Realized Skewness and Kurtosis in Asset Markets

Richard Mawulawoe Ahadzie
(MSc, BSc)

Tasmanian School of Economics and Business

University of Tasmania

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Declaration of Originality

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Richard Mawulawoe Ahadzie
TSBE, University of Tasmania
October 11, 2020

Statement of Authority of Access

The chapters in this thesis represent collaborative efforts with my supervisors. Chapter 2 is based on joint work with Dr. Nagaratnam Jeyasreedharan and the late Prof. Mardi Dungey, which was later co-authored with Dr. Nagaratnam Jeyasreedharan and submitted to '*The Quarterly Review of Economics and Finance*'. Chapter 3 is co-authored with Dr. Nagaratnam Jeyasreedharan and published in '*Quantitative Finance*'. Chapter 4 is co-authored with Dr. Nagaratnam Jeyasreedharan and published in '*Journal of Behavioral and Experimental Finance*'. Chapter 5 is co-authored with Dr. Nagaratnam Jeyasreedharan and submitted to '*Global Finance Journal*'.

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Richard Mawulawoe Ahadzie

TSBE, University of Tasmania

October 11, 2020

Statement of Co-Authorship

The following people and institutions contributed to the published or submitted and under review manuscripts undertaken as part of this thesis:

Candidate – RICHARD MAWULAWOE AHADZIE

Tasmanian School of Business and Economics (TSBE), University of Tasmania.

Co-Author – NAGARATNAM JEYASREEDHARAN

Tasmanian School of Business and Economics (TSBE), University of Tasmania.

Contribution of work by co-authors for each paper:

PAPER 1: Located in Chapter 2

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Author contributions:

Formal analysis, Investigation, Software, Resources, Methodology, Data Analysis and Wrote the manuscript: Candidate; Validation, Visualization, Conceptualization and Review & Editing the manuscript: Candidate, Co-Author; Supervision: Co-Author

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Author contributions:

Formal analysis, Investigation, Software, Resources, Methodology, Data Analysis and Wrote the manuscript: Candidate; Validation, Visualization, Conceptualization and Review & Editing the manuscript: Candidate, Co-Author; Supervision: Co-Author

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Author contributions:

Formal analysis, Investigation, Software, Resources, Methodology, Data Analysis and Wrote the manuscript: Candidate; Validation, Visualization, Conceptualization and Review & Editing the manuscript: Candidate, Co-Author; Supervision: Co-Author

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Author contributions:

Formal analysis, Investigation, Software, Resources, Methodology, Data Analysis and Wrote the manuscript: Candidate; Validation, Visualization, Conceptualization and Review & Editing the manuscript: Candidate, Co-Author; Supervision: Co-Author

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We, the undersigned, endorse the above stated contribution of work undertaken for each of the published or submitted and under review manuscripts contributing to this thesis:

Richard Mawulawoe Ahadzie
TSBE, University of Tasmania
October 11, 2020

Dr Nagaratnam Jeyasreedharan
TSBE, University of Tasmania
October 11, 2020

Dr Satoshi Yamazaki
TSBE, University of Tasmania
October 11, 2020

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Richard Mawulawoe Ahadzie

TSBE, University of Tasmania

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Abstract

The recent advent of high-frequency data has given rise to the notion of realized skewness and realized kurtosis. Unlike sample skewness and sample kurtosis which is normally computed from long samples of low-frequency return series (daily, weekly, monthly return series, and so on), realized skewness and realized kurtosis is computed from high-frequency return series (1-second, 1-minute, 5-minute return series, and so on). The relevance of high-frequency return data has been extensively documented in the extant financial literature. Researchers have shown that with high-frequency return data, realized variance converges to the sample variance and is an efficient estimator of the quadratic variation. However, realized skewness and realized kurtosis do not converge to the sample skewness and sample kurtosis values. This is because the second realized moments depend on both the diffusion and jump components of the observed price, whereas, the third and fourth realized moments depend exclusively on just the jump component. This implies that information embedded in realized skewness and realized kurtosis is different from that of sample skewness and sample kurtosis.

This thesis contributes to the body of literature by adopting, deducing, and following various theoretical methodologies, simulation techniques and empirical procedures to offer a new perspective, which cautions researchers to be observant of the optimal sampling frequency for their country of investigation when using high-frequency return series. Primarily, researchers should also be aware of the effects of sampling-interval and holding-intervals on the estimated realized skewness and realized kurtosis and its implication to high-frequency finance. Researchers need

to be critical of the type of volume used for information flow, and its relationship with realized skewness and realized kurtosis. Finally, it is important to note the significance of high-order moment pricing models in capturing the cross-section of asset returns under various market conditions (upturn and downturn markets) and sample-periods (pre-crisis, crisis, and post-crisis period).

The second chapter of this thesis investigates the optimal sampling frequency for computing realized variance for the DJI30 index and its component stocks, and also whether the obtained sampling frequency could be extended to the Australian framework. To the best of my knowledge, this study is the first to investigate the preferred sampling frequency with a focus on the Australian stocks, and not naively extending the 5-minutes rule of thumb from the US framework. Using 1-second (high-frequency) raw prices downloaded from Thomson Reuters Tick History/Securities Industry Research Centre of Asia-Pacific (TRTH/SIRCA) database from 2010 to 2015, this study computes daily RVs and find that the standard 5-minute interval for the US market holds, a '10-' to '30-minute' sampling frequency is the preferred interval for the Australian framework.

The third chapter of this thesis investigates theoretically and empirically, how realized skewness and realized kurtosis are affected by holding-interval and sampling-interval. In particular, before any computations of realized skewness and realized kurtosis are carried out, one often predetermines the holding-interval and sampling-interval and thus implicitly influencing the actual magnitudes of the computed values of the realized skewness and realized kurtosis (i.e. they have been found to be interval-variant). To-date, little theoretical or empirical studies have been undertaken in the high-frequency finance literature to properly investigate and understand the effects of these two types of intervalings on the behaviour of the ensuring measures of realized skewness and realized kurtosis. This chapter fills this gap by theoretically and empirically analyzing as to why and how realized skewness and realized kurtosis of market returns are influenced by the selected holding-interval and sampling-interval. Using simulated and price index data from the G7 countries, this study then proceeds to illustrate via count-based signature plots, the theoretical and empirical relationships between the realized skewness and realized kurtosis and the sampling-intervals and holding-intervals.

The fourth chapter of this thesis investigates empirically volume-higher order moment relationship by employing various proxies of information flow. The relationship between volume and realized volatility has been extensively documented in the extant financial literature. However, minimal attention has been accorded to volume-realized skewness and volume-realized kurtosis relationships. The insight in this chapter is that these additional higher-order realized moments hold volume-dependent relationships that have been neglected. The empirical analysis employs 142 Australian stocks from 2003 to 2017 downloaded at 15-minute sampling-intervals from the TRTH/SIRCA database and compute their weekly and monthly realized high-order moments. It is found that the volume proxy influences the signage of the ensuring volume-higher-order realized moment regression coefficients. This study then attempts to explain the empirical findings via three common volume-related hypotheses cited in the extant volume literature and conclude that the DOH (Difference of Opinion) hypothesis implicitly encompasses or nests both the SIAH (Sequential Information Arrival) and MDH (Mixture of Distribution) hypotheses. These two subtle but significant findings have yet to be reported in extant volume or trading-related studies.

