05)
> The maths is right, the answer is wrong (Classroom observation, 20/6/05)
> Yes, you're thinking outside the square (15/7/05)

Samantha's students were encouraged to record, on sticky notes, further questions that they themselves came up with when working on tasks. These provided a source of further activities or ideas for the class to consider at a later time. Figure 8.4 is an example of a list of questions one student came up with before proceeding with the open-ended task involving the design of a vegetable garden using 60 metres of fencing wire (Classroom observation, 22/8/05). As the approach to tasks was very much about encouraging students to come up with realistic solutions there was a strong connection to the Context dimension.



Maths Problems!!!!

1. If you want a path way up the centre to actually get to the vegies.
2. How big of an area you have to build the garden.
3. How the fence is to build it strong enough to keep the wallibies out.
4. If the ground isn't to rocky so that you can't dig deep enough to plant the vegies.
5. If you want to plant seed or sprouts.
6. What vegetables you want to plant. for example ~~tom~~ with tomatoes. they grow on a vines but something like potatoes grow underground.
7. If you are going to make it bird proof with netting.
8. If you are going to have a wooden border.

Figure 8.4. Further questions for consideration in solving the vegetable garden problem.

The following conversation occurred when one numeracy question that had been written by a student was introduced to the class. This conversation exemplifies how Samantha would equip her students with the capacity to approach a task, when at first it might seem overwhelming, by having them consider the questions and information that might help them to approach and solve the task.

> Question: On Monday I built a Lego castle out of green Lego blocks. They are 2 cm high and 2 cm long. When I finished it was 20 cm high and 34 cm long. How many blocks did I use? (Classroom observation, 1/8/05)
>
> S1: That's hard.
> T: So you want more information?
>
> S2: Was it solid or hollow?
> S3: Does it have a floor?
> S4: Does it have a roof?
> S5: How deep was it?

Samantha supported her students in the use of many different problem solving strategies, such as guess and check, drawing tables and diagrams, making models, and actually spending time both inside and outside the classroom trialling their ideas. She explicitly described students' strategies and developed the language of reasoning with her students. Although Samantha's students shared their thinking with each other as a whole class, at the end of each numeracy lesson, she also encouraged them to record their ideas, their working out, and their solutions as they were working on tasks. These ideas were recorded in their mathematics workbooks and also on the whiteboard when they were sharing their mathematical thinking with the class (Figure 8.5).

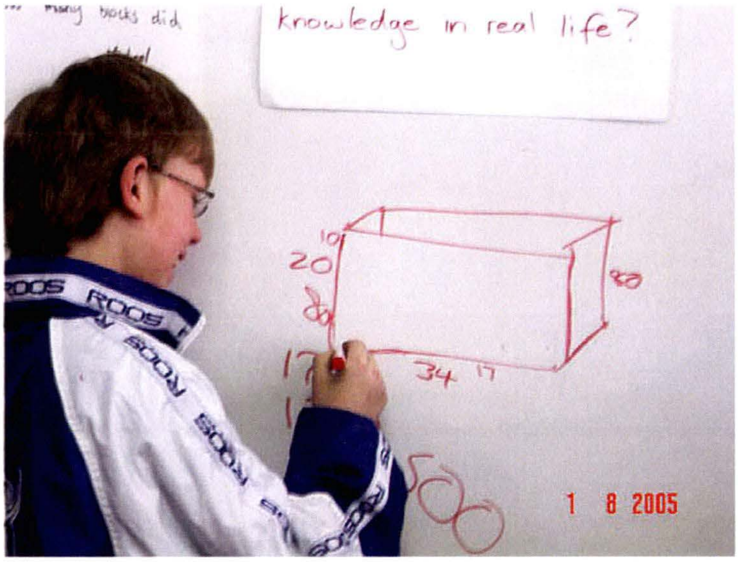


Figure 8.5. Sharing solutions with the class.

Figures 8.6a, 8.6b, and 8.6c show the process one student went through in considering how many 2cm^3 chocolates would fit into a 6cm long, 4cm wide, and 2cm high chocolate box (Classroom observation, 22/7/05). First, the student recorded and drew some ideas, before constructing a model to test these ideas.

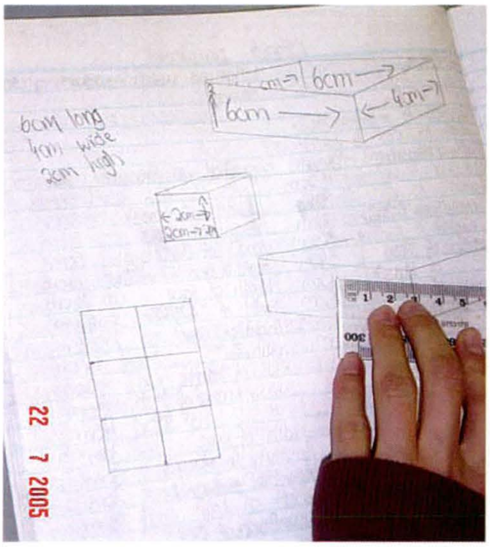


Figure 8.6a. Recording and drawing a model.

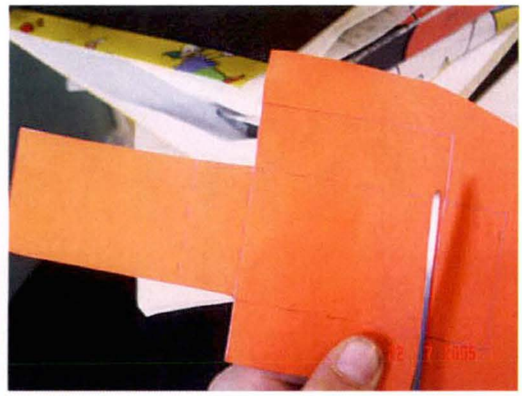


Figure 8.6b. Constructing a model.

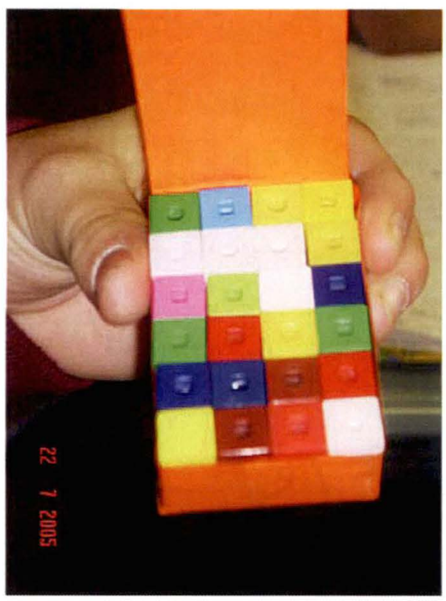
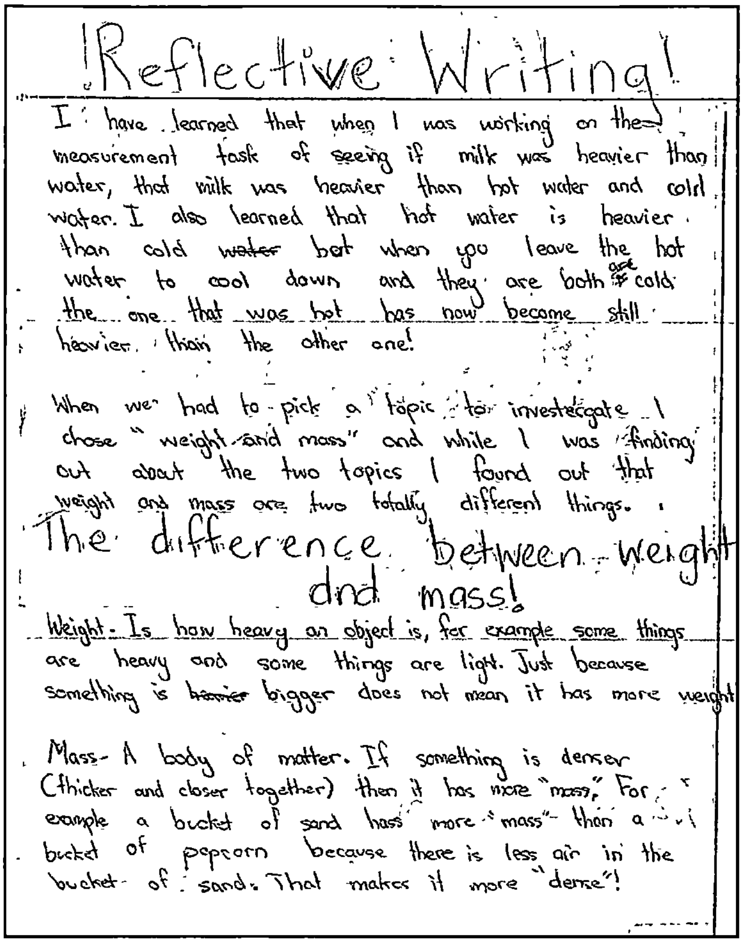


Figure 8.6c. Completing and testing the model (where eight 1 cm cubes were used to represent one 2 cm cube).

Samantha also built reflective thinking into her numeracy classroom. Students were encouraged to apply their ideas, test them, change things based upon the results, and then report these back. By considering other approaches and solutions, they were also reflecting constantly upon their own learning. Samantha often reminded her students of "thinking questions" when they were reflecting upon and sharing their work in order to provide a structure for their sharing. These questions included asking "What you have found out?" and "What do you need to know to go further?" At times students formally recorded their reflections to articulate their learning. Figure 8.7 provides an example of one student's learning during a task to see if milk is heavier than water (Classroom observation, 4/7/05). This also led to her personal interest in finding out the difference between weight and mass.



Reflective Writing!

I have learned that when I was working on the measurement task of seeing if milk was heavier than water, that milk was heavier than hot water and cold water. I also learned that hot water is heavier than cold ~~water~~ but when you leave the hot water to cool down and they are both cold the one that was hot has now become still heavier than the other one!

When we had to pick a topic to investigate I chose "weight and mass" and while I was finding out about the two topics I found out that weight and mass are two totally different things.

The difference between weight and mass!

Weight - Is how heavy an object is, for example some things are heavy and some things are light. Just because something is ~~heavier~~ bigger does not mean it has more weight

Mass - A body of matter. If something is denser (thicker and closer together) then it has more "mass", For example a bucket of sand has more "mass" than a bucket of popcorn because there is less air in the bucket of sand. That makes it more "dense"!

Figure 8.7. Student reflection of learning.

Reasoning in Samantha's classroom was a collective activity. Students worked together and therefore articulated their thoughts and ideas to each other, and collaborated on tasks to come up with solutions. When sharing, the students' ease at questioning each other and contributing to each other's ideas and findings also resulted in shared learning happening in these sessions, progressing their ideas collectively. The following shared conversation involved students sharing their solutions to the chocolate box question.

S1: It can only fit six chocolates in it, because me and [partner] built a box.
T: How?
S1: [Draws a 6 cm by 4cm rectangle on the board] And it was two centimetres high. We then used the centicubes to make two centimetre cubes and put them all in.
T: [S2], you just looked at this and knew the answer. How?
S2: I just knew that it was two centimetres high, so I knew it was just one row so six centimetres long and two goes into six three times and four centimetres wide, so another three, makes six.
S3: I also made a box but added a little lid.
T: How did you work out the solution?
S3: [Draws on the whiteboard]. Also there is forty-eight one centimetre cubes in the chocolate box because there are twenty-four on each level.
T: How can you check?
S3: Count them [goes to desk to count the cubes].
S1: It's 24.
T: Can you explain?
S1: Not really.
S2: You don't even have to count the blocks; there's six, two by two cubes and four in each so six lots of four is twenty-four.
S4: The two by two cube is two centimeters deep, so that's four and four down, which makes eight and six lots of eight are forty-eight.
T: Hands up who thinks twenty-four, and hands up who thinks forty-eight.
S3: Counted and definitely forty-eight.
T: In a two cubic centimeter, Tom was thinking of this [draws 2 by 2 square]. What is it?
S4: That is two centimeters square. It has to also be deep.
T: Needs depth also. What is special about a cube?
S5: Same measurement whichever way you put it.

Samantha took the students' discussion about the box and asked them to explore the relationship among the numbers, 2, 4, 6, 48. Some students explored making another box, 5cm by 8cm by 3cm. Some students discovered the relationship straight away (Length x Height x Depth = Volume of box), while others were challenged to explore other boxes.

Samantha established a classroom environment for her students to think and to be creative in. She openly valued "thinking outside the square" and supported her students to take risks as this was seen as usual practice. In fact, perhaps it could be argued it was not risk-taking at all because variety in the numeracy classroom was standard: variety in tasks, variety in approaching problems, variety in working with materials and space, and variety of solutions.

8.3.2.3 Attitude

Samantha was highly motivated to pass on her enthusiasm for numeracy and wanted her students to develop confidence in their own capabilities as learners and numerate students. Student participation and engagement in the numeracy classroom were evident from the moment this case began. Students often expressed their desire to continue working on numeracy tasks after the two hour time period that was usually allocated, and often in preference to other learning areas (Classroom observation, 11/7/05). Samantha's numeracy classroom was very dynamic as she exploited learning opportunities as they arose, in order to unpack mathematical concepts, and to support students' developing understanding of measurement.

Samantha acknowledged to her students the major shift in her numeracy teaching for those students who may have come from a more traditional mathematics classroom. She made clear her expectations that students would participate actively in their learning and share, explain, and justify their work. She also related this mathematics learning culture to that of mathematicians who "are very creative and don't always have to come up with an answer, [they] work towards it" (Classroom observation, 22/7/05). A few students throughout the case shared their struggle with the open-ended task focus of the numeracy learning because they did not like "thinking." This was rare, however, with the majority of students clearly enjoying the numeracy classroom.

Although the classroom was often "noisy" and active, students were engaged in discussing their learning and going about working on tasks and solving problems. Figure 8.8a and Figure 8.8b show two examples of students engaging positively in their learning both inside and outside the classroom, working together to explore mathematical ideas. Numeracy was not an individual endeavour but a shared experience involving the students working together and developing ideas and solutions together.

Figure 8.8a. Measuring a circle outside.

Figure 8.8b. Measuring jumping distance.

Samantha provided positive assurance to her students by encouraging them to "keep going" and by acknowledging their efforts. She would make the most of every learning opportunity, even a spelling error in measuring distance was turned into an opportunity to explore measurement language and units of measure in a fun way. This was achieved by having the students create drawings of measurement monsters that might represent different units of measurement using the prefix "killer" for kilo (Figure 8.9). Samantha used the notion of a "killermeter" to initiate discussions about standard units of measure and to address students' misconceptions and use of appropriate language.

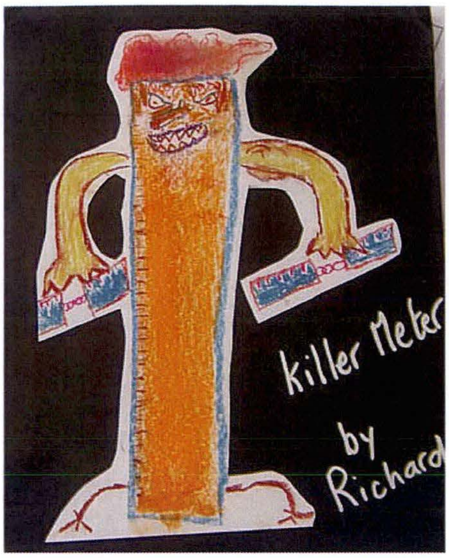


Figure 8.9. A "killermeter."

Students were allowed the freedom to make choices about which tasks they would approach first. They were provided with time to explore more deeply those tasks with which they were more engaged. The nature of the tasks, being open-ended, enabled students to access them at their own level of ability and provided many opportunities for extension for students who showed particular strength and interest in certain areas.

Samantha did not establish herself as the focus of mathematical knowledge, but worked to establish an environment where the class would work together to build understanding, and would look to each other for ideas as well as to Samantha. Samantha expected her students to persevere with tasks and the students were happy to challenge each others' ideas and to see where they themselves might have gone wrong in their own work.

> That's my working out, I could be wrong. (Classroom observation, 1/8/05)
> I did two things wrong. I forgot to times it by two, and also I used sixteen instead of eight. (Classroom observation, 1/8/05)

Samantha also included her students in planning for their own numeracy learning by asking them about their interests in the learning area and enabling them to make choices with particular tasks. Right from the beginning of this unit of work, Samantha asked her students what they wanted to find out about measurement.

Samantha's ability to encourage all students, those who struggled and those who excelled, was evident. The students were always prepared to participate and to share their ideas. Samantha did this in a way that valued each student and without individual students dominating. Furthermore, students were respected for their capabilities by the other students.

> Keep going, you're thinking wider. (20/6/05)
> I like how you've grouped your words. Are you able to explain why you've grouped them like that? (20/6)
> I like the way you have set it out, very logical and systematic. (11/8)
> I love it. Can you measure ability? (Classroom observation, 15/7/05)

Samantha involved her students in their learning, by enabling them to contribute problems, to select tasks to work on, to extend themselves, and to consider a variety of strategies and solutions. All of this was observed to contribute to a highly energetic and positive classroom environment. Samantha gave her students a sense of ownership of their learning by having them write their own assessment of their numeracy learning for the reports that were to be provided to their parents (Figure 8.10). As Samantha had been clear with the learning outcomes for the unit of work and had provided and explained to students the curriculum standards for learning in this curriculum, students were able to consider and discuss their assessment with Samantha and be confident in their self-reporting of learning in this area.

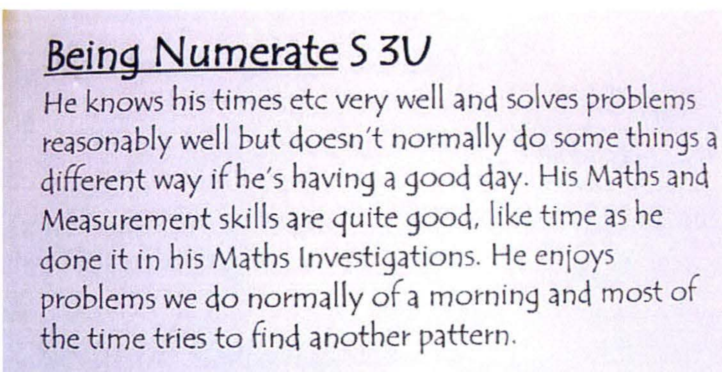

Being Numerate S 3U
He knows his times etc very well and solves problems reasonably well but doesn't normally do some things a different way if he's having a good day. His Maths and Measurement skills are quite good, like time as he done it in his Maths Investigations. He enjoys problems we do normally of a morning and most of the time tries to find another pattern.

Figure 8.10. Student self-reporting on numeracy learning, assessed at standard 3, upper level.

8.3.2.4 Context

The unit of work "How do you measure up?" had two content-focused understanding goals that formed the basis of planning for student learning throughout the term. These were that:

- Objects and events have attributes that can be measured and there are standard units that we use to describe and communicate measures of attributes.
- We use our knowledge and understanding of measurement to answer questions about our world.

(Grade 5/6 planning team documentation, 21/6/05)

Inherent in these learning objectives is the practical and contextual nature of measurement. Samantha, together with her colleagues, planned for students to develop the knowledge and skills that would enable them to measure accurately and to describe and communicate these measurements using formal measurement language and units. Samantha, in particular, wanted her students to see the relevance of measurement in life.

Context was a purposeful element of Samantha's numeracy classroom. In addition to the unit of work "How do you measure up?" being practical and contextual in nature, Samantha herself valued highly the connection of students' mathematics learning to authentic contexts and to their daily experiences both within and outside of school. Samantha wanted her students to value and appreciate the relevance of mathematics to many daily circumstances. She did not use contrived contexts and rarely posed questions that were purely mathematical calculations.

Samantha posed open-ended tasks to her students based upon both her own life and also the lives of the students. Samantha related many daily experiences with an emphasis on the numeracy requirements of making decisions embedded within these experiences. These contexts were many and varied and included spending decisions, travel, planning garden beds, and cutting boards to a set size. She encouraged her students to contribute their own questions to the class as well and these included contexts such as purchasing decisions, sports games, and building with Lego.

As in Ophelia's classroom, due to the nature of the unit of work, the students also undertook many practical and hands-on measuring activities (refer to Figure 8.1).

For Samantha's students these activities included tasks such as estimating and measuring objects, measuring a person, weighing objects, making a square metre, measuring jumping distances, and even constructing a model of a 60 square metre garden outside on the school oval.

Contextual clues were just as important as mathematical clues in Samantha's classroom. Samantha's students were encouraged to consider the contextual implications that might lead to a particular mathematical solution when solving problems. The following are examples of the types of contextual questions students asked when working on open-ended tasks.

> Do you have any irrigation? (22/8/05)
> Does it have a roof? (1/8/05)
> Do you have more jumpers or jackets? (Classroom observation, 27/6/05)
> What did you use to cut [the board]? (Classroom observation 27/6/05)
> Would he have stopped for something to eat? (Classroom observation 4/7/05)

Samantha modelled these types of questions with her students and linked context to the mathematics by having students always consider how the context impacted upon the mathematics.

> How wide are my paths? I need to bring the wheelbarrow in.
> I needed to know what sport? Why did I need to know that? (Classroom observation, 11/7/05)
> Do you think everybody's hair in the world is that thick? (Classroom observation, 18/7/05)
> How do you measure hearing? (Classroom observation, 15/7/05)

The incorporation of authentic contexts in problem solving always added the possibility for many and varied strategies and solutions, and added complexity. This supported student learning across many levels as students could consider assumptions and develop ideas to the level of their own mathematical understanding, or they could challenge themselves further and work together to come up with other solutions.

Samantha implemented a culminating performance task (Figure 8.11) toward the end of the unit of work. The overriding aim of the task was to provide students with an opportunity to demonstrate their learning in a context of their own choosing by posing a "big" or rich question to investigate. The students were provided not only with the task but also with the goals being addressed by the task and an assessment rubric that they had co-constructed with Samantha.

Open Investigation
Thinking about all you have learned during our measurement unit, what are some of the BIG questions related to measurement that you may want to investigate?
You will need to consider:

- What makes a good question?
- What are some big issues in our world that relate to measurement?
- What would you like to find out more about?

Pose the problem, ask the question.
Conduct your investigation
As you discover some possible or partial solutions to your question, make a note of new questions you need to ask as they arise.

Figure 8.11. Culminating performance task.

During the culminating performance task students came up with their own questions from their own interests that they could then answer and evidence their learning in the area of measurement. Students chose contexts that engaged them and these were quite varied. For some students, the contexts were very personal and something in which they were involved such as football, horse-riding, or raising pigeons. For others their contexts were an area of interest such as spiders, planets, and the Great Wall of China. The tasks enabled an in-depth approach to bringing together measurement understandings and evidencing learning.

Figure 8.12 provides the outcomes of one student's culminating performance task in which he considered how long it would take for Tasmania's population to build the Great Wall of China. The solution resulted from complex calculations based upon not only the population of Tasmania but also contextual assumptions made to arrive at a realistic answer. The mathematics, however, was confined to evidencing number estimation and time understandings, rather than the in-depth measurement understandings that the same student exhibited throughout the unit of work.

***What is Tassie's population?**

Tasmania's population was 477,300 in June 30 2003.

***How long did it take for the Chinese to build the great wall?**

It took china 294 years to build the great wall. (770 BC - 476 BC)

***How many people worked on the great wall?**

Around a million people worked on the great wall.

Some things that would contribute to the length of time it took:

- The workers were slave driven.
- The cement used to build it came from the crushed skeletons of the men who died building it.
- Shifts night/ day for Tasmania
- Wages for Tasmanians
- Machinery to excavate the rocks and earth instead of shovels

First I am going to find how long it would take going at the same building rate of the Chinese.
477300 = 47.73% of 1000000.
477300 is approximately 500000
So if there is half the number of workers they would have to take double the time. = 588 years
The Chinese were slave driven and the Tasmanians wouldn't be so I estimate that I should add about 60 years to Tasmania's time. = 648 years
Because the Chinese used a small amount of body bits in their cement I would add about 4 years to how long it would take Tasmania. = 642 years
Tasmanians would have to have night shifts and day shifts so that they wouldn't get sick so I will add about 40 years. = 682 years
Tasmanians would have to have wages, so that would take some time to get the money then pay it. And they wouldn't get anyone working on the wall if they didn't. So I reckon that would slow them down 35 years. = 717 years
The Tasmanian's would have machinery and much more evolved tools and digging equipment even things like better ladders so I think that that would speed Tasmania's building rate up about 210 years. 507 years. **Plus 3 months for council approval!**

Figure 8.12. How long would it take for Tasmania's population to build the Great Wall of China (student work).

Other students made books that provided information in detail of how measurement related to their chosen context. Figure 8.13, provides the introduction to one student's book on "How does measurement apply to horses?" and was followed by one page related to each of five areas about horses and related activities in which the student identified connections with measurement. One of these pages is included (Figure 8.13).

With my new 'BIG' question, I set out a second plan for how I was going to present my research. I decided to find out as many different ways that measurement is used in horses and their surrounding activities. I brainstormed all possible answers that I could think of and then sorted them into 5 groups which include: The Horse, The Rider, Tack and Gear, Riding and Sports and Competition. Then, I turned the rough notes into a full description of how measurement is used around horses.
Because I ride horses and know quite a lot about them and their lifestyle, it wasn't necessary for me to research a lot for what I was looking for. I simply double checked that the information I had brainstormed was correct in a few horse information books.

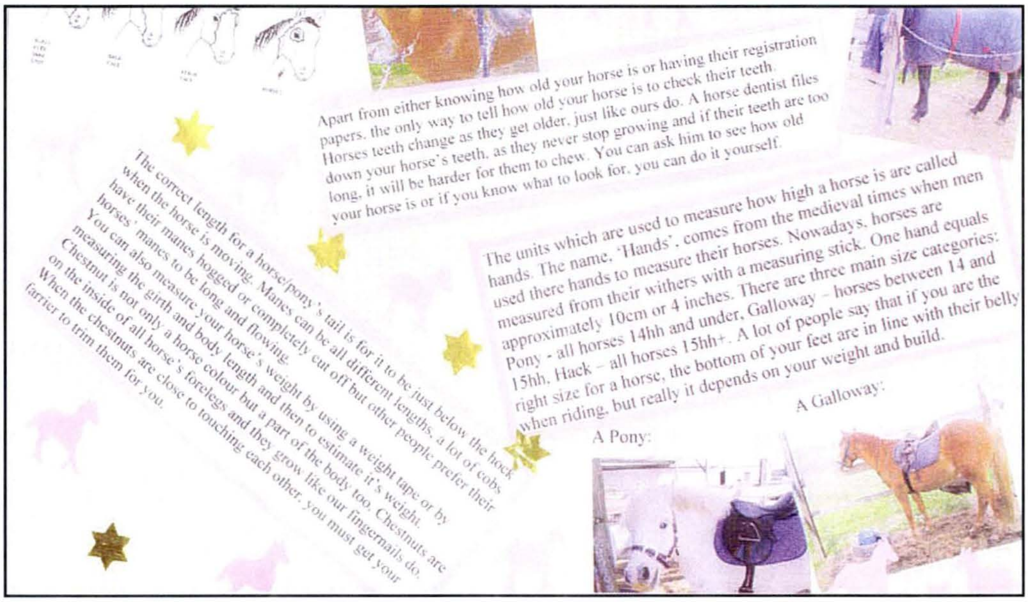


Figure 8.13. Measurement as it relates to horses.

Some of the students demonstrated their understanding of all of the main "big ideas" of measurement as informed by *Mathematics – A curriculum profile for Australian schools*: choosing units, measuring, estimating, time, and using relationships (AEC, 1994). These aspects of measurement were a focus of the twelve week unit of work leading up to the culminating performance task and many students showed that they could transfer their understandings from the broader unit to a context of their own choosing. The incidental evidences of other mathematical topics throughout students' work – number, pattern, space, and chance and data – support the value of the task in encouraging the connectedness of topics within mathematics itself and to contexts socially and culturally grounded in students' lives.

The learning demonstrated in the culminating performance task, by one student who situated her learning in the context of her involvement in the raising and training of pigeons, has been previously reported (Skalicky, 2007a).

Some students found the open nature of this task quite challenging and although they had attempted to produce some work, Samantha had, in the end, given students the option of completing the culminating performance task or compiling a "portfolio of evidence from their measurement work that demonstrated the understanding goals" (Classroom observation, 2/9/05).

8.3.2.5 Equity

Samantha was clearly establishing a numeracy classroom culture where all students could succeed and contribute ideas in the problem-solving environment Samantha created. Alongside this was the expectation that students would be able to explain and justify these ideas and in particular consider the strategies used and the viewpoints of other students in reaching solutions. As a collaborative environment, students in Samantha's classroom appreciated others' ideas and at times celebrated the success of others. This was evidenced throughout the unit of work and is exemplified in one instance when students were developing a conceptual understanding of area. For some students, appreciating that area was about the space that covered a whole shape was a positive achievement (Figure 8.14).

Figure 8.14. Finding the area of a rectangular table.

For one particular student, taking a solid understanding of area further, to how to find out the area of a circular shape was the focus (Figure 8.15a). For this student, learning was not something he undertook on his own; he shared it with his class and along the way other students became involved in the application of this discovery to the planning of a garden which they then modelled outside on the school oval (Figure 8.15b). During this learning, that occurred over many weeks, students worked together, they considered each others' ideas, they shared and tested them, and they ultimately achieved mathematical learning outcomes beyond those which the teacher had originally planned.

