THE AGGREGATE CONSUMPTION FUNCTION - AN ANALYSIS

Including an estimate of the Australian Short-

Run Consumption Function, 1959-1969

CONTENTS

ABSTRACT

In recent years the debate as to the correct form of the aggregate consumption function has continued unabated. Research on the question has been stimulated by the increasing efforts to build up econometric models of national economies. Over the years these models have grown both in size (i.e. in the number of equations and variables employed) and complexity. However, the consumption function has remained an important relationship in these models. The continuing debate has now generated an exceedingly large literature, and it is the first aim of this thesis to review the more important contributions to this literature. Four chapters are devoted to this aim. The first deals with work based essentially upon the original formulation of the consumption function by Keynes, the second deals with Friedman's Permanent Income Hypothesis and the literature which that has inspired, while the final two chapters are given over to a discussion of wealth and the consumption function, including the Life Cycle Hypothesis of Ando, Modigliani and Brumberg.

Research into the nature of the Australian consumption function has not kept pace with overseas work, and the resulting literature is small both in terms of the total number of studies and in terms of the number of successful overseas ideas that have been tried using Australian data. It is the second aim of this thesis to review the work on the Australian consumption function, and a conscientious effort has been made to collect together all available estimates.

The third aim of our work was to provide new estimates of the short-run Australian aggregate consumption function. Eleven different hypotheses were chosen for estimation on the basis of their performance with overseas and Australian data, as revealed by our prior review of the literature. These were estimated first using data on total consumption and then using data on consumption of non-durable goods and services. Attempts were made to introduce lags into the hypotheses by the use of the geometric lag model and the Almon variable technique.

The method of estimation used was single equation least squares (SELS). However, for one of the hypotheses, attempts were made to obtain estimates by the use of two recently proposed methods, both possessing superior properties to SELS.

On the basis of statistical reliability and ex-post forecast performance, the best hypothesis when non-durable consumption data was used, involved current disposable income and lagged consumption as regressors. When total consumption data was used the best hypothesis involved the use of current disposable income, total personal wealth, and lagged consumption as regressors.

CHAPTER ONE

INTRODUCTION

1.1 Introduction   The aims of this thesis are threefold:

(i) To provide a critical review of the more important overseas

literature on the aggregate consumption function; (ii) To critically

review the literature on the Australian aggregate consumption function;

(iii) To attempt to remove some of the deficiencies in the Australian

work, that have been revealed by (i) and (ii), by providing new

estimates of the Australian consumption function using quarterly

data. This chapter is given over to a discussion in section 1.2 of

the nature of the aggregate consumption function and the motivation

lying behind research into its form. This is followed in section 1.3

with some further discussion of the aims and scope of this thesis.

1.2 The Aggregate Consumption Function   While not a specifically

Keynesian idea, the consumption function was not regarded as a

macroeconomic relationship of central importance until the

publication of Keynes' General Theory of Employment, Interest, and

Money. In the chapters of the General Theory dealing with the

consumption function, Keynes grouped the factors influencing the

amount spent on consumption from a given level of income into

subjective factors and objective factors.[1] The subjective factors

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1.   J. M. Keynes, The General Theory of Employment, Interest, and
     Money (London: Macmillan and Coy., 1936), p. 91.

leading a person to refrain from spending out of his income included:
(i) the desire to built up a reserve against unforeseen contingencies;
(ii) preparation for old age, or the education of dependents;
(iii) allowing for greater consumption at a later data; (iv) pure
miserliness. Keynes assumed that these factors could be taken as
given in the short run.[1] The objective factors included: (i) the
level of real income; (ii) offsetting changes in prices and money
income; (iii) windfall gains; (iv) changes in the rate of interest;
(v) changes in the tax structure; (vi) changes in income expectations.
Although Keynes allowed that windfall gains were capable of
influencing the level of consumption expenditure in the short period,
and that substantial changes in the rate of interest and in the tax
structure might have some affect, he concluded that "aggregate
income ... is, as a rule, the principal variable upon which the
consumption-constituent of the aggregate demand function will depend".[2]
Thus, according to Keynes, in the short-run (with subjective factors
taken as given) we can say that

$$C = \phi(X) \tag{1.1}$$

where C = aggregate real desired consumption expenditure

   X = aggregate real income.

Keynes called ϕ' the marginal propensity to consume and assumed that
$0 < \phi' < 1$.

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1. Ibid.

2. Ibid., pp. 95-96.

The consumption function formed an integral part of the model underlying the General Theory, and hence estimation of its parameters was of interest as a test of the model and the conclusions derived therefrom. In early work on estimation, ϕ was taken to be linear in form and the adjustment of consumption to changes in income was taken to be instantaneous. Thus the consumption function was formulated as

$$C_t = a + b X_t \tag{1.2}$$

where b is the marginal propensity to consume. The corresponding aggregate savings function is derived from the identity

$$X_t \equiv C_t + S_t \tag{1.3}$$

Combining (1.2) and (1.3) yields the saving function

$$S_t = -a + (1 - b) X_t \tag{1.4}$$

Throughout the rest of this thesis we shall refer to (1.2) as the basic Keynesian consumption function; similarly for (1.4).

Supporters of the General Theory were encouraged then, when estimates of (1.?) indicated that the consumption function relationship was stable, and that the marginal propensity to consume, b, was indeed a positive fraction. Because of the intimate connection between the size of the marginal propensity to consume and the size of the multiplier, the size of the estimate of b had a further significance.

After the period in which the prime motive for research lay in the testing of the General Theory, impetus for further research came from two sources. Firstly, a number of people thought the analysis by Keynes, leading to (1.1), was unsatisfactory. At the same time, since desired consumption expenditure constituted such a high proportion of aggregate demand, they recognized the benefits to policy makers from having available a correctly formulated and estimated consumption function. Hence these people tackled anew the problem of formulating the aggregate consumption function. The results of this work were the various writings of Duesenberry, Modigliani, Brumberg, Friedman and others.[1]

The second source of motivation for further research came from the effort to build and estimate complete models of the national economy based upon the principles of the General Theory. These econometric models of the economy were designed to serve one or more of the following objectives: (i) To increase understanding of the structure of the economy; (ii) To act as a forecasting device; (iii) To enable quantitative analysis of policy measures.[2] A central feature of these models is the explanation of the determinants of aggregate demand. Since aggregate consumption is the largest component of aggregate demand, the explanation of its determinants is an important feature of an econometric model. Thus econometric work on

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1. Detailed references to this work are given below.

2. J. W. Nevile, "A simple econometric model of the Australian economy," Australian Economic Papers, Vol. 1 (1962), p. 79.

the consumption function has been further stimulated by the various model-building projects that have been undertaken. To improve the overall performance of a model its several parts should be as refined as possible. Hence the continuous work on the improvement of the consumption function, or its complement the savings function. In this drive to improve the consumption function, the model builders have been able to draw upon the writings of the group mentioned in the previous paragraph.

1.3 Aims and scope of this thesis   As noted in section 1.1, the first aim of this thesis is to provide a critical review of the work that has been done on the aggregate consumption function. The next four chapters are devoted to this task. Although the review is reasonably lengthy it has been possible to discuss only a fraction of the available material. However, it is hoped that the reader will find there a discussion of the most important ideas on this topic and of the more relevant empirical work. Wherever possible we have tried to present empirical results that have used recent, quarterly data. We begin the review in chapter two with a discussion of work based substantially upon the Keynesian analysis. Having discovered that the Keynesian function (1.2) was unsatisfactory, researchers modified it in a variety of ways, e.g. by replacing the single income variable with several disaggregated income variables, by attempting to take account of consumers' stocks of liquid assets, by introducing dynamic factors such as trend terms and lagged variables. These various modifications are discussed in chapter two.

Early in the 1950's investigators began a rethink on
the question of the consumption function, and several new
formulations, not explicitly based upon Keynes, were put forward.
These new formulations were based upon an examination of the
behaviour of an individual consumer. From the results obtained an
aggregate consumption function was derived. The two most prominent
examples of this approach, the Permanent Income Hypothesis and the
Life Cycle Hypothesis, are discussed in chapters three and four
respectively. These ideas, in turn, prompted further empirical work
in the same spirit. A discussion of some of this work is also
included in chapters three and four. Finally, some more recent studies
of the consumption function are discussed in chapter five. Throughout
these four chapters an attempt has been made to single out the ideas
that have worked well with the aim of using these, wherever possible,
in our own study of the Australian consumption function.

The second aim of the thesis is to review work on the
Australian consumption function. This is done in chapter six. The
review is of interest in itself, in that it provides an indication of
just how much progress has been made in one field of economic research
in Australia during the last twenty five years. In addition, the
review constitutes essential groundwork for the new estimates of the
Australian consumption function to be presented in chapter seven.
In chapter seven we aim to test various hypotheses which have either
been successfully tested overseas or are based upon ideas that have
been successfully tested, and to provide further tests of promising

Australian ideas which have not been adequately tested. To be
able to do this we must first have carried out a review of overseas
and Australian work on the consumption function. Throughout chapter
seven quarterly data will be used, hence our estimates can be looked
upon as estimates of the short-run consumption function. Finally
some general conclusions and suggestions for further work are given
in chapter eight.

Throughout this thesis an attempt has been made to minimize
any problems that might be caused by notation. For those chapters
in which a large number of variables are discussed an appendix has
been added,listing the symbols used and the variables they represent.

CHAPTER TWO

THE KEYNESIAN CONSUMPTION FUNCTION

AND ITS DERIVATIVE FORMS

2.1 Introduction   In section 2.2 of this chapter some of the
early empirical work on the consumption function will be examined.
The functions to be discussed were all formulated by the investigator
taking the basic Keynesian consumption function, $C_t = a + bX_t$, and
then introducing some extra factor in an attempt to obtain an
improved estimate of the aggregate consumption function. These
derivative formulations were obtained in a variety of ways:
(1) Additional variables, such as a measure of the equality of
income distribution, were introduced; (2) The aggregate income
variable, X_t, of the Keynesian function was sometimes split into two
or three variables representing the aggregate incomes of the separate
factors of production or groups within the community; (3) The static
formulation of the Keynesian function, where current consumption is
determined by current income, was varied by the introduction of
lagged values of income and consumption, trend terms, and changes in
the level of income; (4) Some investigators introduced non-linearities
into the function. It is their derivative nature, their essential
dependence upon the basic Keynesian function, which marks these
studies off from work to be discussed in following chapters, work
which begins with a fresh start, even though the final result may be
an aggregate consumption function very similar in appearance to the
Keynesian consumption function or one of its derivatives.

The literature on the Keynesian consumption function and its derivatives is very extensive and it is not possible to examine any more than a small fraction of the work done. However, it is hoped that the ten studies to be examined will give the reader an indication of how econometric work on the consumption function was developing during the period covered – what methods of estimation were employed, what tests of significance were used, how easily researchers were satisfied with their results, and so on.

An assessment of these studies is carried out in section 2.3. One conclusion drawn there is that the variable X^0, the previous peak value of income, is a very effective explanatory variable. The inspiration for the inclusion of this variable came from work by Franco Modigliani.[1] Both Modigliani and James Duesenberry independently advanced the hypothesis that consumption could be adequately explained by current income and previous peak income. Modigliani's work, like that discussed in section 2.2, was of an essentially derivative nature. Duesenberry, however, advanced his version of the hypothesis only after an examination of the theory of consumer behaviour at the micro level. The Duesenberry-Modigliani hypothesis has been long lived and much tested,[2] and for this reason

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1. Colin Clark, when introducing the variable, termed it the Modigliani factor.

2. The latest test, at the time of writing, was in D. G. Craig, "Predictive accuracy of quarterly and annual aggregate savings functions", Journal of the American Statistical Association, Vol. 65 (1970), pp. 1131-1145. The hypothesis first appeared around 1947.

it will be examined in greater detail in section 2.4 although

(because of Duesenberry's work) the hypothesis cannot be considered

as solely a derivative of the Keynesian consumption function.

Finally, general conclusions are given in section 2.5 along with

implications for Australian empirical work.

2.2 Studies of the Keynesian Consumption Function[1]   The ten

studies to be examined are summarized in table 2.1.[2] The first

estimate, by Staehle,[3] departs from the Keynesian formulation in

a number of ways: (i) The dependent variable, A, is a measure of

the average propensity to consume. The observations on A are

derived by dividing an index of retail sales by an index of labor

income.[4] (ii) In addition to the income variable, X, a measure of

the inequality in the distribution of income is included as an

explanatory variable. This measure, denoted by D, is defined as:[5]

$$D = \frac{\text{cumulative median income} - \text{median income}}{\text{cumulative median income}}$$

1. The notation used below is listed in Appendix 2.1.

2. See p. 22 below.

3. H. Staehle: "Short-period variations in the distribution of incomes", Review of Economics and Statistics, Vol. 19 (1937), pp. 133-143.

4. No attempt will be made for these early studies to describe the income variable used in terms of present day national accounting terms.

5. The cumulative median income is that amount which raises the cumulative total of ranked incomes to half the total of all incomes, where the cumulative total is built up by cumulating from the lowest income upwards.

The value of including D was checked in a second article[1] with the use of bunch map analysis.[2] The income measure used was obtained by deflating an index of labor income by an index of wage rates. The method of estimation was single equation least squares (SELS). No other formulations of the function, or other variables, were tried. With the exception of the bunch map analysis the only test of the estimate was the value of R^2 (0.7343). The work is, however, significant as one of the first estimates of the consumption function using quarterly data.

The second study, by Stone and Stone,[3] is again a quarterly study. Here a trend term has been included "to make allowance for trend influences bearing on the relation of [C'] to [X']".[4] In addition to the estimate shown for Great Britain, estimates were made of the consumption functions of Germany, Holland, Poland, Sweden and the United States. These estimates involved regression of consumption on to income and a trend term, except for those cases

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1. H. Staehle: "New considerations on the distribution of incomes and the 'propensity to consume'", Review of Economics and Statistics, Vol. 20 (1938), pp. 134-141.

2. For an account of bunch map analysis see: S. Valavanis, Econometrics (New York: McGraw-Hill, 1959), pp. 146-150.

3. R. Stone and W. M. Stone: "The marginal propensity to consume and the multiplier, a statistical investigation", Review of Economic Studies, Vol. 6 (1938-1939), pp. 1-24.

4. Ibid., p. 13.

where the correlation between income and the trend term was so high

as to make the resulting parameter estimates unreliable[1], in which

case the trend term was dropped. In each of these latter cases

annual data was used. Only in the case of Poland was deflated data

employed. In all cases the method of estimation was SELS. No other

statistical measures of goodness of fit were calculated except the

adjusted coefficient of determination ($\bar{R}^2$) and the standard error

of the estimate. The authors of this study conclude that

> "It does not appear, in the cases studied, that
> information other than that of changes in income
> is important in explaining changes in the movement
> of consumption except where a change takes place
> in the relation of consumption to income".[2]

Estimate number 3, by Polak,[3] was the result of testing

at least fifteen different formulations of the consumption function.

The final form chosen was

$$C' = 0.95X_1' + 0.70X_2' + X_3' + 0.35G_1 + 0.05G_2 + 0.27t \qquad (2.1)$$

where X_1', X_2' and X_3' are, respectively, "lower" incomes, "higher"

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1. If there are exact, or nearly exact, linear relationships
between regressors the standard errors of the parameter
estimates will be very large. Thus it would be hard to reject
the hypothesis that, for example, the population values of the
parameters are all zero. For an explanation of this problem of
multicollinearity (or intercorrelation) see: A. S. Goldberger,
Econometric Theory (New York: John Wiley and Sons, Inc., 1964),
pp. 192-194.

2. Stone and Stone: "The marginal propensity to consume and the
multiplier", p. 20.

3. J. J. Polak: "Fluctuations in United States consumption,
1919-1932", Review of Economics and Statistics, Vol. 21 (1939),
pp. 1-12, 88.

incomes and farmers' incomes; G_1 and G_2 are capital gains on securities and commodities respectively; t is a trend term. This formulation differs from that of Stone and Stone by splitting the income variable up into a number of categories, the motive for this being that the marginal propensity to consume from the different income types is likely to be different and the distribution of total income amongst the various categories could change over time. Other variables considered were "higher" incomes lagged one year $((X_2')_{-1})$, Pareto's measure of the equality of income distribution (α), and the general level of prices of consumption goods (p). The criteria used to choose between the various estimates were: (i) the magnitude and sign of parameter estimates; (ii) the magnitude of the multiple correlation coefficient R; (iii) the magnitude of the increase in R as extra variables were added into the regressions. Polak assumed that the marginal propensity to consume out of farmers' incomes was unity, hence the coefficient of unity for X_3'. The method of estimation was SELS. No special measures of goodness of fit or statistical significance are shown, but Polak indicates[1] that bunch map analysis was employed. As are the Stones, Polak is content with the Keynesian formulation of the consumption function, concluding that

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1. _Ibid._, p. 9.

"The theory is right in assuming that income is
the most important factor of consumption
expenditure. The influence of all other factors
is practically negligible – even that of the
distribution of income, if the distinction between
[lower] and [higher] income is taken into account".[1]

Estimate number 4, by Ezekiel,[2] is of interest because
of the dynamic variables experimented with. The estimate attempts
to measure the influence of a time trend and the rate of change
of income on consumption. Ezekiel's estimate can be rewritten as

$$C' = 10.256 + 0.622X' - 0.090(X')_{-1} + 0.917t - 0.37t^2 \qquad (2.2)$$

Thus it can be seen that Ezekiel has thought it worth while to
include lagged income as an explanatory variable, as opposed to
Polak[3] who rejected it. The estimate also includes a non-linear
term $-0.37t^2$. However, it has been shown that the coefficient of
t^2 is not significant at the 5% level of significance.[4] Ezekiel
also estimated the same relationship after putting it into per-capita
terms and deflating. However, on the basis of the size of the
coefficient of multiple determination corrected for degrees of
freedom, $\bar{R}^2$, this was rejected. The method of estimation was SELS.

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1. Ibid., p. 10.

2. M. Ezekiel, "Statistical investigations of saving, consumption
    and investment", American Economic Review, Vol. 32 (1942),
    pp. 22-49, 272-307.

3. See p. 13 above.

4. I. Friend, "Ezekiel's analysis of saving, consumption, and
    investment", American Economic Review, Vol. 32 (1942), p. 830.

The study by Smithies[1] has several features which mark
it off from most of the previous work, including that not reported
here. Firstly, constant price data are used.[2] All further studies
to be reported automatically use deflated data. Secondly, the
income variable used represents disposable income. Prior to this
stage it is difficult, in terms of current national income accounting
concepts, to say exactly what is the nature of the income variable
used. Subsequent studies examined sometimes do not use the disposable
income concept,but there is usually a special reason for this. (For
example, Klein in estimates 6.4 to 6.7 does not employ disposable
income data, although he recognizes the desirability of doing so,
because the data is not available.) Thirdly, the standard errors of
the parameter estimates are given, enabling tests of statistical
significance to be performed. Except for these factors the Smithies'
function is not greatly different to that formulated by Stone and
Stone (estimate number 2). Once again the method of estimation is
SELS.

Estimates 6.1 to 6.7 differ from previous estimates in
that each is part of a system of relationships forming an econometric
model of the United States economy.[3] Estimate 6.1 is from a simple

1. A. Smithies, "Forecasting postwar demand: I", _Econometrica_,
    13 (1945), pp. 1-14.

2. Note, however, that constant price data was used by Staehle in
    estimate 1.

3. See L. R. Klein, _Economic Fluctuations in the United States,
    1921-1941_ (New York: John Wiley and Sons, Inc., 1950).
    L. R. Klein, "The use of econometric models as a guide to policy",
    Econometrica, Vol. 15 (1947), pp. 111-151.

three equation model estimated by the method of reduced forms.[1]

The income variable used is disposable income. The formulation of

the function departs from the basic Keynesian function in that it

is cast in a per-capita form, and a lagged value of per-capita

income is included. Standard errors of the parameter estimates

and values of the statistics $\overline{SE}$ and VN were calculated for the

reduced form equation from which 6.1 is derived. The variable

$(\frac{L}{N})_{-1}$, where L represents private holdings of liquid assets, was

also tried,[2] but its coefficient in the reduced form equation was

found to be statistically insignificant. Estimates 6.2 and 6.3

are alternative estimates of the consumption function for a large

16 equation model. In both formulations the income variable is

disposable income. Both formulations are quite simple, the only

special feature being the non-linear term -0.01Xt in 6.3. This term

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1. For an account of the reduced form approach to estimating the
   parameters of a system of equations see Goldberger, Econometric
   Theory, pp. 294-329. The reduced-form approach is employed
   because SELS estimates of the parameters of a relationship,
   where the relationship forms part of a simultaneous system of
   equations, lack the property of consistency (which, roughly
   speaking, means that as the sample size (n) grows the probability
   that the parameter estimate ($\hat{\theta}_n$) differs from the actual parameter

   value (θ) by more than an arbitrarily small amount (ϵ) should be
   decreasing, i.e.

   $$\lim_{n \to \infty} P(|\hat{\theta}_n - \theta| > \epsilon) = 0 \quad).$$

2. Klein, "Econometric models as a guide to policy", p. 123.

is included to test the hypothesis that the marginal propensity to consume is varying over time. (From 6.3 we find that $\frac{dC}{dX}$, the marginal propensity to consume, equals 0.77 − 0.01t, a decreasing function of time). Estimate 6.2 is obtained using the limited-information maximum likelihood method,[1] while SELS is used to derive estimate 6.3. Estimates 6.4 to 6.7 are from a six equation model.[2] In all estimates the only variables used are income variables − wage income before taxation (X_2) and non-wage income before taxation (X_1). Lack of data prevented the use of disposable income type variables. In 6.4 and 6.6 the lagged value of X_1 has been included as an explanatory variable. Estimate 6.4 has been found by use of the full-information maximum likelihood method,[3] while 6.5 and 6.6 have been found by use of the limited information maximum likelihood method. Asymptotic standard errors are in parenthesis below each parameter estimate in 6.5 and 6.6. Finally a SELS estimate of formulation 6.5 is given. The influence of the estimation technique is quite substantial as can be seen by comparing the magnitude of the estimated coefficient of X_1 in 6.5 (0.02) with that in 6.7 (0.25).

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1. See Goldberger, Econometric Theory, pp. 338–345.

2. Klein, Economic Fluctuations in the United States, pp. 58–80. The two previous models referred to above are discussed in both of the publications listed in footnote 3, p. 15 above.

3. See Goldberger, Econometric Theory, pp. 352–356.

A most important feature of this work is the use of estimation techniques (reduced form, full information and limited information maximum likelihood) suitable for the estimation of relationships forming part of a simultaneous system.

Estimates 7.1 and 7.2, by Clark,[1] were obtained in an indirect fashion. On the assumption that total taxation was a stable linear function of gross national product, Clark formulated his consumption function as

$$C = \alpha_0 + \alpha_1 X + \alpha_2 X^0 \qquad (2.3)$$

where X = gross national product and X^0 = the highest previous value of X attained a year or more previously. In addition Clark's import function was:

$$I = \beta_0 + \beta_1 X \qquad (2.4)$$

where I = imports. Subtracting (2.4) from (2.3) gives

$$C - I = \alpha_0' + \alpha_1' X + \alpha_2' X^0 \qquad (2.5)$$

(2.5) was estimated for both (1921-1933) and (1934-1941). These estimates were combined with corresponding estimates of (2.4) to give estimates 7.1 and 7.2. Clark's reason for adopting this approach is to reduce the number of equations and endogenous variables in

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1. C. Clark, "A system of equations explaining the United States trade cycle, 1921 to 1941", _Econometrica_, Vol. 17 (1949), pp. 93-124.

his equation system.[1] The only other variable tried by Clark was

gross national product lagged two quarters. This was rejected on

the basis of bunch map analysis. Clark offers no statistics or

tests in support of his consumption function except the value of

the correlation coefficient. Bunch map analysis is used to support

the inclusion of X^0 in the function. The function does receive

some other support in that it is part of a system of equations

designed to explain the United States trade cycle, which Clark feels

the system does quite well.[2] Although Clark's consumption function

is part of a system of equations, he estimated it by SELS and thus

the parameter estimates will not be consistent.

Estimate 8 was part of a small, highly aggregative quarterly

model of the United States.[3] The consumption function was very

simple; aggregate consumption was made proportional to lagged gross

national product. The proportional character of this consumption

function marks it off from the Keynesian function, since proportionality

implies that the average propensity to consume equals the marginal

propensity (whereas the Keynesian function normally shows the average

propensity greater than the marginal propensity). The system was

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1. Ibid., p. 102.

2. Ibid., pp. 115-116.

3. G. H. Fisher, "A simple econometric model for the United States,
   1947-1950", Review of Economics and Statistics, Vol. 34 (1952),
   pp. 46-48.

estimated by the method of reduced forms, standard errors being given for the estimates of the reduced form parameters.

Estimates 9.1 to 9.3, by Barger and Klein,[1] are noteworthy for including lagged consumption as an explanatory variable. $C_{-3/2}$ is a simple average of the consumption of the two previous quarters, i.e. $\frac{1}{2}(C_{-1} + C_{-2})$. In 9.1 the variable X is gross income before taxes; in 9.2 and 9.3, X_1 is wage income (before taxes) and X_2 is non-wage income (before taxes). Relationship 9.1 was part of a model set up in recursive form, enabling consistent parameter estimates to be obtained by application of the SELS method.[2] The 95% confidence limits for the parameter estimates are:

$$0.931 \leqq \alpha_1 \leqq 0.984 \qquad -0.072 \leqq \alpha_2 \leqq 0.122$$

where α_1 is the coefficient of $C_{-3/2}$ and α_2 is the coefficient of X. Barger and Klein note that the result is defective for the following

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1. H. Barger and L. R. Klein, "A quarterly model for the United States economy", Journal of the American Statistical Association, Vol. 49 (1954), pp. 413-437.

2. Consistent estimates of the parameters of a recursive system can be obtained by application of SELS to each equation of the system, if it is assumed that unlagged disturbances in separate equations are independent. See Goldberger, Econometric Theory, pp. 354-355. Barger and Klein do not make this assumption and use the following procedure to obtain consistent estimates: (1) The equation with only one dependent variable (say Y_1) is estimated by SELS; (2) The equation with two dependent variables (Y_1, Y_2) is estimated by SELS, but actual observations on Y_1 are replaced by calculated values $\hat{Y}_1$. And so on for subsequent equations. See Barger and Klein, "A quarterly model for the U.S. economy", pp. 416-418.

reasons: (i) The statistic VN indicates significant autocorrelation of the disturbances; (ii) The marginal propensity to consume out of current income (α_2) is very low; (iii) The confidence limits for α_2 include negative values.[1] While the study suffers from these defects it is also impressive for the care the authors have taken to obtain consistent parameter estimates and for the calculation of confidence limits. Relationship 9.2 was part of a model which was partially recursive. Limited-information maximum likelihood estimation was required to obtain consistent parameter estimates. A SELS estimate of 9.2 was provided also (see 9.3). Barger and Klein subjected their models to a further test by examining how well they predicted in comparison to three "guesswork" models. In this test the performance of the models both within and without the sample period was checked.

Estimate 10, by Klein and Goldberger,[2] is the consumption function for a 20 equation model of the United States economy. In setting up the equation for testing, the authors have taken account of the functional distribution of income by providing three income variables – X_1 (disposable employee compensation), X_2 (disposable non-wage non-farm income) and X_3 (disposable farm income). In addition, allowance is made for population changes and lags in

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1. Barger and Klein, "A quarterly model for the United States economy", pp. 422-423.

2. L. R. Klein and A. S. Goldberger, An Econometric Model of the United States 1929-1952 (Amsterdam: North-Holland Publishing Coy., 1955), pp. 4-10, 57-66, 90.

TABLE 2.1[a]

Author(s)[b]	Estimate Number	Country	Data[c]	Sample Period	Estimate[d]	Estimation Method[e]	$\bar{R}^2$ etc.[f]
Staehle [1937]	1	Germany	quarterly constant prices	1928–1934	$A = 197.4 - 0.33X - 2.31D$	SELS	$R^2 = 0.7343$
Stone and Stone [1938]	2	Gt. Britain	quarterly current prices	1929(III) –1935(III)	$C' = 408.9 + 0.5210X' - 0.2412t$	SELS	$\bar{R}^2 = 0.909$ SE = 10.0
Polak [1939]	3	U.S.A.	annual current prices	1919–1932	$C' = 0.95X_1' + 0.70X_2' + X_3'$ $+ 0.35G_1 + 0.05G_2 + 0.27t$	SELS	R = 0.994
Ezekiel [1942]	4	U.S.A.	annual current prices	1921–1938	$C' = 10.256 + 0.712X' - 0.090\Delta X'$ $+ 0.917t - 0.37t^2$	SELS	$\bar{R}^2 = 0.986$ SE = 1.044
Smithies [1945]	5	U.S.A.	annual constant prices	1923–1940	$\frac{C}{N} = 76.58 + 0.76\ \frac{X}{N} + 1.15\ t$ $(0.05)\quad(0.58)$	SELS	R = 0.97
Klein [1947, 1950]	6.1	U.S.A.	annual constant prices	1922–1941	$\frac{C}{N} = 84.74 + 0.58\ \frac{X}{N} + 0.15\ (\frac{X}{N})_{-1}$	RF	
	6.2	"	"	1922–1941	$C = 11.87 + 0.73X + 0.04t$	LIML	VN = 1.20 $\overline{SE}$ = 1.36
	6.3	"	"	1921–1941	$C = 9.70 + 0.77X - 0.01Xt + 0.76t$ $(0.04)\quad(0.004)\quad(0.26)$	SELS	VN = 1.46 $\overline{SE}$ = 1.17
	6.4	"	"	1921–1941	$C = 16.78 + 0.02X_1 + 0.23(X_1)_{-1}$ $+ 0.80X_2$	FIML	VN = 1.54 $\overline{SE}$ = 1.14
	6.5	"	"	1921–1941	$C = 17.71 + 0.02X_1 + 0.87X_2$ $(0.07)\quad(0.04)$	LIML	VN = 0.98 $\overline{SE}$ = 1.30
	6.6	"	"	1921–1941	$C = 17.15 - 0.22X_1 + 0.40(X_1)_{-1}$ $(0.19)\quad(0.17)$ $+ 0.82X_2$ (0.05)	LIML	VN = 1.56 $\overline{SE}$ = 1.55
	6.7	"	"	1921–1941	$C = 16.43 + 0.25X_1 + 0.80X_2$ $(0.07)\quad(0.04)$	SELS	VN = 1.34 $\overline{SE}$ = 1.05

Table 2.1 continued

Author(s)	Estimate Number	Country	Data	Sample Period	Estimate	Estimation Method	$\bar{R}^2$ etc.
Clark [1949]	7.1	U.S.A.	quarterly constant prices	1921-1933	$C = -9.64 + 0.622X + 0.322X^0$	SELS	R = 0.966
	7.2	"	"	1934-1941	$C = -2.34 + 0.361X + 0.322X^0$	SELS	R = 0.877
Fisher [1952]	8	U.S.A.	quarterly constant prices	1947-1950	$C = 0.608X_{-1}$	RF	
Barger and Klein [1954]	9.1	U.S.A.	quarterly constant prices	1923-1940	$C = 257 + 0.955C_{-3/2} + 0.035X$	SELS	VN = 1.35
	9.2	"	"	"	$C = 1313 + 0.474C_{-3/2} + 0.655X_1$ $- 0.238X_2$	LIML	VN = 1.67
	9.3	"	"	"	$C = 1375 + 0.443C_{-3/2} + 0.696X_1$ $- 0.257X_2$	SELS	
Klein and Goldberger [1955]	10	U.S.A.	annual constant prices	1929-1941 & 1946-1952	$C = -22.26 + 0.55X_1 + 0.41X_2$ $\quad(9.66)\ (0.06)\quad (0.05)$ $+ 0.34X_3 + 0.26C_{-1} + 0.072L_{-1}$ $\quad(0.04)\quad (0.075)\quad (0.025)$ $+ 0.26N$ $\quad(0.10)$	LIML	VN = 1.98

a.   For definition of notation see Appendix 2.1, p. 50.
b.   For sources see text.
c.   All quarterly studies except that by Stone and Stone employ deseasonalised data.
d.   The figures in parenthesis are the standard errors. In estimate 6.5 and 6.6 the figures are asymptotic standard errors.
e.   SELS = single equation least squares; RF = reduced form; LIML = limited-information maximum likelihood; FIML = full-information maximum likelihood.
f.   SE = standard error of the estimate; $\overline{SE}$ = SE corrected for degrees of freedom; VN = Von Neumann ratio (used for testing for possible autocorrelation of disturbances).

behaviour with the introduction of the variables N and C_{-1}. A variable not encountered before is L_{-1} - end of year liquid assets held by persons. This is the most fundamental modification to the Keynesian function reported so far. Klein and Goldberger argue that the consumer's stock of wealth will affect his expenditure plans for coming periods[1] and that a convenient way to measure wealth stocks is by means of L. The problem of multicollinearity between the three income variables was overcome by the use of survey data. Instead of estimating the coefficients α_1, α_2 and α_3 in $\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$, survey data was used to estimate $\dfrac{\alpha_2}{\alpha_1}$ and $\dfrac{\alpha_3}{\alpha_1}$ and then time series data was used to estimate the coefficient α_1 of

$$\alpha_1 [X_1 + (\frac{\hat{\alpha}_2}{\alpha_1})X_2 + (\frac{\hat{\alpha}_3}{\alpha_1})X_3]$$

where ^ denotes the survey data estimate. The entire model was tested for its forecasting ability both within and without the sample period. The estimation method was limited-information maximum likelihood.

2.3 <u>Assessment</u>   The aim of this section is to provide some assessment of the studies just outlined. The early studies examined (estimates 1 to 5) appear to do quite well when casually examined.

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1. In coming chapters we will be examining other formulations of the consumption function in which the consumer's wealth status is regarded as fundamental.

The hypotheses tested appear to be able to explain more than 90%

of the variation in consumption expenditure. The lowest value for

the multiple correlation coefficient is 0.7343 for estimate number 1.

In addition, the consumption functions finally formulated in estimates

1 to 5 have the virtue of being quite simple in form. However, it

is too much to be asked to accept estimates on the basis of the value

of the correlation coefficient alone. These five estimates, and the

rest also, suffer from a number of deficiencies.

First, in six of the ten cases the estimation method was

SELS. Since the consumption function is, in reality, part of a

system of relationships linking the various economic aggregates

(even if the other relationships are not spelt out), use of SELS

will usually result in inconsistent estimates of its parameters.[1]

Secondly, in only three cases (estimates 6, 9 and 10) was any check

made for possible autocorrelation of disturbances. The consequences

of autocorrelated disturbances are: (1) The conventional least-squares

formulae for calculating the variances (and hence standard errors) of

parameter estimates may seriously underestimate the variances of least-

squares parameter estimates; (2) The usual form of the t-test will

not hold; (3) The classical least-squares estimator is no longer a

best-linear-unbiased-estimator (BLUE) since another linear-unbiased-

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1. See footnote 1, p. 16 above. See also Goldberger, Econometric Theory,
   pp. 288-294 and T. Haavelmo, "Methods of measuring the marginal
   propensity to consume", Journal of the American Statistical
   Association, Vol. 42 (1947), pp. 105-122.

estimator (the generalized least-squares estimator) with smaller sampling variances exists.[1] Thus, the consequences of autocorrelation are that our estimates are no longer BLUE, standard errors are not correctly calculated, and the normal tests of significance are inapplicable. In view of these consequences the lack of a test for autocorrelation is a serious failing. Thirdly, a check on the statistical significance of parameter estimates is almost impossible as in only very few cases were standard errors calculated. Fourthly, only in the case of estimates 9 and 10 was the predictive ability of the function checked against data from outside the sample period. Fifthly, many of the studies used income variables that were not of the "disposable income" type. Sixthly, in many studies the sample period was exceedingly short, estimates being calculated on less than twenty observations. Finally, hypotheses for testing were still being formulated in most cases by taking the Keynesian consumption function and tinkering with it in some way - trying different methods of deflation, splitting up the income variable in a variety of ways etc.

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1. For a discussion of the consequences of autocorrelation and the generalized least squares estimator see Johnston, Econometric Methods, (New York: McGraw-Hill, 1963), Ch. 7 or Goldberger, Econometric Theory, pp. 231-246.

In an attempt to overcome some of these deficiencies Robert Ferber made a systematic study[1] of a number of hypotheses using a common set of annual data. The hypotheses were used to derive seven basic savings functions, namely:

A $S = f(X)$

B $S = f(X, X_{-1})$

C $S = f(X, N)$

D $S = f(X, t)$

E $S = f(X, X_{-1}, t)$

F $S/X = f[(X - X^0)/X]$

G $S/X = f(X/X^0)$

Three separate sample periods were used (1929-1940, 1923-1940, 1923-1930 and 1935-1940), and the data was in turn deflated by a price index, population, or both. The combination of basic hypotheses with different sample periods and different methods of deflating resulted in approximately sixty different savings functions. As well as calculating $\bar{R}^2$, a check was made on each function for the possibility of autocorrelated disturbances. Average absolute percentage forecast-errors were calculated for three different periods outside

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1. R. Ferber, A Study of Aggregate Consumption Functions (New York: National Bureau of Economic Research, Technical Paper No. 8, 1953) and R. Ferber, "The accuracy of aggregate savings functions in the post war years", Review of Economics and Statistics, Vol. 37 (1955), pp. 134-145.

the sample period (four in the case of estimates based on 1923-1930 and 1935-1940 data).[1]

In examining his results Ferber stated that:

"We seek functions that
  (a) will supply the most accurate forecasts when
       fitted to the entire period of observation, and

  (b) are likely to be as accurate in predicting
       savings for prosperous years as for depressed
       years."[2]

Applying the first criterion brought functions of type F and G, and functions of type B when price deflated, to the fore. It was not possible to choose between these by employing the second criterion. Functions of type F and G do, however, perform better relative to a naive forecasting model ($S_t = S_{t-1}$) than do the functions of type B. Of the various functions of type F and G tested, the price deflated form of type F seems the best on the grounds of $\bar{R}^2$ and forecast ability. In later work,[3] Ferber states that the inclusion of lagged income also aids the predictive ability of the savings function.

Thus, to conclude this section, it appears from Ferber's work that the best hypothesis at that time was one involving current income (X), previous peak income (X^0), and possibly lagged income (X_{-1}). As stated in section 2.1 this hypothesis and its variations has been

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1. Further periods were tested in Ferber, "The accuracy of aggregate savings functions in post war years".

2. Ferber, A Study of Aggregate Consumption Functions, p. 46.

3. Ferber, "The accuracy of aggregate savings functions in post-war years", p. 144.

well tested over a considerable period of time and, while not of an entirely derivative nature, it will be convenient at this point to examine it in some greater detail. While doing so, results of several econometric studies more recent in time than that by Ferber will be drawn upon.[1]

2.4   The Duesenberry-Modigliani Hypothesis   The hypothesis that past income levels, and particularly past peak income (X^0), affected consumption expenditure is most strongly associated with the names of Profs. James Duesenberry and Franco Modigliani. Each independently suggested a form of consumption function including X^0.[2]

One source of motivation for the DM hypothesis was the estimates of national income and consumption published by S. Kuznets.[3] These estimates were for overlapping decades (e.g. 1869-78, 1874-83)

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1. One potentially important variable not discussed by Ferber is liquid assets. Its influence will be discussed below.

2. See J. S. Duesenberry, "Income-Consumption Relations and their Implications", reprinted in Readings in Macroeconomics, ed. by M. G. Mueller (New York: Holt, Rinehart and Winston, Inc., 1966), pp. 61-76; J. S. Duesenberry, Income, Saving and the Theory of Consumer Behaviour, (New York: Oxford University Press, 1967); F. Modigliani, "Fluctuations in the Saving-Income Ratio: A Problem in Economic Forecasting". New York: National Bureau of Economic Research, Studies in Income and Wealth, Vol. 11, 1949. The Duesenberry-Modigliani hypothesis will be referred to as the DM hypothesis.

3. S. Kuznets, Uses of National Income in Peace and War, Occasional Paper No. 6 (New York: National Bureau of Economic Research, 1942). Revised estimates of these data were published in S. Kuznets, National Income: A Summary of Findings (New York: National Bureau of Economic Research, 1946).

and, as such, emphasized the long-run characteristics of movements
in national income and consumption. The long-run ratio of consumption
to net national product was shown by these figures to have been
almost constant for some 70 years. Between 1869 and 1928 the ratio
fluctuated between 0.84 and 0.89 only. On the other hand, annual
or quarterly data on consumption and income will display a wide
variation in the consumption-income ratio. Ideally, then, the aggregate
consumption function should be capable of reconciling the secular
(or long-run) constancy of the consumption-income ratio with its
cyclical (or short-run) fluctuations as shown by annual or quarterly
data. The basic Keynesian consumption function cannot do this for

$$C_t = a + bX_t \qquad\qquad a > 0$$

implies a falling consumption-income ratio as income rises. Thus
the vast majority of consumption functions, which were all some
modification of $C_t = a + bX_t$, were inconsistent with long-run data.

One early attempt to explain the discrepancy between the
implications of the Keynesian type function and the long-run data
was made by Smithies.[1] Smithies did this by estimating the Keynesian
consumption function (in per-capita form) with a time trend added.
The estimate was:

$$\frac{C}{N} = 76.58 + 0.76\,\frac{X}{N} + 1.15t$$
$$\qquad\qquad (0.05) \qquad (0.58)$$

Smithies reports that applying this formula to changes in Kuznets
national product figures yielded changes in consumption very close to

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1. Smithies, "Forecasting post war demand".

those given by Kuznets.[1] This attempt was shown to have failed by Modigliani, who re-estimated the Smithies function using revised data and obtained[2]

$$\frac{C}{N} = 71.7 + 0.78 \frac{X}{N} + 0.83t$$

This revised estimate did not reproduce the Kuznets figures.

As an alternative hypothesis Modigliani offered:

$$C_t = 0.773X_t + 0.125X_t^0 \qquad (2.6)$$

where C = real consumption per capita, X = real disposable income per capita, and X^0 = the highest value of X prior to period t. (The constant term was found to be statistically insignificant.) To see how (2.6) helps to reconcile the secular stability of the consumption-income ratio with its cyclical fluctuations, first rearrange (2.6) to give:

$$\frac{C_t}{X_t} = 0.773 + 0.125 \frac{X_t^0}{X_t}$$

In the case of steady growth of income, the highest value of X prior to period t will be X_{t-1}. Hence $X_t^0 = X_{t-1}$. Suppose income is growing steadily at the rate of 2% per period (i.e. $\frac{X_t - X_{t-1}}{X_t} = .02$).

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1. Smithies, of course, was not the first to estimate a consumption function of this form. However, as far as we are aware, he is the first to specifically mention the fact that this form of consumption function may satisfactorily explain Kuznet's figures.

2. Modigliani, "Fluctuations in the saving-income ratio", p. 376.

The ratio $\frac{X_t^0}{X_t}$ will then be equal to 0.98. The consumption income

ratio will be a constant, namely: $0.773 + 0.125 \times 0.98 = 0.895$.

Thus a consumption function of the form suggested by Modigliani

is consistent with a constant long term consumption-income ratio.

On the other hand, suppose income begins to decline. In this case

X_t^0 stays fixed at the previous peak value of income. Function

(2.6) is now of the form

$$C_t \quad = \quad a + 0.773X_t$$

where a is a constant equal to $0.125X_t^0$. The consumption income

ratio is now equal to

$$\frac{C_t}{X_t} \quad = \quad \frac{a}{X_t} \quad + 0.773$$

and will rise as X_t falls. When the recovery begins, and X_t

increases back towards its previous peak, $\frac{a}{X_t}$ will fall and

$\frac{C_t}{X_t}$ will fall back towards its long-run value.

Since 1945, declines in the absolute level of income

have been exceedingly rare, cycles of economic activity being

characterized more by fluctuations in the rate of growth of income

around a long-run upward trend. In these circumstances the DM

hypothesis is still effective in explaining fluctuations in the

consumption-income ratio. Consider

$$\frac{C_t}{X_t} = a + b \frac{X_t^0}{X_t} \qquad (2.7)$$

If income is always rising, but at a fluctuating rate, (2.7) can be written as

$$\frac{C_t}{X_t} = a + b \frac{X_{t-1}}{X_t} \qquad (2.8)$$

Now, if $100\lambda\%$ is the long-run rate of income growth, $\frac{X_{t-1}}{X_t} = (1 - \lambda)$ and (2.8) becomes

$$\frac{C_t}{X_t} = a + b(1 - \lambda) = \left(\frac{C}{X} \right)_{LR}$$

Hence, the long-run $\frac{C}{X}$ ratio is a constant. Suppose that the rate of growth rises <u>above</u> its long run rate to $(\lambda + \Delta\lambda)$. Then (2.8) becomes

$$\frac{C_t}{X_t} = a + b(1 - \lambda - \Delta\lambda)$$

$$= \left(\frac{C}{X} \right)_{LR} - b\Delta\lambda$$

that is, the $\frac{C}{X}$ ratio falls <u>below</u> its long-run value $\left(\frac{C}{X} \right)_{LR}$. Similarly, if the rate of growth falls below its long-run rate, the $\frac{C}{X}$ ratio will rise above its long-run value. Thus, if we

examine data for which the short-run characteristics have been suppressed (such as the Kuznets data), we would expect to find a constant $\frac{C}{X}$ ratio. On the other hand, if we examine short-run data we should expect to find the $\frac{C}{X}$ ratio fluctuating about its long-run value, according as the actual rate of growth fluctuates about the long-run rate of growth. Thus, the DM hypothesis reconciles, in a fairly simple way, the observed long-run and short-run behaviour of the $\frac{C}{X}$ ratio.

The work of Duesenberry resulted in a savings function of the form:

$$\frac{S_t}{X_t} = a \left(\frac{X_t}{X_t^0} \right) + b \qquad (2.9)$$

One argument used by Duesenberry in support of his hypothesis was that consumers would attempt to maintain previously attained consumption standards in the face of a decline in income. As a measure of these standards X^0 was introduced into the consumption (savings) function. An alternative measure suggested was C^0, previous peak consumption,[1] giving:

$$\frac{S_t}{X_t} = a \left(\frac{X_t}{C_t^0} \right) + b$$

We have already seen above that Ferber's work, using

1. T. E. Davis, "The consumption function as a tool for prediction", Review of Economics and Statistics, Vol. 34, (1952), pp. 270-277.

annual data for the United States, gave support to the DM

hypothesis over a series of others tested.[1] T.M. Brown provided

a further check using annual data for Canada for the period

1926-1941, 1946-1949.[2] Brown was investigating the general

question of lags in consumer behaviour and tested some seven

basic hypotheses. The first hypothesis was the simple Keynesian

consumption function:

$$A: \qquad C_t = a_0 + a_1 X_t$$

The next two hypotheses assumed that consumers were slow to react

to changes in their <u>income</u> status:

$$B: \qquad C_t = a_0 + a_1 X_t + a_2 X_{t-1}$$

$$C: \qquad C_t = a_0 + a_1 X_t + a_2 X_t^0$$

Hypothesis C implies that past peak disposable income is the most

important influence on consumption expenditure (besides current income).

On the other hand, hypothesis B implies that the most recent lagged

value of income has the greatest effect of the various past values.

The next two hypotheses assume that consumers are affected by past

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1. Modigliani's consumption function is

$$C_t = a X_t + b X_t^0$$

$$\therefore \quad S_t = (1 - a) X_t - b X_t^0 = (1 - a - b) X_t + b(X_t - X_t^0)$$

$$\therefore \quad \frac{S_t}{X_t} = (1 - a - b) + b \frac{X_t - X_t^0}{X_t}$$

which is a linear version of the type F function chosen by
Ferber as best performer (see p.26-27 above).

2. T.M. Brown, "Habit persistence and lags in consumer behaviour",
<u>Econometrica</u>, Vol. 20 (1952), pp. 355-371.

consumption standards:

$$D: \quad C_t = a_0 + a_1 X_t + a_2 C_{t-1}$$

$$E: \quad C_t = a_0 + a_1 X_t + a_2 C_t^0$$

Hypothesis D implies that the most recent consumption experience is the most important of past experiences, whereas hypothesis E implies that previous peak consumption is the most important – no matter how far back in time that experience was. The final two hypotheses are derived from differing assumptions about the marginal propensity to consume (mpc):

$$F: \quad C_t = a_0 + a_1 X_t + a_2 \frac{X_t^2}{2}$$

$$G: \quad C_t = a_0 + a_1 X_t + a_2 X_t X_{t-1}$$

Hypothesis F implies that the mpc is a linear function of income, since F implies that

$$\frac{dC_t}{dX_t} = a_1 + a_2 X_t$$

Hypothesis G, on the other hand, implies that the mpc is a linear function of lagged income, since G implies that

$$\frac{dC_t}{dX_t} = a_1 + a_2 X_{t-1}$$

Each hypothesis was varied by including the shift variable A^C, which took the value zero for 1926-1941 and unity for 1946-1949. Brown attempted by this device to take account of the increase during World War II in personal holdings of liquid assets, data for which did not exist. The 14 hypotheses resulting were estimated by SELS.

Brown judged the DM hypothesis with shift variable as the best

performer, i.e.[1]

$$C_t = a_0 + a_1 X_t + a_2 X_t^0 + a_3 A_t^C$$

Brown repeated the 14 regressions, this time with

disposable income split into disposable wage and disposable non-wage

income. The best result was the estimate of

$$C_t = a_0 + a_1 X_{1t} + a_2 X_{2t} + a_3 C_{t-1} + a_4 A_t^C$$

where X_{1t}, X_{2t} are disposable wage income and disposable non-wage

income respectively. Thus, the DM hypothesis is not directly

confirmed, but Brown's results do provide further support for the

inclusion of lagged values of variables in the consumption function.[2]

The work done by Brown has been repeated by Arnold Zellner,

this time using quarterly data for the United States.[3] Since

Zellner had observations on liquid asset holdings, the shift variable

A^C was not used but was replaced by L_{t-1} – beginning of quarter liquid

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1. The presence of the intercept a_0 means that this is not strictly
   the DM hypothesis. A statistically significant estimate for a_0
   would mean that $\frac{C_t}{X_t}$ will decline (assuming $a_0 > 0$) even if X_t is
   growing at some steady rate.

2. In assessing the various regression results Brown appears to have
   used the following statistics: (i) The t-ratio (i.e. the ratio
   of the parameter estimate to its standard error); (ii) The
   standard error of the estimated (corrected for degrees of freedom);
   (iii) The coefficient of variation (i.e. the ratio of the standard
   error of the estimate to the mean value of the dependent variable);
   (iv) The Non-Neumann ratio; (v) $\bar{R}^2$. See ibid., p.361-362.