The fifth chapter follows a set of methodologies documented in the extant literature for investigating the higher-order co-moment risk-return relationship for the Australian stock market. Using 142 stocks from 2003 to 2017 downloaded from TRTH/SIRCA database. For this study, monthly realized return and monthly higher-order co-moment estimates are computed from 15-minute series. The high-frequency return data will ensure robust estimates for the empirical analysis. The empirical results show that the average return for standard beta and kappa risks are asymmetric and diametrically opposite in upmarket and downmarket periods, while gamma risks yield significant gains to the investor regardless of the market condition (the results are consistent for the three methodologies considered: (i) the single sorting of excess return on risk measures, (ii) double sorting of excess returns on risk measures, and (*iii*) the Fama-MacBeth cross-sectional regression). Additional results from the Fama-MacBeth cross-sectional regression shows that gamma and kappa risk factors remain priced, even in the presence of continuous beta and jump beta. It is found that not only the normalized co-variance risk factor is important in asset pricing but also normalized co-skewness and co-kurtosis risk factors are also priced separately. This study further splits the full-sample data into sub-periods and observes that the level of significance of the risk premium varies across the sub-periods. The results contribute to the debate on whether systematic realized higher-order co-moments can explain the cross-sectional Australian stock returns.

To conclude, this thesis brings to light some research questions and answers related to realized skewness and realized kurtosis that are yet to be considered in the existing high-frequency finance literature and hence contributes to the body of knowledge in field of finance.

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Chapter 1

Introduction

The multi-moment capital asset pricing model (CAPM) has received considerable attention in the existing financial literature (see Adcock, 2010; De Athayde and Flôres Jr, 2004; Jondeau and Rockinger, 2003, etc.). Several researchers have shown that the notion of asset return being normally distributed should be relaxed because asset returns are affected by skewness and kurtosis (see Greene and Fielitz, 1977; Cowan and Sergeant, 2001; Chen et al., 2001; Chung et al., 2006; Desmoulins-Lebeault, 2012, etc.). Such arguments hold that financial models like CAPM, which depend on the normality of the asset returns may neglect relevant risk components.[1] As such, the standard CAPM does not converge to the total higher-order systematic co-moment risk in the marketplace, and it may, therefore, represent an imperfect measure of risk.[2] Rubinstein (1973); Ingersoll (1975);

[1]The standard CAPM, which was proposed by Sharpe (1964); Lintner (1975); Mossin (1966), can be regarded as one of the most important theories in modern finance. The CAPM suggests that the expected return of an asset depends entirely on the asset's systematic risk and that for this reason, the only relevant metric is co-variance risk. The model implies a linear relationship between expected returns and systematic risk (beta). For this relationship to hold, the CAPM places strong restrictions on (i) the asset return distribution (assuming Gaussian distribution) and (ii) the agent's utility function (by employing the quadratic utility function), which does not correspond to rational agent behavioural characteristics. The assumption of Gaussian distribution is normally made for reasons of convenience in theoretical models; however, it is less likely to hold in the high-frequency paradigm.

[2]Fama and French (1992, 1995); Carhart (1997) has shown that fundamental variables, such as size, value and momentum, can explain the cross-sectional variation in expected returns. Ang et al. (2006) shows that asymmetric betas explain the cross-section of asset returns, while Amihud (2002); Pástor and Stambaugh (2003); Acharya and Pedersen (2005) show that liquidity risk

Kraus and Litzenberger (1975) were the first to consider relaxing the normality assumption of the CAPM, whereby several aspects of the model fall short in the context of realistic settings. The authors incorporated a higher-order moment into the pricing model by considering the unconditional asymmetric characteristics of asset return distributions. Kraus and Litzenberger (1975) shows the significance of positive skewness to investors even in circumstances in which risk aversion did not increase. Consequently, researchers and practitioners have focused on implementing the higher-order pricing model as an appropriate tool for capturing systematic co-moment risk (see Harvey and Siddique, 2000a; Dittmar, 2002; Lambert and Hübner, 2013; Poti and Wang, 2010; Moreno and Rodríguez, 2009; Kostakis et al., 2012, for more details).[3]

Harvey and Siddique (2000a) outlines the economic significance of incorporating the co-skewness risk factor into the two-moment CAPM. Barone Adesi et al. (2004) shows that accounting for co-skewness causes the three-factor model of Fama and French (1993) to lose its predictive power. Chung et al. (2006) suggests that higher-order co-moments are relevant to risk-averse investors who are wary of 'extreme events'. The same authors also show that for low-frequency return data, the Fama and French (1993) risk factors converge to higher-order co-moment risk measures. Vanden (2006) suggests that the three-factor model of Fama and French (1993) are an imperfect proxy of co-skewness risk. Consequently, Smith (2007) shows the predictive power of co-skewness risk factor over the conditional two-moment CAPM and the conditional Fama and French (1993) three-factor model. Brunnermeier et al. (2007); Barberis and Huang (2008) shows that a security's skewness

factors explain the cross-section of asset returns. Roll and Ross (1994); Kandel et al. (1995) show that even minor deviations in efficiency could lead to an insignificant risk-return relationship. Breen and Korajczyk (1993); Kothari et al. (1995) states that the presence of survivorship bias in empirical data could lead to measurement errors during asset pricing testing. Amihud et al. (1992); Kim (1995) find that the predictive power of the empirical asset pricing test is conditional on the effects of the errors-in-variables problem.

[3]Fang and Lai (1997); Dittmar (2002) were the first to present a pricing model framework that accounted for both the asymmetric and leptokurtic characteristics of asset return distributions. Prior research has shown that high-order moment risk premiums are priced separately across various stock markets, market conditions and sample periods. Such heterogeneity indicates that the results from one particular market may not apply to other markets. This finding forms the motivation for the empirical investigations undertaken in this thesis.

could also impact on investors' decisions concerning their portfolio. This is in line with the theory of cumulative prospect and consistent with the empirical findings of (Mitton and Vorkink, 2007; Boyer et al., 2010).[4] Scott and Horvath (1980) shows that rational investors are averse to negative skewness and excess kurtosis. Similarly, Ranaldo and Favre (2005); Liow and Chan (2005); Sharpe (1964); Jurczenko and Maillet (2006) note that investors display a preference for odd moments (i.e., mean and skewness) and an aversion to even moments (i.e., variance and kurtosis). Investors also tend to display a preference for positive skewness and an aversion to negative skewness (Do et al., 2014). Jurczenko and Maillet (2006) shows that investors must consider not only the mean-variance decision criterion of their portfolios but also the mean-variance-skewness-kurtosis decision criterion. This is indicative of the importance of skewness and kurtosis in asset allocation and asset pricing models. This mean-variance-skewness-kurtosis decision criterion is achieved through the Taylor series expansion of the utility function, which is contrary to the quadratic utility function of the mean-variance space proposed by Markowitz and Todd (2000). Campbell et al. (2002) shows that the quadratic utility function does not accurately depict investors' preferences, while Jurczenko et al. (2005) proposes a nonparametric portfolio optimization criterion for the portfolio selection problem in the mean-variance-skewness-kurtosis space. The same authors constructed a general approach for obtaining an efficient portfolio in a nonconvex mean-variance-skewness-kurtosis framework that outperforms the mean-variance approach. Also, Sears and Wei (1988) shows that ignoring co-skewness risk may bias the results obtained when investigating the risk-return relationship trade-off.