Figure 8.15a. Finding the area of a circle. *Figure 8.15b*. Applying this knowledge to planning a garden.

Samantha challenged her students to use their mathematical knowledge and understanding in a thoughtful and critical manner when considering strategies and approaches to solving the open-ended tasks that were posed. They were constantly making decisions and resolving problems and investigations. This was part of the active classroom environment in which mathematics was applied to real-world situations for students to consider.

Students were encouraged to take risks and were comfortable in acknowledging when they may have gone wrong in an approach or a solution. They were encouraged to explain and justify their strategies and their reasoning. A normal part of the classroom culture was the challenging of their on and others' strategies and thinking, with the goal of considering reasonable solutions. Both Samantha and her students asked questions to understand better the tasks they were working on. This was done to ensure that their approach and solutions were reasonable and that they had uncovered the underlying assumptions embedded within tasks or, if not part of the task, that they were clear about the assumptions they made in reaching any particular solution.

Samantha occasionally exposed her students to contexts within the media that raised questions about social issues and were presented using mathematical information. One problem presented to students involved considering the validity or otherwise of some data presented on a television sports program about the use of non-performance enhancing drugs.

> Should AFL players who test positive for non-performance enhancing drugs be suspended? 59% voted NO, and 41% voted YES. On SBS World of Sports (Classroom observation, 22/7/05)

In discussing this report, students came up with many issues that they felt made the data potentially meaningless without other information, and they also alluded to potential biases that could underpin the data [a variety of student responses].

> The statistic is meaningless.
> Can't find out how many people voted.
> Some people could have voted more than once.
> I wanted to know who voted, if they didn't like football they might say yes.
> I wanted to know the number of people that voted.

The types of mathematical tasks Samantha posed and the reasoning capacities she was developing with her students were all important preparation for when students may be presented with social, political, and cultural situations which mathematics influences. A classroom culture was established in which challenging each others' ideas was commonplace. This was achieved respectfully and involved all students and students with very diverse mathematical capabilities. The students themselves were interested in each others' strategies and solutions, contributed their own ideas to others and shared viewpoints – because all of this valued was by Samantha. The students actually moved each others' thinking forward by questioning each other and considering alternatives and others' viewpoints. Samantha's motive was to empower her students for their futures as learners and as citizens.

8.3.3 Student learning

The six students interviewed in this phase of the study were asked to describe and discuss the tasks they had undertaken during the unit of work, "How do you measure up?" The interviews were analysed with respect to the five dimensions of numeracy. Chapter 3 presented how each dimension was categorised and described. The following subsections detail how the students' learning was evidenced across the five dimensions of numeracy. The reporting of the results is intended to provide representative evidence of the range of responses for each category within the dimension. In doing this, at least one comment from each student is included for each of the five dimensions.

8.3.3.1 Mathematics

The numeracy unit of work in this case was designed to "get [students] using measurement, understanding how formal units of measurement work, and how they apply to our real-world situation" (Samantha, 14/12/05). Samantha involved her students in many open-ended, authentic tasks to support student learning across a range of measurement attributes. Students in this case also completed a culminating performance task in which they applied their understanding of measurement to answer a question of their choice. Within the context of discussing both the open-ended tasks and the culminating performance, the students demonstrated specific mathematics understandings in the area of measurement, across a range of levels. Table 8.1 summarises the observed categories of Mathematics for each student, as evidenced by the student interview.

Table 8.1
Student learning: Mathematics dimension (Stringybark Primary School - Samantha)

Category / Student	S13	S14	S15	S16	S17	S18
Reading and describing measurement	✓	✓	✓	✓	✓	✓
Making meaning from measurement	✓	✓	✓	✓	✓	✓
Analysing and interpreting measurement	✓	✓	✓	✓		✓
Evaluating and transferring measurement understandings to new or different contexts			✓	✓		✓

All of the six students were able to *Read and describe measurement.* They did this by identifying appropriate measurement attributes and units of measure and by identifying the measurement tools that they used during the unit of work. The following are examples.

> I've learnt how to measure! And how to use the measuring equipment and using different types of measuring tools, like the scales, the metre rule and the metre clicky thing, the trundle wheel. [S15]

> My mum sent me an email on how you could measure the amount of seconds she has lived, in minutes and in hours and in days and months and years. [S17]

The six students could also *Make meaning from measurement.* Student 17, who had found the unit of work very challenging, was able to provide a reason for having measurement in the context of "how much money" as "you wouldn't know how much change you would get [if no measurement]." The other five students provided strong evidence of their capacity to relate the measuring tasks they were completing to a developing understanding of the purpose of measurement and measurement concepts such as estimation, conservation of area, the relationship between area and perimeter, and the use of appropriate formal units of measure and appropriate measuring tools. The following are examples.

> We did have a square metre, which just because it's longer or different shapes doesn't mean it has more area. If you have like a square metre and cut it in half and put it together it is still a metre square, but the perimeter is different. [S13]

> We thought we were getting the main things, like weight, height, and age and then we went from arm to fingertip, and we were going to measure temperature

except we couldn't get the gear, and instead of measuring your foot in metres we decided to measure it in centimetres ... then we measured the distance between our eyes from there to there, and we just used a measuring tape, the little measuring thing … the main thing we used was the height stick, the scales and that measuring tape, depending on what we were measuring. [S16]

Five of the six students evidenced learning at the level of *Analysing and interpreting measurement*. These students were able to consider multiple aspects of the task they were undertaking and shared their interpretations and findings. Three examples are presented.

Once, [another student] and I were in class and you had to bring your own problem, and he made a building of a house, but with no roof, and he used one centimetre cubes and you had to find out how many centimetre cubes he had used. At first I thought he did the whole thing [a solid building] … but it was just the outside … and then I figured out after a while. We got a few blocks, a lot of blocks, and just built it.
Int: Oh I see, you've got 80 plus 80 which equals [the area of two walls of the building] 160.
S14: And the two 70's. It was pretty easy in the end. [S14]

Okay! So [the teacher] wanted us to draw a garden and she wanted to have corn, potatoes, peas, carrots, and strawberries and then she wanted a certain amount of rows in each of them and she wanted I think it was about five rows of corn, and I had to put in a bit of extra so it could grow. She wanted really big ones. And then for the potatoes she wanted a space of 6 by 4 or something, yeah, and then I put them in over here, and then, the peas. There were three rows of peas, and so I put them in over here, and then the carrots, I put them, I think she wanted four or five rows of them and I put them in over there, and then the strawberries. I put them, about five rows of them.
Int: So how big was the garden altogether?
S15: 60 metres around the outside … we had to think about what the perimeter was of everything, so we had to try out 15 by 15 and all this and it came out that the square was the most, but now we have found out that a circle is bigger. [S15]

Well, with my big question, Tasmania has 477,300 people in it, I knew that was 47.73% of a million and I somehow want to work out how long it would take [for the population of Tasmania to build the Great Wall of China], but [the exact number] was too hard … and it was around half, 50%, so I used that. [S18]

Three of the six students were able to *Evaluate and transfer measurement understandings to new or different contexts*. Student 15 described how she made and tested predictions regarding possible differences between the weight of milk and water. She also inferred that cold water was lighter than hot water and went on to test her theory.

We were making statements, and I thought that milk would be heavier than water and we had to get these measuring containers and we had to measure them. [The teacher] brought in a litre of milk and we put that in. Then we had to put a litre of water in there, cold water, and the milk was heavier by, I don't know how much

it was now, but it was heavier, and then we made another statement, that cold water was lighter than hot water and that was right.
Int: So any reason why you thought that in the first place, why hot water would be heavier?
S15: Well we thought that the heat would make a difference, because you've got pressure building down on it.
Int: So what happened as it cooled down?
S15: As it cooled down it went equal. [S15]

Student 18 described the learning he undertook and the discoveries he made about pi and the area of a circle when he began exploring the area of a circular table after one student was using counters to calculate the area. Student 18 went on to apply this new understanding to the design of a vegetable garden later in the unit of work. He used his knowledge about the area of a circle to consider a circular garden, using calculations to discover that, using the same perimeter, a circular garden would provide a larger area than a square or rectangular garden.

> I thought because people started to measure things and like put counters on the table [for area] so then I thought like well, it's not going to be like a measurement. It's not going to be a number of counters. What I did was I got big pieces of paper and put them on the table and cut out the shape of the top of the table, and then I put it in five centimetre squares and then, the bits that weren't quite square around the edges, I cut out and put them in a box and then I counted the ones that were full squares and then the ones in the box I put them together to make it just around about a square. I added them all up and then when I got that I counted them all up and times it by 25 'cos there's 25 squares in a five centimetre square, so then I got 12,650 centimetres squared and then [the teacher] pointed out pi.
> Int: What did you find out about pi [π] ?
> S18: Well in the end I needed the formula, with the πr^2. I used it with the table and the diameter was 120, but to make sure I did it the right way, I did it across the other way and it was good and then I halved it to get the radius. I did that and it ended up being 11,304 square centimetres. [S18]

> Well the question we did was, well, first it was the rectangle, using the rectangle, what would be the biggest area using sixty metres of fencing wire, and they got the square 15 by 15 and then later on I just thought well, I'll just do a circle and see what the circle comes up with and the square was 224 square centimetres and the circle was 286.62419 square centimetres. [S18]

8.3.3.2 Reasoning

Table 8.2 summarises the observed categories of Reasoning for each student, as evidenced by the student interview.

Table 8.2

Student learning: Reasoning dimension (Stringybark Primary School - Samantha)

Category \ Student	S13	S14	S15	S16	S17	S18
Remember	✓	✓	✓	✓	✓	✓
Comprehend	✓	✓	✓	✓	✓	✓
Analyse	✓	✓	✓	✓		✓
Evaluate	✓	✓	✓	✓		✓
Create						✓

All of the students were able to *Remember* and recall information. In this case, this level of learning related not only to measurement data, but also to other areas of mathematics and to contexts that were at times relevant to an individual's path of inquiry. The following are examples.

> A composite number is one that you can times more than one composite. You can't just times it by itself. It has lots of different things that connect and a prime only you can only times it by itself. [S13]
>
> Some venom of spiders affects the nerves. [S17]

All six students could also *Comprehend* with a common theme of students' discussions relating their understanding of mathematical ideas to the practical and investigative nature of the classroom. Examples from Student 13 and Student 14 demonstrate this.

> So I held the book and I tried to imagine that I had a kilo of sugar. [S13]
>
> Int: How did you work on the step distance?
> S14: Well I got [another student] and we went outside and got a bit of chalk and just did five normal steps and he just marked them on the back of his foot every time we did it ... [S14]

Student 18 captures the overall sentiment of "knowing," that is just answering questions without comprehension, as compared with "knowing" and understanding by "doing."

> With worksheets you might know lots about maths and it will ask you these questions and just write the answers, whereas say, you might know the answers but yet you don't know how far it might actually be ...whereas if we are actually doing it, then we know. [S18]

Five of the six students went on to demonstrate a capacity to *Analyse.* Evidence for this level of Reasoning matched closely the student examples for the *Analysing and interpreting measurement* category in the Mathematics dimension (refer Section 8.3.3.1). Other areas of mathematics learning were evidenced in this category, beyond the goals of the measurement unit of work. In the following example, Student 18 discusses his detailed diagram in which he analysed the relationship between the numbers in Pascal's Triangle and discovered patterns related to divisibility by eleven, and the sum of each row.

> It's what I figured out, 'cos it's got all the ones down the side and it's got twos, three, four, five and six and all that. The first line is one and then I went one times eleven equals the second line, which is eleven and then eleven times eleven equals the next line and that line times eleven equals the next line, and that line times eleven equals the next line, but then it didn't work for the next one.... And then if you add all of them together [across each line] the answer doubles, like 2, 4, 6, 8, 16, they kept on adding up until I went down to about the sixteenth line and finished there. [S18]

Four of the six students demonstrated learning at the level of *Evaluate,* comparing and discriminating amongst ideas to make decisions about how to proceed with a task or an idea. In the first example, Student 13 considered critically teaching contexts in which students work to synthesise knowledge, skills, and process from different contexts as compared with her current classroom where she believed that her teacher brought these aspects of learning together. Although it is not possible to provide the whole student interview, evidence was provided from the rest of the interview that lead the author to believe that this was a genuine evaluation.

> You might learn one thing from one teacher and another from another and keep them both in mind, and when you need to do it, I don't know. Here, [the teacher] teaches us all the things together. [S13]

> Int: And why did the garden end up being 15 by 15 m?
> S16: Cos that happened to be the most area and that was 225 I think. But then later I realised that you could have a bigger one if you used the circle and that's when we went outside.
> Int: So what did you go and do with that?
> S16: We got five wickets and we put one in the middle and then we measured out was, because nine and a half was the radius, nine and a half and then we put another stick there and then we made a string that was that long and then tied that around the pole so we could swing it around and then we put poles around it and that.
> Int: So you put your mark right around the edge of the circle. So why did you want to make that?"
> S16: So that we could figure out how we could measure exact circles. We thought if we are going to do it, like little, we might as well have done it outside. [S16]

> I asked if I could do, because we were designing the vegetable garden, I asked if I could do one for a circle and she had five things she wanted ... but there were two she wanted to do bigger ... the corn and potatoes and so what I did was I drew my circle and then I drew another circle inside it pretty close to the edge and then I drew a circle around the outside close to that one for the path that goes around and one to go in and then around the outside I split it into thirds like that and then ...in the middle I just split it straight in half like corn on one side and potatoes on the other. [S18]

Five of the six students interviewed evidenced high level reasoning by their capacity to naturally move from analysing to evaluating and making informal inferences as they described the tasks they undertook. Although these same students demonstrated their ability to bring together ideas and at times hypothesise, only one of the six students provided explicit evidence of his ability to *Create* new and original ideas when discussing the measurement unit of work. Student 18 did this on numerous occasions. When considering an effective and accurate way to find out the area of a circular table, he discovered the relationship between the area and the diameter of the table and went on to complete and evidence clear understanding of the area of a circle and its connection with diameter and circumference.

> I thought 'cos people started to measure things and like put counters on the table [for area] so then I thought like well, it's not going to be like a measurement, it's not going to be a number of counters. What I did was I got big pieces of paper and put them on the table and cut out the shape of the top of the table, and then I put it in five centimetre squares and then, the bits that weren't quite square around the edges, I cut out and put them in a box and then I counted the ones that were full squares and then the ones in the box I put them together to make it just around about a square. I added them all up and then when I got that I counted them all up and times it by 25 'cos there's 25 squares in a five centimetre square, so then I got 12,650 centimetres squared and then [the teacher] pointed out pi.
> Int: What did you find out about pi [π]?
> S18: Well in the end I needed the formula, with the πr^2, I used with the table and the diameter was 120, but to make sure I did it the right way, I did it across the other way and it was good and then, I halved it to get the radius and I did that and it ended up being 11,304 square centimetres. [S18]

> Well the question we did was, well, first it was the rectangle, using the rectangle, what would be the biggest area using sixty metres of fencing wire, and they got the square 15 by 15 and then later on I just thought well, I'll just do a circle and see what the circle comes up with and the square was 224 square centimetres and the circle was 286.62419 square centimetres ... And then [the teacher] asked me how to do a perfect circular fence because I thought it was hard.
> Int: So what did you think?
> S18: I [drew] poles around the outside, and then I got the fence and put it around the outside, then I got a thing that was probably like the middle, like a pole but

you can bend it, and then put a pole in the middle of those two poles to try and make it circular [describing a model].
Int: And then [two other students] went outside to test your theory. [S18]

8.3.3.3 Attitude

As students discussed the different tasks they had undertaken during the measurement unit of work, their attitudes toward their learning were revealed by both the comments made and also by the manner in which they conveyed them. Table 8.3 summarises the observed categories of Attitude for each student, as evidenced by the student interviews.

Table 8.3
Student learning: Attitude dimension (Stringybark Primary School - Samantha)

Student / Category	S13	S14	S15	S16	S17	S18
Confidence/ self-efficacy	✓	✓			✓	✓
Interest	✓		✓	✓		✓
Enjoyment	✓	✓	✓	✓	✓	✓
Intellectual stimulation	✓	✓	✓	✓	✓	✓
Diligence/Perseverance	✓	✓		✓		✓
Appreciates value of mathematics	✓	✓	✓	✓	✓	✓

Overall, this was a very strong dimension for the students in this class, with all of the six students interviewed evidencing four or more of the affective factors deemed to contribute to successful numeracy learning. In terms of *Confidence*, four of the six students displayed a positive self-concept about their ability to learn and succeed in the measurement unit of work. The following are two examples.

> Yep, [I know a lot about time] … It was pretty easy in the end. [S14]

> [I just had] a bit of a head start really and like we did this mind map and did different types of measurements and I already knew what different types were so I was able to put those in. [S18]

Students' *Interest* in their learning was predominantly related to engaging in a particular task or area of the measurement unit of work, as exemplified by Student

13 and Student 18. In the last example, Student 15 shares an interest in the inherent nature of the numeracy lessons themselves.

> I picked poverty [for culminating performance] because it sort of interests me. If it didn't interest me I might go and find something else that interests me. [S13]
>
> Pi, that was really interesting and I would like to learn more about that sort of stuff, just about everything really. [S18]
>
> We do a lot more interesting, different maths, and we do a different variety. [S15]

All of the students experienced *Enjoyment* in the numeracy classroom. Their enjoyment came from the problem-solving nature of the tasks and the opportunities the students had to work on authentic tasks that they could explore and model. The following example is representative of the students' views.

> It's like everyone used to hate maths, but now it is fun solving the problems and all that. [S16]

Student 17, although he had found many aspects of the learning difficult, was still engaged at some level and enjoyed the practical nature of the learning.

> I like using my hands … because I can have a visual of it. [S17]

The following comment by Student 18 evidenced the connection between the enjoyment experienced in the classroom and the problems that Samantha posed from her own life.

> Yeah, it's good … 'cos it's sometimes funny and stuff [referring to problems posed by the teacher from her own life]. [S18]

Not only was the category *Intellectual stimulation* evidenced by all of the students in this case, but also it was evidenced by a variety and depth of very rich quotes from the students. One example from each student is included here to provide evidence of the breadth of experiences that resulted in students being challenged and intellectually stimulated during the unit of work.

> You might learn one thing from one teacher and another from another and keep them both in mind, and when you need to do it, I don't know. Here, [the teacher] teaches us all the things together. [S13]
>
> You have more chances, problems not worksheets. You can go explore further and [the teacher] doesn't mind.
> Int: How do you think that helps you with your learning?
> S14: Just like different ways and like information you can keep in your head if someone asks you. [S14]
>
> You get more involved in it and most of the time, you learn more stuff … and you want to learn more and more. [S15]

> Int: So why did you want to make it [circle outside]?
> S16: So that I could figure out how we could measure exact circles. [S16]

> Like you're using your hands and you might be using your brain more. [S17]

> Well the men who died building [the Great Wall] they used to crush the skeletons and mix it with cement and stuff. ... I talked about that once at home for a long time. [S18]

Four of the six students showed their *Diligence* by discussing times when they persevered on tasks. Again many instances were provided during the student interviews and three examples are included that represent the tendency for students to take time to work through problems and re-work possible solutions. The examples also evidence the connection between this category and students' enjoyment, interest, and intellectual stimulation in their learning.

> They might just print it out and stick it in, but you can't do that because you have to put it in your own words and make it more interesting to read and fun to learn. [S13]

> So I did 3 divided by 2, like that and 5 divided by 3, and then 8 over 5, and kept going.
> Int: All the way up to my goodness, like a trillion, 346 billion.
> S14: I think a billion ... If you like the question you can keep going on it. [S14]

> It's what I figured out, 'cos it's got all the ones down the side and it's got twos, three, four, five and six and all that. The first line is one and then I went one times eleven equals the second line, which is eleven and then eleven times eleven equals the next line and that line times eleven equals the next line, and that line times eleven equals the next line, but then it didn't work for the next one.... And then if you add all of them together [across each line] the answer doubles, like 2, 4, 6, 8, 16. They kept on adding up until I went down to about the sixteenth line and finished there. [S18]

Student 16 also reflected on receiving assessment feedback that would motivate him to work harder next time.

> If I got [my work] back and I get a three and a half [out of five] ... then I would have a look at it really hard in the next one and that ... and in here [the rubric] it describes everything you need to do. [S16]

All of the students in this case evidenced an *Appreciation for the value of mathematics* when discussing the very practical nature of measurement and its relevance to every aspect of life. The following are examples.

> Measuring helps me, because I like building. I like building stuff with my hands and I normally go outside every weekend and measure with my Dad and try to help him with stuff and build stuff. [S13]

> I would like to put all my, what I call maths problems that we did, I would like to put it in a book, all the questions that we had to do, and have a look at that, and give it to another class and they could do them. [S15]

> I'd like to be able to remember it all. It could help me in the future like, going for a job and college and getting a driver's licence. [S16]

> Like say speed. You say, see that car was going fast, and they would be able to say that car was going about 110 kilometres per hour because when you say fast anyone could think like 200 or things like that. [S18]

8.3.3.4 Context

As discussed in Section 8.2.3.4, Context was a purposeful and highly valued element of student learning in Samantha's numeracy classroom. Students were provided with many opportunities to explore the relevance of measurement in both in-school and out-of-school contexts. During the student interviews, students discussed the many tasks they had undertaken during the unit of work, and Table 8.4 summarises the observed categories of Context for each student, as evidenced by the interviews.

Table 8.4
Student learning: Context dimension (Stringybark Primary School - Samantha)

Student / Category	S13	S14	S15	S16	S17	S18
Personal experience of context					✓	
Context integrated with mathematics as presented	✓	✓	✓	✓	✓	✓
Context integrated with mathematics, from both prior knowledge and as presented	✓	✓	✓	✓		✓
Relational understanding of the mathematics and the context and can transfer to new contexts	✓	✓*	✓*	✓		✓

* Evidence of this level of learning was emerging during the interview, but as these students had not yet completed their culminating performance task they did not fully evidence this category.

The *Personal experience of context* category was not representative of the students interviewed in this case. Student 17, who had difficulty sharing his learning during the interview, did however express his thoughts that it was easier to understand some measurement concepts when he was "using [his] hands" or was able to "have a visual of it." These instances were deemed to be evidence of Student 17's preference for practical application of measurement and therefore the use of personal experience in a practical sense to develop some level of mathematical understanding.

There were times during the interviews when five of the six students remained focused on the *Context integrated with the mathematics as presented.* The following examples are representative of the types of responses in which students discussed both the measurement and the context as represented within a task but without demonstrating understanding or considering implications of the relationship between the mathematics and the context beyond these parameters. The following are examples.

> [When measuring a person] I just had to note how tall they are and then like, put it in metres, kilometres and centimetres, feet, yards ... and we did a few different ones, like head to knee ... and like each side of the body, if they are symmetrical or different. [S14]

> Okay! So [the teacher] wanted us to draw a garden and she wanted to have corn, potatoes, peas, carrots, and strawberries and then she wanted a certain amount [sic] of rows in each of them and she wanted I think it was about five rows of corn, and I had to put in a bit of extra so it could grow, she wanted really big ones. And then for the potatoes she wanted a space of 6 by 4 or something, yeah, and then I put them in over here, and then, the peas, there were three rows of peas, and so I put them in over here, and then the carrots, I put them, I think she wanted four or five rows of them and I put them in over there, and then the strawberries, I put them, about five rows of them.
> Int: So how big was the garden altogether?
> S15: 60 metres around the outside. [S15]

> My big question was what is the longest road in the world and I found out that that's the Eyre Highway which starts in Adelaide, Port Augusta and goes all the way through to Western Australia, to Perth to Lawson ... It's 1687 kilometres. [S16]

> Well, my big question is how long would it take Tasmania's population to build the Great Wall of China? I have nearly finished it and I worked out that over one million people worked on the Great wall of China and Tasmania's population is 477,300 and that's nearly about half a million so I knew that that would take twice as long as it's half the people. [S18]

Although Student 17's mathematics learning was not as high as the other students' demonstrated, in this dimension he evidenced his ability to move beyond his personal experience of context when he shared his decisions in choosing the font size in a book he was making because of the age of the potential audience.

> Like the size of the writing, so it's not way too big, it's like one that I can give my Dad for a present. You don't have to do too big writing because it's not for kids or anything like that. [S17]

Five of the six students, demonstrated a capacity to integrate both the mathematics and the context at a higher level in order to make sense of measurement concepts. There were times during their interviews when *Context was integrated with the mathematics, from both prior knowledge and as presented.* As this category was well evidenced for the students in this case, one example from each student is given. In all of the examples, the students were bringing together their measurement understandings with the respective context, as well as their own knowledge of these contexts to make sense of the task and to extend their learning. For Student 13 it was her experience of how heavy some other objects that weighed one kilogram felt and how tall she could imagine the height of the school principal to be.

> Well, I held the book and I tried to imagine that I had a kilo of sugar or so and I sort of varied them, and I thought it weighed a bit more than a kilo or so, I thought 1.5 kilograms.
> Int: So you were trying to think of something you already knew the weight of?
> S13: Yes and compare it.
> Int: And did you do that with most of them?
> S13: Oh [Principal's height] I just thought he would be about up to there [points to a place on the door] I thought of the door and I thought it was 2 metres and he is not quite that tall. [S13]

For Student 14 it was his knowledge of the relative sizes of different vegetables that influenced his decisions to design a vegetable garden with realistic spacing between the vegetables. Student 16 also considered other knowledge when designing a vegetable garden, but for him it was making room for compost bins and paths for wheelbarrows that were important.

> [The teacher] had 60 metres of wire, so that she was building a garden, so I had to try and figure out what was the biggest area you can get, which was fifteen by fifteen, but if the house was there [referring to bordering on one side of the garden] you could have it bigger, but she couldn't do that. So each, like peas are smaller than spuds, and we had to work out how many rows she wanted … yeah how much room she wanted between so that she could walk through. [S14]

> She wanted three rows of corn, three rows of peas, three rows of potatoes, four rows of carrots and five rows of strawberries, but she wanted them corn and the carrots to be in a bigger space, because they take up more room … and then I figured out all the dimensions. I measured out and then I had some space left so I put a compost in.
> Int: So how did you decide to place them the way you have?

> S16: I am not sure, I thought, I will put them two there and I was going to have three there but there was space either way, so that one's probably a bit bigger ... and the smallest path is a metre ... for the wheelbarrows and that. [S16]

Student 18 considered quite complex contextual implications when working on his culminating performance task about how long it might take Tasmania's population to build the Great Wall of China.

> [Student continues to discuss his Great Wall of China investigation] ...then I added things to that that would contribute to how long it would take, like Tasmania has modern technology and stuff plus excavators, so I just thought a round about in my head just how long that would make the time and then I got my answer. [S18]

The students in this case were provided with many opportunities to make connections between their learning in different tasks and a culminating performance task was also implemented for students to choose their own context and develop a piece of work that demonstrated their understanding of measurement concepts. All of the students discussed their culminating performance in the interview, but only three students were at a stage of completion of this task that enabled them to evidence an ability to apply their measurement understandings completely in this particular new context. Students 14 and 15 discussed the early stages of their investigations and were moving towards sound evidence of a *Relational understanding of the context and the mathematics and an ability to transfer this to new contexts*. Students 13, 16, and 18 evidenced this category of Context, not only when discussing the culminating performance tasks that they were still working on at the time of the interview, but also in their discussions about other tasks that they undertook during the unit of work.

> Like if you put mulch on a garden, a person might tell you that so much mulch you can fit in one square metre or something and then you could measure the garden and say I want so much mulch and you know exactly how much mulch. [S13 – referring back to an earlier task using newspaper to make a square metre]

Student 18 transferred his earlier learning about calculating the area of a circle to a new task on designing a vegetable garden by hypothesising and testing that a circular garden would create more area (p. 246). When Student 16 came up with the idea to go outside and make a circle the actual size of that posed by Student 18 and to check previously calculated results (p. 245) this student was taking the learning of his peer and applying it within a new context.

8.3.3.5 Equity

In the context of the student interviews, in which students shared and discussed the tasks they had undertaken during the measurement unit of work, Table 8.5 summarises the observed categories of Equity for each student. This dimension was evidenced when students revealed a propensity to be open to others' mathematical ideas, and to consider alternate strategies when solving problems and undertaking investigations. It was further evidenced when these same capabilities were applied to social and political contexts.

Table 8.5
Student learning: Equity dimension (Stringybark Primary School - Samantha)

Student / Category	S13	S14	S15	S16	S17	S18
Personal social engagement	✓			✓		✓
Awareness of issues	✓					
Considering viewpoints	✓					
Relating mathematical information with social and political consequences	✓					
Challenging inequity						

Three of the six students demonstrated *Personal social engagement* by their awareness of others' contributions in the learning environment and their disposition to question and make decisions based on their own and others' strategies. In the following example, Student 13 described a task involving the design of a vegetable garden using 60 metres of fencing wire. She revealed the shared learning that took place in coming to a decision that a square provided the largest area. She then mentioned the other shapes that students considered, before realising that a circle provided the greatest area. In this particular example, she does not, however, incorporate any evidence of her own or others' reasons for their decisions.