3. A. Zellner, "The short-run consumption function", Econometrica,
   Vol. 25 (1957), pp. 552-567.

assets. Zellner did not bother with the non-linear hypotheses (F and G) of Brown, and thus tested a total of 10 formulations of the consumption function (hypotheses A to E with and without L_{t-1}). Each formulation was estimated by SELS and by the method of Reduced Forms. In the case of hypotheses B to E the reduced form estimate yielded a negative mpc and the results were not presented by Zellner. The results obtained are shown in table 2.2 (p.38). The data used was for 1947 (I) to 1955 (I), excluding 1950 (III) and 1951 (I).

These results were subjected to a much more careful examination than for any previous work. Each estimate was first considered in the light of the following three criteria: (i) the consistency of the signs of the parameter estimates with prior expectations, (ii) the statistical significance of the parameter estimates, (iii) the possible auto correlation of disturbances. If the estimate passed all three of these tests it was included in a group of acceptable hypotheses. These were then ranked according to their predictive ability, both within and without the sample period, and to the value of $\bar{R}^2$. In judging their predictive ability Zellner employed three "naïve" or "guess work" models. After subjection to this array of tests functions A.2 (SELS estimate) and D.2 were chosen, i.e.

$$A.2(SELS): \quad C = -21.91 + 0.708X + 0.368L_{-1}$$
$$(0.021) \quad (0.054)$$

TABLE 2.2[a]

Function	Estimation Method[b]	Estimate	$\bar{R}^2$	d[c]
A.1	SELS	$C = 38.09 + 0.747\ X$ $\quad\quad\quad\quad (0.033)$	0.944	0.75*
	RF	$C = 63.53 + 0.620\ X$	–	0.41*
A.2	SELS	$C = -21.91 + 0.708\ X + 0.368\ L_{-1}$ $\quad\quad\quad\quad (0.021)\quad\quad (0.054)$	0.979	1.79
	RF	$C = -18.52 + 0.665\ X + 0.396\ L_{-1}$	–	1.44**
B.1	SELS	$C = 36.52 + 0.553\ X + 0.203\ X_{-1}$ $\quad\quad\quad\quad (0.172)\quad\quad (0.176)$	0.944	0.61*
B.2	SELS	$C = -21.96 + 0.582\ X + 0.133\ X_{-1}$ $\quad\quad\quad\quad (0.104)\quad\quad (0.108)$ $\quad\quad\quad + 0.362\ L_{-1}$ $\quad\quad\quad\quad (0.052)$	0.979	1.34**
C.1	SELS	$C = 30.37 + 0.417\ X + 0.369\ X^0$ $\quad\quad\quad\quad (0.157)\quad\quad (0.172)$	0.950	0.54*
C.2	SELS	$C = -22.58 + 0.532\ X + 0.199\ X^0$ $\quad\quad\quad\quad (0.099)\quad\quad (0.110)$ $\quad\quad\quad + 0.347\ L_{-1}$ $\quad\quad\quad\quad (0.051)$	0.981	1.39**
D.1	SELS	$C = 0.10 + 0.128\ X + 0.870\ C_{-1}$ $\quad\quad\quad\quad (0.093)\quad\quad (0.127)$	0.978	1.53
D.2	SELS	$C = -18.96 + 0.375\ X + 0.489\ C_{-1}$ $\quad\quad\quad\quad (0.110)\quad\quad (0.160)$ $\quad\quad\quad + 0.219\ L_{-1}$ $\quad\quad\quad\quad (0.067)$	0.984	1.77
E.1	SELS	$C = 0.35 + 0.165\ X + 0.825\ C^0$ $\quad\quad\quad\quad (0.112)\quad\quad (0.156)$	0.971	0.99*
E.2	SELS	$C = -23.02 + 0.458\ X + 0.369\ C^0$ $\quad\quad\quad\quad (0.114)\quad\quad (0.165)$ $\quad\quad\quad + 0.272\ L_{-1}$ $\quad\quad\quad\quad (0.065)$	0.982	1.54**

* Autocorrelation at 5% level.
** Test inconclusive at 5% level.
a Derived from table II p. 560 of A. Zellner "The Short-Run Consumption Function".
b SELS = single equation least squares; RF = reduced form.
c d = Durbin-Watson test statistic.

$$D.2: \quad C = -18.96 + 0.375X + 0.489C_{-1} + 0.219L_{-1}$$
$$(0.110) \quad (0.160) \quad (0.067)$$

Thus the quarterly work of Zellner, like that of Brown, does not confirm the DM hypothesis directly but favours a function employing the variables disposable income and liquid assets or disposable income, lagged consumption and liquid assets.

For later purposes it will be useful to include here a mention of some work by Z. Griliches et al.[1] In this work Griliches produced several new estimates of functions D.1 and D.2 using revised data. For the same sample period as Zellner, the new estimate of D.2 was:

$$C_t = -24.7 + 0.576X_t + 0.228C_{t-1} + 0.319L_{t-1}$$
$$(0.113) \quad (0.150) \quad (0.070)$$
$$(R^2 = 0.983, \ d = 1.68)$$

Estimates were also calculated for 1952-1960 and 1947-1960. The estimate for 1947-1960 was:

$$C_t = -13.1 + 0.539X_t + 0.265C_{t-1} + 0.258L_{t-1}$$
$$(0.077) \quad (0.102) \quad (0.044)$$
$$(R^2 = 0.997, \ d = 1.45)$$

These estimates would appear to confirm Zellner's conclusions about the worth of liquid assets and lagged consumption as explanatory variables.

Estimates of D.1 were also calculated. The estimate for

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1.   Z. Griliches et.al., "Notes on estimated aggregate quarterly consumption functions", Econometrica, Vol. 30 (1962), pp. 491-500.

1947-1960 was

$$C_t = 3.1 + 0.300X_t + 0.670C_{t-1}$$
$$(0.085) \quad (0.097)$$

$$(R^2 = 0.994, \ d = 1.09)$$

Zellner's estimate of D.1 was regarded by him as unacceptable since the estimated coefficient of X_t was not statistically different from zero. This is not the case for this new estimate, however, and this formulation is worthy of further testing.

A further test of the DM hypothesis was made by Duesenberry and others when constructing a quarterly econometric model of the United States economy.[1] The actual hypothesis tested was a variation of Duesenberry's original hypothesis[2], namely:

$$\frac{C_t}{X_{t-1}} = a + b\frac{X_{t-1}}{X_{t-1}^0} + c\frac{C_{t-1}}{X_{t-2}} \qquad (2.10)$$

Duesenberry et. al. chose this form after experimentation. However, Griliches et. al. suggest the following derivation[3]

$$R_t = \frac{C_t}{X_{t-1}} \qquad (2.11)$$

$$Z_t = \frac{X_{t-1}}{X_{t-1}^0} \qquad (2.12)$$

$$R_t^* = \alpha + \beta Z_t \qquad (2.13)$$

$$R_t - R_{t-1} = \gamma(R_t^* - R_{t-1}) \qquad (2.14)$$

---

1. J.S. Duesenberry, O. Eckstein, and G. Fromm, "A Simulation of the United States economy in recession", Econometrica, Vol. 28 (1960), pp. 749-809 esp. pp. 800-807.

2. See(2.9) p. 33 above.

3. Griliches et. al., "Notes on estimated aggregate quarterly consumption functions", pp. 491-492.

R_t is the consumption-income ratio; Z_t is the ratio of income

in period (t-1) to peak income prior to period (t-1); R_t^* is the

desired value of R_t; α, β and γ are constants. The following

restrictions are imposed upon the parameters of the model: $\alpha>0$,

$\beta<0$, and $0<\gamma<1$. Equation (2.13) states that the desired consumption-

income ratio is equal to a constant plus an adjustment term βZ_t. If

income is growing steadily at $100\lambda\%$ then $Z_t = \dfrac{X_{t-1}}{X_{t-2}} = 1 + \lambda$[1] and

$R_t^* = \alpha + \beta(1 + \lambda)$, a constant. But, if there is a decline in income

Z_t will fall (X_{t-1}^0 will stay fixed, while X_{t-1} will fall), leading to

a rise in R_t^* (given that $\beta<0$). The further Z_t falls, the higher R_t

rises. As the economy enters into the recovery stage, and income

begins to rise, Z_t will increase and the desired consumption ratio

will fall. This process is identical with that described above[2]

when the DM hypothesis was first discussed except that now we

are talking of the _desired_ consumption ratio. Equation (2.14) links

the actual and desired consumption ratios. This equation implies that

the actual value of the ratio changes by some fraction of the

discrepancy between the desired value of the consumption ratio and

the actual value previously attained.[3] Substituting (2.13) into

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1. The rate of growth is here defined as $\lambda = \dfrac{X_{t-1} - X_{t-2}}{X_{t-2}}$

2. See pp. 30-31.

3. Note the correspondence between equations (2.13) and (2.14) and the
   partial adjustment or rigidity model for generating distributed lags.
   See M. Nerlove, _Distributed Lags and Demand Analysis for Agricultural
   and Other Commodities_, (Washington: U.S. Dept. of Agriculture, 1958),
   pp. 14-20 or K.F. Wallis, "Some recent developments in applied
   econometrics", _Journal of Economic Literature_, Vol. 7 (1969),
   pp. 771-782.

(2.14) gives:

$$R_t = R_{t-1} + \gamma(\alpha + \beta Z_t) - \gamma R_{t-1}$$

i.e.
$$\frac{C_t}{X_{t-1}} = \alpha\gamma + \beta\gamma \frac{X_{t-1}}{X_{t-1}^0} + (1 - \gamma)\frac{C_{t-1}}{X_{t-2}}$$

i.e.
$$\frac{C_t}{X_{t-1}} = a + b\frac{X_{t-1}}{X_{t-1}^0} + c\frac{C_{t-1}}{X_{t-2}} \qquad (2.10)$$

where $a = \alpha\gamma$, $b = \beta\gamma$, $c = (1-\gamma)$ and parameter restrictions imply

that $a>0$, $b<0$ and $0<c<1$.

The commonsense of the parameter restrictions can now be

easily demonstrated. Suppose the economy enters a slump. As X

falls consumers could be expected to aim at consuming an increasing

proportion of their falling income (so as to protect previously

attained consumption standards) i.e., we would expect to see R_t^* rising.

Since Z_t is falling during this time, R* will only rise if $\beta<0$. Hence

the restriction on β. To see the need for the restriction on α,

suppose income were constant over a period of time. Z_t would then

be unity and R_t^*, the desired consumption ratio, would equal $(\alpha + \beta)$.

Since R_t^* must be positive, the restriction on β implies that $\alpha>0$.

To derive the restriction on γ, first write (2.14) as:

$$R_t = \gamma R_t^* + (1- \gamma)R_{t-1} \qquad (2.15)$$

Lagging (2.15) one period gives:

$$R_{t-1} = \gamma R_{t-1}^* + (1- \gamma)R_{t-2} \qquad (2.16)$$

Substituting (2.16) into (2.15) yields:

$$R_t = \gamma R_t^* + (1-\gamma)\gamma R_{t-1}^* + (1-\gamma)^2 R_{t-2}$$

Repeated lagging and substitution gives:

$$R_t = [R_t^* + (1-\gamma) R_{t-1}^* + (1-\gamma)^2 R_{t-2}^* + \ldots] \qquad (2.17)$$

That is, R_t is a weighted average of current and past values of

the desired consumption ratio. So that the most recent values

of R* have the greatest effect in determining R_t the weights should

decline, i.e. we must have $0 < 1 - \gamma < 1$, i.e. $0 < \gamma < 1$. The 'old'

Duesenberry function (2.9) is a special case of (2.10) when $\gamma = 1$,

i.e. when $R_t = R_t^*$.

The model formulated by Duesenberry et. al. was a

recursive model for which consistent parameter estimates could be

obtained by the use of SELS.[1] The model was fitted to per capita

data for 1930-'38, 1948-'57. The variables $\dfrac{D_t}{X_{t-1}}$ and $\dfrac{A_t}{X_{t-1}}$, where

D_t = per capita consumer debt and A_t = per capita liquid assets,

were also tried. These, however, did not have significant coefficients.

The final result for the period 1948-1957 was:

$$\frac{C_t}{X_{t-1}} = 0.8218 - \underset{(0.0819)}{0.6250} \frac{X_{t-1}}{X_{t-1}^0} + \underset{(0.0526)}{0.7843} \frac{C_{t-1}}{X_{t-2}}$$

$$R^2 = 0.866$$

$$SE = 0.0069$$

$$VN = 1.84.$$

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1. See footnote 2, p. 20 above.

All parameter estimates are significant, and signs conform with

expectations. The value of R^2 is very high considering that the

equation is being estimated in ratio form. Griliches et. al.[1]

have re-estimated this equation using data for three periods

1948-1957, 1958-1960 and 1948-1960. The result for 1948-1960 was:

$$\frac{C_t}{X_{t-1}} = 0.839 - \underset{(0.134)}{0.565} \frac{X_{t-1}}{X_{t-1}^0} + \underset{(0.099)}{0.705} \frac{C_{t-1}}{X_{t-2}}$$

$$R^2 = 0.533$$

$$d = 2.47$$

Once again the parameter estimates are significant and conform

in sign and magnitude with expectations. The value of R^2 is still

quite high.

2.5 Conclusions   As stated in Chapter one, there are three

aims of this work: (1) To provide a critical review of the

literature on the aggregate consumption function; (2) To examine

Australian work in this area; (3) To make some contribution towards

eliminating deficiencies in Australian work that have been shown up

by (1) and (2). Clearly, the content of this chapter is of relevance

to the first and third of these aims, and in this section we will

summarize our review of this part of the literature and then consider

the implications of the work examined for an empirical study employing

Australian data.

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1. Griliches et. al., "Notes on estimates aggregate quarterly
    consumption functions", p. 493.

Firstly, the <u>formulation</u> of the consumption function
has changed in a number of ways: (a) While early research workers
were quite confident that current income was the only important
variable, as time passed extra variables were tried in an attempt
to improve the fit of the function. Many of these variables
were not very great departures from the Keynesian function (indeed
variables such as income distribution measures had been suggested
by Keynes). The greatest departure was in the use of a liquid assets
variable. Originally added in to attempt to measure any influence
on expenditure caused by the build up of liquid assets during World
War II, the liquid asset position of consumers came to be regarded
as a continuing part of the consumer's decision making process.
(b) The static formulation of the consumption function has been
dispensed with. Time trends and lagged values of income and consumption
have been incorporated. The lag structures used have been obtained
in a fairly arbitrary way in the main. (c) The single income variable
has on occasion been replaced with variables measuring the incomes
of sub-groups of consumers. (d) There has been some work on non-linear
consumption functions (Brown, Klein, Ezekiel), but in the main this
area has been ignored. (e) The formulation by Griliches <u>et. al.</u> of
the DM function[1] is evidence of a trend towards more careful derivation
of the final function to be tested. Arriving at the final function
by combining together a series of relationships enables more careful

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1. See p. 40 above.

prior estimates of parameter signs and magnitudes.

Secondly, methods of estimation have become more

sophisticated. Originally the only technique employed was SELS.

However, it is now recognized that the consumption function is

often one of a set of simultaneous relationships and, so as to

obtain consistent estimates, methods such as reduced form, full

information maximum likelihood, limited information maximum likelihood

have been employed. Some workers (Barger and Klein, Duesenberry et. al.)

have taken advantage of the recursive nature of their models to enable

them to use SELS and still obtain consistent estimates.

Thirdly, the estimated function is now exposed to a very

much more powerful battery of acceptance tests before being regarded

as satisfactory. In the early part of the period discussed above

investigators were content to accept an estimate if it gave a high

R^2. Tests employed on the later studies include: (1) The value of

$\bar{R}^2$; (2) A test for autocorrelated disturbances(based upon either the

Durbin-Watson statistic or the Von Neumann ratio); (3) Tests of

significance on the parameter estimates; (4) The consistency of the

sign and magnitude of parameter estimates with prior expectations;

(5) The predictive ability of the estimate both within and without

the sample period compared with that of "naive" or "guess work"

models.

Finally, certain questions as to the nature and type of

data employed appear to have been settled. Investigators now

automatically employ price deflated data and, unless prevented by special data problems, use income variables of a disposable income type. There is also increasing emphasis on quarterly studies for employment in short-run econometric models.

Turning now to the material of relevance to the third aim, the best source of working hypotheses for an Australian study is the paper by Zellner. Zellner's work is the most thorough of all in the selection of the final hypothesis and also employs post-war quarterly data. The best performers in that study were:

$$C = f (X, L_{-1})$$
$$C = f (X, L_{-1}, C_{-1}).$$

The recomputations of Zellner's functions, done by Griliches et. al., suggest that consideration also be given to:

$$C = f (X, C_{-1}).$$

From the earlier work by Brown on annual data, we get the following possibilities:

$$C = f (X, X^0, A^C)$$
$$C = f (X_1, X_2, C_{-1}, A^C)$$

The work done by Duesenberry et. al. (and the recomputations by Griliches et. al.) suggest that a consumption function in ratio form, involving several lagged variables

$$\frac{C}{X_{-1}} = f \left(\frac{X_{-1}}{X^0_{-1}} , \frac{C_{-1}}{X_{-2}} \right)$$

may be quite effective. Thus, the variables that we might consider for an Australian study are:

1.   X   = disposable income

   or X_1, X_2 = disaggregated disposable income

2. X^0 = previous peak income

3. X_{-1} = lagged income

4. C_{-1} = lagged consumption

5. L_{-1} = beginning of period liquid assets.

The work done by Griliches et. al.[1] suggests that there may

be benefits to be reaped, in the form of aid in building up

expectations as to the sign and magnitude of parameter estimates,

by combining these variables together from a series of sub

relationships.[2]

    We have now finished examining the contribution to

the literature made by work based on the Keynesian consumption

function. In the next three chapters we will look at work which

ignores this body of literature and sets out to make a fresh start.

Most of the studies to be looked at base their approach, at least

in theory, on some process of utility maximization at the micro

level. We shall begin in chapter three with the Permanent Income

Hypothesis and some of the work based upon its ideas.

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1.   See pp. 40-42 above.

2.   As (2.11), (2.12), (2.13), (2.14) were combined to give (2.10).
     See p. 40 above.

APPENDIX 2

2.1 Notation used in chapter 2

A   = ratio of retail sales to labor income

X   = aggregate real income

X_1

X_2 }   = disaggregated real income

X_3

C   = aggregate real consumption

D   = (cumulative median income − median income)/cumulative median income

t   = time

G_1 = capital gains on securities

G_2 = capital gains on commodities

N   = population

X_{-k}, C_{-k} = income and consumption lagged k periods

ΔX = $X_t - X_{t-1}$

X_t^0, C_t^0 = peak income and consumption prior to period t

$C_{-3/2}$ = $\frac{1}{2}(C_{-1} + C_{-2})$

L_{-1} = beginning of period liquid assets holdings of persons

S   = aggregate real savings

A^C = shift variable (= 0 for 1926-1941, =1 for 1946-1949)

X', C' = income and consumption in current prices

CHAPTER THREE

THE PERMANENT INCOME HYPOTHESIS

3.1 Introduction   In this chapter various aspects of Friedman's

Permanent Income Hypothesis (PIH)[1] will be examined. The PIH has

had a very large influence on the work on the consumption function

and also on various other areas of macroeconomics.[2] It departs

completely from the Keynesian consumption function in its initial

formulation though, as we shall see, it can lead to an aggregate

consumption function that is quite Keynesian in appearance. The

hypothesis is based upon an examination of the theory of consumer

behaviour. A summary of this examination is given in section 3.2

which ends with the presentation of Friedman's aggregate consumption

function. Throughout this section Friedman's distinctive notation

will be employed. In section 3.3 the aggregate consumption function

will be incorporated into a model and various tests on the model will

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1. See M. Friedman, A Theory of the Consumption Function (New York:
    National Bureau of Economic Research, 1957). Some aspects of
    the work appeared earlier in M. Friedman and S. Kuznets, Income
    from Independent Professional Practice (New York: National Bureau
    of Economic Research, 1945), Ch. VII.

2. See, for example, M. Friedman and A. J. Schwartz, "Money and
    business cycles", Review of Economics and Statistics, Vol. 45
    (1963), Sup., pp. 32-64; J. Tobin and C. Swan, "Money and
    Permanent Income", American Economic Review (Papers and Proc.),
    Vol. 59 (May, 1969), pp. 285-295; D. Laidler, "The Permanent-
    Income concept in a macro-economic model", Oxford Economic Papers,
    Vol. 20 (1968), pp. 12-23; R. Eisner, "A Permanent Income theory
    for investment", American Economic Review, Vol. 57 (1967),
    pp. 363-390; G. C. Chow, "On the long-run and short-run demand
    for money", Journal of Political Economy, Vol. 74 (1966),
    pp. 111-131.

be examined. It is this model which is referred to as the PIH.

Tests of the PIH are by no means unanimous that, as formulated by Friedman, the hypothesis is valid. Also, the hypothesis leads to what is essentially a long-run consumption function. Yet, in spite of its long-run nature and doubt as to its formal validity, the PIH has been the basis of a number of short-run consumption function studies, two of which are examined in sections 3.4 and 3.5. Finally, we conclude with section 3.6.

3.2 <u>Basic Theory</u>[1]   Friedman begins his analysis with the consideration of an individual consumer unit operating under conditions of certainty. Under such conditions the consumer will know his income stream over future periods, the price structure, and the interest rate structure. In these circumstances there are only two reasons why the unit's consumption in any period will differ from its income in that period: (1) the pattern of income receipt over time may not coincide with the preferred pattern of consumption, in which case the consumer will have to borrow (or lend) to enable consumption plans to be carried out; (2) the consumer may wish to earn interest on any loans that are made (assuming a positive interest rate).

Suppose utility (u') is given by a function of current and future consumption, i.e.

$$u' = h(C_1, C_2, \ldots)$$

where C_i is planned consumption in period i (i = 1, 2, ...) in the

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1. This section follows chapter II of Friedman, <u>A Theory of the Consumption Function</u>.

prices of period i. Suppose only two periods are considered, i.e.

$$u' = h(C_1, C_2)$$

Let R_1 and R_2 be the unit's income in those periods, and suppose that the interest rate at which the consumer can borrow or lend is i. Assuming that the unit can borrow against its future income, the maximum possible consumption in the first period is the present value of current and future income, i.e. $[R_1 + \frac{R_2}{1 + i}]$. Consumption in the second period will then be zero. On the other hand, if no consumption is undertaken in the first period, it will be possible to consume $[R_1(1 + i) + R_2]$ in the second period. The figure $[R_1 + \frac{R_2}{1 + i}]$ is the unit's wealth at the beginning of period 1 (W_1), while the figure $[R_1(1 + i) + R_2]$ is the unit's wealth at the beginning of period 2 if no consumption is undertaken in period 1 $(W_2 = W_1(1 + i))$. The problem facing the consumer can now be represented on an indifference map (see figure 3.1). The indifference curves represent equally acceptable combinations of planned consumption in the first period and in the second period. The line AB is the locus of attainable consumption combinations; it cuts the vertical axis at W_1 and the horizontal axis at $W_2 = W_1(1 + i)$. (Friedman assumes that all income is disposed of, hence combinations inside the area OAB will not be chosen.)

Planned consumption in year 1 and year 2 will be found at the point of tangency of the 'budget' line AB and an indifference curve. Given the set of indifference curves, this point is determined

Figure 3.1
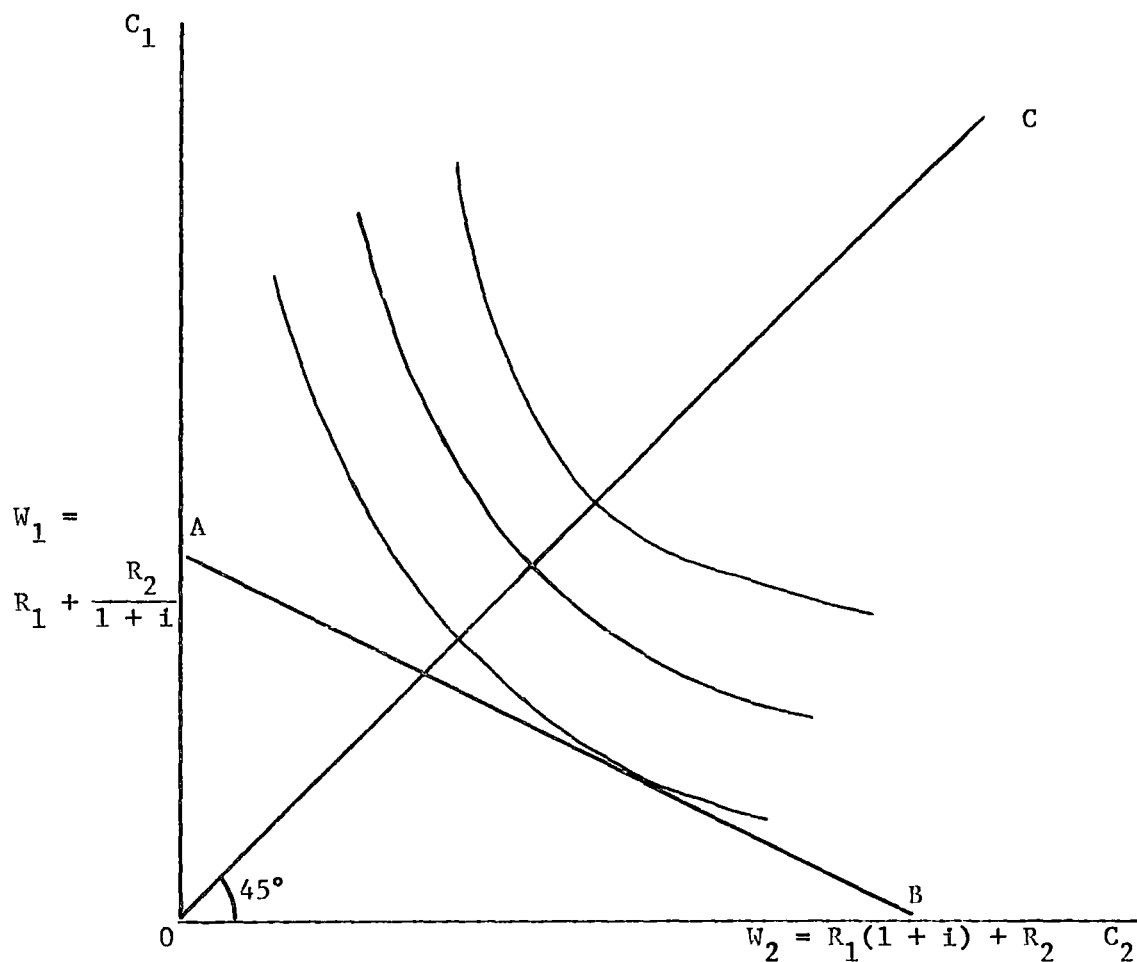
by the intercept and slope of AB, i.e. by W_1, wealth in period 1, and i, the rate of interest.[1] So C_1 is a function of W_1, i, and the shape of the indifference curves, i.e.

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1. The slope of AB is $-\dfrac{1}{1+i}$.

$$C_1 = f(W_1, i, u) \tag{3.1}$$

where u is a vector of all those factors determining the shape of the indifference curves.

To enable closer specification of (3.1), Friedman introduces some additional assumptions concerning the utility function. Implicit in figure 3.1 are the usual assumptions about the utility function which ensure that the indifference curves are negatively sloped and convex to the origin. In addition it is assumed that the utility function is a homogeneous function of C_1 and C_2. This implies that the curves all have a common slope where they intersect any line passing through the origin. To see this suppose that u' is homogeneous of degree k, i.e.

$$u'(\lambda C_1, \lambda C_2) = \lambda^k u'(C_1, C_2)$$

Thus, the partial derivatives of u' (marginal utilities) will be homogeneous of degree (k-1),[1] i.e.

$$u_1'(\lambda C_1, \lambda C_2) = \lambda^{k-1} u_1'(C_1, C_2)$$

And similarly for u_2'. Now a line from the origin to an arbitrary point (C_1^0, C_2^0) is the locus of all points (tC_1^0, tC_2^0), $t \geq 0$. The marginal rate of substitution (MRS) at (tC_1^0, tC_2^0) is equal to the ratio of the marginal utilities at that point, i.e.

$$\text{MRS} = \frac{u_1'(tC_1^0, tC_2^0)}{u_2'(tC_1^0, tC_2^0)} = \frac{t^{k-1} u_1'(C_1^0, C_2^0)}{t^{k-1} u_2'(C_1^0, C_2^0)} = \frac{u_1'(C_1^0, C_2^0)}{u_2'(C_1^0, C_2^0)}$$

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1. See R.G.D. Allen <u>Mathematical Analysis for Economists</u> (London: Macmillan, 1938), p. 319.

Thus the MRS is constant all along the line. Since the slope of an indifference curve cutting the line is equal in magnitude to the MRS, we can see that all indifference curves cutting the line have a common slope. Secondly, it is assumed that the curves are symmetric about OC, which requires that their slopes in the vicinity of OC are numerically equal to 1. This symmetry implies that the consumer in the vicinity of OC is not perturbed by a reduction of C_1 by one unit, so long as C_2 is increased by exactly one unit. That is, the consumer does not ask to be compensated for postponing consumption by asking for more in period 2 than was given up in period 1.

Now suppose $i = 0$. Hence, the slope of AB is numerically equal to one.[1] The assumption of symmetry then implies that the point of tangency will be somewhere on OC, regardless of the size of initial wealth. From figure 3.2 it is clear that, for $i = 0$, C_1 is always one half of W_1, regardless of the size of W_1. The homogeneity assumption means that for an interest rate other than zero (i.e. for a slope of AB such as in figure 3.1), points of tangency between AB and the indifference curves will all occur along some other straight line through 0, regardless of the level of initial wealth W_1. It is clear, then, that the optimal value of C_1 is always some constant fraction of W_1 for a particular value of i. The determinants of this fraction are the slope of the budget line, i, and

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1.   See footnote 1, p. 53 above.

Figure 3.2 (i = 0)
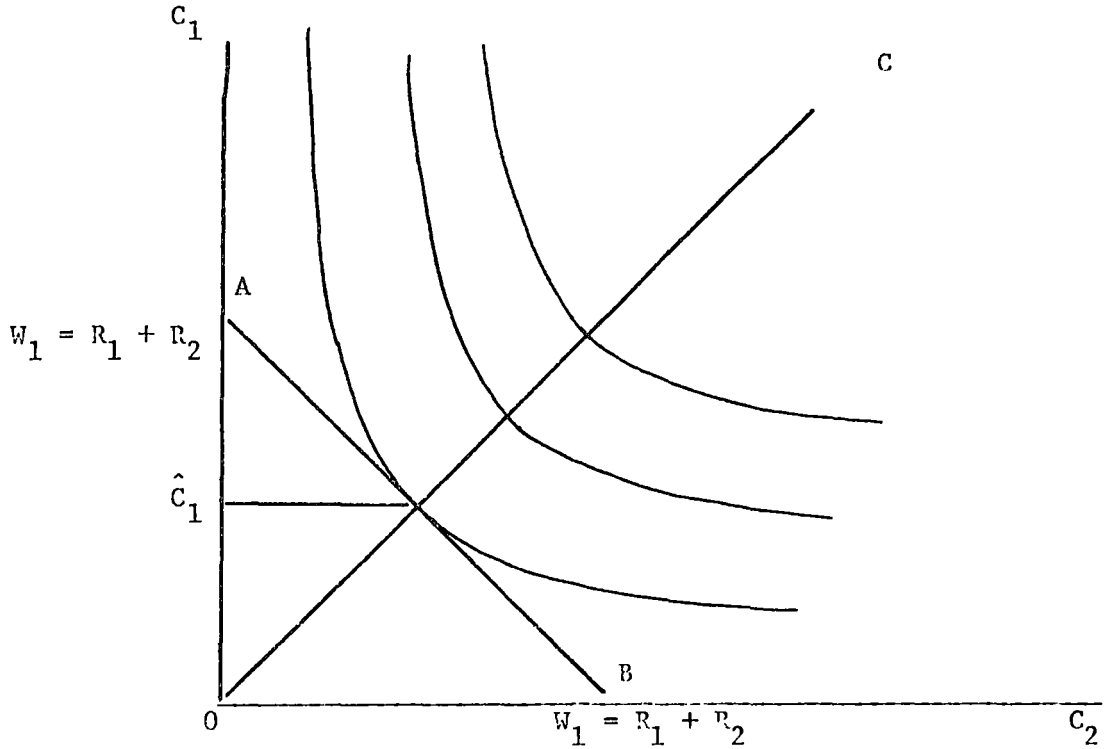
the shape of the indifference curves (u). So (3.1) becomes

$$C_1 = k'(i, u)W_1 \qquad (3.2)$$

Friedman introduces new terminology to describe income and consumption. Theoretically, income is that amount the consumer may spend while maintaining its wealth intact. Given initial wealth W_1 and an interest rate i, the theoretical income of the consumer is iW_1. This theoretical measure Friedman calls __permanent income__ (denoted by y_{p1}). Also, C_1 is the value of services it is planned to consume. Statistics of consumption expenditure give actual expenditure on goods and services. That is, they include expenditure on durables whereas, ideally, consumption statistics should only include

a measure of the use the consumer derives from his stock of durables. To emphasize this interpretation of consumption Friedman replaces C_1 by C_{p1}, which he calls <u>permanent consumption</u>. Thus (3.2) becomes:

$$C_{p1} = k'(i, u)W_1$$

$$= k'(i, u)\frac{y_{p1}}{i}$$

i.e. $\qquad C_{p1} = k(i, u)y_{p1}$ $\hfill$ (3.3)

where $k = k'/i$.

The introduction of uncertainty does not greatly alter the result so far derived. One effect of uncertainty will be to give rise to a new reason for holding wealth, and that is to build up a reserve for future emergencies. The higher is this reserve the less the need to add to it, and vice versa. So, the permanent consumption/permanent income ratio will be an increasing function of the size of the reserve. In a situation of uncertainty human wealth (i.e. expected future wage income) is not very valuable as a reserve. Indeed one purpose of the reserve will be to cover situations of diminished earning power. So, the stock of non-human wealth (i.e. income producing assets) is more critical. This can be measured by the proportion of permanent income derived from non-human wealth. For a fixed interest rate this proportion is measured by the ratio of non-human wealth to permanent income, denoted by w. The ratio is of more value, as a measure of the required reserve, than would be

the absolute value of the reserve. A given reserve may be making

a small or a large contribution to the achievement of a given

permanent income and, hence, the way to check the adequacy of the

reserve against uncertainty is to compare it with the permanent

income that is to be protected. So, Friedman concludes his discussion

of uncertainty by modifying $k(i, u)$. This is replaced with $k(i, w, u)$.

Dropping time subscripts (3.3) thus becomes

$$C_p = k(i, w, u)y_p \qquad (3.4)$$

Friedman now regards (3.4) as holding even if the consumer's planning

period is indefinitely longer than two periods.[1]

This function, applying to only a single consumer unit for

a single time period, Friedman now applies to all units collectively,

obtaining:

$$c^1 = K x^1 \qquad (3.5)$$

where c^1 = aggregate permanent consumption,

x^1 = aggregate permanent income,

and K "depends on the form of the function k for a single consumer

unit as well as on the distribution of consumer units by the variables

entering into k".[2] Friedman now assumes that (3.5) applies for all

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1. "It makes much more sense if [relation (3.4)] is regarded as a
    generalization from this special case to a longer horizon",
    ibid., p. 11.

2. Ibid., p. 115. See also ibid., pp. 18-19 for the process of
    aggregation leading from (3.4) to (3.5).

time units and that K can be taken as "roughly" constant for all time units.[1] Thus we can write (3.5) as:

$$c_t^1 = K X_t^1 \qquad t = \ldots -1, 0, 1, 2 \ldots \qquad (3.6)$$

This is the aggregate consumption function of the PIH.

3.3 <u>The PIH Model and Tests</u>   Friedman next combined his aggregate consumption function (3.6) with two equations linking measured and permanent income and measured and permanent consumption:

$$c_t^1 = K X_t^1 \qquad (3.6)$$

$$X_t = X_t^1 + X_t^2 \qquad (3.7)$$

$$C_t = c_t^1 + c_t^2 \qquad (3.8)$$

In (3.7) X_t is measured income of period t and X_t^2 is <u>transitory</u> income. In (3.8) C_t is measured consumption of period t while C_t^2 is <u>transitory</u> consumption. Friedman is not clear as to how to split X_t into X_t^1 and X_t^2. He states that X_t^1 is "analogous to the 'expected' value of a probability distribution" and that the transitory component X_t^2 is "to be interpreted as reflecting all 'other' factors, factors that are likely to be treated by the unit affected as 'accidental' or 'chance'

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1. <u>Ibid</u>., p. 115. Friedman points out (<u>Ibid</u>., p. 119) that the constancy of K is not required by the PIH but is consistent with it. The main justification used by Friedman for assuming K constant is the long-run stability of the savings-income ratio shown by the Goldsmith savings study. (<u>Ibid</u>., p. 116)

occurrences ...".[1]   And again he says that "the precise line to be drawn between permanent and transitory components is best left to be determined by the data themselves, to be whatever seems to correspond to consumer behaviour".[2]   However, it seems in keeping with the spirit of the PIH to look upon X^1 as reflecting the consumer's long-run income expectations, which may be either above or below current measured income.

As formulated in (3.6) to (3.8) the PIH is untestable since observations on X^1 and C^1 are not available, making testing of (3.6) impossible.   To allow estimation, and hence, testing, Friedman went on to specify some of the stochastic characteristics of the transitory terms C^2 and X^2.   The specifications laid down by Friedman were:

$$E(C_t^2) \; = \; E(X_t^2) \; = \; 0 \tag{3.9}$$

$$\rho_{X_t^1 X_t^2} \; = \; \rho_{C_t^1 C_t^2} \; = \; \rho_{C_t^2 X_t^2} \; = \; 0 \tag{3.10}$$

where $E(C_t^2)$ is the mean (expected) value of transitory consumption, $\rho_{X_t^1 X_t^2}$ is the correlation coefficient between permanent and transitory income etc.[3]   (3.9) states that the mean values of the observations on transitory income and transitory consumption are equal to zero. (3.10) states that transitory elements are uncorrelated and are

1. Ibid., pp. 21-22.

2. Ibid., p. 23.

3. Ibid., p. 26, p. 30.

uncorrelated with the corresponding permanent component. For example, $\rho_{x_t^1 x_t^2} = 0$ means that the transitory component of income does not vary systematically with the permanent component. Similarly, $\rho_{c_t^2 x_t^2} = 0$ means that there is no systematic link between transitory income and transitory consumption, in contrast to that which exists between permanent income and permanent consumption.

The model as now formulated is very close to the classical "errors in variables" model. This last mentioned model requires a more complete stochastic specification of the transitory terms, namely that c^2 and x^2 are normally and independently distributed variables with zero means and constant variances.[1] If formulated this way, it is possible to obtain a maximum-likelihood estimate of K using statistics on X_t and C_t only. To do this, however, it is necessary to assume that

$$\sigma_1^2 \, / \, \sigma_2^2 \; = \; \lambda$$

where σ_1^2 is the variance of X^2 and σ_2^2 is the variance of C^2, and λ is some known number.[2]

A more fruitful, and more convenient, approach, however, is to actually measure permanent income. The model is then not an example of "errors in both variables" but an example of "errors in

1. See J. Johnston, Econometric Methods pp. 148-176 esp. pp. 150-156. See also Goldberger, Econometric Theory, pp. 282-284.

2. See Johnston, Econometric Methods, p. 152 and Friedman, A Theory of the Consumption Function f.n. 13, p. 36.

the dependent variable". This model is indistinguishable from the usual "error in equations" model of econometrics. Hence, we can consistently estimate K, in equation (3.11) by least squares regression of measured consumption, C, on permanent income, X^1.[1]

Substituting (3.6) into (3.8) we get:

$$C_t = K X_t^1 + c_t^2 \tag{3.11}$$

By appropriate stochastic specification of the disturbance, c_t^2, statistical inference tests on the parameter estimates can be carried out.

The rest of this section will be devoted to an examination of some measures of permanent income, and associated tests of the PIH. The work to be looked at has been arranged into three broad groups: (1) The first group of studies relates to the PIH's implication that the consumption function has a statistically insignificant intercept. The group includes Friedman's original estimate of the PIH consumption function. (2) The second group discusses the appropriateness of Friedman's specification that $\rho_{X^1X^2} = \rho_{X^2C^2} = 0$. (3) The final group contains evidence relating to a further implication of the PIH, namely that the marginal propensity to consume out of transitory income is zero.

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1. See Johnston, Econometric Methods, p. 156; Goldberger, Econometric Theory, p. 284; C. Christ, Econometric Models and Methods (New York: John Wiley and Sons, Inc., 1968), footnote 13, p. 251. This conclusion does,however, ignore any problems caused by methods used to measure permanent income (such as distributed lag models) and possible simultaneous equation bias.

Friedman's measure of permanent income can be obtained by starting with the following adjustment model:[1]

$$\frac{dX^1(t)}{dt} = \alpha X^1(t) + \beta[X(t) - X^1(t)] \qquad (3.12)$$

where α = the trend rate of growth of income, β = a constant. In this model permanent income is <u>continuously</u> adjusted, firstly by supposing that permanent income is growing at the trend rate (hence giving $\alpha X^1(t)$ as an initial estimate of the change in X_t^1), and then adjusting for the deviation of measured income from permanent income (giving the added term $\beta[X(t) - X^1(t)]$). Assuming permanent income is zero at $t = -\infty$, (3.12) can be solved to give:[2]

$$X^1(t) = \frac{\beta}{e^{(\beta-\alpha)t}} \int_{-\infty}^{t} e^{(\beta-\alpha)x} X(x)\, dx \qquad (3.13)$$

That is, X^1 is estimated by taking a weighted average of current and past values of actual income. So long as $\beta > \alpha$, the most recent values of X have the highest weights. For this particular model the sum of the weights is greater than 1.[3] Friedman aims by this fact to ensure

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1. Friedman derived the measure somewhat differently by starting with the expression

$$X^1(t) = X_0 e^{\alpha t} + \beta \int_{-\infty}^{t} e^{\alpha(x-t)}[X(t) - X_0 e^{\alpha x}]e^{\alpha(t-x)} dx$$

   (See expression (5.16) p. 144 Friedman, <u>A Theory of the Consumption Function</u>.) That is, permanent income was put equal to the trend value of actual income ·<u>plus</u> a weighted average of past deviations of actual income from the trend value of income.

2. See Appendix 3.1 p.106 below for the solution.

3. <u>Ibid.</u>

that permanent income will not be underestimated during periods when measured income is undergoing steady growth.

Income estimates are only made for discrete periods of time and Friedman therefore could not fit (3.13) directly to time series data. Expression (3.13) was approximated by use of 17 weights: 0.330, 0.221, 0.148, ..., 0.001.[1] The weights, rounded to three decimal places, summed to unity. Applying these weights to observations 1 to 17 on X yielded an observation on permanent income, X^1, for period 17. Applying them to observations 2 to 18 yielded an observation on X^1 for period 18. Continuing in this way Friedman generated the time series for permanent income. Regressing real per capita measured consumption onto real per capita permanent income yielded:[2]

$$C_t = .88 \ X_t^1 + C_t^2 \qquad\qquad R^2 = 0.96$$

$$\text{coeff. of variation} = 4.0$$

This regression equation was fitted using time series for C_t and X_t^1 for 1905 to 1951. The equation reproduced the time profile of real per capita consumption quite well, particularly for the years prior to 1941.[3] Friedman also estimated the equation with a constant term

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1. See Friedman, A Theory of the Consumption Function, pp. 145–147 and P. Cagan, "The monetary dynamics of hyper-inflation" in Studies in the Quantity Theory of Money, ed. by M. Friedman (Chicago: Uni. of Chicago Press, 1956), pp. 39–41 for a description of the method for calculating the weights.

2. See Friedman, A Theory of the Consumption Function, p. 147, Table 15

3. Ibid., p. 148.

added so as to be able to check the zero intercept implied by the PIH. The value obtained for the constant was -4.0 with a t-ratio of 0.24, indicating support for the homogeneous form of the consumption function. Friedman reported the second estimate of K as being little different from the first.[1]

An attempt to check this conclusion (and others) was made by R. L. Moore[2] who used the 17 weights (0.330, 0.221, 0.148, etc.) calculated by Friedman to obtain permanent income estimates for the years 1949 to 1967 from data on real per capita disposable income. Moore derived a measure of real per capita permanent consumption by adding expenditure on services and single-use goods to the value of services obtained from consumer durables. The service value was obtained by applying a pattern of declining weights, based upon depreciation rates, to past expenditure on durables. The resulting regression equation was:

$$c_t^1 = \begin{matrix} -228.3 \\ (23.8) \end{matrix} + \begin{matrix} 1.07 \\ (0.03) \end{matrix} x_t^1 \qquad R^2 = 0.987$$

The parameter estimates are all highly significant, indicating rejection of the homogeneous form of the consumption function. However, there are some grounds for not considering this evidence as satisfactory The first is that the weights calculated by Friedman were used by Moore

1. _Ibid._, p. 147.

2. R. L. Moore, "The Permanent Income Hypothesis: evidence from post-war time series data", _American Economist_, Vol. 15 (1971), pp. 62-70.

to calculate permanent income. Since the actual pattern of these weights was determined by Friedman's original data, there is no a priori reason for believing that they apply to Moore's data. Secondly, the marginal propensity to consume out of permanent income is greater than unity (1.07). While it is conceivable that the mpc could be unity, it is wholly inconsonant with the PIH for the consumer to be consuming more than 100% of each increment in long-run expected income (X^1). This problem could disappear if the permanent income estimating weights were recalculated for the new set of data.

A second study, supporting Friedman's hypothesis of homogeneity, was made by Holmes.[1] Using the same data series and weights as Friedman, Holmes obtained time series of X_t^1 and X_t^2 (by subtracting X_t^1 from X_t).[2] Regressing C_t on X_t^1 and X_t^2 gave

$$
C_t = 0.017 + 0.895 \, X_t^1 + 0.267 \, X_t^2 + C_t^2 \tag{3.14}
$$
$$
\quad\;\; (0.028) \quad (0.049) \qquad (0.101)
$$
$$
R^2 = 0.954
$$
$$
d = 0.61
$$

The Durbin-Watson statistic indicates a significant level of serial correlation. As is often done in such cases, Holmes then assumed that the disturbance term (C_t^2) is generated by a first order autoregressive process, i.e.

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1. J. M. Holmes, "A direct test of Friedman's Permanent Income theory", Journal of the American Statistical Association, Vol. 65 (1970), pp. 1159-1162.

2. There is one difference between the series for X^1 obtained by Holmes and Friedman. Friedman extrapolated the X series back in time to obtain a longer series for X and hence for X^1. Since Holmes did not bother with this, his X^1 series is shorter.

$$c_t^2 = \rho c_{t-1}^2 + u_t \tag{3.15}$$

where $|\rho| < 1$ and u_t is normally distributed with zero mean and constant variance. Since $d = 0.6$, the first order autocorrelation coefficient of residuals, $\hat{\rho}$, is approximately equal to 0.7.[1] This may be taken as an estimate of the parameter ρ.[2] Combining $\hat{\rho}$ with (3.14) yielded the following alternative estimate:[3]

$$c_t = \underset{(0.052)}{0.057} + \underset{(0.083)}{0.826} x_t^1 + \underset{(0.089)}{0.361} x_t^2 + c_t^2 \tag{3.16}$$

$$R^2 = 0.853$$

At the 5% level of significance the hypothesis that the intercept is zero is acceptable.

The work done by Holmes can also throw some light on the validity of Friedman's specification that $\rho_{x_t^1 x_t^2} = 0$, i.e. that permanent and transitory incomes are uncorrelated. Having derived a series for permanent income, and having found a corresponding series for transitory income by subtracting it from measured income, Holmes estimated the correlation coefficient between x^1 and x^2 at 0.452, which was significant at the 1% level.[4] This significant correlation,

1. By definition d is approximately equal to $2(1 - \hat{\rho})$. See Goldberger, Econometric Theory, p. 244.

2. Ibid., p. 245.

3. For the method of estimation see ibid., pp. 245-246.

4. Holmes, "A direct test of Friedman's Permanent Income theory", p. 1161.

however, should not come as a surprise. Transitory income is given
as $X_t^2 = X_t - X_t^1$ by the PIH.[1] Since Holmes, like Friedman, estimates
X_t^1 by a weighted average of current and past values of X_t, it is
clear that X_t^2 will be some weighted average of current and past values
of X_t. Since X_t^1 and X_t^2 are both given by weighted averages of the
same variable it would be most surprising if they were not correlated.
Indeed Holmes has shown that where

$$X_t^1 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i X_{t-i} \qquad (3.17)$$

i.e. where permanent income is a weighted average of current and past
values of income (with geometrically declining weights), permanent and
transitory income are bound to be positively correlated.[2] Friedman
does concede the possibility that transitory income, in the form of
windfalls, may

> "introduce a positive correlation between transitory
> and permanent component of income; it is possible
> also that they introduce a negative correlation."[3]

Holmes' work, however, seems to have determined that the correlation
will be positive where the Friedman technique for estimating permanent
income is used.

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1. See (3.7) on p. 59 above.

2. J. M. Holmes, "A condition for independence of permanent and
   transitory components of a series", Journal of the American
   Statistical Association, Vol. 66 (1971), pp. 13-15.

3. M. Friedman, "Windfalls, the 'Horizon', and related concepts in
   the Permanent Income Hypothesis", in Measurement in Economics,
   C. Christ et. al. (Stanford, California: Stanford Uni. Press,
   1963), p. 20.

Friedman also laid down that $\rho_{X_t^2 C_t^2} = 0$, i.e. that there

is no systematic connection between transitory income and consumption.

D. Suits has tested this specification and concluded that it is

correct.[1] One method used by Suits was to measure X^1 and C^1 by an

average of the four previous quarters observations on X and C, i.e.

$$X_t^1 = \frac{1}{4} \sum_{i=1}^{4} X_{t-i}, \qquad C_t^1 = \frac{1}{4} \sum_{i=1}^{4} C_{t-i}$$

Then $X_t^2 = X_t - X_t^1$ and $C_t^2 = C_t - C_t^1$. Regressing C_t^2 on X_t^2 yields a

coefficient of determination of only 0.30. It can be argued[2] that

it is more meaningful to regress C_t^2 on to transitory saving,

$S_t^2 = X_t^2 - C_t^2$. For this regression Suits obtained:

$$C_t^2 = 4.510 - \underset{(0.143)}{0.358} S_t^2 \qquad r^2 = 0.11$$

Not only has r^2 fallen to 0.11 but the estimated relationship implies

that as transitory income _increases_, transitory consumption falls.