Skewness and kurtosis play a key role in various fields of finance (i.e. asset pricing, portfolio management, derivatives market, mutual funds, etc.) and can offer new perspectives on the way data is approached in empirical research, as well as

[4] Barberis and Huang (2008) argue that the positive skewness of a firm could be overpriced and could, therefore, decrease the expected returns. Elsewhere, using a model of optimal beliefs, Brunnermeier et al. (2007) shows that investors are attracted to highly right-skewed assets that tend to have lower subsequent expected returns. However, the authors highlight the limitations of their model when the rationality of the investor is questioned. Mitton and Vorkink (2007) modelled a rational investor's heterogeneous preference for skewness and show that idiosyncratic skewness significantly impacts marketplace prices.

help form new theories that rely on the distributional properties of return data. In the context of derivatives pricing, Corrado and Su (1996, 1997); Brown and Robinson (2002) employ skewness and kurtosis to relax the restrictive Gaussian distribution assumption of the popular Black and Scholes (1973) option pricing model. Xing et al. (2010) find a positive relationship between option-implied skewness and future returns, while Conrad et al. (2013) finds a negative relationship between option-implied skewness and future returns. Additionally, Conrad et al. (2013) finds a strong relationship between individual firms' risk-neutral volatility, skewness and kurtosis to future returns. Chang et al. (2013) shows that market skewness risk premium is priced in the presence of other common risk factors, including market excess returns, volatility, size, value and momentum. In relation to mutual funds, Klemkosky (1973); Ang and Chua (1979) show that ignoring the third moment of the return distribution in performance evaluation bias the conclusions drawn, which could impact investors directly in their portfolio creation and asset allocation strategies. Similarly, Prakash and Bear (1986); Leland (1999) also developed performance measures by incorporating skewness. Stephens and Proffitt (1991) went one step further to generalize the performance measure to incorporate any number of higher-order moments. The authors show that ignoring higher-order moments could impact a fund's performance ranking test. Moreno and Rodríguez (2006) argues that co-skewness is priced, even when size, value and momentum risk factors are controlled for. Liu et al. (1992); Vines et al. (1994); Liow and Chan (2005); Lee et al. (2008) show the importance of co-skewness in the pricing of real estate. Christie-David and Chaudhry (2001) shows that the second, third and fourth moments are important in explaining returns in the futures market. The authors also investigate the relevance of co-skewness and co-kurtosis in explaining the return-generating process in futures markets. Knif et al. (2020) employs higher-order co-moments to characterize the returns of hedge fund indices. They find that co-skewness and co-kurtosis are priced at the tail distribution of returns, which implies that co-skewness and co-kurtosis risk measures capture significant tail risks.

High-frequency data have given rise to the notion of realized skewness and realized kurtosis. Unlike sample skewness and sample kurtosis, which are normally computed from long samples of low-frequency return series (e.g., daily, weekly and monthly return series), realized skewness and realized kurtosis are computed from high-frequency return series (e.g., 1-second, 1-minute and 5-minute return series, etc.).[5] The relevance of high-frequency return data has been discussed at length in the financial literature on the subject. It has been shown that with high-frequency return data, realized variance converges to sample variance and is an efficient estimator of quadratic variation. However, Amaya et al. (2015); Ahadzie and Jeyasreedharan (2020) show that the values of realized skewness and realized kurtosis do not converge to the values of sample skewness and sample kurtosis because the second realized moment depends on both the diffusion and jump components of the observed price, while the third and fourth realized moments depend exclusively on the jump component. This suggests that information embedded in the realized skewness and realized kurtosis differs from that of the sample skewness and sample kurtosis.

As information embedded in realized skewness and realized kurtosis differs from that of sample skewness and sample kurtosis, employing high-frequency return data could offer new perspectives on the following questions: (*i*) Does the optimal sampling frequency differ between the US and Australian equity markets? Could the 5-minute rule of thumb be extended from US markets to Australian markets? (*ii*) How do the sampling-interval and holding-interval affect the estimated realized

[5] Sample skewness is the third normalized higher-order moment, while sample kurtosis is the fourth normalized higher-order moment of the probability distribution. Sample skewness measures the asymmetry of a distribution, while sample kurtosis measures the heavy tails (i.e., elongation) of a distribution. A positive (negative) sample skewness indicates a distribution with an asymmetric tail extending towards more positive (negative) values, while sample kurtosis characterizes how steep or flat a return distribution is. The combination of sample skewness and sample kurtosis with mean and standard deviation are used to describe the overall shape of the probability distribution of a variable. The relevance of sample skewness and sample kurtosis has been extensively documented in the financial literature as it can help measure the full characteristics of the asset return behaviour (see Samuelson, 1975; Kirchler and Huber, 2007). Hwang and Satchell (1999) suggests that the skewness and kurtosis in asset returns could be due to non-stationarity or non-economic factors. Damodaran (1985) shows that the skewed distribution of asset returns mirrored investors' asymmetrical responses to good and bad news from firms. According to Rubinstein (1973); Ingersoll (1975); Kraus and Litzenberger (1975), the significance of the higher-order moment in asset pricing models cannot be ignored.

variance, realized skewness and realized kurtosis, and what are the implications for investors' trading strategies? (*iii*) Can the signals from information flow (trading volume) be explained by realized high-order moments as a means of observing the dynamics of this relationship across holding periods and sample periods? (*iv*) How should realized higher-order co-moment risk be measured and priced to capture the differences between systematic co-variance, co-skewness and co-kurtosis risks? Further, are the risk premia the same across different market conditions and sample periods? This thesis adopts various theoretical methodologies, simulation techniques and empirical procedures to address these questions.

In Chapter 2, we explore the optimal sampling frequencies for realized variance of US and Australian stocks and indices. While prior research focuses predominately on the US framework, in this study, an empirical approach that cautions against blindly extending the standard and preferred 5-minute sampling frequency of the US market to other markets (i.e., the Australian equity framework) is outlined. Unlike other developed countries, such as the US and UK, the Australian equity market is unique in both its size and characteristics (see Alles and Murray, 2017). This means that the Australian equity market should be specifically investigated, rather than assuming that certain rules and relationships that hold in US or UK translate linearly to the Australian market. Indeed, estimating realized variance from very high-frequency return series could bias the estimated realized variance. However, such a problem could be resolved by either sampling the return series at a finer sampling-interval (see Bandi and Russell, 2008; Bollerslev et al., 2008; Oomen, 2006; Aït-Sahalia et al., 2005; Lahaye et al., 2011; Hansen and Lunde, 2006; Andersen et al., 2003) or by employing any of the various error-reduction techniques documented in the extant financial literature (see Hansen and Lunde, 2004, 2003; Zhang et al., 2005; Ebens et al., 1999; Andersen et al., 2001). Although the 5-minute rule has been theoretically and empirically justified by Bandi and Russell (2008), the empirical results in the financial literature are conflicting. As suggested by Bandi and Russell (2008), when estimating realized variance with intra-day return series, it is necessary to sample at an interval at which autocorrelation is less of an issue to ensure that an unbiased estimate of realized volatility is obtained. The same authors also recommend that when sampling very illiquid stocks, a 15-minute interval may constitute a preferred sampling frequency for computing realized variance, which should be lowered for very high liquid stocks. Consequently, in this study, the optimal sampling frequency for Australian stocks is expected to differ significantly from the 5-minute sampling rule of thumb.

This study aims to determine whether the 5-minute optimal unbiased sampling frequency rule of thumb employed in the US framework for realized variance also holds for all DJI30 stocks and DJI30 index (US equity framework). We further determine whether this rule of thumb could be extended to S&P/ASX20 stocks and S&P/ASX20 index (Australian equity framework). We use 1-second raw prices to estimate the daily realized variance at different sampling-intervals. The frequency-based signature-plot of Andersen et al. (2000) is used to report the empirical results. The results showed that, for the US framework, both the DJI30 index and stocks realized variance can be sampled at 30-seconds; therefore, the 5-minute rule of thumb holds. In the case of S&P/ASX20 index and stocks, we observe that the 30-second and 5-minute rule of thumb of the US framework cannot be extended to the Australian market. We note a '10-' to '30-minute' sampling frequency is the preferred interval for the Australian framework. The present study contributes to the existing finance literature, which lacks detailed research on the preferred sampling frequency for the Australian equity market.