> [Samantha] had 60 metres of wire and ... she had a big enough space to do something, to grow vegies, and she wanted the biggest area possible to grow her vegies, and we had to design all different shapes that she get a bigger amount of area with the 60 metres of wire. It ended up that we had a 15 by 15 perfect square

> ... but then [another student] thought of a solution that if you built it up near, against a wall you could have 20 metre sides, but [teacher] said she was not doing that and then [another student] thought we could do it in a circle. We also did a triangle, and we tried a circle and it seemed a lot bigger than the 15 by 15 square, and [the teacher] thought 'Oh, I wish I did the circle. [S13]

The example responses of Student 16 and Student 18 revealed not only a disposition to question their own work in relation to the work of their peers in considering what might be the most effective solution, but also the incorporation of reasons and opinions for their decision. For Student 18, this category of the Equity dimension was evidenced early in the interview when he described his discovery of how to accurately calculate the area of a circle, by considering the viewpoints of his fellow students, who were trying to calculate area by covering a circular table with counters. He then discussed applying this understanding to the vegetable garden design task, when he came up with the original idea of a circular vegetable garden, again after his peers had designed a square vegetable garden.

> Well the question we did was, well, first it was the rectangle, using the rectangle, what would be the biggest area using sixty metres of fencing wire, and they got the square 15 by 15 and then later on I just thought well, I'll just do a circle and see what the circle comes up with and the square was 224 square centimetres and the circle was 286.62419 square centimetres. [S18]

In the second example Student 16 had taken into account the solution strategy of Student 18 in designing a circular garden, and went outside to the school oval to make a model of the garden to see if it really did have a larger area than a square garden of the same perimeter. He enacted his classmates' ideas with a particular purpose in mind, to see if he could make and measure an exact circle (p. 245)

Although social and political contexts were not a common feature of the measurement unit of work, one of the six students demonstrated an ability to *Relate mathematical information with social and political consequences*. When Student 13 was providing an example of the benefits of measurement units, she referred to the possible consequences of wasting resources and money by not knowing how to measure accurately.

> If you put mulch on a garden, a person might tell you that so much mulch you can fit in one square metre or something and then you could measure the garden and say I want so much mulch and you know exactly how much mulch, [and if you couldn't measure it] you would have to estimate it and then you would waste money, or get too much and you would have to go back or something. [S13]

In exemplifying this connection between the benefits of measuring accurately in a social context and with economic implications, Student 13 is implicitly evidencing her understanding of the issues involved and the options she refers to, of measuring accurately compared to estimating. For this reason, a tick has also be included in the two previous categories, *Awareness of issues* and *Considering viewpoints*, as these are assumed.

None of the six students interviewed demonstrated the highest level of Equity, *Challenging inequity.*

8.4 Chapter summary

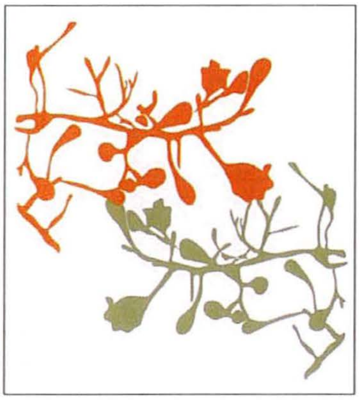
This chapter has shared the beliefs and practices of Samantha and the learning experiences and outcomes of her students. Samantha was an experienced and confident upper primary school teacher and a mentor to her colleagues at Stringybark. She revealed a strong sense of the Tasmanian curriculum reforms matching her own philosophy of teaching and learning. She was passionate about the capacity of the curriculum to support teaching for deep understanding and she was also passionate about numeracy and its relevance to students' lives both now and in the future. Samantha's teaching was informed by thoughtful planning based upon her own pedagogy and upon curriculum documentation. She positioned numeracy alongside literacy, inquiry, and reflective thinking as the core areas of the curriculum that informed her planning across all areas. As a member of the same collaborative planning team as Ophelia, Samantha implemented the same unit of work, "How do you measure up?," during this case study.

The dimensions of Mathematics, Reasoning, Attitude, Context, and to a lesser degree Equity featured in this case. The engaging and enjoyable nature of the numeracy classroom was centred on Samantha's use of open-ended questions that were based upon authentic life experiences. These tasks enabled a natural connection among mathematical concepts, encouraged deeper engagement with the embedded mathematics within tasks, and provided a purposeful inclusion of contexts relevant to students. The tasks could also be solved using a variety of strategies and this required student to consider assumptions, ask questions, and explain and justify their solutions. Samantha supported her students in their use of problem-solving strategies and their abilities to share and challenge each others' ideas. Samantha's numeracy classroom was a very dynamic, active, and collaborative environment as she exploited learning opportunities as they arose and worked toward her goal of creating independent and creative thinkers.

> *The goals of the measurement unit were to get kids using measurement, understanding how formal units of measurement work, and how they apply to our real-world situation and I feel generally that the kids got a lot out of it, that we largely achieved those goals with most kids. We got part way to achieving them with all the kids. Some obviously took it on board and obviously did a lot better than others, but I can confidently say that all of the kids came out with a much better understanding of our formal measurement system and a much better understanding of real life applications. ... I would like to refine the planning so that I am catering more to actually scaffolding skills. ... With the more capable kids, I would run it basically the same, because I could watch them just pick up and go really. ...*
>
> *The biggest surprise was the stuff that the kids came up with. The things they thought of that had not crossed my mind at all, a lot of those completely blew me away. There were surprises in the really creative ways they approached a lot of the measurement tasks. They were very different but worked and they could tell me why they were doing it that way. Probably the biggest surprise was running that unit of work for a full term very intensively and they didn't get sick of it. It lasted for a full term and they remained immersed and excited and interested. ... It involved every strand of mathematics. They weren't aware they were doing it, but they were using very complex problem solving skills and strategies, they were using all four mathematical processes in working out and playing with measurements. They were using the whole decimal system in playing with measurements. There was a lot of data collection and recording, the patterning, looking at patterns. They have touched on every single strand of mathematics using the measurement focus.*
>
> *I saw with a lot of kids, apart from the obvious huge increase in confidence in dealing with the area of measurement, I could see mathematical skills being used and practiced which they weren't aware they were doing, but I could see the maths in it and get very excited by this. ...*
>
> *I planned quite deliberately, for Thinking. The whole purpose, not purpose but the way the unit was put together was as an Inquiry based unit, referring to the Essential Learnings standards as to what sort of skills do I need to be developing in the kids, what sort of*

thinking strategies do I need to be encouraging. A lot of the activities that were given to them and the open-ended tasks and the explorations were aimed at getting them to use specific thinking strategies. Throughout the teaching and also the reporting back from the kids I am learning to focus on thinking strategies, to be able to say "That is good mathematical thinking," "You have chosen a really useful strategy for that and it was very efficient," "I like the way you have evaluated that data." So using the language very consciously with them and encouraging them to use the language. I also deliberately incorporated a certain amount of reflective thinking. Throughout the unit they were often asked to stop, go back, reflect on what they had done, reflect on what you had found out, could there have been a better way to do it. Then getting the kids to actively share the thinking processes that they had used and identify those processes. By the end of the unit, the class were then getting pretty good at telling me what process they had used in order to get that solution, so they started to get an understanding of the whole language. ... It's just getting them used to using words like judge, analyse, evaluate, respond to, react to, ask questions about. I made that very very explicit and the kids become very comfortable with that and actively develop those strategies. They taught each other as much as I taught them. (Samantha, 14/12/05)

In the final Results chapter, Chapter 9, the findings of the research study for the teachers Ange and Jen, from Tanglefoot School, and for their students, is presented.



Chapter Nine

Results: *Tanglefoot School – Ange and Jen*

> *Numeracy has its place as a stand alone discipline with mathematics, well I believe it does, and I think it is important to keep that. We can't afford to lose that at all, but it also needs to continue to be integrated. ... It's vital [the connection between mathematics and other concepts] because otherwise they lose it, the information goes because it has no meaning to them, which happens with a lot of maths. ... I think it is always going to be important, that connection and I guess the way that we can do that is looking at what is current, so here we chose the G8 summit.* (Ange, 2/11/06)

9.1 Introduction

The setting for this case study, Tanglefoot School, is described in Section 5.4. Tanglefoot's middle school constructed its curriculum using Tasmania's Essential Learnings framework (D*o*ET, 2002), but in a manner unique to the school. The 18 key element outcomes were positioned across the curriculum, not only supporting interdisciplinary learning, but also within a discipline-based framework (see Table 5.1).

The middle school teachers scheduled two or three transdisciplinary units of work each year, when timetabling enabled them to work collaboratively. These units of work were designed to complement the discipline-based educational framework and to encourage targeted learning outcomes in authentic contexts. In this case, the two Grade 8 teachers, Ange and Jen, worked together to implement a five week interdisciplinary unit of work, "Live 8," with the Grade 8 students at

Tanglefoot School. Ange and Jen allocated three 50-minute lessons and one 100-minute lesson per week for the integrated unit.

Ange, a Grade 8 home tutor teacher and teacher of Mathematics and Science for the middle school, had been teaching for seven years. After teaching in Canada and England, Ange spent three years in Tasmania's State Government system, teaching Physical Education and general middle school subjects, before commencing at Tanglefoot School in 2003. Jen, also a Grade 8 home tutor teacher and teacher of English, History, and Studies of Society and the Environment (SOSE), had been teaching for three years. After graduating in 2002, Jen spent 15 weeks teaching in North Western Tasmania before being employed at Tanglefoot School. Both Ange and Jen had been planning and assessing for teaching and learning informed by the Essential Learnings for three years.

Data analysed and reported in this chapter are two teacher interviews with Ange and Jen, field observations taken during fifteen classroom visits with Ange and Jen and their 34 Grade 8 students, six individual student interviews, together with documents and photographs collected across the four phases of the research. It is noted that the case of Ange and Jen with respect to their beliefs about numeracy and subsequent student learning according to the five dimensions of practice conceptualised in Chapter 3 has been reported previously (Skalicky, 2007b).

9.2 Curriculum

This section presents Ange and Jen's beliefs about curriculum and summarises the unit of work that Ange and Jen were teaching during this study.

9.2.1 Ange and Jen constructing curriculum

Ange and Jen, in discussing and constructing curriculum, revealed two key themes to their conception of curriculum. First, an interwoven view was presented, in which both disciplinary knowledge and generic capacities – such as communication, teamwork, and research skills – were valued. Second, an inclusive view emerged, in which students' individual learning needs and self-esteem were valued, together with curriculum planning that caters for individual differences.

Ange and Jen felt that the Essential Learnings supported "real-life learning goals" and "[sat] nicely with integrated units of work." They gave examples of when they had planned for mathematical learning in previous integrated units of work in which a number of subject disciplines were brought together to investigate a "core topic." The disciplines remained central to their teaching and both teachers had expertise in particular subject areas: Mathematics and Science (Ange) and English and SOSE (Jen). As middle years' teachers they were aware of the broader picture of their students' whole education and were concerned with ensuring that the learning they provided connected with, and prepared students for, senior secondary and more particularly, "post-compulsory" education. They saw discipline-based learning as an important part of catering for the future needs of their students.

In planning and organising inter-disciplinary units of work, Ange and Jen looked at what disciplines would best inform investigation of the concept they and/or their students wished to explore. They also considered the structures of the middle school, including teachers and timetabling, that would enable the resources and time to implement the unit of work effectively. This placed some restrictions on inter-disciplinary learning, but Ange and Jen appreciated the curriculum construction of Tanglefoot's middle school that enabled this learning to occur.

The opportunity to engage the students in group work and inquiry learning, in which tasks were more open-ended, was identified by Ange and Jen as being an important part of the inter-disciplinary units they provided. They wanted their students to develop information literacy and communication capabilities, planning and organisational capabilities, problem-solving skills, and abilities to work with others. These generic capacities were planned for and assessed in inter-disciplinary units of work alongside relevant discipline-based learning outcomes.

Ange and Jen sought to provide an inclusive learning environment for their students. They saw curriculum as playing an important role in contributing to the development of students' "self-esteem" by catering for the "different learning needs" of students and allowing students to "go in and show exactly what they do know and what they can achieve." Ange also identified students' "motivation levels" as an important factor in their achievement. The integrated units of work were planned and implemented to provide students with autonomy to investigate a core or negotiated topic and the opportunity to achieve to "their full potential." Ange and Jen did this through the use of open-ended inquiry tasks and by encouraging the students to negotiate aspects of their work, from topic choice to presentation style.

Ange's and Jen's personal views were aligned with the school's construction of curriculum and they had autonomy over how they implemented integrated units of work with their students.

9.2.2 The unit of work: "Live 8"

The "Live 8" unit of work provided the situated activity in which the teachers chose to bring together the disciplines of Mathematics and SOSE. The aim was to enhance students' numeracy capabilities, their abilities to work collaboratively, their skills in information literacy and communication, and ultimately their understandings of the concept of poverty. The unit of work was inspired by the 2005 G8 world summit held in Gleneagles, Scotland in July 2005. G8 summits have been held annually since 1975 and involve the political leaders of the eight member countries (United States, United Kingdom, Japan, France, Germany, Italy, Canada, and Russia) discussing important world and economic issues. The two main themes for the 2005 summit were Africa and climate change. Prior to the summit, music concerts were held across the world to coincide with the United Kingdom's Make Poverty History campaign. These concerts, "Live 8," were designed to increase awareness about poverty, particularly in Africa, and to pressure governments into taking specific action towards relieving poverty.

Ange and Jen were motivated by this world event and a belief that the "Live 8" concerts would provide the Grade 8 students with an engaging, "real-world" context in which to learn about the contrasting nature of developed and developing, or third-world, countries and issues related to the broader concept of poverty. In addition, prior learning in mathematics would be extended and drawn upon to enhance students' understandings of the issues explored. Generic learning outcomes targeted and assessed in inter-disciplinary units of work were specified in Tanglefoot's Middle School Assessment Booklet. For the "Live 8" unit of work these included Being information literate, Creating and pursing goals, and Acting democratically.

9.3 Numeracy

This section presents the results for this case within three main sections: Ange's and Jen's beliefs about numeracy, the enactment of numeracy in the classroom with Ange and Jen and their students, and the student learning outcomes of six individual students.

9.3.1 Ange's and Jen's beliefs about numeracy

Ange and Jen expressed a view about numeracy that did not preference the role of numeracy across the curriculum above the role of mathematics as a "discrete subject." Ange, in particular, spoke of the importance of mathematics for providing some students with "pathways" for their future learning. She saw the Essential Learnings, and its incorporation into the middle school curriculum framework, as a means of providing opportunities to "get the girls interested in maths" and of supporting the students in learning to be numerate across disciplines. The following subsections detail how the two teachers' conversations about their teaching practice could be described according to the five dimensions as developed in the Conceptual Framework (Chapter 3).

9.3.1.1 Mathematics

The foundational role that mathematics plays in developing numeracy was evident when Ange described numeracy: "I think numeracy is applying, the application of those mathematics skills into different areas." Her content knowledge was evident in many of the comments she made as she discussed her teaching. Aspects of the content of algebra and number were mentioned in describing the importance of teaching and assessing for numeracy. For example, in discussing how numeracy was developed with her students, Ange spoke about looking for ways to demonstrate to her students the importance of "relationships between variables in algebra" and the relevance this might have outside of pure mathematics.

As the SOSE teacher, Jen mentioned the importance of students having the opportunity to apply their knowledge of concepts related to culture, community, society, and the environment, to build their understanding of important mathematical concepts.

> In SOSE we use numeracy in graphing, reading tables, analysing statistics and things like that. I make sure they can relate it to [life] ... If they have to apply it they can actually grasp the concept. (Jen, 12/4/05)

9.3.1.2 Reasoning

In discussing the role of numeracy in the middle school curriculum, Ange referred to the language of "thinking" as forming an important part of student assessment. Both teachers mentioned the middle school assessment booklet (Tanglefoot, 2005) on numerous occasions and the important role it had in bringing the Essential Learnings into assessment in the middle school and therefore in informing their teaching. In this booklet, strategies such as posing questions, recalling strategies and relationships, conjecturing, justifying, explaining, and drawing conclusions were listed as important elements of working mathematically.

The teachers talked about wanting to see evidence of how the students were thinking and problem solving. Ange highlighted the value of students "showing their working out" as it helped the teachers to "really know how they [the students] are going" as opposed to "working in class out of books." She felt that text books did not provide her with information on how students were thinking when solving problems.

9.3.1.3 Attitude

The importance of a positive disposition toward numeracy in contributing to positive numeracy outcomes, although not explicitly mentioned by either of the two teachers, was implicit in their comments. Ange mentioned her aim to "get the girls interested in maths" through the teaching of numeracy. Jen said she wanted to "make sure the girls can relate to it" and tried to engage the students with tasks that would be of interest to them. For Ange and Jen, the role of numeracy, as mathematics in context, was the key to developing this engagement, "interest," and positive disposition.

9.3.1.4 Context

Both Ange and Jen expressed a belief that numeracy was very much about using mathematics in context. They saw numeracy as "something that is taught in lots of subjects" and involving the "application of mathematical skills into different areas." Jen gave examples from when she spent a short time teaching Grade 7 mathematics, not her usual teaching area, in which she would provide the students with opportunities to "try to apply that knowledge too… to real-life situations."

The contexts valued by the teachers were authentic, real-world contexts, as evidenced in this comment by Ange.

> I think that is the way that Maths will probably be going in the future. It is going to be real-world context and I think that is important. … and I think the Essential Learnings, with Being Numerate as a focus, will sit quite nicely with integrated units of work. Hopefully that will develop over the years. (Ange, 12/4/05)

When talking about their teaching they provided examples of contexts they had used with students. Contexts such as crime, health, design, and decorating were mentioned.

9.3.1.5 Equity

Ange discussed how important it was for mathematics education to cater for "the needs of all students." She described numeracy in its role across the curriculum as being the way "to get the people who struggle." Neither Ange nor Jen expressed in the interview aspects of numeracy teaching that would equip students with the ability to consider information critically, or consider inequities in society. Despite this, the "Live 8" unit of work implemented after the interview provided an example of their underlying beliefs in this area.

9.3.2 Numeracy as enacted in the "Live 8" unit of work

In addition to their usual discipline-based mathematics classes, the Grade 8 students at Tanglefoot School were provided with interdisciplinary learning opportunities to encourage connections among disciplines and the development of generic skills. These units of work also formed part of Tanglefoot's philosophy of encouraging a shift to independent learning for middle school students. In this case study, Ange and Jen set the expectations for learning in the real-world context of poverty and planned for discipline-based, as well as interdisciplinary learning outcomes, specifically, Being information literate, Creating and pursuing goals, and Acting democratically.

In this case, four lessons per week, ranging from 50 to 100 minutes, were allocated for the *Live 8* unit of work. During the fifteen classroom visits undertaken in this case, detailed observation notes were recorded and later analysed, enabling a picture to be created of Ange and Jen and their students' numeracy learning in an interdisciplinary context, according to the five dimensions of numeracy as proposed in this thesis.

9.3.2.1 Mathematics

During the first few lessons Ange and Jen introduced tasks that enabled the students to revisit mathematical knowledge and skills related to data and graphing, as these would be important tools for the students when undertaking their major assignment on poverty. These tasks (Figure 9.1a and Figure 9.2a) involved data collection and recording, completing frequency distribution tables (Figure 9.1b), constructing graphs (Figure 9.2b), and writing interpretive statements about the graph and data presented (Figure 9.2b).

When working through these foundational tasks, Ange and Jen revisited the basic skills and concepts related to collecting and representing data such as the use of a title, key and axes labels, and calculating percents and averages; as well as clarifying with students the meaning of terms such as range, dependent and independent variables, and distribution. Students' statistical and graphing understanding became important in enabling them to understand the context in which the *Live 8* unit of work was situated, considering the concept of poverty as it applied to their particular country of study.

Frequency Distribution Tables

In order to gain a better understanding about poverty and to be able to compare Australia's wealth with third world countries, you are required to gain raw data on the following topics:

1. The number of televisions per household
2. The number of mobile phones per household
3. The number of computers per household
4. The number of cars per household.

Conduct a survey of the class to determine the results.

Once you have the data, you need to arrange it into frequency distribution tables. These tables will allow you to answer the following questions:

1. How many people had three or more televisions?
2. What fraction of responses had 4 or more televisions?
3. What percentage is this?
4. What was the total number of phones in the 34 households?
5. What is the average number of phones per household?
6. What was the highest number of phones in one household?
7. How many people had less than 2 computers?
8. What was the range of scores for computers?

Please draw your tables and answer the questions on A4 loose-leaf paper and hand in either Miss or Miss at the end of the lesson.

Assessment
Criteria 5 - Understanding Chance + Data.

Figure 9.1a. Frequency distribution and survey task.

Television frequeny distribution

Score	Tally	frequency
1	////	3
2	~~||||~~ ~~||||~~ |	11 14
3	~~||||~~ [illegible]	[illegible] 22
4	~~||||~~ [illegible]	[illegible] 26
5	~~||||~~	5 31
6	|	1 32
7	|	1 33
8	|	1 34
	Total	34

Figure 9.1b. Frequency distribution task: Student table for number of televisions per family.

※ OUR WONDERFUL WORLD — COLLIN SALE (1994) p. 341

PATTERNS OF DEVELOPMENT 341

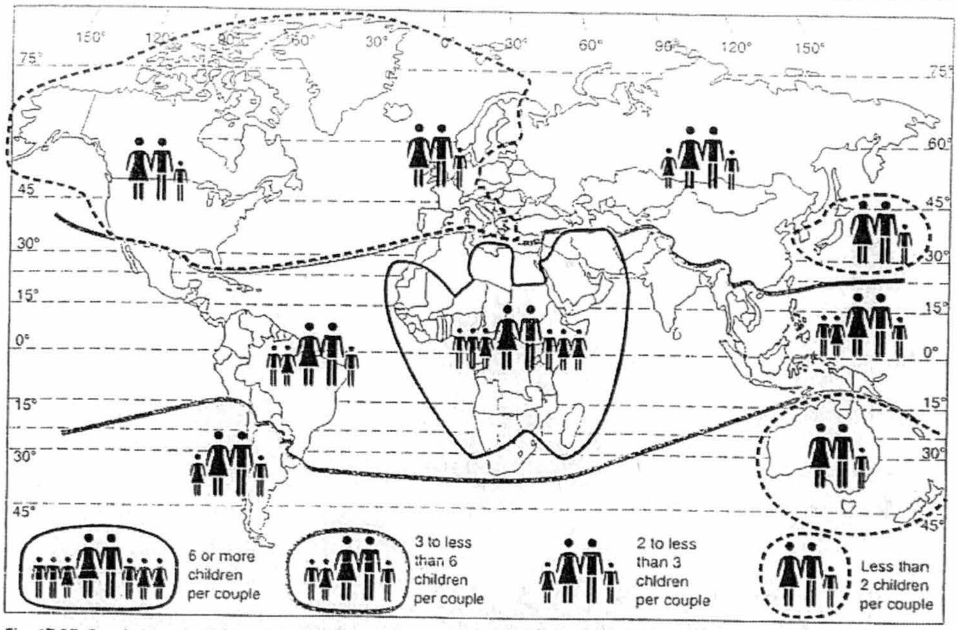


Fig. 17.27 Population growth rates throughout the world as indicated by the average number of children per couple in each country.

Table 17.8 Births per woman. The average number of children every woman bears during her life time as calculated by each country's age-specific fertility rates *

Average number of children per woman	***Countries with the highest birth rates (61 nations already with 760 million people, but more than doubling in less than a generation)***
7 or more	Ethiopia, Malawi, Mali, Niger, Oman, Saudi Arabia, Uganda, Rwanda, Yemen Republic (9 nations with a present population totalling 132 million)
6.5 or more but less than 7	Angola, Burkina Faso, Burundi, Comoros, Congo, Cote d'Ivoire, Djibouti, Gambia, Guinea, Kenya, Laos, Libya, Mauritania, Mozambique, Senegal, Sierra Leone, Somalia, Swaziland, Syria, Tanzania, Togo, Zambia (22 nations with a present population totalling 171 million)
6 or more but less less than 6.5	Benin, Chad, Ghana, Guinea-Biseau, Iran, Iraq, Jordan, Liberia, Madagascar, Maldives, Sudan, Zaire (12 nations with a present population totalling 157 million)
5.5 or more, but less than 6	Bhutan, Cameroon, Central African Republic, Equatorial Guinea, Gabon, Lesotho, Namibia, Nepal, Nigeria, Pakistan, Qatar, Solomon Islands (12 nations with a present population totalling 277 million)
5 or more but less than 5.5	Cape Verde, Guatemala, Honduras, Nicaragua, Papua New Guinea, Sao Tome & Principe (6 nations with a present population totalling 23 million)
	Countries with the lowest birth rates (30 nations with 882 million people, and populations remaining static)
Less than 2	Antigua & Barbuda, Australia, Austria, Barbados, Belgium, Bulgaria, Canada, Cuba, Denmark, Finland, France, Germany, Greece, Hong Kong, Hungary, Italy, Japan, Korea Republic (South Korea), Luxembourg, Mauritius, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, United States of America.

* Based on statistics in the *International Yearbook, 1993–94*, Reed Information Services, UK, 1993

Figure 9.2a. Data presented about population density and average number of births.

In considering how to reflect accurately the data on the average number of children born to women across the world, the students discussed the best way to represent the data when a minimum average and a maximum average were provided for most countries (Classroom observation, 24/6/05). Figure 9.2b provides an example of how most students chose to reflect this using a different colour in their bar graphs.

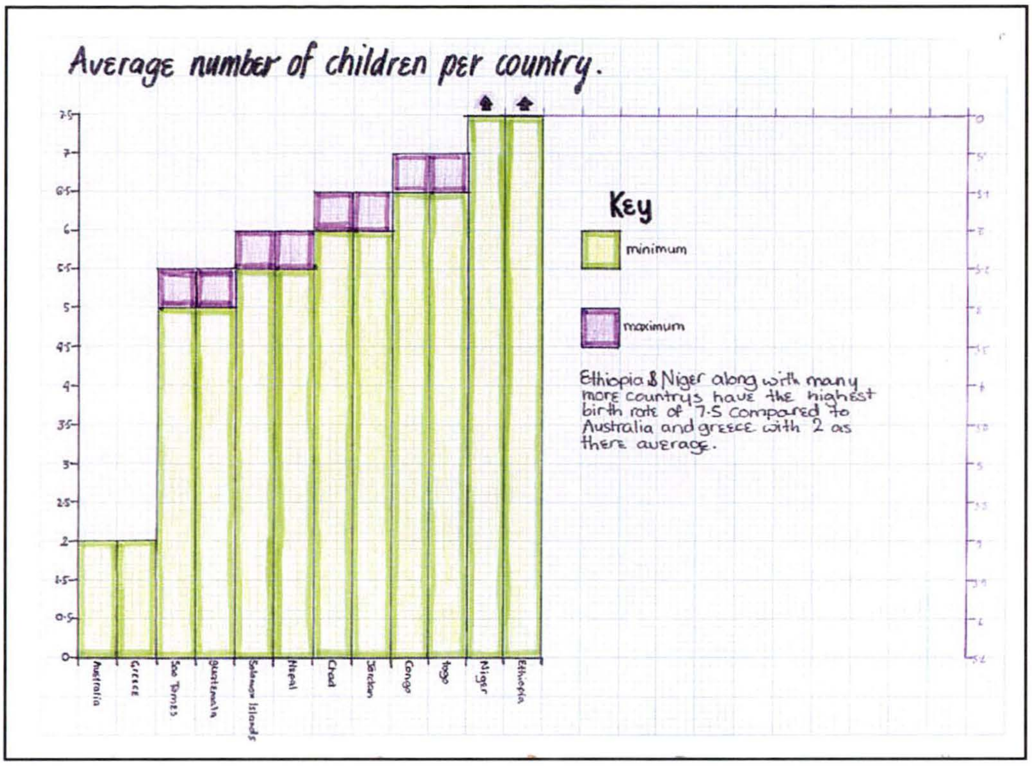


Figure 9.2b. Example of student graph and interpretive statement on average birth rates.

As the students moved to working more independently, Ange and Jen provided them with guidelines about the specific mathematical areas that needed to be included in their poverty assignment when they addressed the overall impact of poverty on their countries of study. In researching their countries, students worked in pairs and made and recorded decisions regarding the selection and representation of many statistics, including population data, literacy rates, life expectancy rates, birth rates, and incidences of poverty. In all cases the students were required to compare their country with Australia, to enable them to make meaningful comparisons and, therefore, interpretations of the data. The students constructed tables and graphs and these mathematical elements of the poverty assignment were assessed as a separate criterion in the students' final submitted assignment.

Students applied their knowledge of graphing in many different ways and with the large amounts of information they accessed. Ange and Jen encouraged the students to consider what data to use and how to represent data to best provide a meaningful and accurate story. These decisions included which type of graph to use, what each axis would represent, and choice of scale. The students would discuss these decisions with each other and at times with the teachers, particularly if they wanted to clarify or if they were unsure. Figure 9.3 shows one student working on her choice of scale which she had discussed with Ange because she was "not sure what scale to use because Nepal is 21 and Australia is 800," referring to average income per week in these countries (Classroom observation, 14/7/05).