Laumas performed a similar test by first estimating permanent income

and permanent consumption, and then calculating the transitory figures

as a residual.[3] Permanent income was estimated from

1. D. B. Suits, "The Determinants of Consumer Expenditure: A Review
   of Present Knowledge", in Impacts of Monetary Policy, Commission
   on Money and Credit (Englewood Cliffs, N.J.: Prentice-Hall, 1963),
   pp. 36-39.

2. Ibid., pp. 23-26, or G. Ackley, Macroeconomic Theory (New York:
   Macmillan, 1961), pp. 233-236.

3. P. S. Laumas, "A test of the Permanent Income Hypothesis",
   Journal of Political Economy, Vol. 77 (1969), pp. 857-861.

$$X^1(t) = \beta \int_{-\infty}^{t} e^{\beta(x-t)} X(x) \, dx \qquad (3.18)$$

Similarly for permanent consumption. Annual Canadian data were used. Two estimates of permanent consumption were made, one based on consumption figures including durable goods expenditure and the other based on consumption figures excluding durable goods expenditure. Correlating the resulting transitory income and consumption figures yielded a correlation coefficient of 0.96 for the estimate based upon data including durable goods, and 0.95 for the estimate based upon data excluding durable goods. These results are more consistent with the specification $\rho_{X_t^2 C_t^2} = 1$ than $\rho_{X_t^2 C_t^2} = 0$.

As mentioned above, a further test of the PIH is to measure the marginal propensity to consume out of transitory income. The PIH states that $C_t = KX_t^1 + C_t^2$, i.e. consumption is determined only by the variables permanent income and transitory consumption. These variables, however, are not connected in any systematic way to transitory income.[1] Hence, consumption is not connected to, or determined by, transitory income. Thus, the marginal propensity to consume out of transitory income (mpc_t) will be zero.[2]

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1. Since $\rho_{X_t^1 X_t^2} = \rho_{C_t^2 X_t^2} = 0$, see (3.10) p. 60 above.

2. If X^1 is determined by an expression like (3.17) we would expect $\rho_{X_t^1 X_t^2} > 0$, according to the work of Holmes, and therefore for consumption to be partly determined by X^2 giving a positive mpc_t.

Laumas[1] estimated the mpc_t to be between 0.37 and 0.57, and the marginal propensity to consume permanent income (mpc_p) to be between 0.87 and 0.97. He also found that the difference between the estimates of mpc_t and those of mpc_p to be significant at the 1% level. Results of Holmes already presented[2] give alternative estimates of 0.267 and 0.361 for the mpc_t. Both estimates are significant at the 1% level, and are less than the corresponding estimates for the mpc_p (0.895 and 0.826 respectively). Estimates of both marginal propensities have also been made by Mrs. Jean Crockett.[3] Crockett estimated permanent income by fitting a semi-log time trend to real per capita disposable income. The trend values were defined as permanent income, and deviations from the trend as transitory income. Rather than fitting the function

$$C_t = a + bX_t^1 + cX_t^2$$

Crockett substituted $(X_t - X_t^2)$ for X_t^1, obtaining

$$C_t = a + bX_t + (c - b)X_t^2$$

The estimate of $(c - b)$ will be an estimate of the difference between mpc_t and mpc_p. Using annual, constant price data for 1929-41, 1946-60

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1. Ibid., pp. 859-860.

2. See (3.14) and (3.16) above.

3. J. Crockett, "Income and Asset Effects on Consumption: Aggregate and Cross Section", in Models of Income Determination, Vol. 28 of Studies in Income and Wealth (New York: National Bureau of Economic Research, 1964), pp. 124-128.

the following estimate was obtained:

$$C_t = 104 + \underset{(0.020)}{0.752} X_t - \underset{(0.058)}{0.153} X_t^2$$

$$\bar{R}^2 = 0.9924$$

The estimate of the difference between the two marginal propensities is significant, and indicates a mpc_t of 0.599 (= 0.752 - 0.153). A further regression, using postwar data for 1948-1960, yielded:

$$C_t = 135 + \underset{(0.018)}{0.732} X_t - \underset{(0.090)}{0.458} X_t^2$$

$$\bar{R}^2 = 0.9925$$

The estimate of the mpc_t has now fallen to 0.274 (= 0.732 - 0.458) and Crockett suggests that the higher postwar liquid asset level has reduced the importance of X^2 as an explanatory variable.[1]

From the tests examined the following conclusions can be drawn:

(1) The weight of evidence would seem to support the homogeneous form of the consumption function, the contrary conclusions of Moore being regarded as unsatisfactory on several grounds.[2]

(2) All evidence examined refutes the derived hypothesis of the PIH that the marginal propensity to consume out of transitory income is zero. This derived hypothesis is built upon the specification that

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1. Ibid., p. 126.

2. See p. 65 above.

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$$\rho_{X_t^1 X_t^2} = \rho_{C_t^2 X_t^2} = 0$$

and, as we have seen, evidence exists showing that both correlation coefficients may not be zero.[1] In the light of this evidence (that $mpc_t \neq 0$) some investigators have postulated what they term the "loose" variant of the PIH, which requires that "the marginal propensity to consume (MPC) out of transitory income, though possibly greater than zero, be appreciably smaller than the MPC out of permanent income."[2] The various time-series studies examined above all support this "loose" variant.

In spite of the failure under test of particular aspects of the PIH its basic ideas have been a stimulus to much other work on the consumption function. This stimulus may have been provided more by the intuitively attractive idea that (i) income and consumption can be split into permanent (long-run) and transitory components and (ii) that permanent consumption is principally determined by permanent income, than by any theoretical niceties of the PIH. The next two sections are an examination of two studies based upon the division of

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1. See pp. 67-70 above.

2. See p. 499 of R. C. Bird and R. G. Bodkin, "The National Service Life-Insurance Dividend of 1950 and Consumption: A further test of the "strict" Permanent-Income Hypothesis", _Journal of Political Economy_, Vol. 73 (1965), pp. 499-515. This study is one of several by Bodkin using cross-sectional data.

income and consumption into permanent and transitory components.

We shall use these studies to assess the "operational worth" of the

Permanent-Income Hypothesis.

3.4 A recent study for the United States

In this section an analysis by Zellner, Huang and Chau (ZHC)[1] of the short-run United States consumption function will be examined. Like Friedman, ZHC split consumption into permanent and transitory components:

$$C_t = c_t^1 + c_t^2 \qquad\qquad (3.19)$$

where C_t = real quarterly consumption expenditure, c_t^1 = real quarterly permanent consumption, and c_t^2 = real quarterly transitory consumption. Permanent consumption is explained by the PIH, namely

$$c_t^1 = k_1 x_t^1 \qquad\qquad (3.20)$$

where $0 < k_1 < 1$, i.e. permanent consumption is some positive fraction of permanent income. Earlier, (3.20) was treated as an exact relationship and, hence, in the regression

$$C_t = k_1 x_t^1 + w_t$$

the disturbance term, w_t, could be interpreted as transitory consumption, c_t^2, giving[2]

$$C_t = k_1 x_t^1 + c_t^2$$

1. A. Zellner, D. S. Huang, and L. C. Chau, "Further analysis of the short-run consumption function with emphasis on the role of liquid assets", Econometrica, Vol. 33 (1965), pp. 571-581.

2. See p. 62 above.

However, it seems more reasonable to assume a stochastic relationship between C_t^1 and X_t^1 such as:

$$C_t^1 = k_1 X_t^1 + u_t \qquad (3.20')$$

Using $C_t = C_t^1 + C_t^2$ we then get

$$C_t = k_1 X_t^1 + C_t^2 + u_t$$

ZHC suggest that

$$C_t^2 = \alpha(L_{t-1} - L_t^d) \qquad (3.21)$$

That is, C_t^2 is a component of C_t which bears a <u>systematic</u> relation to other variables, namely L_{t-1} = actual holdings of real liquid assets at the end of quarter (t-1) and L_t^d = desired holdings of real liquid assets at the end of quarter t. If (3.21) is also an inexact relationship, it can be written as

$$C_t^2 = \alpha(L_{t-1} - L_t^d) + v_t \qquad (3.21')$$

Combining (3.19) with (3.20') and (3.21') gives:

$$C_t = k_1 X_t^1 + \alpha(L_{t-1} - L_t^d) + u_t + v_t$$

where, say, the $\varepsilon_t = u_t + v_t$ are independently distributed with zero mean and constant variance.[1] Ignoring the disturbance term the aggregate consumption function of ZHC can be written as:

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1. In fact ZHC specify no particular stochastic properties for the disturbance term.

$$C_t = k_1 X_t^1 + \alpha(L_{t-1} - L_t^d) \qquad (3.22)$$

Going back to (3.21) for a moment, ZHC give two possible reasons for the effectiveness of this hypothesis.[1] Firstly, unexpected increments to income of a windfall or transitory nature may manifest themselves in a build up of liquid assets above the desired level, prompting consumers to remove this imbalance by expenditure of a transitory nature. Similarly, unexpected decrements to income may lead, by this process, to negative transitory consumption. Secondly, consumers desiring to undertake expenditure of a major kind (such as the purchase of new consumer durables) may build up an imbalance in liquid asset holdings prior to undertaking the expenditure. Regardless of whether the imbalance results from, or results in, a decision to purchase, a positive imbalance (actual holdings exceeding desired holdings) would be expected to lead to an increase in consumption expenditure. Thus, prior expectations are that $\alpha > 0$.

Before examining ZHC's estimates of (3.22) return for a moment to (3.20). Two alternatives to this were examined, namely:

$$C_t^1 = k_2(X_t^1 + \beta X_{t-1}^1 + \beta^2 X_{t-2}^1 + \ldots) \qquad (3.23)$$

$$C_t^1 = \pi C_{t-1}^1 + k_3 X_t^1 \qquad (3.24)$$

In (3.23) C_t^1 is determined by past as well as current values of X_t^1. The weight β is such that the most recent values of X^1 have the greatest influence on C^1, i.e. $0 < \beta < 1$. Whereas (3.20) postulates an

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1. Zellner et. al., "Further analysis of the short-run consumption function", p. 574.

instantaneous adjustment of C^1 to changes in X^1, (3.23) postulates

a lagged response possibly due to inertia on the part of consumers.

Hypothesis (3.24) implies that consumption is affected by current

permanent income and by the most recently experienced level of

permanent consumption, i.e. (3.24) is a habit persistence hypothesis.

To discriminate between these three competing hypotheses, they were

first expressed in terms of observable variables. Firstly, actual

consumption data were used in place of permanent consumption.

Secondly, permanent income was measured by a weighted average of

current and past values of actual income.[1]

$$X_t^1 = (1 - \lambda)[X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots] \qquad (3.25)$$

Combining (3.20) and (3.25) gives:

$$C_t = k_1(1 - \lambda)[X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots] \qquad (3.26)$$

Lagging (3.26) by one period and mutliplying through by λ gives:

$$\lambda C_{t-1} = k_1(1 - \lambda)[\lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots] \qquad (3.27)$$

Subtracting (3.27) from (3.26), and rearranging gives[2]

$$C_t = \lambda C_{t-1} + k_1(1 - \lambda)X_t \qquad (3.28)$$

---

1. (3.25) is the discrete analogue to the measure of permanent income
   used by Laumas. See (3.18) p. 70 above.

2. Compare (3.28) with the hypotheses tested by Barger and Klein
   (see p. 22 above), Brown (p. 35), Zellner (p. 38) and
   Griliches et. al. (p. 40).

The transformation used to reduce (3.26) to (3.28) is called a Koyck transformation after its originator.[1] Employing this transformation in a similar fashion, (3.23) and (3.25) yield:

$$C_t = (\lambda + \beta)C_{t-1} - \lambda\beta C_{t-2} + k_2(1 - \lambda)X_t \qquad (3.29)$$

and (3.24) and (3.25) yield:

$$C_t = (\lambda + \pi)C_{t-1} - \lambda\pi C_{t-2} + k_3(1 - \lambda)X_t \qquad (3.29')$$

Formulations (3.29) and (3.29') involve identical variables.[2] Since we expect that $0 < \lambda < 1$, $0 < \beta < 1$, $\pi > 0$, the coefficient of C_{t-2} in both (3.29) and (3.29') should be negative. However, regressing C_t on C_{t-1}, C_{t-2} and X_t resulted in a positive coefficient less than its standard error.[3] Hence ZHC rejected (3.23) and (3.24) in favour of (3.20).

Returning now to the aggregate consumption function (3.22). Denoting it by H(1), we have

$$H(1) \qquad C_t = k_1 X_t^1 + \alpha(L_{t-1} - L_t^d)$$

There are two unobservable variables in H(1): X_t^1 and L_t^d. To relate X_t^1 to observable magnitudes the following hypotheses are suggested by ZHC:

1. L. M. Koyck, Distributed Lags and Investment Analysis, (Amsterdam: North-Holland Publishing Coy., 1954).

2. Which should not be surprising since (3.24) is just a Koyck transformation of (3.23).

3. Zellner et. al., "Further analysis of the short-run consumption function", p. 572.

$$H(2) \qquad X_t^1 = (1 - \lambda)[X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \dots]$$

$$H(3) \qquad X_t^1 = (1 - \lambda)[X_t + (\lambda + \gamma)X_{t-1} + (\lambda + \gamma)^2 X_{t-2} + \dots]$$

In both, X_t^1 is a weighted average of current and past values of actual income. In H(2) the weights sum to unity, while in H(3) they sum to $\frac{1 - \lambda}{1 - \lambda - \gamma}$, which is $\gtrless 1$ according as $\gamma \gtrless 0$. H(3) is derived from the model:

$$X_t^1 - X_{t-1}^1 = \gamma X_{t-1}^1 + (1 - \lambda)(X_t - X_{t-1}^1)$$

This is the discrete analogue of the model used by Friedman to estimate permanent income.[1] γ is the trend rate of growth of permanent income and, as such, is positive. Thus the sum of the weights in H(3) is greater than unity.

Concerning L_t^d, two hypotheses are offered to explain its determination:

$$H(4) \qquad L_t^d = \eta X_t^1$$

$$H(5) \qquad L_t^d = \eta X_t^1 - \delta i_t$$

where i_t is a measure of the short term interest rate. H(4) is in accordance with Friedman's view that permanent income is the principal determinant of the demand for money.[2] A priori expectations are that

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1. See (3.12) p. 63 above.

2. See M. Friedman and A. J. Schwartz, "Money and business cycles", Review of Economics and Statistics, Vol. 45 (1963), Sup. pp. 32-64 esp. pp. 56-59 or J. Tobin and C. Swan, "Money and permanent income: some empirical tests", American Economic Review (Papers and Proc.), Vol. 59 (1969), pp. 285-295 for an outline of the permanent income theory of the demand for money.

$\eta > 0$. In H(5) the influence of interest rates on L_t^d is included. Since a rising interest rate implies an increasing opportunity cost of holding money and other liquid assets, L^d and i should move in opposite directions. Hence, a priori expectations are that $\delta > 0$.

By combining H(1) with one of H(2), H(3) and one of H(4), H(5), four different consumption functions can be formed. For example, consider the combination of H(1) with H(2) and H(4):

$$\text{H(1)} \qquad C_t = k_1 X_t^1 + \alpha(L_{t-1} - L_t^d)$$

$$\text{H(2)} \qquad X_t^1 = (1 - \lambda)(X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots)$$

$$\text{H(4)} \qquad L_t^d = \eta X_t^1$$

Substitute H(4) into H(1) and rearrange:

$$\therefore \qquad C_t = (k_1 - \alpha\eta)X_t^1 + \alpha L_{t-1}$$

Substituting H(2) into the last expression gives:

$$C_t = (k_1 - \alpha\eta)(1 - \lambda)(X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots) + \alpha L_{t-1}$$

Lagging this expression one period and multiplying through by λ gives

$$\lambda C_t = (k_1 - \alpha\eta)(1 - \lambda)(\lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots) + \alpha\lambda L_{t-2}$$

Subtracting from the previous expression, and rearranging, gives:

$$\text{H(1, 2, 4)} \quad C_t = \lambda C_{t-1} + \alpha L_{t-1} - \alpha\lambda L_{t-2} + (k_1 - \alpha\eta)(1 - \lambda)X_t$$

In a similar way the other three functions can be obtained, giving:

$$H(1, 3, 4) \qquad C_t = (\lambda + \gamma)C_{t-1} + \alpha L_{t-1} - \alpha(\lambda + \gamma)L_{t-2}$$
$$+ (k_1 - \alpha\eta)(1 - \lambda)X_t$$

$$H(1, 2, 5) \qquad C_t = \lambda C_{t-1} + \alpha[L_{t-1} - \lambda L_{t-2}] + \alpha\delta[i_t - \lambda i_{t-1}]$$
$$+ (k_1 - \alpha\eta)(1 - \lambda)X_t$$

$$H(1, 3, 5) \qquad C_t = (\lambda + \gamma)C_{t-1} + \alpha[L_{t-1} - (\lambda + \gamma)L_{t-2}]$$
$$+ \alpha\delta[i_t - (\lambda + \gamma)i_{t-1}] + (k_1 - \alpha\eta)(1 - \lambda)X_t$$

These composite hypotheses are non-linear in the structural parameters, the result being that SELS estimates of the coefficients of the regressors may yield several different estimates of the structural parameters. For example, in H(1, 2, 4) an estimate of α is provided by the coefficient of L_{t-1}, while a second estimate is provided by dividing the coefficient of C_{t-1} into the coefficient of L_{t-2} (and changing the sign). Except by chance, these two estimates will not be the same. This problem can be eliminated by estimating H(1, 2, 4) subject to the (non-linear) restriction that:

the coefficient of L_{t-2} = - the coefficient of L_{t-1}

x the coefficient of C_{t-1}

To do this, non-linear estimating techniques are required. ZHC provided non-linear single equation least squares (NL-SELS) and non-linear two

stage least squares (NL-2SLS) estimates of all composite hypotheses.[1]

Estimates of the last two hypotheses [H(1, 2, 5) and H(1, 3, 5)] showed δ to be small with respect to its standard error, thereby favouring H(4) over H(5). So attention will be concentrated on the estimates of H(1, 2, 4) and H(1, 3, 4). Non-linear estimation of H(1, 2, 4) subject to the restriction mentioned above will provide estimates of λ, α and $(k_1 - \alpha\eta)(1 - \lambda)$. Similarly, non-linear estimation of H(1, 3, 4) subject to the same restriction will provide estimates of $(\lambda + \gamma)$, α and $(k_1 - \alpha\eta)(1 - \lambda)$. Since the list of regressors in both H(1, 2, 4) and H(1, 3, 5) is the same (namely C_{t-1}, L_{t-1}, L_{t-2} and X_t), one non-linear regression of C_t on C_{t-1}, L_{t-1}, L_{t-2} and X_t, subject to the above mentioned restriction, will provide estimates which can be interpreted as estimates of λ, α and $(k_1 - \alpha\eta)(1 - \lambda)$ for H(1, 2, 4) or as estimates of $(\lambda + \gamma)$, α and $(k_1 - \alpha\eta)(1 - \lambda)$ for H(1, 3, 5). ZHC carried out this regression both with and without a constant term. For both NL-SELS and NL-2SLS the estimate of the constant was many times its standard error, and the results for the regressions including the constant are therefore shown in table 3.1 below.

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1. 2SLS is a method for obtaining consistent parameter estimates for a relationship which is part of a simultaneous system of equations, as the consumption function usually is.
   See Goldberger, Econometric Theory pp. 329-336 or H. Theil, Principles of Econometrics (Amsterdam: North-Holland Publishing Coy., 1971), pp. 451-458 for a description of two stage least squares.

TABLE 3.1[a]

Parameter estimated			
H(1, 2, 4)	H(1, 3, 4)	NL–SELS	NL–2SLS
λ	$\lambda + \gamma$	0.2419 (0.1051)	0.5241 (0.1258)
α	α	0.3758 (0.0502)	0.4992 (0.0968)
$(k_1 - \alpha n)(1 - \lambda)$	$(k_1 - \alpha n)(1 - \lambda)$	0.5381 (0.0805)	0.3064 (0.0968)
constant	constant	−12.470 (3.299)	−10.884 (3.669)

a. Derived from Table I p. 575 Zellner et. al. "Further Analysis of the Short-Run Consumption Function".

The estimates of λ and $\lambda + \gamma$ are in the region (0, 1), as they must be if the weights in H(2) and H(3) are to decline. The estimates of α are also in agreement with prior expectations as to sign. All parameter estimates are high with respect to their standard errors. The disappointing feature of the results is the presence of the apparently significant constant term. If the consumption function is in fact of the form

$$C_t = a + k_1 X_t^1 + \alpha(L_{t-1} - L_t^d) \qquad a < 0$$

then in the long-run (when $L_{t-1} = L_t^d$), we have

$$C_t = a + k_1 X_t^1 \qquad a < 0$$

which implies an increasing ratio of consumption to permanent (long-run) income, as permanent income grows. This is not born out by facts such as the data produced by Kuznets and referred to in the previous chapter.[1]

Note that ZHC have made no test for autocorrelation of disturbances. Indeed, they go ahead to assume that the disturbances are autocorrelated and are generated by a first-order autoregressive process, i.e.

$$v_t = \rho v_{t-1} + \varepsilon_t$$

where ε_t is assumed to be non autocorrelated with zero mean and constant variance σ^2. Take $H(1, 2, 5)$

$$C_t = \lambda C_{t-1} + \alpha[L_{t-1} - \lambda L_{t-2}] + \alpha\delta[i_t - \lambda i_{t-1}]$$
$$+ (k_1 - \alpha\eta)(1 - \lambda)X_t + v_t$$

Lagging one period, and multiplying through by ρ, gives

$$\rho C_{t-1} = \rho\lambda C_{t-2} + \rho\alpha[L_{t-2} - \lambda L_{t-3}] + \rho\alpha\delta[i_{t-1} - \lambda i_{t-2}]$$
$$+ \rho(k_1 - \alpha\eta)(1 - \lambda)X_{t-1} + \rho v_{t-1}$$

Subtracting this expression from the previous, and rearranging we get:

$$C_t = (\lambda + \rho)C_{t-1} - \rho\lambda C_{t-2} + \alpha[L_{t-1} - \lambda L_{t-2}] - \rho\alpha[L_{t-2} - \lambda L_{t-3}]$$
$$+ \alpha\delta[i_t - \lambda i_{t-1}] - \rho\alpha\delta[i_{t-1} - \lambda i_{t-2}]$$
$$+ (k_1 - \alpha\eta)(1 - \lambda)(X_t - \rho X_{t-1}) + \varepsilon_t$$

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1. See p. 28 above.

Note that $v_t - \rho v_{t-1}$ has been replaced with ε_t, assumed to be non-autocorrelated. NL-SELS and NL-2SLS estimates of this function were made. The NL-2SLS estimates were preferred because of the more reasonable average lag in the adjustment of permanent income to actual income implied by the 2SLS estimate of λ.[1] In addition the NL-2SLS estimate of the function with constant term resulted in an insignificant estimate of the constant. Hence in table 3.2 the NL-2SLS estimates of the function without constant are presented. Once again δ is insignificant, indicating support for

$$H(4) \qquad L_t^d = \eta X_t^1$$

over

$$H(5) \qquad L_t^d = \eta X_t^1 - \delta i_t$$

The coefficient of the liquid asset imbalance term (α) is highly significant, indicating the potential worth of such a term in an Australian study. A piece of corroborative evidence on the liquid asset imbalance term was provided by Mrs. Jean Crockett.[2] As noted in the previous section Crockett estimated X^1 by the trend values of income obtained by fitting a semi-log time trend to actual income

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1. The average lag is given by $\lambda/(1 - \lambda)$. For the NL-2SLS estimate, the lag averaged 6.9 quarters, while for the NL-SELS estimate the lag averaged less than one quarter. For the formula $\lambda/(1 - \lambda)$ see p. 772 of K. F. Wallis, "Some recent developments in applied econometrics: dynamic models and simultaneous equation systems", Journal of Economic Literature, Vol. 7 (1969), pp. 771-798.

2. See Crockett, "Income and Asset Effects on Consumption", p. 127.

TABLE 3.2[a]

Parameter Estimated	Estimate
λ	0.8732 (0.0410)
α	0.6974 (0.1531)
δ	10.168 (7.251)
$k_1 - \alpha\eta$	0.4045 (0.1195)
ρ	-0.3213 (0.1241

a. Taken from table IV Zellner et. al. "Further analysis of the short-run consumption function".

data. "Equilibrium" liquid asset holdings were then found from a regression of liquid assets on to permanent income (X^1) with a time trend added. The actual expression used was

$$L^1_{t-1} = -190 + 0.956 \, X^1_t + 6.028 \, t$$

where L^1_{t-1} = equilibrium liquid assets at the end of period t-1. The following two regressions (using annual data) were calculated:

$$C_t = 75 + \underset{(0.014)}{0.766} \, X_t - \underset{(0.045)}{0.238} \, X^2_t + \underset{(0.017)}{0.083} \, (L_{t-1} - L^1_{t-1})$$

$$\bar{R}^2 = 0.9961$$

$$C_t = 163 + 0.716 \underset{(0.009)}{X_t} - 0.219 \underset{(0.045)}{\Delta X_t} + 0.027 \underset{(0.017)}{(L_{t-1} - L^1_{t-1})}$$

$$\bar{R}^2 = 0.9958$$

The significant coefficients of $(L_{t-1} - L^1_{t-1})$, particularly in the first regression, indicate further support for the Zellner, Huang and Chau thesis.

3.5 <u>A model for Britain</u>   In a series of papers,[1] Stone and Rowe have developed and tested a consumption function for Britain based upon the PIH. The Stone-Rowe (S-R) model splits income and consumption into permanent and transitory components, the separate components of consumption each being explained by different variables. In addition, the S-R model introduces wealth into the consumption function on the grounds that from time-to-time "abnormal" accumulations of wealth will lead consumers to undertake abnormal levels of expenditure. "The general proposition ... is that whereas we should expect more income to be accompanied by more spending and more saving, we should expect more wealth to be accompanied by more spending and less saving."[2]

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1. R. Stone and D. A. Rowe, "A post-war expenditure function", The Manchester School, Vol. 30 (1962), pp. 187-201, R. Stone; "Private saving in Britain, past, present and future", The Manchester School, Vol. 32 (1964), pp. 79-112; R. Stone, "Spending and saving in relation to income and wealth", L'Industria, (No. 4 1966), pp. 471-499 reprinted in R. Stone, Mathematical Models of the Economy and other Essays (London: Chapman and Hall Ltd., 1970).

2. Stone, Mathematical Models of the Economy and other Essays, pp. 121-122. Stone and Rowe give the credit for the idea of including wealth in the consumption function to Modigliani and R. Brumberg, whose work will be examined in chapter four below.

Stone and Rowe split beginning-of-period wealth into two parts, normal or permanent wealth, W_t^1, and transitory wealth, W_t^2. Thus the first three relationships of the S-R model are:

$$C_t = C_t^1 + C_t^2 \tag{3.30}$$

$$X_t = X_t^1 + X_t^2 \tag{3.31}$$

$$W_t = W_t^1 + W_t^2 \tag{3.32}$$

Permanent consumption is a homogeneous linear function of the permanent components of income and wealth:

$$C_t^1 = \alpha_1 W_t^1 + \beta_1 X_t^1 \tag{3.33}$$

Transitory consumption is proportional to transitory income:[1]

$$C_t^2 = \beta_2 X_t^2 \tag{3.34}$$

So as to be able to replace the unobservable variable, X^1, the S-R model postulates that permanent income is a weighted average of current and past values of income, i.e.

$$X_t^1 = \lambda [X_t + (1 - \lambda)X_{t-1} + (1 - \lambda)^2 X_{t-2} + \ldots]$$

Applying a Koyck transformation[2] to this gives

$$X_t^1 = \lambda X_t + (1 - \lambda)X_{t-1}^1 \tag{3.35}$$

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1. This is contrary to Friedman's "strict" variant of the PIH but is in accord with the "loose" version, supporting evidence for which is given above. See p. 73 above.

2. See p. 78 above.

Permanent wealth is similarly defined by a weighted average of current and past values of actual wealth. Hence we can write

$$W_t^1 = \lambda W_t + (1 - \lambda)W_{t-1}^1 \qquad (3.36)$$

Note that the weights used to formulate W^1 are identical with those used to formulate X^1.[1] The seven relationships (3.30) to (3.36) constitute the basic S-R model.

To derive the S-R consumption (saving) function substitute (3.33) and (3.34) into (3.30), giving:

$$C_t = \alpha_1 W_t^1 + \beta_1 X_t^1 + \beta_2 X_t^2$$

Using (3.31) this gives

$$C_t = \alpha_1 W_t^1 + (\beta_1 - \beta_2)X_t^1 + \beta_2 X_t \qquad (3.37)$$

Now, substitute (3.35) and (3.36) into (3.37):

$$C_t = \alpha_1 \lambda W_t + \alpha_1(1 - \lambda)W_{t-1}^1 + (\beta_1 - \beta_2)\lambda X_t$$
$$+ (\beta_1 - \beta_2)(1 - \lambda)X_{t-1}^1 + \beta_2 X_t \qquad (3.38)$$

Lagging (3.37) by one period and multiplying through by $(1 - \lambda)$ gives

$$(1 - \lambda)C_{t-1} = \alpha_1(1 - \lambda)W_t^1 + (\beta_1 - \beta_2)(1 - \lambda)X_{t-1}^1$$
$$+ \beta_2(1 - \lambda)X_{t-1} \qquad (3.39)$$

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1. Had different weights been used in (3.35) and (3.36) more than one Koyck transformation would then have been required to eliminate the unobserved variables. Alternatively the method outlined in Nerlove, _Distributed Lags and Demand Analysis_ pp. 25-29 could have been used. In either case the number of lagged values of variables would have been larger.

Subtracting (3.39) from (3.38) and rearranging gives:

$$C_t = \alpha_1 \lambda W_t + [\beta_1 \lambda + \beta_2 (1 - \lambda)] X_t - \beta_2 (1 - \lambda) X_{t-1}$$
$$+ (1 - \lambda) C_{t-1} \qquad (3.40)$$

Stone and Rowe chose to estimate (3.40) in ratio form, the dependent variable being the ratio of savings to income. Estimating in ratio form will help to suppress any affects of common trends in the variables, and hence reduce the problem of multicollinearity. This transformation is sometimes useful in dealing with problems caused by heteroskedastic disturbances.[1] Since savings, S_t, equals income _minus_ consumption, (3.40) yields:

$$\frac{S_t}{X_t} = [1 - \beta_1 \lambda - \beta_2 (1 - \lambda)] - \alpha_1 \lambda \frac{W_t}{X_t} + \beta_2 (1 - \lambda) \frac{X_{t-1}}{X_t}$$
$$- (1 - \lambda) \frac{C_{t-1}}{X_t} \qquad (3.41)$$

That is:

$$\frac{S_t}{X_t} = b_0 + b_1 \frac{W_t}{X_t} + b_2 \frac{X_{t-1}}{X_t} + b_3 \frac{C_{t-1}}{X_t} \qquad (3.42)$$

where $b_0 = [1 - \beta_1 \lambda - \beta_2 (1 - \lambda)]$

$b_1 = -\alpha_1 \lambda$

$b_2 = \beta_2 (1 - \lambda)$

$b_3 = -(1 - \lambda)$

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1. Heteroskedastic disturbances do not have a constant variance, in which case the ordinary least squares estimator is not a BLUE estimator. See Goldberger, _Econometric Theory_, pp. 235-236.

From least squares estimates of the four regression coefficients $(b_0, \ldots, b_3)$ estimates of the four structural parameters $(\alpha_1, \beta_1, \beta_2, \lambda)$ can be found as follows

$$\alpha_1 = -b_1/(1 + b_3)$$

$$\beta_1 = (1 - b_0 - b_2)/(1 + b_3)$$

$$\beta_2 = -b_2/b_3 \tag{3.43}$$

$$\lambda = (1 + b_3)$$

Prior expectations are that all parameters lie between 0 and 1, since λ is the weighting factor in a distributed lag, and since the other factors are marginal propensities to consume. Since a given increment to the _flow_ of permanent income should have a greater effect on consumption than would a similar increment to the _stock_ of permanent wealth, β_1 should be larger than α_1.

Using seasonally adjusted price deflated quarterly data for the period 1955(II) to 1963(I), the following estimate of (3.42) was obtained:[1]

$$\frac{S_t}{X_t} = \begin{array}{c} 0.472 \\ (0.121) \end{array} - \begin{array}{c} 0.0130 \\ (0.0060) \end{array}\frac{W_t}{X_t} + \begin{array}{c} 0.394 \\ (0.111) \end{array}\frac{X_{t-1}}{X_t} - \begin{array}{c} 0.684 \\ (0.145) \end{array}\frac{C_{t-1}}{X_t}$$

$$R^2 = 0.893$$
$$d = 2.17$$

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1. Stone, "Private saving in Britain, past, present and future", p. 87.

All parameters are at least twice their standard errors. Using (3.43) the following estimates of the structural parameters can be found:

$$\alpha_1 = 0.0411, \quad \beta_1 = 0.411, \quad \beta_2 = 0.583, \quad \lambda = 0.316$$

All estimates of structural parameters conform with prior expectations about sign and magnitude. The value of λ implies an average lag of 2.1 quarters, considerably shorter than the 6.9 quarters obtained for the U.S. economy by Zellner, Huang and Chau.[1] The time pattern of the savings ratio was quite closely reproduced.[2] The value of R^2 was quite remarkably high for an equation in ratio form.

The basic S-R model has been varied in a number of ways. Possibly the most interesting variation was the inclusion of a government policy variable G.[3] The policy variable G was an index of the measures used by the government to dampen down, or stimulate, consumption spending. The deviation of G from its normal level, G^1, was included as a factor determining the level of transitory consumption. That is, (3.34) was replaced with:

$$C_t^2 = \beta_2 X_t^2 + \gamma(G_t - G_t^1) \tag{3.44}$$

where G_t^1 was measured by a weighted average of current and past values of G, i.e.

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1. See p. 85 above. Note that because of the notational difference between H(2) and (3.35) the appropriate formula for calculating the average lag is now $\dfrac{1 - \lambda}{\lambda}$ rather than $\dfrac{\lambda}{1 - \lambda}$.

2. Stone, op. cit., p. 88 diagram 2 or p. 109 Table 2.

3. Ibid., pp. 103-104, and Stone and Rowe, "A post-war expenditure function", pp. 187-188.

$$G_t^1 = \lambda G_t + (1 - \lambda)G_{t-1}^1 \qquad (3.45)$$

Note that once again the same weighting factor λ has been used to build up the weighted average. When G is above its normal level a restraint on spending should be felt, and hence prior expectations were that $\gamma < 0$. Manipulating the model in the same way as before, and expressing in ratio form gave:

$$\frac{S_t}{X_t} = b_0 + b_1 \frac{W_t}{X_t} + b_2 \frac{X_{t-1}}{X_t} + b_3 \frac{G_t - G_{t-1}}{X_t} + b_4 \frac{C_{t-1}}{X_t} \qquad (3.46)$$

where $b_0 = 1 - \beta_1 \lambda - \beta_2(1 - \lambda)$

$b_1 = -\alpha_1 \lambda$

$b_2 = \beta_2(1 - \lambda)$

$b_3 = -\gamma(1 - \lambda)$

$b_4 = -(1 - \lambda)$

Estimating (3.46) by SELS using seasonally adjusted, price-deflated quarterly data for the period 1955(II) to 1961(I) gave:[1]

$$\frac{S_t}{X_t} = \underset{(0.104)}{0.541} - \underset{(0.0104)}{0.0220} \frac{W_t}{X_t} + \underset{(0.125)}{0.366} \frac{X_{t-1}}{X_t}$$

$$+ \underset{(0.718)}{0.505} \frac{G_t - G_{t-1}}{X_t} - \underset{(0.172)}{0.648} \frac{C_{t-1}}{X_t}$$

$$R^2 = 0.9916$$
$$d = 2.12$$

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1. Stone and Rowe, "A post-war expenditure function", p. 192.

This estimate implies that the structural parameters are:

$$\alpha_1 = 0.0625, \quad \beta_1 = 0.265, \quad \beta_2 = 0.565, \quad \gamma = -0.779, \quad \lambda = 0.352$$

These estimates conform with all prior expectations as to sign and magnitude. The value of R^2 is higher than for the previous estimate, while the value of the Durbin-Watson statistic is consistent with the hypothesis of no serial correlation. The coefficient b_3 is insignificant, implying that either $\gamma = 0$, i.e. that the government policy variable is insignificant, or that $\lambda = 1$. The second alternative implies that $X_t^1 = X_t$ and $W_t^1 = W_t$,[1] i.e. that transitory components of income and wealth are zero. The first of these alternatives is the more reasonable. The apparent failure of the policy variable may be due to the relatively unsophisticated device used to measure G, namely the percentage downpayment on the hire-purchase of radio and electrical goods. The variable is intuitively attractive and should not be disregarded on the basis of this much evidence alone.

Other variations of the model tried were:

(1) To attempt to measure the influence of transitory wealth on transitory consumption by replacing (3.34) with:

$$c_t^2 = \alpha_2 W_t^2 + \beta_2 X_t^2$$

This hypothesis has not been tested with quarterly data, but

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1. See (3.35) and (3.36).

for eight different regressions using annual data the estimates of α_2 all had the wrong sign (negative).[1]

(2) To allow for potentially different responses of wage and profit earners the income variable was split. The relationships of the model are then:

$$C_t = C_t^1 + C_t^2$$

$$W_t = W_t^1 + W_t^2$$

$$X_t = X_{at} + X_{bt}$$

$$X_{at} = X_{at}^1 + X_{at}^2$$

$$X_{bt} = X_{bt}^1 + X_{bt}^2$$

$$X_{at}^1 = \lambda X_{at} + (1 - \lambda)X_{a,t-1}^1$$

$$X_{bt}^1 = \lambda X_{bt} + (1 - \lambda)X_{b,t-1}^1$$

$$W_t^1 = \lambda W_t + (1 - \lambda)W_{t-1}^1$$

$$C_t^1 = \alpha_1 W_1 + \beta_{1a}X_{at}^1 + \beta_{1b}X_{bt}^1$$

$$C_t^2 = \beta_{2a}X_{at}^2 + \beta_{2b}X_{bt}^2$$

where X_{at} and X_{bt} are the disposable incomes of, say, wage and non-wage income earners respectively. Note that in building up the estimates of

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1. See Stone, <u>Mathematical Models of the Economy</u>, p. 129, Table 1.

permanent wealth and permanent income for wage and non-wage income earners, the same weighting factor λ has been used. Solving these equations together gives:

$$\frac{S_t}{X_t} = [1 - \beta_{1a}\lambda - \beta_{2a}(1 - \lambda)] \frac{X_{at}}{X_t} + [1 - \beta_{1b}\lambda - \beta_{2b}(1 - \lambda) \frac{X_{bt}}{X_t}$$

$$- \alpha_1\lambda \frac{W_t}{X_t} + \beta_{2a}(1 - \lambda) \frac{X_{a,t-1}}{X_t} + \beta_{2b}(1 - \lambda) \frac{X_{b,t-1}}{X_t}$$

$$- (1 - \lambda) \frac{C_{t-1}}{X_t}$$

This equation has not been estimated with quarterly data, but an estimate using annual data has been calculated and shows a significant difference between β_{1a} and β_{1b}.[1]

Before concluding this section there is one further point to consider – the long-run consumption (savings) function implied by the S-R model. As noted above, when quarterly data was used the model reproduced the time path of the savings-income ratio quite well. There is, however, the question of the model's ability to reproduce the (constant) savings-income ratio shown by long-run data such as the Kuznet's data. To examine this question Stone and Rowe supposed income to be growing exponentially from a base $\bar{X}$; i.e.

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1. Stone, "Private saving in Britain, past, present and future", pp. 84, 85.

$$X_t = \bar{X} e^{\rho t}$$

where ρ is the rate of growth of income.[1] Hence

$$X_{t-k} = \bar{X} e^{\rho(t-k)} = \bar{X} e^{\rho t} e^{-k\rho}$$

$$= X_t e^{-k\rho} \tag{3.46a}$$

Now, permanent income is given by a weighted average of current and past values of actual income:

$$X_t^1 = \lambda[X_t + (1 - \lambda)X_{t-1} + (1 - \lambda)^2 X_{t-2} + \ldots]$$

Using (3.46a) this can be written as:

$$X_t^1 = \lambda[X_t + (1 - \lambda)X_t e^{-\rho} + (1 - \lambda)^2 X_t e^{-2\rho} + \ldots]$$

$$= \lambda X_t \left[\frac{1}{1 - (1 - \lambda)e^{-\rho}} \right]$$

since $0 < (1 - \lambda)^k e^{-k\rho} < 1$ for $k > 0$. Hence

$$\frac{X_t^1}{X_t} = \frac{\lambda}{1 - (1 - \lambda)e^{-\rho}} = \xi \tag{3.47}$$

where ξ is a constant for any particular value of ρ. Now, if wealth is also assumed to be growing at the rate ρ, a similar argument yields:

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1. That is, $\dfrac{dX_t}{dt} / X_t = \rho$. An assumption more in keeping with the discrete analysis of the S-R model above is

$$\frac{X_t - X_{t-1}}{X_{t-1}} = \rho$$

Since the results to be derived are not altered by this assumption, and the derivation is a little more complex, the Stone-Rowe assumption has been used.

$$\frac{W_t^1}{W_t} = \xi \tag{3.48}$$

Dropping time subscripts (for convenience), (3.37) implies:

$$C = \alpha_1 \frac{W^1}{W} \cdot W + [(\beta_1 - \beta_2) \frac{X^1}{X} + \beta_2]X$$

which gives, using (3.47) and (3.48),

$$C = \alpha_1 \xi W + [(\beta_1 - \beta_2)\xi + \beta_2]X$$

Now $\frac{dW}{dt}$, the rate of change of wealth, equals savings. Therefore

$$\frac{dW}{dt} = S = X - C$$

$$= [1 - \beta_1\xi - \beta_2(1 - \xi)]X - \alpha_1\xi W$$

That is,

$$\frac{dW}{dt} = \phi X - \psi W$$

where $\phi = [1 - \beta_1\xi - \beta_2(1 - \xi)]$, $\psi = \alpha_1\xi$. Prior expectations are that ϕ and ψ are positive, since "the general proposition ... is that whereas we should expect more income to be accompanied by more spending and more saving, we should expect more wealth to be accompanied by more spending and less saving."[1] Since the rate of growth of wealth is ρ, $\frac{dW}{dt} = \rho W$. Hence

$$\rho W = \phi X - \psi W$$

$$\therefore \quad \frac{W}{X} = \frac{\phi}{\rho + \psi} \tag{3.49}$$

---

1. Stone, <u>Mathematical Models of the Economy and other Essays</u>, pp. 121-122.

The savings ratio is now obtained directly from (3.49).

$$\frac{S}{X} = \frac{dW}{dt} / X = \frac{\rho W}{X}$$

which gives, from (3.49):

$$\frac{S}{X} = \frac{\rho\phi}{\rho + \psi} \tag{3.50}$$

Since ϕ and ψ are functions of ρ only, (3.50) shows that the savings ratio is a function of the rate of growth of income only. For $\rho = 0$, i.e. for a situation of no growth in income, (3.50) implies that $S/X = 0$. That is, in the case of zero growth in income the marginal (and average) propensity to consume is unity. As the rate of income growth increases so does the savings ratio.[1] For all rates of income growth, however, savings (and hence consumption) are proportional to income. Thus the long-run consumption function of the S-R model is

$$C_t = k\, X_t$$

where k is a function of the long-run rate of growth of income. Hence the model is not inconsistent with the (constant) long-run savings-income ratio revealed by historical data such as that of Kuznets.

For comparison, take the Zellner-Huang-Chau consumption function and assume income is growing exponentially at the rate ρ. From (3.22) we have

$$C_t = k_1 X_t^1 + \alpha(L_{t-1} - L_t^d)$$

---

1. $\dfrac{d(S/X)}{d\rho} = \dfrac{\psi\phi}{(\rho + \psi)^2} > 0$, since a priori $\psi, \phi > 0$.

Suppose also that in the long-run $L_{t-1} = L_t^d$. Hence the long-run consumption function is

$$C_t = k_1 X_t^1$$

Now X_t^1 is given by[1]

$$X_t^1 = (1 - \lambda)[X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots] \qquad (3.25)$$

$$= (1 - \lambda)[1 + \lambda e^{-\rho} + \lambda^2 e^{-2\rho} + \ldots]X_t$$

$$= \frac{1 - \lambda}{1 - \lambda e^{-\rho}} X_t$$

since $X_t = \bar{X} e^{\rho t}$. That is

$$X_t^1 = \xi X_t$$

where $\xi = \dfrac{1 - \lambda}{1 - \lambda e^{-\rho}}$.[2] Thus the long-run savings ratio for ZHC is

$$\frac{S_t}{X_t} = 1 - \frac{C_t}{X_t}$$

i.e. $$\frac{S_t}{X_t} = 1 - k_1 \xi \qquad (3.51)$$

---

1. ZHC also estimated X_t^1 from

$$X_t^1 = (1 - \lambda)[X_t + (\lambda + \gamma)X_{t-1} + (\lambda + \gamma)^2 X_{t-2} + \ldots]$$

   We have used (3.25) in preference to this, as (3.25) was equivalent to the formula used by Stone and Rowe. No derived conclusions are materially affected by this choice.

2. Comparing (3.25) and (3.35) shows that $(1 - \lambda)$ in the ZHC model is equivalent to λ in the S-R model. Hence the expression for ξ just derived for the ZHC model is identical with the corresponding expression (3.47) for the S-R model.

(3.51) is constant for a particular value of ρ, and hence the ZHC

model is consistent with long-run data of the Kuznets' type. For

the situation of no growth ($\rho = 0$) $\xi = 1$ and hence $\frac{S_t}{X_t} = 1 - k_1$.

Since $0 < k_1 < 1$ (k_1 is the marginal propensity to consume out of

permanent income), we can see that $0 < \frac{S_t}{X_t} < 1$ in a situation of no

growth. Hence the marginal propensity to consume is less than unity

in a no growth situation. This contrasts with the S-R model which

implies a long-run mpc of unity in a no growth situation.[1]

3.6 <u>Conclusions</u>   This chapter has been concerned with a discussion

of Friedman's Permanent Income Hypothesis and some examples of

subsequent research which have made use of it. Ignoring time subscripts,

we saw from section 3.2 and 3.3 that the PIH can be stated as:

$$c^1 = K X^1 \tag{3.6}$$

$$X = X^1 + X^2 \tag{3.7}$$

$$C = c^1 + c^2 \tag{3.8}$$

$$E(c^2) = E(X^2) = 0 \tag{3.9}$$

$$\rho_{X^1 X^2} = \rho_{c^1 c^2} = \rho_{c^2 X^2} = 0 \tag{3.10}$$

Substituting (3.6) into (3.8) yields:

$$C = K X^1 + c^2 \tag{3.11}$$

---

1. See p. 99 above.

Friedman estimated X^1 by a weighted average of current and past values of actual income (X). An estimate of K was then obtained by regressing actual consumption on to permanent income (X^1) (C^2 being treated as though it were a disturbance term).

In section 3.3, evidence relating to the various parts of the PIH was examined. Specifically, the following hypotheses were discussed:

(1) The consumption function (3.11) is homogeneous (i.e. has zero intercept).

(2) The two income components are uncorrelated (i.e. $\rho_{X^1 X^2} = 0$)

(3) The two consumption components are uncorrelated (i.e. $\rho_{C^1 C^2} = 0$)

(4) The marginal propensity to consume transitory income is zero (i.e. $mpc_t = 0$).

The evidence examined supported (1) but was against (2), (3) and (4), though in the case of (2) the conclusion $\rho_{X^1 X^2} \neq 0$ was dictated by the method used to estimate permanent income.[1] Thus, the PIH is judged to be defective in the version (3.6) to (3.10). However, the real worth of the PIH is perhaps best judged by the success or otherwise of subsequent research which has used the concepts and framework of the PIH as its basis. To this end, two well known studies of the aggregate consumption function employing the notions of the PIH were discussed in sections 3.4 and 3.5.

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1. See p. 68 above.

In section 3.4 work by A. Zellner, D. S. Huang and

L. C. Chau was examined. Two alternatives to (3.6) above were

tested, namely an inertia hypothesis

$$C_t^1 = k_2(X_t^1 + \beta X_{t-1}^1 + \beta^2 X_{t-2}^1 + \ldots)$$

and a habit persistence hypothesis

$$C_t^1 = \pi C_{t-1}^1 + k_3 X_t^1$$

Both alternatives were rejected after testing. Transitory consumption

was made proportional to a liquid asset imbalance term, giving

$$C_t = k_1 X_t^1 + \alpha(L_{t-1} - L_t^d) \tag{3.22}$$

for the aggregate consumption function. The weight of evidence

supported the following hypotheses about the determination of X^1 and L^d:

$$X_t^1 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i X_{t-i}$$

$$L_t^d = \eta X_t^1$$

Function (3.22) performed well in terms of (1) the significance of

parameter estimates, (2) the agreement of signs and magnitudes of

parameter estimates with prior expectations. An interesting feature

of the work was the use of a non-linear estimation method to avoid

conflicting estimates of structural parameters.

In section 3.5 work by R. Stone and D. A. Rowe was examined.

A very important feature of the Stone-Rowe model is the introduction

of wealth into the consumption function on the hypothesis that the
consumer has some notion of an equilibrium wealth-income ratio and
his savings behaviour is partly determined by attempts to adjust
towards this. Wealth, like income and consumption, is split into
a permanent component and a transient component. A series of
estimates of the basic S-R model and its variants has been made using
both annual and quarterly data. Judged by conventional tests – size
of R^2, value of Durbin-Watson statistic, significance of the parameter
estimates and their agreement with a priori expectations as to sign
and magnitude – the model does well. One variation of the model is
to include a government policy variable G as a determinant of the
level of transitory consumption giving:

$$C_t^2 = \beta_2 X_t^2 + \gamma(G_t - G_t^1)$$

While the estimate of γ is significant when annual data is employed,
it is insignificant when quarterly data is used. Both the S-R model,
and the ZHC model, have a long-run consumption function consistent
with that implied by long-run data of the Kuznets type.

In the formulation of both models the respective groups of
authors have drawn heavily upon ideas first put forward by Friedman
in the Permanent Income Hypothesis. The satisfactory performance of
both models can be taken as validation of the "operational" worth of
the PIH.