Chapter 3 tests the effects of the sampling-interval (e.g., 1-minute, 5-minute, 30-minute) and holding-interval (e.g., daily, weekly, monthly) on the estimated realized higher-order moments, and their implications to asset pricing. The chapter argues that unlike sample skewness and sample kurtosis, which is a function of the holding-interval over which returns are estimated (see Hawawini, 1980; Smith, 1978; Francis, 1975; Fogler and Radcliffe, 1974), realized skewness and realized kurtosis is a function of both the sampling-interval and the holding-interval (see Amaya et al., 2015; Mei et al., 2017). We investigate these two intervaling effects (i.e., the holding effect and the sampling effect) and expect differing effects on the estimated realized skewness and realized kurtosis. We show analytically, theoretically, and empirically, the relationship between realized skewness and realized kurtosis and the two types of intervals. To present and discuss the results we motive and justify a new type of signature-plot (i.e., the count-based signature-plot). Our empirical findings provide graphical evidence for the analytical findings of Amaya et al. (2015) that realized skewness and realized kurtosis do not converge to the sample skewness and the sample kurtosis. This chapter contributes to the body of literature as the previous literature on this subject focus on the effects of the holding-interval using low-frequency return data. We focus on both intervaling effects on realized skewness and realized kurtosis by employing high-frequency return data.

Further, Chapter 3 also determines the theoretical limits of the expected values of realized skewness and realized kurtosis for the high-frequency paradigm, and highlights the contribution of diffusion and the jump component. Using a Monte Carlo simulation of an assumed independently and identically distributed (iid) jump-diffusion price generation process, we estimate the realized skewness and realized kurtosis for three holding-intervals (i.e., days, weeks and months) and a number of sampling-intervals (i.e., 1-, 2-, 3-, 4-, 5-minute return series, and so on). We note that the features of these signature plots differ for diffusion processes and jump-diffusion processes. In the empirical testing, we use 1-minute market index data for the G7-countries. The results for the simulated estimates are comparable to the empirical results but are not identical. It is shown that the central limit theorem for realized skewness and realized kurtosis of high-frequency data only holds at the limit when jumps are few and far between or are implicitly assumed away, such as when the size of the sampling-interval approaches the holding-interval. For pure diffusion processes, we observe that all realized moments implicitly converge to their corresponding sample moments as asymptotically indicated by the limiting realized moment equations and simulations. However, the presence of jumps in the price series at high-frequency influences the estimated realized skewness and realized kurtosis, hence they do not converge to the sample moments. Such findings imply that high-frequency pricing models are conditional on the holding-interval and sampling-interval and that the relevance of high-frequency pricing models is driven by the direction and magnitude of jumps in the price process. This underscores why the predictive power of asset pricing models tends to vary across holding-intervals. In-short, the choice of sampling-interval and holding-interval affects the relevance of high-frequency pricing models and influence the conclusions that can be drawn.

Having obtained the preferred sampling-interval for the Australian equity market, Chapter 4 uses a 15-minute return series to investigate the relationship between information flow (trading volume) and realized higher-order moments. Although the trading volume-volatility relationship has been the subject of considerable attention in prior literature, research on the relationships between the volume-realized skewness and volume-realized kurtosis is scarce. Ideally, the arrival rate of information flow to the marketplace and its relationship with volatility can be highly beneficial to investors' trading strategies. This has motivated researchers to formulate theoretical hypotheses that aim to explain the volume-volatility relationship. The popular mixture of distribution hypothesis (MDH) of Clark (1973) was the first hypothesis proposed. It explained the contemporaneous positive volume-volatility relationship reported in the financial literature. According to Chan and Fong (2006), the positive relationship is driven by the rate of information flow into the marketplace. Apart from the MDH hypothesis, the sequential information arrival hypothesis (SIAH) of Copeland (1976) was also proposed to explain the volume-volatility relationship. The SIAH hypothesis assumes that all traders receive new information in sequential time. This implies that the sequential nature of the information arrival process creates time-asymmetric trading. Finally, the difference of opinion hypothesis (DOH) by Shalen (1993); Harris and Raviv (1993) also helps explain the positive volume-volatility relationship. This hypothesis suggests as investors trade based on their subjective belief's volatility will increase as trade increases.

The SIAH and DOH seek to explain not only the volume-volatility relationship but also the relationships between the volume-skewness and volume-kurtosis. The main purpose of Chapter 4 is to empirically investigate the relationship between the volume-skewness and volume-kurtosis and examine to what extent the SIAH and DOH can be used to explain these relationships. Do et al. (2014) attempts to investigate volume-realized higher-order moments. However, the authors focused on the spillover effects of higher-order realized moment risks and trading volume across 18 countries considered for stock and FX markets and used the number of trades as their only proxy of information flow. The investigations in this present study go beyond the number of trades proxy as various proxies of information flow are considered. Do et al. (2014) also generalizes the US-specific 5-minute optimal sampling frequency to the other 17 countries. As shown in Chapter 2, this narrow approach to sampling high-frequency return series can yield inaccurate results. In Chapter 4, we compute high-order realized moments (i.e., realized variance, realized skewness and realized kurtosis) for weekly and monthly holding periods. Using various proxies of information flow, we show that the volume proxy influences the sign of the volume-higher-order realized moment regression coefficients. We then attempt to relate our empirical findings to the MDH, SIAH and DOH hypotheses and note that the DOH hypothesis implicitly encompasses or nests both the SIAH and MDH hypotheses. The dynamic and significant link between volume and higher-order moments is shown by highlighting the significance of the regression coefficients across holding periods and various market conditions.

In Chapter 5, we also use the 15-minute return series for Australian stocks to examine the relationship between monthly realized return and monthly realized higher-order moment risk, as well as to determine how investors price systematic co-skewness and systematic co-kurtosis according to different market conditions and sample periods. The particular approach incorporated in the analysis stems from criticisms of the standard CAPM of (Sharpe, 1964; Lintner, 1975; Mossin, 1966). The CAPM reinforces the notion that co-variance risk (systematic co-variance) is the only metric that matters in the asset pricing model and that asset returns are normally distributed. This limitation of the CAPM has led researchers to develop alternative theoretical methodologies or to extend the original CAPM model to improve its predictive capacity (see Fama and French, 1992, 1995; Carhart, 1997; Bollerslev et al., 1988; Jagannathan and Wang, 1996; Shalit and Yitzhaki, 1984; Okunev, 1990, just to mention a few). Rubinstein (1973); Ingersoll (1975); Kraus and Litzenberger (1975) were the first to propose higher-order moment pricing models. Since then, the predictive power of the high-order moment pricing model over the CAPM model has been extensively documented in financial literature (see Harvey and Siddique, 2000a; Dittmar, 2002; Lambert and Hübner, 2013; Poti and Wang, 2010). However, investigations into the realized higher-order co-moment risk-return relationship are comparably scarce in relation to the Australian equity framework specifically. In addition, we believe that using high-frequency return data will yield more robust empirical results. The present study aims to explain how Australian stock returns are influenced by normalized realized higher-order co-moment risks. We anticipate that the realized higher-order co-moment risks to be priced separately across different market conditions and sample periods.

We begin the empirical test in Chapter 5 by employing the single sorting of excess realized returns on the risk measures approach. We construct 10-decile equally weighted portfolios sorted on the risk measures. Using the high-minus-low spread of the portfolio averages, we find the strength and significance of the risk-return relationships. In addition, the monotonic relation outlined by Patton and Timmermann (2010) is utilized to identify monotonic relationships across the entire portfolio, as knowledge of such relationships can benefit investors' trading strategies. While the single sorting approach has some value, it does not control for other variables that may explain the cross-section of the asset return. The double sorting approach seeks to address this limitation and shows that the risk premium can be priced when explicitly controlling for other variables. A cross-sectional Fama–MacBeth regression, which is capable of accounting for more than one control variable is also employed at the firm level to further investigate the predictive power of the risk measures. In the analysis, we test the predictive power of the two-, three- and four-moment CAPM models documented in the literature. A unique feature of these models is that they can capture investors' risk arising from the volatility, skewness and kurtosis of the marketplace. Additionally, the predictive power of the higher-order co-moment risk is examined in the presence of continuous and jump beta.[6] As a result, we are able to estimate the various risk exposures of the different risk factors and their respective risk premiums. We also identify the most important systematic risk components (i.e., the systematic co-variance, co-skewness and co-kurtosis) that explains the Australian stock returns. The empirical testing conducted in this chapter shows that the realized higher-order co-moment pricing model is superior to both the standard CAPM model and the jump-diffusive two-beta CAPM model.