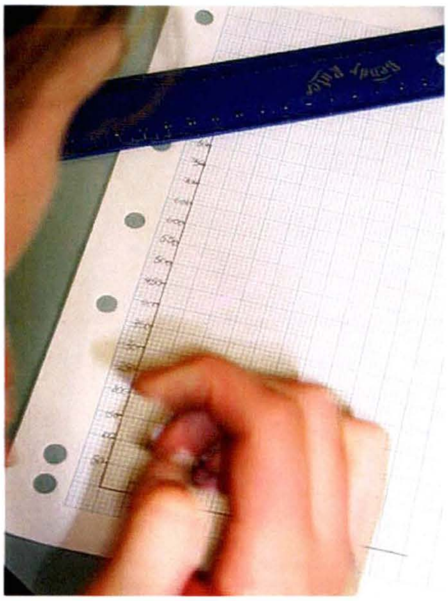


Figure 9.3. Working on selection of scale when constructing a graph.

Bar graphs were the most common type of graph used by students to represent their findings. This choice of graph was not unexpected as in all cases the students had to compare their country of study with Australia in order to consider the implications for issues of poverty and, therefore, comparative data were easily depicted and interpreted using bar graphs. Although Ange and Jen did not specifically discuss the different types of graphs and which ones might be best suited to representing different types of statistical information, they did encourage the students to think about their choice of graph and to represent their work in different forms of graphs (Classroom observation, 12/7/05).

Figures 9.4a and 9.4b demonstrate the variety of presentation choices that students made when preparing and presenting bar graphs to represent information. Although most students drew their own graphs, a few chose to use technological tools such as Microsoft Excel and Power Point to support and present their work.

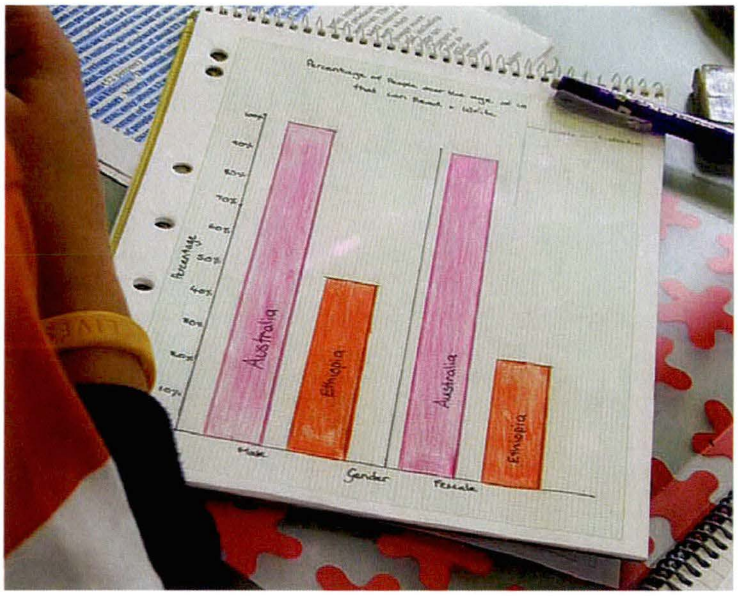


Figure 9.4a. Literacy rates for males and females in Australia and Ethiopia.

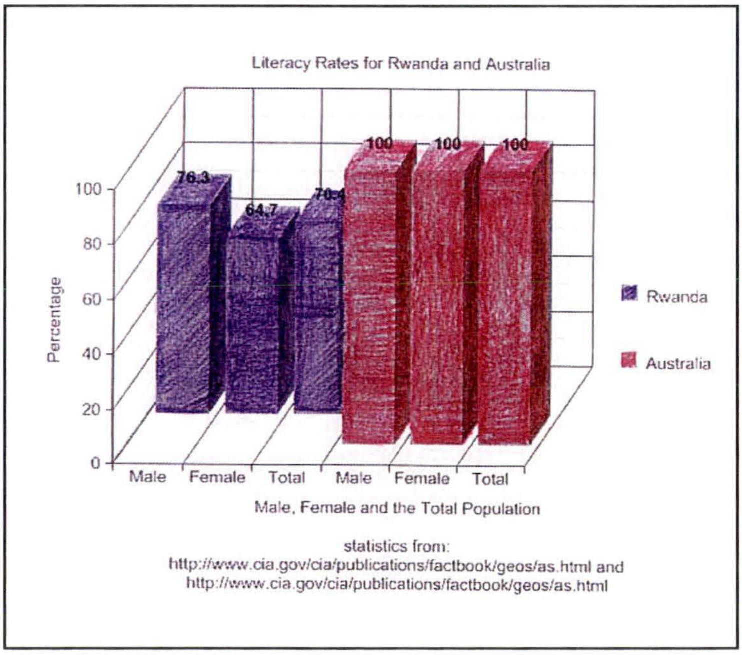


Figure 9.4b. Graph of literacy rates for Rwanda and Australia.

The students also used line graphs when representing data that involved something changing over time, as exemplified in Figure 9.5a and 9.5b, both graphs reporting on an aspect such as population growth and life expectancy and how they have changed over time. Figures 9.5c and 9.5d also show the use of line graphs when quite complex data were being presented. In these two examples are shown mortality rates in Italy for different age groups and over different years, and the incidence of poverty in rural and urban areas of Sudan, based upon household size.

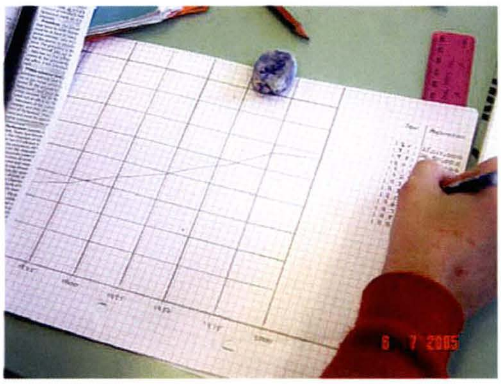
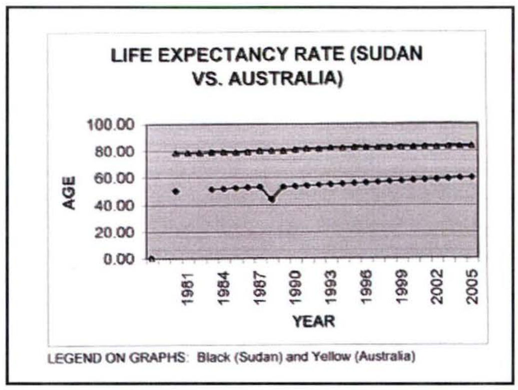


Figure 9.5a. Population growth in Italy from 1850 to 2000.



. *Figure 9.5b.* Life expectancy of people in Sudan and Australia from 1980 to 2005.

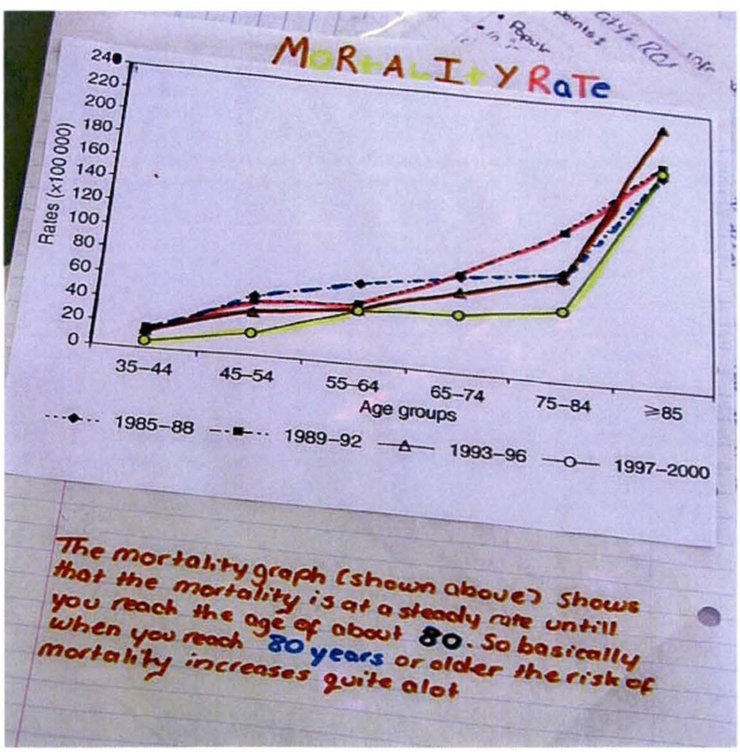
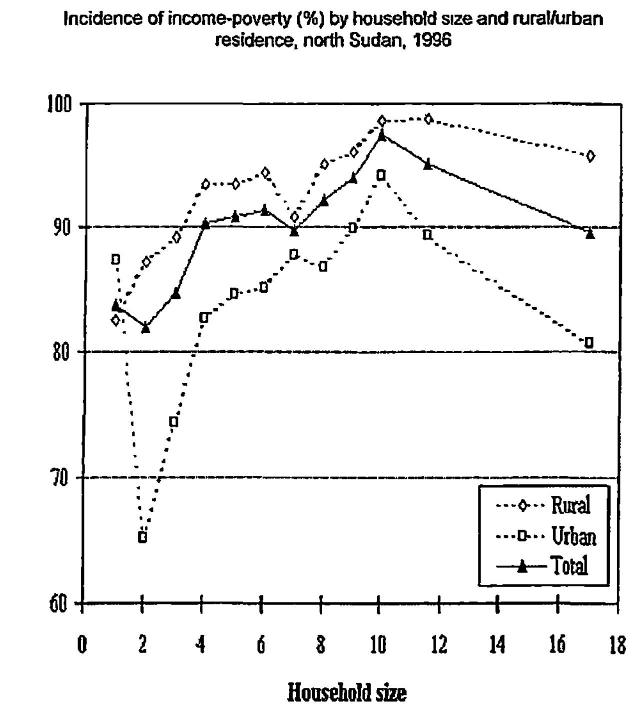


Figure 9.5c. Mortality rates in Italy.



The above graph shows, the regional disintegration of the income-poor from 1996 in urban areas as a whole, or rural areas, it is estimated that the largest proportion of income-poor is close to 29% and other region has the lowest less than 3%. Around 16-17% in other region such as the Eastern, Middle and Kordofan have roughly the same share of the income-poor in rural north Sudan About 12% of the rural income-poor reside in the Gezira region while the Northern region's share is about 7%.

Figure 9.5d. Incidence of poverty in Sudan.

At times students were innovative in the way they chose to represent data. In Figure 9.6 the student used a number line to show, simply but clearly, the Gini index of both Australia and Somalia and then discuss the distribution of income in a country. In Figure 9.7 the student had found information on the percentages of people living in poverty across Rwanda, and chose to show this on a regional map of the country.

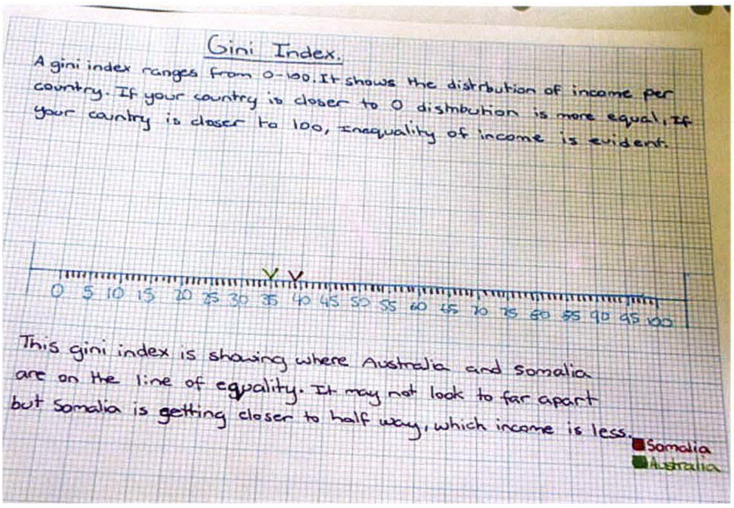


Figure 9.6. Gini index for Somalia and Australia.

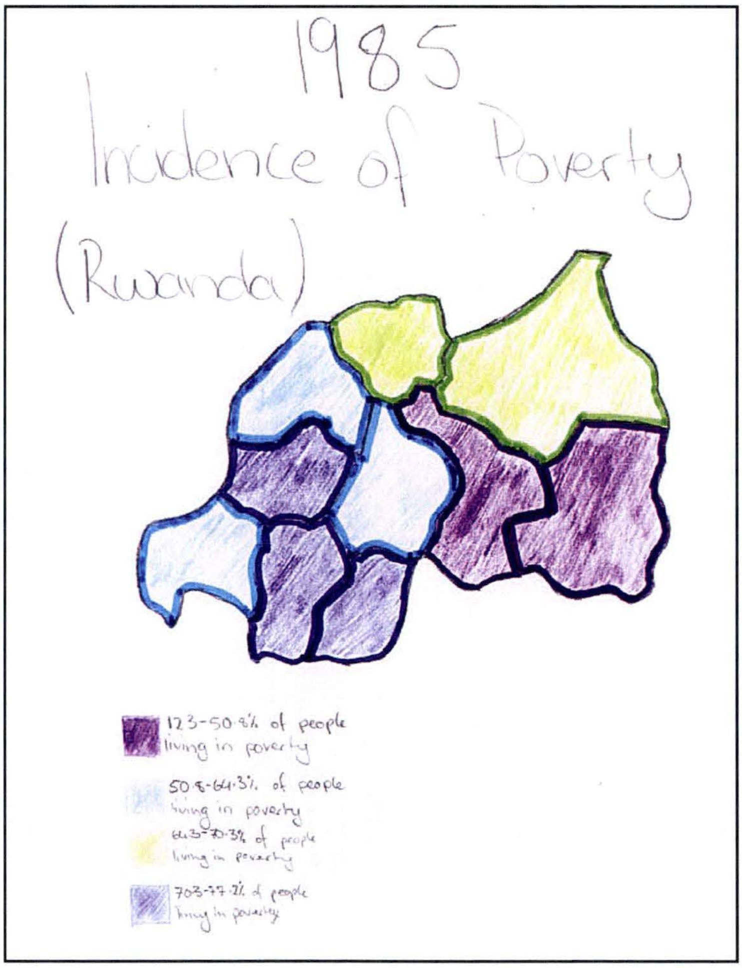


Figure 9.7. Incidence of poverty in Rwanda by region.

As the students worked to research, summarise, synthesise, and present their information, they were generally able to describe their graphs and make meaning from them in relation to their developing understanding of the historical and current situation of their country of study. This was evidenced by the discussions they had with each other, with their teachers and, at times, with the researcher, throughout the unit of work.

> S: It [graph on mortality] is comparing Canada to [sic] Australia, so the same age for males and one year different for females.
> Int: Do you think it is significant?
> S: I am not sure.
> Int: If you were comparing it with one of the poorer nations what do you think it might look like?
> S: It would be way down here [pointing to a lower age on the graph]
> Int: So Canada and Australia seem quite similar?
> S: Yes, they are very close
> (Classroom observation, 5/7/05)
>
> S: My graph shows the population change in Italy from 1850 and it was about 20 million then and in 2000 more than 60 million.
> Int: So it has more than doubled?
> S: Yes and it looks like now it is starting to plateau.
> Classroom observation, 6/7/05)

Toward the end of the case study, students were more clearly relating the statistical data and graphs they had created to the contextual information they had also researched on their country of study.

> S: The Gini index [for Ethiopia] is not actually that different to [sic] Australia, so even though it is an undeveloped country its wealth is not distributed evenly. Perhaps there are a lot more poor people in Ethiopia but there must be some with more income. (Classroom observation, 14/7/05)

The students' understanding of the work they had researched and put together over the weeks was ultimately evidenced in their final assignments, which were both handed in for assessment and shared with the whole class as presentations at the end of the unit of work. Students demonstrated their abilities to synthesise the mathematical data they had collected, represented, and interpreted, with the contextual information for their country of study. Each pair of students presented for approximately five minutes and incorporated into their presentations clear explanations of their graphs, using the statistical information to make the data meaningful and link the mathematics to the standard of living and issues of poverty in the respective countries (Classroom observation, 28/7/05 and 29/7/05). An example is provided.

> ... The average mortality rate is 8.97 deaths per thousand people and the average birth rate is 27.08 per thousand people. Out of the total population, 69.4% are over 15 years of age and can read and write. ... Of the 69.4%, 80.8% of the males

can read and write and 53.3 % of the females can read and write. The lowest 10% of the population struggle on only 2.9% of household income and the top 10% of the population have over one third of the household income, being 33.8% …
(Classroom observation, 28/7/05)

The completed assignments that students handed in to Ange and Jen for assessment formed the basis of the student interviews, the results of which are reported in Section 9.3.3. Although a variety of areas of mathematics formed part of the students' discussions, it is the chance and data outcomes that were the focus of analysis of the students' learning.

9.3.2.2 Reasoning

Although the Grade 8 students at Tanglefoot would undertake many independent individual tasks they were also encouraged to work together and share ideas as part of their learning. This was particularly the case during interdisciplinary units of work in which a shift toward more independent learning was valued, but within the context of students still working in groups and being encouraged to share, plan, and present ideas.

In this case study, students' levels of reasoning deepened as the unit of work proceeded. During the early stages of the unit of work, students were revising mathematical knowledge and skills, and their conversations were very much reiterating their knowledge or asking questions to remember or understand an aspect of their previous mathematics learning that would help them complete the task.

T: How do you work out the score?
S: Write down the numbers from 0 to the highest, so 8.
T: What do we do with the tally?
S: Count how many ones first.
T: What next?
S: Frequency.
T: How do we work out the frequency?
S: Get the total of number with one television, and so on.
(Classroom observation, 23/6/05)

S: I am not sure what to call it? [referring to title for a graph]
T: What are you showing?
S: Average number of children around the world.
T: Is it around the world? Look at the sheet and be precise in what you are showing, what about birth rate?

S: Average birth rate of children per country.
(Classroom observation, 24/6/05)

Students were also brainstorming prior knowledge about the concepts of "developed" and "underdeveloped" countries and considering the implications when watching the SBS video "World's Apart" that followed the story of a middle class American family living for nine days in a third world village in Kenya. The following are a variety of individual student responses.

I heard Africa owes Britain thirty-two million dollars.
What did the money do, that they borrowed?
There's not enough work for everyone.
Children work to survive.
In China they are only allowed to have two children.
(Classroom observation, 23/6/05)

As the unit of work moved to an interdisciplinary assignment on poverty, students were collecting information, in relation to both the mathematics skills of data and graphing and also the contextual knowledge they were researching about their country of study. The student discussions were centred on what they found out as they used books and the internet to search for information relevant to the questions they wanted to answer. Ange and Jen remained available to students to answer questions and clarify ideas.

The students were expected to make their own decisions about what information was relevant to their inquiry and how they would represent it. The students, who had been allocated partners with whom to work, planned out their assignments and divided up tasks amongst themselves. At times Ange and Jen questioned the students about their work and their decisions. With the students often working in the library throughout the unit of work, much of their reasoning was internalised and note-taking was an important aspect of recording information and ideas during this stage of the unit of work. Examples of the students' thinking were gleaned from their discussions with each other and the questions they asked their teachers. These discussions and questions were usually based on the clarification of ideas about the data collected and about their interpretations of the data. The following students' questions are examples.

What is another word for temperate? (1/7/05)
What is the name of that organisation for aid with the thorns? ... I am trying to find out about aid programs in Russia. (1/7/05)
What does government system mean? (5/7/05)

> I am trying to work out what the Djourab depression is. It is Chad's lowest point and the internet describes it as a basin at the northern end of the lake. (12/7/05)
> What scale should I use? (14/7/05)

Students applied their knowledge and skills about graphing when representing the statistical data they had found in different forms of graphs. During this stage of the unit of work, many students shared their ideas about how they might present the statistical information and at times sought clarification from their teachers about particular aspects so as to make sense of the data, and understand how ideas related to each other. It was at this time that students' reasoning moved to a more analytical level as they organised the data and considered the relationships among variables.

> I am not sure what scale to use because Nepal is 21 and Australia is 800. [referring to average income per week in these countries]
> (Classroom observation, 14/7/05).

In Figure 9.8. the same student interpreted the distribution of income data for Nepal and Australia but did not relate these data to the very large differences between average income in the two countries.

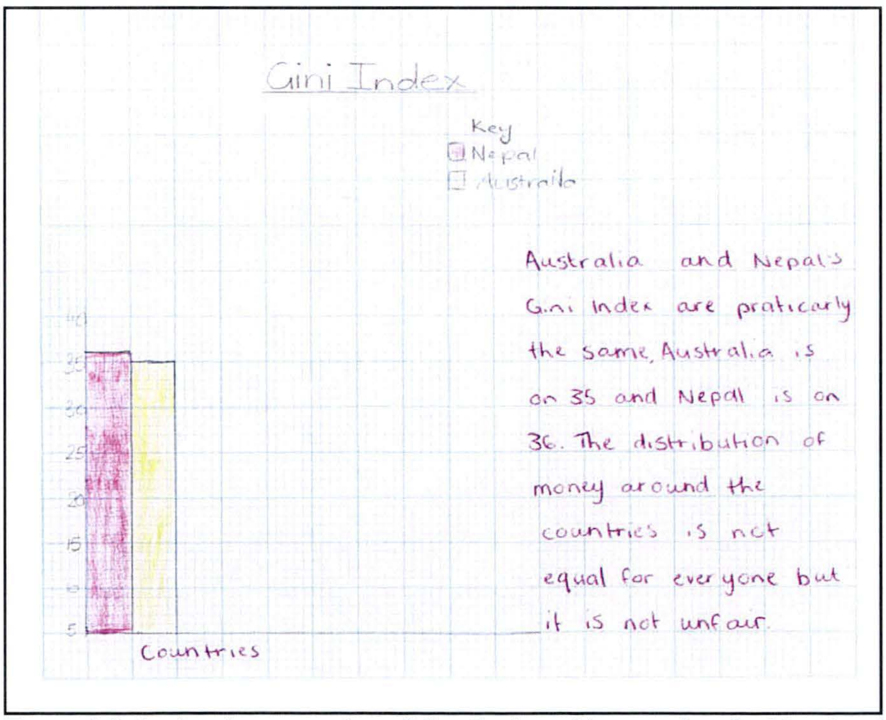


Figure 9.8. Student interpretation of distribution of income data for Nepal and Australia.

As students began to synthesise their mathematical learning with their contextual understandings of their country of study, there was some evidence of students evaluating and making judgements about the deeper issues of poverty that impacted upon the country. This higher level of reasoning was particularly evident when the students presented their work to each other during the last few lessons of the unit of work. Their explanations and understandings of the complex issues impacting upon poverty and life in the countries they were studying were critical and supported by the mathematics and graphical data they presented. An excerpt from one of the student presentations on their country of study, Afghanistan, is included to demonstrate the students' ability to bring together their analysis of their learning and provide clear information based upon statistical data and incorporate their own judgments related to the interpretation of these data based upon the context.

> Afghanistan is in southern Asia, east of Iran and northwest of Pakistan. Afghanistan has an area of 647,000 square kilometres which is a bit more than Texas in America … The population of Afghanistan is a massive 29, 928, 978 with the capital being Kabul and they have a population of one million. 44.7% are aged between 0 and 14, 53.9% are aged between 15 and 64 and 2.4% are over 64, so there are not very many people who reach over 64. The birth rate is 47 children per 1000 births. The infant mortality rate is 163.07 per 1000 live births with a life expectancy of 42.9 years so they don't live for very long and there is a fertility rate of 6.75 children per woman.
>
> The literacy rate in Afghanistan is around 36% out of the whole population, 51% being male and 21% being female. This would be because in Afghanistan it is more important for the male to get the education and the majority of women stay at home and look after the children and do the household chores so it is not all that important for the females to get the education. Like Australia you are not an adult until you are 18. Although the law says you must go to school between the ages of 7 and 14, most Afghanistan people cannot afford to send their children to school.
>
> Australia plays a major role in helping Afghanistan with their aid programs, and since September last year raised a massive 110 million for their aid programs and another 18 million was claimed to be given between last year and this year. The government is Islamic republic and they gained their independence in 1919 …
> Water is a major problem in rural and urban areas. Water is precious to the people and there is not much around and it is poor quality. … Some people have to walk kilometres to get water from a well but even then the water isn't that clean. Water in Afghanistan is scarce and so they use it very carefully.
>
> Up to 8 million people face severe food shortages or starvation. The reality is that the end to this nightmare is nowhere in sight. The US estimate about 500 000 disabled orphans in Afghanistan. 53% of Afghanistan's people are below the poverty line. Below the poverty line means they are worse off, so they do not have enough money to cope each day …

9.3.2.3 Attitude

Ange and Jen had established a classroom environment in which students worked both individually (Figure 9.9a) and in groups (Figure 9.9b) during this case. As the unit progressed, Ange and Jen removed themselves from a central role in guiding the learning, to one in which the students were working independently with Ange and Jen available if needed.

Figure 9.9a. Students working autonomously on tasks.

Figure 9.9b. Students working in pairs on major assignment.

As the majority of the unit of work involved the students researching and putting together their ideas, and ultimately their final poverty assignment, they largely worked autonomously with their partners, and had a significant degree of flexibility in how they structured and went about their work. The students were very comfortable asking questions of each other and of the teachers to clarify their ideas or to seek feedback on their work and make improvements (Figure 9.10).

Figure 9.10. A student discussing her work with Ange.

As the unit proceeded, students became more engaged in the tasks. This occurred as they developed particular interests in their country of study and life in that country, and this was evidenced by their discussions and the work they were producing. There were, however, times when the students were off-task, with their conversations not involving their school work, although these were not predominant. As students were working in pairs they were able to negotiate the allocation of tasks required to complete the assignment. Most students chose to share both the mathematical and contextual requirements of the assignment.

The students at Tanglefoot School were also particularly fortunate to have access to a wide range of resources to support their learning, including a well-resourced library, internet access, and the use of computers. The students enjoyed working together to decide how they would present their final assignment with most students completing posters (Figure 9.11). Other students presented their work in A4 display folders and one pair provided a Power Point presentation.

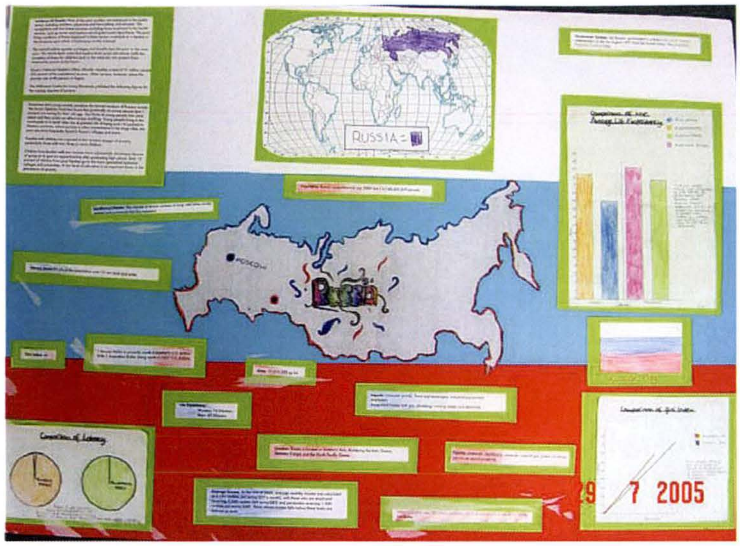


Figure 9.11. Student poster presenting a final assignment on Russia.

Developing students' capacities to communicate their learning was valued at Tanglefoot School. Ange and Jen provided students with opportunities to do this not only with their written work but also by giving students opportunities to present their work orally (Figure 9.12). The students particularly enjoyed the final stage of the unit of work when they presented their assignments to each other, including their learning about their particular country and the impact of poverty on those countries. Students spoke confidently and were interested in each other's learning. They were also involved in peer assessment of this culminating performance.

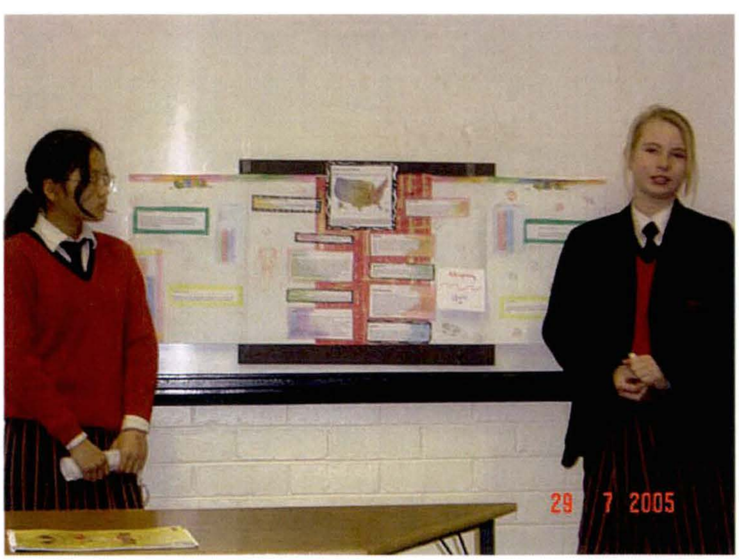
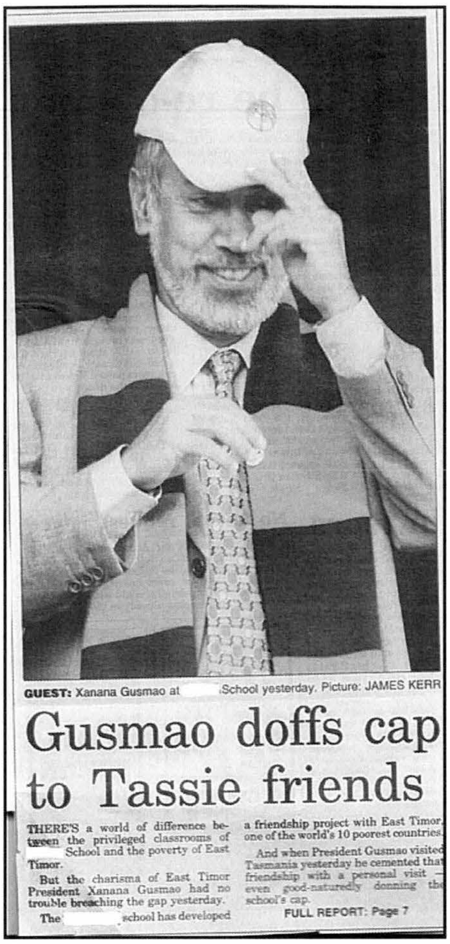


Figure 9.12. Students presenting their work to their class.