In addition to the general idea that measured income and
measured consumption can be divided into permanent and transitory
components, and that permanent and transitory consumption can be

separately explained, there are a number of other ideas potentially useful for a study using Australian data to be gained from the discussion of this chapter: (1) The first is that there is benefit to be derived from the structural model approach to deriving the final function to be tested. Firstly, this approach makes the job of forming prior expectations about the sign and magnitude of regression parameters much easier, and secondly, if there is some defect in the final equation the source of the problem can be more easily found. For example, the quarterly estimate of the S-R model showed that the estimate of the coefficient of $\frac{G_t - G_{t-1}}{X_t}$ was insignificant. Immediately we can say that this implies that either $\gamma = 0$ (and hence $(G_t - G_t^1)$ is not a useful term) or $\lambda = 1$ (that is, permanent components are equal to measured amounts). (2) A second point is the possible use of a liquid asset imbalance term and a government policy variable. The first of these worked well, while the policy variable did not do well with quarterly data. However, the notion of a policy variable is attractive and may prove useful with Australian data. (3) The final idea is that total wealth (rather than just liquid asset holdings) may be important. In the next two chapters this idea will be examined more thoroughly, beginning with the work by Modigliani, Brumberg and Ando known as the Life Cycle Hypothesis.

APPENDIX 3

3.1 Solution of (3.12)

$$\frac{dX^1(t)}{dt} = \alpha X^1(t) + \beta[X(t) - X^1(t)] \tag{3.12}$$

Rearranging, and multiplying through by $e^{(\beta-\alpha)t}$, gives:

$$e^{(\beta-\alpha)t}\frac{dX^1(t)}{dt} + (\beta-\alpha)e^{(\beta-\alpha)t}X^1(t) = \beta e^{(\beta-\alpha)t}X(t)$$

That is

$$\frac{de^{(\beta-\alpha)t}X^1(t)}{dt} = \beta e^{(\beta-\alpha)t}X(t)$$

$$\therefore \quad e^{(\beta-\alpha)}X^1(t) = \int_{-\infty}^{t} \beta e^{(\beta-\alpha)x} X(x)\ dx + H$$

where H is a constant.

$$\therefore \quad X^1(t) = \frac{\beta}{e^{(\beta-\alpha)t}} \int_{-\infty}^{t} e^{(\beta-\alpha)x} X(x)\ dx + He^{-(\beta-\alpha)t}$$

If $X^1(t = -\infty) = 0$, then

$$X^1(t = -\infty) = He^{-(\beta-\alpha)t} = 0$$

$$\therefore \quad H = 0$$

Hence

$$X^1(t) = \frac{\beta}{e^{(\beta-\alpha)t}} \int_{-\infty}^{t} e^{(\beta-\alpha)x} X(x)\ dx \tag{3.13}$$

The sum of the weights is

$$\frac{\beta}{e^{(\beta-\alpha)t}} \int_{-\infty}^{t} e^{(\beta-\alpha)x} \, dx = \frac{\beta}{e^{(\beta-\alpha)t}} \left[\frac{1}{\beta-\alpha} e^{(\beta-\alpha)x} \right]_{-\infty}^{t}$$

$$= \frac{\beta}{e^{(\beta-\alpha)t}} \left[\frac{1}{\beta-\alpha} e^{(\beta-\alpha)t} \right]$$

(as long as $\beta - \alpha > 0$)

$$= \frac{\beta}{\beta-\alpha}$$

Thus the weights will sum to more than unity if income is steadily growing (i.e. $\alpha > 0$), and to less than unity if income is falling (i.e. $\alpha < 0$).

CHAPTER FOUR

WEALTH AND THE CONSUMPTION FUNCTION I:

THE LIFE CYCLE HYPOTHESIS

4.1 Introduction   As seen in the previous chapter, the

consumer's wealth position is an important element in the

development of the PIH. The individual's consumption function

for the PIH was shown to be:[1]

$$C_1 = k'(i, u) W_1$$

where C_1 denotes current consumption and W_1 the consumer's

current net worth. In this chapter and the next further

theoretical work, leading to empirically testable hypotheses,

involving the consumer's wealth position will be considered.

Unlike the PIH the work to be looked at in this chapter (and

most of that in the next chapter) involves wealth explicitly

in the final equation to be tested. The lack of data on consumers'

net worth has made these various hypotheses harder to test than

the PIH which can be reduced to a relationship between variables

for which statistics have been collected on a quarterly basis

for a considerable period of time.

In this chapter, the Life Cycle Hypothesis (LCH)

developed by A. Ando, R. Brumberg and F. Modigliani will be

examined. The theoretical basis of the LCH will be discussed in

section 4.2. During this discussion it should become clear that

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1. See (3.2) p. 56 above.

there are many points of contact between the LCH and the PIH.
In that section we will also examine the long-run versus short-run implications of the hypothesis. Empirical work on the aggregate consumption function implied by the LCH will be looked at in section 4.3. Conclusions are written up in section 4.4.

4.2 <u>The theoretical basis of the LCH</u>[1]   As in the case of the PIH, the derivation of the LCH consumption function is based upon a process of utility maximization subject to a resources constraint.[2] Assuming that the price of consumables is constant over time, utility will be a function of the value of current and future consumption, and any assets bequeathed by the consumer. If the value of assets bequeathed is assumed to be zero, the utility function can be written as:

$$U = U(c_t^T, c_t^{eT(T+1)}, c_t^{eT(T+2)}, \ldots c_t^{eTL}) \qquad (4.1)$$

where c_t^T is the value of consumption in the current year (t) by a consumer aged T years, c_t^{eTi} is the consumption expenditure an individual of age T plans (in year t) for year i of his life (i > T)

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1. This section is based upon F. Modigliani and R. Brumberg, "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data", in K. K. Kurihara (ed.), <u>Post-Keynesian Economics</u> (New Brunswick, N.J.: Rutgers University Press, 1954) and A. Ando and F. Modigliani, "The Life-Cycle Hypothesis of saving: aggregate implications and tests", <u>American Economic Review</u>, Vol. 53 (1963), pp. 55-84. The authors of these papers will be referred to collectively in the text as MBA.

2. See section 3.2 above.

and L years is the consumer's life span (assumed to be equal to

N years earning life plus M years retirement). As with the PIH,

consumption is defined as expenditure on single-use goods and

services plus the service derived from stocks of consumer durables.

The resources constraint is:

$$w_{t-1}^T + y_t^T + \sum_{i=T+1}^{N} \frac{y_t^{eTi}}{(1 + r_t)^{i-T}} = c_t^T + \sum_{i=T+1}^{L} \frac{c_t^{eTi}}{(1 + r_t)^{i-T}} \qquad (4.2)$$

where w_{t-1}^T = the consumer's net worth at the end of year (t-1),

y_t^T = the consumer's labour income in year t (i.e. income other than

earnings from net wealth), y_t^{eTi} = the labour income an individual

of age T expects (in year t) to earn in year i of his life (i > T)

and r_t = the rate of interest in year t. The left hand side of (4.2)

is the consumer's lifetime resources (equal to his existing stock of

assets plus the present value of future labour income), while the

right hand side is the present value of current and future consumption.

Note that (4.2) implies that consumers do not expect to receive any

inheritance during the remainder of their life cycle. For future

convenience denote the left hand part of (4.2) by v_t^T, i.e.

$$v_t^T = w_{t-1}^T + y_t^T + \sum_{i=T+1}^{N} \frac{y_t^{eTi}}{(1 + r_t)^{i-T}} \qquad (4.3)$$

MBA make the further assumption that the utility function is

homogeneous with respect to consumption at different points in time.[1]

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1. See Ando and Modigliani, "The 'Life Cycle' Hypothesis of Saving",
   p. 56. cf. with a similar assumption in the PIH p. 54 above.

Maximizing (4.1) subject to (4.2) yields:[1]

$$c_t^T = \gamma_t^T v_t^T \tag{4.4}$$

That is, consumption is proportional to life resources v_t^T.[2]

The coefficient of proportionality depends on the form of the

utility function, the rate of return on assets and the current age

of persons.[3] Now, denote by y_t^{eT} the average annual labour income

that a consumer of age T years in year t expects to earn over the

remainder of his earning life. Hence

$$y_t^{eT} = \frac{1}{N - T} \sum_{i=T+1}^{N} \frac{y_t^{eTi}}{(1 + r_t)^{i-T}}$$

Thus (4.3) can be written as

$$v_t^T = w_{t-1}^T + y_t^T + (N - T) y_t^{eT} \tag{4.5}$$

Substituting (4.5) into (4.4) gives:

$$c_t^T = \gamma_t^T y_t^T + \gamma_t^T (N - T) y_t^{eT} + \gamma_t^T w_{t-1}^T \tag{4.6}$$

---

1. See Appendix 4.1 for the derivation of this result for the special case N = T + 1.

2. cf. (4.4) with the corresponding expression for the PIH (see (3.2) p. 56 above).

3. See Appendix 4.1 for the specific form of γ_t^T for the special case N = T + 1.

MBA then postulate an aggregate consumption function of the same form as (4.6), namely:[1]

$$C_t = \alpha_1' Y_t + \alpha_2' Y_t^e + \alpha_3' W_{t-1} \qquad (4.7)$$

where C_t = aggregate consumption, Y_t = aggregate labour income, Y_t^e = aggregate expected labour income, and α_1', α_2' and α_3' are constants. Obtaining (4.7) involves first aggregating over all consumers of age T, and then aggregating the result over all age groups. MBA have described the aggregation process, and a set of conditions sufficient to ensure the time stability of the parameters of (4.7).[2] However, they state that this has been done only to exhibit "the conditions for existence of a particular form of the aggregate consumption function".[3] In effect, having obtained the consumption function for an individual MBA then aggregate by analogy to obtain the aggregate consumption function.

Before going ahead to consider empirical tests of the LCH, the long-run/short-run characteristics of the LCH consumption function will be examined. In order to examine the long-run implications of (4.7), MBA suppose that labour income is growing at a steady rate.[4]

---

1. Ando and Modigliani, "The 'Life-Cycle' Hypothesis of Saving", p. 58.

2. Ibid. pp. 58-59.

3. Ibid. p. 56 f.n. 2.

4. See Ando and Modigliani, "The 'Life-Cycle' Hypothesis of Saving", p. 76. For a very full analysis of the long-run/short-run implications of the LCH the reader is referred to F. Modigliani, "The Life Cycle Hypothesis of Saving, the Demand for Wealth and the Supply of Capital", Social Research, Vol. 33 (1966), pp. 160-217.

In this circumstance Y_t^e is made proportional to Y_t yielding:

$$C_t = \alpha Y_t + \alpha_3' W_{t-1} \qquad \alpha = \alpha_1' + \alpha_2'\beta$$

where β is the constant of proportionality.[1] Now total income (X) equals labour income plus non-labour income. Non-labour income will be equal to rW_{t-1}, where r is the average rate of return on assets held. In the long-run r is supposed constant. Hence, savings, S_t, are given by

$$
\begin{aligned}
S_t &= X_t - C_t \\
&= X_t - \alpha_3' W_{t-1} - \alpha Y_t \\
&= X_t - \alpha_3' W_{t-1} - \alpha(X_t - rW_{t-1}) \\
&= (1 - \alpha)X_t - (\alpha_3' - \alpha r)W_{t-1} \\
\therefore \quad S_t &= aX_t - bW_{t-1}
\end{aligned}
\tag{4.8}
$$

---

1. As a possible justification for making Y_t^e proportional to Y_t, suppose Y_t^e is being estimated by a distributed lag of current and past values of Y_t, such as

$$Y_t^e = (1 - \lambda)[Y_t + (\lambda + \rho)Y_{t-1} + (\lambda + \rho)^2 Y_{t-2} + \ldots]$$

where ρ is the trend rate of growth of Y_t^e (see H(3) p. 79 above). If labour income, Y, is also growing at the rate ρ, this can be written as:

$$
\begin{aligned}
Y_t^e &= (1 - \lambda)[1 + \frac{\lambda + \rho}{1 + \rho} + (\frac{\lambda + \rho}{1 + \rho})^2 + \ldots]Y_t \\
&= (1 - \lambda) \frac{1}{1 - \frac{\lambda + \rho}{1 + \rho}} Y_t \\
&= (1 + \rho)Y_t \\
&= \beta Y_t \qquad\qquad \beta = 1 + \rho
\end{aligned}
$$

Thus, the constant of proportionality will be greater than unity, thereby allowing for the long term upward trend in actual income.

where $a \equiv 1 - \alpha$, $b \equiv \alpha_3' - \alpha r$. Assuming capital gains zero over the long run, we can say that

$$\Delta W_t = S_t \qquad (4.9)$$

where $\Delta W_t = W_t - W_{t-1}$. Adding (4.8) and (4.9) we get

$$\Delta W_t = aX_t - bW_{t-1}$$

$$\therefore \quad \frac{\Delta W_t}{W_{t-1}} = a \frac{X_t}{W_{t-1}} - b$$

Adding and subtracting ρ, and then rearranging, gives:

$$\frac{\Delta W_t}{W_{t-1}} = \rho + a \left[\frac{X_t}{W_{t-1}} - \frac{\rho + b}{a} \right]$$

That is,

$$\frac{\Delta W_t}{W_{t-1}} = \rho + a \left[\frac{X_t}{W_{t-1}} - h \right] \qquad (4.10)$$

where $h = \frac{\rho + b}{a}$. Now suppose $\frac{X_t}{W_{t-1}} > h$. Therefore the term in brackets is positive and hence $\frac{\Delta W_t}{W_{t-1}} > \rho$. Hence the rate of growth

of total income X is less than that of wealth W.[1] Thus $\dfrac{X_t}{W_{t-1}}$ must

fall towards h as time passes. Similarly, if $\dfrac{X_t}{W_{t-1}} < h$, total income

X must grow faster than wealth W, and $\dfrac{X_t}{W_{t-1}}$ will increase towards h as

time passes. If $\dfrac{X_t}{W_{t-1}} = h$, then $\dfrac{\Delta W_t}{W_{t-1}}$ and $\dfrac{\Delta X_t}{X_{t-1}}$ will both equal ρ

(the rate of growth of labour income), and $\dfrac{X_t}{W_{t-1}}$ will remain steady

at h. Hence the LCH implies a constant long-run wealth-income ratio

equal to

$$\frac{W_{t-1}}{X_t} = \frac{1}{h} = \frac{a}{\rho + b} \tag{4.11}$$

---

1. Since $X_t = Y_t + rW_{t-1}$ $\tag{1}$

$$\therefore \frac{\Delta X_t}{X_{t-1}} = \frac{\Delta Y_t}{Y_{t-1}} \cdot \frac{Y_{t-1}}{X_{t-1}} + r\frac{\Delta W_{t-1}}{W_{t-2}} \cdot \frac{W_{t-2}}{X_{t-1}}$$

$$= \frac{\Delta Y_t}{Y_{t-1}} \cdot \frac{Y_{t-1}}{X_{t-1}} + \frac{\Delta W_{t-1}}{W_{t-2}} \cdot \frac{X_{t-1} - Y_{t-1}}{X_{t-1}} \quad \text{from (1)}$$

$$= \frac{\Delta W_{t-1}}{W_{t-2}} + \frac{Y_{t-1}}{X_{t-1}} [\frac{\Delta Y_t}{Y_{t-1}} - \frac{\Delta W_{t-1}}{W_{t-2}}]$$

$$\therefore \frac{\Delta X_t}{X_{t-1}} = \frac{\Delta W_{t-1}}{W_{t-2}} + \frac{Y_{t-1}}{X_{t-1}} [\rho - \frac{\Delta W_{t-1}}{W_{t-2}}]$$

Hence if $\dfrac{\Delta W_{t-1}}{W_{t-2}} > \rho$, $\dfrac{\Delta X_t}{X_{t-1}} < \dfrac{\Delta W_{t-1}}{W_{t-2}}$

The corresponding savings-income ratio is:

$$\frac{S_t}{X_t} = \frac{W_t - W_{t-1}}{W_{t-1}} \cdot \frac{W_{t-1}}{X_t}$$

$$= \rho \cdot \frac{1}{h}$$

$$\therefore \qquad \frac{S_t}{X_t} = \frac{\rho a}{\rho + b} \qquad\qquad (4.12)$$

Thus the LCH implies a <u>constant</u> long-run savings ratio.[1] Both the savings ratio and the wealth-income ratio are functions of the rate of growth of income. Since

$$\frac{d(S/X)_t}{d\rho} = \frac{ab}{(\rho + b)^2} > 0$$

and

$$\frac{d^2(S/X)_t}{d\rho^2} = \frac{-2ab}{(\rho + b)^3} < 0$$

we can see that the long-run savings ratio rises with the rate of growth of income, but at a decreasing rate. In the same way, we have

$$\frac{d(W_{t-1}/X_t)}{d\rho} = \frac{-a}{(\rho + b)^2} < 0$$

$$\frac{d^2(W_{t-1}/X_t)}{d\rho^2} = \frac{2a}{(\rho + b)^3} > 0$$

Hence the wealth ratio declines as the rate of growth of income rises, but at an increasing rate. For a zero rate of income growth the

1. cf. (4.11) and (4.12) with (3.49) and (3.50) of the Stone-Rowe model of the previous chapter.

savings ratio is zero, i.e. the long-run marginal propensity to consume is unity.

To exhibit the short-run characteristics of their model, MBA continue to assume that Y_t^e is proportional to Y_t,[1] giving:

$$C_t = \alpha Y_t + \alpha_3' W_{t-1}$$

from which the savings function

$$S_t = aX_t - bW_{t-1} \tag{4.8}$$

can be derived.[2] The long-run savings function is

$$S_t = \frac{\rho a}{\rho + b} X_t \tag{4.12}$$

Now, in any particular year W_{t-1} is known at the beginning of the year, since W_{t-1} is the stock of wealth at the end of year $(t-1)$. So (4.8) can be shown on the S – X plane as a line with intercept $-bW_{t-1}$ and slope a. In figure 4.1 below, (4.8) is shown for several years and (4.12) is also shown.

Now, suppose that income for year 1, X_1, is equal to the trend value of income. Thus savings for year 1 can be calculated from either S_1 or (4.12). Thus S_1 and (4.12) must intersect at P_1. Positive savings in year 1 cause the stock of wealth to grow and the

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1. See Ando and Modigliani, "The Life-Cycle Hypothesis of Saving", p. 78.

2. See p. 113 above.

Figure 4.1

$$S_t = \frac{\rho a}{\rho + b} X_t$$
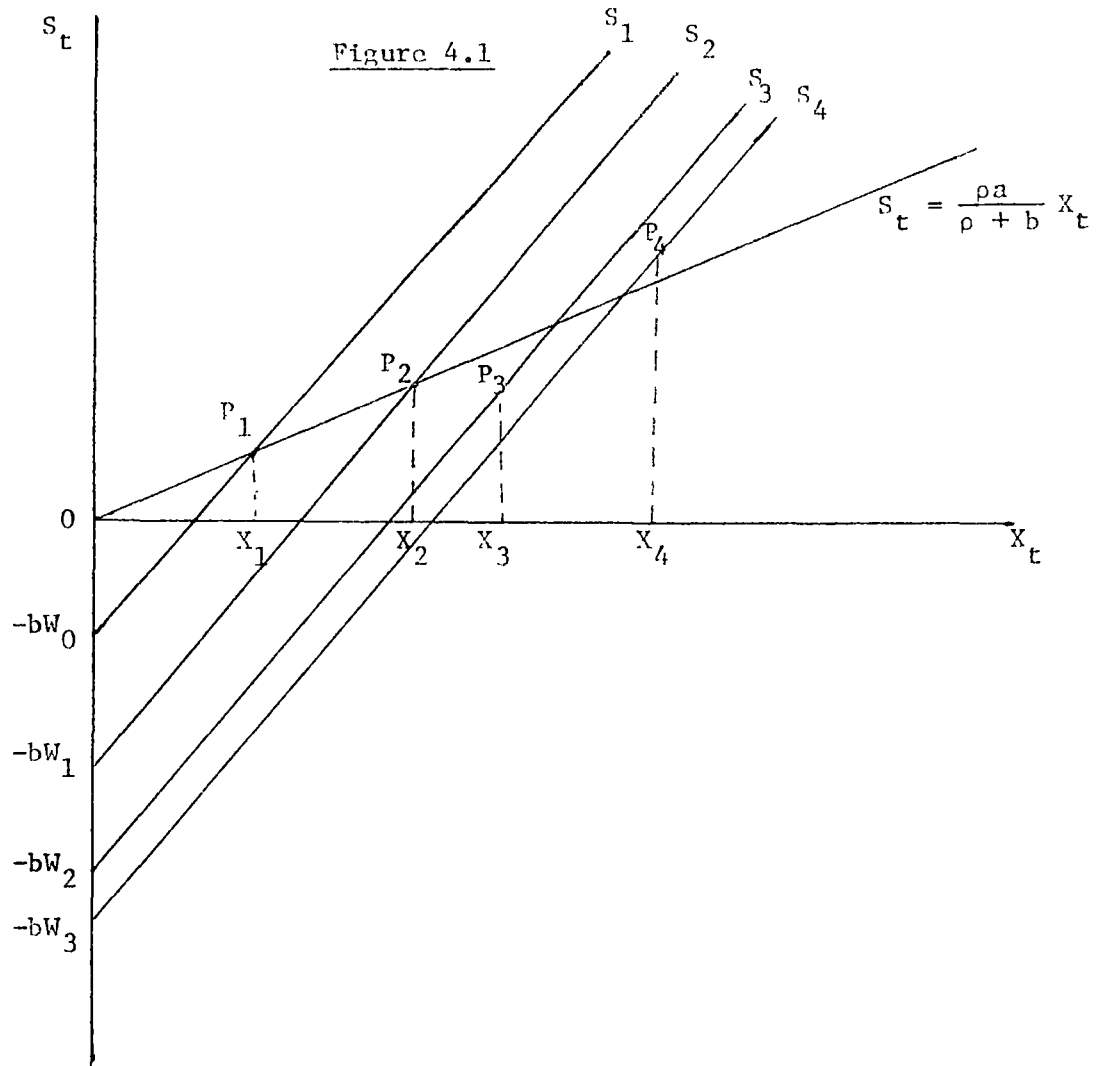
short-run savings function (4.8) to move to the right to S_2. If income grows at the trend rate, then S_2 will intersect (4.12) at P_2. Once again, the savings function will move to the right. Suppose that in year 3 the rate of income growth falls below the trend rate (i.e. $\frac{OX_3}{OX_2} < \frac{OX_2}{OX_1}$). In these circumstances, savings in year 3 will be given by $X_3 P_3$. Since the slope of OP_3 is less than that of the long-run savings function, it is clear that the savings ratio has fallen along with the cyclical decline in the rate of growth of income. The smaller savings will result in a smaller than otherwise

shift in the savings function to S_4. If the rate of growth of income increases sufficiently, it is possible in year 4 for savings to exceed $\frac{\rho a}{\rho + b} X_4$. That is, an upward swing in the rate of growth of income will produce an upward swing in the savings ratio.

Thus, the LCH consumption (savings) function implies long-run constancy and cyclical variability of the savings ratio.

4.3 Empirical tests of the LCH

The first published test of the LCH aggregate consumption function was by R. Brumberg.[1] Denoting non-labour income by R_t, Brumberg obtained the following saving function:

$$S_t = Y_t + R_t - C_t$$

$$= Y_t + R_t - (\alpha_1'Y_t + \alpha_2'Y_t^e + \alpha_3'W_{t-1})$$

$$\therefore \quad S_t = (1 - \alpha_1')Y_t + R_t - \alpha_2'Y_t^e - \alpha_3'W_{t-1} \quad (4.13)$$

To overcome the lack of data on Y^e and W, Brumberg first put (4.13) into first difference form:

$$\Delta S_t = (1 - \alpha_1')\Delta Y_t + \Delta R_t - \alpha_2'\Delta Y_t^e - \alpha_3'\Delta W_{t-1} \quad (4.14)$$

Brumberg then hypothesized that ΔY^e was a linear function of the change in labour income, i.e.

$$\Delta Y_t^e = \alpha_0 + K\Delta Y_t \quad (4.15)$$

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1. R. Brumberg, "An approximation to the aggregate saving function", Economic Journal, Vol. 66 (1956), pp. 66-72.

On the other hand, ΔW_{t-1} will equal the sum of savings in period (t-1) plus capital gains, i.e.

$$\Delta W_{t-1} = S_{t-1} + G_{t-1} \qquad (4.16)$$

Substituting (4.15) and (4.16) into (4.14) we obtain:

$$\Delta S_t = (1 - \alpha_1')\Delta Y_t + \Delta R_t - \alpha_2'(\alpha_0 + K\Delta Y_t) - \alpha_3'(S_{t-1} + G_{t-1})$$

$$= -\alpha_0\alpha_2' + (1 - \alpha_1' - \alpha_2'K)\Delta Y_t + \Delta R_t - \alpha_3'S_{t-1} - \alpha_3'G_{t-1}$$

i.e. $\Delta S_t = b_0 + b_1\Delta Y_t + b_2\Delta R_t + b_3 S_{t-1} + b_4 G_{t-1}$ $\qquad (4.17)$

where $b_0 = -\alpha_0\alpha_2'$, $b_1 = (1 - \alpha_1' - \alpha_2'K)$, $b_2 = 1$, $b_3 = b_4 = -\alpha_3'$.

Since consumption expenditure under the LCH includes only the service value of consumer durables, savings statistics had to be revised to include expenditure on durables. Changes in an index of share prices were used as a proxy for capital gains. Using constant price data for the years 1929-'49, (4.17) was estimated by least-squares to give:[1]

$$\Delta S_t = -610 + 0.89\,\Delta Y_t + 0.38\,\Delta R_t - 0.14\,S_{t-1} - 27\,G_{t-1}$$
$$\quad\;\; (380) \quad (0.08) \qquad (0.18) \qquad\; (0.03) \qquad\; (14)$$

$$\qquad (4.18)$$

$$R^2 = 0.97$$

All signs conform to expectations, and all coefficients are statistically significant at the 10% level. However, the coefficients

---

1. The constant term and the coefficient of G_{t-1} in (4.18) are apparently in the wrong units. For example, the correct value of the coefficient of G_{t-1} is -0.027. See P. Newman, "Factor shares and the savings function", Economic Journal, Vol. 66 (1956), footnote 1, p. 734.

of ΔR_t and G_t are not significant at the 5% level, nor does the hypothesis $b_2 = 1$ look acceptable.[1] The value of R^2 is quite high, especially so when it is remembered that the hypothesis has been tested in first-difference form.

The hypothesis, without the capital gains variable, was tested using data for 1929-1941, giving:

$$\Delta S_t = \begin{array}{cccc} 104 & + \ 0.53 \ \Delta Y_t & + \ 0.69 \ \Delta R_t & - \ 0.14 \ S_{t-1} \\ (330) & (0.14) & (0.28) & (0.08) \end{array} \qquad (4.19)$$

The most noticeable feature of this is the great change in the value of parameter estimates; aside from the coefficient of S_{t-1} all other parameters have drastically changed values (and, in one case, changed sign). The intercept and coefficient of S_{t-1} no longer appear to be significant. Brumberg used (4.19) to forecast annual savings for the period 1942 to 1949. The mean percentage forecast error was only 1.4%. However, the standard deviation of the percentage forecast errors was 14.2%, with the actual percentage error ranging from -26.7% to 17.3%. Brumberg also forecast with the Duesenberry-Modigliani hypothesis[2] and two naive models ((i) $S_t = S_{t-1}$, (ii) $S_t = S_{t-1} + \Delta S_{t-1}$). The LCH, as tested by Brumberg, showed itself superior to all of these alternatives. (The Duesenberry-Modigliani function, for example, gave a mean forecast error of 48.1%). While these forecast results for the

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1. Note that since G_{t-1} has been replaced with a proxy variable we can no longer expect b_3 to equal b_4.

2. See p. 28 above.

LCH do not look impressive, except in comparison to the other models examined, it must be remembered that the forecasts are for the years of the second world war and immediately afterwards, years which most researchers do not even attempt to discuss.

Further estimates of the LCH were published by Ando and Modigliani.[1] Two basic hypotheses were derived from the function:

$$C_t = \alpha_1' Y_t + \alpha_2' Y_t^e + \alpha_3' W_{t-1} \qquad (4.7)$$

Hypothesis I is obtained by supposing that Y_t^e is proportional to Y_t, giving:[2]

$$C_t = \alpha Y_t + \alpha_3' W_{t-1} \qquad (4.20)$$

Both α and α_3' should be positive and less than unity, with α (the marginal propensity to consume current labour income) significantly higher than α_3' (the marginal propensity to consume wealth stocks). The second hypothesis is obtained by supposing that the per-capita expected income of those currently employed (y_t^e) is proportional to their current per capita incomes, i.e.

$$y_t^e = \beta_1 \frac{Y_t}{E_t}$$

where E_t is the total number employed. The per-capita expected income

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1. Ando and Modigliani, "The Life-Cycle Hypothesis of saving". Revised results are given in A. Ando and F. Modigliani, "The 'Life Cycle' Hypothesis of saving: a correction", American Economic Review, Vol. 54 (1964), pp. 111-113. The revised results are used below.

2. See p.113 above for one possible derivation of (4.20) from (4.7).

of the unemployed (y_t^{eu}) is, in turn, proportional to y^e. Hence

$$y_t^{eu} = \beta_2' y_t^e = \beta_2 \frac{Y_t}{E_t}$$

where $\beta_2 = \beta_2' \beta_1$. Aggregate expected income is then given by:

$$Y_t^e = E_t y_t^e + (N_t - E_t) y_t^{eu}$$

where N_t is the total labour force, and hence $(N_t - E_t)$ is the number

of unemployed. Substituting for y_t^e and y_t^{eu} and rearranging, we get

$$Y_t^e = E_t \cdot \beta_1 \frac{Y_t}{E_t} + (N_t - E_t) \cdot \beta_2 \frac{Y_t}{E_t}$$

i.e.
$$Y_t^e = (\beta_1 - \beta_2) Y_t + \beta_2 \frac{N_t}{E_t} Y_t \qquad (4.21)$$

Substituting (4.21) into (4.7) gives

$$C_t = \alpha_1 Y_t + \alpha_2 \frac{N_t}{E_t} Y_t + \alpha_3' W_{t-1} \qquad (4.22)$$

where $\alpha_1 = [\alpha_1' + \alpha_2'(\beta_1 - \beta_2)]$, $\alpha_2 = \alpha_2' \beta_2$. (4.22) will be referred to

as hypothesis II. A priori expectations are that $0 < \alpha_3' << \alpha_1 < 1$.

   Using United States current price data for the years 1929-1959

(excluding 1941-1946), both hypotheses were fitted by SELS. The constant

terms obtained were significant, but in view of their small size Ando

and Modigliani did not abandon the homogeneous form of the function.[1]

Table 4.1 shows the results of five tests of hypothesis I and II in

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1. See "The Life Cycle hypothesis of saving: aggregate implications
    and tests", p. 63.

TABLE 4.1[a]

Hypothesis tested	Mode of[b] Regression	Estimated coefficients and their standard errors			SE	R^2	d
		Y_t	$Y_t \dfrac{N_t}{E_t}$	W_{t-1}			
I	A	0.640 (0.039)	–	0.077 (0.008)	2.860	0.999	0.89
I	B	0.550 (0.116)	–	0.079 (0.021)	2.335	0.921	2.00
I	C	0.634 (0.020)	–	0.080 (0.030)	0.018	0.962	0.99
II	A	0.430 (0.108)	0.287 (0.139)	0.058 (0.012)	2.690	0.999	0.85
II	B	0.444 (0.124)	0.274 (0.147)	0.051 (0.025)	2.215	0.929	1.82

a: adapted from TABLE A p. 112 of Ando and Modigliani, "The 'Life Cycle' Hypothesis of saving: a correction", American Economic Review, Vol. 54 (1964), pp. 111-113.

b: A: regression in which the variables are in original form.

   B: regression in which the variables are in terms of first differences.

   C: regression in which the variables are in ratio form.

which the constant term was forced to equal zero. Consider

hypothesis I first. The first regression yielded:

$$C_t = 0.640Y_t + 0.077W_{t-1}$$

Using the published standard errors it can be shown that the

parameter estimates are significantly different from zero at the

1% and 5% levels. However, the Durbin-Watson statistic shows

evidence of positive serial correlation at both the 1% and 5% levels

of significance, casting some doubt on the conclusions drawn from the

significance tests.[1] A common device for overcoming the problem of

serially correlated disturbances is to first transform the variables

into first differences.[2] After making this transformation the

estimated consumption function was

$$\Delta C_t = 0.550\Delta Y_t + 0.079\Delta W_{t-1}$$

The parameter estimates are significant at the 1% and 5% levels, and

the Durbin-Watson test shows no sign of serial correlation at the 5%

level. Finally, the function was estimated in ratio form. In this

form the common trends of the variables will be eliminated, thereby

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1. See p. 24 above.

2. This device would overcome the problem of autocorrelation if
   the disturbances ε_t were generated by a first order autoregressive
   process:

   $$\varepsilon_t = \rho\varepsilon_{t-1} + u_t$$

   where u_t is non-autocorrelated, and ρ is approximately unity.
   See Goldberger, Econometric Theory pp. 236-238.

eliminating problems of multi-collinearity and perhaps
heteroskedasticity.[1] The result was:

$$\frac{C_t}{Y_t} = 0.634 + 0.080 \, \frac{W_{t-1}}{Y_t}$$

Again the estimates are significant at the 1% and 5% levels. The
value of the Durbin-Watson statistic (0.99), however, indicates the
presence of positive serial correlation at both the 1% and 5% levels.
Ando and Modigliani also tested the possibility that there had been
a shift in the coefficient of either labour income or net worth since
the world war by estimating

$$C_t = (\alpha + \alpha X_t)Y_t + (\alpha_3' + \alpha_3' X_t)W_{t-1}$$

where X_t took the value zero for 1929-1940 and unity for 1947-1959.
This was estimated in a variety of forms, but in no case were the
coefficients of X_t statistically significant.

Thus in summary we have for hypothesis I:

(i)   The hypothesis is supported by a consistently high value of R^2

(ii) The estimated coefficient of net worth is statistically
      significant and all estimates lie in the fairly narrow range
      0.077 - 0.080

(iii) On the other hand the coefficient of labour income is not so
      stable, ranging from 0.550 - 0.640. In each case the coefficient
      is statistically significant.

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1. See p. 90 above.

(iv)   All estimates agree with prior expectations as to sign and
magnitude.

(v)   For all except the first difference regression the Durbin–
Watson test shows evidence of positive serial correlation.

(vi)   There is no evidence of a shift in the coefficient of either
labour income or net worth from the pre-war to the post-war
period.

In the case of hypothesis II, a SELS fit of the original
hypothesis gave:

$$C_t = 0.430Y_t + 0.287Y_t \frac{N_t}{E_t} + 0.058W_{t-1}$$

The hypothesis in this form does not pass the Durbin–Watson test at
either the 1% or 5% levels, nor is the coefficient of $Y_t \frac{N_t}{E_t}$ significant.
Estimating the equation in first difference form gave:

$$\Delta C_t = 0.444\Delta Y_t + 0.274 \, \Delta(Y \frac{N}{E})_t + 0.051\Delta W_{t-1}$$

The Durbin–Watson test now shows no sign of serial correlation at
either the 1% or 5% levels. The coefficient of $\Delta(Y \frac{N}{E})_t$ is still
insignificant however. Remaining results presented by Ando and
Modigliani were directed at testing for shifts in the coefficients of
labour income and net worth. No significant shifts were detected.
Given the insignificant coefficient of $\Delta(Y \frac{N}{E})$, it would seem that
hypothesis I was the preferable of the two hypotheses.

So far, the tests of the LCH that have been reported have
used annual data. While the hypothesis has performed reasonably well

with this data, a more critical test involves the use of quarterly data. Evans has tested hypothesis I with U.S. quarterly current price data for the period 1947-1962.[1] The estimate obtained was:

$$\frac{C_t}{Y_t} = 0.8152 + \underset{(0.0141)}{0.0456} \left(\frac{W_{t-1}}{Y_t} \right) \qquad \bar{R}^2 = 0.130$$
$$d = 0.68$$

The net worth coefficient is statistically significant, but the Durbin-Watson test indicates serial correlation of residuals. The value of $\bar{R}^2$, however, indicates that the hypothesis has little power to explain quarter-to-quarter changes in the $\frac{C}{Y}$ ratio. In the light of this result Evans modified the basic hypothesis by including a lagged $\frac{C}{Y}$ term, implying some form of habit persistence effect in the adjustment of $\frac{C}{Y}$ to changes in the wealth-income ratio. The result for this form was:

$$\frac{C_t}{Y_t} = 0.2013 + \underset{(0.0095)}{0.0108} \frac{W_{t-1}}{Y_t} + \underset{(0.0780)}{0.7661} \frac{C_{t-1}}{Y_{t-1}}$$
$$\bar{R}^2 = 0.658$$
$$d = 2.09$$

While the value of $\bar{R}^2$ has improved quite markedly, and the Durbin-Watson statistic is consistent with the hypothesis of no serial

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1. See M. K. Evans, "The importance of wealth in the consumption function", Journal of Political Economy, Vol. 75 (1967), pp. 335-351.

correlation,[1] the estimated coefficient of $\frac{W_{t-1}}{Y_t}$ is no longer

significant. Since the net-worth term is one of the distinctive

features of the LCH, it seems from this evidence that the hypothesis

may not be of great use in a quarterly study of the aggregate

consumption function.

Another quarterly study, with bearing on the LCH, is one

by Branson and Klevorick.[2] This study begins with the following

variant of Ando and Modigliani's hypothesis I:

$$(\frac{C}{N})_t = b_0' (\frac{Y}{N})_t^{b_1} (\frac{W}{N})_{t-1}^{b_2} (P_t)^{b_3} \qquad (4.23)$$

where C_t = aggregate _real_ consumption expenditure on non-durables and

services plus the use value of durable goods; Y_t = aggregate _real_

labour income after tax, W_{t-1} = aggregate _real_ net worth of households,

N_t = population, P_t = the price level.[3] Taking natural logarithms

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1. This favourable result must however be discounted to some extent since, as the regression now involves lagged values of the dependent variable, the Durbin-Watson test is no longer strictly applicable. (See M. Nerlove and K. F. Wallis, "Use of the Durbin-Watson Statistic in Inappropriate situations", _Econometrica_, Vol. 34 (1966), pp. 235-238.

2. W. H. Branson and A. K. Klevorick, "Money illusion and the aggregate consumption function", _American Economic Review_, Vol. 59 (1969), pp. 832-849.

3. Branson and Klevorick have included the price level to test the hypothesis that consumers suffer from money illusion. If this hypothesis is correct then $\sum_{k=0}^{K} \eta_k$ in equation (4.25) below will be positive. Estimates of equation (4.25) show this term to be significantly positive.

of (4.23) yields:

$$\ln \left(\frac{C}{N} \right)_t = b_0 + b_1 \ln \left(\frac{Y}{N} \right)_t + b_2 \ln \left(\frac{W}{N} \right)_{t-1} + b_3 \ln P_t \quad (4.24)$$

To allow for possible lags in the adjustment of consumption to changes in Y, W and P, (4.24) is rewritten in a distributed lag format:[1]

$$\ln \left(\frac{C}{N} \right)_t = b_0 + \sum_{i=0}^{I} \gamma_i \ln \left(\frac{Y}{N} \right)_{t-i} + \sum_{j=0}^{J} \delta_j \ln \left(\frac{W}{N} \right)_{t-1-j}$$

$$+ \sum_{k=0}^{K} \eta_k \ln P_{t-k} \quad (4.25)$$

This equation was estimated using U.S. quarterly data for the period 1955(1) to 1965(4). The distributed lag weights were estimated by the Almon variable technique.[2] After experimenting with various patterns of weights, Branson and Klevorick chose the following estimate:

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1. Ando and Modigliani have also considered the introduction of lags in the LCH consumption function, see A. Ando and F. Modigliani, "Econometric analysis of stabilization policies", _American Economic Review (Papers and Proc.)_, Vol. 59 (1969), pp. 296-314.

2. S. Almon, "The distributed lag between capital appropriations and expenditures", _Econometrica_, Vol. 33 (1965), pp. 178-196. The Almon variable technique is explained more fully in appendix 7.1 below, but it essentially involves the transformation of the observations on $\ln (X/N)$, $\ln (W/N)$ and $\ln P$ into observations on a set of "Almon" variables. The dependent variable is then regressed onto these "Almon" variables. From the result the weights γ_i, δ_j, η_k can then be determined. The technique assumes that each set of weights lies on a polynomial of some specified degree. By varying the lag lengths and the degree of the polynomials the research worker can choose the best weighting pattern.

$$\ln \left(\frac{C}{N} \right)_t = -1.953 + \sum_{i=0}^{7} \gamma_i \ln \left(\frac{Y}{N} \right)_{t-i} + 0.127 \ln \left(\frac{W}{N} \right)_{t-1}$$
$$\qquad (0.114) \qquad\qquad\qquad\qquad\qquad (0.036)$$

$$+ \sum_{k=0}^{7} \eta_k \ln P_{t-k} \qquad\qquad (4.26)$$

$R^2 = 0.9984$, $d = 1.757$, $\Sigma \gamma_i = 0.661$ (with standard error of 0.043),

$\Sigma \eta_k = 0.418$ (0.036). That is, the weighting pattern chosen involves

the current value of $\ln \left(\frac{W}{N} \right)$ only, while quite long distributed lags

of $\ln \left(\frac{Y}{N} \right)$ and $\ln (P)$ are used. The actual weights γ_i and η_k are

given in table 4.2. With the exception of the last weight, each value

of γ_i is at least twice its standard error.

In contrast to the work by Evans, this modified version of

the LCH does lend some support to the ideas of the Life Cycle

Hypothesis, if not to the actual form put forward by Ando and

Modigliani. As well as introducing the variable P and allowing for

a more complicated lag structure, Branson and Klevorick have

transformed the basic data by expressing it in real per-capita terms

and taking natural logarithms of the results. Nevertheless, net

worth does appear to be a significant variable in this quarterly study.

4.4 <u>Conclusions</u>   In this chapter we have examined another

formulation of the aggregate consumption function based upon a process

of utility maximization, subject to a resource constraint, at the

micro level. The outcome of this maximization process is a consumption

function for an individual of the form:

$$C_t^T = \gamma_t^T v_t^T \qquad\qquad (4.4)$$

TABLE 4.2[a]

lag (i, k)	Coefficient of	
	$\ell n \left(\frac{Y}{N}\right)_{t-i}$	$\ell n \, P_{t-k}$
0	0.274 (0.046)	0.100 (0.082)
1	0.151 (0.015)	0.093 (0.029)
2	0.081 (0.021)	0.080 (0.047)
3	0.048 (0.018)	0.063 (0.039)
4	0.039 (0.013)	0.044 (0.023)
5	0.038 (0.019)	0.026 (0.029)
6	0.030 (0.019)	0.010 (0.032)
7	0	0
Sum	0.661	0.418
Standard Error of Sum	(0.043)	(0.036)

a: Taken from Branson and Klevorick, "Money illusion and the aggregate consumption function", Table 2 p. 841.

where v_t^T is the value in year t of the lifetime resources of a

consumer of age T years. v_t^T is replaced with an expression involving

the consumer's net worth, his current labour income and the average

expected labour income over the remainder of his life cycle. Hence

$$c_t^T \; = \; \gamma_t^T \; [w_{t-1}^T + y_t^T + (N - T)y_t^{eT} \;] \tag{4.6}$$

There is one feature of the work at this stage which calls for comment

and that is, that current labour income (y_t^T), current stocks of

assets (w_{t-1}^T), and the present value of expected labour income

$[(N - T)y_t^{eT}]$ all have the same coefficient in (4.6). This carries

the implication, for example, that a given increment to y_t^T will have

no greater effect on consumption expenditure than the same increment

to wealth stocks. A priori, it seems more reasonable to expect an

increase of, say, \$1,000 in y_t^T to lead to a much bigger increase in

consumption than would the same increase in the value of assets.

Certainly, at the aggregate level, the empirical evidence supports a

much larger coefficient for current labour income than for wealth.

Relationship (4.6) is then used to derive an aggregate

consumption function. As in the case of the PIH, the aggregation

procedure amounted to little more than aggregation by analogy.

Nevertheless, the theoretical work behind the LCH does provide some

rationale for including a net worth term in the aggregate consumption

function.

When estimated with annual date, the hypothesis performs

reasonably well, particularly the first difference form of hypothesis I.

Since hypothesis I does not explicitly include a measure of the

variable Y^e, it cannot be regarded as providing a test of the worth of a human capital variable. Hypothesis II does make some test of the human capital variable by including the term $\frac{N_t}{E_t} Y_t$. However, all estimates of hypothesis II that were examined showed this variable to be insignificant. Thus, either this measure of Y^e is not a good proxy, or Y^e itself is not a useful variable. In so far as the latter possibility is true, we can conclude that empirical evidence does not support the inclusion of a human capital variable.

The evidence from estimates based upon quarterly data is mixed. The estimate of the ratio form of hypothesis I made by Evans shows the LCH to be a failure – even on the criterion of achieving a high R^2, which most functions seem capable of doing with ease, the LCH does not perform well. Evans' modification of the LCH, by the inclusion of a lagged value of the dependent variable, was not successful as the coefficient of the net worth variable was then no longer significant. Against this evidence must be set the results of Branson and Klevorick. After modifying hypothesis I in a number of ways (using real per capita data transformed into logs, and employing a distributed lag format), the net worth term was found to be significant. A wealth variable was also successfully used by Stone and Rowe in the model discussed in the previous chapter. The S-R model combined ideas from both the LCH and the PIH, introducing the notion of permanent wealth as a determinant of the permanent component of consumption.

While modifications of the LCH, or models employing ideas from the LCH, can be found to work well, the LCH formulated by Ando and Modigliani does not perform satisfactorily with quarterly data, and for this reason we shall not attempt to apply it to Australian data. However, the central idea of the hypothesis, that the consumer's net worth is an important determinant of the level of consumption, is a potential contributor to a study of the Australian consumption function.

4.1 Obtaining (4.4) when L = T + 1

We wish to maximize

$$U = U(c_t^T, c_t^{eT(T+1)}) \tag{1}$$

subject to the resource constraint

$$v_t^T = c_t^T + \frac{c_t^{eT(T+1)}}{1 + r_t} \tag{2}$$

Form the expression

$$L = U(c_t^T, c_t^{eT(T+1)}) + \lambda(v_t^T - c_t^T - \frac{c_t^{eT(T+1)}}{1 + r_t}) \tag{3}$$

where λ is a Lagrange multiplier. U is assumed to be homogeneous. Suppose U is homogeneous of degree 1. Then (3) can be written as

$$L = c_t^T \phi(\frac{c_t^{eT(T+1)}}{c_t^T}) + \lambda(v_t^T - c_t^T - \frac{c_t^{eT(T+1)}}{1 + r_t}) \tag{4}$$

where $\phi = U(1, \frac{c_t^{eT(T+1)}}{c_t^T})$. Differentiating (4) with respect to c_t^T, $c_t^{eT(T+1)}$ and λ, and setting the results equal to zero gives

$$\frac{\partial L}{\partial c_t^T} = \phi + c_t^T \cdot \phi' \cdot (-\frac{c_t^{eT(T+1)}}{(c_t^T)^2}) - \lambda = 0 \tag{5}$$

$$\frac{\partial L}{\partial c_t^{eT(T+1)}} = c_t^T \cdot \phi' \cdot \left(\frac{1}{c_t^T} \right) - \frac{\lambda}{1 + r_t} = 0 \tag{6}$$

$$\frac{\partial L}{\partial \lambda} = v_t^T - c_t^T - \frac{c_t^{eT(T+1)}}{1 + r_t} = 0 \tag{7}$$

Solving (6) for λ and substituting into (5) gives:

$$\phi - (c_t^T)^{-1} \phi' c_t^{eT(T+1)} - \phi'(1 + r_t) = 0 \tag{8}$$

Substituting from (7) into (8) gives

$$\phi - (c_t^T)^{-1} \phi' (1 + r_t)(v_t^T - c_t^T) - \phi'(1 + r_t) = 0$$

i.e. $\qquad \phi - (c_t^T)^{-1} \phi' (1 + r_t) v_t^T = 0$

$$\therefore \qquad c_t^T = [\frac{\phi'}{\phi} (1 + r_t)] v_t^T \tag{9}$$

$$\therefore \qquad c_t^T = \gamma_t^T v_t^T \tag{10}$$

where $\qquad \gamma_t^T = [\frac{\phi'}{\phi} (1 + r_t)] \tag{11}$

Substitute (9) into (8)

$$\therefore \qquad \phi - \frac{\phi}{\phi'(1 + r_t)v_t^T} \phi' c_t^{eT(T+1)} - \phi'(1 + r_t) = 0$$

$$\therefore \qquad c_t^{eT(T+1)} = [(1 + r_t) - \frac{\phi'}{\phi} (1 + r_t)^2] v_t^T$$

i.e. $\qquad c_t^{eT(T+1)} = \gamma_t^{T+1} v_t^T \tag{12}$

where $\qquad \gamma_t^{T+1} = [(1 + r_t) - \frac{\phi'}{\phi} (1 + r_t)^2] \tag{13}$

Relaxing the assumption that U is homogeneous of degree 1, and replacing it with the assumption that U is homogeneous of degree k, results in k entering the expressions for γ_t^T, γ_t^{T+1}. Similarly, generalizing from the case L = T + 1 results in the expressions for γ_t^T etc. becoming more complex.

Before the terms γ_t^T etc. can be spelt out in any more detail the utility function must be specified. Suppose that the individual consumer's utility function is:[1]

$$U(C_T, C_{T+1}, \ldots, C_L) = \alpha [C_T^{-\beta} + (1 + \gamma)C_{T+1}^{-\beta} + \ldots$$
$$\ldots + (1 + \gamma)^{L-T} C_L^{-\beta}] \tag{14}$$

i.e.
$$U = \alpha \sum_{j=T}^{L} (1 + \gamma)^{j-T} C_j^{-\beta} \tag{15}$$

(14) is a monotonic transformation of a CES function;[2] α, β, and γ are parameters. $\sigma \equiv \dfrac{1}{1 + \beta}$ is the elasticity of substitution between consumption in any pair of periods. γ is interpreted as the consumer's subjective rate of discount for future consumption. The consumer's resource constraint is:

$$v_T = \sum_{j=T}^{L} \left(\frac{1}{1 + r} \right)^{j-T} C_j \tag{16}$$

---

1. This example is taken from W. E. Weber, "The effect of interest rates on aggregate consumption", _American Economic Review_, Vol. 60 (1970), pp. 591-600. To reduce the burden of notation somewhat, the following convention will be followed: $C_T \equiv C_t^T$, $C_{T+1} \equiv C_t^{eT(T+1)}$, $v_t^T \equiv v_T$, $r_t^T \equiv r$.