Lastly, Chapter 6 summarizes the implications of the research findings of this thesis, outlines the limitations of the research and provides suggestions for further research. The results of the present thesis show that researchers utilizing the high-frequency return data must be cognisant of the optimal sampling frequency for the country under investigation. Primarily, researchers in the field of high-frequency finance should note the effects of the sampling-interval and holding-intervals on the estimated realized skewness and realized kurtosis. In addition, they should be cautious of the type of proxy used for information flow and its relationship with realized skewness and realized kurtosis. Researchers should also not neglect asset pricing models that account for the non-normality of the asset return distribution when investigating the risk-return relationship across asset returns. The findings outlined in this thesis contribute to the body of financial research that examines the high-frequency finance paradigm.

[6]The continuous and jump beta are estimated by splitting the standard beta into diffusive and jump components (see Todorov and Bollerslev, 2010; Dungey and Yao, 2013; Bollerslev et al., 2016; Chowdhury et al., 2018).

Chapter 2

Optimal Sampling Frequencies for Realized Variance of American and Australian Stocks and Indices

2.1 Introduction

Realized variance (RV) has been extensively documented in the financial forecasting literature as an empirical and theoretical measure of realized volatility. Under ideal conditions, the RV approaches the integrated variance (IV; see Andersen and Bollerslev, 1998; Barndorff-Nielsen and Shephard, 2002; Meddahi, 2002, for more details). According to Andreou and Ghysels (2002), this consistency is based on an assumption that does not hold in practice. They infer that the RV is biased, and hence inconsistent, for the IV. The main drawback of the RV is that of an error-in-variables problem (see Hansen and Lunde, 2004). Preferably, one should construct the RV from high-frequency intra-day return data of the true price process (p^*). However, p^* is unobservable, and hence, the RV is obtained from observed prices, which are contaminated with market microstructure noise. The contamination of the observed prices can be attributed to serial correlation generated by bid-ask bounce, price discreteness and price reporting error. Therefore, using the observed price instead of the true price in computing the RV makes the RV biased and inconsistent for the IV (Andreou and Ghysels (2002); Oomen Roel (2002)).

The biased nature of the RV constructed from intra-day return data offsets any efficiency gains that could have been derived from sampling at such high frequencies. This phenomenon has been well documented in financial time series literature (see Ebens et al., 1999; Bai et al., 2001; Oomen Roel, 2002). Nevertheless, computing the RV from daily squared-return data (low frequency) is not the solution for obtaining an efficient measure of volatility, because at a low frequency, valuable information is lost. Consequently, some researchers have come up with techniques aimed at reducing the measurement error of the RV. For example, Zhang et al. (2005) employs a sub-sampling technique that generates a more efficient estimator for the RV. Oomen (2006) propose alternative sampling schemes (calendar-, business- and transaction-time sampling), and show that transaction time sampling is superior to the popular practice of calendar time sampling. Hansen and Lunde (2004) shows that a bias correction of the RV is a more efficient estimator for the RV. They use a weighting scheme that corrects the q_m autocorrelation terms in the intra-day returns. Hansen and Lunde (2003) shows that a Newey-West modified RV is an unbiased estimator for the IV. Finally, Ebens et al. (1999) and Andersen et al. (2001) construct RV from first-order moving average filtered returns.

The stylized procedure of estimating RV by summing intra-day return series in the holding period relative to employing any of the error-correction techniques documented in the extant literature may result in obtaining biased RVs. However, the following advantages could still be derived from RVs constructed from high-frequency data without employing the aforementioned techniques: (i) RV is a good proxy for the unobserved conditional variance when evaluating the predictability of autoregressive conditional heteroskedasticity (ARCH)-type models (Hansen and Lunde (2004)), (ii) estimations based on RVs are more precise compared to squared daily returns in an out-of-sample forecast of ARCH-type models (Andersen and

Bollerslev (1998)), (*iii*) time series analysis of RVs yields useful information about the dynamic model of RV (Andersen et al. (2003)) and (*iv*) RVs constructed from intra-day returns possess strong predictive power in comparison to using squared daily returns (Hansen and Lunde (2003); Andersen et al. (2004)). This is consistent with Amaya et al. (2015), who shows that RV converges to the total quadratic variation, and using higher-frequency returns in computing RV yields more efficient estimates of quadratic variation. According to Bandi and Russell (2008), it is paramount to compute RV with intra-day return data where autocorrelation is less of an issue, since computing RV with contaminated return data results in the significant accumulation of noise. Therefore, one can obtain a good estimate of RV by using the stylized approach when a favourable sampling frequency is known. The nonparametric nature of the stylized approach gives an added advantage when compared with error-reduction techniques.

In the US framework, it is typical to construct RV using returns sampled at a 5-minute frequency. The rationale is that this can partially offset the bias of the RV (Andersen and Bollerslev, 1997). The theoretical and empirical justification for this approach is formally presented by Bandi and Russell (2008), who derive the optimal sampling frequency under a mean squared error criterion. Liu et al. (2015) show that it is difficult to significantly beat the 5-minute RV. However, the 5-minute optimal sampling frequency creates a dichotomy in the existing literature: some researchers find the optimal sampling frequency for computing RV to be at a lower frequency than the standard 5-minute frequency. Bandi and Russell (2008) recommend that, when sampling very illiquid stocks, a 15-minute sampling frequency could be preferred for computing RV, which should be lowered for very high liquid stocks. Bollerslev et al. (2008), using signature plots, shows that the optimal sampling frequency for 40 US equities is 17.5 minutes. Oomen (2006) shows that the optimal sampling frequency for realized volatility for IBM stock to be 20 minutes, while it changes to about 3 minutes with a first-order bias correction. Aït-Sahalia et al. (2005) observe the optimal sampling interval for computing RV for the day to be 22 minutes. Hansen and Lunde (2006), using

DJI30 stocks, show that the noise in RV may be ignored when intra-day returns are sampled at low frequencies, such as 20 minutes. Lahaye et al. (2011) using signature plots show that RV starts to stabilize at about 15 minutes. Andersen et al. (2003) employs a 30-minute return series to compute RV.

The objective of this study is twofold. The first is to determine whether the 5-minute optimal unbiased sampling frequency rule of thumb, employed in the US framework for RVs (as reported by Bandi and Russell (2008)) holds for all DJI30 stocks and the DJI30 index. The second is to determine if this can be extended to S&P/ASX20 stocks and the S&P/ASX20 index.[1]

The motivation of the extension to the Australian framework stems from Alles and Murray (2017), who have shown that, although Australia is a developed country, its equity market requires separate investigation, as some aspects differ from major international equity markets. They show that, relative to the US and UK market, the trading volume for the Australian equity market is less than 5% of that recorded on the New York Stock Exchange. The Australian equity market is concentrated in a small number of sectors, with the materials sector dominating, making the market highly weighted in one sector. Alles and Murray (2017) also report that the Australian equity market is mainly represented and weighted by domestic firms, with less than 2% being overseas companies. This leads to investors not having a wide range of investment opportunities, which in turn might result in cyclical economic patterns. We emphasize that the illiquidity effects in the Australian framework have a significant probability of determining the observed optimal sampling frequency. This is in line with the findings of Bandi and Russell (2008); thus, we expect the 5-minute optimal sampling frequency to hold for liquid markets (such as the US equity market) while illiquid markets (such as Australia) to have an optimal sampling frequency less than the standard 5 minute. The Australian equity market is unique, and its share market behaviour/patterns might be different from that of the major international equity markets (e.g., the

[1]We use signature plots employed by Andersen et al. (2000) to evaluate the optimal unbiased sampling frequency of RV. The nonparametric nature of the signature plots makes it a significant tool used in the literature.