9.3.2.4 Context

As an integrated unit of work, in which the disciplines of mathematics and SOSE were brought together to develop students' understandings of the concept of poverty, context was a major element and the key motivation for the unit on which this case study was based. Ange and Jen selected a context that was current, that they thought would engage the students, and ultimately that enabled them to develop targeted learning outcomes in both mathematics and SOSE. The context was introduced at the beginning of the unit of work, with the G8 summit being held and the *Live 8* concerts providing the initial motivation for the unit of work. Ange and Jen presented the students with some broad statistics regarding poverty around the world in order for them to begin to consider their own ideas about developed and underdeveloped countries. Just as Ange and Jen had implemented "tuning in" activities related to data and graphing prior to the major poverty assignments, they also implemented "tuning in" tasks concerning the context of poverty, by having the students consider their own ideas and their own lifestyles, and by watching a video of life in a Kenyan village.

The students were encouraged to read newspapers and other media outside the classroom, during the period of the G8 summit. In addition, a visit to Tanglefoot School by the President of East Timor, Xanana Gusmao, and his family, provided very real and confronting contextual information relevant to their unit of work (Figure 9.13).



GUEST: Xanana Gusmao at School yesterday. Picture: JAMES KERR

Gusmao doffs cap to Tassie friends

THERE'S a world of difference between the privileged classrooms of School and the poverty of East Timor.

But the charisma of East Timor President Xanana Gusmao had no trouble breaching the gap yesterday.

The school has developed a friendship project with East Timor, one of the world's 10 poorest countries.

And when President Gusmao visited Tasmania yesterday he cemented that friendship with a personal visit — even good-naturedly donning the school's cap.

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Figure 9.13. East Timor President, Xanana Gusmao visits Tanglefoot School.

The major assignment required students to investigate government systems, landforms, exports, the economy, the water supply, and other areas that impacted upon living conditions in each country. As the students began to construct their graphs, they made links with their contextual learning, based upon the research they had undertaken on the history, culture, government, and economy of their country of study. It was at this stage that they began to construct meaning around the data they had collected as they considered the reasons for the statistics they were reporting, such as life expectancy, mortality rates, and literacy rates.

> Nepal is a lot less because they are malnourished. [when discussing life expectancy results for Nepal and Australia] (Classroom observation, 14/7/05)

> The Gini index [for Ethiopia] is not actually that different to [sic] Australia, so even though it is an undeveloped country its wealth is not distributed evenly. Perhaps there are a lot more poor people in Ethiopia but there must be some with more income. (Classroom observation, 14/7/05)

During the final presentations the students shared their learning about their country. They did this by presenting some purely mathematical information and some purely contextual information, but mostly they integrated their understanding of context with the mathematics embedded in their work. It was during these presentations that the synthesis of mathematical and contextual knowledge enabled students to think critically and make informed evaluation of the impact of poverty in different countries. Their contextual knowledge broadened even further as they listened to each other's learning and the evidence presented in graphical form. The first two quotes exemplify student learning that focused mostly on context, whilst the last quote exemplifies the way students synthesised their understanding of the context and the mathematics to explain the data they were presenting (Classroom observations, 28/7/05).

> In the 1970's farm production was high enough to feed the country and be exported to other countries. The main crops were rice and corn. Large amounts of rubber were also produced. During the Vietnam War many farms and other plantations were destroyed and food production decreased dramatically.

> The population is over eight million ... Somalia has a dark history and has been besieged by a lot of countries. The Arabs took over and then Italy did and a lot of countries. There was civil war and that was pretty horrible. It's history has been violent and there was a militia who were violent towards the civilians and thousands and thousands of lives lost. In 1990 the aftermath of the civil war was starvation and drought. Even more people died from there being not enough food and not clean water and today they are still struggling for independence today.

> This graph is the percentage of people over the age of 15 that [sic] can read and write. There are more people in Australia that can read and write than in Nepal. This would be because Nepal does not have enough money to support schools and education. ... The life expectancy for males and females is a lot higher in Australia than in Nepal. This would be because the Nepalese are malnourished and are one of the poorest countries in the world. Males live up to the age of 60 and females to 69.

9.3.2.5 Equity

This dimension of numeracy acknowledges that mathematics can be morally, socially, and politically loaded. The very nature of this unit of work, being centred on the issue of poverty, had a social, moral, and political context. The use of mathematics to present data about an under-developed country had the power to make clear the impact that poverty had on the lives of people, particularly as Ange and Jen had asked their students to compare their country of study with Australia, whose lifestyle the students themselves experienced.

In Ange's and Jen's classrooms, students were provided with many opportunities to present and share their own ideas. They were also encouraged to put forward these ideas for critiquing by both their peers and their teachers. These capabilities enabled students to develop important dispositions that could then be used to question inequities in society and consider possible alternatives.

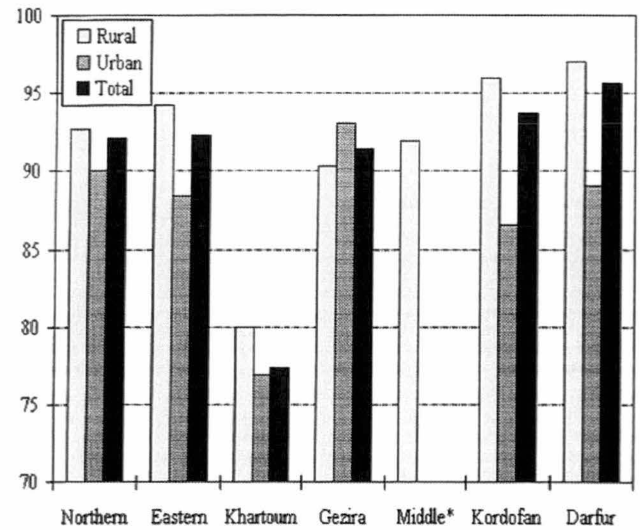
This dimension of student learning became evident during the students' presentations. As they presented their own work and engaged in the learning of their peers, the students demonstrated their awareness of issues involved in the complex nature of poverty. They used the mathematics to support their viewpoints and also asked questions of each other to understand better the issues impacting upon poverty in different countries. The students also gave reasons, both social and political, for the many circumstances that were influenced by poverty in their country of study and some examples have been presented previously (Context dimension – Section 9.3.2.4).

Some students incorporated possible solutions to these circumstances or action that could be taken that might alleviate poverty. There were times when students were emotive about the implications of the data and related it to how it might impact on them, if they were living in such circumstances, as for example in this excerpt from the student presentation on Somalia.

> [This graph shows] the average life expectancy and in both countries females are expected to live for longer but as you can see there is a huge difference between them. In Australia females are expected to live for about 84 years and males are expected to live to about 76 and then in Somalia females are only expected to live to about 50 and my Mum would be dead by now. Males are expected to live to about 46, so it is really difficult for children because like us some of us have older parents and so you would be living on your own from about 15. …
> (Classroom observations, 28/7/05)

Figures 9.14a and 9.14b present a section from an assignment presented by two students who had studied Sudan. These students used both mathematical and contextual data to discuss the incidence of poverty in Sudan and relate this to the levels of education of its people. The students concluded that although poverty does reduce with higher levels of education, poverty overall is still high and that "eradicate[ing] poverty in a country such as Sudan is not only about reducing inequality but equal distribution, justice, growth and development are needed to raise the overall standard of living" (Figure 9.14b). These students brought together many aspects of their learning about Sudan in considering what action might need to be taken to have any possible hope of changing the circumstances for Sudan's people.

Incidence of income-poverty (%) by regions and rural/urban residence, north Sudan, 1996

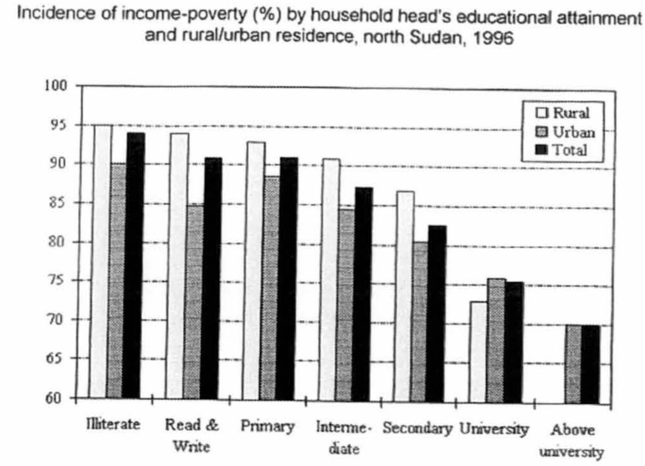


[Note: No estimates for the incidence of poverty in the urban segment of the Middle region.]* –

Incidence of income-poverty according to the poverty lines shows regional disparity essentially between Khartoum on one side and all other regions. With the exception of the Gezira region, the incidence of income-poverty is estimated to be higher in the rural segments of the regions.

The incidence of income-poverty is estimated to decline steadily with increasing educational attainment of the household head, at a much faster pace past completion of primary education. However, the incidence of poverty is still considerable in households headed by university graduates in both rural and urban areas see figures below.

Figure 9.14a. Student graph of incidence of poverty in Sudan across rural and urban areas.



Fascinatingly, the impact of education on reducing income-poverty is estimated to be much more consistent in rural areas. In urban areas, for example, households of heads with limited educational attainment (primary and intermediate) are not better off than those whose head can only read and write.

Larger households are estimated to suffer a higher incidence of income-poverty up to ten members per household. Beyond that size a reduction in income-poverty ensues, to about average levels, especially in urban areas. The lowest incidence of income-poverty is estimated for urban households with two or three members only. These are bound to be recently formed households of young couples *[see graph on household size]*.

For north Sudan as a whole, a little more than half the income-poor household are estimated to be headed by illiterates. Almost three quarters of the heads of income-poor households have not completed primary education. About 3% of the income-poor households are headed by individuals with university education or higher. The share of successively higher level of educational attainment of the household head of income-poor households declines steadily except for a slight increase in the share of secondary education over intermediate. It's important to start with to eradicate poverty in a country such as Sudan is not only about reducing inequality, equal distribution, justice, growth and development are needed to raise the overall standard of living.

Figure 9.14b. Student graph of incidence of poverty in Sudan related to educational attainment.

9.3.3 Student learning

The six students interviewed in this phase of the study were asked to describe and discuss specific graphs that they had completed during the unit of work. In particular, the students' major assignments on poverty formed the focus for the interviews. The students were happy to participate in the interview and were forthcoming in telling the stories of their graphs. The conversations started with specific mathematical content displayed in the graphs but, as the interview progressed, the comments encompassed broader issues about how the graphs helped them understand poverty. The interviews were analysed with respect to the five dimensions of numeracy. Chapter 3 detailed how each dimension was categorised and described. The following subsections detail how the students' learning was evidenced across the five dimensions of numeracy. The reporting of the results is intended to provide representative evidence of the range of responses for each category within the dimension. In doing this at least one comment from each student is included for each of the five dimensions.

9.3.3.1 Mathematics

Table 9.1 summarises the observed categories of Mathematics for each student, as evidenced by the student interview.

Table 9.1
Student learning: Mathematics dimension (Tanglefoot School)

Student / Category	S19	S20	S21	S22	S23	S24
Reading and describing graphs	✓	✓	✓	✓	✓	✓
Making meaning from graphs	✓	✓	✓	✓	✓	✓
Analysing and interpreting graphs	✓	✓	✓	✓		✓
Evaluating and making inferences		✓				

All the students demonstrated specific mathematics understandings in discussing their graphs and used the mathematics to help them when comparing their country of investigation with Australia. They also used the language of mathematics, specifically as it related to chance and data, and number. Some students also mentioned how their previous mathematics learning had helped during this interdisciplinary unit of work, as exemplified by Student 20's comment about frequency tables.

> We have done frequency tables before in maths and it helped here because I understood. With graphs I always forget which way the x and y axes go. It helped me to remember and how to set it out and what they're for. [S20]

The students were all able to *Read and describe graphs*, with typical responses in this category involving the students focusing on specific numbers or points on their graphs related to data that they had been asked to represent in their major assignments, such as average income, literacy rates, and life expectancy.

> literacy [rate] we're 100 and they're 70. [S20]

> It's 77 [life expectancy in years for males] and then 84 for the Canadian female and then 82 for Australia. [S23]

The students were also able to *Make meaning from graphs* by making specific comparisons among the data in their graphs. They were able to explain what the data was representing, as exemplified by the comment from Student 23.

> It tells us that females are expected to live longer and even longer when they are in Canada. The average life expectancy of both countries is high. [S23]

Two students, S21 and S24, connected the comparisons they were making with the type of graph they had chosen to represent the data.

> I thought it would be easier to show them because you can see more clearly when they are in bar graphs and easy to understand for other people… you see the big difference there is between Australia and Nepal. [S21]

Five of the six students were able to *Analyse and interpret graphs.* Student 19 in considering an average income of $280 for families in Afghanistan, commented on the effect of hearing about the average income for other countries studied by her peers, and realised further the wide gap in income between developed and under-developed countries.

> With ours it was just comparing Afghanistan to [sic] Australia, but hearing others …we could compare Afghanistan to [sic] other countries to see whether they were better off than Afghanistan. Like someone did France or Italy and obviously they are not third world countries. Comparing Italy to [sic] Australia and then to [sic] Afghanistan it was still quite a big difference. And third world countries, some of them the average income was less than $280 a month. [S19]

The following examples also demonstrate those students that moved from the information in the graph itself, to interpreting this data based upon the broader learning from their research project.

> The weekly average income ...when I looked at this it really shocked me a bit because you don't really realise how much money goes in and out of your house and for Australia 800 dollars a week is really a lot of money and when I saw Somalia which is one dollar it was really … amazing, the graph, when you look at it you can see how much difference between the two countries. [S22]

> The literacy rate … it shows that the females, as in probably most countries in that region or area are less educated than males probably because of priorities in the system and religious beliefs and the literacy in Australia is obviously amazingly higher than Sudan but in Sudan they have a program … they give free education and I think it is for the first six years and the government is focusing on eliminating illiteracy in the country. [S24]

Although most of the students interviewed were able to compare and critique some of their ideas, only one of the students was able to *Evaluate and make inferences* from their graphs. Student 20 not only interpreted her graphs clearly using the mathematical data she had organised and represented in her graphs, but she also made judgments about the implications of what this data represented and used mathematical data to inform these judgments. The following is an example.

> The Gini index, I thought that was quite interesting because they didn't have a much lower, they had a lot lower income, because they are poorer than us, but [the money] was fairly distributed. ... Australia is really rich but it doesn't mean that their organisation is that good. ...The Gini index for Australia is 35.2 and then Rwanda 28.9 ... and [I thought] it would probably [be] higher considering all the violence and stuff... basically zero is all the money in the country is completely fairly distributed and one hundred means completely unevenly distributed. [S20]

9.3.3.2 Reasoning

Students evidenced their learning in this dimension both in discussing the mathematics of their graphs and also as they became more engaged in sharing the reasons for the results and considering other information they had researched about their country of study, such as the people and their culture, the geography, the water supply, and the government systems. Table 9.2 summarises the observed categories of Reasoning for each student, as evidenced by the student interview.

Table 9.2
Student learning: Reasoning dimension (Tanglefoot School)

Student / Category	S19	S20	S21	S22	S23	S24
Remember	✓	✓	✓	✓	✓	✓
Comprehend	✓	✓	✓	✓	✓	✓
Analyse	✓	✓	✓	✓		✓
Evaluate	✓	✓	✓	✓	✓	
Create						

All of the students demonstrated their capacity to *Remember* and recall information. The students did this in relation to specific statistics, such as is exemplified by Student 19, who identifies the average life expectancy for males in Australia and in Afghanistan.

> The Australian male is 76 and the Afghanistan [male] is the same as the female, so about 41 to 42. [S19]

They also remembered other basic knowledge, such as specific characteristics of their country of study, or knowledge related to the over-riding concept of poverty.

> Australia is a developed country and it doesn't have as much poverty in it. [S21]

> Canada is the second biggest country in the world. [S23]

All six students showed their ability to *Comprehend* from not only the data and graphical displays they had presented in their work, but also from the written information and the oral student presentations that they had shared upon completion of their major assignments. Three examples are presented.

> If we hadn't done this we wouldn't have known. It has made us more aware of the way we live to the way for example that Afghanistan lives. [S19]

> I knew that there was poverty in the world, but I didn't realise it was this extreme. [S22]

> Well it actually gives you an exact amount [when graphing] as opposed to saying, you could just say that Australia is much more advanced, a better country than Sudan because of its population, birth rates, literacy rates, but when you actually see it you realise how big the problem is. [S24]

Five of the six students evidenced their capacity to *Analyse* information and understand how components relate to one another. In this case, analysis was relevant for the mathematics within the unit of work and, at times, the broader context being considered. Student 20 when discussing the life expectancy of people living in Rwanda, attributed such a low life expectancy to living conditions, war, and diseases. She also distinguished the low life expectancy in Rwanda from that in Australia where living conditions are so distinctively different.

> They only live to their forties ... It would have been two hundred years ago for us when we lived that long, but now in the modern world it's different. ... Because again all the violence, heaps and heaps and heaps of people were killed and the water and the hygiene and disease and stuff like cholera and dysentery. In the refugee camps they have a lot of that and everything and yeah. [S20]

Student 24, in explaining literacy rates for Sudan, provided an analysis of the relationship between the literacy rates and aspects of life in Sudan, such as education, religious beliefs, and government priorities.

> The literacy rate … it shows that the females, as in probably most countries in that region or area are less educated than males probably because of priorities in the system and religious beliefs and the literacy in Australia is obviously amazingly higher than Sudan but in Sudan they have a program … they give free education and I think it is for the first six years and the government is focusing on eliminating illiteracy in the country. [S24]

Although only one student demonstrated her ability to *Evaluate* within the Mathematics dimension, where mathematical data as presented within graphs were required to justify ideas, decisions, and inferences within the broader unit of work, five of the six students evidenced this level of thinking within the Reasoning dimension. The following are two examples in which students used reasoned arguments to justify their views.

> Because they've borrowed money from all these countries. The other countries, since they're so rich, they shouldn't worry about it because they have a lot of money and the aid programs are good but they probably need to do more to help the country out like bring it more food supplies and more fresh water. [S21]

> I think they are doing the right thing raising money, but I don't think it will help in the long run, because sometimes the money doesn't even get to the countries to help. I am not really sure what they could do, just getting aid into the countries and helping get clean water and clean food and resources and I think all the countries like America and Australia and Asia, I think should rally put in to help out these countries that are not as well off because Africa is a struggling country and I think they really do need some help and we are being a bit selfish with our resources. … I think Live 8 was a really good way of raising money and they're now looking at making a better preventative for AIDs in Africa from all that money, using it in labs, some kind of drug to help but yes it is a difficult issue. [S22]

> I think that the richer and more developed countries in the world need to offer money and support, like they are at the moment but I also think like in Africa they have got a really corrupt government, so they give them money and all that sort of thing but often the government takes it for themselves rather than using it to help the people like I think that something needs to be done about the governments, but even if their government is overthrown they are still going to need support from richer countries. [S23]

The six students interviewed did not evidence Reasoning at the highest level, *Create*. Students at this level of Reasoning would use data and information to suggest their own solutions or bring together the suggestions of others and synthesise them. Although not evidenced within the context of the student interviews, it is noted that during the student presentations given at the conclusion of the unit of work, a number of students showed that they were moving toward this level of thinking.

9.3.3.3 Attitude

The students' personal disposition toward numeracy became evident as they discussed their major assignments from the interdisciplinary unit on poverty. Their attitudes toward their learning were revealed by both the comments made and at times by the manner in which they conveyed them. Table 9.3 summarises the observed categories of Attitude of each student.

Table 9.3
Student learning: Attitude dimension (Tanglefoot School)

Student / Category	S19	S20	S21	S22	S23	S24
Confidence/ self-efficacy	✓			✓	✓	✓*
Interest	✓	✓		✓	✓	✓
Enjoyment		✓				✓
Intellectual stimulation	✓			✓		
Diligence/Perseverance	✓					✓
Appreciates value of mathematics	✓	✓	✓	✓	✓	✓

* not explicit, but observed in the interview

Of the six students, Students 19 and 24 evidenced a high degree of overall positive attitude to numeracy by demonstrating five of the six affective factors deemed to contribute to successful numeracy learning. Another two students evidenced four of the six factors. In terms of *Confidence*, four of the six students displayed a positive self-concept about their capabilities in the context of the unit of work. For two of the students this confidence related to working with and creating graphs as exemplified by the comment from Student 23.

> When you have a comparative graph it makes it easier to show the difference and it doesn't have as much impact when you just hear it. [S23]

For Student 19 however, her confidence was situated within the context of the unit of work and the knowledge that she could discuss it with her sister who was living in the United Kingdom where the main Live 8 event was being staged.

> It made it a lot easier for me just because I understood it more. [S19]

There was one student, Student 24, who conveyed her confidence throughout the whole interview by the manner in which she discussed her work.

Five of the six students evidenced *Interest* by sharing a distinct engagement with the contextual nature of the learning and by expressing a preference for using mathematics in real-life settings. Student 20, for example, focused on the application of skills in engaging her in learning. In this particular comment, Student 20 is also providing evidence of her *Appreciation of the value of mathematics*.

> It is more interesting and you actually put it to use rather than just learning it so we actually put it there and have to come back to the skills we've learnt and stick it on. I prefer to use it in real life because it's interesting and it's so much better because it is for something you're learning about and not learning how to do it. [S20]

By "stick it on" the student here was referring to applying the mathematics. For Student 24 her interest connected with the real-life nature of the unit of work, specifically because her mother was born in the Philippines, itself an underdeveloped country. For this student her initial interest in the context led to intellectual stimulation and perseverance in researching and representing the statistics in order to satisfy her interest in her study of Sudan.

> Well my Mum comes from the Philippines and over there it is quite crowded and the medical system is quite bulked up because of the population so we discuss it. It is not really a poor country but we discuss it as it is a general interest between all of us. [S24]

In addition to the overriding interest by the five students in the contextual nature of the unit of work, two of these five students also demonstrated an engagement with a particular task, for example when Student 22 discussed her learning about the variety of ways that the Gini index could be represented, including the use of a number line.

> It is more interesting … I think maybe we should do more of this because it did teach me about making graphs. Like I didn't even know that you could do something like Alex did with the Gini index, it was really good. …and I remember [student] did a graph, I think it was a line graph to show the Gini index and that was really good too so I learnt about that too. [S22]

Student *Enjoyment* went beyond students discussing areas of learning that they found interesting, and incorporated students being clearly excited about their learning. Two of the six students displayed this sense of fun when sharing their work, and two examples are included.

> It was a lot more fun [using mathematics in poverty assignment] than doing it out of a text book. [S20]

> I focused a lot on the statistics and graphs ... I might have got a bit side-tracked by all the statistics. [excitedly] [S24]

Two of the six students demonstrated the *Intellectual stimulation* they gained by working on their major poverty assignment. For these students the important factor in motivating them intellectually to consider the real impact of poverty was the nature of the task given, in which they had to compare their country of study with life in Australia. The following are examples.

> If we hadn't done this we wouldn't have known. It has made us more aware of the way we live to the way for example that Afghanistan lives ... we actually had to think about it. [S19]

> Another idea about poverty and making it sink into us a bit how difficult it would be and how easy it is for us to live. Comparing it sort of, instead of just studying a country and saying there's a war. Comparing it sort of makes it a bit more personal and thinking about children there who can't read and write when they're fifteen. [S22]

In terms of *Diligence,* two of the six students evidenced a willingness to persevere and engage with tasks. Examples include a comment by Student 24 about the time put into gathering correct statistics, and by Student 19 about the discussions she had with family members.

> Well it was actually quite hard to compile all the correct statistics and data. I had to go to several different websites and collect different numbers for each year and then I had to put them all together. ... I went and tried every format to put it in and which suited the task best. [S24]

> I talked to my sister about it on the phone and she is in London, so it helped me to understand how *Live 8* was working and what was happening and that had an influence on the way I did my assignment. [S19]

All of the six students evidenced an *Appreciation of the value of mathematics* when sharing the value of the graphs in supporting a clear understanding of the impact of poverty on their country of study. The following are examples.

> Well you see the big difference there is between Australia and Nepal, the big difference between the countries ... when it is in a graph and you can see it. [S21]

> I think that with the graphs and everything it was important to use that because if you didn't have the knowledge of Maths you wouldn't be able to do the graphs and also for understanding the things like mortality rates. I didn't have any idea of what, on the side it said 26.06 and then slash 1000 ... and [Ange] told us about understanding Maths ways of writing and I think it is important with a country like this to learn about ...and with the economy as well to have the [Maths] knowledge. [S22]

> It is just statistics and numbers but if we compare it to how our life is then it makes it more real. [S23]

Student 24 shared her thoughts on the overall value of mathematics in everyday life.

> Maths is an essential part of life and I suppose to include it in this it is quite important. [S24]

9.3.3.4 Context

The context of student learning, in this case the country of investigation and wider issues of poverty, featured prominently when students were explaining their graphs. As discussed above in the dimension of Attitude, students expressed a preference for applying their learning of mathematics to real-life contexts. There were many times, when explaining their graphs, that the students related their learning to the context of their country of study, and to poverty. Table 9.4 summarises the observed categories of Context for each student, as evidenced by the student interview.

Table 9.4
Student learning: Context dimension (Tanglefoot School)

Student / Category	S19	S20	S21	S22	S23	S24
Personal experience of context			✓		✓	
Context integrated with mathematics as presented	✓		✓	✓	✓	
Context integrated with mathematics, from both prior knowledge and as presented	✓	✓		✓		✓
Relational understanding of the mathematics and the context and can transfer to new contexts	✓					

For two of the six students the first level of this dimension, *Personal experience of context*, was demonstrated when discussions about their work were based more on personal opinion than an informed knowledge of the context as it related to the mathematics embedded within the graph. In the following example, Student 23 considered why the life expectancy of Canadian males might be less than that of Canadian females.

> I think probably because, I know it sounds sexist and all that but because the men would be like worn out more because they tend to do more work outside the home and spend time outside the home. [talking about Canadian life expectancy] [S23]

When discussing her graph of literacy rates in Nepal, Student 21 did consider some reasons based upon her learning of the context and also included some of her own personal opinion.

> … [and] females do most of the housework and men are the ones that [sic] get the jobs and supplies. [S21]

Students 21 and 23 did, at other times, remain focused on the *Context integrated with the mathematics as presented.* Three other students also provided evidence of learning at this level when they remained focused on the data and context presented within their graphs, without being distracted or influenced by personal experience or opinion, but also without thinking or discussing beyond the graph itself. The following are examples.

> If you have got the graph you can see the difference like the literacy rate the massive difference between the Afghanistan female to the Australian female. [S19]

> They're one of the poorest countries in the world and most of the people live under the poverty line and children, most children under five die of malnourishment before they reach the age of five. [S21]

> It's 77 [life expectancy for males] and then 84 for the Canadian female and then 82 for Australia … It tells us that females are expected to live longer and even longer when they are in Canada. The average life expectancy of both countries is high. [S23]

Four of the six students demonstrated a capacity to integrate both the mathematics and the context at a higher level when discussing the graphs they had created during the unit of work. There were times during their interviews when *Context was integrated with the mathematics, from both prior knowledge* beyond the graph *and as presented.* These students were able to make connections between their learning about the context of their country of study from other sources with the data they were collecting and representing in their graphs.

> The big difference in the way that Australia live to the way that Afghanistan live, like the average income in Australia, can earn $30,000 a year and in Afghanistan it is $280 a month [emphasised] that they have to live off and so that would be

really hard if you've got a big family and if you only have one income, usually it's only the male who is working and the wife has to stay home. [S19]

It would have been two hundred years ago for us when we lived that long [not past 50] but now in the modern world it's different … [In Rwanda] all the violence, heaps and heaps and heaps of people were killed and the water and the hygiene and disease and stuff like cholera and dysentery. In the refugee camps they have a lot of that. [S20]

Well it shows that pretty much everyone in Australia can read who are over the age of fifteen, but in Nepal they don't have much literacy options to read. The men are just over 60% and the females just over 25% and I think it is probably because they don't have enough money to fund education. [S21]

At the highest level of this dimension, students can bring together a *Relational understanding of the context and the mathematics and can transfer this to new contexts.* One student, Student 19, demonstrated her ability to draw together her learning about life in Afghanistan based upon both mathematics and the context that she had studied, and transfer this to her consideration of Afghanistan's debt. She made some informal inferences and considered implications of the large debt in Afghanistan and how other countries might be able to assist. It is also noted that due to the nature of the context in this case having social, cultural, economic, and political implications there are connections with the Equity dimension when students evidenced this highest category in the Context dimension.