2. For another study employing a CES type utility function in the context of the LCH see M. J. Farrell, "The magnitude of 'rate-of-growth' effects on aggregate savings", _Economic Journal_, Vol. 80 (1970), pp. 873-894 esp. p. 890 ff.

Maximizing (15) subject to (16) gives for current consumption:

$$C_T = \gamma_T \ v_T$$

where

$$\gamma_T = \{ \sum_{j=T}^{L} [(1 + \gamma)(1 + r)^{-\beta}]^{\sigma(j-T)} \}^{-1}$$

Clearly γ_T depends upon the consumer's age (T), the rate of interest (r) and the form of the utility function (β and γ).

CHAPTER FIVE

WEALTH AND THE CONSUMPTION FUNCTION II:

FURTHER EVIDENCE

5.1   Introduction   In this final chapter reviewing work on the

consumption (saving) function, three further attempts to formulate

a successful aggregate consumption function will be examined. The

unifying element in each attempt is the introduction, at some stage

in the formulation, of consumer's net worth. In section 5.2, a paper

by Lydall[1] is discussed which assumes that saving takes place so as

to bring wealth stocks into some desired relationship with consumption.

For example, the consumer may adopt a pattern of consumption which

aims to accumulate wealth stocks equal to five times current consumption

expenditure. The aggregate saving function derived includes net

personal wealth as an explanatory variable. In section 5.3, work by

Ball and Drake, and Spiro is examined.[2] While both these functions

involve wealth in the initial stages of their formulation, transform-

ations are carried out which result in the variable's exclusion from

the final form of the function tested. Ball and Drake reject the

1. H. F. Lydall, "Saving and wealth", Australian Economic Papers,
    Vol. 2 (1963), pp. 228-250.

2. R. J. Ball and P. S. Drake, "The Relationship between aggregate
    consumption and wealth", International Economic Review, Vol. 5
    (1964), pp. 63-81; A. Spiro, "Wealth and the consumption function",
    Journal of Political Economy, Vol. 70 (1962), pp. 339-354.

approach of the PIH and the LCH, which have consumers attempting to choose an optimum pattern of consumption over the remainder of their life cycle subject to a life-time resources constraint,[1] and postulate instead that consumers' utility depends on current consumption and wealth, and that consumers set out to maximize utility subject to a budget restraint involving current income only. Finally, Spiro proposes that "savings are the result of a discrepancy between the actual and the desired stock of wealth", and that "when there is no discrepancy, savings equal zero".[2] In section 5.4, conclusions about the usefulness of wealth in the consumption function are presented.

5.2 <u>The work of Lydall</u>   As noted in the previous section, Lydall begins by supposing that "an individual ... has ... some view about the 'ideal' relation between the level of his consumption expenditure during a given period of time and the amount of accumulated wealth which he would like to pass on to subsequent periods".[3] The relation used by Lydall is:

$$w_i^* = kc_i \qquad (5.1)$$

where w_i^* = the consumer's desired level of wealth at the end of period i,[4] c_i = consumption expenditure during period i, k = a constant.

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1. See sections 3.2 and 4.2 above.

2. Spiro, "Wealth and the consumption function", p. 339.

3. Lydall, "Saving and wealth", p. 231.

4. The period i is assumed to be very much shorter than one year or one quarter.

All variables are measured in current prices. The individual aims, by his savings in the current period, to bring about some _partial_ adjustment of his actual wealth stocks to their desired level. Hence

$$s_i = m(w_i^* - w_{i-1}) \qquad 0 < m < 1 \qquad (5.2)$$

where s_i = the individual's current saving, w_{i-1} = wealth at the end of period i-1. Finally, we have the budget identity

$$x_i = c_i + s_i \qquad (5.3)$$

where x_i = disposable income during period i. Substituting (5.3) into (5.1) and the result into (5.2) gives:

$$s_i = mkx_i - mw_{i-1} - mks_i$$

$$\therefore \qquad s_i = \alpha x_i - \gamma w_{i-1} \qquad (5.4)$$

where $\alpha = \dfrac{mk}{1 + mk}$, $\gamma = \dfrac{m}{1 + mk}$. To obtain the longer period consumption function (i.e. the quarterly or annual function), Lydall supposes that there are n short periods per long period. Therefore, aggregating over time gives

$$\sum_{i=1}^{n} s_i = \alpha \sum_{i=1}^{n} x_i - \gamma \sum_{i=1}^{n} w_{i-1} \qquad (5.5)$$

Lydall next proposes that average wealth per short period ($\dfrac{1}{n} \sum_{i=1}^{n} w_{i-1}$) is approximately equal to initial wealth holdings (w_0) plus half of

total savings during the long period ($\frac{1}{2} \sum\limits_{i=1}^{n} s_i$), i.e.[1]

$$\frac{1}{n} \sum_{i=1}^{n} w_{i-1} = w_0 + \frac{1}{2} \sum_{i=1}^{n} s_i$$

$$\therefore \quad \sum_{i=1}^{n} w_{i-1} = nw_0 + \frac{n}{2} \sum_{i=1}^{n} s_i \qquad (5.6)$$

Substituting (5.6) into (5.5) gives:

$$\sum_{i=1}^{n} s_i = \alpha \sum_{i=1}^{n} x_i - n\gamma w_0 - \frac{n\gamma}{2} \sum_{i=1}^{n} s_i$$

$$\therefore \quad \sum_{i=1}^{n} s_i = \frac{2\alpha}{2 + n\gamma} \sum_{i=1}^{n} x_i - \frac{2n\gamma}{2 + n\gamma} w_0$$

i.e. $\quad s' = \lambda x' - \mu w'_{-1} \qquad (5.7)$

where $s' = \Sigma s_i$, $x' = \Sigma x_i$, $w'_{-1} = w_0$, $\lambda = \dfrac{2\alpha}{2 + n\gamma}$, $\mu = \dfrac{2n\gamma}{2 + n\gamma}$.

(5.7) is the individual saving function: savings (s') depend upon current disposable income (x') and the stock of wealth held by the consumer at the end of the previous period (w'_{-1}). The parameters of (5.7), λ and μ, are functions of n, m and k. Assuming then that n, m and k are constant over time, λ and μ will also be constant over time.

---

1. Suppose n = 4, w_0 = 100 and saving in each short period is s_i = 10. Then $\frac{1}{n} \Sigma w_{i-1} = \frac{1}{4}(100 + 110 + 120 + 130) = 115$, while

    $w_0 + \frac{1}{2} \Sigma s_i = 100 + \frac{1}{2}(10 + 10 + 10 + 10) = 120.$

To obtain the aggregate saving function, it is now necessary to aggregate (5.7) over all individuals. First, write (5.7) for individual j:

$$s_j' = \lambda_j x_j' - \mu_j w_{-1,j}'$$

Aggregating over all individuals gives:

$$\sum_{j=1}^{N} s_j' = \sum_{j=1}^{N} \lambda_j x_j' - \sum_{j=1}^{N} \mu_j w_{-1,j}' \qquad (5.8)$$

To simplify this expression, Lydall assumes[1] that there are only M distinct values of λ_j and μ_j in the community, and that the ratio of the mean income of each group of persons with parameters λ_k and μ_k to the mean income of the community is constant. A similar assumption is made about wealth. Hence

$$\bar{x}_k' = \theta_k \bar{x} \qquad k = 1, \ldots, M \qquad (5.9a)$$

$$\bar{w}_k' = \eta_k \bar{w} \qquad k = 1, \ldots, M \qquad (5.9b)$$

where θ_k and η_k are constants, $\bar{x}_k'$ and $\bar{w}_k'$ are the mean income and the mean wealth respectively of group k, and $\bar{x}$ and $\bar{w}$ are mean community income and wealth. Now, taking note of the fact that there are only M distinct values of λ_j we can write

$$\sum_{j=1}^{N} \lambda_j x_j' = \sum_{k=1}^{M} \lambda_k n_k \bar{x}_k' \qquad (5.10)$$

where n_k = the number of persons with parameter λ_k. Substituting

1. Lydall, "Saving and wealth", p. 234.

(5.9a) into the right hand side of (5.10) gives

$$\sum_{j=1}^{N} \lambda_j x'_j = \sum_{k=1}^{M} \lambda_k n_k \theta_k \bar{x}$$

$$= (\sum_{k=1}^{M} \lambda_k n_k \theta_k)(\frac{\sum_{j=1}^{N} x_j}{N})$$

i.e.
$$\sum_{j=1}^{N} \lambda_j x'_j = aX \qquad\qquad (5.11)$$

where $a = \dfrac{\sum\limits_{k=1}^{M} \lambda_k n_k \theta_k}{N}$ and $X = \sum\limits_{j=1}^{N} x_j$, i.e. X = aggregate disposable

income. In a similar fashion (5.9b) gives

$$\sum_{j=1}^{N} \mu_j w'_{-1,j} = bW_{-1} \qquad\qquad (5.12)$$

where $b = \dfrac{\sum\limits_{k=1}^{M} \mu_k n_k n_k}{N}$ and $W_{-1} = \sum\limits_{j=1}^{N} w'_{-1,j}$, i.e. W_{-1} = aggregate wealth

held by the community at the end of the previous period. Thus, the

aggregate savings function is (from (5.8), (5.11) and (5.12)):

$$S_t = aX_t - bW_{t-1} \qquad\qquad (5.13)$$

For a and b to be constant over time, θ_k and n_k and the size of each

group (with respect to the whole population) must be constant over time.

Before considering the statistical tests of this hypothesis,

the long-run/short-run characteristics of the model will be examined.

To find the long-run implications of the model, suppose disposable

income is growing at the rate ρ. Then, ignoring capital gains, the long-run savings and wealth ratios are[1]

$$\frac{S_t}{X_t} = \frac{\rho a}{\rho + b} \qquad (5.14)$$

$$\frac{W_{t-1}}{X_t} = \frac{a}{\rho + b} \qquad (5.15)$$

Thus, as in the case of the Stone-Rowe model and the LCH, the long-run savings and wealth ratios are <u>constant</u> for any particular value of ρ. Hence Lydall's work implies that a stable trend rate of growth of income will result in a constant long-run savings ratio. In addition (5.14) and (5.15) imply, as a first approximation, that cyclical fluctuations in the rate of growth of income around the trend rate will produce fluctuations in the savings ratio around its long-run value.

The implied cyclical behaviour of the savings ratio can be demonstrated fairly simply. From above we know that, in any year, savings are determined by

$$S_t = aX_t - bW_{t-1} \qquad (5.13)$$

The long-run savings function is

$$S_t = \frac{\rho a}{\rho + b} X_t \qquad (5.14)$$

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1. These can be derived by an argument almost identical to that used to obtain the corresponding expressions for the LCH (see the derivation of (4.11) and (4.12) on pp.113-116. **above**).

These savings functions are identical to the corresponding

expressions for the LCH, namely (4.8) and (4.12). Hence the

graphical treatment used in the previous chapter for the LCH applies

here,[1] and the conclusions about the cyclical characteristics of the

LCH also apply. That is,the savings function (5.13) implies that

the savings ratio $\frac{S}{X}$ will rise and fall along with cyclical increases

and decreases in the rate of growth of income.[2]

Thus the savings function proposed by Lydall is consistent

with both secular constancy and cyclical variability in the savings

ratio.

To conclude this section,we will now consider the

empirical evidence offered by Lydall. The savings function was

estimated in ratio form, i.e.

$$\frac{S_t}{X_t} = a - b \frac{W_{t-1}}{X_t}$$

to eliminate any problems due to multicollinearity. This may also

help to deal with any problems due to heteroskedastic disturbances.[3]

Since a is the marginal propensity to save out of the _flow_ variable X,

and b is the marginal propensity to save out of the _stock_ variable W_{-1},

we expect both a and b to be positive fractions with a considerably

greater than b. Savings (and hence wealth) were defined to include

expenditure on durables net of depreciation (i.e. consumption included

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1. See pp. 117-119 **above.**

2. See pp. 118-119 above.

3. See p. 90 above.

the use value of durables only). Using United States annual current-price data for the years (1920-41), Lydall obtained

$$\frac{S_t}{X_t} = 0.4098 - \underset{(0.004)}{0.0656} \frac{W_{t-1}}{X_t} \qquad \begin{array}{l} R^2 = 0.923 \\[2mm] d = 1.765 \end{array}$$

United Kingdom data for the period 1948-1960 gave the estimate

$$\frac{S_t}{X_t} = 0.1668 - \underset{(0.004)}{0.0368} \frac{W_{t-1}}{X_t} \qquad \begin{array}{l} R^2 = 0.883 \\[2mm] d = 1.340 \end{array}$$

The estimation method was SELS. The estimates are quite satisfactory, as judged by the accompanying statistics: the value of R^2 for each estimate is quite high, the values of d are consistent with absence of serial correlation, and the implied t-ratios are quite high. The signs and magnitudes of the estimates conform to prior expectations. The estimates for the two countries are quite different. This may perhaps be explained by the fact that the U.S. data was entirely pre-World War II while the U.K. data was entirely post-war. On the other hand, there does not seem to be any necessary reason why the estimates for these two countries should be identical, or even very close.[1]

5.3 <u>The Ball-Drake-Spiro consumption function</u>   Ball and Drake, and Spiro independently derived and tested an aggregate consumption function of the form[2]

$$C_t = aX_t + bC_{t-1} \qquad a + b = 1 \qquad (5.16)$$

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1. A suitable statistical test of the equality of the U.S. and U.K. coefficients is given in Johnston, <u>Econometric Methods</u>, pp. 136-138.

2. Spiro did not express his function in this form, but a Koyck transformation of the function tested would yield (5.16).

Ball and Drake begin by supposing that utility is a function of current consumption and end-of-period wealth, i.e.

$$u_{it} = f_i(w_{it}, c_{it}) \tag{5.17}$$

where w_{it} = end-of-period wealth for consumer i, c_{it} = current period consumption. f_i is assumed to be homogeneous of degree 1, hence

$$u_{it} = c_{it} \, h_i \left(\frac{w_{it}}{c_{it}} \right) \tag{5.18}$$

where $h_i \left(\frac{w_{it}}{c_{it}} \right) = f_i \left(\frac{w_{it}}{c_{it}}, 1 \right)$. The budget restraint is

$$x_{it} = c_{it} + w_{it} - w_{i,t-1} \tag{5.19}$$

Maximizing (5.18) subject to (5.19) yields:

$$w_{it} = k_i \, c_{it} \tag{5.20}$$

where $k_i = (1 - h_i' / h_i)$.[1] Aggregating (5.20) over all individuals yields

$$\sum_i w_{it} = \sum_i k_i \, c_{it}$$

$$= \left(\sum_i k_i \, \frac{c_{it}}{C_t} \right) C_t$$

$$\therefore \quad W_t = k \, C_t \tag{5.21}$$

where W_t = aggregate end-of-period wealth, C_t = aggregate consumption,

1. See Appendix 5.1.

$k = \sum_i k_i \dfrac{c_{it}}{C_t}$. Obviously, k is constant over time only if $\dfrac{c_{it}}{C_t}$ and k_i are constant over time. k, being the ratio of aggregate wealth to consumption expenditure, will be positive. The aggregate form of the budget identity is

$$X_t = C_t + W_t - W_{t-1} \tag{5.22}$$

Substituting (5.21) into (5.22) yields

$$X_t = C_t + kC_t - kC_{t-1}$$

$$\therefore \quad C_t = (1 - \sigma)X_t + \sigma C_{t-1} \tag{5.23}$$

where $\sigma = \dfrac{k}{1 + k}$. Since $k > 0$, $0 < \sigma < 1$.

So as to be able to examine the long-run implications of (5.23), Ball and Drake first obtain an expression for current saving[1] as follows:

$$S_t = W_t - W_{t-1}$$

which, from (5.22) gives,

$$S_t = X_t - C_t \tag{5.24}$$

$$= X_t - (1 - \sigma)X_t - \sigma C_{t-1} \qquad \text{from (5.23)}$$

$$= \sigma X_t - \sigma C_{t-1}$$

$$= \sigma[X_t - X_{t-1} + S_{t-1}] \qquad \text{from (5.24)}$$

1. Ball and Drake, "The relationship between aggregate consumption and wealth", p. 69.

$$\therefore \quad \frac{S_t}{X_t} = \sigma \left[\frac{X_t - X_{t-1}}{X_{t-1}} + \frac{S_{t-1}}{X_{t-1}} \right] \frac{X_{t-1}}{X_t} \qquad (5.25)$$

Suppose now that income is growing at the rate ρ. Therefore (5.25) gives

$$\frac{S_t}{X_t} = \sigma \left[\rho + \frac{S_{t-1}}{X_{t-1}} \right] \frac{1}{1 + \rho}$$

i.e. $\qquad \dfrac{S_t}{X_t} = \dfrac{\sigma\rho}{1 + \rho} + \dfrac{\sigma}{1 + \rho} \dfrac{S_{t-1}}{X_{t-1}}$

The solution of this difference equation is[1]

$$\frac{S_t}{X_t} = \frac{S_0}{X_0} \left(\frac{\sigma}{1 + \rho} \right)^t + \left(\frac{\overline{S}}{X} \right)$$

where $\left(\dfrac{\overline{S}}{X} \right)$ is the solution of

$$\left(\frac{\overline{S}}{X} \right) = \frac{\sigma\rho}{1 + \rho} + \frac{\sigma}{1 + \rho} \left(\frac{\overline{S}}{X} \right)$$

i.e. $\qquad \left(\dfrac{\overline{S}}{X} \right) = \dfrac{\sigma\rho}{(1 - \sigma) + \rho} \qquad (5.26)$

As we have noted above, $0 < \sigma < 1$. Hence, for $\rho > 0$ we have $0 < \dfrac{\sigma}{1 + \rho} < 1$, and $\dfrac{S_t}{X_t} \to \dfrac{\sigma\rho}{(1 - \sigma) + \rho}$ as $t \to \infty$. Thus the equilibrium savings ratio is a function of the rate of growth. If the long-run trend rate of growth is stable, the long-run savings ratio should be approximately constant at $\dfrac{\sigma\rho}{(1 - \sigma) + \rho}$. The same graphical technique used to explore the cyclical behaviour of the savings ratio that was

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1. See W. J. Baumol, Economic Dynamics (New York: Macmillan, 2nd ed., 1959), pp. 175-181.

implied by the Lydall model could also be used here.[1] Hence, as in

the case of Lydall, we can conclude that cyclical fluctuations in

the growth rate of income will produce corresponding fluctuations

in the savings ratio. Thus, the Ball-Drake consumption function

(5.23) is consistent with both long-run stability and short-run

variability in the savings ratio.

The wealth ratio corresponding to (5.26) can be easily found.
From (5.21) we can write

$$W_{t-1} = kC_{t-1}$$

$$= k(X_{t-1} - S_{t-1})$$

$$\therefore \quad \frac{W_{t-1}}{X_t} = k(1 - \frac{S_{t-1}}{X_{t-1}})(\frac{X_{t-1}}{X_t})$$

In equilibrium $\frac{S_{t-1}}{X_{t-1}} = \frac{\sigma\rho}{(1 - \sigma) + \rho}$, and $\frac{X_{t-1}}{X_t} = \frac{1}{1 + \rho}$.

$$\therefore \quad (\overline{\frac{W_{-1}}{X}}) = k(1 - \frac{\sigma\rho}{(1 - \sigma) + \rho})(\frac{1}{1 + \rho})$$

$$= k \cdot \frac{(1 - \sigma)(1 + \rho)}{(1 - \sigma) + \rho} \cdot \frac{1}{1 + \rho}$$

But $k = \frac{\sigma}{1 - \sigma}$.

$$\therefore \quad (\overline{\frac{W_{-1}}{X}}) = \frac{\sigma}{(1 - \sigma) + \rho} \qquad (5.27)$$

---

1. See pp. 117-119 above.

(5.26) and (5.27) are similar to the results derived for the S-R

model, LCH, and Lydall.[1]

Since the consumption function tested by Spiro is identical

with that of Ball and Drake, its derivation will be presented before

the empirical evidence bearing upon the Ball-Drake function is

discussed. Spiro begins with the aggregate consumption function[2]

$$C_t = BW_{t-1} + BX_t + \sum_{i=0}^{\infty} a_i X_{t-i} \qquad B > 0 \qquad (5.28)$$

where W_{t-1} = aggregate (non-human) wealth at the end of period (t-1),

X_t = disposable income (which Spiro assumes is received at the

beginning of each time period), and B, a_i are constants. After the

distribution of incomes (i.e. just at the beginning of each period),

assets held are $(W_{t-1} + X_t)$. The coefficient B represents the asset

effect upon consumption, and it is assumed that an increase in asset

holdings will produce an increase in consumption. The coefficients

a_i represent the influence of current income (beside its asset effect)

and all past incomes upon consumption. By a process of algebraic

manipulation, Spiro obtains the following expression for W_{t-1} in terms

of past incomes:[3]

$$W_{t-1} = \sum_{i=1}^{\infty} [(1 - B)^i - \sum_{n=0}^{i-1} (1 - B)^n a_{i-n-1}] X_{t-i} \qquad (5.29)$$

---

1. (3.49) and (3.50), pp. 98-99 above, (4.11) and (4.12), pp. 115-116, above, and (5.14) and (5.15), p. 146 above.

2. Spiro, "Wealth and the consumption function", p. 340.

3. See Appendix 5.2 pp. 167-168 below for the derivation of (5.29).

Substituting (5.29) into (5.28) gives:

$$C_t = BX_t + \sum_{i=0}^{\infty} a_i X_{t-i} + B \sum_{i=1}^{\infty} [(1 - B)^i - \sum_{n=0}^{i-1} (1 - B)^n a_{i-n-1}] X_{t-i}$$

$$= (B + a_0)X_t + \sum_{i=1}^{\infty} [a_i + B(1 - B)^i - B \sum_{n=0}^{i-1} (1 - B)^n a_{i-n-1}] X_{t-i}$$

$$\therefore \quad C_t = b_0 X_t + \sum_{i=1}^{\infty} b_i X_{t-i} \tag{5.30}$$

where $b_0 = B + a_0$

$$b_i = a_i + B(1 - B)^i - B \sum_{n=0}^{i-1} (1 - B)^n a_{i-n-1} \qquad i = 1, 2, 3, \ldots$$

To reduce the estimation problem to manageable proportions, Spiro assumed that the b_i were a set of geometrically declining weights summing to unity. In support of the unit sum of the weights, Spiro considered the case where income was constant at, say, $\bar{X}$. (5.30) can then be written as

$$C_t = D\bar{X}$$

where $D = \sum_{i=0}^{\infty} b_i$. Obviously, in this situation consumption is constant from period to period. Now if D is less than unity, savings are positive and hence wealth will increase. But an increase in wealth will produce an increase in consumption,[1] contradicting the earlier conclusion that consumption is constant. Hence D ∤ 1. Similarly D ∤ 1. Hence, the sum of the weights, D, must equal unity. Thus, (5.30) is written as:

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1. See (5.28) above.

$$C_t = K \sum_{i=0}^{\infty} (1 - K)^i X_{t-i} \qquad 0 < 1-K < 1 \qquad (5.31)$$

leaving only one parameter to be estimated from data on income and consumption. Spiro also shows that if the weights a_i in (5.28) are themselves geometrically declining weights of the form

$a_i = (K - B)(1 - K)^i$, then (5.31) follows exactly, rather than as an approximation from (5.30).[1]

The implied long-run relationships follow immediately. Suppose income is growing at rate $\rho > 0$. (5.31) can then be rewritten as

$$C_t = (K \sum_{i=0}^{\infty} (\frac{1 - K}{1 + \rho})^i) X_t$$

Since $0 < 1-K < 1$, we have $0 < \frac{1 - K}{1 + \rho} < 1$ for $\rho > 0$. Hence

$$C_t = \frac{K(1 + \rho)}{\rho + K} X_t$$

$$\therefore \qquad \frac{S_t}{X_t} = \frac{\rho(1 - K)}{\rho + K} \qquad (5.32)$$

That is, the long-run savings ratio is a constant for any particular trend rate of growth. The long run wealth-income ratio is a little more difficult to obtain. First from (5.29) we have:

$$W_{t-1} = \sum_{i=1}^{\infty} d_i X_{t-i} \qquad (5.33)$$

where $d_i = (1 - B)^i - \sum_{n=0}^{i-1} (1 - B)^n a_{i-n-1}$. From the definition of b_i,[2]

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1. Spiro, "Wealth and the consumption function", pp. 350-352.
2. See p. 154 above.

it is clear that

$$b_i = a_i + Bd_i$$

Now, as stated above, if the weights a_i are of the form $(K - B)(1 - K)^i$, b_i will equal $K(1 - K)^i$. Hence

$$K(1 - K)^i = (K - B)(1 - K)^i + Bd_i$$

$$\therefore \quad d_i = (1 - K)^i$$

Thus (5.33) can be written as

$$W_{t-1} = \sum_{i=1}^{\infty} (1 - K)^i X_{t-i}$$

$$= \left[\sum_{i=1}^{\infty} \left(\frac{1 - K}{1 + \rho} \right)^i \right] X_t$$

$$= \frac{1 - K}{K + \rho} X_t \qquad \text{since } 0 < \frac{1 - K}{1 + \rho} < 1$$

$$\therefore \quad \frac{W_{t-1}}{X_t} = \frac{1 - K}{K + \rho} \qquad\qquad (5.34)$$

Thus the long-run wealth-income ratio, as in previous models examined, is constant for any particular rate of growth.[1]

Having examined the formulation of the Spiro-Ball-Drake consumption function and its implied long-run/short-run properties, it remains to discuss the empirical evidence for the function.

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1. (5.32) and (5.34) are identical to the corresponding expressions for Ball and Drake, namely: (5.26) and (5.27), a result which is hardly surprising, since a Koyck transformation of (5.31) - the Spiro consumption function - yields (5.23) - the Ball and Drake function.

Spiro estimated his function using constant price data for the years 1905-1949 (excluding war years). The consumption data measured expenditure on non-durables, services and the use value of durables. Income was equal to personal disposable income plus retained corporation profits. Retained profits were included on the grounds that individuals will "take into account the savings of corporations (and hence the increase in value of their certificates of ownership) when determining how much they (the individuals) will save."[1] The estimated function was:

$$C_t = 0.162X_t + 0.162(0.838)X_{t-1} + 0.162(0.838)^2X_{t-2} + \ldots$$
$$R^2 = 0.98$$

This result is unsatisfactory for a number of reasons. First, no information is given as to how the estimate was obtained. Secondly, no standard errors are provided, making significance tests impossible. Thirdly, a visual examination only was used to check for possible autocorrelation of disturbances. (This check seemed to indicate the presence of auto correlation.) Finally, the estimate does not give much indication as to the likely performance of the hypothesis if quarterly data were to be employed.

While still using annual data, Ball and Drake tested the hypothesis more thoroughly. Firstly, they fitted the function

$$C_t = aX_t + bC_{t-1}$$

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1. Spiro, "Wealth and the consumption function", p. 344.

with intercept term suppressed. The values of a and b obtained

enabled a check upon the theory which implied that $a + b = 1.$[1] Three

sets of consumption data were employed: Total consumption figures

(for the U.S. and the U.K.), consumption of non-durable goods and

services (for the U.K.), and consumption of non-durables plus the

use value of durables (for the U.S.). The income variable used was

personal disposable income. Both SELS and 2SLS estimates were

provided. The U.S. estimates, using post war data (1946-1960), for

total consumption were

$$\text{SELS:} \quad C_t = \underset{(0.13)}{0.66} \; X_t + \underset{(0.15)}{0.31} \; C_{t-1} \qquad \bar{R}^2 = 0.99$$

$$\text{2SLS:} \quad C_t = \underset{(0.30)}{0.23} \; X_t + \underset{(0.38)}{0.78} \; C_{t-1} \qquad \bar{R}^2 = 0.98$$

The sum of a and b in both estimates does not differ significantly

from unity.[2] In the case of the 2SLS estimate the coefficient of X_t

is not significant. On the other hand, in the case of the SELS

estimate the coefficients of X_t and C_{t-1} are, respectively, much

higher and much lower than their theoretical values.[3] The results for

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1. See (5.23) p. 150 above.

2. For the statistical test employed here see Theil, Principles of Econometrics, p. 138.

3. For example, from (5.23) the coefficient of X_t is $1 - \sigma = \dfrac{1}{1 + k}$, where k is the wealth-consumption ratio. Available data on wealth and consumption suggest that $1 - \sigma$ should be approximately 0.17 - 0.20. See Ball and Drake, "The relationship between aggregate consumption and wealth", Table 3, p. 78.

the U.K. are somewhat better. Using data for the years 1950-1960, both SELS and 2SLS estimation yielded

$$C_t = \underset{(0.10)}{0.45} X_t + \underset{(0.11)}{0.54} C_{t-1} \qquad \bar{R}^2 = 0.98$$

While the coefficients are statistically significant, they do not coincide well with their theoretical values. (The coefficient of X should be in the region of 0.28, while that of C_{t-1} should be approximately 0.72.)

The results for consumption of non-durables etc. for the U.S., using postwar data were

$$\text{SELS:} \quad C_t = \underset{(0.14)}{0.50} X_t + \underset{(0.16)}{0.44} C_{t-1} \qquad \bar{R}^2 = 0.99$$

$$\text{2SLS:} \quad C_t = \underset{(0.22)}{0.36} X_t + \underset{(0.26)}{0.61} C_{t-1} \qquad \bar{R}^2 = 0.98$$

For the 2SLS estimate the sum of the coefficients is not significantly different from unity. However, the coefficient of X_t is less than twice its standard error. The coefficients in the SELS estimate are significant, and have moved closer to their theoretical values[1] (although still a long way off). The sum of the coefficients is significantly different from unity. The results for non--durable consumption for the U.K. were

$$\text{SELS:} \quad C_t = \underset{(0.06)}{0.29} X_t + \underset{(0.06)}{0.68} C_{t-1} \qquad \bar{R}^2 = 0.99$$

$$\text{2SLS:} \quad C_t = \underset{(0.06)}{0.29} X_t + \underset{(0.07)}{0.68} C_{t-1} \qquad \bar{R}^2 = 0.98$$

---

1. Which are 0.14 to 0.17 for a and 0.83 to 0.85 for b.

The sum of the coefficients in each case is not significantly different from unity. Both coefficients are significantly different from zero. The coefficients are also close to their theoretical values.[1] The authors state that none of the estimates presented above show signs of serial correlation. In addition, various estimates, including an intercept term, were made using U.S. data and 2SLS. In all cases, the constant term was not statistically significant.

The estimates by Ball and Drake suggest that their hypothesis is not unreasonable for non-durable consumption, particularly for U.K. data. However, the hypothesis still has to pass the more exacting test of quarterly data. Results by Griliches et. al., already presented above,[2] give some support to it. Using total consumption data (1947-1960) for the U.S. Griliches et. al. obtained (by SELS):

$$C_t = 3.1 + \underset{(0.085)}{0.300} X_t + \underset{(0.097)}{0.670} C_{t-1} \qquad \begin{array}{l} R^2 = 0.994 \\ d = 1.09 \end{array}$$

Both coefficients are significant, and sum close to unity (0.970). The Durbin-Watson test, as far as it is valid in this situation, suggests the presence of serial correlation and, hence, the significance tests are not strictly applicable.[3] Evans has also tested a variation of the Spiro-Ball-Drake function.[4] Using United States quarterly data

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1. Ball and Drake, "The Relationship between aggregate consumption and wealth", p. 81.

2. See p. 39 above. Note that the estimate by Griliches is not a strict test of the hypothesis, since the intercept term is not restricted to zero. The estimated value of the intercept is, however, very close to zero.

3. See p. 24 above for the implications of autocorrelated disturbances

4. M. K. Evans, "The importance of wealth in the consumption function", Journal of Political Economy, Vol. 75 (1967), pp. 335-351.

for 1947–1962 Evans obtained for <u>total consumption</u>:

$$C_t = \underset{(0.082)}{0.466} X_t + \underset{(0.091)}{0.590} \frac{1}{4} \overset{4}{\underset{i=1}{\Sigma}} C_{t-i} \qquad R^2 = 0.993$$
$$d = 0.99$$

and for <u>non-durable consumption</u> (including use value of durables):

$$C_t = \underset{(0.046)}{0.286} X_t + \underset{(0.053)}{0.695} \frac{1}{4} \overset{4}{\underset{i=1}{\Sigma}} C_{t-i} \qquad R^2 = 0.998$$
$$d = 1.83$$

The lag term ($\frac{1}{4} \overset{4}{\underset{i=1}{\Sigma}} C_{t-i}$) rather than C_{t-1} is used on the grounds

"that additional information can be obtained about consumption

patterns by including the consumption of the previous four quarters".[1]

Estimation was by SELS. All coefficients are significant, and the

value of R^2 is very high. The value of d, however, indicates the

presence of serial correlation in the case of total consumption.

Restricting attention to the non-durables estimate, we see that the

coefficients sum to 0.981. This value is significantly less than

unity.[2] In addition, Evans estimated this function with an intercept

term, obtaining

$$C_t = \underset{(1.28)}{3.31} + \underset{(0.045)}{0.302} X_t + \underset{(0.050)}{0.691} \frac{1}{4} \overset{4}{\underset{i=1}{\Sigma}} C_{t-i}$$

The intercept term here is significant, a further point against the

Spiro-Ball-Drake formulation of the aggregate consumption function.

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1. <u>Ibid</u>., p. 345.

2. <u>Ibid</u>., p. 342–343.

From the above then, we conclude that there is not strong support for the hypothesis when <u>annual</u> data on <u>total consumption</u> is used. The hypothesis does perform better when <u>annual data</u> on <u>non-durable consumption</u> is employed, particularly in the case of the United Kingdom, where the parameter estimates are significant, agree closely with their theoretical values, and sum close to unity. The <u>quarterly</u> estimates by Griliches <u>et. al.</u> (using total consumption data) yield significant estimates of the coefficients, which sum close to unity. There is insufficient information, however, to test the statistical significance of the sum of the coefficients. In addition, the Durbin-Watson test indicates that the residuals are serially correlated. The estimate by Evans employing data on consumption of non-durables and services (plus the use value of durables) passes the Durbin-Watson test, but the estimated coefficients are significantly different from unity. In addition, when estimated with an intercept term, it was found that the intercept was significant.

5.4 <u>Conclusions</u>   In this chapter, and the previous one, a number of hypotheses involving wealth have been discussed, and it is the purpose of this section to assess their relative merits as indicated by the empirical evidence cited.

Turning first to the work which does not include wealth in the final function tested, i.e. that of Spiro and Ball and Drake. While the hypothesis performs reasonably well for annual data on non-durable consumption, its performance with quarterly data is not satisfactory. Evidence has been given which runs contrary to the

homogeneous form of the function.[1] In addition, further evidence

has been cited which is inconsistent with the theoretical implication

that the function is homogeneous of degree one.[2] However, since

Spiro et. al. do not include wealth in the function actually tested,

the results just referred to cannot be regarded as a decisive test

against the wealth hypothesis[3] in general, but just as a test

unfavourable to their particular form of the wealth hypothesis.

The remaining work examined is that by Lydall, and that

based upon the Life Cycle hypothesis. The version of the wealth

hypothesis put forward by Lydall performed reasonably well with annual

data. However, it has not been subjected to the more exacting test of

estimation by quarterly data. Without this further test it is

difficult to feel more than "neutral" towards Lydall's hypothesis.

Thus, we are left with the LCH. As noted above,[4] while the LCH performs

reasonably well with annual data, its performance with quarterly data

is not good. A quarterly estimate of the LCH consumption function,

made by Evans,[5] showed evidence of significant serial correlation

and gave a value of $\bar{R}^2$ of only 0.130.

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1. See p. 161 above.

2. See p. 161 above.

3. The wealth hypothesis is taken to mean any hypothesis which
   introduces wealth, either explicitly or implicitly, into the
   aggregate consumption function.

4. Sect. 4.4 p. 133 above.

5. See p. 128 above.

The discrepancy between the satisfactory performance of the wealth hypothesis using annual data and its unsatisfactory performance when quarterly data is used, may be due to the use of the same formulation of the consumption function for both annual and quarterly data. For example, the LCH is tested in the form

$$C_t = a_1 Y_t + a_2 Y_t^e + a_3 W_{t-1}$$

regardless of whether the test data is quarterly or annual. It seems reasonable, however, to expect a relationship involving quarterly data to be more complicated especially in its dynamic characteristics, than one involving annual data. Two studies referred to previously tend to confirm this expectation. One is the study by Branson and Klevorick,[1] which tests the LCH by an aggregate consumption function of the form:

$$\ell n \left(\frac{C}{N} \right)_t = b_0 + \sum_{i=0}^{I} \gamma_i \, \ell n \left(\frac{Y}{N} \right)_{t-i} + \sum_{j=0}^{J} \delta_j \, \ell n \left(\frac{W}{N} \right)_{t-1-j}$$

$$+ \sum_{k=0}^{K} \eta_k \, \ell n \, P_{t-k}$$

The authors of this study employ the Almon technique for estimating the distributed lag weights, thereby ensuring great flexibility in the choice of weights. This version of the LCH performs well in terms of the statistical significance of the parameters and a high value of R^2. Another quarterly study which performs quite well is the

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1. See pp. 129-131 above.

mixed PIH/LCH model of Stone and Rowe. In the Stone-Rowe model

$$C_t = \alpha_1 \sum_{i=0}^{\infty} \lambda(1 - \lambda)^i W_{t-i} + (\beta_1 - \beta_2) \sum_{i=0}^{\infty} \lambda(1 - \lambda)^i X_{t-i} + \beta_2 X_t$$

While the lag structure chosen here is more restrictive than in the

Branson and Klevorick study (W_{t-i} and X_{t-i} have identical weights,

and the weights are specified to be geometrically declining summing

to one) the model nevertheless performs well with quarterly data.[1]

Thus, rather than reject the use of a wealth variable in a quarterly

Australian study, because of the poor performance of the LCH, Ball-

Drake-Spiro etc. with quarterly data, it would be more useful to

experiment with dynamic formulations of these hypotheses along the

lines suggested in the Stone-Rowe and Branson-Klevorick studies.

Our examination of the principal overseas work on the

aggregate consumption function is now complete. The next chapter

will consist of a discussion of work done on the Australian consumption

function. Following this discussion,it is intended, where possible,

to try for Australian quarterly data those ideas which have been

found successful overseas and which have not yet been tested with

Australian data, or have been inadequately tested. From the results

of this work it is hoped to be able to suggest a satisfactory form

for the short-run Australian aggregate consumption function.

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1. See pp. 91-92 above.

5.1 Deriving (5.20) from (5.18) and (5.19)

From (5.18) and (5.19) form the expression

$$L = c_{it} h_i \left(\frac{w_{it}}{c_{it}} \right) + \lambda(x_{it} - c_{it} - w_{it} + w_{i,t-1}) \tag{1}$$

$$\therefore \quad \frac{\partial L}{\partial c_{it}} = h_i + c_{it} h_i' \left[-\frac{w_{it}}{c_{it}^2} \right] - \lambda = 0 \tag{2}$$

$$\frac{\partial L}{\partial w_{it}} = c_{it} h_i' \left[\frac{1}{c_{it}} \right] - \lambda = 0 \tag{3}$$

From (3) $\lambda = h_i'$. Substitute into (2).

$$\therefore \quad h_i - \frac{w_{it}}{c_{it}} h_i' - h_i' = 0$$

$$\therefore \quad \frac{w_{it}}{c_{it}} = \left[1 - \frac{h_i'}{h_i} \right]$$

$$\therefore \quad w_{it} = k_i c_{it} \tag{5.20}$$

where $k_i = \left(1 - \frac{h_i'}{h_i} \right)$. Since h_i is itself a function of $\frac{w_{it}}{c_{it}}$, further

specification of the utility function is needed before k_i can be said

to be independent of time.

5.2 Derivation of (5.29)

(5.28) can be written as:

$$C_t = BW_{t-1} + x_t \tag{1}$$

where

$$x_t = BX_t + \sum_{i=0}^{\infty} a_i X_{t-i} \tag{2}$$

Now by definition:

$$S_{t-1} = X_{t-1} - C_{t-1} \tag{3}$$

$$\therefore \quad S_{t-1} = X_{t-1} - BW_{t-2} - x_{t-1} \qquad \text{from (1)} \tag{4}$$

Again, by definition:

$$W_{t-1} = W_{t-2} + S_{t-1} \tag{5}$$

Therefore, from (4) we have

$$W_{t-1} = W_{t-2} + X_{t-1} - BW_{t-2} - x_{t-1}$$

$$\therefore \quad W_{t-1} = (1 - B)W_{t-2} + X_{t-1} - x_{t-1} \tag{6}$$

Lagging (6) one period, and substituting the result into (6) gives:

$$W_{t-1} = (1 - B)^2 W_{t-3} + (X_{t-1} - x_{t-1}) + (1 - B)(X_{t-2} - x_{t-2})$$

Repeated lagging and substitution gives:[1]

$$W_{t-1} = \sum_{j=0}^{\infty} (1 - B)^j (X_{t-1-j} - x_{t-1-j}) \tag{7}$$

---

1. The term in lagged $W \to 0$ since $(1 - B)^j \to 0$ as $j \to \infty$.

Substituting (2) into (7) gives

$$W_{t-1} = \sum_{j=0}^{\infty} (1 - B)^j [(1 - B)X_{t-1-j} - \sum_{i=0}^{\infty} a_1 X_{t-1-j-i}] \qquad (8)$$

Expanding out (8) gives

$$
\begin{aligned}
W_{t-1} = \; & [(1 - B)X_{t-1} - a_0 X_{t-1} - a_1 X_{t-2} - a_2 X_{t-3} - \ldots] \\
& + (1 - B)[(1 - B)X_{t-2} - a_0 X_{t-2} - a_1 X_{t-3} - a_2 X_{t-4} - \ldots] \\
& + (1 - B)^2 [(1 - B)X_{t-3} - a_0 X_{t-3} - a_1 X_{t-4} - a_2 X_{t-5} - \ldots] \\
& + \ldots \\
& \;\; \vdots \\
= \; & [(1 - B) - a_0]X_{t-1} + [(1 - B)^2 - (1 - B)a_0 - a_1]X_{t-2} \\
& + [(1 - B)^3 - (1 - B)^2 a_0 - (1 - B)a_1 - a_2]X_{t-3} + \ldots
\end{aligned}
$$

i.e. $W_{t-1} = \sum_{i=1}^{\infty} [(1 - B)^i - \sum_{n=0}^{i-1} (1 - B)^n a_{i-n-1}]X_{t-i} \qquad (5.29)$

CHAPTER SIX

STUDIES OF THE AUSTRALIAN CONSUMPTION FUNCTION

6.1 Introduction   In this chapter we turn to the second of our

aims, namely: To critically review work on the Australian aggregate

consumption function. This review is essential ground work for the

empirical work on the Australian consumption function to be done in

chapter seven below. In chapter seven we are aiming (i) to test

various ideas that have been proven useful elsewhere,and (ii) to test

promising Australian ideas that have as yet not been adequately tested.

To be able to do this,we must have previously carried out a review of

both the overseas and the Australian literature on the consumption

function. In the previous four chapters we have carried out a review

of the principal overseas work,and we now turn to a discussion of the

Australian work.

The discussion has been split into two main parts. The

first part, section 6.2, deals with single equation studies, i.e. those

where an attempt has been made to find one relationship to explain

total consumption. A number of features of this work deserve comment

here. Firstly the number of studies is exceedingly small. Only six

published studies were found, and within these there is only one

really "Australian" idea.[1] A further feature is the absence of any

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1. Another Australian study (B. L. Bentick, "Foreign borrowing,
    wealth, and consumption: Victoria 1873-1893", Economic Record,
    Vol. 45 (1969), pp. 415-431) has been left out because of the
    time period covered by the study,and also because it uses data
    covering Victoria only.

effort, such as that by Ferber referred to previously,[1] to test a whole series of hypothetical consumption functions using a common set of (Australian) data. Thirdly, Australian authors have not bothered to seek out the long-run implications of their hypotheses. There are possibly two reasons for this. Firstly, some of the consumption functions tested have been of such a simple nature that the long-run implications can be discerned without any special effort. Secondly, and perhaps more importantly, there is no set of long-run Australian data corresponding to that of Kuznets for the United States. Thus it is not clear exactly what long-run characteristics it is desirable for an Australian consumption function to have.[2] A fourth feature of this work is the lack of any attempt to build up to an aggregate consumption function from an examination of consumer behaviour at the micro level. Instead, researchers have proceeded by putting forward various lists of aggregate variables which might prove useful. In this sense these studies correspond to those discussed in chapter two above.[3]

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1. See footnote 1 p. 26 above.

2. These comments do not apply unqualified to the study by Lydall ("Saving and Wealth"). However, when discussing these points, Lydall does so in the context of the stable United States savings ratio discovered by Kuznets (See "Saving and Wealth", p. 243).

3. Once again, these remarks are not meant to apply to the study by Lydall referred to in the previous footnote.

The second part of our discussion, sections 6.3 and 6.4, deals with multiple equation studies, i.e. studies in which relationships are sought to explain separate components of total consumption, such as consumption of non-durables, consumption of durables. Almost all of this work has been associated with various efforts to build a complete econometric model of the Australian economy. Since this model-building effort has only gotten under way in recent times, the total amount of work is not very extensive, and it will be possible to present the consumption equations from almost all available econometric models of Australia. The models employing annual data are discussed in section 6.3, while those using quarterly data are dealt with in section 6.4. In discussing these disaggregated consumption functions, we are departing from the practice of previous chapters, where only single equation studies were examined. However, we feel justified in doing this in the light of the small amount of published work on the Australian consumption function. Conclusions are presented in section 6.5.

6.2 <u>Single Equation Studies</u>[1]   The six studies to be examined in this section involve four different formulations of the aggregate consumption function and each formulation will be dealt with in the order of its publication. The principal discussion, however, will centre on the

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1.   See appendix 6.1 below for a list of the notation to be used in this chapter. Since the list of variables is quite extensive, alphabetized symbols have been employed, e.g. CMV for consumption of motor vehicles.

suggestion by Arndt and Cameron that consumption is a linear function
of non-farm disposable income.[1]

Naturally, the first estimate of the consumption function
using Australian data was based upon the Keynesian function:
$C_t = a + bX_t$. This estimate was made by P. J. Lawler in 1949.[2] By
combining current and constant price data with aggregate and per-
capita data, four estimates were obtained. Although no correlation
coefficients were given, recomputation using Lawler's data shows that
the per-capita constant price equation gives the highest value for
the correlation coefficient.[3] Lawler's estimate for this equation was:

$$\frac{C_t}{N_t} = 25.6 + 0.672 \frac{X_t}{N_t} \tag{6.1}$$

The sample data were in £'s, for the years 1928-29 to 1939-40,
expressed in 1938-39 prices. Estimation was by SELS. Lawler himself
chose (6.1) as the preferable estimate, although providing no statistical
measures upon which to base the choice. Lawler's other estimates are
shown in table 6.1.[4]

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1. H. W. Arndt and B. Cameron, "An Australian consumption function,
   Economic Record, Vol. 33 (1957), pp. 108-115, 268.

2. P. J. Lawler, "The consumption function", Economic Record,
   Vol. 25 (1949), pp. 93-122.

3. It is not possible to discriminate between the equations on the
   basis of the statistical significance of the parameters, since
   all parameter estimates were highly significant.

4. See p. 183 below.

The second estimate to appear was that by Arndt and Cameron in 1957.[1] The Arndt-Cameron consumption function was simply the Keynesian function, but with non-farm personal disposable income (XNF) used as the income variable rather than personal disposable income (X). The reasoning advanced to support this change was:

> "that farmers' consumption levels are in various ways (such as direct personal contacts, availability of goods and services, advertising) influenced by consumption levels in the rest of the community. If farm consumption is a function of non-farm consumption, which in turn is a function of non-farm disposable income, total consumption will be a function of non-farm disposable incomes."[2]

That is: $\qquad C = C^F + C^{NF}$

where C^F and C^{NF} are farm and non-farm consumption respectively. Since C^F is influenced by C^{NF}, we can rewrite this as:

$$C = f(C^{NF}) + C^{NF}$$

Arndt and Cameron have further argued that C^{NF} is determined by non-farm disposable income, XNF. Hence

$$C = f[g(XNF)] + g(XNF)$$

i.e. $\qquad C = f^*(XNF)$

Arndt and Cameron suppose that f^* is a linear function giving:

$$C_t = a + b\ XNF_t \qquad\qquad\qquad (6.2)$$

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1. Arndt and Cameron, "An Australian consumption function".

2. Ibid., p. 109.

Essentially, there are two steps in the argument put forward by Arndt and Cameron: (i) $C^F = f(C^{NF})$ and (ii) $C^{NF} = g(XNF)$. Step (i) implies that the farming community adjusts its consumption standards to those of the non-farming community with such a short lag that C^F is determined by the non-farm consumption of the <u>same</u> period. While this may be plausible for long periods, it seems more reasonable for adjustment to take place over several periods when these periods are as short as 3 months or twelve months.[1] With respect to step (ii), it should be clear from previous chapters that a relationship attempting to explain the level of consumption by the level of disposable income <u>alone</u> will be inadequate. Thus, neither step in the argument seems very convincing on <u>a priori</u> grounds. Nor is the empirical evidence put forward very satisfactory. Using annual data for 1946-47 to 1955-56, Arndt and Cameron obtain (by SELS):

$$C_t = 288.2 + 0.8875 \ XNF_t \qquad r = 0.9926$$

The data are in £m. in 1953-54 prices. Similar regressions were carried out for the 10 years 1928-29 to 1937-38 and the 20 years 1928-29/1937-38 <u>and</u> 1946-47/1955-56. In all cases, the correlation coefficient was very high. However, the correlation coefficients by

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1. To allow for a more gradual adjustment, the model could be reformulated as:
$$C = C^F + C^{NF}, \ C^F = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i \ C_{-1}^{NF}, \ C^{NF} = \alpha + \beta XNF$$
Rearranging, we obtain:
$$C = 2\alpha(1 - \lambda) + (2 - \lambda)\beta XNF - \lambda\beta XNF_{-1} + \lambda C_{-1}$$
A test of this hypothesis is included in chapter seven below.

themselves are not very strong support for the hypothesis. Both

variables (i.e. C_t and XNF_t) showed a strong upward trend in the

post-war period,[1] which of itself would lead to a high value of r.

Recognizing this, Arndt and Cameron regressed year-to-year <u>changes</u>

in consumption onto both year-to-year changes in XNF and year-to-year

changes in X. The correlation coefficients obtained were 0.97 and

0.36 respectively.[2] <u>Current price</u> data were used here. While this

is a piece of more impressive evidence in favour of the Arndt-Cameron

hypothesis,there is no reason to believe that the same result would

be obtained when using quarterly data.[3] In general,it must be

concluded that the hypothesis has not been rigorously tested: The

estimates were unsupported by standard errors, making tests of

significance impossible. No test was made for possible autocorrelation

of disturbances, nor was any effort made to test the predictive power of

the function against alternatives. In short, at this stage, the

function seems little more than a superficially attractive idea that

has been inadequately tested.