US). Mujtaba Mian and Adam (2001) show that volatility estimated at different sampling frequencies behave differently, and one should be aware of when to use high- or low-frequency return data in estimating volatility.

The second motivation arises from the fact that most researchers blindly employ the so-called 5-minute optimal sampling frequency, popularly used in the US framework, without investigating whether the said frequency is country-specific or extendable to their country of investigation. For example, Do et al. (2014), in investigating volume-realized higher-order moment relationships, generalize the standard US 5-minute unbiased sampling frequency to the other 17 countries they consider, which includes Australia. This naive approach of using high-frequency return series has the potential to result in significant accumulation of noise, which might impact the results and conclusions obtained. We contribute to the debate on whether the 5-minute sampling frequency for the US holds for other international markets, specifically Australia. To the best of our knowledge, we are the first to investigate preferred sampling frequency with a focus on the Australian share market. We aim to obtain a sampling interval where the autocorrelation is relatively less of an issue, to prevent estimating RV with contaminated intra-day return series.

Thus, in this study, we use 1-second raw prices from 2010 to 2015, downloaded from the Thomson Reuters Tick History/Securities Industry Research Centre of Asia-Pacific (TRTH/SIRCA) database, in computing the relevant daily RVs at various sampling frequencies. Our empirical results show that, for the US framework, both the DJI30 index and stock RVs can be computed with the intra-day return series sampled at 30 seconds. It is worth mentioning that the standard 5-minute sampling frequency also holds for the results of the DJI30 index and stocks. However, for S&P/ASX20 index and stocks, we note that the observed 30 seconds and the 5-minute rule thumb cannot be extended to the Australian framework. We observe that, for the S&P/ASX20 index, 10 minutes is the preferred sampling frequency. However, when we take a closer look at the constituent stocks of the index, we observe a 20-minute sampling frequency to be favourable for computing RV. We infer that, perhaps on an average, a 10- to 30-minute window could be the preferred sampling frequency for the Australian equity framework.[2] This creates a 10-minute lower bound and a 30-minute upper bound, within which researchers can carefully select a frequency where there is a reasonable trade-off between the number of observations and microstructure noise. Consequently, researchers in the high-frequency finance paradigm must be observant and wary of extending US rules of thumb to other financial markets, such as the Australian share market.

The remainder of this chapter is organized as follows. Section 2.2 contains a brief review of relevant theory used in estimating realized variance. Section 2.3 describes the high-frequency data and the sampling procedure employed. The signature plots and empirical findings are discussed in Section 2.4 and Section 2.5 concludes.

2.2 Realized variance

We begin a concise review of the theory relating to estimating realized variance from high-frequency return series. As mentioned earlier, suppose p_t^* is the latent price in a continuous time and p_t is the observed price.[3] We construct $p_{t\equiv[0,\infty)}$ artificially from the observed price using previous tick method that was proposed by Wasserfallen and Zimmermann (1985). Let $a = t_0 < \cdots < t_m = b$ be the time at which raw prices $p_{t,i}$ are observed, where $i = 0, 1, ..., m$, then the artificial continuous time process at any point in time $\tau \in [t_0, t_m)$ can be defined as,

$$p_\tau \equiv p_{t,i}, \quad [t_i, t_{i+1}) \tag{2.1}$$

[2]S&P/ASX20 comprises the 20 largest ASX-listed stock and accounts for about 47% of the Australian equity market capitalisation. The constituent stocks are highly liquid. Perhaps, the preferred sampling frequency for constituent stocks of S&P/ASX200, ASX300 and All Ordinaries indices, which are less liquid than ASX20, would be 15 minutes; this is consistent with the findings of Bandi and Russell (2008).

[3]The noise process can be defined as $u_t \equiv p_t - p_t^*$. The noise component u_t may be attributed with microstructure noise that arise from bid-ask bounce, price discreteness, rounding errors and price reporting error during trading of the asset and documentation of the price process (see Bai et al., 2001; Andreou and Ghysels, 2002; Oomen Roel, 2002, for more details).

The artificial prices enable us to construct equidistant intra-daily returns for computing the realized variance at any frequency. We define the discrete-time intra-day returns on trading day t by;

$$r_{t,i} = P_{t,i} - P_{t,i-1}, \quad i = 1, 2, .., m; \quad t = 1, 2, ..., T \tag{2.2}$$

where $P_{t,i}$ is the i-th intra-day log price for day t, T is the total number of days in the sample and m is the number of equally spaced intra-day returns over trading day t partitioned into equal length $\triangle_m \equiv (b - a)/m$ and $[a, b] \subset t$. Suppose the observed price follows a semi-martingale process on some filtered probability space (Ω, $\mathcal{F}$, $(\mathcal{F}_t)_{t\geq 0}$,$\mathsf{P}$) in a frictionless market where there are no arbitrage opportunities (see Back, 1991). Then in the presence of jumps, the observed price can be modelled as a continuous time semi-martingale jump-diffusion process;

$$p_t = \int_0^t \mu_D dt + \int_0^t \sigma_D dW_t + \sum_{k=1}^{N(t)} J(Q_k), \tag{2.3}$$

where μ_D is the diffusive mean, σ_D is a diffusive volatility process and dW_t is the increments to a Brownian motion W_t, $N(t)$ is a counting process and $J(Q_k)$ are the non-zero jump increments (see Fleming and Paye, 2011, for more details). The quadratic variation for the jump-diffusion process is defined as,

$$QV_t = \int_0^t \sigma_D^2 dt + \sum_{k=1}^{N(t)} J^2(Q_k), \tag{2.4}$$

the first term on the right-hand side of Equation 2.4 is the integrated variance and the second term is the sum of the squared jumps (variance of the jump component). We observe that Equation 2.4 reduces to a 'pure' diffusion model with continuous sample paths when there are no jumps in the price process (i.e. the jump component is set to zero). For this jump-diffusion process to hold, it is assumed that μ_D and σ_D are jointly independent of W_t. The integrated variance (IV) for this type of process is defined $IV_t \equiv \int_0^t \sigma_D^2 dt$ and equals to the quadratic variance (QV).

In high-frequency finance, the proxy for sample variance is the realized variance (RV); replacing the traditional use of squared returns at low frequencies. It is well documented that realized variance is a more robust estimate of volatility (see Andersen and Bollerslev, 1998; Andersen et al., 2003; Hansen and Lunde, 2004, 2003; Barndorff-Nielsen and Shephard, 2004; Andersen et al., 2007). The RV is defined as the sum of squared high-frequency returns as given by;

$$RV_{t,i} = RM(2)_{t,i} \equiv \sum_{i=1}^{N} r_{t,i}^2 \rightarrow \int_0^t \sigma_D^2 dt + \sum_{k=1}^{N(t)} J^2(Q_k), \quad as \quad N \rightarrow \infty \tag{2.5}$$

The RV is an efficient estimator of the quadratic variation, it converges to the QV as the number of observations (N) goes to infinity ($RV_{[a,b]}^{(N)} \rightarrow QV_{[a,b]}$ as $N \rightarrow \infty$ (see Andersen and Bollerslev, 1998; Barndorff-Nielsen and Shephard, 2002). It is also apparent from Equation 2.5 that in the absence of jumps, RV converges to the IV.

2.3 Data

The data consist of high-frequency (intra-day) asset prices of DJI30 and S&P/ASX20 indices and stocks. We download the raw price with a 1-second sampling frequency from the TRTH/SIRCA database. Our data sample is from 4 January 2010 - 31 December 2015, and between 10 am and 4 pm of each trading day. The 1-second sampling frequency results in 21,600 sample points in a day; this translates into 32.62 million sample points over the 5 years considered (the span of the data sample yields 1,510 trading days for the 5 years).