Australia is a well off country; we are not in debt at all, and if we lent money to countries, like America has plenty of money and lends money and are still well off with nothing wrong without that money. So Afghanistan for example, are 1.5 million in debt and I reckon a quarter of that would be interest so by the time they have borrowed the money they have got to pay the same amount probably back in interest so it is not helping them borrow the money. If anything it is making their case worse. So I reckon for countries like America or even if they just lent money to a few countries, not having as high an interest rate would have a massive improvement because they wouldn't be having to pay nearly the exact amount back in interest. [S19]

9.3.3.5 Equity

In this particular unit of work, exploring the nature and impact of poverty, the mathematics enabled students to consider and explore social, cultural, and political contexts beyond their own context and beyond the classroom learning environment as well. Table 9.5 summarises the observed categories of Equity for each student.

Table 9.5
Student learning: Equity dimension (Tanglefoot School)

Category / Student	S19	S20	S21	S22	S23	S24
Personal social engagement						
Awareness of issues	✓	✓	✓	✓	✓	✓
Considering viewpoints	✓	✓	✓	✓		✓
Relating mathematical information with social and political consequences	✓	✓	✓			
Challenging inequity	✓	✓	✓			

During the interviews, discussions were based upon the major poverty assignment students had completed. For this case therefore, the first category of this dimension, *Personal social engagement* is not evidenced. Although within the other three case studies, the students were discussing distinct tasks and how they approached them and came up with solutions, in this case, the final assignment on poverty formed the focus of the student interviews. The students, therefore, were actually talking about their country of study and the mathematics that had informed their developing understanding of poverty. Evidences of student learning in this case, therefore, begin from the second category.

All of the six students interviewed demonstrated their *Awareness of issues* related to the social, cultural, economic, and political contexts of their particular country of study. The following comment from Student 24 is an example of awareness focused on the country she had studied.

> I was quite concerned about the birth rates and the health supplies there. [S24]

The following two examples show how the students also revealed their awareness by making broader statements about poverty in general.

> To realise how much of a difference there is to [sic] the way we live to the way other countries live because if we hadn't done this we wouldn't have known. It has made us more aware of the way we live to [sic] the way for example that Afghanistan lives. [19]

> I understand more how poverty occurs like and how much poverty there is. How many people live in poverty … I do understand it more. [S23]

Five of the six students evidenced their ability to *Consider viewpoints* when they expressed opinions about the circumstances of those living in poverty. For these students, the awareness they had gained from studying a country in depth and from comparing it with life in Australia had resulted in a response to people's circumstances. They used language such as "extreme," "a big difference," and "how far behind they are." The following are examples.

> With ours it was just comparing Afghanistan to [sic] Australia, but hearing others ...we could compare Afghanistan to [sic] other countries to see whether they were better off than Afghanistan. Like someone did France or Italy and obviously they are not third world countries. Comparing Italy to Australia and then to Afghanistan it was still quite a big difference. And third world countries, some of them the average income was less than $280 a month. [S19]

> Because the [water systems] were destroyed ... because of all the violence and stuff, it is quite hard for them. [S20]

> [Sharing graph of literacy rates in Nepal] In Nepal they don't have much literacy options to read ... and I think it is probably because they don't have enough money to fund education. [S21]

> There is a civil war going on there at the moment and it's quite ... a corrupt government there and their education system as far as I can see is building back but what I was primarily concerned about is how far behind they are. They have aid resources from other countries but I was quite concerned about the birth rates and the health supplies there. [S22]

> You could just say that Australia is a much more advanced better country than Sudan because of its population, birth rates, literacy rates, but when you actually see it you realise how big the problem is. [S24]

Three of the six students demonstrated a capacity to *Relate mathematical information with social and political consequences*. They used their understanding of the mathematical data they had collected, represented, and interpreted in their graphs to consider and express social consequences for the people living in different countries across the world. The following examples incorporate a consideration of issues such as low income, lack of water and food, disease, and violence, and some of the implications of these issues including malnourishment, challenging daily life circumstances, and even early death.

> The average income, people in Australia can earn $30,000 a year easily and in Afghanistan it is $280 a month [emphasised] that they have to live off and so that would be really hard if you have got a big family and usually it's only the male that [sic] is working ... [S19]

> It would have been two hundred years ago for us when we lived that long [referring to life expectancy for Rwandan males, not living past 50] but now in

> the modern world it's different … [In Rwanda] all the violence, heaps and heaps and heaps of people were killed and the water and the hygiene and disease and stuff like cholera and dysentery. In the refugee camps they have a lot of that. [S20]

> Well I thought because Nepal is a pretty small country but had a population of over 27 million so that was pretty big and they're one of the poorest countries in the world and most of the people live under the poverty line and children, most children under five die of malnourishment before they reach the age of five. [S21]

In the final category of the Equity dimension, students are able to bring together their mathematical learning and use mathematics to operate more powerfully in the world, by *Challenging inequity* in society, and at times even considering or taking action to resolve inequities. In this particular unit of work, the mathematics enabled the students to question societal structures. Student 23 described the tensions between the importance of wealthier countries providing financial support and the difficulty in ensuring the money goes to where it is needed.

> I think that the richer and more developed countries in the world need to offer money and support, like they are at the moment but I also think like in Africa they have got corrupt governments and so they give them money and all that sort of thing but often the government takes it for themselves rather than using it to help the people. I think something needs to be done about the governments but even if their government is overthrown they are still going to need support from richer countries. [Student 23]

Student 22 focused on the basic needs and important resources needed in underdeveloped countries.

> I think they are doing the right thing raising money but I don't think it will help in the long run, because sometimes the money doesn't even get to the countries to help. I am not really sure what they could do, just getting aid into the countries and helping get clean water and clean food and resources and I think all the countries like America and Australia and Asia, I think should really put in to help out these countries that are not as well off because Africa is a struggling country and I think they really do need some help and we are being a bit selfish with our resources. … I think the Live 8 was a really good way of raising money and they're now looking at making a better preventative for AIDs in Africa from all that money, using it in labs, some kind of drug to help but yes it is a difficult issue. [S22]

All six students considered personal contributions that they felt would, in some small way, assist in alleviating poverty. The following excerpts represent the range of suggestions, from supporting local fundraising opportunities and raising awareness, to considering a career in overseas aid work.

> I feel when I am older I would like to sponsor a child because … they are not getting the education they need and they are dying when they are young. [S21]

> Well because the poorer countries have borrowed money from developed countries. The other countries since they are so rich they shouldn't worry about it because they have a lot of money and the aid programs are good but they probably need to do more to help the countries, like bring in more food supplies and more fresh water and stuff like that. [S21]

> I think the government should listen more to the people because we do have lots of ideas and with refugees here it is quite a big issue because some people don't want to let them in because they didn't go through the proper forms that you have to go through, but you have to look at it from their point of how the government keeps them here and sends them back sometimes when their country is better. [S24]

> Well ever since I was little, because my Dad does all this stuff in different countries, I have wanted to make a difference in those kinds of places so I think, like I want to study medicine and help out there. I have never thought about being in the army or anything like that because that just makes more problems. I was thinking of going and being a doctor or an anaesthetist or paediatrician in somewhere like Somalia or probably in Iraq but maybe some other countries in Africa. [S22]

For Students 22, 23, and 24, however, there was no evidence throughout the interview to indicate they were using the statistical data they had collected and represented to support their opinions about inequity. Learning evidenced in the previous category, *Relating mathematical information with social and political consequences* is important here. With no other indications of using mathematics to support their arguments, these students only evidenced their awareness of issues and a sense of inequity, not a comprehensive understanding informed by mathematics.

For Students 19, 20, and 21, their comments involving challenging inequity itself and considering taking action were also considered in the context of the whole conversation. Due to the evidence of these students' understandings of the relationship between the mathematics and the social and political consequences, their comments regarding *Challenging inequity* are counted as evidence of this category of learning. An example is the comment shared by Student 19 at the highest category of the Context dimension (p. 299).

9.4 Chapter summary

This chapter has shared the beliefs and practices of Ange and Jen and the learning experiences and outcomes of their students. As Grade 8 teachers responsible for not only a Grade 8 class but also the teaching of a discipline area, Ange (Mathematics/Science teacher) and Jen (SOSE/English teacher) saw discipline-based learning as an important part of the students' education. They also valued interdisciplinary learning and the opportunities it provided for students to apply their disciplinary knowledge; to engage in group work; and to develop more generic capacities such as problem-solving skills, communication capabilities, and information literacies. The incorporation of the Essential Learnings into the Tanglefoot middle school curriculum was seen as supporting the students to be numerate across the curriculum.

The dimensions of Mathematics (to a lesser degree), Reasoning, Context, and Equity featured in this case, as Ange and Jen planned and implemented an interdisciplinary unit of work, "Live 8," inspired by the 2005 G8 world summit. The unit was designed to provide the students with an engaging and authentic context in which to develop an understanding of the concept of poverty and to develop further the students' skills in graphing and data analysis. Although independent learning was encouraged throughout the unit of work, this occurred within the context of students working in pairs and being encouraged to share and present their ideas to each other. As students synthesised their mathematical learning with their contextual understandings they were able to engage more deeply with the complex issues impacting upon poverty and life in the countries they were studying.

> *Ange: I think the good thing was that because we were actually studying [data and graphing] in maths at the time the unit of work gave it meaning, so it wasn't just numbers, it actually has meaning. I think that most of the girls really took on the graphs well and looked at the statistics because they had to compare [their country of study] to Australia so it had meaning for them and they really wrote really good summaries about it. But some of the others just did the graphs. ... I mean it added a new dimension to it, it wasn't just learning about the culture it was also that higher order thinking, "We have the statistics what do we do with this?" ... Some of the girls got their statistics and they could incorporate them into their comments.*

Instead of just doing their graphs they were using the data they collected and incorporating that data into their summaries. ... It was good because they had to really think about what type of graph would really suit the information, which they did to varying degrees.

Interviewer: And how important do you think those connections are - using the mathematics to connect with an element of another concept?

Ange: It's vital because otherwise they lose it. The information goes because it has no meaning to them, which happens with a lot of maths. I try and teach about why algebra is so useful, "I know how to do this, but why, when am I ever going to use it?" So if there's a relationship there and they can think, "Yea I might need that" so yes it's definitely vital. ... I think it is always going to be important that connection and I guess the way that we can do that is looking at what is current, so here we chose the G8 summit. So what is going on around you that will help the girls make that connection, with what's happening. (Ange and Jen, 2/11/05)

This chapter has concluded the presentation of the results of the study. A discussion of the outcomes associated with the teachers and the students is presented in the next chapter, in relation to the numeracy framework developed and conceptualised through five dimensions of practice. The diverse possibilities available both for teaching and for student learning are also discussed, as well as implications for curriculum design and professional learning, and suggestions for future research.

Chapter Ten

Discussion and Conclusion

10.1 Introduction

The aim of this study was to investigate the positioning of numeracy by teachers and the numeracy experiences and learning outcomes of students, within the context of curriculum reform. Wilkins (2000) argued that "the effects of different curricula and instructional methods on the development of quantitative literacy should be examined" (p. 416). The practice of five teachers, who were engaging with the Tasmanian curriculum reforms and who also had an interest in numeracy teaching and learning, provided a unique opportunity to gain insight into the nature of numeracy within a transdisciplinary curriculum. Firstly, a conceptual framework for numeracy, incorporating five dimensions of practice, was developed (Chapter 3). The five dimensions – *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity* – provided the lens through which the beliefs and practices of the teachers who participated in the study were considered and the learning exhibited by individual student participants was examined. By using the framework to bring together the teachers and the students in these ways a unique understanding was gained as to the nature of teachers' and students' engagement with numeracy as well as the nature of numeracy itself within naturalistic classroom settings. The study was conducted through four phases of inquiry and the results presented by case with data for teachers and their respective students reported together (Chapters 6 to 9).

Two objectives underpinned the research:

- First, a theoretical objective, to deepen understanding of the construct of numeracy in the context of current reform agendas, through the development of a conceptual framework for numeracy that aligns with and extends current research about numeracy and its capacity to equip students for their current and future lives as democratic citizens.
- Second, a practical objective, to contribute to an understanding of the complex nature of the teaching and learning of numeracy. The study was concerned with the enactment of curriculum in the classroom, in which the roles and experiences of teachers and students are equally important.

The following research questions were posed:

1. How are teachers positioning numeracy in reform-based learning environments according to five dimensions of practice?
2. How are students experiencing numeracy in these reform-based learning environments according to five dimensions of practice?
3. How does a five-dimensional framework for numeracy, developed to align with a transdisciplinary curriculum context, contribute to an understanding of numeracy teaching and learning?

The findings of the research in relation to these three questions are discussed in this final chapter. Sections 10.2 and 10.3 summarise the findings of the results that have been comprehensively presented in Chapters 6 to 9. Section 10.4 revisits the conceptual framework developed for this study based upon the findings and in light of the literature. The chapter goes on to consider implications of the study for both curriculum design and professional learning. Limitations of the study and recommendations for further research are also discussed. The thesis is concluded by bringing together the motivation for the research and its significance.

10.2 Teachers positioning numeracy

The five teachers who participated in the study, although teachers of varying professional backgrounds and length of teaching experience, were all teachers who were supportive of the curriculum reforms being implemented in their schools. They had all also been using the Essential Learnings framework to inform their planning for two to three years. They were teachers for whom numeracy was a valued part of the overall curriculum and who wanted to provide their students with worthwhile and effective learning experiences in this area. In keeping with the curriculum context, the teachers saw numeracy as having a cross-curricular relevance and a role to play in students' lives beyond the classroom. Although they were all middle years' teachers, three of the teachers (Alice, Ophelia, and Samantha) were teaching in primary school settings, where they had responsibility for the whole curriculum, whereas the two teachers at Tanglefoot School (Ange and Jen) were both discipline-specific teachers and middle years' home tutor teachers in a K-12 setting and who worked together from time to time to implement interdisciplinary units of work.

The units of work planned and implemented by the teachers in this study were each motivated by different learning objectives. Alice, from Snowgum, taught a focused unit of work on graphing over a four week period. She had identified a gap in students' understanding and skills during a previous unit of work, and planned for and provided her students with specific tasks to learn specific ideas of graphing. Ophelia and Samantha, both from Stringybark, were members of the same collaborative planning team. Although the two teachers had planned the unit of work "How do you measure up?" together, they implemented it individually with their own Grade 6 classes over a two month period. It had specific mathematical learning goals as well as goals related to the relevance of measurement in the real world, and to the processes of inquiry and reflective thinking. Ange and Jen, from Tanglefoot School, brought their two Grade 8 classes together for the five-week interdisciplinary unit of work "Live 8." The teachers' aim was to integrate the disciplines of Mathematics and SOSE to enhance students' numeracy capabilities, their abilities to work collaboratively, their skills in information literacy and communication, and ultimately their understandings of the concept of poverty.

Although there were clear similarities between the teachers' overall views about numeracy and its relevance for students, the teachers presented distinctive numeracy pedagogies. For Samantha, numeracy was a passionate endeavour and an area in which she felt she could achieve a wide range of learning outcomes across the whole curriculum. For Ange and Jen, numeracy was clearly distinct from the still important discipline-based mathematics lessons that Ange taught separately to any interdisciplinary units. Ophelia, being relatively new to teaching within the upper primary context, was relying on the curriculum reforms to shape her own numeracy teaching and was enjoying the constant reflection that this brought. For Alice, the curriculum reforms presented a challenging environment within which to bring her numeracy teaching more in line with her broader philosophy about teaching and learning. This broader philosophy was about promoting the development of student understanding about key concepts and ideas in transdisciplinary contexts.

The teachers in this study had clearly articulated learning objectives that they planned for and monitored throughout the duration of the units of work. They had high expectations of their students and used questioning to challenge students and to uncover their thinking. Connections both within and between mathematics concepts and in different contexts were valued and all the classroom learning environments were interactive, with students involved in regular class discussions with each other and with their teachers. In this way the teachers in this study demonstrated many of the characteristics of effective teachers of numeracy (Askew et al., 1997; Clarke et al., 2002). Alice, Ophelia, Samantha, Ange, and Jen were teaching for numeracy rather than purely teaching mathematics. They were also reflecting Bishop's (2000) claim that teachers interested in teaching numeracy are more likely to teach mathematics from a real-life contextual perspective and to use activities, investigations, or projects that enable this learning.

The case of Samantha and Ophelia, who planned the same unit of work, but taught it separately within their own classrooms, provided a unique opportunity to explore the differing ways that numeracy teaching and learning can occur from the same planning documentation and stipulated learning goals. The collaborative nature of their planning and ongoing discussions about their work enabled Ophelia to try new ideas with her students and to move to using different types of tasks with her students. Stringybark Primary School, where Ophelia and Samantha were teaching, had a very strong sense of leadership and support structures to equip teachers to implement the reforms and enable professional growth. This has been recognised as important by Fullan and Miles (1992), Hargreaves (1994), Knapp (1997), and Little (1999) and was recognised in these case studies. Due to the active nature of Samantha's classroom, a surface look might raise the question of the mathematical learning that was occurring beyond a clearly enjoyable experience. Observation over the extended period of this case study, together with the documentation collected and the interviews with Samantha and her students, revealed just how purposeful her teaching was. Samantha was informed by thoughtful planning based upon her own pedagogy and curriculum documentation. Samantha took time to interpret learning outcomes expected for her grade level across a range of abilities, continually reflected upon these, and shared them with her students.

Examining the beliefs and practices of the teachers according to the five dimensions of numeracy uncovered the distinctive and often unique ways that teachers enact their own beliefs about teaching and learning. Although all of the five dimensions were evident to some degree in each case, some featured more than others, not only based upon where the teachers were at with their own numeracy pedagogy, but also based upon the particular learning objectives of each of the units of work. This is discussed in more detail in Section 10.4. The beliefs of the five teachers informed their planning, instructional decisions, task selection, and assessment practices. This supports the findings of previous work relating teacher beliefs to classroom practice (Fullan, 1993; Prawat, 1992; Richardson, 1996).

10.3 Students experiencing numeracy

Across all of the four cases students engaged with a plethora of numeracy experiences, from mathematically-focused discrete tasks to open-ended inquiry tasks that required students to consider many factors beyond the mathematics, and where they had significant time to undertake their investigations. It was rare that worksheets were used in these cases, as students were mostly engaged in active learning with a wide variety of resources available to them. In Ophelia's and

Samantha's cases, learning was very hands-on and practical, with students provided with opportunities to explore measurement concepts and develop measuring skills with real-world objects. Students in each of the cases were expected to inquire, to ask questions, and to share their ideas with each other. The real-world relevance of mathematical ideas was valued in each classroom and in Alice's, Ophelia's, and Samantha's cases, students at times could select their own contexts. Overall, the 112 students who participated in the study were engaged in their learning environments. There were no serious behavioural concerns and students were observed to undertake the activities to the best of their abilities. There was diversity of academic ability in each case, and the selection of the six students interviewed in each case, reflected the spread of students in each classroom. Variability of student learning outcomes was demonstrated within each case, as would be expected with a spread of students. There was, however, general evidence across the 24 students interviewed (six in each case) that student learning within the five dimensions reflected the beliefs and what was planned for by the respective teachers.

Collectively across the four cases, student learning was demonstrated across all five dimensions of numeracy. In the case of Alice, where the dimensions of *Mathematics*, *Reasoning*, and *Attitude* featured, student learning was variable across the six students although learning in each of these dimensions was still well represented. Notably, two students, Students 3 and 5, who evidenced six categories and five categories respectively of the *Attitude* dimension, also evidenced a higher level of learning within each of the dimensions of *Mathematics*, *Reasoning*, and *Context*. It is also noted that for two students, Students 1 and 2, there was some evidence of a negative attitude toward their mathematics learning in terms of confidence and interest. As Alice herself had only very recently turned her own attitude around, it is likely that it may take a longer time to support those students who are less confident and engaged in this area to change their attitudes. The dimension of *Equity* was not evident for the students in this case, apart from one student demonstrating learning at the lowest category. This again, is likely to be because Alice was focusing very much on creating a comfortable learning environment and working on her own practice, and therefore students were not exposed to mathematics embedded within complex social situations. Also, the nature of the interview itself did not easily allow students to demonstrate the first category of *Equity*, "Personal social engagement," as the interview was concerned with discussing the work that the students had completed. This category was however evidenced in the classroom in the ways in which students worked together.

In the case of Ophelia, again diverse student learning outcomes were demonstrated across the five dimensions of numeracy. In this case, the connections between students' demonstration of a significantly positive *Attitude* toward numeracy did not necessarily relate strongly to students who were high on the other dimensions. Of interest is that Student 11, who had the highest category of learning in the dimension of *Context*, did not demonstrate learning as strongly in the dimensions of *Mathematics* and *Reasoning* as Students 8 and 12. As in the case of Alice, the *Equity* dimension was not demonstrated in this case, based upon the student interview, except with one student, Student 11, who evidenced the first category of *Equity*, "Personal social engagement," when discussing his measurement work.

In looking at the case of Samantha, where all of the dimensions (although *Equity* to a lesser extent) were evidenced in her beliefs and observed in her practice, overall, students demonstrated high level learning. Three of the six students demonstrated their abilities to "Evaluate and transfer their measurement understandings to new or different contexts" within the *Mathematics* dimension. Students' capacities for high level *Reasoning* was also evidenced with five of the six students demonstrating learning to the second highest level, "Evaluate," and one student forming new ideas and understandings of his own when investigating the area of circular objects. In this case, students' attitudes toward their numeracy learning were particularly strong, with all students evidencing four or more of the six categories of the *Attitude* dimension. Two of the six students demonstrated five of the six categories and another two, all six of the affective factors deemed to contribute to successful numeracy learning. The nature of this classroom clearly engaged the students and even Student 17 who struggled with many areas of the curriculum, was still positive and worked to achieve his best in the learning environment. Again, the *Context* dimension was strong in this case, with most students demonstrating their ability to "Relate their understanding of the mathematics to context, and transfer these understandings to new contexts," the highest category in this dimension. Samantha, was constantly challenging her students to work with many varied contexts. The culminating performance task, involving students selecting their own context within which to demonstrate their learning throughout the unit of work, had supported students in achieving this ability to transfer their learning.

Within the dimension of *Equity*, three of the six students demonstrated their "Personal social engagement" in the interview setting, by sharing their awareness of others' contributions in the learning environment and their disposition to question and make decisions based on their own and others strategies. Furthermore, one student demonstrated her capacity to "Relate mathematical information to social consequences" (the second highest category). Although the learning objectives of the unit of work were not related to social and political contexts, this evidence was found in Samantha's case because of the nature of the classroom that she had created. Students were engaged continuously in shared learning and were challenged to explore new ideas and consider other perspectives and ways of approaching tasks.

In the case of Ange and Jen, the unit of work was socially and politically driven. The teachers had purposefully planned for learning outcomes in relation to understanding of the concept of poverty and its impact upon many social, economic, and political factors. The students in this case, as well as demonstrating diverse learning outcomes across the four dimensions of *Mathematics*, *Reasoning*, *Attitude*, and *Context*, also provided clear evidence of the *Equity* dimension. All six students shared their "Awareness of issues" related to the social, cultural, economic, and political contexts of their particular country of study, and five of the six evidenced their ability to "Consider other viewpoints" when expressing their opinions about these contexts and the circumstances in which people lived. Three of the six students demonstrated high level learning in using their mathematical understandings to provide evidence of the social consequences of people in underdeveloped countries. These same three students were also providing emerging evidence of their capacities to "Challenge these inequities."

Ange and Jen had explicitly planned for learning across mathematics, reasoning, and context, and this resulted in students evidencing learning within all of these areas. Students' inherent attitudes toward numeracy were revealed and five of the six students shared that they found mathematics more interesting within interdisciplinary units of work. All of the students showed an "Appreciation for the value of mathematics" within this case, as they were very clear about how the mathematics had enabled them to see the impact of poverty.

Student learning across the five dimensions of numeracy was influenced by how the Essential Learnings was interpreted into the teaching and learning sequences planned and implemented by each case study teacher. The teachers' emphases on learning goals, mathematical topics, tasks selected, questions asked, expectations set, and the nature of classroom discussions strongly influenced the capacity of students to demonstrate learning not only across the five dimensions but also across multiple levels of learning within each dimension. It is evident that there is a strong relationship between what is planned for by teachers, what is observed in the classroom, and the learning outcomes achieved. Learning outcomes are not evidenced if teachers are not clear and purposeful with their teaching. The National Research Council (2001) noted that "opportunity to learn is widely considered the single most important predictor of student achievement" (p. 334). This opportunity is defined as "circumstances that allow students to engage in and spend time on academic tasks" (p. 333). In this study, the case study teachers provided students with a multitude of "opportunities to learn" and this was evidenced by the learning outcomes exhibited by the students.

In the case of Samantha, for whom all of the dimensions were at the fore of her pedagogy, her students exhibited high level learning across most of the dimensions, and with respect to *Equity* this dimension was beginning to be evidenced as well. The teaching and learning of numeracy in this case produced a very strong interconnection among the mathematics, development of thinking skills, a positive attitude toward numeracy, its relevance by use of authentic context, and building an inclusive and equitable classroom environment. Samantha tried to provide an environment in which all students could be creative at their own levels and the researcher is convinced this was successful. It may be possible that where numeracy teaching is strong across all of the five dimensions of numeracy, student learning will not only feature across all of the dimensions, but also be truly creative and exhibited to a higher level than would occur otherwise. The case of Samantha provides a glimpse of this possibility.

10.4 Revisiting the framework

A conceptual framework for numeracy was developed for this study based upon a synthesis of the literature. The resultant five dimensions – *Mathematics, Reasoning, Attitude, Context,* and *Equity* – recognise the complex nature of numeracy and align with the underpinning ideas of a social constructivist view of knowledge acquisition. Descriptions of each dimension were drawn from the literature relevant to each area. The conceptual framework was critical to this study in that it provided a lens through which to examine the positioning of numeracy by teachers and the learning experiences of students within reform-based classroom learning environments. It was used to situate the data and enabled analysis and rich description of the phenomenon of numeracy within four classroom learning environments in a way that has not been done before. Aspects of each of the five dimensions were evidenced in all of the four case studies, and where most of the five dimensions were at the fore of the teachers' beliefs, planning, and practices; students experienced richer and arguably improved learning across all of the five dimensions of numeracy. Applying the framework in the case studies in this way has demonstrated its usefulness and also the usefulness of the conceptualisation of numeracy proposed.

Notably, the dimensions of *Mathematics* and *Reasoning* featured in the four cases, with the dimensions of *Attitude, Context,* and *Equity* being evidenced more variably across the beliefs of teachers, the enactment of numeracy within the classroom learning environments, and individual student learning outcomes. This section revisits the framework according to each of the five dimensions and summarises the evidence of its value and some insights from the research.

10.4.1 Mathematics

Mathematics is clearly the foundation of numeracy. Numeracy has been interpreted as the development of mathematical skills (e.g., Brown, 2000; Clarke, 2000; Gould, 2000), as the use of mathematics in context (e.g., Hammond & Beesey, 1999; Kuss, 2000; Siemon, 2000), or as more about the promotion of mathematics learning across all areas of the curriculum (e.g., Chapman, Kemp, & Kissane, 1990; Goos, 2001; Thornton & Hogan, 2004). Whatever the conceptualisation of numeracy, importantly, mathematics – as a discipline involving developmental learning within the strands of number, measurement, space, chance and data, and pattern and algebra – is the undisputed foundational dimension of numeracy.

The *Mathematics* dimension was clearly visible in each of the four classrooms and was exemplified by all of the students as they discussed their learning during the student interviews. The role of mathematics in the development of numeracy was reflected by each of the case study teachers. They often referred to the "knowledge," "skills," and "processes" that students needed to learn in order to become numerate. The teachers were all focusing on a targeted area of the mathematics curriculum and had articulated specific learning outcomes in the respective mathematical strands in their planning. Choice of tasks, questioning of students, and aspects of assessment were predominantly centred on ensuring that students experienced opportunities to explore the mathematical concepts and to further their understanding in these areas. Within the three primary school settings, there was a common focus on development of students' number sense, irrespective of the broader mathematical goals of the unit of work. A unique aspect of the middle school setting of Ange and Jen was the shift from the mathematics being at the fore of the learning during the commencement of the unit of work, when students' prior knowledge was being revisited, to the mathematics becoming a tool for understanding the nature of poverty during their major assignment. In this case, the mathematics was not taught, but rather revisited and then applied within an authentic context. In contrast, Alice, who was focusing on the same area of mathematics – graphing and data analysis – provided students with discrete tasks to support them in developing targeted skills in reading, creating, and interpreting a variety of graphs and in considering the most appropriate graph for each purpose. Although in different ways, all of the teachers were working to build conceptual understanding (Hiebert & Carpenter, 1992).