A second estimate of this consumption function, in <u>current</u>

<u>price</u> form, was made by C.E.V. Leser[4] using annual data for 1948-49

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1. Indeed,XNF should be an even steadier series than X, since the more
   volatile component - farm disposable income - has been removed.

2. Arndt and Cameron, "An Australian consumption function", pp. 111, 268.

3. Indeed,see equation (6.9) below,where the regression of ΔC_t onto ΔX_t
   yields a squared correlation coefficient of 0.973.

4. C. E. Leser, "The pattern of Australian demand", <u>Economic Record</u>,
   Vol. 34 (1958), pp. 220-221.

to 1956-57. Leser began by supposing that consumption was a linear function of non-farm disposable income and farm disposable income:

$$C_t = a + b\ XNF_t + c(X_t - XNF_t)$$

i.e. $$C_t = a + (b - c)XNF_t + cX_t$$

To help reduce the problem of multicollinearity which might exist due to the similar trend in both XNF and X, the function was estimated in ratio form:

$$\frac{C_t}{X_t} = \frac{a}{X_t} + (b - c)\frac{XNF_t}{X_t} + c$$

Using SELS, Leser obtained

$$\frac{C_t}{X_t} = \frac{148.8}{X_t} + 0.99768\frac{XNF_t}{X_t} - 0.0718 \tag{6.3}$$

Leser reported a coefficient of multiple determination of approximately 0.95. (6.3) can be rewritten as:

$$C_t = 148.8 + 0.9259\ XNF_t - 0.0718\ (X_t - XNF_t) \tag{6.4}$$

As with the study by Arndt and Cameron, the great lack of statistical measures makes a proper assessment of this estimate impossible. One disturbing feature is the incorrect sign on the coefficient of farm disposable income (X - XNF). However, in the absence of standard errors it is not possible to determine whether or not this coefficient is in fact statistically significant.

Cameron, in a later work,[1] has provided both annual and quarterly estimates using more up-to-date figures. Using annual current price figures for 1953-54 to 1963-64, Cameron obtained by SELS the estimate:

$$C_t = 71.7 + \underset{(0.019)}{0.9792} \ XNF_t + \Delta HP_t \tag{6.4a}$$

$$r = 0.998$$

where ΔHP is the "current net increase in instalment credit balances outstanding". Presumably, to obtain this estimate Cameron regressed $(C_t - \Delta HP_t)$ onto XNF_t. The value of the correlation coefficient is high, and the coefficient of XNF is highly significant. Using deseasonalized current price quarterly data for the period 1958 (III) to 1964 (II), Cameron obtained the following two estimates:

$$CNF_t = 78.5 + \underset{(0.028)}{0.8374} [YNF_{t-1} - T_t] + \Delta HP_t \tag{6.5}$$

$$r = 0.9876$$

$$CNF_t = 68 + \underset{(0.0257)}{0.8241} \ XNF_t + \Delta HP_t \tag{6.6}$$

$$r = 0.989$$

where CNF_t = consumption of non-farm products, YNF = non-farm personal income, T = non-farm personal tax (both direct and indirect).[2] While the number of observations used (24) is not large, it is more than twice

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1. B. Cameron, _Production, Employment and Prices in Australia, 1958-59 to 1963-64_ (Melbourne: F. W. Cheshire, 1967).

2. Note that by definition $XNF_t = YNF_t - T_t$.

the number that had been used to obtain the annual estimates mentioned above. Once again, the value of r is high and parameters are highly significant. Estimate (6.4) was also subjected to a further test - namely its ex-post forecast ability. Forecasts for CNF for 1965 (II) and 1966 (II) were made, the forecast errors being approximately 1.4% and 2% respectively.

While the results discussed above do lend support to XNF being a useful income variable, they have not established its clear (or even marginal) superiority over personal disposable income as the relevant income variable.

There remain only two studies of the aggregate consumption function to look at. The first is Lydall's study of saving and wealth. Since this has been extensively discussed above,[1] it is now only necessary to present Lydall's results for Australian data. Using current price data for the years 1951-52 to 1961-62, SELS estimation gave:

$$\frac{S_t}{X_t} = 0.7300 - \underset{(0.013)}{0.2407} \frac{W_{t-1}}{X_t} \tag{6.7}$$

$$R^2 = 0.974$$
$$d = 1.753$$

Statistically speaking, the estimate is good: the value of R^2 is high, the value of d is consistent with absence of serial correlation and the coefficient of $\frac{W_{t-1}}{X_t}$ is highly significant. However, while the signs of both parameter estimates are consistent with expectations,

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1. See section 5.2 pp. 141-148 above.

their magnitudes are not. In particular, both estimates seem far too high. To see this, consider the consumption function corresponding to (6.7), namely:

$$C_t = 0.2700 \ X_t + 0.2407 \ W_{t-1}$$

From this we can see that an increase of, say, $10m. in the <u>stock</u> of wealth at a particular point in time will result in a perpetual increase in the annual flow of consumption expenditure of $2.4m. On the other hand, a perpetual increase of $10m. in the <u>flow</u> of income will result in a perpetual increase in the annual flow of consumption expenditure of only $2.7m. Lydall cites the manner in which the wealth data were obtained as a possible reason for this result. The series for W_{t-1} was constructed by assuming a base stock of wealth of £5,000m. in 1948-49 and cumulating net personal saving in subsequent years. To obtain an alternative series for W_{t-1}, Lydall arbitrarily cut £2,000m. from the base stock, obtaining the following estimate as a result:

$$\frac{S_t}{X_t} = 0.46 - 0.125 \ \frac{W_{t-1}}{X_t}$$

Until a better series for wealth is available, it cannot be said that the worth of a total personal wealth variable has been decided one way or another.

The final estimates of the consumption function to be looked at in this section arose as a result of an attempt to evaluate the

effects of taxation policy on consumption expenditure.[1] The results presented below use <u>current price</u> quarterly data; annual results were also presented by Auld. All equations were estimated by SELS. The basic quarterly consumption function tested by Auld is:

$$\Delta C_t = \alpha_0 + \alpha_1 \Delta X_t \tag{6.8}$$

The seasonal element in the data was removed by adding in three dummy variables S_{it}, i = 1, 2, 3 to give

$$\Delta C_t = \alpha_0 + \alpha_1 \Delta X_t + \alpha_2 S_{1t} + \alpha_3 S_{2t} + \alpha_4 S_{3t}$$

where S_{it} = 1 in period i and zero at other times. The estimated coefficients of the seasonal dummy variables are not presented in the text here, but are included in table 6.1. Auld's estimate of (6.8) is:

$$\Delta C_t = 162.2 + \underset{(0.033)}{0.1991} \, \Delta X_t \qquad R^2 = 0.973 \tag{6.9}$$

The coefficient of ΔX_t is significant. Amongst other variables tried were ΔHP – the net increase in instalment credit balances outstanding, ST^r – the estimated <u>reduction</u> in Commonwealth <u>sales tax revenue</u> due to a change in legislation, and ST^i – the estimated <u>increase</u> in Commonwealth <u>sales tax revenue</u> due to a change in legislation. The resulting estimates were:

$$\Delta C_t = 156.0 + \underset{(0.037)}{0.1875} \, \Delta X_t + \underset{(0.570)}{0.9139} \, \Delta HP_t \tag{6.10}$$

$$R^2 = 0.972$$

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1. D.A.L. Auld, "An application of econometrics to evaluate fiscal tax policy", <u>Economic Record</u>, Vol. 45 (1969), pp. 147-157.

$$\Delta C_t = 174.1 + \underset{(0.036)}{0.1869} \Delta X_t + \underset{(1.71)}{2.255} ST^r_{t-1} - \underset{(5.37)}{12.87} ST^i_{t-1} \quad (6.11)$$

$$R^2 = 0.978$$

In both (6.10) and (6.11) the coefficient of ΔX_t is significant. For the other variables, only the coefficient of ST^i_{t-1} is very significant. The negative coefficient of ST^i_{t-1} indicates that an increase in Commonwealth sales tax revenue (and hence a decrease in disposable income) in one quarter leads to a drop in consumption spending in the next quarter. Over all, the additional variables tried do not seem to improve the basic function significantly.[1]

The extra variables tried were more successful when ΔCD, the change in consumer expenditure on durables, was used as dependent variable.[2] The results obtained were

$$\Delta CD_t = 24.51 + \underset{(0.014)}{0.0497} \Delta X_t + \underset{(0.202)}{1.002} \Delta HP_t \quad (6.12)$$

$$R^2 = 0.947$$

$$\Delta CD_t = 29.55 + \underset{(0.018)}{0.0381} \Delta X_t + \underset{(0.868)}{0.3692} ST^r_{t-1} - \underset{(2.73)}{5.218} ST^i_{t-1} \quad (6.13)$$

$$R^2 = 0.912$$

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1. Indeed, the addition of ΔHP seems to cause a <u>fall</u> in the value of R^2 from 0.973 to 0.972. Since the value of R^2 cannot fall as extra variables are added into the regression (see Christ, <u>Econometric Models and Methods</u>, p. 510-511) this would seem to indicate the possibility of a computing error on the part of Auld, or that his reported results are values of $\bar{R}^2$ and not R^2.

2. By including a consumption function for durables, we are departing somewhat from the guidelines laid down in section 6.1, which said that this section was to be concerned with relationships explaining <u>total</u> consumption. However, as this work by Auld is not part of a multiple equation study (in the sense used in section 6.1), it does not fit exactly into sections 6.3 or 6.4 either. Hence, we have chosen to discuss it here.

This time both ΔHP_t and ST^i_{t-1} have significant coefficients. ST^r_{t-1} does not have a significant coefficient, indicating that consumers react to increases in sales tax, but not to reductions. That is, there is an asymmetry in the behaviour of consumers.

Another approach adopted by Auld was to split ΔX_t into two components: ΔX^T_t - the change in disposable income resulting from personal income tax changes,[1] and ΔX^A_t - changes in X due to other factors (in effect, $\Delta X^A_t = \Delta X_t - \Delta X^T_t$). The result was:

$$\Delta CD_t = 12.86 + \underset{(0.017)}{0.0267} \; \Delta X^A_t + \underset{(0.299)}{0.7056} \; \Delta X^T_t \qquad (6.14)$$

$$R^2 = 0.882$$

An attempt to refine this approach was made by using the variables ΔX^{Tr}_t - the change in X due to a personal income tax reduction, and ΔX^{Ti}_t - the change in X due to a tax increase. The resulting estimate was:

$$\Delta CD_t = 28.07 + \underset{(0.018)}{0.0243} \; \Delta X_t + \underset{(0.698)}{2.392} \; \Delta X^{Tr}_{t-1} - \underset{(0.429)}{0.5658} \; \Delta X^{Ti}_{t-1}$$

$$(6.15)$$

$$R^2 = 0.924$$

In (6.14) the coefficient of ΔX^T_t is significant, but that of ΔX^A_t is not. Since it is highly unlikely that changes in X^T are the principal determinants of changes in CD, (6.14) must be rejected as a serious

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1. Estimates for this variable were obtained from estimates of the revenue effect of income tax changes reported in the Budget Speech.

TABLE 6.1

SINGLE EQUATION STUDIES OF THE AUSTRALIAN CONSUMPTION FUNCTION

Author[a]	Data	Sample Period	Estimate[b]	R^2 etc.
Lawler [1949]	annual, current prices (£m.'s)	1928-29 to 1939-40	$C = 103.2 + 0.772\,X$	
	annual, constant prices (£m.'s)	"	$C = 136.0 + 0.726\,X$	
	annual, current prices (£'s)	"	$\frac{C}{N} = 16.2 + 0.764\,\frac{X}{N}$	
	annual, constant prices (£'s)	"	$\frac{C}{N} = 25.6 + 0.672\,\frac{X}{N}$	
Arndt and Cameron [1957]	annual, constant prices (£m.'s)	1928-29 to 1937-38	$C = -40.8 + 1.071\,XNF$	$r = 0.9952$
	"	1946-47 to 1955-56	$C = 288.2 + 0.8875\,XNF$	$r = 0.9926$
	"	1928-29 to 1937-38 plus 1946-47 to 1955-56	$C = 160.8 + 0.9342\,XNF$	$r = 0.9988$
	annual, current prices (£m.'s)	1946-47 to 1955-56	$C = 67.9 + 0.9598\,XNF$	
	"	1928-29 to 1937-38	$C = 0.8 + 1.0473\,XNF$	
Leser [1958]	annual, current prices (£m.'s)	1948-49 to 1956-57	$\frac{C}{X} = 0.99768\,\frac{XNF}{X} + \frac{148.8}{X}$ $- 0.07178$	$R^2 = 0.95$
Cameron [1967]	annual, current prices ($m.)	1953-54 to 1963-64	$C = 71.7 + \underset{(0.019)}{0.9792}\,XNF + \Delta HP$	$r = 0.998$
	quarterly, current prices ($m.) deseasonalized	1958(III) to 1964(II)	$CNF_t = 78.5 + \underset{(0.028)}{0.8374}[YNF_{t-1}$ $- T_t] + \Delta HP_t$	$r = 0.9876$
	"	"	$CNF = 68.0 + \underset{(0.0257)}{0.8241}\,XNF + \Delta HP$	$r = 0.989$
Lydall [1962]	annual, current prices	1951-52 to 1961-62	$\frac{S_t}{X_t} = 0.7300 - \underset{(0.013)}{0.2407}\,\frac{W_{t-1}}{X_t}$	$R^2 = 0.974$ $d = 1.753$

			Estimate	S_1	S_2	S_3
Auld [1969]	quarterly, current prices	1958 to 1965	6.9	-175.5 (22.5)	-36.84 (27.2)	-309.6 (13.4)
			6.10	-175.7 (26.6)	-37.53 (32.9)	-276.2 (26.8)
			6.11	-178.8 (26.9)	-39.15 (30.5)	-297.6 (16.8)
			6.12	-27.55 (11.2)	-20.67 (13.1)	-33.92 (9.70)
			6.13	-15.66 (13.7)	-4.58 (15.5)	-66.67 (8.56)
			6.14	-7.829 (11.3)	+13.59 (13.7)	-63.67 (8.84)
			6.15	3.455 (12.8)	+4.197 (14.6)	-71.41 (8.12)

Notes:

a: For sources see text.

b: All estimates were obtained by SELS.

estimate of the Australian consumption function. Similarly, in (6.15) the only variable with a significant coefficient is ΔX_{t-1}^{Tr}, and once again it seems most implausible that this variable is the principal determinant of changes in CD. Certainly, without more evidence than that offered by Auld there are no good grounds for accepting these estimates as serious estimates of the Australian consumption function.

This concludes the examination of single equation studies of the Australian consumption function. In the next two sections we shall examine a number of studies which use several equations to explain total consumption. For example, Nevile[1] divides total consumption (C) into two parts: consumption expenditure (excluding motor vehicles) (CEMV) and consumption expenditure on motor vehicles (CMV). Separate relationships to explain CEMV and CMV are then estimated. Those studies using annual data are discussed in section 6.3, while those using quarterly data are discussed in section 6.4.

6.3 Multiple Equation Studies using Annual Data   There have been six multiple equation studies using annual data. Three of the studies[2] proceed by dividing consumption into consumption excluding motor vehicles (CEMV) and consumption of motor vehicles (CMV). The various

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1. J. W. Nevile, "A simple econometric model of the Australian economy", Australian Economic Papers, Vol. 1 (1962), pp. 79-94.

2. Nevile, "A simple econometric model of the Australian economy"; J. Kmenta, "An econometric model of Australia, 1948-61", Australian Economic Papers, Vol. 5 (1966), pp. 131-164; and J. H. Duloy, "Structural changes due to immigration: an econometric study", Australian Economic Papers, Vol. 6 (1967), pp. 223-233.

relationships estimated are fairly straight forward extensions of the Arndt-Cameron consumption function. The estimated relationships are shown in table 6.2A below.[1] The remaining three studies[2] divide consumption expenditure into expenditure on durables (CD) and expenditure on non-durables (CND). These three studies have attempted to introduce some more modern ideas by the inclusion of lagged values of the dependent variable in each equation. The estimated relationships are shown in table 6.2B below.[3]

In his model of the Australian economy, Nevile[4] adopts the Arndt-Cameron consumption function to explain CEMV, obtaining (by SELS):

$$CEMV = -56.4 + 0.978 \ XNF \qquad R = 0.992 \qquad (6.16)$$
$$(0.041)$$

To explain motor vehicle expenditure, Nevile adds the percentage sales tax on motor vehicles (STMV) to the Arndt-Cameron function, obtaining (by SELS):

$$CMV = -396.4 + 0.130 \ XNF - 2.134 \ STMV \qquad R = 0.983 \qquad (6.17)$$
$$(0.014) \qquad (0.757)$$

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1. See p. 192 below.

2. N. Podder, "Forecasting with an econometric model" (paper presented at the 41st congress of ANZAAS, Adelaide, 18-22 Aug., 1969); J. A. Zerby, "An econometric model of monetary interaction in Australia", Australian Economic Papers, Vol. 8 (1969), pp. 154-177; J. W. Nevile, Fiscal Policy in Australia (Melbourne: F. W. Cheshire, 1970).

3. See p. 200 below.

4. Nevile, "A simple econometric model of the Australian economy".

The data used were in £ m. at constant prices and covered the years
1947-48 to 1959-60.   Consumption is divided in this way between CEMV
and CMV because, in Nevile's words "expenditure on motor cars ... is
important and distinctive enough to warrant a separate equation".[1]
Kmenta suggests that the distinctive character of motor vehicle
expenditure is emphasized by the fact "that governmental measures
to regulate aggregate demand have at times singled out the demand
for motor-cars as one of the factors requiring special attention".
The inclusion of sales tax on vehicles in the motor vehicle expenditure
function is justified by virtue of its being "the main regulator" of
the demand for vehicles.[2]

The main function of the model estimated by Kmenta was
"to serve as a means of analysing the cyclical effects of immigration
on the Australian post-war economy".   Kmenta assumes that the influx
of migrants leaves the parameters of the various relationships unaltered
but causes shifts in the relationships.   To measure these shifts, Kmenta
adds the variable Δ_t - net annual immigration - to each relationship.[3]
Using constant price data for the years 1947-48 to 1960-61, Kmenta
obtained:[4]

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1.   Ibid., p. 88.

2.   Kmenta, "An econometric model of Australia", p. 134.

3.   Ibid., pp. 131-132.

4.   Ibid., p. 144.

$$CEMV_t \;=\; 219.0122 \;+\; \underset{(0.0486)}{0.9129}\; XNF_t \;-\; \underset{(0.6275)}{0.4027}\; A_t \tag{6.18}$$

$$R^2 = 0.9698$$
$$d \;= 1.737$$

$$CMV_t \;=\; -166.5387 \;+\; \underset{(0.0164)}{0.11866}\; XNF_t \;-\; \underset{(0.9535)}{2.2804}\; STMV_t$$

$$+\; \underset{(0.1053)}{0.1362}\; A_{t-1} \tag{6.19}$$

$$R^2 = 0.9105$$
$$d \;= 2.993$$

The coefficients of XNF are significant at the 1% level, while the coefficient of the immigration variable is insignificant in both equations. The coefficient of STMV is significant at the 5% level. The value of the Durbin-Watson statistic in both cases is consistent with the hypothesis of no serial correlation of disturbances at the 5% level. Two-stage least squares was used as the method of estimation. This work by Kmenta is superior to much of that going before it, in the sense that proper significance tests on parameters have been carried out and residuals tested for serial correlation. Some earlier studies[1] do not even provide the information needed for the reader to carry out the tests. The net result of the study, however, is that we have still not advanced much beyond the ideas of Arndt and Cameron, and certainly not beyond those of Nevile in 1962.

The third study of this group was another attempt to discover whether or not immigrants had had any effect upon the Australian

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1. e.g. the 1958 study by Leser.

economy.[1] The author of the study, _Duloy_, validly criticized the

work by Kmenta on the grounds that

> "it assumes only an 'impact effect' of migration.
> To take the consumption function as an example,
> the impact effect hypothesis implies that migrants
> shift the consumption function only in the year of
> their arrival, and that they have a consumption-
> income relationship identical to that of the locals
> in all subsequent years. But if migrants differ from
> locals in some important economic characteristics,
> such as asset holdings or age structure, then they
> are likely to operate on a consumption function
> different from that of the locals for a number of
> years after the year of arrival."[2]

Duloy begins by assuming that per-capita consumption of migrants and

locals is determined by their respective non-farm disposable incomes,

and that the m.p.c. of both groups is equal. Thus we have

$$\frac{CEMV_\ell}{L} = \alpha_\ell + \beta \frac{XNF_\ell}{L} \tag{6.20}$$

$$\frac{CEMV_m}{MI} = \alpha_m + \beta \frac{XNF_m}{MI} \tag{6.21}$$

where the subscripts ℓ and m refer to locals and migrants respectively,

and L and MI are the number of locals and migrants respectively. Note

that, since Duloy defines migrants as "those who have migrated to

Australia and for whom behavioural parameters differ from the rest of

the population",[3] MI is not necessarily equal to the number living in

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1. Duloy, "Structural changes due to immigration".

2. _Ibid._, p. 223.

3. _Ibid._, p. 225.

the community who immigrated to the country at one time or another.
From (6.20) and (6.21) we can derive an expression for per-capita
consumption:

$$\frac{CEMV}{N} = [\ L \cdot \frac{CEMV_\ell}{L} + MI \cdot \frac{CEMV_m}{MI}\ ]\ /\ N$$

$$= [L\alpha_\ell + \beta XNF_\ell + MI\alpha_m + \beta XNF_m]\ /\ N$$

$$= [N\alpha_\ell + (\alpha_m - \alpha_\ell)MI + \beta XNF]\ /\ N$$

Adding time subscripts, and rearranging, we get

$$\therefore \quad \frac{CEMV_t}{N_t} = \alpha_\ell + (\alpha_m - \alpha_\ell)\frac{MI_t}{N_t} + \beta\frac{XNF_t}{N_t} \qquad (6.22)$$

Duloy next assumes that the consumption function for a
group of newly arrived migrants lies <u>above</u> that of the local community,
but that over time it adjusts **down** towards the local community function.
This adjustment takes place through the intercept parameter. Suppose
$\alpha_m^*(\tau)$ is the intercept parameter of the group of migrants who arrived
τ years ago, and that the adjustment process is given by:

$$\alpha_m^*(\tau) = \begin{cases} \alpha_\ell + k(1 - \gamma^{T-\tau}) & 0 \le \tau \le T \\ \alpha_\ell & \tau > T \end{cases} \qquad (6.23)$$

where $0 \le \gamma \le 1$. Thus, i $\not{t}$ it assumed that after T years the adjustment
is complete. Duloy's assumption about the direction of adjustment of
the migrant consumption function requires that $k > 0$.

The intercept parameter α_m must now be viewed as some sort of average of the individual intercepts $\alpha_m^*(\tau)$. Thus α_m will be a function of time, $\alpha_m(t)$. In particular, it is assumed that

$$\alpha_m(t) = \frac{\displaystyle\sum_{\tau=0}^{T-1} \alpha_m^*(\tau)\Delta_{t-\tau}}{\displaystyle\sum_{\tau=0}^{T-1} \Delta_{t-\tau}}$$

That is, $\alpha_m(t)$ is a weighted arithmetic average of the $\alpha_m^*(\tau)$, where the weights are net permanent arrivals $\Delta_{t-\tau}$.[1] Using (6.23), this can be rewritten as

$$\alpha_m(t) = \frac{\displaystyle\sum_{\tau=0}^{T-1} [\alpha_\ell + k(1 - \gamma^{T-\tau})]\Delta_{t-\tau}}{\displaystyle\sum_{\tau=0}^{T-1} \Delta_{t-\tau}}$$

$$\therefore \quad \alpha_m(t) = \alpha_\ell + k\frac{\displaystyle\sum_{\tau=0}^{T-1} (1 - \gamma^{T-\tau})\Delta_{t-\tau}}{\displaystyle\sum_{\tau=0}^{T-1} \Delta_{t-\tau}} \tag{6.24}$$

Also $\dfrac{MI_t}{N_t} = \displaystyle\sum_{\tau=0}^{T-1} \Delta_{t-\tau} \Big/ N_t$, which with (6.24) implies that

$$\therefore \quad (\alpha_m - \alpha_\ell)\frac{MI_t}{N_t} = k\frac{\displaystyle\sum_{\tau=0}^{T-1} (1 - \gamma^{T-\tau})\Delta_{t-\tau}}{N_t} \tag{6.25}$$

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1. Since adjustment is complete after T years, there will be only T terms in the average.

Substituting (6.25) into (6.22) gives:

$$\frac{CEMV_t}{N_t} = \alpha_\ell + k \frac{\sum_{\tau=0}^{T-1} (1 - \gamma^{T-\tau})\Delta_{t-\tau}}{N_t} + \beta \frac{XNF_t}{N_t} \qquad (6.26)$$

The parameters to be estimated are α_ℓ, k, γ, T, and β. A priori, we require $0 \leq \gamma \leq 1$. In addition, Duloy's assumptions imply $k > 0$. A number of different estimates were obtained by placing various restrictions on γ and/or T. The preferred estimate was obtained by setting $\gamma = 0$ and employing a non-linear iterative least squares procedure.[1] The estimated parameters were

$$\alpha_\ell = 6.56, \quad k = -353.95, \quad T = 7, \quad \beta = 1.01$$

with $R^2 = 0.99$ and $d = 2.37$. Hence the estimated consumption function has the form:

$$\frac{CEMV_t}{N_t} = 6.56 - 353.95 \frac{\sum_{\tau=0}^{6} \Delta_{t-\tau}}{N_t} + 1.01 \frac{XNF_t}{N_t} \qquad (6.27)$$

One apparently unsatisfactory feature of this estimate is that the estimated value of k is negative, the consequence of this being that the intercept parameter for each migrant group, $\alpha_m^*(\tau)$, lies <u>below</u> instead of above the intercept parameter of the local consumption function.[2] This implies that, at a particular income level, migrants spend a <u>lower</u> proportion of their income than do locals on consumption,

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1. Duloy, "Structural changes due to immigration", p. 229 footnote 7.

2. $\alpha_m^*(\tau) - \alpha_\ell = k(1 - \gamma^{T-\tau}) < 0$ if $k < 0$ and $0 \leq \gamma \leq 1$.

TABLE 6.2A

MULTIPLE EQUATION STUDIES OF THE AUSTRALIAN CONSUMPTION FUNCTION USING ANNUAL DATA: I

Author[a]	Data	Sample Period	Estimate	Estimation Method	R^2 etc.
Nevile [1962]	constant prices (£m.)	1947-48 to 1959-60	$CEMV_t = -56.4 + \underset{(0.041)}{0.978} XNF_t$	SELS	$R = 0.992$
			$CMV_t = -396.4 + \underset{(0.014)}{0.130} XNF_t - \underset{(0.757)}{2.134} STMV_t$	SELS	$R = 0.983$
Kmenta [1966]	constant prices (£m.)	1947-48 to 1960-61	$CEMV_t = 219.0122 + \underset{(0.0486)}{0.9129} XNF_t - \underset{(0.6275)}{0.4027} \Delta_t$	2SLS	$R^2 = 0.968$ d = 1.737
			$CMV_t = -166.5387 + \underset{(0.0164)}{0.11866} XNF_t - \underset{(0.9535)}{2.2804} STMV_t$ $+ \underset{(0.1053)}{0.1362} \Delta_{t-1}$	2SLS	$R^2 = 0.9105$ d = 2.993
Duloy [1967]	constant prices (£m.)	1947-48 to 1960-61	$\dfrac{CEMV_t}{N_t} = 6.56 - 353.95 \dfrac{\sum\limits_{\tau=0}^{6} \Delta_{t-\tau}}{N_t} + 1.01 \dfrac{XNF_t}{N_t}$	NLILS[*]	
			$\dfrac{CMV_t}{N_t} = -44.0 + 87.43 \dfrac{\sum\limits_{\tau=0}^{2} \Delta_{t-\tau}}{N_t} + 0.19 \dfrac{XNF_t}{N_t}$ $- 0.16 STMV_t$	NLILS[*]	

Notes:

a: For sources see footnote 2 p. 134.

*: NLILS = non-linear iterative least squares.

excluding motor vehicles, whereas Duloy had assumed the opposite

behaviour. Duloy did,however, provide no evidence to support his

contention, and the econometrics seem to show otherwise.

To obtain a vehicle consumption function, Duloy supposed

that per-capita vehicle consumption for both migrants and locals was

a function of their respective per-capita non-farm disposable incomes

and the sales tax on vehicles. The parameters of both migrant and

local consumption functions were assumed identical except for the

intercept parameter. Making the same manipulations as before gave:

$$\frac{CMV_t}{N_t} = \alpha_\ell + k \frac{\sum\limits_{\tau=0}^{T-1} (1 - \gamma^{T-\tau})\Delta_{t-\tau}}{N_t} + \beta \frac{XNF_t}{N_t} + \delta STMV_t$$

The preferred estimate was obtained by setting $\gamma = 0$, the estimates

of the remaining parameters being

$$\alpha_\ell = -44.00, \quad k = 87.43, \quad T = 3, \quad \beta = 0.19 \quad \text{and} \quad \delta = -0.16$$

with $R^2 = 0.94$ and $d = 2.68$. Thus, the motor vehicle consumption

function has the form

$$\frac{CMV_t}{N_t} = -44.00 + 87.43 \frac{\sum\limits_{\tau=0}^{2} \Delta_{t-\tau}}{N_t} + 0.19 \frac{XNF_t}{N_t} - 0.16 STMV_t \quad (6.28)$$

A point to note about (6.28) is that the estimate of k is positive,

indicating that, initially, the migrant function lies above the local

function and that, therefore, migrants spend a higher proportion of

their incomes on motor vehicles than do locals.

A further interesting feature of the results is that the rather complicated lag term, $\frac{1}{N_t} \left[\sum_{\tau=0}^{T-1} (1 - \gamma^{T-\tau}) \Delta_{t-\tau} \right]$, reduces to a very simple unweighted lag, $\frac{1}{N_t} \Sigma \Delta_{t-\tau}$, in both equations since $\gamma = 0$ in both preferred estimates. Finally, the estimates of the standard errors (using SELS formulae) indicate that the coefficient of the migration factor ($\frac{\Sigma \Delta_{t-\tau}}{N}$) was statistically significant in both (6.27) and (6.28). It appears, then, that Duloy's econometric work has been able to detect differences in the pattern of migrant/local spending.

While the results of this study are quite interesting, its principal aim (like that of Kmenta), was to examine the influence of immigration. As a consequence, the consumption functions were formulated to test for any possible "migration effect". Many other potentially useful variables have therefore been disregarded by these authors. Nevile and Kmenta were also restricted by the fact that they were building a model of interdependent relationships, and to keep the model simple each relationship had to be reasonably simple. Hence we cannot expect lavish experimentation with the form of the consumption function by researchers attempting to build models of the whole economy within the framework of a small number of equations.

The next three studies to be examined divided total consumption expenditure into two parts: expenditure on durables (CD) and expenditure on non-durables (CND). Total consumption was divided in this way because, in Nevile's words, "in the case of consumer durables,

the purchase is as much akin to savings as to consumption [and] economic theory, therefore, would lead us to expect the two types of consumption to behave differently".[1] The estimates by _Podder_[2] were part of a simple seven equation model of the Australian economy. Both relationships are based upon a simple form of the Permanent Income Hypothesis:

$$C = cY^p$$

where Y^p = permanent income, and is approximated by a simple geometric lag of past incomes:

$$Y^p = (1 - b)[Y_t + bY_{t-1} + b^2Y_{t-2} + \ldots]$$

Clearly, a Koyck transformation will result in:

$$C = c(1 - b)Y + bC_{-1}$$

To keep his model simple, Podder used Gross National Product (Y) as his income variable. Using deflated annual data for the years 1952/53-1967/68, Podder obtained:

$$CND_t = 578.9433 + 0.3434 \, Y_t + 0.3159 \, CND_{t-1} \quad (6.29)$$
$$ (0.0483) (0.1863)$$
$$R^2 = 0.994$$

$$CD_t = 37.2685 + 0.0613 \, Y_t + 0.2332 \, CD_{t-1} \quad (6.30)$$
$$ (0.0218) (0.2036)$$
$$R^2 = 0.910$$

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1. Nevile, _Fiscal Policy in Australia_, p. 81.

2. Podder, "Forecasting with an econometric model".

The estimation method used was 2SLS. While the equations tested were exceedingly simple, they are interesting as an example of one of the first attempts to introduce the Permanent Income Hypothesis to Australian data.[1] One point to note before going on is the relatively low level of significance of the estimated coefficient of the lagged variable in both (6.29) and (6.30). As far as durable consumption is concerned, the poor performance of the lagged variable is not very surprising since a high expenditure on durables in the previous period will raise the stock of durables held at the beginning of the current period and thereby tend to depress current period expenditure. Similarly for a low expenditure on durables in the previous period. Hence there is not necessarily a strong positive correlation between CD_t and CD_{t-1} and any positive coefficient of CD_{t-1} (such as we have in (6.30)) is unlikely to be significant.

Zerby[2] began like Podder, by assuming that current consumption was determined by current income and lagged consumption. In an attempt to disaggregate, he replaced the single income term by personal non-farm income (YNF), personal farm income (YF), and personal taxes (TT), hoping by this to achieve the same effect as using non-farm disposable income and farm disposable income, observations on which were said to be unavailable.[3] In addition, a number of monetary

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1. The forms tested are, of course, also compatible with Brown's habit persistence hypothesis. See p. 35 above.

2. Zerby, "An econometric model of monetary interaction".

3. Ibid., pp. 158-159.

variables were tried in each equation. Three methods of estimation were used — SELS, 2SLS, and three-stage least squares (3SLS).[1] The SELS estimates, using deflated annual data for the years 1948/49-1964/65, were:[2]

$$CD_t = \begin{array}{l} -0.2274 + 0.1957 \ YNF_t + 0.1822 \ YF_t - 0.4741 \ TT_t \\ (0.1920) \quad (0.0351) \qquad\quad (0.0824) \qquad\quad (0.1914) \end{array}$$

$$\begin{array}{l} - 0.0456 \ r_t - 0.0180 \ CD_{t-1} \\ (0.0366) \qquad (0.3005) \end{array} \qquad (6.31)$$

$$\bar{R}^2 = 0.9422$$
$$SE^2 = 0.0033$$
$$VN = 2.1692$$

$$CND_t = \begin{array}{l} 0.3597 + 0.8241 \ YNF_t + 0.0902 \ YF_t - 0.7223 \ TT_t \\ (0.2419) \quad (0.1025) \qquad\quad (0.1474) \qquad\quad (0.2784) \end{array}$$

$$\begin{array}{l} - 0.0703 \ CND_{t-1} \\ (0.1059) \end{array} \qquad (6.32)$$

$$\bar{R}^2 = 0.9963$$
$$SE^2 = 0.0088$$
$$VN = 1.6187$$

Of the monetary variables tried, the short term rate of interest performed best with the CD equation, while none performed satisfactorily with the CND equation. In the case of (6.31), the tax variable and both income variables are significant, and for (6.32) the tax variable

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1. 3SLS is a method of estimation whereby consistent estimates of the parameters of the structural equations of an interdependent model may be obtained. In this respect it resembles 2SLS. It differs from 2SLS in that all estimates are obtained simultaneously, whereas in 2SLS they are obtained equation by equation. For an outline of the method see Goldberger, Econometric Theory, pp. 346-352.

2. VN is the Von Neumann ratio for testing for the presence of autocorrelated disturbances.

and the non-farm income variable are significant. The lagged dependent variable performed poorly in both equations. The 2SLS and 3SLS estimates of (6.31) agree quite well with the SELS estimate in terms of signs, magnitudes and t-ratios, with the one exception that the coefficient of CD_{t-1} has a different sign and is approximately 8-9 times larger in magnitude (with a much higher t-ratio) for both 2SLS and 3SLS estimates. In the case of (6.32), there is little agreement between the results of the three methods. For comparison, the 3SLS estimate is:

$$CND_t = \begin{array}{c} -0.3836 \\ (0.2184) \end{array} + \begin{array}{c} 1.0710\ YNF_t \\ (0.0699) \end{array} + \begin{array}{c} 0.6612\ YF_t \\ (0.1147) \end{array}$$

$$\begin{array}{c} -\ 1.6050\ TT_t \\ (0.2139) \end{array} - \begin{array}{c} 0.0994\ CND_{t-1} \\ (0.0651) \end{array} \qquad (6.33)$$

The greater-than-unity coefficient of YNF makes this particular estimate unacceptable.

The last study to be examined is that by J. W. Nevile.[1] In searching for an equation to explain non-durable consumption expenditure, Nevile begins by assuming that all cash benefits (CB) to persons (i.e. pensions, unemployment relief, student grants, etc.) will be completely spent on non-durables. Hence, a relationship to explain $(CND_t - CB_t)$ is needed. Nevile adopts the relation:

$$(CND_t - CB_t) = a + b(XNF_t - CB_t) + c(CND_{t-1} - CB_{t-1}) \qquad (6.34)$$

suggesting the Permanent Income Hypothesis or the habit persistence

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1. Nevile, Fiscal Policy in Australia, pp. 81-84, 109.

hypothesis as rationale. To see the connection between the PIH and (6.34), denote $(CND_t - CB_t)$ by C_t and $(XNF_t - CB_t)$ by X_t. Then, a simple form of the PIH is[1]

$$C_t = \alpha X_t^p$$

where X^p is permanent income. Suppose X^p is explained by

$$X_t^p = (1 - \lambda)[X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots]$$

Therefore,

$$C_t = \alpha(1 - \lambda)[X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots]$$

giving

$$C_t = \alpha(1 - \lambda)X_t + \lambda C_{t-1}$$

i.e. $$CND_t - CB_t = \alpha(1 - \lambda)(XNF_t - CB_t) + \lambda(CND_{t-1} - CB_{t-1})$$

Using SELS, and deflated annual data for the years 1954/55–1966/67, the following estimate is obtained:

$$
\begin{aligned}
CND_t = \ &CB_t + 616 + \underset{(0.0589)}{0.3980} \ (XNF_t - CB_t) \\
&+ \underset{(0.0975)}{0.4250} \ (CND_{t-1} - CB_{t-1}) \quad\quad\quad (6.35) \\
&\hspace{10cm} R^2 = 0.999
\end{aligned}
$$

Unlike the previous two studies, the lagged term $(CND_{t-1} - CB_{t-1})$ now has a highly significant coefficient. Nevile begins the search

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1. Nevile actually assumed that C_t was a non-homogeneous function of X_t^p (i.e. $C_t = \alpha + \beta X_t^p$) with the result that (6.34) includes an intercept term.

TABLE 6.2B

MULTIPLE EQUATION STUDIES OF THE AUSTRALIAN CONSUMPTION FUNCTION USING ANNUAL DATA: II

Author[a]	Data	Sample Period	Estimate	Estimation Method	R^2 etc.
Podder [1969]	constant prices ($m.)	1952–53 to 1967–68	$CND_t = 578.9433 + \underset{(0.0483)}{0.3434} Y_t + \underset{(0.1863)}{0.3159} CND_{t-1}$	2SLS	$R^2 = 0.994$
			$CD_t = 37.2685 + \underset{(0.0218)}{0.0613} Y_t + \underset{(0.2036)}{0.2332} CD_{t-1}$	2SLS	$R^2 = 0.910$
Zerby [1969]	constant prices ($m.)	1948–49 to 1964–65	$CND_t = \underset{(0.2419)}{0.3597} + \underset{(0.1025)}{0.8241} YNF_t + \underset{(0.1474)}{0.0902} YF_t$ $\qquad - \underset{(0.2784)}{0.7223} TT_t + \underset{(0.1059)}{0.0703} CND_{t-1}$	SELS	$\bar{R}^2 = 0.9963$ $SE^2 = 0.0088$ $VN = 1.6187$
			$CD_t = \underset{(0.1920)}{-0.2274} + \underset{(0.0351)}{0.1957} YNF_t + \underset{(0.0824)}{0.1822} YF_t$ $\qquad - \underset{(0.1914)}{0.4741} TT_t - \underset{(0.0366)}{0.0456} r_t - \underset{(0.3005)}{0.0180} CD_{t-1}$	SELS	$\bar{R}^2 = 0.9422$ $SE^2 = 0.0033$ $VN = 2.1692$
Nevile [1970]	constant prices ($m.)	1954–55 to 1966–67	$CND_t = CB_t + 616 + \underset{(0.0589)}{0.3980} (XNF_t - CB_t)$ $\qquad + \underset{(0.0975)}{0.4250} (CND_{t-1} - CB_{t-1})$	SELS	$R^2 = 0.999$
			$CD_t = 159 + \underset{(0.0569)}{0.3766} (XNF_t - CB_t)$ $\qquad - \underset{(0.0643)}{0.3439} (XNF_{t-1} - CB_{t-1}) + \underset{(0.1666)}{0.5801} CD_{t-1}$	SELS	$R^2 = 0.935$

Notes:

a: For sources see footnote 2 p. 185.

for a relationship to explain consumption of durables (CD) by supposing that CD is determined by disposable income and the stock of durable goods at the end of the previous year (KD_{t-1}). The measure of disposable income used is, as for non-durables, non-farm disposable income _minus_ cash benefits. Hence:

$$CD_t = a + c(XNF_t - CB_t) - jKD_{t-1} \qquad (6.36)$$

To eliminate KD, for which no data existed, Nevile assumed that depreciation was a constant proportion d of the initial stock of durables. Hence the closing stock will be equal to the depreciated opening stock plus current expenditure,[1] i.e.

$$KD_t = (1 - d)KD_{t-1} + CD_t$$

$$\therefore \qquad jKD_t = (1 - d)jKD_{t-1} + jCD_t \qquad (6.37)$$

From (6.36) we get

$$jKD_{t-1} = a + c(XNF_t - CB_t) - CD_t \qquad (6.38)$$

Substituting (6.38) into (6.37) gives

$$jKD_t = (1 - d)[a + c(XNF_t - CB_t) - CD_t] + jCD_t \qquad (6.39)$$

Lagging (6.39) one period and substituting into (6.36), we obtain after rearrangement:

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1. This implies that there is no depreciation on durables in the period of their purchase.

$$CD_t = \alpha_0 + \alpha_1(XNF_t - CB_t) + \alpha_2(XNF_{t-1} - CB_{t-1})$$

$$+ \alpha_3 CD_{t-1} \qquad\qquad (6.40)$$

where $\alpha_0 = ad$, $\alpha_1 = c$, $\alpha_2 = -c(1 - d)$, $\alpha_3 = (1 - d - j)$. SELS
estimation yields:

$$CD_t = 159 + \underset{(0.0569)}{0.3766} (XNF_t - CB_t) - \underset{(0.0643)}{0.3439} (XNF_{t-1} - CB_{t-1})$$

$$+ \underset{(0.1666)}{0.5801} CD_{t-1} \qquad\qquad (6.41)$$

$$R^2 = 0.935$$

From the estimates of α_i it is possible to deduce that $a = 1832$,
$c = 0.377$, $d = 0.087$, $j = 0.333$. The value of d implies that the
average life of durable goods is approximately 11 years, which is not
unreasonable.

We now go on to consider those multiple equation studies
which used quarterly data.

6.4 <u>Multiple Equation Studies using Quarterly Data</u>[1] There are two
well known multiple equation studies of the Australian consumption
function, one by the Reserve Bank of Australia and the other by the
Commonwealth Treasury and the Bureau of Census and Statistics. Both
studies are parts of continuing projects by the bodies mentioned and

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1. In this section the figures in parenthesis below parameter estimates
   are t-ratios, i.e. the ratio of the standard error to the parameter
   estimate. A t-ratio of approximately 2 indicates a significant
   coefficient at the 5% level of significance.

the results given below were the latest available at the time of writing.[1]

Both studies are similar in that they disaggregate consumption into three components - consumption of motor vehicles (CMV), consumption of other durables (COD), and consumption of non-durables (CND). A type of stock adjustment model is employed by both to explain consumption of durables. Basically, the stock adjustment model consists of the following relationships[2]

$$q_t = d_t + \Delta k_t \qquad (6.42)$$

$$d_t = \rho k_{t-1} \qquad (6.43)$$

$$\Delta k_t = k_t - k_{t-1} = \theta(k_t^d - k_{t-1}) \quad 0 < \theta < 1 \qquad (6.44)$$

$$k_t^d = \alpha + \beta y_t + \gamma p_t \qquad (6.45)$$

Relationship (6.42) states that expenditure, q_t, is made up of two parts: d_t = replacement expenditure, and Δk_t = net investment in new stocks. Replacement expenditure is supposed to be proportional to the

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1. The estimates for the Reserve Bank are from W. E. Norton and J. F. Henderson, "A model of the Australian economy: a further progress report" (paper presented at the Australasian Conference of Econometricians, Monash University, 9th - 13th August, 1971), while those for the Treasury are from C. I. Higgins, "A wage-price sector for a quarterly Australian model" (paper presented at the Australasian Conference of Econometricians, Monash University, 9th - 13th August, 1971).

2. This outline follows C. Christ, "Econometrics in economics: some achievements and challenges", Australian Economic Papers, Vol. 6 (1967), pp. 164-165.

opening stock (relationship (6.43)). Meanwhile,it is assumed that the consumer has in mind some desired closing stock, k_t^d, which is determined by such factors as current income (y_t) and relative prices (p_t) (relationship (6.45)). Finally,it is supposed that current net investment will raise the total stock of durables by some _fraction_ θ of the difference between the opening stock and the desired closing stock (relationship (6.44)). Substituting (6.43), (6.44) and (6.45) into (6.42) yields

$$q_t = \alpha\theta + \beta\theta y_t + \gamma\theta p_t + (\rho - \theta)k_{t-1} \qquad (6.46)$$

The Reserve Bank study modifies this model in several ways. First, the stock of liquid assets is introduced into (6.44) and (6.45). That is, liquid wealth is taken to be a determinant of the desired closing stock of durables, and the level of liquid wealth acts as a constraint determining the level of net investment. Secondly, some allowance is made for depreciation of current purchases. Hence (6.43) is replaced with

$$d_t = \rho k_{t-1} + \epsilon q_t \qquad 0 < \epsilon < \rho < 1$$

The inequality $0 < \epsilon < \rho < 1$ is based upon the assumptions that the rate of depreciation is constant throughout the life of the durable and that the purchases of durables take place at a steady rate throughout each period.[1] With these changes the final equation for

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1. See R. Stone and D. A. Rowe, "The market demand for durable goods", _Econometrica_, Vol. 25 (1957), p. 424.

testing is:

$$q_t = a_0 + a_1 y_t + a_2 p_t + a_3 \ell_{t-1} + a_4 k_{t-1} \qquad (6.47)$$

where ℓ_{t-1} is the stock of liquid wealth at the end of period (t-1).

Using <u>constant price</u> data for the period 1959 (1) to 1969 (4), the following estimates were obtained by SELS:[1]

$$CMV_t = 197 + \underset{(2.03)}{11.00} S_3 - \underset{(1.48)}{0.0150} KMV_{t-1} + \underset{(3.59)}{0.0305} X_t$$
$$+ \underset{(2.40)}{0.0119} M_{t-1} - \underset{(2.07)}{207} RP_t \qquad (6.47a)$$

$$SE = 15.48$$
$$R^2 = 0.847$$
$$d = 0.89$$

$$COD_t = 84 - \underset{(2.72)}{0.0033} S_1 \cdot KOD_{t-1} - \underset{(1.58)}{0.0017} S_2 \cdot KOD_{t-1}$$
$$- \underset{(1.79)}{0.0250} KOD_{t-1} + \underset{(7.40)}{0.0684} X_t$$
$$+ \underset{(2.07)}{0.0148} M_{t-1} - \underset{(0.79)}{114} RP_t \qquad (6.48)$$

$$SE = 11.39$$
$$R^2 = 0.950$$
$$d = 1.05$$

where KMV_{t-1}, KOD_{t-1} are stocks of motor vehicles, other durables at the end of period (t-1); M_{t-1} is the stock of money at the end of period (t-1); RP_t is a measure of relative prices;[2] S_1, S_2, S_3 are

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1. Note that the subscript for M in (6.48), and (6.51) below, is shown as t in Norton and Henderson, "A model of the Australian economy".

2. For example, in the case of (6.48) RP_t is equal to the ratio of the implicit deflator for COD to the consumer price index.

seasonal dummy variables. Seasonally unadjusted data were used,
the seasonal adjustment being made via some pattern of additive or
multiplicative seasonal dummy variables. In (6.47a), one additive
seasonal dummy, S_3, has been included, while in (6.48) two
multiplicative seasonal dummies, S_1 and S_2, have been used. Norton
and Henderson state that the estimated relationships shown were chosen
from amongst a number of estimates, and that the criteria used to
select the final estimate were

> "the sign and significance of the coefficient of the
> explanatory variables suggested by economic theory;
> where relevant, the shape and mean of the lag
> distribution on the explanatory variables; the
> results of ex post forecasts; and the values of R^2,
> the standard error of estimate and the Durbin-Watson
> and Durbin statistics".[1]

The Treasury study modifies the simple stock adjustment
model by assuming that the desired closing stock depends upon current
non-farm disposable income, the maximum rate of interest on fixed
deposits (RMFD), the rate of sales tax on durables [either STMV
(sales tax on motor vehicles) or STOD (sales tax on other durables],
and the volume of money (M_{-1}).[2] Using seasonally adjusted <u>current</u>

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1. Norton and Henderson, "A model of the Australian economy", p. 3.
   For earlier estimates of the consumption functions, see W. E. Norton
   and J. R. Broadbent, <u>Equations for Personal Consumption
   Expenditure</u> (Reserve Bank of Australia, Occasional Paper No. 3B,
   1970). For the Durbin statistic see J. Durbin, "Testing for serial
   correlation in least squares regression when some of the regressors
   are lagged dependent variables", <u>Econometrica</u>, Vol. 38 (1970),
   pp. 410-421.

2. See C. I. Higgins, "A model for National Income forecasting"
   (paper presented to the Economic Soc. of Aust. and N.Z., Canberra,
   1970), appendix, p. 6.