The true price is unobservable; this implies that the downloaded data may be contaminated with market microstructure noise. It is easy to recognise prices that are recorded as 0, but the same cannot be said for other misrecorded prices. To deal with 0 values, the previous-tick method proposed by Wasserfallen and Zimmermann (1985) is used. The data was filtered for outliers, and we winsorise the data at 99.995% and 0.005% for the upper and lower percentiles, respectively.

This ensures that extreme outliers are dealt with without any significant loss of information. We also exclude weekends and overnight returns from the data before estimating the intra-day returns. The intra-day returns are computed as the change in the logarithm of the closing prices of successive days, as shown in Equation 2.2.

Suppose we want to compute daily RVs for any of the stocks or indices relative to a specific sampling frequency; we employ the procedure adopted in Ahadzie and Jeyasreedharan (2020). We carefully select values of the sampling intervals to prevent the length of the intra-day returns being a fraction. For example, to construct daily RVs with 5-minute return series, we divide the holding period of data points (360—1 minute each trading day) by 5 minutes, and the number of observations will be 72 (thus $\triangle_m \equiv (b-a)/m$, $\triangle_{5-minute} = 360/5$, where $a =$ 10 am, and $b = 4$ pm). Following Hansen and Lunde (2003, 2004, 2006), we use a 30-minute sampling frequency estimate as the benchmark.

2.4 Empirical Results

In this section, we discuss the behaviour of RVs across sampling intervals using time-based signature plots. From the 1-second price data, we construct various daily RVs at different sampling frequencies (e.g., 30 seconds, 1 minute, 30 minutes and so on).

Figure 2.1 is the signature plot of the RV of the DJI30 price-weighted index computed with 1-second to 60-minute return series. The horizontal red line is the RV computed with 30-minute return series (thus $\hat{\sigma^2} \equiv \overline{RV}^{30min}$). Following Hansen and Lunde (2003, 2004, 2006), the RV computed with the 30-minute return series is the benchmark to which the RVs at different sampling frequencies are compared. Using RVs computed with the 30-minute return series, with 1000 bootstrapping replication, we compute the 95% confidence interval for the preferred sampling frequency. The average RVs are reported on the vertical axis, and the sampling frequency on the horizontal axis.

In Figure 2.1, we observe that, at higher frequencies, such as 1 second and 15 seconds, the average RVs deviate from the target RV, which is depicted by the red horizontal line. According to Hansen and Lunde (2004), this deviation may be due to the presence of significant serial correlation at such sampling frequencies. This is consistent with the autocorrelation plot in Figure $A.4$. By visual inspection, the popular 5-minute sampling frequency widely used for US high-frequency return series seems to hold. This suggests that one can safely sample US equity returns at 5-minute sampling, as is the norm.

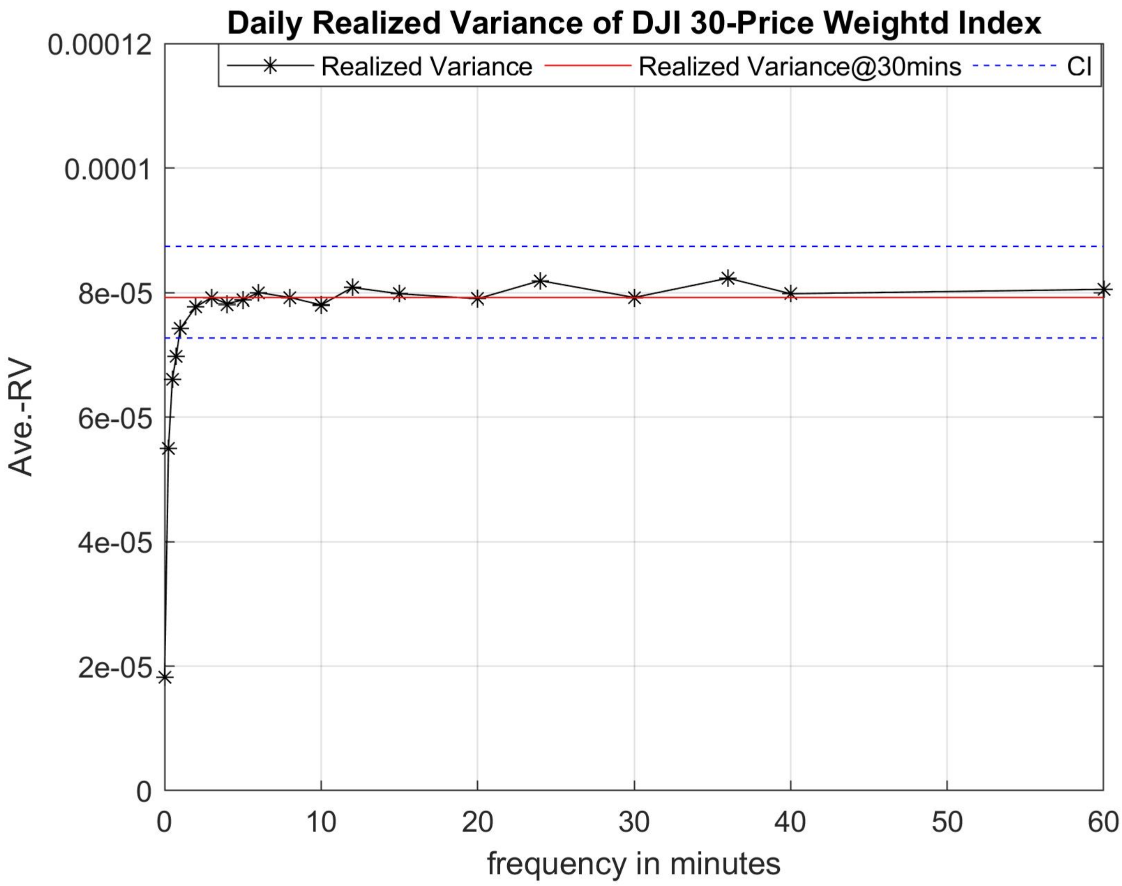

Figure 2.1: Average daily realized variance for US (DJI30 index)

From Figures 2.2 and 2.3, we take a closer look at each stock of the DJI30. We observe that the standard 5-minute sampling frequency still holds for all constituent stocks. However, considering the challenges with visual inspection, we employ the two-sample t-test for equal means to verify the results obtained for the time-based signature plots. Here, we compare the RVs computed at each sampling frequency to our target RV. The results are reported in Table 2.1, where '1' means the computed RV diverges from our target RV, and '0' otherwise. We note that stocks such as General Electric Company (GE), Nike Inc. (NKE) and Pfizer Inc. (PFE) can be even sampled as high as a 1-second frequency. Additionally, we observe that about 53% (16 out of 30 stocks converge to the target RV) of the constituent stocks of DJI30 can be sampled at a 15-second sampling frequency—such stocks include Apple Inc. (AAPL), Coca-Cola Company (KO) and Intel Corporation (INTC), just to name a few. At 30 seconds, all stocks can be sampled without the result generating a biased estimate. This is consistent with Amaya et al. (2015), who use Monte Carlo techniques and 1-second data to show that estimates of the realized higher moments (RV inclusive) are reliable in finite samples and that, at 1-minute return series are robust to the presence of market microstructure noise. The results for the two-sample t-test are consistent with the results from the time-based signature plots reported earlier and suggest that, in the US framework, the standard 5-minute sampling frequency is robust to market microstructure noise and ideal for computing RVs.