Both Ophelia and Samantha taught a unit of work on measurement, however, each teacher enacted the unit in her classroom in a way that reflected her own beliefs about numeracy, and about teaching and learning. Ophelia used a mix of structured and open-ended tasks, with the learning of a particular attribute or skill of measurement in mind. As the unit of work progressed she began to trial more open-ended tasks, to see what this brought to students' experiences and their mathematical understanding of concepts. Samantha, however, used all open-ended tasks with many coming from everyday life. Samantha, as with Ophelia, often selected tasks with the purpose of targeting a key idea or attribute of measurement, for example, area. Many of the tasks that Samantha used led to new tasks that extended the students' learning in the same conceptual area of mathematics. In this way there were many connections among the mathematical ideas being considered and also often involving other areas of mathematics without students explicitly being aware of the connections. Samantha involved her students in contributing mathematical questions to the classroom for others to solve.

Despite the many varied approaches to the mathematics underpinning numeracy within the cases, all of the teachers were thoughtful in ensuring a progression of mathematical concepts and ideas in the tasks they chose in order to build student understanding appropriate to the mathematics within each unit. The role of the discipline of mathematics as it relates to the middle school curriculum incorporates a much greater diversity of mathematical concepts and ideas beyond those that were explored in this thesis. The thesis only considers the *Mathematics* dimension as it relates to graphing and measurement, whereas exploring the dimension of *Mathematics* as it might relate to other important middle school concepts such as proportional reasoning, algebraic thinking, and geometry would be of interest in future research.

10.4.2 Reasoning

Perhaps not surprisingly, *Reasoning* featured for all of the teachers. With Thinking being one of the five Essential Learnings, the teachers had a sound understanding of the value of students developing high level thinking skills beyond recall and comprehension (Bloom, 1956) and of developing skills of inquiry and reflective thinking. In the three cases set within the primary school context, a problem solving approach along the lines of Van de Walle (2004) was evident. The teachers valued student questioning, used of a variety of strategies and representations, and followed up with opportunities for sharing of strategies, explaining thinking, and justifying solutions. In these three cases there were times of whole class sharing often at the end of a lesson. In all of the cases the teachers themselves thoughtfully questioned their students, modelling the thinking that they valued, and using questioning to push their students further. Alice used many formal thinking tools such as mind maps, Venn diagrams, and reflective journals, to uncover student thinking. All of the case study teachers used questions such as "Can you explain why?" or "How did you do that?" In addition to this type of questioning, Samantha challenged her students to think deeply by being very specific with her questioning, for example, "What strategy could you use to be fairly sure your values are right?" and to push them to seek more information, "What do you need to know to go further?" Ophelia and Samantha also had clearly articulated learning objectives in their planning that described outcomes desired within the areas of inquiry and reflective thinking.

At Tanglefoot, reasoning was not something that was explicitly taught to students. As adolescent learners, the role of reasoning was more about the opportunities provided to the students to work both independently and together. The teachers scaffolded the learning so that as the unit of work progressed, the students were required to apply their knowledge and skills and synthesise their mathematical learning with their contextual understandings. The teachers provided more formal presentation opportunities at the end of the unit of work to enable them to share their work as well as their capacities to bring together their ideas and interpretations of their learning. In some respects these more formal presentations mirror the more regular informal whole class sharing that occurred in the other case studies.

There was some overlap observed between the dimension of *Reasoning* and that of *Mathematics* and this is discussed in Section 10.4.6.

10.4.3 Attitude

All of teachers recognised the importance of a positive attitude toward mathematics on learning outcomes and therefore purposefully worked to create comfortable and open learning environments for their students. It was for Alice and Samantha, however, that the dimension of *Attitude* featured. For Alice this was because she was experiencing a major shift in her own attitude toward numeracy. She was therefore very consciously working on creating a supportive classroom learning environment that instilled confidence in students so as they were comfortable to share, take risks, and persevere with their learning. She had, in the recent past, not been a confident teacher of numeracy herself, and for her this major shift in her pedagogy was very important. Samantha was clearly a confident and passionate teacher of numeracy who saw its relevance in all areas of the curriculum and continually motivated her students in their own numeracy learning. She was purposeful in providing an engaging, enjoyable, and dynamic learning environment where students had the freedom to make choices about which tasks they would approach and how they would go about solving them.

Much of the research in the area of student attitudes toward mathematics learning has been based upon self-report scales (Beswick, Watson, & Brown, 2006; Galbraith & Haines, 2000; Tapia & Marsh, 2004). This study has brought insight to the impact and nature of attitudes upon student learning, by using students' conversations about their learning to reveal their attitudes. Although some students evidenced high level learning without having displayed many of the categories of *Attitude* (for example, Student 12 in the case of Ophelia, and Student 20 in the case of Ange and Jen), the converse was rarely true. Where students displayed five or six of the categories of the *Attitude* dimension, their learning within the other dimensions of numeracy was usually strong. Although no clear correlation can be drawn, it appears that students' positive attitudes toward their numeracy learning are related to their capacity to engage and be successful. This is exemplified with Students 3 and 5 in the case of Alice. There are, however, students who are still capable and successful learners no matter what the circumstances.

In the case of Samantha in particular, where the *Attitude* dimension was strong overall, students' learning across the five dimensions of numeracy was notable. As a passionate teacher for numeracy it is argued therefore that the teacher's own positive attitudes toward numeracy can influence positively her students' attitudes and their learning. This supports the research of Karp (1991) who found that "the daily experiences of students in mathematics classes of teachers with positive attitudes were found to be substantially different from those of students in classrooms of teachers with negative attitudes" (p. 266).

In the four case study classrooms, students were engaged in their learning. By observing and interviewing students, it was found that students' perceiving their learning as at times "hard," "difficult," or "tricky" was not related to negative attitude as can occur on self-report scales. Likert scale items as used in self-report scales often include items where students rate mathematics in terms of how "easy" or "difficult" they find it (e.g., Beswick, Watson, & Brown, 2006). Often therefore, students reporting mathematics to be difficult is attributed to negative attitudes. In this study, however, although students often shared their perseverance on tasks in terms of its difficulty, in being able to question them and gauge their feelings, this was nearly always related to positive attitudes toward their learning, and was also observed within the classrooms as students persevered and engaged with their learning. The benefits of students engaging with and expending effort when working to understand mathematical concepts has been considered by Hiebert and Grouws (2007). In a recent comprehensive and international review of the affects of teaching on student learning in mathematics education, Hiebert and Grouws discuss two features of classroom teaching that facilitate the development of conceptual understanding: "explicit attention to connections among ideas, facts, and procedures, and engagement of students in struggling with mathematics" (p. 391). They assert that these features are also likely to influence positively students' mathematical proficiency.

Leder and Grootenboer (2005) found that there are "few studies in which the difficult task was attempted of exploring the relationship between affect and a range of other important factors including cognition, learning and achievement … yet there are opportunities to investigate these issues within classrooms" (p. 5). This study, although not a study about attitudes, has contributed insight to student attitudes through classroom observations and student interviews. It is deemed important that teachers consider the development of a positive classroom learning environment as part of what they explicitly plan for and in the opportunities they provide their students.

10.4.4 Context

Context is a well recognised aspect of numeracy in so far as numeracy is about the application or use of mathematics (e.g., AAMT, 1997; Johnstone, 1994; Kemp & Hogan, 2000; Willis, 1998). The embedding of mathematics within contexts provides relevance for students. In this study, the use of context was varied across the cases. Although the dimension of *Context* was important to all the teachers, there was a degree of variability in the authentic nature of the contexts within which mathematics was embedded.

For Alice, the graphing tasks that students worked with were related to contexts, but these did not form a part of Alice's learning objectives. The contexts within the tasks were sometimes contrived, and sometimes related to students' everyday lives. In the cases of Ophelia and Samantha, context was more purposefully used to enable students to relate measurement to the real world by the use of practical measuring tasks and tasks where measurement was applied in variety of real-world contexts. In Ophelia's case these contexts were mostly related to the practicalities of measurement and the occasional context such as how many students might fit on a bus of a certain mass.

It was within the cases of Samantha, and Ange and Jen, however, that the dimension of *Context* featured most prominently. Samantha, in particular, wanted her students to see the relevance of measurement in life and she valued highly the connection of students' mathematics learning to authentic contexts and daily experiences. Samantha, therefore, posed questions daily in which authentic contexts formed the basis of the questions. These open-ended tasks were based upon her own life and the life of her students and it was this authentic aspect that enabled students to explore many possible strategies and solutions for solving problems. Numeracy learning in this classroom was not confined within the four walls of the classroom. Students were provided with the freedom to take their investigations outside of the classroom if it would support their thinking and problem solving. The culminating performance task that both Ophelia and Samantha had planned for, but only Samantha enacted in her classroom, was an in depth investigation undertaken by students in a context of their own choosing. Her numeracy teaching was driven by the relevance of context, and mathematical concepts were applied in new contexts daily, both posed by the teacher and considered and pursued by the students themselves. In this classroom, student learning for this dimension was strong, with three of the six students demonstrating a "Relational understanding of the mathematics and the context," and an additional two students demonstrating emerging evidence at this highest category.

In the three primary school settings, with one teacher teaching all of the areas of the curriculum, the units of work, although incorporating a variety of contexts and informed by the transdisciplinary nature of the Essential Learnings, were still planned with numeracy being a core focus. For Ange and Jen, being in a middle school environment, context was a major element of the unit of work that they had planned. It was the key motivation for the unit in which students investigated the concept of poverty within underdeveloped countries across the world. The mathematical area of graphing and data analysis provided the means by which students could investigate in detail the impact of poverty on the lives of people living in different countries. In this case, students' contextual understandings were assessed alongside their mathematical understandings at the end of the unit of work.

For this dimension of numeracy, the development of the four categories of student learning is new, being informed by ideas of Anderson, Reder, and Simon (1996), Fosnot and Dolk (2001), Griffin (1995), Hughes-Hallett (2001), and Kemp and Hogan, 2000. This contrasts with the dimensions of *Mathematics*, *Reasoning*, and *Attitude* that have been more fully developed by other researchers. In all four cases in this study, the categories described for the *Context* dimension (Table 3.6) were useful in distinguishing between students who were able to move beyond their own personal experience of context, to consider how the mathematics informed their sense-making, and ultimately be effective in transferring mathematical understandings to new contexts.

10.4.5 Equity

This dimension of numeracy has not previously been incorporated into numeracy research studies that have not had a critical mathematical focus (e.g., Boaler, 2008; Frankenstein, 1998; Gutstein, 2003; Skovsmose, 2004). The results of the study provide evidence of the value of bringing the dimension of *Equity* into a balanced and contemporary conception of numeracy. Educators are considering the need for classrooms to target individual learning needs and differentiated curricula (e.g., Brown, 1999; Rose & Meyer, 2002; Small, 2009). It can be argued that *Equity* becomes a core dimension of numeracy that emphasises the importance of numeracy learning that is accessible to all students and that equips them to question assumptions and use mathematics in an analytical and critical manner to make decisions, resolve problems, and ultimately challenge inequities in society.

The five teachers in this study held student-centred beliefs about teaching and learning and this resulted in classroom environments in which learning was shared and individual approaches were valued. In many ways the teachers' beliefs and practices aligned with the view that mathematics education is less of an instructional process and more of a socialisation process (Resnick, 1989; Schoenfeld, 1992). In the four case studies, students were provided with opportunities to work together, to share ideas, and to consider a variety of viewpoints. This resulted in students being supported to access tasks at their own level and to extend themselves by working with others. In all of the cases, classroom cultures were being established by the teachers where students were being supported to develop the dispositions that form foundations with which to go on to consider mathematics and its implications in social, political, and economic contexts.

Boaler (2008) in describing "relational equity" argues for a pedagogical approach that encourages "respect for others' ideas, commitment to the learning of others, and learned methods of communication and support" (p. 174). The first category of *Equity* as defined in this study, "Personal social engagement," aligns well with Boaler's "relational equity" and was found throughout all of the cases in the classroom observations. Evidence of this category, based upon the student interviews, was not as strong. The nature of the interview context, with students discussing their learning after completion of the unit of work, meant that the learning they demonstrated in the interview was largely focused on outcomes of the unit of work, rather than their dispositions to engage with others during the unit of work.

In the cases of Samantha, and Ange and Jen, the dimension of *Equity* featured beyond the level of encouraging "Personal social engagement" with the ways students were enabled to work together. This is not unexpected. First, in the case of Samantha, students were constantly challenged to use their mathematical knowledge in a critical manner when considering approaches to tasks and problem solving strategies. They were always questioning assumptions to uncover more about the questions posed and to consider ways of going about their learning. Samantha, herself, was motivated to empower her students for their futures as learners and as citizens and therefore always exposed her students to authentic contexts that at times involved social issues in life or in the media. Although only Student 13 evidenced learning within the *Equity* dimension to a high level during the interview setting, three of the six students evidenced their "Personal social engagement."

In the case of Ange and Jen, the unit of work itself, exploring the nature and impact of poverty, was highly socially, culturally, politically, and economically relevant. Throughout the unit students became "Aware of issues" that impacted upon the nature of poverty in the countries in which they were studying and it was the data they obtained and the graphs they created that enabled them to understand these issues. In putting together a major assignment and presenting these to the class, students evidenced, both within the classroom observations and during their interviews with the researcher, their abilities to "Consider and express a variety of viewpoints" based upon their work. Three students in this case also made critical interpretations of the mathematical information they gleaned and went on to "Challenge inequities" that they had identified during the unit of work.

The student learning categories developed and described for this dimension (Table 3.7) were helpful in considering the relevance of the *Equity* dimension for today's students. The categories enabled close examination of how teachers might support students beyond the first category where students are able to question and make decisions based on own and others' mathematical strategies in the social context of the learning environment, to consider critically the relationship between mathematics and social, cultural, and political consequences.

The framework developed and verified in this study captures the rich tapestry of numeracy and its capacity to empower teachers to equip their students with high quality numeracy learning, reflecting the views of Ernest (1998):

> What is needed is differentiated school mathematics curricula to accommodate different aptitudes, attainments, interests, and ambitions. Such differentiation must depend on balanced educational and social judgements rather than exclusively on mathematicians' views of what mathematics should be included in the school curriculum. (Ernest, 1998, p. 273)

10.4.6 Relationships between and within the dimensions

Numeracy, conceptualized in this thesis through five dimensions of practice, is a complex construct. The five dimensions are not considered to be totally distinct from each other, but rather have complex interconnections and complementarities in combining to support the development of students' numeracy capabilities. Although these interconnections do not form the focus of this thesis, in Section 10.4.6.1 two examples are discussed that demonstrate the complexities involved and the potential for further research that would prove valuable in understanding the complex relationship between the five dimensions. Further, in Section 10.4.6.2, the notion of hierarchy in relation to each of the five dimensions is considered on the basis of the results.

10.4.6.1 Interconnections between the dimensions

The dimension of *Reasoning* is closely related to the *Mathematics* dimension, in that students will not achieve high level learning outcomes within *Mathematics* without the same capacities to reason at a high level. It may therefore be considered that these two dimensions might not be distinguishable and in the case of Alice, the two dimensions strongly mirrored each other in terms of student learning outcomes. In the other cases, however, although there were also close connections between these two dimensions in terms of student learning, outcomes were not identical. The case of Ange and Jen demonstrates the distinctions that can be drawn out between the dimensions of *Mathematics* and *Reasoning* when all of the five dimensions of numeracy are valued equally, and with targeted learning outcomes, as there were in this case, for both contextual and mathematical learning. Fundamentally, reasoning is a capacity that is relevant to all learning, and it is therefore not surprising that there are connections between students' learning outcomes for the *Reasoning* dimension and the other hierarchical dimensions, as all learning is underpinned by some thinking.

Only one student, Student 18 in the case of Samantha, evidenced learning at the highest level of the *Reasoning* dimension, "Create." Based upon the student interviews, this category, in which students demonstrate the ability to put together ideas from their learning to create new ideas, was only found in Student 18. This student was very engaged with tasks within the learning environment and also evidenced all of the six categories of the *Attitude* dimension. Although no firm conclusions can be drawn, as only one student is involved, it would not be surprising if a highly positive attitude toward numeracy learning would be important for a propensity and capacity to be creative.

In conceptualising a framework with five dimensions of practice, it was not intended that they should be considered completely distinct and disconnected areas of numeracy. Although the interconnections are complex and important, by looking closely at teacher beliefs and practices and student learning for each of the five dimensions, as has been done in this study, more can be learned about the nature of numeracy.

10.4.6.2 The hierarchical nature of the five dimensions of practice

As has been discussed throughout Sections 10.4.1 to 10.4.5, the categories developed and described to reflect the main aspects for each of the five dimensions of numeracy were found to be helpful in exploring the nature of each dimension within the classroom learning environment both for teachers and for students. For the dimension of *Mathematics*, the categories were drawn from the work of existing research in the fields of graphing and measurement, and based upon this research this dimension was assumed to be developmentally hierarchical in nature. Given that it is such research that informs curriculum as well as teachers' planning in mathematics, it is not unexpected that the students' learning outcomes in this study supported the hierarchical nature of the *Mathematics* dimension. The teachers in each case were thoughtful in ensuring a progression of mathematical concepts and ideas in the tasks they chose in order to build student understanding. Similarly, for the dimension of *Reasoning*, the hierarchical nature was supported in the four case studies where teachers were often explicitly teaching and assessing the thinking strategies of their students.

For the dimension of *Attitude*, its non-hierarchical nature was borne out in the results of each case study. Of particular interest was the finding that where students displayed five or six of the categories of the *Attitude* dimension, their learning within the other dimensions of numeracy was usually strong and in Section 10.4.3 this relationship was discussed and examples provided. For the two dimensions, *Context* and *Equity*, the results of the study support the assumption that there is some sense of hierarchy or increasing complexity in the way in which students are able to integrate and relate their mathematical understanding to contexts, and in the case of *Equity*, social, economic, and political consequences.

Although the data in this thesis did not challenge the assumed hierarchical nature or otherwise of the dimensions, in taking the framework forward, this aspect together with issues of interconnections between the dimensions may be an interesting focus of further research.

10.5 Relationship to and implications for curriculum design

The framework for numeracy with five dimensions of practice was used with both teachers and students and points to a more complete way to design numeracy curriculum in the future. This section considers the relationship of the framework to curriculum, both locally and nationally, and considers implications of the research findings for curriculum design both at the classroom and school level, as well as more broadly.

Although this study was situated within a transdisciplinary curriculum context, numeracy, and its relevance across the curriculum, remains at the fore of recent national documents and in the context of the introduction of a national curriculum in Australia.

> While the major responsibility for the enhancement of numeracy outcomes resides within school mathematics, numeracy outcomes for students will be enhanced by an across the curriculum focus premised on the principle that numeracy education is everybody's business. (DEEWR, 2008, p. 6)

> Numeracy knowledge, skills and understanding need to be used and developed in all learning areas. Initial and major continuing development of numeracy will be in mathematics but the national curriculum will ensure that this competency is used and developed in all learning areas. (National Curriculum Board [NCB], 2009b, p. 12)

The *National Numeracy Review Report* emphasises students' capacities to use mathematics in a variety of contexts both in and out of school as the core component of numeracy. The report also discusses the role of strategic thinking, fostering positive student motivation, and the importance of addressing the numeracy needs of particular student groups (DEEWR, 2008). Although not as specifically articulate, these areas broadly align with the five dimensions of numeracy conceptualised in this thesis.

During 2007, with a change in the state government ministerial portfolio for Education, the local reform curriculum was re-branded as the *Tasmanian curriculum* and resulted in a move back to discipline-based learning areas. The restructured *Tasmanian curriculum* (D*o*ET, 2007a) still explicitly acknowledges the centrality of thinking and teaching for understanding within every learning area. It remains underpinned by the same values, purposes, and goals that were formulated in 2002 to guide quality education for all students (D*o*ET, 2002; D*o*ET, 2007a). These goals closely align with the national goals for young Australians (MCEETYA, 2008a) and place value on developing students' capacities to communicate, reason, question, make decisions, and solve complex problems, as well as to be creative and well prepared to participate actively in democratic society as global citizens.

Tasmanian curriculum documentation that describes the teaching and learning of "mathematics-numeracy" continues to emphasise the relevance of numeracy across the curriculum and its importance for developing active, life-long learners.

> Mathematics–numeracy is a mandated area of study in the Tasmanian curriculum from Kindergarten to Year 10. The skills, knowledge and understanding acquired in mathematics are central to the learning and development of students and impact on learning in all curriculum areas. Developing proficiency in mathematics–numeracy enables students to take their place in Australian society as confident problem solvers, critical thinkers, confident communicators, active participants and lifelong learners. (D*o*ET, 2007b, p. 5)

The Essential Learnings message was a student-focused one and more about teaching for numeracy as opposed to teaching numeracy. This can continue to be promoted within the restructured *Tasmanian curriculum* and the results of this study provide insights into how this might be supported.

The numeracy framework expounded in this thesis has the potential to drive curriculum design at a whole school level or at the classroom level. It is a framework with which teachers can collaboratively or individually consider their own practice and the learning experiences that they want to provide for their students. If we understand what numeracy looks like, as has been shown in this thesis, then curriculum can be designed, and learning experiences for students can be developed, with the aim of achieving outcomes across all dimensions of the framework.

In terms of the Mathematics curriculum, previous documents such as the K-8 Guidelines (DEAT, 1993) focus on the strands of mathematics whilst current documents (DEEWR, 2008; NCB, 2009a) place value on the importance of numeracy but do not explicitly provide examples of what this might look like in the classroom learning environment. This thesis has provided clear examples of how numeracy with respect to each of the five dimensions – *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity* – can be developed.

10.6 Implications for professional learning

The conceptual framework for numeracy developed in this study provided an opportunity to be explicit about the dimensions that contribute to effective numeracy practice from both a teaching and a learning perspective. The outcomes of the research have shown that it is possible to describe teachers' beliefs and practices according to the five dimensions as well as to look closely at student learning in these areas.

If the conceptual framework for numeracy is considered to inform effective practice, as argued in the findings of this study, it is important to consider how we can use it to inform teaching. Professional learning opportunities for teachers are recognised as being key. Learning opportunities for teachers are one of six areas highlighted by Borko et al. (2003) as having the potential to influence school capacity and student learning in the context of reform. Further, Sowder (2007) purports the value of engaging teachers with research through professional learning that has a focus on "building teachers' capacities including strong content knowledge and a variety of ways to build practice on children's thinking and clear learning goals" (p. 215).

In this study the framework itself was not used with the teachers or the students, but rather the teaching and learning that occurred within the natural classroom learning environment was analysed based upon the five dimensions of practice.

The outcome of this and the way that the framework is able to provide insight into the work of teachers and students across all the five dimensions has potential for professional learning with teachers in two ways. First, in being able to be explicit about the relationship among elements of teaching practice and how it promotes the development of numeracy, teachers might better understand their own practice and how their beliefs, planning, teaching strategies, and assessment practices have the power to influence and support student learning in this area. Second, in showing how student learning can be exemplified and critiqued across each of the five dimensions, the framework has the potential to support teachers to understand better the learning of their students and also to target specific learning outcomes.

In enabling teachers to examine their numeracy teaching and learning in depth, both from their own perspectives and from the perspectives of their students, teacher "pedagogical content knowledge," and "knowledge of learners and their characteristics" would be enhanced. Supporting teachers to build these areas of knowledge as described by Shulman (1987) can only be of benefit. Shulman's framework has been used in mathematics education in a variety of contexts for assessing teachers (e.g., Mayer & Marland, 1997; Watson, 2001). Shulman's domains have also been used to design professional learning programs for teachers and to capture the breadth of teachers' experiences, an example of which is detailed by Watson, Beswick, Caney, and Skalicky (2006).

Borko, Mayfield, Marion, Flexer, and Cumbo (1997) found that professional development experiences that provided opportunities for teachers to explore new instructional strategies and ideas in the context of their own classroom practice were among the most effective for promoting and supporting teacher change. The framework for numeracy used in this study, with examples, could be used with teachers to reflect upon their own practice and such reflection is essential to building, maintaining, and developing the capacities of teachers (Day, 1999). The thesis has also demonstrated the capacity for the framework for numeracy to provide an in-depth look at student work and student learning and to provide a lens to support teacher reflection. Supporting teachers to reflect is an important element of professional learning (e.g., Borko, Davinroy, Bliem, & Cumbo, 2000; D'Ambrosio, Harkness, & Boone, 2004; Hill, 2002; Martinez & Mackay, 2002).

The tasks that teachers select to use with their students imply much about the importance or otherwise to them of the dimensions of numeracy. A worksheet of algorithms to solve, for example, imparts a message about mathematics involving memorisation and rote learning. The use of open-ended and inquiry tasks demonstrates that teachers place value on student reasoning and using a variety of strategies to solve problems. It may also demonstrate the role of context within numeracy. In looking at tasks themselves and their relationships to the five dimensions of practice, teachers could be encouraged to consider the messages that are implied about what mathematics is about.

In times of reform, teachers themselves may change specific behaviours or teaching strategies, while still missing key understandings about their students' ways of thinking and working and their own pedagogical practices (Spillane & Zeuli, 1999). Consequently, Schorr, Warner, Gearhart, and Samuels (2007) assert that when considering professional learning, "parallel assumptions regarding students' knowledge acquisition and learning must also be applied to teachers' knowledge acquisition and learning" (p. 432). The outcomes from this research study have demonstrated that both teachers' beliefs and practices and student learning can be examined in depth according to the same framework for numeracy. It therefore provides a unique opportunity to be used in professional learning as Schorr et al. encourage – with teachers examining both assumptions about their students' knowledge and learning as well as their own.

10.7 Limitations

In revisiting the nature of the research, the question of its quality and potential impact within the education research community is important. This study was planned and implemented with sensitivity to the potential limitations of qualitative case study research and this was reflected in both the design of the study and the choice of methods selected (Chapter 4). The research considered all aspects of the classroom, incorporating both teachers and students across three school settings. Interview, observation, document, and photographic data were collected and analysed in order to consider the research questions and represent the findings of the research comprehensively. The research was evaluated against five criteria:

credibility, *dependability*, *transferability, relevance*, and *communicability* (cf. Section 4.11). These criteria were informed by both the qualitative education research community in general and the mathematics education research community more specifically regarding what constitutes quality research (Fritzell, 2006; Lincoln & Guba, 2000; Malara & Zan, 2002; Schoenfeld, 2002).

As emphasised by the NCTM Research Committee (2009), this thesis also argues that "numerous kinds of high-quality research are needed for multiple uses and multiple users in mathematics education" (p. 237). This ideal, supported by Schoenfeld (2000), was a driving force in developing the research framework and in designing the research process. In making methodological choices in the research, it was the research questions themselves that drove the design and methodologies selected. Qualitative case study research has the ability to "evoke images of the possible" (Shulman, 1983) and case studies play an important role in developing "systemic understanding of patterns of practice in classrooms where teachers are trying to enact reform" (Spillane & Zeuli, 1999, p. 20)

As discussed in Section 10.5, the nature of the curriculum changed during 2007 when political pressures resulted in the curriculum framework moving back to a more traditional discipline-based construction. The curriculum shift might be perceived as a limitation of the study in terms of its relevance, and certainly had the potential to undermine the research outcomes. It did, however, become a strength of the research, through enabling examination of the nature of numeracy both within the reform-learning environments in which the data were collected and also by considering numeracy more abstractly irrespective of curriculum construction. Whether values-based or disciplined-based curricula documentation exists, teachers in classrooms remain influenced by their own beliefs about the nature of numeracy. Equally, as discussed in Section 10.5, the cross-curricular relevance of numeracy remains, together with the mathematics education reform ideas of the importance of conceptual understanding, the centrality of thinking, and the importance of authentic learning experiences for students in order to support them to develop a set of skills and capabilities that will equip them for their future lives as citizens.

It is acknowledged that this study was situated within the middle years of schooling. It is recognised that examining numeracy in other contexts across levels of schooling and different curriculum and cultural contexts would be most valuable and these possibilities are discussed further in Section 10.8.

10.8 Directions for further research

The very nature of the research, being a qualitative collective case study, required that clear boundaries were set and decisions made in order to investigate the research questions. A number of potential directions for further research have been generated by this study beyond the boundaries and scope of the study. Three main areas of further research are considered in this section. First, use of the conceptual framework to examine numeracy teaching and learning at other levels of schooling, in other curriculum contexts, and in other social and cultural settings is considered. Second, possibilities for profiling teachers according to the five dimensions and in a more quantifiable way, is explored. Third, implications of the research for the development of assessment that more clearly aligns with a multi-dimensional perspective of numeracy, is shared.

10.8.1 Numeracy at other levels of schooling and in different curricula, social, and cultural settings.

This study was set within the middle years of schooling (Grades 5-8) in the state of Tasmania, Australia. Examining the role of each of the five dimensions of numeracy within other levels of schooling would be of interest. In the early years of schooling it might be anticipated that the dimensions of *Reasoning*, *Attitude* and *Context* are emphasised, as young children are encouraged to explore mathematical concepts and their relevance to the children's everyday activities. In senior secondary schooling, however, the dimensions of *Mathematics* and *Reasoning* might be emphasised over the other three dimensions as students undertake pathways to higher level mathematics. Creating an awareness of which dimensions are at the fore at different stages of schooling would provide opportunities for educators to investigate ways in which teachers and students can be supported to consider the relevance of all of the dimensions at every level.