<u>price</u> data, the following estimates were obtained:

$$CMV_t = -100.1889 + 0.0923 \ XNF_t + 0.0754 \ M_{t-1}$$
$$ (1.193) \quad (2.088) \qquad (3.282)$$

$$- 0.2033 \ KMV_{t-1} - 15.7888 \ RMFD_t$$
$$(4.014) \qquad\qquad (1.591)$$

$$- 43.1723 \ STMV_t + 0.934 \ \hat{u}_{t-1} \qquad\qquad (6.49)$$
$$(0.456)$$

$$SE = \$8.83m.$$
$$\bar{R}^2 = 0.955$$
$$d = 2.41$$

$$COD_t = 128.9906 + 0.0447 \ XNF_t + 0.0395 \ M_{t-1}$$
$$ (6.528) \quad\ (2.903) \qquad (5.377)$$

$$- 0.0773 \ KOD_{t-1} - 12.2456 \ RMFD_t$$
$$(4.760) \qquad\qquad (3.439)$$

$$- 23.9254 \ STOD_t + 0.831 \ \hat{u}_{t-1} \qquad\qquad (6.50)$$
$$(0.379)$$

$$SE = \$3.09m.$$
$$\bar{R}^2 = 0.995$$
$$d = 1.60$$

One noticeable feature of these estimates is the relatively poor performance of the sales tax variables compared to the models using annual data.[1]

Both studies approach the question of obtaining a relationship to explain consumption of non-durables in a formally similar way, although the final equations obtained are quite different. The Reserve Bank study assumes that there is an equilibrium level of

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1. See for example (6.17) and (6.19) above.

expenditure (CND^e) related to disposable income and relative prices, and that quarterly changes in expenditure are some positive fraction of the discrepancy between the current period's equilibrium expenditure and the last period's actual expenditure. Quarterly changes in expenditure are also affected by holdings of liquid wealth.[1]

$$CND^e_t = m + nX_t - vRP_t$$

$$CND_t = CND_{t-1} + w(CND^e_t - CND_{t-1}) + xM_{t-1} \quad 0 < w < 1$$

Substituting the first of these expressions and rearranging yields:

$$CND_t = wm + wnX_t - wvRP_t + xM_{t-1} + (1 - w)CND_{t-1}$$

Using seasonally unadjusted <u>constant price</u> data for the period 1959 (1) - 1969 (4), the following SELS estimate was obtained:

$$
\begin{aligned}
CND_t = \ & 1691 - \underset{(14.15)}{0.1040} S_1 \cdot CND_{t-1} - \underset{(10.15)}{0.0447} S_3 \cdot CND_{t-1} \\
& + \underset{(8.81)}{0.7760} CND_{t-1} + \underset{(9.47)}{0.1576} X_t \\
& + \underset{(1.59)}{0.0294} M_{t-1} - \underset{(3.04)}{1830} RP_t \quad\quad (6.51)
\end{aligned}
$$

$$SE = 22.05$$
$$R^2 = 0.996$$
$$d = 1.90$$

Once again multiplicative seasonal dummies appear to have performed best.[2]

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1. See Norton and Broadbent, <u>Equations for personal consumption expenditure</u>, p. 6.

2. Norton and Broadbent report that the use of several measures of permanent income resulted in a poorer fit. See <u>ibid.</u>, pp. 10-11.

The Treasury study uses a consumption function for non-durables similar to that devised by Duesenberry and others.[1] One interpretation of this function is that consumers attempt a gradual adjustment of their consumption-income ratio to its desired level, which in turn depends upon the ratio of income to its previous peak value.[2] That is

$$R_t^* = \alpha + \beta Z_t$$

where R_t^* is the desired value of $R_t = \dfrac{CND_t}{XNF_t}$, and $Z_t = \dfrac{XNF_t}{XNF_t^0}$, where

XNF_t^0 is the peak value of XNF in the previous eight quarters. The adjustment process is described by

$$R_t - R_{t-1} = \gamma(R_t^* - R_{t-1})$$

Substituting the first relationship into the second and rearranging gives:

$$R_t = \alpha\gamma + \beta\gamma Z_t + (1 - \gamma)R_{t-1}$$

i.e. $\qquad \dfrac{CND_t}{XNF_t} = a + b \dfrac{XNF_t}{XNF_t^0} + c \dfrac{CND_{t-1}}{XNF_{t-1}}$ $\qquad\qquad$ (6.52)

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1. See p. 40 above.

2. This interpretation is just a simple modification of that suggested by Griliches et. al. for the original Duesenberry, Eckstein, Fromm function (see pp. 40-42 above).

Using seasonally adjusted <u>current price</u> data, the following estimate

was obtained:

$$\frac{CND_t}{XNF_t} = \underset{(8.338)}{0.5769} - \underset{(8.460)}{0.5364} \frac{XNF_t}{XNF_t^0} + \underset{(43.146)}{0.9560} \frac{CND_{t-1}}{XNF_{t-1}} \tag{6.53}$$

$$SE = \$11.28m.$$
$$\bar{R}^2 = 0.979$$
$$d = 2.28$$

Parameter estimates here conform with a priori expectations as to sign

and magnitude.[1] All parameter estimates are significant and the

Durbin-Watson test gives no indication of serial correlation of

disturbances.[2]

6.5 <u>Conclusions</u>   As the prime concern of this thesis is with the

aggregate consumption function, the conclusions will be presented first

with respect to the single equation studies and then with respect to

the studies of the disaggregated consumption function that were

discussed in sections 6.3 and 6.4.

The first conclusion that can be drawn from our examination

of the single equation studies is that very little work on the

Australian aggregate consumption function has been done. There has

been no systematic study, using Australian data, of the many hypotheses

discussed in previous chapters and elsewhere. In fact only two

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1. The restrictions discussed on p. 42 above apply equally here.

2. Note however that the test is not strictly applicable due to the
    presence of a lagged value of the dependent variable amongst the
    regressors.

hypotheses have been examined at all, namely (a) the Keynesian function (discussed by Lawler) and the Arndt-Cameron and Auld modifications of it, and (b) the wealth hypothesis (discussed by Lydall).

The second point to make is that what work has been done, has not been of a very rigorous nature: (1) The sample data used has generally consisted of a very small number of observations. In the case of the studies using annual data, the average number of observations is approximately ten. For the studies using quarterly data, the position is much improved, but even here the number of observations is only 24-26. (2) Most of the work done has used annual data. (3) Only one investigator (Lydall) checked for possible autocorrelation of residuals. (4) Only three of the estimates (those by Lydall, Cameron [1967], and Auld) included standard errors to enable significance tests to be performed. (5) With one exception,[1] no check was made on the forecast ability of the estimated relationships using data from outside the sample period, nor was any comparative study with naive models done. (6) Many of the estimates were carried out using current price data, although some investigators acknowledged this deficiency.[2] (7) Finally, SELS was used in every case, with the result that parameter estimates will not be consistent. In general,

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1. Cameron [1967].

2. In addition, Lydall's wealth hypothesis is formulated in such a way that it is not clear that constant price data must be used.

functions have been estimated by simple least squares and then judged acceptable or not on the basis of the value of the squared correlation coefficient.

The multiple equation studies examined fall naturally into three groups: (1) The work of Nevile [1962], Kmenta [1966], and Duloy [1967]; (2) the work of Podder [1969], Zerby [1969], and Nevile [1970]; (3) the work done at the Reserve Bank and the Commonwealth Treasury and Bureau of Census and Statistics. The first group consists basically of further modifications of the Keynesian function. The Arndt-Cameron income variable has been used with the addition of a sales tax variable to improve the fit of the motor vehicles equation. Kmenta and Duloy have also investigated the effects of an immigration variable. So there is nothing radically new about the approach of the first group. The sample size is still very modest, consisting of 13-14 observations. Standard errors are, however, provided. In addition, Kmenta employs 2SLS to obtain consistent estimates. Constant price data are used throughout. On balance, the statistical work is slightly better than that used in the single equation studies.

The second group consists of work from the late 1960's, and represents quite a departure from earlier work in the way functions to be tested were formulated. Prior to this work, investigators, with the exception of Lydall, relied entirely upon simple modifications of the basic Keynesian consumption function. This group does, however, give an indication that overseas work (of quite long standing) had finally been

noticed. The main ideas behind the work here are drawn from the
Permanent Income Hypothesis or the inertia hypothesis. The measure
of permanent income used has been an average of current and past
values of disposable income, with geometrically declining weights.
There has been a further increase in the sample size with an average
number of observations of approximately 15.[1] Two of the three studies
obtained 2SLS estimates, and one of these also employed 3SLS methods
of estimation. Standard errors are given for all estimates, but only
Zerby checked for autocorrelation.

The third group consists of the work done at the Reserve
Bank and the Treasury to construct a quarterly model of the Australian
economy. Since quarterly data are used, the number of observations
employed is much higher than for previous studies, being almost fifty
for the Reserve Bank. Current price data are used by the Treasury;
however, their model is to be re-estimated using constant price data.[2]
Both models are apparently estimated by SELS, while 2SLS estimates by
the Reserve Bank are to be made.[3] All estimates are accompanied by
correlation coefficients, Durbin-Watson statistic, t-ratios, standard
errors. The Treasury study also includes an estimate of the

1. This is an improvement which comes naturally with time and does
   not, of course, reflect virtue upon the part of the author of the
   study.

2. See Higgins, "A wage-price sector for a quarterly Australian
   model", p. 41.

3. See Norton and Henderson, "A model of the Australian economy:
   a further progress report", p. 4.

Duesenberry-Eckstein-Fromm (DEF) consumption function. These two
studies seem to be more carefully formulated, and backed up with more
statistical measures than any other study.

In summary,then,there has not been a great deal of work done
on the Australian consumption function. Approximately four
hypotheses have been tested:

(i)   The basic Keynesian hypothesis, plus modifications

(ii)   A wealth hypothesis

(iii) A version of the P.I.H.

(iv)   The DEF function.

As shown in previous chapters, each of these hypotheses has been
tested many times and in many different forms using overseas data.
In addition,many other hypotheses, some of which have been discussed
before, have also been tested. It is the aim of our next chapter to
formulate a number of hypotheses about the Australian aggregate
consumption function, in the light of the preceding review chapters,
and test these with quarterly Australian data. It is hoped that this
work will go some way towards making up the leeway in the study of this
important macroeconomic relationship in the Australian context.

6.1 Notation used in Chapter Six

C	=	total personal consumption expenditure
CB	=	cash benefits to persons
CD	=	consumption expenditure on durables
CEMV	=	total consumption excluding motor vehicle expenditure
CMV	=	consumption expenditure on motor vehicles
CND	=	consumption expenditure on non-durables
CNF	=	consumption of non-farm products
COD	=	consumption of other durables
C^F	=	consumption expenditure by the farm sector
C^{NF}	=	consumption expenditure by the non-farm sector
Δ_t	=	net permanent immigrants in year t
ΔHP	=	net increase in instalment credit balances outstanding
KD_{-1}	=	stock of durable consumer goods at the end of previous period
KMV_{-1}	=	stock of motor vehicles at the end of the previous period
KOD_{-1}	=	stock of other durables at the end of the previous period
M_{-1}	=	stock of money at the end of the previous period
N	=	population
r	=	short term rate of interest
RMFD	=	maximum interest rate on fixed deposit
RP	=	measure of relative prices
S	=	total personal savings

S_i = seasonal dummy variable

STMV = sales tax on motor vehicles

STOD = sales tax on other durables

ST^i = estimated increase in Commonwealth sales tax revenue following a change in legislation

ST^r = estimated reduction in Commonwealth sales tax revenue following a change in legislation

T = non-farm personal income tax

TT = total personal taxes

W_{-1} = net personal wealth at the end of the previous period

X = personal disposable income (PDI)

XNF = non-farm PDI

ΔX^T = the change in X resulting from changes in personal income tax

ΔX^A = the change in X resulting from changes in factors other than personal income tax

ΔX^{Ti} = the change in X resulting from an increase in personal income tax

ΔX^{Tr} = the change in X resulting from a reduction in personal income tax

Y = G.N.P.

YF = personal farm income before tax

YNF = personal non-farm income before tax

CHAPTER SEVEN

THE AUSTRALIAN AGGREGATE CONSUMPTION FUNCTION: 1959-1968

7.1 <u>Introduction</u>   This chapter aims to cover the third of the
objectives laid down in chapter one, i.e. to make some contribution
towards eliminating deficiencies in the Australian work on the
aggregate consumption function that have been shown up by our review
of the general literature on the aggregate consumption function and
of the Australian estimates of the function. With this aim in mind,
we draw up in section 7.2 below a list of hypotheses about the
consumption function which have themselves been tested successfully
using overseas data, or are based upon ideas that have been
successfully tested with overseas data. These hypotheses in turn
have either not been tested with Australian data at all, or have
been inadequately tested. For the sake of "completeness" a new
estimate of the strict Keynesian function, $C = a + bX$, is provided.
It was also our intention to obtain new estimates of any uniquely
Australian functions that have proved successful. From the previous
chapter it should be clear that the only formulation that could fall
into this category is that by H. W. Arndt and B. Cameron. While
this "Australian" consumption function cannot be said to be wholly
successful either on a priori grounds or on the basis of the empirical
evidence, a modified version of it has been included in the list for
estimation.

In section 7.3 questions relating to methods of estimation and treatment of data are dealt with. The estimated relationships are presented and discussed in section 7.4. As will be clear from section 7.2, more than one estimate of some of the hypotheses is possible. The criteria used to select the reported estimate are included in section 7.4. After the results are discussed, an attempt is made to distinguish the better of the estimates. Conclusions are given in section 7.5.

7.2 Hypotheses to be tested   The first hypothesis to be tested, which will be denoted by H(1), is the basic Keynesian consumption function:

$$H(1): \quad C = a + bX \tag{7.1}$$

where C is aggregate real consumption and X is aggregate real personal disposable income. A priori, we expect $0 < b < 1$. The second hypothesis has been tested successfully with quarterly data by Griliches et. al.:[1]

$$H(2): \quad C = a + bX + cC_{-1} \tag{7.2}$$

This type of function is suggested by Brown's habit persistence hypothesis, or alternatively by supposing that consumption is a linear

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1. See p. 40   above.

function of permanent income of the type

$$C = k_1 + k_2 X^1 \qquad\qquad (7.3)$$

where

$$X^1 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i X_{-i} \qquad\qquad (7.4)$$

(7.2) is obtained by substituting (7.4) into (7.3) and applying a Koyck transformation. A priori expectations are that $0 < b < 1$ and $0 < c < 1$.

The third and fourth hypotheses performed well in the quarterly study by Zellner.[1] Hypothesis three is

$$H(3): \quad C = a + bX + cL_{-1} \qquad\qquad (7.5)$$

where L_{-1} is the stock of liquid assets at the end of the previous period. Two series for L_{-1} were tried:

L_{-1}^1 = notes and coins in the hands of the public <u>plus</u>
current deposits of the public with trading banks.

$L_{-1}^2 = L_{-1}^1$ <u>plus</u> fixed deposits of the public with trading
banks <u>plus</u> deposits with all savings banks.

A further series was also tried, namely:

W_{-1} = total private sector wealth.[2]

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1. Zellner, "The short-run consumption function". See also p. 38 and p. 39 above.

2. For actual data used and sources see Appendix pp. 277-284 below.

This variable (W_{-1}) has performed well in the Stone-Rowe model and also in various tests of the Life Cycle Hypothesis. A priori expectations are that $0 < b < 1$ and $0 < c < 1$. Hypothesis four includes lagged consumption as an explanatory variable:

$$H(4): \quad C = a + bX + cL_{-1} + dC_{-1} \tag{7.6}$$

A priori expectations are that $0 < b < 1$, $0 < c < 1$ and $0 < d < 1$. The study by Brown showed the following formulation to be a good performer for Canadian data:

$$C = a + bX + cX^0 + dA^c$$

where X^0 is the highest previous value of X and A^c is Brown's proxy variable for the stock of liquid assets.[1] We shall test this using the three series L_{-1}^1, L_{-1}^2 and W_{-1} in place of A_t^c. Thus we have for hypothesis five:

$$H(5): \quad C = a + bX + cX^0 + dL_{-1} \tag{7.7}$$

where L_{-1} is approximated by L_{-1}^1, L_{-1}^2 or W_{-1}. A priori expectations are that $0 < b < 1$ and $0 < d < 1$. The sixth hypothesis is:

$$H(6): \quad \frac{C}{X_{-1}} = a + b\,\frac{X_{-1}}{X_{-1}^0} + c\,\frac{C_{-1}}{X_{-2}}$$

This has been tested successfully by Duesenberry et. al.[2] A priori

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1. See p. 36 above.

2. See p. 43 above.

expectations are that $a > 0$, $b < 0$ and $0 < c < 1$.[1]

Hypothesis (7) is a reformulation of the Arndt-Cameron consumption function. First it is assumed that non-farm consumption, C^{NF}, is determined by non-farm disposable income, XNF:

$$C^{NF} = \alpha + \beta XNF \tag{7.8}$$

On the other hand farm consumption levels are influenced, by a lagged adjustment process, by non-farm consumption:

$$C^F = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i C_{-i}^{NF} \tag{7.9}$$

$$\therefore \quad C = C^F + C^{NF}$$

i.e.
$$C = (1 - \lambda)[C^{NF} + \lambda C_{-1}^{NF} + \ldots] + \alpha + \beta XNF \tag{7.10}$$

Applying a Koyck transformation to (7.10) yields:

$$C = (1 - \lambda)C^{NF} + \alpha(1 - \lambda) + \beta XNF - \beta \lambda XNF_{-1} + \lambda C_{-1} \tag{7.11}$$

Substituting (7.8) into (7.11) and rearranging gives

$$C = 2\alpha(1 - \lambda) + \beta(2 - \lambda)XNF - \beta \lambda XNF_{-1} + \lambda C_{-1} \tag{7.12}$$

Observations on the variable XNF are not published, and indeed cannot be obtained since it is impossible to find out what proportion of *indirect* taxation is paid by the farm (or non-farm) sector. Arndt

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1. See p. 42 above.

and Cameron obtained their series for XNF via a breakdown of personal income tax.[1] A similar breakdown for recent data is, however, not possible,[2] and therefore an alternative procedure was required. To obtain an estimate of XNF, suppose that the share of non-farm disposable income in total disposable income is equal to the pre-tax share of non-farm income in total personal income, i.e.

$$\frac{XNF}{X} = \frac{YNF}{Y}$$

where Y = personal income before taxes, YNF = non-farm personal income before taxes. Therefore we can write

$$XNF = (\frac{YNF}{Y}) X \qquad (7.13)$$

This assumption is unlikely to be true, but there is no way of knowing whether it will result in a more or less accurate measure of XNF than any other assumption. Substituting into (7.12) gives

$$H(7): \quad C = a + b (\frac{YNF}{Y})X + c(\frac{YNF}{Y})_{-1} X_{-1} + dC_{-1} \qquad (7.14)$$

Since it is reasonable to suppose that $0 < \lambda < 1$ and $0 < \beta < 1$, expectations are that $0 < b < 2$, $-1 < c < 0$, $0 < d < 1$.

Hypothesis (8) is based upon various ideas from the Permanent Income Hypothesis and other associated work.[3] First we assume that

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1. Arndt and Cameron, "An Australian consumption function", footnote 1, pp. 108-109.

2. See Zerby, "An econometric model of monetary interaction in Australia", footnote 14, p. 159.

3. See chapter 3 above.

total consumption can be split into two components – permanent

consumption and transitory consumption:

$$C = c^1 + c^2 \qquad (7.15)$$

The simple PIH is adopted to explain c^1, i.e.

$$c^1 = \alpha_1 X^1 \qquad (7.16)$$

The Zellner, Huang and Chau explanation of c^2 by a liquid asset

imbalance[1] is adopted:

$$c^2 = \alpha_2(L_{-1} - L^d) \qquad (7.17)$$

where L^d is desired holdings of liquid assets at the end of the current

period. Substituting (7.16) and (7.17) into (7.15) yields:

$$C = \alpha_1 X^1 + \alpha_2(L_{-1} - L^d) \qquad (7.18)$$

To explain X^1 and L^d we use

$$X^1 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i X_{-i} \qquad (7.19)$$

$$L^d = \eta X^1 \qquad (7.20)$$

(7.19) is the most commonly used device for estimating permanent

income, while (7.20) was the relationship chosen by Zellner, Huang

and Chau.[2] Substituting (7.19) and (7.20) into (7.18) and applying

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1. See p. 75 above.

2. See p. 82 and p. 85 above.

a Koyck transformation yields:

$$C = aX + bL_{-1} + cC_{-1} \tag{7.21}$$

where $a = (\alpha_1 - \alpha_2 \eta)(1 - \lambda)$

$b = \alpha_2$

$c = \lambda$

This was further varied by the addition of the variable A_t: an index of consumer attitudes, and $(G_t - \bar{G}_t)$: a government policy variable along the lines suggested by Stone and Rowe.[1] $\bar{G}_t$ is the normal level of G_t. If G_t is above $\bar{G}_t$ it is assumed that the government is attempting to depress consumer spending. Thus the coefficient of $(G_t - \bar{G}_t)$ can be expected to be negative. The attitudes variable is introduced in the belief that consumers' attitudes towards such factors as expected price and income movements can influence aggregate spending.[2] With the addition of these variables (7.21) becomes:

$$H(8): \quad C = aX + bL_{-1} + cC_{-1} + dA + e(G - \bar{G}) \tag{7.22}$$

Four different forms of this were obtained: (i) by putting $d = e = 0$; (ii) by putting $d = 0$; (iii) by putting $e = 0$; (iv) by leaving the values of d and e unspecified. Three series for L_{-1} were tried:

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1. See p. 92 above. A description of the construction of series for A_t, G_t and $\bar{G}_t$ is given in section 7.3 below.

2. Much of the work on the influence of consumer attitudes has been done by G. Katona and E. Mueller. A summary of some of this work is given in G. Ackley, Macroeconomic Theory (New York: Macmillan, 1961), pp. 288-295 and D. B. Suits, "The Determinants of Consumer Expenditure: A Review of Present Knowledge", pp. 18-19.

L^1_{-1}, L^2_{-1} and W_{-1}. Thus, in total, 12 regressions were performed. All estimates by Zellner, Huang and Chau showed $a = (\alpha_1 - \alpha_2 n)(1 - \lambda)$ and $b = \alpha_2$ to be positive fractions.[1] In addition, $c = \lambda$ should be a positive fraction, and, as said above, e should be negative. Hence, a priori expectations about signs and magnitudes are

$$0 < a < 1, \quad 0 < b < 1, \quad 0 < c < 1, \quad e < 0.$$

Hypothesis 9 was reached by replacing the geometric lag in (7.19) with

$$X^1 = \sum_{i=0}^{n-1} w_i X_{-i} \qquad (7.19')$$

Substituting (7.19') and (7.20) into (7.18) yields:

$$C = (\alpha_1 - \alpha_2 n) \sum_{i=0}^{n-1} w_i X_{-i} + \alpha_2 L_{-1}$$

After adding A and $(G - \bar{G})$ we have for H(9)

$$H(9): \quad C = \sum_{i=0}^{n-1} a_i X_{-i} + bL_{-1} + cA + d(G - \bar{G}) \qquad (7.23)$$

where $a_i = (\alpha_1 - \alpha_2 n)w_i$ and $b = \alpha_2$. Since it is reasonable to suppose that the marginal propensity to consume out of both current and lagged income is some positive fraction, we specify a priori that $0 < a_i < 1$ for all i. In addition, we would expect $0 < b < 1$ and $d < 0$.

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1. See Zellner et. al., "Further analysis of the short-run consumption function", Tables I to IV.

The weights a_i in (7.23) were estimated by the Almon variable technique.[1] This technique involves transforming the observations on X into a corresponding set of observations on a number of "Almon" variables. (7.23) is then estimated with the Almon variables in place of the X_{-i}. From the estimated coefficients of the Almon variables the weights a_i can be calculated. Since the technique assumes that the weights lie on a polynomial of some specified degree, it is necessary, before estimation is possible,to specify both the degree of the polynomial (q + 1) and the length of the lag (n). In the estimates of H(9), polynomials of degree 2, 3 and 4 were used. Lags of length 2 to 10 were used with the second degree polynomial, 3 to 10 with the third degree polynomial, and 4 to 10 with the fourth degree polynomial. Two considerations were used in determining the lag length: (i) the requirement q < n had to be obeyed,[2] and (ii) it is unlikely that values of X more than 10 periods in the past will exert an influence on consumption spending.

Three different series for L_{-1} were used: L_{-1}^{1}, L_{-1}^{2} and W_{-1}. In addition four variations of (7.23) were obtained: (i) by putting c = d = 0, (ii) by putting c = 0, (iii) by putting d = 0, (iv) by leaving c and d unrestricted. Hence, twelve different forms of (7.23) were tried. As has been seen in the previous paragraph, 24 different combinations of n and q were used with each regression. Thus, in total, 288 regressions were performed.

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1. See appendix 7.1 below for a brief outline of this method.

2. See appendix 7.1, p. 267 below.

Hypothesis (10) consists of the following set of relationships

$$C = C^1 + C^2 \tag{7.15}$$

$$C^1 = \alpha_1 X^1 \tag{7.16}$$

$$C^2 = \alpha_2(W_{-1} - W^d) \tag{7.24}$$

$$X^1 = \sum_{i=0}^{n-1} w_i X_{-i} \tag{7.19'}$$

$$W^d = \sum_{i=0}^{m-1} v_i W_{-1-i} \tag{7.25}$$

That is, transitory consumption C^2 is now made proportional to the imbalance in total wealth stocks of the private sector. The unobserved variables, X^1 and W^d, are both explained by distributed lag relationships involving past values of W, in the case of W^d, and current and past values of X, in the case of X^1. W^d is thus approximated by a relationship similar to that used by Stone and Rowe to estimate permanent wealth[1] and the term $(W_{-1} - W^d)$ is therefore similar to their transitory wealth term.[2] Substituting (7.16), (7.24), (7.19') and (7.25) into (7.15) yields:

$$C = \alpha_1 \sum_{i=0}^{n-1} w_i X_{-i} + \alpha_2 W_{-1} - \alpha_2 \sum_{i=0}^{m-1} v_i W_{-1-i} \tag{7.26}$$

1. See p. 89 above.

2. See p. 94 above.

i.e. $\quad C = \sum_{i=0}^{n-1} a_i X_{-i} + bW_{-1} + \sum_{i=0}^{m-1} c_i W_{-1-i}$ (7.27)

where $a_i = \alpha_1 w_i$, $b = \alpha_2$ and $c_i = -\alpha_2 v_i$. After adding in A and $(G - \bar{G})$ we have for H(10):

H(10): $\quad C = \sum_{i=0}^{n-1} a_i X_{-i} + bW_{-1} + \sum_{i=0}^{m-1} c_i W_{-1-i} + dA + e(G - \bar{G})$

(7.28)

A priori expectations are that $0 < a_i < 1$, $0 < b + c_0 < 1$, $0 < c_i < 1$ $(0 < i \leq m - 1)$, and $e < 0$.

Four different forms of H(10) were obtained: (i) First by putting $d = e = 0$, (ii) second by putting $d = 0$, (iii) third by putting $e = 0$, and finally by leaving d and e unrestricted. The two sets of weights in (7.28) were estimated by the Almon variable technique. If each lag structure had been estimated in the same way as was the one structure in H(9)[1] there would have been a total of 24^2 different Almon specifications of the lag structure. Combined with the four different forms of H(10), this would have resulted in over two thousand estimates of H(10). To make the problem of choosing a best estimate feasible, it was decided to limit the longest lag to 4. Polynomials of degree 2, 3 and 4 were used, but it was specified that both lag structures were to be estimated using polynomials of the same order. These restrictions resulted in 14 different Almon specifications.[2]

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1. i.e. with polynomials of degree 2, 3, 4 each with lags of appropriat length up to a length of 10 - giving a total of 24 different specifications.

2. e.g. with polynomials of degree 3 there were four possible combinations of lags: (3, 3), (3, 4), (4, 3), (4, 4), where the first number of each pair refers to the length of the lag on X, and the second number to the length of the lag on W_{-1}.

Combined with the four different forms of H(10) this gave a total of 56 estimates.

The final hypothesis for testing is:

$$H(11): \quad C = aX + bC_{-1} \qquad\qquad (7.29)$$

This is derived from the simple PIH

$$C = \alpha X^1 \qquad\qquad 0 < \alpha < 1 \qquad (7.30)$$

where X^1 is estimated by a geometric lag

$$X^1 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i X_{-i} \qquad 0 < \lambda < 1 \qquad (7.31)$$

Substituting (7.31) into (7.30) and applying a Koyck transformation yields

$$C = \alpha X + \lambda C_{-1}$$

i.e. H(11), where $a = \alpha$ and $b = \lambda$. Hence, given the likely values of α and λ a priori expectations are that $0 < a < 1$ and $0 < b < 1$. It is to be noted that this is identical to H(2) except that the intercept term of H(2) is suppressed to give H(11).

It may be informative at this point to recall just how much work on the above eleven hypotheses has been done with Australian data. In the case of H(1): $C = a + bX$, two estimates have been reported above, one by Lawler using annual constant price data and one by Auld (in first difference form) using quarterly current price data.[1]

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1. See table 6.1, p. 183 above.

H(2) introduces lagged consumption into the consumption function. Podder has estimated this function with annual constant price data, but using gross national product in place of personal disposable income.[1] Zerby and Nevile have also estimated versions of H(2). In the estimate by Zerby YF, YNF and Total Taxes were used in place of disposable income. Nevile employed non-farm disposable income in place of disposable income. Both Zerby and Nevile used annual constant price data.[2]

A version of H(3): $C = a + bX + cL_{-1}$ has been tested by Lydall using annual current price data.[3] H(4): $C = a + bX + cL_{-1} + dC_{-1}$ has been estimated by the Reserve Bank of Australia using quarterly constant price data.[4] The Duesenberry hypothesis H(6): $\dfrac{C}{X_{-1}} = a + b\,\dfrac{X_{-1}}{X_{-1}^0} + c\,\dfrac{C_{-1}}{X_{-2}}$ has been tested in the Commonwealth Treasury study using quarterly current price data.[5] H(7): $C = a + b(\dfrac{YNF}{Y})X + c(\dfrac{YNF}{Y})_{-1}X_{-1} + dC_{-1}$ has not been tested at all, but the simple Arndt-Cameron function has been tested by Arndt and Cameron using annual constant price data,[6] Cameron using quarterly current price data,[7] Nevile using annual constant

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1. See table 6.2B, p. 200 above.

2. Ibid.

3. See table 6.1, p. 183 above.

4. See (6.51), p. 208 above.

5. See (6.53), p. 210 above.

6. See table 6.1, p. 183 above.

7. Ibid.

price data,[1] and a number of others. On the other hand, hypotheses
H(5), H(8), H(9), H(10) and H(11) have not been tested at all, even
indirectly.

Thus, the number of hypotheses tested directly is only
five (H(1), H(2), H(3), H(4) and H(6)), and the number of estimates
of these hypotheses is not more than one or two. In addition, as was
stated in chapter one, our prime interest at this stage is in
estimates employing quarterly data.[2] Further, it will be argued below
that the constant price form of the function is to be preferred over
that using undeflated data.[3] In all the above mentioned studies of
the Australian consumption function there is but one quarterly study
using deflated data, namely the Reserve Bank estimate of H(4). It is
safe to conclude then that there is a most substantial deficiency in
Australian research in this area, and we hope in the rest of this
chapter to make some contribution towards improving the situation.
We begin in section 7.3 with the consideration of certain questions
relating to treatment of data and methods of estimation.

7.3 Statistical questions relating to data and estimation   The
material in the section is dealt with under two sub-headings. Under
the sub-heading "data" the following points are discussed: (i) What
definition of consumption are we to use? (ii) What devices are needed

1. See table 6.2A, p. 192 above.
2. See p. 7 above.
3. See pp. 235-236 below.

to overcome any lack of data? (iii) What methods are to be used
(a) to deflate data, and (b) to take account of seasonality? Under
the sub-heading "estimation methods", the estimation methods are
discussed, along with any likely statistical problems. Finally, a
list of the various statistics calculated with each estimate is given.

7.3.1 Data   The first question relating to data concerns the
definition of consumption to be employed. Ideally, consumption
expenditure is equal to aggregate expenditure on single-use goods
and services plus the use value of consumers' stocks of durables.
Purchases of durable goods by consumers would then be classed as an
investment. However, statistics of the use value of stocks of consumer
goods are not available. Rather than attempt to construct such
statistics[1] it was decided to estimate each hypothesis first with
statistics on total consumption (C) and then with statistics covering
expenditure on non-durable goods and services (CND). The first set
of estimates are shown in table 7.1,[2] while those using CND as
dependent variable are shown in 7.4.[3]

The next question that had to be considered was the
availability of data. Most statistics required were obtained from the
regular publications of the Commonwealth Bureau of Census and Statistics

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1. Such statistics have been constructed by, for example, R. L. Moore,
   "The Permanent Income Hypothesis: evidence from post-war time
   series data".

2. See p. 245 below.

3. See p. 256 below.

or the Reserve Bank of Australia. Data for the four variables XNF, A, G - $\bar{G}$, and W_{-1} did, however, pose a problem. As noted in section 7.2 a proxy, ($\frac{YNF}{Y}$)X, had to be used in place of XNF. The series for total private sector wealth, W, was obtained from a Reserve Bank of Australia discussion paper.[1] This paper gave estimates of closing stocks of wealth for the third quarter 1958 through to the fourth quarter 1969.

The rate of sales tax on motor vehicles (STMV) was used as a measure of the government policy variable G. STMV has been used many times in the past as a measure of government efforts to affect consumer expenditure on motor vehicles.[2] For comparison, Stone and Rowe in their work employed the percentage downpayment on the hire-purchase of radio and electrical goods as a measure for G.[3] Since there is no officially specified downpayment percentage in Australia, this measure was inapplicable. $\bar{G}$ was put equal to an unweighted average of G in the previous four quarters. The only difficulty associated with the use of STMV concerned its value in those quarters in which the rate of sales tax was altered. In such quarters G was put equal to a weighted average of the old and the new rates, with the weights being the number of days in the quarter for which the

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1. J. Helliwell et. al. Quarterly Estimates of Private Sector Wealth (Reserve Bank of Australia, Research Discussion Paper No. 18, 1971).

2. See Nevile, "A simple econometric model of the Australian economy"; Kmenta, "An econometric model of Australia 1948-61".

3. See Stone and Rowe, "A post-war expenditure function", p. 199.

old rate applied and the number of days for which the new rate applied respectively.

The index of consumer attitudes, A_t, presented the most difficult problem. Ideally, of course, such an index ought to be based upon some survey of consumers. Such an approach was out of the question for two reasons however. Firstly, the index should reflect the attitude of Australian consumers and the time and expense involved in constructing and carrying out an Australia wide, statistically reliable survey is obviously too great for a study of this nature. Secondly, and far more critically, even if such a survey were carried out it would provide only a single observation on A_t. In the United States, on the other hand, surveys of consumer attitudes have been carried out on a semi-annual or quarterly basis since the beginning of the 1950's.[1] In place of the survey approach we looked for a number of proxy variables for A which (i) may have values reflecting current consumer attitudes, or (ii) may in themselves influence consumer attitudes. Four variables were chosen: (a) The quarter-to-quarter percentage change in the consumer price index, (b) The quarter-to-quarter percentage change in average weekly earnings, (c) The quarter-to-quarter changes in outstanding instalment credit balances financed by all businesses, (d) total registered unemployed.

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1. See E. Mueller, "Ten years of consumer attitude surveys: their forecasting record", Journal of the American Statistical Association Vol. 58 (1963), pp. 899-917.

It was felt that (a), (b) and (d) may _influence_ consumer attitudes, while (c) _reflects_ consumer attitudes. To attempt to overcome the problem of different units, each series was first divided through by its value in the four quarters of 1963. The resulting four series were then added together to give the attitudes index. From the nature of the construction of the attitudes index it is difficult (or impossible) to form any expectations as to the likely sign or magnitude of its coefficient in a regression.

Quarterly price deflated data for the period 1959(2) to 1969(1) were used throughout, giving a total of 40 observations. Quarterly data was used because we were looking for estimates of the _short-run_ consumption function. Quarterly data also has the advantage of substantially increasing the number of observations available for use.[1] One reason for using deflated data is that at the micro level demand functions for goods are homogeneous of degree zero in prices and income,[2] i.e. there is absence of money illusion. Hence, we would expect at the aggregate level a similar absence of money illusion.[3]

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1. While the use of quarterly rather than annual data increases the number of observations by a factor of 4, it does not increase the amount of statistical information by the same amount. See L. R. Klein, A Textbook of Econometrics (Evanston, Illinois: Row, Peterson and Coy., 1953), p. 314 where Klein points out that the use of quarterly data will not lead to an increase in the number of degrees of freedom by a factor of 4.

2. See J. M. Henderson and R. E. Quandt, Microeconomic Theory (New York: McGraw-Hill, 1958), pp. 21-22.

3. However, see Branson and Klevorick, "Money illusion and the aggregate consumption function", for one study in which results indicate that money illusion is a significant factor at the aggregate level.

This requires that the variables in the consumption function be expressed in real terms. In the results below all data, where appropriate, have been deflated by the consumer price index, base 1966-67 = 100. The consumer price index has been used as it seems more sensible to deflate consumption and income data etc. by an index relevant to the decisions of consumers. Carl Christ also provides arguments for deflating all variables by a single price index.[1]

Since we are using quarterly data some allowance must be made for seasonality. There are two ways of treating seasonal effects: (i) Use deseasonalized data obtained by the use of, for example, four-quarter moving averages; (ii) Use undeseasonalized data while attempting to take account of seasonal effects by introducing additive or multiplicative seasonal dummy variables. If deseasonalized data were available (i) would be preferable to (ii) since it is computationally less burdensome. However, deseasonalized data were not available for all variables and hence there was little to choose between the methods on grounds of computational ease. There are some theoretical reasons for preferring (ii) over (i). Firstly, the inclusion of seasonal dummies gives the opportunity for testing hypotheses about the affect of seasonality upon parameters. Secondly, the inclusion of the seasonal dummies makes quite clear that some degrees of freedom are lost by attempting to take account of seasonality

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1. C. F. Christ, "Aggregate econometric models", American Economic Review, Vol. 46 (1956), p. 396.

In addition to these theoretical reasons there is some empirical evidence that seasonal dummies is the preferable method. Using covariance analysis G. W. Ladd found in five out of six studies examined that the additive seasonal dummies model was preferable to the use of seasonally adjusted data.[1] Norton and Broadbent made a number of estimates of the Australian consumption function using quarterly deflated data. The equations were estimated using both seasonal dummies and seasonally adjusted data. They concluded that

> "In the equations based on seasonally adjusted data the performances of the relative price and liquid wealth variables are poorer and the Durbin-Watson ratios are markedly lower [than in the equations using seasonal dummies]."[2]

As a consequence they presented the estimates based upon the seasonal dummies. In view of these theoretical and empirical results the various hypotheses, with the exception of H(11), were estimated with additive seasonal dummies. Thus, for example, H(1) with seasonal dummies becomes

$$C = a_0 + a_1 Q_1 + a_2 Q_2 + a_3 Q_3 + bX \qquad (7.32)$$

where Q_i = 1 during quarter i and = 0 during other quarters. This treatment implies that seasonality affects the intercept term, but leaves the slope coefficient, b, unaltered.

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1. G. W. Ladd, "Regression analysis of seasonal data", Journal of the American Statistical Association, Vol. 59 (1964), p. 420.

2. Norton and Broadbent, Equations for Personal Consumption Expenditure, fn. 1, p. 7.

7.3.2 <u>Estimation methods</u>   Hypotheses one to ten were estimated by single equation least squares. Since the consumption function is in reality part of a simultaneous system of relationships, the parameter estimates obtained will be biased (<u>simultaneity</u> bias) and indeed will not even be consistent.[1] A consequence of this lack of consistency will be that estimates will be biased even for large samples.[2] For rather specialized cases it is possible to show that the estimated coefficient of disposable income will be biased upwards from its true value.[3]

Even if this simultaneity problem were not present least squares estimates of some hypotheses will be biased due to the presence of lagged values of the dependent variable among the regressors.[4] This bias is known as <u>distributed lag</u> bias. In addition the estimates may also be inconsistent, and therefore biased even for large samples.[5] Note that in the case of H(9) and H(10), where the distributed lag weights are estimated using the Almon variable method, lagged values of the dependent variable do not occur amongst the regressors and, hence, the estimates will be free of distributed lag bias.

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1. See, for example, Johnston, <u>Econometric Methods</u>, pp. 231-234.

2. <u>Ibid.</u>

3. <u>Ibid.</u>, p. 234.

4. <u>Ibid.</u>, pp. 211-215.

5. Goldberger, <u>Econometric Theory</u>, pp. 276-277.

The form of H(11) was such that it was possible to apply two recently proposed estimation methods as alternatives to least squares. These methods, one proposed by Liviatan[1] and an iterative procedure discussed by Dhrymes, Klein and Steiglitz (DKS),[2] have the property that they give consistent estimates under a weaker set of assumptions than is required to obtain consistent estimates by least squares. For the application of both of these methods it was necessary to work with deseasonalized data. This data was obtained by the use of a four-quarter moving average.

With H(1) to H(8) the following statistics were calculated: $\bar{R}^2$, $\overline{SE}$, t-ratios, and either the Durbin-Watson or Durbin statistic. The last two statistics mentioned are designed to test for possible autocorrelation of disturbances, the Durbin statistic being designed for regressions including lagged values of the dependent variable amongst the regressors. This statistic is defined as[3]

$$h = (1 - \frac{1}{2} d) \sqrt{\frac{n}{1 - n\hat{V}(\lambda)}}$$

1. N. Liviatan, "Consistent estimation of distributed lags", International Economic Review, Vol. 4 (1963), pp. 44-52.

2. P. J. Dhrymes, L. R. Klein, and K. Steiglitz, "Estimation of distributed lags", International Economic Review, Vol. 11 (1970), pp. 235-250, esp. pp. 244-245. Both the Liviatan and DKS estimators are discussed in P. J. Dhrymes, Distributed Lags: Problems of Estimation and Formulation (San Francisco: Holden-Day, Inc., 1971), Ch. 5.

3. See Durbin, "Testing for serial correlation in least squares regression when some of the regressors are lagged dependent variables", p. 419.

where d is the value of the Durbin-Watson statistic, (n + 1) is the sample size and $\hat{V}(\lambda)$ is the estimated variance of the SELS estimate of the coefficient of y_{t-1}, where y_t is the dependent variable. The statistic is tested as a standard normal deviate. Clearly the test based upon this statistic is inapplicable whenever $n\hat{V}(\lambda) \geq 1$. In such cases the Durbin-Watson statistic, although not strictly appropriate, is used. For H(9) and H(10) the following statistics were calculated: $\bar{R}^2$, $\overline{SE}$, t-ratios for the estimated coefficients of the Almon variables, and the Durbin-Watson statistic. t-ratios for the actual distributed lag weights have not been calculated. In the case of H(11) our principal interest lay in the size of the estimates given by the alternative estimation methods, and consequently no accompanying statistics were calculated. We shall now present and discuss the results obtained.

7.4 Estimates of the short-run consumption function   From section 7.2 above it should be clear that for some hypotheses more than one regression has been carried out for each definition of the dependent variable. In the case of H(1), H(2), H(6) and H(7) only one result was obtained.[1] But for H(3), H(4) and H(5) three regressions were carried out - one using L^1_{-1}, a further one using L^2_{-1}, and another using the wealth series W_{-1}. For H(8), twelve estimates were obtained by the use of the three series L^1_{-1}, L^2_{-1}, W_{-1} in conjunction with the four forms of H(3) obtained by the

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1. i.e. one result for total consumption, C, and one result for consumption of non-durables, CND. The results are separately reported in tables 7.1A and 7.4A.

(i) putting d = e = 0, (ii) putting d = 0, (iii) putting e = 0, (iv) leaving d and e unrestricted.[1] Finally, for H(9) and H(10) 288 and 56 estimates respectively were calculated.[2] Before considering the results we shall go over the methods used to select the reported estimate from the available alternatives.

In the case of H(3), H(5) and H(4, 8) the criteria used were: (i) The estimate must be accompanied by a satisfactory value of the statistic d or h, i.e. the null hypothesis of zero first-order autocorrelation of disturbances must be acceptable at the 5% level; (ii) The accompanying t-ratios must show the estimated parameter to be statistically different from zero at the 5% level of significance; (iii) Parameter estimates should conform with prior expectations as to sign and magnitude. For each hypothesis there was no more than one estimate satisfying all three criteria. For some hypotheses no estimate satisfied all three criteria. In these circumstances a subjective choice had to be made with the aim of getting "reasonable" values for t-ratios and d or h. In the case of H(4, 8) three results are presented in table 7.1A. Of these H(4, 8): 3 seems the best in terms of t-ratios and the statistic d or h. However, the implied long-run marginal propensity to consume is only 0.357.[3]

1. See p. 224 above. Note that the form of H(8) obtained by putting d = e = 0 is precisely H(4). For this reason H(4) and H(8) have been treated together and results are reported as H(4, 8).

2. See p. 226 and p. 229 above.

3. The long-run marginal propensity to consume is found by suppressing time subscripts in the consumption function and solving for C. The resulting coefficient of disposable income is then described as the long-run m.p.c. Hence, the long-run m.p.c. corresponding to

$$C_t = a + bX_t + cC_{t-1} \text{ is } \frac{b}{1-c} \text{ .}$$

Estimates H(4, 8): 1 and H(4, 8): 2 are not significantly poorer

in terms of t-ratios etc. but have more reasonable values for the

long-run marginal propensity to consume, namely 0.591 and 0.892

respectively.

In the case of H(9), 12 separate regressions were performed

for each of 24 lag structures. To find the best estimate we began

by looking for the lag structure giving the lowest value for $\overline{SE}$, the

adjusted standard error of the estimate, for each list of regressors.

The resulting structures were all based upon polynomials of degree 4,

and the weights took on a rather implausible U-shape. Attention was

then directed to structures based upon polynomials of degree 3, since

these appeared to give the next best set of values for $\overline{SE}$. From these,

the estimates with satisfactory t-ratios were chosen. The final

estimate was then **selected**, on a somewhat more subjective basis,

taking into account (i) the value of the Durbin-Watson statistic,

and (ii) the signs and magnitudes of parameter estimates. This

procedure was applied both for the estimates involving the variable C

and those involving the variable CND.

In the case of H(10), 4 regressions for each of 14 lag

structures were carried out. With C as dependent variable we began

by looking for the lag structure which minimized $\overline{SE}$ for each list of

regressors. Since the distribution of $\overline{SE}$ was rather "flat", a fairly

wide choice amongst structures based upon polynomials of degree 3 and

4 was still possible. However, the variables A and $(G - \overline{G})$ were

insignificant whenever included, and the reported result was chosen

from amongst the remaining regressions on the basis of good values for t-ratios. The results with CND as dependent variable were in general unsatisfactory. The minimum $\overline{SE}$ criterion resulted in choice being restricted to structures based upon polynomials of degree 2 with a relatively long lag. Amongst this set of estimates none could be found with a satisfactory set of t-ratios. The result presented is typical of those available.[1]

The selected estimates of the various hypotheses are presented in tables 7.1 and 7.4 below.[2] In table 7.1A the results are given for the case where total consumption, C, is the dependent variable. For convenience the estimates of the four intercept parameters[3] are given separately in table 7.1B. This convention has not been followed in the case of H(6) - the Duesenberry/Eckstein/ Fromm function - because of the special significance of the intercept term in that hypothesis. The estimated coefficients of the Almon variables and the associated distributed lag weights for H(9) and H(10) are given in table 7.2. Tables 7.4A, 7.4B and 7.5 contain the same information for the regressions using CND (consumption of non-durables) as dependent variable.

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1. It should be noted that outside the minimum $\overline{SE}$ choice set, the t-ratios were equally bad.

2. See p. 245 and p. 256 respectively.

3. i.e. the estimates of the intercept term, a_0, and the coefficients of the three seasonal dummies, a_1, a_2, a_3.

7.4.1 The results using C   We will first discuss the results

using C as dependent variable, after which an attempt will be made

to select the best of the estimates from table 7.1A.

Firstly, the Keynesian consumption function does quite

well on the criteria of good t-ratios and a high value for $\bar{R}^2$. The

estimates of the intercept parameters indicate that seasonal factors

are significant. However, the Durbin-Watson statistic indicates that

serial correlation of the disturbances may be a problem.

If lagged consumption is introduced as a dependent variable

the problem of serial correlation, as indicated by the Durbin **statistic**,

disappears. There is also an increase in the value of $\bar{R}^2$ and a very

large reduction in $\overline{SE}$. Both parameter estimates in H(2) are highly

significant. The coefficient of disposable income falls very

substantially and takes on a similar value whenever lagged consumption

is a regressor.[1] The question arises then as to whether or not serial

correlation bias is adversely affecting the estimate. This would not

seem to be the case since the consistent estimate of the coefficient

obtained by using Liviatan's method is in the same region as the other

estimates.

A variable of interest is the liquid asset variable. L^1_{-1}

appears twice in table 7.1A, in the estimate of H(3) and of H(4, 8): 2.

In neither case is it statistically different from zero at the 5% level.