As presented in Table 2.1, at the time of the study, Australia's curriculum landscape presented a diverse range of perspectives from a strongly discipline-based focus formulated around key learning areas (as was the case in New South Wales and the Australian Capital Territory) to States where more integrated approaches were, and remain, the focus (for example, in South Australia and Queensland). The framework for numeracy explored within this study has the potential to be applied to other teachers and students and within other curriculum contexts. Investigating the prevalence of each of the five dimensions in different curricula would enable an increased awareness of what is valued and provide a foundation for recommendations for change.

Considering the teaching and learning of numeracy more broadly and its positioning within a diverse range of social and cultural contexts, both within Australia and internationally, would be of interest. It may also contribute ideas to how we can best support student learning relevant for individuals. As a framework that has brought *Equity* in as an equal and supporting partner to the other four dimensions, it may support teachers and researchers who already know it is important and might be lacking in other perspectives of numeracy. In being able to be explicit about *Equity*, teachers and researchers can be supported in advocating the importance of equity for all students, for example, indigenous cultures, rural and remote students, and students from low socio-economic backgrounds. Examining the teaching and learning of numeracy according to the five dimensions may hold educators more accountable for dealing with all aspects of numeracy and for promoting or discouraging inequity. In addition, outcomes might inform curriculum design for teachers in differing cultural contexts that support the practices that are inherent in what they are doing by being able to be more explicit about them.

10.8.2 Profiling teachers and students according to five dimensions of numeracy.

In considering the implications of the research for professional learning (Section 10.6), ways of making the conceptual framework practical and applicable for teachers are of interest. The framework has the ability to be explored further as a means of profiling teachers and students and providing them with feedback on their respective teaching and learning. This study has examined both teachers and students and further work in this area would enable powerful comparisons to be made among the dimensions as evidenced by teachers and their students. Not only can teachers use the framework themselves, but also they could assess their students in relation to a particular activity, a unit of work, or even a term or year's work. Further work could explore this in a more quantifiable way.

The author has begun to think about this area, with the case of one of the teachers, Alice, who participated in the study, already reported (Skalicky, 2008). Alice's beliefs about numeracy according to the five dimensions were described and a radar chart was used as a means of visually representing Alice's numeracy pedagogy at a point in time and in the context of a particular unit of work, particularly the degree to which each dimension was evidenced. Skalicky (2008) recognised that teacher pedagogy is a changing construct and therefore this work has potential to be taken further and developed in a more rigorous way. In addition, initial thinking about what a rubric for evaluating teachers against each of the five dimensions might look liked based on broad perspectives that aligned with Bloom (1956) was explored and this could be a starting point for more work in this area. It would require a rigorous approach, as was done in this study with the students, where categories of learning were described across each of the dimensions of practice. Such research would provide a practical component to go alongside the framework in the future and assist in making the framework more usable for teachers, educators, and researchers.

In relation to student learning, the categories developed for each dimension and used with the students in this study could be taken and used to represent student learning visually, where the data might be translated into visual or graphical representations. This would provide a practical snapshot that would enable teachers to look at for a piece of work, or a collection of learning. It is noted that in this study the dimension of *Mathematics* was considered only in relation to graphing and measurement as these were the focus of units of work planned and implemented by the case study teachers. This provides a model for examination of other strands of mathematics.

10.8.3 Implications for assessment

It was evident from the beginning of the research that assessment was an important aspect of the teaching and learning cycle that intersected with the curriculum reforms. This came through in the broader school contexts, as well as with the individual teachers and their students. Teachers were grappling with the reporting and assessment implications and mandates that were being implemented alongside the curriculum changes. Assessment is one of the areas of research interest in the last ten years that sits alongside the reform movements both broadly within teaching and learning and within the mathematics education context. (e.g., Black, Harrison, Lee, Marshall, & Wiliam, 2003; Wiggins, 1998; Wiske, 1998).

Educators interested in the bringing together of disciplines for the purpose of teaching concepts in a contextual framework are grappling with the issue of how best to assess understanding within rich contexts (Frykholm, 2002). Changes in curriculum and accompanying pedagogy need to be matched with appropriate changes in assessment practices (Resnick & Resnick, 1985) and this poses considerable challenges for the devising of assessment tasks in a transdisciplinary framework (Skalicky, 2004). As activities become more complex, the need to assess many aspects of the expected outcomes becomes important. To have a different task for each area of interest, however, may create a plethora of instruments or questions for students to answer.

In examining assessment issues within the context of reform-based curricula, it is necessary to look further at the issues surrounding the assessment of multiple objectives. "The assessment of multiple solutions or multiple paths to a single solution, will occur only when we have an approach to assessment that has the same principles as contemporary approaches to mathematics education" (Van den Heuvel-Panhuizen & Becker, 2003). This study has demonstrated that different dimensions of numeracy can be evaluated for student learning. The potential exists therefore to take this further and use these findings to consider how best to assess the multiple dimensions that exist within the types of tasks that were used by teachers in this study. Studies such as Kastberg and Ambrosio (2004), Skalicky (2005; 2006), and Van den Heuvel-Panhuizen and Becker (2003) have begun to consider some of the issues surrounding the assessment of contextually-rich problems and the assessment of multiple learning objectives.

10.9 Conclusion

Interest in the field of numeracy has spanned 50 years. The research presented in this thesis was motivated by an interest in the influence of curriculum reform on the teaching and learning of numeracy. The thesis has demonstrated the potential for numeracy learning when teacher beliefs and practices align with an understanding of numeracy as a complex construct that extends across foundational mathematical concepts and skills, strategic thinking, disposition, recognition of context, and critical practice. It establishes that middle years' numeracy learning could be strengthened through an increased emphasis on all five dimensions as proposed in this thesis – *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity*.

The research, in being inspired by social constructivist thought, values multiple aspects of knowledge construction. The study was theoretically framed and thoughtfully designed to explore the positioning of numeracy through multiple lenses and from the perspective of both teachers and learners. A literature review conducted by Jones (2004) on the impact of twenty years of research in mathematics education within Australia, points to such a shift in mathematics education research:

> Mathematics education research in the next 20 years will take a more multifaceted yet integrated approach to learning and teaching: one that uses multiple lenses to look at teachers and learners from cultural, social, political, and psychological perspectives. In essence, the research problematique will unit objects of research like equity, access, and context with extant objects like learning and teaching … future research designs will need to accommodate theoretical frameworks that examine equity and access and incorporate cultural, social, political, and psychological perspectives. (p. 372)

Numeracy conceptualisations such as those articulated by Steen (2001) and the AAMT (1997), as well as curriculum interpretations of numeracy within transdisciplinary frameworks (DoET, 2002; QSCC, 1999), enabled the researcher to identify five dimensions of practice that are deemed important in developing numeracy for the future. Through providing a comprehensive synthesis of the literature related to both curriculum and numeracy, and relevant to the development of a national curriculum (NCB, 2009a), this study contributes to the current theoretical knowledge in this area and adds to the limited existing knowledge about how numeracy is shaped and enacted in reform-based learning environments. Furthermore, the framework that has been developed through the study presents a new way of examining teachers' beliefs and practices, as well as student learning, and contributes to the practical knowledge in this area. It is anticipated that the framework may prove useful to other researchers and teachers in interpreting and understanding the teaching and learning of numeracy.

Chapters 6 to 9 provide vivid descriptions of four classrooms where teachers and students were engaging in numeracy. These four different glimpses of what numeracy might look like in classrooms if numeracy is conceptualised in this way have demonstrated the variation in pathways to successful learning outcomes within a multi-dimensional approach. By investigating these learning environments in depth, using multiple sources of data, the research has extended understanding of what numeracy is and what it has the potential to be.

It seems appropriate that this thesis concludes by taking a forward thinking view as shared by Samantha.

> *I think in ten years time we will see a team of teachers and educators who are using numeracy much more as an integral part of their whole day, the whole week, the whole term – planning and realising that it can be incorporated everywhere else. My hope is that we will continue to move forward from where we are now.* (Samantha, 14/12/05)

The relationship between numeracy and mathematics remains a highly debated topic for experts in mathematics education and this is an issue that has not been resolved and is not likely to be in the near future (NCB, 2009a). As Australia moves to frame a national mathematics curriculum that acknowledges the importance of mathematics as a discipline, as well as finding its place in a cross-curricula context, the challenges for the conceptualisation and positioning of numeracy remain. Within this context, preparing students to become active, thinking citizens within democratic society is becoming increasingly valued. Noss (1998) reinforces the view that new numeracies may continue to evolve to represent the ever changing social and economic needs of society. The framework for numeracy that has been used as a lens with both teachers and students in this study has uncovered a broader and richer picture of what is going on in classrooms that has not been uncovered previously. Such a framework, bringing the dimensions of *Mathematics*, *Reasoning*, *Attitude*, *Context*, and *Equity* together, has the potential to support teachers and students in looking toward the next 50 years of numeracy.

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Appendices

Appendix A - Queensland School Curriculum Council's Numeracy Framework

Appendix B – Semi-structured interview schedule: Phase 1 teacher interviews

Appendix C – Semi-structured interview schedule: Phase 3 student interviews

Appendix D – Semi-structured interview schedule: Phase 4 teacher interviews

Appendix E – Ethics approval letter from the University of Tasmania

Appendix F – Ethics approval letter from the Department *of* Education, Tasmania

Appendix G – Data collection timeline for the research, including ethical requirements for each phase

The following appendices are contained in the CD-ROM that is included at the back of the thesis:

Appendix H – Teacher interview transcripts from Phase 1 and Phase 4 teacher interviews; and including Principal interview transcripts

Appendix I – Student interview transcripts from Phase 3 of the study

Appendix J – Researcher reflective note transcripts related to Phase 2 of the study

Appendix K – Coding of the teacher interviews: Example of Alice

Appendix L – Scanned copies of classroom observation records from Phase 2 of the study

Appendix M – Coding of the student interviews

Appendix A – Queensland School Curriculum Council's Numeracy Framework (QSCC, 2001. p. 7)

<table>
<tr>
<td>Resources for FOUNDATIONAL PRACTICE
Uses the knowledge and skills of mathematics</td>
<td>Resources for LINKING PRACTICE
Uses strategic processes and strategies</td>
</tr>
<tr>
<td>Entails developing and drawing upon a rich web of interrelated mathematical concepts and skills developing.
Number sense: appreciation and use of numbers and number relationships
Data sense: use of statistical and measurement information
Spatial sense: use of spatial, visual and location information
Pattern sense: appreciation of patterns
Formula sense: use of formulae and algebraic expressions.
The emphasis is on the concepts and skills associated with the strands of the mathematical syllabus:
Number
Measurement
Space concepts and visualisation
Chance and data
Patterning and algebra.</td>
<td>Entails developing and drawing upon a repertoire of strategic processes and strategic skills to enhance transfer from one context to another.

The emphasis is on thinking mathematically using a range of:
Strategic processes
Planning, organising, analysing, synthesising, visualising, justifying, generalising, evaluating, reflecting, communicating, problem-solving, explaining, validating, inferring, comparing, classifying and estimating.
Strategic skills
exhibits familiarity with a wide range of contexts, and comprehends similarities and differences between them
identifies key features of problem
identifies features of contexts which may be applied to mathematical procedures
makes assumptions about the context and relates to previous experiences to guide the selection of appropriate mathematical concepts and techniques
asks clarification questions
poses sub-questions
decides precision of solution required.</td>
</tr>
<tr>
<td>Resources for PRAGMATIC PRACTICE
Applies idiosyncratic knowledge, strategies and affects</td>
<td>Resources for CRITICAL PRACTICE
Understands the intent of the problem or investigation</td>
</tr>
<tr>
<td>Entails developing and drawing upon personal approaches to choosing and using mathematics, in context with a disposition for their use.

The emphasis is on idiosyncratic strategies, proficiency with mathematics, and affective components. The numerate person:
chooses and uses mathematical tools appropriate for purpose
uses mathematics confidentally and fluently in familiar and unfamiliar contexts
takes risks mathematically – willing to 'have a go'
uses a range of informal and idiosyncratic strategies based on features of context
adapts approaches to context and perseveres
demonstrates orientation towards transferring between contexts
checks for reasonableness of results in relation to context
demonstrates working with others
demonstrates a positive response to the use of mathematics in a variety of contexts</td>
<td>Entails developing a critical understanding of mathematics.

The emphasis is on knowing what mathematics is and how it can be used. The numerate person:
recognises when mathematics may be useful
recognises which strategic processes and strategic skills are appropriate
demonstrates an orientation to use mathematics in an analytical and critical manner
interprets and analyses mathematics presented in a range of media
understands perspective (bias) in mathematical representations
understands how mathematical representations can manipulate people's views
uses appropriate mathematical ideas to communicate with others
recognises the mathematical structure that underlies problem solving and investigations
uses initiative and creativity</td>
</tr>
</table>

Appendix B – Semi-structured interview schedule: Phase 1 teacher interviews

Phase 1 teacher interviews

- *How long have you been teaching and across what grades? What grade are you teaching at the moment?*
- *How are you finding implementing the Essential Learnings curriculum into your classroom*
  - *Have you found it has changed your teaching practice? How?*
  - *What do you see are the main benefits of the Essential Learnings?*
  - *What disadvantages stand out for you?*
- *The Essential Learnings provide opportunities to integrate learning.*
  - *Can you describe what an integrated or transdisciplinary curriculum means for you?*
  - *In what ways do you incorporate this into your teaching program?*
  - *Any specific examples?*
- *What types of tasks do you use in your teaching? e.g. Do you use structured/ inquiry learning/open-ended investigative tasks in your teaching? [connected to the previous question]*
  - *What are the benefits of this way of teaching?*
  - *What are the challenges?*
- *What are some of the ways that you find useful to assess students' numerical understanding? Any specific examples?*
- *How are you including numeracy and mathematics into your program?*
  - *Do you see them as different or the same?*
  - *Can you give some practical examples ?*
- *What is your view on where numeracy and mathematics are best placed in the Essential Learnings framework?*
- *Lynn Arthur Steen in his book* Mathematics and Democracy: the case for quantitative literacy *discusses the importance of numeracy for today's citizens and he defines it as including the following elements : confidence with mathematics, cultural appreciation, interpreting data, logical thinking, making decisions, mathematics in context, number sense, practical skills, prerequisite knowledge, and symbol sense. For Steen quantitative literacy is "inseparable from its context" and requires that the skills of arithmetic, data, chance, computing, modelling, statistics, and reasoning are all developed within real-world contexts?*
  - *What do you think of Steen's views?*
  - *How do you think this might sit with the Essential Learnings?*

Appendix C – Semi-structured interview schedule: Phase 3 student interviews

Alice's Students: Interview Questions

- *Tell me everything you know about graphing·*
  - *What are some of the main things you learnt about graphing? What surprised you?*
  - *Did you talk about any of the graphing work at home with your family? What sorts of things did you talk about?*
  - *Have you learnt about data and graphing before? Where? What did you already know about data and graphs?*
- *Tell me about some of the graphing work you have included in your journal.*
  - *Choose some graphing activities and tell me about them?*
  - *Graphs tell a story, can you tell me the story of this graph?*
  - *Tell me how you organised all the information? What did you do with the numbers? Why did you decide to represent your graph this way? Use this scale?*
  - *Why do you think it is helpful to represent the data in this way, graphically? Why not just write down all the statistics in words?*
  - *Why do you think it is helpful to use graphing to describe things in the real world? Can you imagine and tell me how the answer to your question might have looked without graph?*
- *[Alice] told me that she chose to do the graphing work in numeracy after your unit of work on Australia.*
  - *Can you tell me about that unit of work What you did? What you learnt?*
  - *Did you do much graphing of statistical information in that unit of work?*
  - *What difference did it make putting the graphs in? How did it help you understanding the topic?*
  - *Why was it important to use mathematics in that project?*
  - *Have you gone back and talked about any of the work on Australia since you finished the graphing work? Tell me about that. How did the graphing skills help?*
- *Have you been taught about representing data in tables and graphs before? Where? How did this prior learning help you in this numeracy work?*
  - *Do you think it is different learning about data and graphing in numeracy to using them in a unit of work like the one on Australia?*
  - *What do you think about learning to do graphs here in your numeracy time compared to using those mathematical skills in a unit of work?*
- *You've finished your graphing work.*
  - *Is there anything more you would have liked to have done? Anything you would have done differently or like to change?*
- *Reflective comment*
  - *Now you know all these things about data and graphing, what would you like to learn more about or do with this information? What areas of data and graphing would you like to go on and look at if you had the chance? [can you justify thinking? Explain?]*

Ophelia's Students: Interview Questions

- *Tell me about your project:*
  - *What are some of the main things you learnt aboutmeasurement? What surprised you?*
  - *Did you talk about your work/learning at home with your family? What sorts of things did you talk about? Had you learnt about your topic before? Where?*
  - *Have you learnt about measurement before? Where? What did you already know about measurement?*
- *Tell me about measurement information you have included in your work:*
  - *Choose some measurement tasks and tell me about how would went about solving them? Choose some measurement information and tell me about it?*
  - *Tell me how you organised all the information? What did you do with the numbers? Why did you decide to put this information in a table? Use these sorts of measurements?*
  - *Why do you think it is helpful to use measurement to describe things in the real world? Can you imagine and tell me how the answer to your question might have looked without the measurement information?*
- *Have you been taught about measurement in other classes? In previous grades? Where? How did this prior learning help you in this unit of work?*
  - *Do you think it is different learning about measurement by doing lots of measurement questions in numeracy to using it to also explore topics like you did here? And like you did along the way exploring lots measurement in real-life like the body measurements, weighing the objects, the bus problem?*
  - *You have also been publishing a book this term and Mr [ ] came and spoke to you about publishing as well. How did you use what you learnt in the measurement unit to create your book? Can you tell me how it helped you make decisions? [size and shape of book, margins, font size, illustrations?]*
- *You've finished your measurement unit of work now.*
  - *Is there anything more you would have liked to have done?*
  - *Anything you would have done differently or like to change?*
- *What do you think your measurement work that you have shared shows me that you have learnt?*
  - *Now you know all these things about the topic you studied, what would you like to learn more about or do with this information? What areas of measurement would you like to go on and look at if you had the chance? [can you justify thinking? Explain?]*

Samantha's Students: Interview Questions

- *Tell me about your project:*
  - *What are some of the main things you learnt about your [big question]? What surprised you?*
  - *Did you talk about your topic or question at home with your family? What sorts of things did you talk about? How did your family help you with your project?*
  - *Had you learnt about your topic before? Where?*
  - *Have you learnt about measurement before? Where? What did you already know about measurement?*
- *Tell me about measurement information you have included in your work:*
  - *Choose some measurement tasks and tell me about how would went about solving them?*
  - *You went and got lots of information about your question from websites, books, when you were doing your assignment. Tell me how you organised all the information? What did you do with the numbers? Why did you decide to put this information in a table? Use these sorts of measurements?*
  - *Why do you think it is helpful to use measurement to describe things in the real world? Can you imagine and tell me how the answer to your question might have looked without the measurement information?*
  - *Why was it important to use mathematics in this project? What difference did it make putting the measurement details in? How did it help you understanding the topic?*
- *Have you been taught about measurement in other classes? In previous grades? Where? How did this prior learning help you in this unit of work?*
  - *Do you think it is different learning about measurement by doing lots of measurement questions in numeracy to using it to also explore topics like you did here? And like you did along the way exploring lots measurement in real life like the body measurements, the tables, [Samantha's] garden?*
  - *You have also been publishing a book this term and Mr [ ] came and spoke to you about publishing as well. How did you use what you learnt in the measurement unit to create your book? Can you tell me how it helped you make decisions? [size and shape of book, margins, font size, illustrations?]*
- *You've finished your measurement unit of work now.*
  - *Is there anything more you would have liked to have done?*
  - *Anything you would have done differently or like to change?*
- *What do you think your work and your big question shows me that you have learnt?*
  - *Now you know all these things about the topic you studied, what would you like to learn more about or do with this information? What areas of measurement would you like to go on and look at if you had the chance?[can you justify thinking? Explain*

Ange's and Jen's Students: Interview Questions

- *Tell me about your project:*
  - *What are the key things you learnt about your country? What surprised you, had an impact on you?*
  - *Did you follow the Live 8 or G8 summit outside of class time? How, newspaper, TV? Did you talk about it with your family?*
  - *Describe if and how this influenced your assignment?*
  - *Had you studied your country previously? Where?*
  - *Have you looked at the issue of poverty before? Where?*
- *Tell me about your data, the graphs and tables you have included in your assignment:*
  - *Graphs tell a story, can you tell me the story of this graph?*
  - *You went and got lots of information about your country from websites, books, when you were doing your assignment. Tell me how you organised all the information? What did you do with the numbers? Why did you decide to put this information in a table? This in a graph?*
  - *Why do you think it is helpful to represent the data in this way, graphically? Why not just write down all the statistics in words?*
  - *Why was it important to use mathematics in this project? What difference did it make putting the information in tables? graphs? How did it help you understanding the issue of poverty in your country? What do you think was the reason for comparing your country to Australia?*
- *Have you been taught about representing data in tables and graphs in other classes? In previous grades? Where? How did this prior learning help you in this unit of work?*
  - *Do you think it is different learning about data and graphing in mathematics class to learning about them in an integrated unit like this Live 8 one?*
  - *What do you think about using those mathematical skills here in this integrated unit compared to learning to do graphs in your mathematics class?*
- *You've finished your Live 8 unit of work and your major assignment.*
  - *Is there anything more you would have liked to have done? Anything you would have done differently or like to change?*
  - *Now that you have seen all the other presentations, did that make you think about what you have done in your assignment? How?*
- *What do you think your assignment shows me that you have learnt?*
  - *Now you know all these things about the country you studied, what do you think should happen to alleviate/reduce/get rid of poverty? Government level? Personal level?*

Appendix D – Semi-structured interview schedule: Phase 4 teacher interviews

Final Teacher Interview: Alice

- *Can you tell me your thoughts on the unit of work and how it went in relation to your original goals?*
  - *What were original goals? [Australia unit]*
  - *What was great?*
  - *Were there any surprises?*
  - *Anything you would have like to have done differently?*
  - *Or refined?*
- *What do you think the graphing unit of work and activities brought to the students' mathematical skills and understandings?*
- *How do you think these skills and understandings would have helped in the Australia unit, or units like this in the future?*
- *Where would you have liked to have taken the graphing work, if it wasn't the end of the year?*
- *As a teacher how do you feel you are progressing with situating numeracy in the Essential Learnings?*
  - *What would support you as a teacher to be able to move forward in this area?*
- *If we imagine say five or even ten years from now, where would you like to see numeracy/mathematics [in the curriculum]?*
- *What impact has being the EL's coordinator for the school had on your teaching practice this year?*

Final Teacher Interview: Ophelia

- *Can you tell me your thoughts on the unit of work and how it went in relation to your original goals?*
  - *What were original goals?*
  - *What was great?*
  - *Were there any surprises?*
  - *Anything you would have like to have done differently?*
  - *Or refined?*
- *What do you think the measurement activities brought to the students' mathematical skills and understandings?*
- *Can you tell me how you specifically planned for Thinking within the unit of work.*
- *How does the way you run your classroom contribute to the development of students' thinking abilities.*
- *As a teacher how do you feel you are progressing with situating numeracy in the Essential Learnings?*
  - *What would support you as a teacher to be able to move forward in this area?*
  - *If we imagine say five or even ten years from now, where would you like to see numeracy/mathematics [in the curriculum]?*

Final Teacher Interview: Samantha

- *Can you tell me your thoughts on the unit of work and how it went in relation to your original goals?*
  - *What were original goals?*
  - *What was great?*
  - *Were there any surprises?*
  - *Anything you would have like to have done differently?*
  - *Or refined?*
- *What do you think the measurement activities brought to the students' mathematical skills and understandings?*
- *Can you tell me how you specifically planned for Thinking within the unit of work.*
- *How does the way you run your classroom contribute to the development of students' thinking abilities.*
- *As a teacher how do you feel you are progressing with situating numeracy in the Essential Learnings?*
  - *What would support you as a teacher to be able to move forward in this area?*
- *If we imagine say five or even ten years from now, where would you like to see numeracy/mathematics [in the curriculum]?*
- *What did the culminating performance, the Big Question reveal to you about the students' understandings?*
- *What impact has being the numeracy coordinator for the school had on your teaching practice this year?*

Final Teacher Interview: Ange and Jen

- *Can you tell me your thoughts on the unit of work and how it went in relation to your original goals?*
    - *What were original goals?*
    - *What was great?*
    - *Were there any surprises?*
    - *Anything you would have like to have done differently?*
    - *Or refined?*
- *What do you think the numeracy skills of chance and data brought to the unit of work?*
    - *What was the purpose of the frequency distribution task and birth rate task to start with?*
    - *How did the chance and data enhance the girls' understandings of poverty?*
    - *How did it extend their mathematical skills and understandings?*
    - *How important do you think connections like these are? And Why?*
- *As a teacher how do you feel you are progressing with situating numeracy in the Essential Learnings? What would support you as a teacher to be able to move forward in this area?*
- *If we imagine say five or even ten years from now, where would you like to see numeracy/mathematics [in the curriculum]?*

Appendix E – Ethics approval letter from the University of Tasmania

Tasmania
DEPARTMENT *of* HEALTH *and* HUMAN SERVICES

Human Research Ethics Committee (Tasmania) Network

UNIVERSITY OF TASMANIA

Southern Tasmania Social Sciences Human Research Ethics Committee (HREC)
APPLICATION APPROVAL

To:	Dr Jane Watson School of Education University of Tasmania Private Bag 66 HOBART
From:	Amanda McAully (Executive Officer)
Date:	16 July 2004
Subject:	**H7988:** Quantitive literacy in a reform-based curriculum.

The Southern Tasmania Social Sciences Human Research Ethics Committee has recommended approval of this project. You are required to report immediately anything that might affect ethical acceptance of the project, including:

- serious or unexpected adverse effects on participants;
- proposed changes in the protocol;
- unforeseen events that might affect continued ethical acceptability of the project.

You are also required to inform the Committee if the project is discontinued before the expected date of completion, giving the reasons for discontinuation.

Ethics approval is subject to annual review, therefore not completing a report could affect the project's continuing ethics approval. Please submit your first report on this project by 15 July 2005. The Annual report form can be found on our website: http://www.research.utas.edu.au/rdo/ethics/human.htm

Important: If research on the project has finished, please complete the above form selecting the "Final Report" option, and return as soon as possible for audit purposes.

Amanda McAully (Executive Officer)

University of Tasmania
Research and Development Office
Private Bag 1 Hobart Tas 7001
Ph: 6226 2763
Fax: 6226 2765
Amanda.McAully@utas.edu.au

Appendix F – Ethics approval letter from the Department *of* Education, Tasmania

Tasmania

DEPARTMENT *of* EDUCATION

18 August 2004

Jane Watson
Faculty of Education
University of Tasmania
Locked Bag 66
SANDY BAY TAS 7006

Dear Jane

RE: QUANTITATIVE LITERACY IN A REFORM-BASED CURRICULUM

I have been advised by the Departmental Consultative Research Committee that the above research study adheres to the guidelines established and that there is no objection to the study proceeding.

Please note that you have been given permission to proceed at a general level, and not at individual school level. You must still seek approval from the principals of the selected schools before you can proceed in those schools.

A copy of your final report should be forwarded to the Director, Office for Educational Review, Department of Education, GPO Box 169, Hobart 7001 at your earliest convenience and within six months of the completion of the research phase in Department of Education schools.

Yours sincerely

Alison Jacob
DEPUTY SECRETARY
(SCHOOL EDUCATION)

116 Bathurst Street, Hobart GPO Box 169, Hobart, 7001, Tasmania, Australia
Telephone: (03) 6233 8011 *Facsimile:* (03) 6231 1576

Appendix G – Data collection timeline for the research, including ethical requirements for each phase

School & Teacher	Term 1 - 2005				Term 2 - 2005			Term 3 - 2005			
	Feb	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Tanglefoot Ange and Jen	Gain principal support	Invite teachers to participate	Initial teacher interview		Daily classroom observations		Parent consent for student interviews	Collect student work samples	Collect teacher planning & assessment records	Final teacher interview	
		Project details sent to parents					Student interviews				
Stringybark Samantha	Gain principal support	Initial teacher interview		Project details sent to parents	Daily classroom observations Attend collaborative planning and staff meetings				Collect student work samples	Collect teacher planning & assessment records	Final teacher interview
	Invite teacher to participate						Parent consent for student interviews				
							Student interviews				
Stringybark Ophelia	Gain principal support			Invite teacher to participate	Daily classroom observations Attend collaborative planning and staff meetings				Collect student work samples	Collect teacher planning & assessment records	Final teacher interview
				Initial teacher interview			Parent consent for student interviews				
				Project details sent to parents			Student interviews				
Snowgum Alice			Gain Principal support				Project details sent to parents		Daily classroom observations		Collect teacher planning & assessment records
			Invite teacher to participate							Parent consent for student interviews	Final teacher interview
										Student interviews	
			Initial teacher interview							Collect student work samples	

Key: Ethical procedures; Phase 1 – Initial teacher interview; Phase 2 – Classroom-based unit of work; Phase 3 – Student interviews; Phase 4: Final teacher interview