1.   See H(4, 8): 1, H(4, 8): 2, H(4, 8): 3 and H(11).

TABLE 7.1A

ESTIMATES OF SHORT-RUN CONSUMPTION FUNCTIONS, USING QUARTERLY AUST. DATA 1959(2) – 1969(1): SLOPE ESTIMATES[a]

Hypothesis	Method of Estimation[b]	Estimate[c]	$\bar{R}^{2*}$	$\overline{SE}^*$	d^*	h^*
H(1)	SELS	$C = 0.882\,X$ (37.20)	0.975	65.82	1.43	
H(2)	SELS	$C = 0.203\,X + 0.781\,C_{-1}$ $(3.11)\quad(10.55)$	0.994	32.31		0.94
H(3)	SELS	$C = 0.856\,X + 0.102\,L^1_{-1}$ $(27.97)\quad(1.30)$	0.976	65.18	1.39	
H(5)	SELS	$C = 0.419\,X + 0.101\,X^0 + 0.023\,W_{-1}$ $(4.64)\quad(0.97)\quad(4.66)$	0.988	45.84	.87	
H(6)	SELS	$\dfrac{C}{X_{-1}} = 1.132 - 0.153\,Q_1 + 0.001\,Q_2 - 0.078\,Q_3$ $\quad\;(8.11)\;\;(-7.21)\quad(0.02)\quad(-8.69)$ $\quad - 0.589\,\dfrac{X_{-1}}{X^0_{-1}} + 0.421\,\dfrac{C_{-1}}{X_{-2}}$ $\quad(-4.84)\qquad(3.17)$	0.967	0.017		2.19
H(7)	SELS	$C = 0.505\left(\dfrac{YNF}{Y}\right)X - 0.075\left(\dfrac{YNF}{Y}\right)_{-1}X_{-1} + 0.522\,C_{-1}$ $\quad(5.50)\qquad\qquad\quad(-0.60)\qquad\qquad\qquad(3.11)$	0.996	26.84	1.84	**
H(4, 8): 1	SELS	$C = 0.204\,X + 0.007\,W_{-1} + 0.655\,C_{-1}$ $(3.20)\quad(1.64)\quad(6.22)$	0.994	31.54		1.23
H(4, 8): 2	SELS	$C = 0.199\,X + 0.049\,L^1_{-1} + 0.777\,C_{-1} - 5.925(G - \bar{G})$ $(3.14)\quad(1.10)\quad(10.54)\quad(-2.11)$	0.994	31.20		0.18
H(4, 8): 3	SELS	$C = 0.155X + 0.069\,L^2_{-1} + 0.566\,C_{-1} - 5.382\,(G - \bar{G})$ $(2.48)\quad(2.27)\quad(4.66)\quad(-2.29)$	0.995	29.51		1.40
H(9)	SELS	$C = \displaystyle\sum_{i=0}^{5} a_i\,X_{-i} + 0.020\,W_{-1}$ $\qquad\qquad(4.69)$	0.988	41.59	1.04	
H(10)	SELS	$C = \displaystyle\sum_{i=0}^{3} a_i\,X_{-i} - 0.026\,W_{-1} + \sum_{i=0}^{2} c_i\,U_{-1-i}$ $\qquad\qquad(-0.84)$	0.989	41.92	0.94	
H(11)	L DKS	$C = 0.166\,X + 0.826\,C_{-1}$ ***				

Notes:

a: estimates of the intercept parameters are given in table 7.1B.

b: SELS = Single equation least squares, L = method of Liviatan, DKS = method of Dhrymes, Klein, Steiglitz.

c: figures in parenthesis below parameter estimates are t-ratios.

*: $\bar{R}^2$ = adjusted coefficient of multiple determination, $\overline{SE}$ = adjusted standard error of the estimate, d = Durbin-Watson statistic, h = Durbin statistic.

**: Durbin statistic could not be calculated $(n\hat{V}(\lambda) > 1)$.

***: DKS procedure did not converge.

TABLE 7.1B

ESTIMATES OF SHORT-RUN CONSUMPTION FUNCTIONS, USING QUARTERLY

AUST. DATA 1959(2) - 1969(1): INTERCEPT ESTIMATES

Hypothesis	Parameter[a]			
	a_0	a_1	a_2	a_3
H(1)	-132.195	236.765	327.402	253.935
	(-1.40)	(7.23)	(9.66)	(7.76)
H(2)	150.201	-390.599	-32.794	-140.777
	(2.81)	(-6.34)	(-0.86)	(-3.46)
H(3)	-433.872	196.750	286.170	235.831
	(-1.74)	(4.41)	(6.21)	(6.69)
H(5)	-38.624	-92.984	18.725	-15.890
	(-0.57)	(-1.22)	(0.27)	(-0.27)
H(6)	See Table 7.1A			
H(7)	265.604	-259.380	-140.706	-120.946
	(5.17)	(-4.38)	(-11.18)	(-3.89)
H(4, 8): 1	127.136	-362.354	-44.136	-138.824
	(2.35)	(-5.79)	(-1.17)	(-3.49)
H(4, 8): 2	-14.136	-404.887	-52.996	-148.640
	(-0.094)	(-6.72)	(-1.38)	(-3.76)
H(4, 8): 3	287.737	-386.412	-103.874	-166.546
	(3.55)	(-6.80)	(-2.33)	(-4.36)
H(9)	100.007	-346.456	-221.744	-204.285
	(1.54)	(-17.52)	(-10.35)	(-9.11)
H(10)	162.476	-361.677	-227.305	-188.512
	(2.20)	(-12.06)	(-3.50)	(-2.51)
H(11)	Not Applicable			

Notes:

a: figures in parenthesis below estimates are t-ratios.

TABLE 7.2

ESTIMATED COEFFICIENTS OF ALMON VARIABLES AND DISTRIBUTED LAG WEIGHTS

Hypothesis	H(9)		H(10)			
Lagged Variable	X		X		W	
	Almon[a] Coefficient	Distributed Lag Weight	Almon[a] Coefficient	Distributed Lag Weight	Almon[a] Coefficient	Distributed Lag Weight
	0.169 (4.99)	0.1248	0.207 (3.71)	0.1699	0.039 (1.65)	0.0432
	0.068 (2.29)	0.1682	0.136 (2.94)	0.2087	-0.008 (-1.31)	0.0174
		0.1541		0.1627		-0.0171
		0.1064		0.0777		
		0.0489				
		0.0055				
Sum of Weights		0.6079		0.6190		0.0435

Notes:

a: figures in parenthesis below the estimate of the Almon variable coefficient are t-ratios.

The variable L_{-1}^2 appears once, in the estimate of $H(4, 8)$: 3.

Its estimated coefficient is statistically significant at the 5%

level. An alternative to L_{-1}^1 and L_{-1}^2 tried in most regressions was

W_{-1}. This variable has been selected in preference to L_{-1}^1 and L_{-1}^2

in the case of $H(5)$, $H(4, 8)$: 1, and $H(9)$. In the case of $H(5)$ and

$H(9)$ the estimated coefficient of W_{-1} is strongly significant.

However, as the Durbin-Watson test indicates autocorrelation of

disturbances in both estimates, there may be some overstatement of

the t-ratios.[1] In the case of $H(4, 8)$: 1 the Durbin statistic

indicates absence of serial correlation, in which case the conventional

t-test holds. The estimated coefficient of the wealth variable is

statistically significant there at the 10% level. The wealth variable

appears also in $H(10)$, first as a single term $-0.026W_{-1}$ and secondly

as part of a distributed lag term $\sum_{i=0}^{2} c_i W_{-1-i}$. The coefficient of

the single term is clearly not significant, having a t-ratio of only

-0.84. While t-ratios have not been calculated for the distributed

lag weights, the t-ratios of the Almon variable coefficients (from

which the weights are calculated) indicate that these coefficients

are not significant at the 5% level.[2] In addition, the Durbin-Watson

statistic indicates presence of serial correlation in $H(10)$. Thus

there does appear to be strong evidence for wealth or liquid assets

being a significant variable in the consumption function.

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1. See p. 24 above.

2. See table 7.2 p. 247.

Our results also throw some light on the farm, non-farm break up of disposable income suggested by Arndt and Cameron. When testing H(7) (our version of the Arndt-Cameron function) it was not possible to calculate the Durbin-statistic, since $n\hat{V}(\lambda)$ was greater than unity; however, the Durbin-Watson statistic indicates absence of serial correlation. In addition, H(7) has a very high value for $\bar{R}^2$ and the lowest value of $\overline{SE}$ of all the estimates in table 7.1A. The sign and magnitude of all parameter estimates conforms with prior expectations. Thus far the evidence in favour of H(7) is very good. However, the coefficient of $(\frac{YNF}{Y})_{-1} X_{-1}$ is not statistically significant. This coefficient is equal to $\beta\lambda$ where β is the coefficient of XNF in

$$C^{NF} = \alpha + \beta XNF \qquad (7.8)$$

and λ is the adjustment coefficient from[1]

$$C^F = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i C^{NF}_{-i} \qquad (7.9)$$

Hence we have $\beta\lambda = 0$, i.e. $\beta = 0$ or $\lambda = 0$. Since it does not seem reasonable to have β in (7.8) equal to zero, the first reaction to this result is to conclude that $\lambda = 0$, i.e. that there is an instantaneous adjustment of farm consumption levels to non-farm levels.

This evidence would tend to support the original formulation by Arndt and Cameron. However, the coefficient of lagged consumption

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1. See p. 221 above.

in H(7) is strongly significant. This coefficient is equal to λ.[1]

Thus we have two contrary pieces of evidence concerning the value

of λ; one supports the Arndt-Cameron thesis while the other implies

a lag in the adjustment of farm consumption standards to non-farm

standards.[2] Since the coefficients of $(\frac{YNF}{Y})X$, $(\frac{YNF}{Y})_{-1}X_{-1}$ and

C_{-1} are subject to a non-linear restriction in the parameters β and

λ, a proper test of H(7) will require the use of an estimation method,

such as that employed by Zellner, Huang and Chau,[3] which incorporates

this constraint.

A further question concerns the usefulness of the variable

X^0. This has been tested in H(5) and H(6). In H(5) the estimated

coefficient of X^0 is clearly insignificant. The formulation in H(6)

works much better. Parameter estimates are significant and conform

in sign to prior expectations. However, the value of h indicates the

presence of serial correlation and hence casts doubt upon any

conclusions as to the significance of parameter estimates.

The government policy variable was tried in the testing of

H(8), H(9) and H(10). In the various regressions performed it

generally appeared with the correct sign and a fairly high t-ratio.

It has been chosen in two out of the three reported estimates of

H(4, 8). In general it seems to have worked more satisfactorily than

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1. See (7.12), p. 221 above.

2. However, the length of the lag implied by the coefficient of C_{-1}
   is only slightly longer than one quarter.

3. See p. 81 above.

in the Stone-Rowe model where the parameter estimate was insignificant when quarterly data were used.[1]

A final point of interest is the average lag in the adjustment of permanent income to actual income implied by the Liviatan estimate of H(11). This result is an estimate of the PIH function

$$C = \alpha X^1 \tag{7.30}$$

where
$$X^1 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i X_{-i} \tag{7.31}$$

The average lag is given by $\frac{\lambda}{1 - \lambda}$, and in this case is equal to 4.75, i.e. almost 5 quarters.[2] Using United States data Zellner et. al. found an average lag of 6.9 quarters in the adjustment of permanent income to actual income.[3]

Having examined the results generally, we now aim to find the best of the estimates in table 7.1. This will be done mainly on the basis of the ex-post forecast ability of the various estimates for the four quarters 1969(2) until 1970(1) following the sample period. Rather than carry out ex-post forecasts for all estimates, we began by selecting those estimates: (i) which pass the Durbin or Durbin-Watson test, (ii) which have significant parameter estimates, and (iii) whose

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1. See p. 94 above.

2. See footnote 1 p. 85 above.

3. Ibid.

parameter estimates conform in sign and magnitude with prior
expectations. Only two estimates complied with all of these
requirements, namely H(2) and H(4, 8): 3. So as not to unduly
restrict choice, three additional estimates were examined – H(4, 8): 1,
H(4, 8): 2, and H(9). These three had been previously excluded by a
narrow margin only; in the case of H(4, 8): 1 and H(4, 8): 2 the
t-ratios for the variables W_{-1} and L_{-1}^1, respectively, were too low,
while H(9) had been excluded on the basis of the value of its Durbin-
Watson statistic. Three "naive" or "guesswork" models were employed
for comparison. Naive model I (NM I) states that next period's
consumption is equal to this period's, i.e. $C_{t+1} = C_t$. Naive model
II (NM II) puts next period's consumption equal to the current period's
plus the change from last period to this period, i.e. $C_{t+1} = C_t +$
$(C_t - C_{t-1})$. The third model (NM III) calculates next period's
consumption by applying the current consumption-income ratio to next
period's income, which is assumed known, i.e. $C_{t+1} = \dfrac{C_t}{X_t} \cdot X_{t+1}$. Two
measures of predictive performance were used: (i) the average
absolute error (AAE), and (ii) the square root of the mean square error
of prediction, i.e.

$$M = \sqrt{\frac{\Sigma(C_a - C_p)^2}{n}}$$

where C_a = the actual value of consumption, C_p = the corresponding
predicted value of consumption, and n = the number of periods involved
in the prediction.

The actual value of consumption, the predicted values, and the values for M and AAE are shown below in table 7.3. All five consumption functions are clearly superior to the naive models.[1] Ranking the five consumption functions according to the values of AAE and M gives

AAE	M
H(9)	H(9)
H(4, 8): 1	H(4, 8): 3
H(2), H(4, 8): 3	H(4, 8): 1
	H(2)
H(4, 8): 2	H(4, 8): 2

Of the five estimates tested then, H(4, 8): 1, H(4, 8): 3 and H(9) are clearly superior to H(2) or H(4, 8): 2. Of the first three mentioned H(9) is the best ex-post forecaster, but its superiority here is not dramatic.[2] On the other hand H(4, 8): 1 and H(4, 8): 3 pass the Durbin-test for autocorrelated disturbances while H(9) fails the corresponding Durbin-Watson test. Confining attention, therefore, to H(4, 8): 1 and H(4, 8): 3, there is little to choose between them on the basis of the accompanying statistic (h, $\bar{R}^2$, t-ratios, etc.).

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1. Since we are using undeseasonalized data, the naive models are at an automatic disadvantage with respect to the consumption functions (which include facilities for taking care of seasonality), and it would be most surprising if any of the naive models had proved superior.

2. The difference between the average absolute error of H(4, 8): 1 and H(9), for example, is only $7m. per quarter compared with a figure for actual consumption of some $3,900m. per quarter.

TABLE 7.3

ACTUAL AND PREDICTED CONSUMPTION, 1969(2) - 1970(1)

Actual Consumption	Predicted Consumption							
	H(2)	H(4, 8):1	H(4, 8):2	H(4, 8):3	H(9)	NM I	NM II	NM III
3817	3810	3824	3818	3793	3857	3647	3322	3773
3843	3847	3842	3860	3834	3861	3817	3987	3874
4236	4137	4131	4153	4125	4154	3843	3869	4420
3852	3947	3930	3968	3913	3874	4236	4629	3782
Average Error	1.75	5.25	12.75	20.75	0.50	51.25	14.75	-25.25
A.A.E.	51.25	47.75	54.25	51.25	40.50	243.25	445.75	82.25
M	68.72	65.50	71.82	64.61	47.78	287.87	501.04	102.05

However, the value for the long-run m.p.c. implied by H(4, 8): 1 is more plausible than the value implied by H(4, 8): 3,[1] and we therefore choose H(4, 8): 1 as the best estimate.

7.4.2 The results using CND

The results obtained when using CND as dependent variable are very similar to those just discussed, and for this reason the discussion here will be brief. To facilitate comparison with the earlier results, topics will be discussed in the same order as before.

Once again the simple Keynesian function performs well on the criteria of good t-ratios, high $\bar{R}^2$ etc. However, the Durbin-Watson statistic indicates that serial correlation could be a problem. The addition of lagged CND removes any doubt about serial correlation of disturbances. The coefficient of disposable income drops most markedly from 0.786 to 0.096. Estimate H(4, 8) also includes lagged CND as a regressor and the resulting estimate of the coefficient of X is 0.094. In both cases the estimate is statistically significant. Liviatan's estimator yielded

$$CND = 0.004X + 1.006CND_{-1}$$

A priori expectations were that the estimated coefficient of CND_{-1}, $\hat{\lambda}$, would satisfy the restriction $0 < \hat{\lambda} < 1$. This restriction is needed because λ is the adjustment coefficient in

$$X^1 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i X_{-i}$$

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1. See pp. 241-242 above.

TABLE 7.4A

ESTIMATES OF SHORT-RUN CONSUMPTION FUNCTIONS, USING QUARTERLY AUST. DATA 1959(2) – 1969(1): SLOPE ESTIMATES[a]

Hypothesis	Method of Estimation[b]	Estimate[c]	$\bar{R}^{2*}$	$\overline{SE}^{*}$	d^{*}	h^{*}
H(1)	SELS	$CND = 0.786\ X$ (33.61)	0.969	64.99	1.35	
H(2)	SELS	$CND = 0.096\ X + 0.890\ CND_{-1}$ $(1.98) \qquad (14.52)$	0.996	24.56		−0.46
H(3)	SELS	$CND = 0.785\ X + 0.004\ L^{1}_{-1}$ $(25.37) \qquad (0.05)$	0.968	65.93	1.35	
H(5)	SELS	$CND = 0.334\ X + 0.141\ X^{0} + 0.020\ W_{-1}$ $(3.53) \qquad (1.29) \qquad (3.84)$	0.983	48.09	0.69	
H(6)	SELS	$CND = 0.913 - 0.129\ O_{1} + 0.015\ O_{2} - 0.063\ O_{3}$ $(7.11) \quad (-6.75) \qquad (0.73) \qquad (-7.61)$ $\qquad - 0.458\ \dfrac{X_{-1}}{X^{0}_{-1}} + 0.440\ \dfrac{CND_{-1}}{X_{-2}}$ $\quad (-4.21) \qquad (3.22)$	0.962	0.015		2.21
H(7)	SELS	$CND = 0.309\ (\frac{YNF}{Y})\ X - 0.024\ (\frac{YNF}{Y})_{-1}\ X_{-1} + 0.649\ CND_{-1}$ $(4.06) \qquad\qquad (-0.26) \qquad\qquad\qquad (4.52)$	0.997	21.50		−0.20
H(4, 8)	SELS	$CND = 0.094\ X + 0.002\ W_{-1} + 0.857\ CND_{-1}$ $(1.93) \qquad (0.54) \qquad (9.85)$	0.995	24.83		−0.39
H(9)	SELS	$CND = \sum\limits_{i=0}^{5} a_{i}\ X_{-i} + 0.015\ W_{-1} - 6.010\ (G - \bar{G})$ $\qquad\qquad (3.77) \qquad (-1.92)$	0.987	38.30	0.72	
H(10)	SELS	$CND = \sum\limits_{i=0}^{3} a_{i}\ X_{-i} + 0.022\ W_{-1} + \sum\limits_{i=0}^{2} c_{i}\ W_{-1-i}$ $\qquad\qquad (1.83)$	0.986	41.88	0.75	
H(11)	L DKS	$CND = 0.004\ X + 1.006\ CND_{-1}$ $** $				

Notes:

a: estimates of the intercept parameters are given in Table 7.4B.
b: SELS = single equation least squares, L = method of Liviatan, DKS = method of Dhrymes, Klein and Steiglitz.
c: figures in parenthesis below estimates are t-ratios.
*: $\bar{R}^{2}$ = adjusted coefficient of multiple determination, $\overline{SE}$ = adjusted standard error of estimate,
  d = Durbin-Watson statistic, h = Durbin statistic.
**: DKS estimate not calculated.

TABLE 7.4B

ESTIMATES OF SHORT-RUN CONSUMPTION FUNCTIONS, USING QUARTERLY

AUST. DATA 1959(2) – 1969(1): INTERCEPT ESTIMATES

Hypothesis	Parameter[a]			
	a_0	a_1	a_2	a_3
H(1)	-219.383	255.831	334.873	251.005
	(-2.36)	(7.91)	(10.01)	(7.77)
H(2)	138.459	-360.630	-52.578	-167.014
	(3.22)	(-8.17)	(-1.78)	(-5.34)
H(3)	-230.977	254.293	333.288	250.309
	(-0.92)	(5.63)	(7.15)	(7.02)
H(5)	-129.017	-73.499	28.811	-14.799
	(-1.82)	(-0.92)	(0.40)	(-0.24)
H(6)	See Table 7.4A			
H(7)	181.974	-259.225	-97.117	-136.377
	(5.55)	(-5.53)	(-8.66)	(-5.32)
H(4, 8)	130.795	-355.940	-55.518	-166.892
	(2.86)	(-7.83)	(-1.83)	(-5.28)
H(9)	-33.854	-262.460	-153.573	-169.239
	(-0.55)	(-14.40)	(-7.69)	(-8.19)
H(10)	54.820	-297.872	-200.725	-192.132
	(0.78)	(-14.12)	(-9.52)	(-9.68)
H(11)	Not Applicable			

Notes:

    a: figures in parenthesis below estimates are
        t-ratios.

TABLE 7.5

ESTIMATED COEFFICIENTS OF ALMON VARIABLES AND DISTRIBUTED LAG WEIGHTS

Hypothesis	H(9)		H(10)			
Lagged Variables	X		X		W	
	Almon Coefficient	Distributed Lag Weight	Almon Coefficient	Distributed Lag Weight	Almon Coefficient	Distributed Lag Weight
	0.133 (4.21)	0.0838	0.170 (6.61)	0.1086	−0.002 (−0.36)	−0.0016
	0.100 (3.65)	0.1265		0.1629		−0.0021
		0.1361		0.1629		−0.0016
		0.1203		0.1086		
		0.0872				
		0.0444				
Sum of Weights		0.5983		0.5430		−0.0053

If $\lambda > 1$ it is then not possible to carry out a Koyck transformation to give H(11). Nor does it make good economic sense to have smaller weights on the recent values of X than on all other values.[1] Thus it was not possible to check the estimates of the X coefficient against consistent estimates. The estimates obtained are, at least, of the same order of magnitude as that by the Reserve Bank of Australia.[2]

Turning now to the liquid asset variable. This is much less prominent than in the previous set of regressions. L^1_{-1} appears once, in H(3), with a t-ratio of only 0.05. L^2_{-1} does not appear at all. Hence liquid assets do not seem to be an important explanatory variable. This result is quite consistent with various economic arguments put forward above. For example, liquid assets have some-times entered our consumption functions via a liquid asset imbalance term designed to explain _transitory_ consumption. By its nature non-durable consumption expenditure corresponds more to permanent consumption than to transitory consumption, and hence we need not expect a strong tie up between CND and L^1_{-1} or L^2_{-1}.

On the other hand W_{-1} appears four times, in H(5), H(4, 8), H(9) and H(10). The estimated coefficient of W_{-1} is insignificant in

1. Without obtaining an alternative consistent estimate to the Liviatan estimate it was not possible to obtain a consistent estimate by the DKS method.

2. In a regression for CND, where CND_{-1} was amongst the regressors, the coefficient of X was found to be 0.1576. See p. 208 above.

H(4, 8) and in the distributed lag term in H(10).[1] The estimated coefficient is,however, strongly significant in H(5) and H(9). It may be that these formulations have captured some of the permanent wealth effect put forward by Stone and Rowe as a determinant of permanent consumption.[2]

The previous comments concerning the significance of X^0 and the division of X into farm and non-farm components apply here without amendment. The government policy variable does not get quite as strong support this time. It appears only once, in H(9), accompanied by a Durbin-Watson statistic indicating the presence of serial correlation.

Finally, the performance of H(10) was uniformly abysmal for all lag structures and lists of regressors tried.

We now conclude this section by attempting to find the best of the estimates in table 7.4 on the basis of their ex-post forecast ability. Once again the forecast period is the four quarters 1969(2) - 1970(1) immediately following the end of the sample period. Estimates H(2) and H(9) were chosen from the available estimates. The choice criteria used were (i) good t-ratios, (ii) a good value for d or h, and (iii) conformity of parameter estimates with prior expectations as to sign and magnitude. The actual value

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1. To be precise the Almon coefficient generating the weights is not significant. See Table 7.5.

2. See p. 88 above.

of consumption, the predicted value, and the values of M and AAE

are given below in table 7.6. Once again the estimated consumption

TABLE 7.6

ACTUAL AND PREDICTED CONSUMPTION 1969(2) - 1970(1)

Actual Consumption	Predicted Consumption				
	H(2)	H(9)	NM I	NM II	NM III
3334	3321	3357	3187	2935	3297
3337	3343	3360	3334	3481	3384
3654	3574	3608	3337	3340	3838
3372	3445	3408	3654	3971	3262
Average Error	3.50	-9.00	46.25	-7.50	21.00
AAE	43.00	33.43	187.25	364.00	94.50
M	54.62	32.00	224.52	399.17	111.28

functions are clearly superior to the naive models. Of the two

consumption functions H(9) is the better on both the value of AAE

and M. However, the average absolute error (AAE) associated with

H(2) is only $10m. per quarter higher than that for H(9), compared

with actual consumption of approximately $3,400m. per quarter. At

the same time, H(2) passes the Durbin test while H(9) fails the

Durbin-Watson test. In addition, the values of $\bar{R}^2$ and $\overline{SE}$ for H(2)

are better than those for H(9). Since there is nothing to choose

between the two on the basis of t-ratios, we therefore select $\underline{H(2)}$ as the best estimate.

7.5 <u>Conclusions</u>   In this chapter we have formulated and tested with Australian data a number of short-run consumption functions. Each was tested, first with data on total consumption (C) and then with data on the consumption of non-durable goods and services (CND). The results were similar for both sets of tests and the following general conclusions are offered:

1. The marginal propensity to consume out of current income is quite low when lagged consumption is included in the list of regressors.

2. The liquid asset variables, L_{-1}^{1} and L_{-1}^{2}, have not been particularly successful (especially with regressions using CND).

3. A total net personal wealth variable has featured several times with a strongly significant coefficient and appears in the chosen equation for total consumption.

4. Conflicting evidence exists as to the worth of splitting disposable income into farm and non-farm components, and as to the manner of the split up.

5. As presently constructed the index of consumer attitudes is not a useful variable. On the other hand some success has been achieved with the government policy variable G.

6. Generally speaking, the use of the Almon variable technique for estimating distributed lag weights results in poor values of the Durbin-Watson statistic.

7. The attempts to employ consistent methods of estimation were unsuccessful.

8. The preferred consumption functions are:

$$C = 127.136 - 362.354 \ Q_1 - 44.136 \ Q_2 - 138.824 \ Q_3$$
$$(2.35) \quad (-5.79) \qquad (-1.17) \qquad (-3.49)$$

$$+ \ 0.204 \ X + 0.007 \ W_{-1} + 0.655 \ C_{-1}$$
$$(3.20) \qquad (1.64) \qquad (6.22)$$

$$\bar{R}^2 = 0.994$$
$$\overline{SE} = \$31.54m.$$
$$h = 1.23$$

$$CND = 138.459 - 360.630 \ Q_1 - 52.578 \ Q_2 - 167.014 \ Q_3$$
$$(3.22) \quad (-8.17) \qquad (-1.78) \qquad (-5.34)$$

$$+ \ 0.096 \ X + 0.890 \ CND_{-1}$$
$$(1.98) \qquad (14.52)$$

$$\bar{R}^2 = 0.996$$
$$\overline{SE} = \$24.56m.$$
$$h = -0.46$$

7.1 Estimation of distributed lags by the Almon variable method

This appendix aims to give a brief outline of the Almon variable method for estimating distributed lags.[1] Suppose that we wish to estimate Friedman's PIH consumption function

$$C_t = K \, X_t^1 \qquad (1)$$

and suppose that permanent income is approximated by the n period lag

$$X_t^1 = \sum_{i=0}^{n-1} w_i' \, X_{t-i} \qquad (2)$$

Substitute (2) into (1)

$$\therefore \qquad C_t = K \sum_{i=0}^{n-1} w_i' \, X_{t-i}$$

$$\therefore \qquad C_t = \sum_{i=0}^{n-1} w_i \, X_{t-i} \qquad (3)$$

where $w_i = K w_i'$, $i = 0, 1, \ldots, n-1$. It is our desire to estimate the w_i.

The Almon method begins by supposing that the weights w_i are the values of some polynomial $w(x)$ evaluated at the points $x = 0, 1, \ldots, n-1$.[2] That is

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1. For a fuller account of the polynomial lag model and the Almon variation of it, the reader is referred to Dhrymes, Distributed Lags: Problems of Estimation and Formulation, Chapters 3 and 8.

2. As the previous footnote hinted, this approach is taken by a whole class of distributed lag models, and is not unique to the Almon model.

$$w_i = w(i) \qquad i = 0, 1, \ldots, n-1 \qquad (4)$$

Now, if it is known that

$$w(x_j) = b_j \qquad j = 0, 1, \ldots, q+1 \qquad (5)$$

i.e. if the value of the polynomial at each of (q+2) points x_j is known, then we can say[1] that

$$w(x) = \sum_{j=0}^{q+1} \phi_j(x)\, b_j \qquad (6)$$

where

$$\phi_j(x) = \prod_{\substack{k=0 \\ k \neq j}}^{q+1} \frac{x - x_k}{x_j - x_k} \qquad j = 0, 1, \ldots, q+1 \qquad (7)$$

Since $\phi_j(x)$ is a polynomial of degree (q+1), $w(x)$ in (6) is clearly of the required degree. Also $\phi_j(x_i) = \delta_{ij}$, where δ_{ij} is the Kronecker delta. Hence $w(x_j)$ has the value b_j at the point x_j, $j = 0, 1, \ldots, q+1$. Thus $w(x)$ in (6) possesses the required properties.

Before the weights can be calculated from (6) two further pieces of information are required: (i) the values b_j (j = 0, ..., q+1), and (ii) the (q+2) points x_j (j = 0, ..., q+1).

Turning to (ii) first. The Almon model assumes that $x_0 = -1$ and $x_{q+1} = n$, and that the value of $w(x)$ at these points is zero (i.e. $b_0 = b_{q+1} = 0$). Hence, from (5) we have

$$w(-1) = w(n) = 0$$

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1. G. Birkoff and S. MacLane, _A Survey of Modern Algebra_ (New York: Macmillan, 1953), p. 64.

i.e. zero weights before time 0 and after time n. In addition (6) becomes

$$w(x) = \sum_{j=1}^{q} \phi_j(x) \, b_j \qquad (8)$$

The remaining q points x_i are distributed arbitrarily over the interval [0, n].

Turn now to (i), the determination of the values b_i. b_0 and b_{q+1} have just been given values. To find the remaining q b_i's first substitute (8) into (3). Therefore

$$C_t = \sum_{i=0}^{n-1} \sum_{j=1}^{q} \phi_j(i) \, b_j \, X_{t-i}$$

$$= \sum_{j=1}^{q} b_j \, [\sum_{i=0}^{n-1} \phi_j(i) \, X_{t-i}]$$

$$\therefore \qquad C_t = \sum_{j=1}^{q} b_j \, A_{tj}$$

where $A_{tj} = \sum_{i=0}^{n-1} \phi_j(i) \, X_{t-i}$. Now each A_{tj} is just a certain linear combination of the X's. Hence, observations on the A_{tj}'s can be built up. Estimation of the b_j's can then be made by a least squares regression of C_t on to the q Almon variables A_{tj}.

Once the b_j are estimated the weights are given by

$$w_i = w(i) \qquad\qquad i = 0, 1, \ldots, n-1$$

where from (8)

$$w(i) = \sum_{j=1}^{q} \phi_j(i) \, b_j$$

The estimates of w_i based upon the Almon model are efficient relative to those based upon a least squares fit of (3)[1] so long as $n > q$.

This exposition has been carried out for the case of a single lagged variable. The method extends in an obvious way to handle the situation of more than one lagged variable and/or other unlagged variables. It should also be clear that the only decisions that have to be made by those working with the Almon method are the determination of n and q, i.e. the lag length and the degree of the polynomial generating the weights.

1. That is, the Almon estimator, like the least squares estimator based upon (3), is unbiased but has a smaller variance than the least squares estimator based upon (3). For a proof of this result see Dhrymes, Distributed Lags, pp. 225-226.

CONCLUSIONS

8.1 Introduction   In chapter one the aims of this thesis were
stated to be: (i) To provide a critical review of the principal
literature on the aggregate consumption function; (ii) To critically
review the work done on the Australian aggregate consumption function;
(iii) To make some contribution towards removing deficiencies in
Australian work, revealed by (i) and (ii), by obtaining new estimates
of the quarterly Australian aggregate consumption function. We
pursued our first aim in chapters two to five, while aims (ii) and
(iii) were covered in chapters six and seven respectively. It is the
purpose of this final chapter to attempt to draw up some conclusions
relating to these three aims. These conclusions are presented in
section 8.3 below, after a discussion in section 8.2 of the
limitations of our work. Finally, in section 8.4 some suggestions for
further work are advanced.

8.2 Limitations   In the case of aim (i) the limitations on our
work arise from the necessary incompleteness of our review. The
literature on the aggregate consumption function is so extensive that
only a highly selective review was possible. In making our selection
from the literature we aimed to find, and discuss, those ideas which
seemed the most important, either from the point of view of popularity
or empirical success. A selection of work employing these ideas was
also made. Thus, for example, we began chapter three with a discussion
of the original work by Friedman on the Permanent Income Hypothesis

and concluded with a discussion of the Stone-Rowe, and Zellner-Huang-Chau work which employed **PIH** ideas. The selective nature of our review will have been a **limiting factor**, therefore, if important ideas have been overlooked, or if important empirical work employing these ideas has been omitted. The same problem was not encountered in the case of aim (ii). The Australian literature on the consumption function is far from extensive and it was decided to discuss all that was available.

In the case of our third aim, to provide estimates of various short-run consumption functions using Australian data, the limitations were of a statistical nature. Firstly our conclusions are limited by the fact that the estimation method used was SELS. Therefore, as mentioned above,[1] the resulting parameter estimates will suffer from simultaneity bias and in some cases from distributed lag bias. An attempt was made to check the results obtained by using the estimators put forward by Liviatan and Dhrymes, Klein and Steiglitz, but generally the attempt was unsuccessful. A further problem experienced was associated with the lack of certain data, in particular data on consumer attitudes. Actual observations on non-farm disposable income could not be obtained, and a proxy variable ($\frac{YNF}{Y}$)X was used instead.

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1. See section 7.3.2 above.

8.3 <u>Conclusions</u>   With the above limitations in mind the following conclusions relating to our <u>first aim</u> are offered:

1.   Firstly, it is clear that we cannot speak of <u>the</u> consumption function very meaningfully except as a general term covering a wide variety of hypotheses. The original Keynesian formulation, C = a + bX, has been recognized as unsatisfactory for a long time. However, none of the competing hypotheses that have subsequently been put forward can be said to have proved itself over its rivals.

2.   The early approach of investigators of the consumption function was to add to aggregate income various lists of other macro-economic variables and then to estimate the parameters of the resulting equation. This approach has been abandoned to some extent, and attempts to derive an aggregate function often begin with an examination of the behaviour of the individual consumer. The most obvious examples of this approach are the Permanent Income and Life Cycle Hypotheses.

3.   There is an increasing tendency to derive the consumption function by fitting together several other macro relationships. The most obvious example of this approach is the work of Stone and Rowe or Zellner, Huang and Chau discussed in chapter three. This approach has the benefit of facilitating the formation of a priori expectations about the sign and magnitude of parameter estimates. If the estimate of the resulting consumption function is unsatisfactory the job of tracing the source of the trouble is also made more easy.[1]

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1.   See p. 249 above for example.

4. There has been increasing attention paid to the dynamic structure of various lagged variables. There has been frequent use of the geometric lag model (for example by Stone and Rowe, Zellner, Huang and Chau, and Griliches et. al.[1]), and one example of the Almon variable lag model has been discussed.[2]

5. The statistical problems associated with least squares estimation have been recognized and tackled by many researchers. Two and three stage least squares, limited-and full-information maximum likelihood methods have all been employed at one time or another. Methods for handling autocorrelated disturbances have been used by Zellner, Huang and Chau.[3]

Our conclusions concerning the Australian work examined are:

1. Firstly the amount of work done is very small. Only six single equation studies of the aggregate consumption function could be found,[4] one of which had as its primary aim an investigation of the effectiveness of fiscal tax policy. Only two of these used quarterly data. In neither case was constant

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1. See p. 88 , p. 77, and p. 43 above respectively.

2. See p.129 above.

3. See p. 84 above.

4. See section 6.2 above.

price data used. The remaining studies examined[1] were part
of some model building project and involved the use of
partially disaggregated consumption functions.

2. The only specifically Australian idea in the work examined
is that advanced by Arndt and Cameron which suggests that
in the Australian context the appropriate income variable for
the basic Keynesian function is non-farm disposable income
(XNF), giving $C = a + b$ XNF. While this can hardly be termed
a very radical idea, it was adopted for most early estimates
of the Australian consumption function.[2] Our conclusion on
this idea is that the empirical evidence does not clearly
establish the superiority of this variable, and that the
formulation $C = a + b$ XNF is implausible. An alternative
formulation was offered for later testing.

3. Overseas ideas such as the PIH and the LCH did not receive
any attention until the last few years. As yet only one
quarterly constant price study reporting on the testing of
these ideas has been published.

4. The "statistical" quality of the Australian work has improved
over the years with the latest quarterly estimates being
accompanied by t-ratios, $\bar{R}^2$, the standard error of estimate
and the Durbin-Watson statistic. The estimates have usually
been obtained by SELS, but estimates using 2SLS and 3SLS have
also been published.

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1. See sections 6.3 and 6.4 above.
2. Those by Arndt-Cameron, Leser, Nevile (1962), Kmenta, Duloy, and, finally, Cameron (1967).

With regard to the _new estimates of the Australian_ _consumption function_ presented in chapter seven, the following conclusions apply:

1. In general, liquid assets does not seem to be an important variable, especially for regressions involving CND as dependent variable. On the other hand total personal wealth appeared in several places with a strongly significant coefficient, including the estimate judged to be best when total consumption (C) was the dependent variable.

2. The estimates of the reformulated Arndt-Cameron function were not completely satisfactory and it was not possible to come to a specific conclusion about the worth of the non-farm disposable income variable.

3. Some success was achieved with the government policy variable $(G - \bar{G})$, although it did not figure in the final equations chosen. On the other hand the consumer attitudes index A was uniformly unsuccessful.

4. The preferred equations were:

$$C = \underset{(2.35)}{127.136} - \underset{(-5.79)}{362.354}\ Q_1 - \underset{(-1.17)}{44.136}\ Q_2 - \underset{(-3.49}{138.824}\ Q_3$$

$$+ \underset{(3.20)}{0.204}\ X + \underset{(1.64)}{0.007}\ W_{-1} + \underset{(6.22)}{0.655}\ C_{-1} \qquad (8.1)$$

$$\bar{R}^2 = 0.994,\ \overline{SE} = \$31.54m.,\ h = 1.23$$

$$CND = \underset{(3.22)}{138.459} - \underset{(-8.17)}{360.630}\ Q_1 - \underset{(-1.78)}{52.578}\ Q_2 - \underset{(-5.34)}{167.014}\ Q_3$$

$$+ \underset{(1.98)}{0.096}\ X + \underset{(14.52)}{0.890}\ CND_{-1} \qquad (8.2)$$

$$\bar{R}^2 = 0.996,\ \overline{SE} = \$24.56m.,\ h = -0.46$$

The long run marginal propensity to consume implied by (8.1)
is 0.591, and by (8.2) is 0.873. Both results can be taken
to imply some support for the PIH.[1]

8.4 Suggestions for further work   One area of work involves the
construction of special series of data. For example, tests of
certain aspects of the PIH require that a series for consumption of
single-use goods and services plus the use value of stocks of
consumer durables be available. At the moment no such series exists
for Australia. A test of the Life Cycle Hypothesis for Australia
would also be possible if a series for labour income was derived.
A third set of series which might be constructed is one for factor
disposable incomes. The single income variable has been disaggregated
in this way in many overseas studies.[2] A fourth possibility is a
series for consumer attitudes.

A second obvious area for further work lies in the use of
new estimation methods. For example, to properly test H(7) in the
previous chapter, some non-linear estimation technique is needed.
There are also a large number of methods for estimating distributed
lags, which have not yet been tried with Australian data.[3] A further

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1. See pp.222-4 above for the derivation of the form of (8.1) and
   pp.218-9 above for the derivation of the form of (8.2)

2. See, for example, the estimate by Klein and Goldberger, Table 2.1
   p. 22   above.

3. See for example L. R. Klein, "The estimation of distributed
   lags," Econometrica, Vol. 26 (1958), pp. 553-565.

possibility lies in the use of new methods for introducing lags into the consumption function. With the exception of the work presented in the previous chapter utilising the Almon variable method, the geometric lag structure has been the only lag generating device used in Australian work. Another well known possibility is the rational distribution lag structure.[1]

To adequately test the PIH, alternative estimators of permanent income should be tried. In chapter seven above, permanent income was estimated by a weighted average of current and past values of actual income with weights determined either by a simple geometric lag structure with weights summing to unity, or by the Almon method with the sum of the weights determined by the data. Other possibilities are: (i) The geometric lag structure used by Zellner, Huang and Chau, with weights summing to more than unity;[2] (ii) Jorgenson's rational distributed lag; (iii) Friedman's estimate of permanent income;[3] (iv) Crockett's estimate found by fitting a semi-log trend curve to data on measured income.[4]

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1. See D. W. Jorgenson, "Rational distributed lag functions," Econometrica, Vol. 34 (1966), pp. 135-149.

2. See H(3), p. 79 above.

3. See (3.13) p. 63 above. A similar method has been used in a Reserve Bank study with no apparent success. (See Norton and Broadbent, Equations for Personal Consumption Expenditure, pp. 10-11).

4. See p. 71 above.

It may also be possible to utilize in some way the cross-sectional data being generated by the Australia-wide survey of consumer finances and expenditure carried out by Drane, Edwards and Gates.[1] An example of the use of cross-section data in a time series study by Klein and Goldberger has been discussed above.[2]

A final suggestion is that a study similar to that undertaken by Ferber be done using Australian data. This study would involve the estimation of a number of the better known hypotheses about the aggregate consumption function, and the subjection of the estimates to various statistical tests. One question that could be tackled within such a study concerns the relative merits of the variable non-farm disposable income (XNF) over total personal disposable income (X). By estimating all hypotheses, first using XNF and then using X, some judgement as to the relative performance of the two variables may be possible.

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1. See H. R. Edwards and R. C. Gates, "The Sydney survey of consumer finances" (paper presented to the 38th Congress of ANZAAS in Hobart, August, 1965) for a preliminary report on this project.

2. See p. 23 above.

DATA APPENDIX

I. Variables Used

(1) C = Total personal consumption expenditure ($m.)

     [Statistical source = (3)]

(2) CND = Consumption expenditure on non-durable goods and

     services ($m.) = C minus (expenditure on (i) electrical

     goods, (ii) other household durables, (iii) purchase of

     motor vehicles). [(3)]

(3) Q_1

(4) Q_2 = Seasonal dummies (Q_i = 1 in quarter i and = 0 at other

(5) Q_3 times).

(6) X = Personal disposable income ($m.) [(3)]

(7) L^1_{-1} = Notes and coin in the hands of public plus current

     deposits of the public with all trading banks, at the

     end of the previous period ($m.). [(8)]

(8) L^2_{-1} = L^1_{-1} plus fixed deposits of the public with all trading

     banks plus savings bank deposits, at the end of the

     previous period ($m.). [(8)]

(9) W_{-1} = Total private sector wealth at the end of the previous

     period ($m.). [(7)]

(10) X^0 = Highest previous value of X ($m.).

(11) Y = Total personal income ($m.) [(3)]

(12) YNF = Non-farm personal income = Y minus income of farm

     unincorporated enterprises ($m.). [(3)]

(13) A = Consumer attitudes index, based upon:

      (i) The consumer price index [(2), No. 53 and (4)].

      (ii) The total balances outstanding for instalment

          credit for retail sales (all businesses). [(1)]

      (iii)Total registered unemployed. [(4)]

      (iv) Average weekly earnings per employed male unit

          [(2), No. 50 and (5)].

(14) $G-\bar{G}$ = Government policy variable based upon the rate of sales

      tax on motor vehicles (STMV). [(6)]

(15) cpi = The consumer price index (base: 1966-67 = 100).

      [(2), No. 53 and (4)]

II. Statistical Sources

(1) Commonwealth Bureau of Census and Statistics, Instalment
          Credit for Retail Sales.

(2) _____ Labour Report.

(3) _____ Quarterly Estimates of National Income and
          Expenditure.

(4) _____ Quarterly Summary of Australian Statistics.

(5) _____ Wage Rates and Earnings.

(6) Commonwealth of Australia, Sales Tax (Assessment) Act (1930-1970),
          and Sales Tax (Exemptions and Classification) Act
          (1935-1970).

(7) J. Helliwell, et. al. Quarterly Estimates of Private Sector
          Wealth. Reserve Bank of Australia, Research Discussion
          Paper No. 18, 1971.

(8) Reserve Bank of Australia, Statistical Bulletin.

III. Data Used

	(1)	(2)	(3)	(4)	(5)
	C	CND	O_1	O_2	O_3
1958:4					
1959:1	1925	1689			
1959:2	2037	1781	0	1	0
1959:3	2074	1781	0	0	1
1959:4	2318	1971	0	0	0
1960:1	2137	1841	1	0	0
1960:2	2273	1960	0	1	0
1960:3	2293	1956	0	0	1
1960:4	2495	2125	0	0	0
1961:1	2230	1956	1	0	0
1961:2	2311	2040	0	1	0
1961:3	2313	2034	0	0	1
1961:4	2519	2192	0	0	0
1962:1	2329	2037	1	0	0
1962:2	2464	2148	0	1	0
1962:3	2471	2121	0	0	1
1962:4	2724	2333	0	0	0
1963:1	2491	2162	1	0	0
1963:2	2623	2285	0	1	0
1963:3	2684	2297	0	0	1
1963:4	2931	2504	0	0	0
1964:1	2663	2312	1	0	0
1964:2	2839	2451	0	1	0
1964:3	2888	2475	0	0	1
1964:4	3183	2721	0	0	0
1965:1	2886	2496	1	0	0
1965:2	3052	2641	0	1	0
1965:3	3092	2680	0	0	1
1965:4	3363	2924	0	0	0
1966:1	3047	2679	1	0	0
1966:2	3228	2836	0	1	0
1966:3	3280	2875	0	0	1
1966:4	3609	3138	0	0	0

	(1)	(2)	(3)	(4)	(5)
	C	CND	Q_1	Q_2	Q_3
1967:1	3301	2912	1	0	0
1967:2	3486	3068	0	1	0
1967:3	3563	3119	0	0	1
1967:4	3918	3384	0	0	0
1968:1	3606	3162	1	0	0
1968:2	3779	3301	0	1	0
1968:3	3789	3322	0	0	1
1968:4	4198	3635	0	0	0
1969:1	3880	3391	1	0	0
1969:2	4092	3574	0	1	0
1969:3	4143	3597	0	0	1
1969:4	4604	3972	0	0	0
1970:1	4229	3703	1	0	0

	(6)	(7)	(8)	(9)	(10)
	X	L_{-1}^1	L_{-1}^2	W_{-1}	X^0
1958:4	2549				
1959:1	2076				2549
1959:2	2098	3393	6728	38826	2549
1959:3	2333	3274	6676	39802	2549
1959:4	2715	3342	6866	41136	2549
1960:1	2242	3577	7148	42484	2715
1960:2	2362	3639	7238	43096	2715
1960:3	2441	3521	7192	43896	2715
1960:4	2939	3470	7241	44506	2715
1961:1	2473	3551	7401	43955	2939
1961:2	2439	3467	7401	45061	2939
1961:3	2550	3293	7301	46112	2939
1961:4	3001	3248	7439	46166	2939
1962:1	2511	3457	7753	46821	3001
1962:2	2604	3518	7915	47603	3001
1962:3	2675	3379	7845	47181	3001
1962:4	3241	3332	8026	47386	3001
1963:1	2685	3525	8367	48607	3241
1963:2	2764	3587	8542	49824	3241
1963:3	2961	3464	8517	50288	3241
1963:4	3579	3493	8749	51550	3241
1964:1	2950	3783	9242	52781	3579
1964:2	3049	3942	9541	54425	3579
1964:3	3187	3732	9556	54903	3579
1964:4	3764	3760	9880	55720	3579
1965:1	3266	3991	10313	57170	3764
1965:2	3283	4034	10488	56872	3764
1965:3	3400	3791	10337	56727	3764
1965:4	3885	3689	10458	57608	3764
1966:1	3344	3907	10846	59544	3885
1966:2	3450	3933	11000	60210	3885
1966:3	3601	3816	10938	60526	3885
1966:4	4247	3881	11218	61293	3885

	(6)	(7)	(8)	(9)	(10)
	X	L^1_{-1}	L^2_{-1}	W_{-1}	X^0
1967:1	3767	4203	11686	62833	4247
1967:2	3738	4291	11895	63419	4247
1967:3	3780	4074	11824	65603	4247
1967:4	4487	4113	12155	68438	4247
1968:1	3763	4495	12739	70310	4487
1968:2	4016	4507	12824	71824	4487
1968:3	4009	4414	12805	75484	4487
1968:4	5009	4421	13096	74825	4487
1969:1	4281	4752	13635	78076	5009
1969:2	4462	4885	13930	78959	
1969:3	4553	4750	13974	77684	
1969:4	5281	4756	14267	78896	
1970:1	4762	5205	14934	81368	

	(11)	(12)	(13)	(14)	(15)
	Y	YNF	A	G – $\bar{\text{G}}$	cpi
1958:4					83.5
1959:1	2274	2033			83.8
1959:2	2304	2268	2.42	0.0	84.2
1959:3	2566	2295	1.78	0.0	84.5
1959:4	2998	2527	2.97	0.0	85.0
1960:1	2465	2281	2.02	0.0	85.8
1960:2	2593	2566	5.39	0.0	87.3
1960:3	2699	2521	3.22	0.0	88.3
1960:4	3260	2770	2.99	5.0	88.9
1961:1	2734	2441	0.45	4.5	89.5
1961:2	2703	2686	2.08	-2.7	90.1
1961:3	2811	2581	-0.42	-2.7	89.9
1961:4	3321	2850	0.27	-2.7	89.6
1962:1	2767	2560	-1.08	-5.9	89.4
1962:2	2866	2861	1.08	-6.4	89.4
1962:3	2954	2721	0.79	-4.5	89.6
1962:4	3591	3019	1.63	-2.7	89.7
1963:1	2964	2705	-0.45	-0.8	89.7
1963:2	3049	3017	1.89	0	90.0
1963:3	3279	2941	1.10	0	90.2
1963:4	3978	3324	1.46	0	90.1
1964:1	3270	2942	0.75	0	90.7
1964:2	3372	3318	3.04	0	91.5
1964:3	3559	3282	3.39	1.4	92.6
1964:4	4219	3634	4.07	2.1	93.7
1965:1	3649	3266	0.74	1.5	94.3
1965:2	3672	3645	3.12	0.9	95.2
1965:3	3814	3598	2.81	0.3	96.2
1965:4	4396	3846	3.57	0.0	97.4
1966:1	3764	3503	-0.39	0.0	97.6
1966:2	3886	3867	2.52	0.0	98.4
1966:3	4069	3854	1.41	0.0	98.8
1966:4	4832	4161	3.05	0.0	99.7

	(11)	(12)	(13)	(14)	(15)
	Y	YNF	A	$G - \bar{G}$	cpi
1967:1	4264	3822	0.19	0.0	100.1
1967:2	4220	4214	3.77	0.0	101.3
1967:3	4298	4153	3.63	0.0	102.7
1967:4	5160	4559	1.94	0.0	103.0
1968:1	4305	4189	0.61	0.0	103.4
1968:2	4606	4642	3.03	0.0	104.2
1968:3	4721	4531	1.38	0.0	104.6
1968:4	5791	5029	4.13	0.0	105.7
1969:1	4931	4624	1.17	0.0	106.4
1969:2				0.0	107.2
1969:3				0.0	107.8
1969:4				0.0	108.7
1970:1				0.0	109.8

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