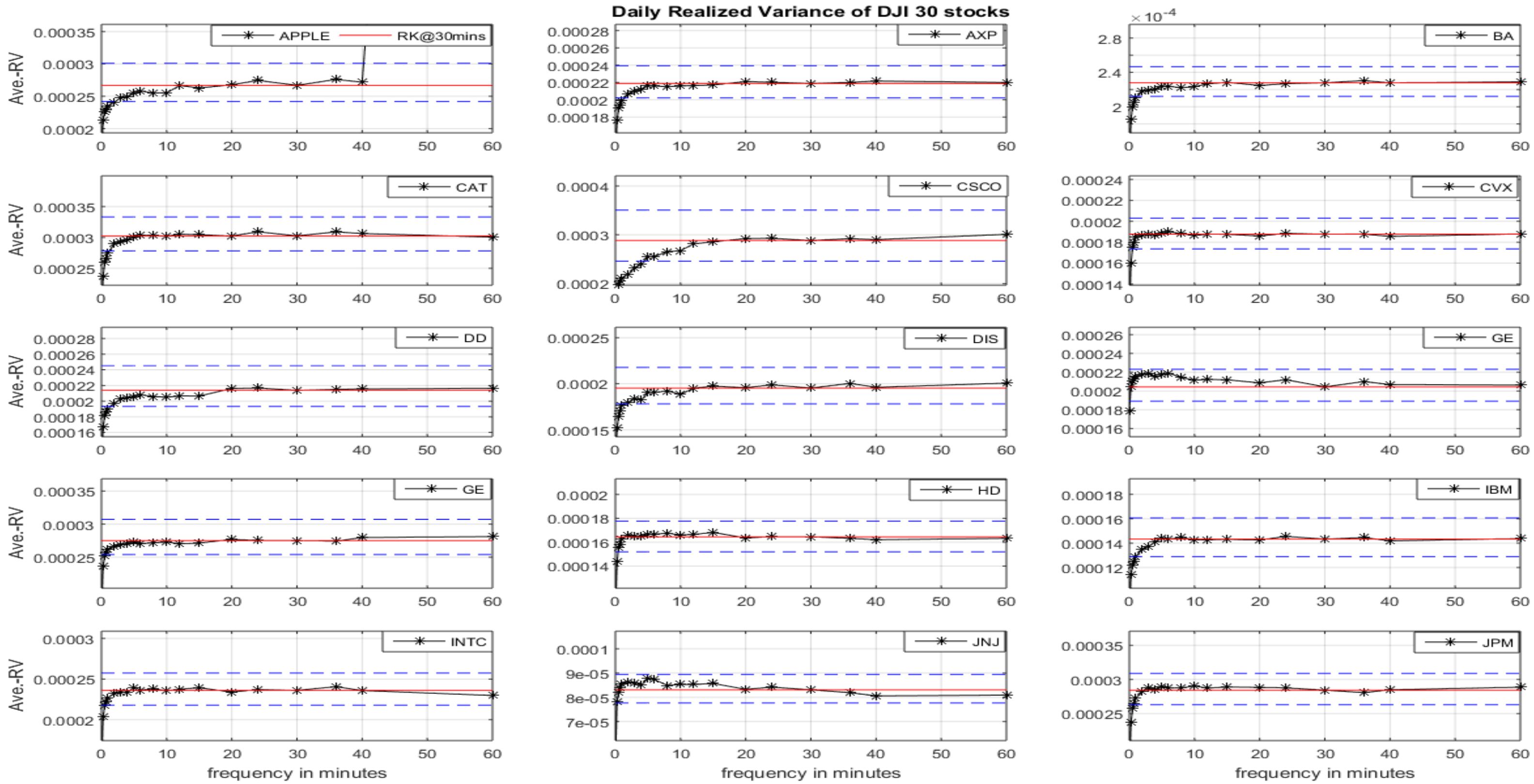

Figure 2.2: Average daily realized variance for US (DJI30 stocks)

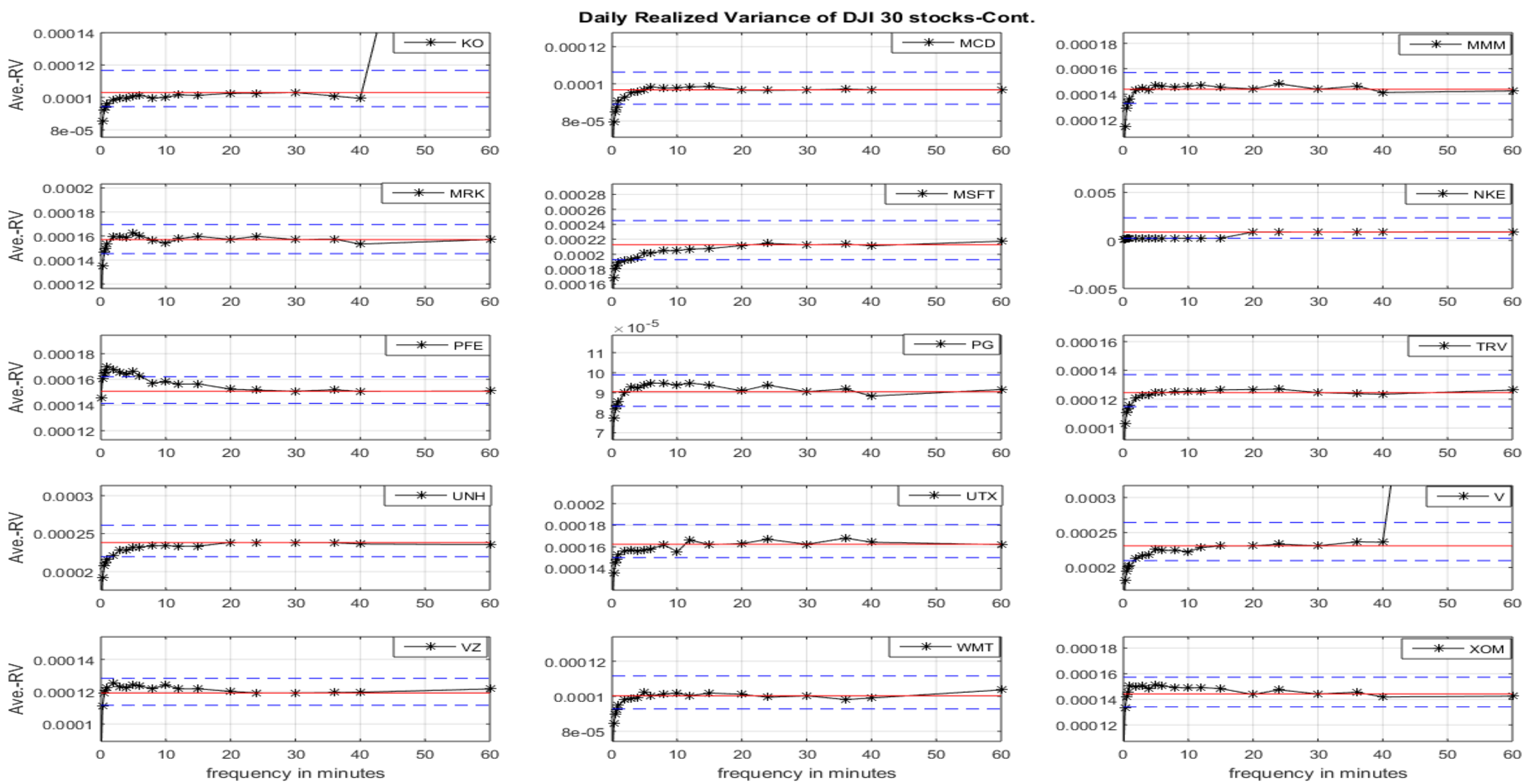

Figure 2.3: Average daily realized variance for US (DJI30 stocks cont.)

Table 2.1: Two-sample t-test for equal means for DJI 30 stocks-RV

Stocks	$1s$	$15s$	$30s$	$45s$	$1m$	$2m$	$3m$	$4m$	$5m$	$6m$	$8m$	$10m$	$12m$	$15m$	$20m$	$24m$	$30m$	$36m$	$40m$	$60m$
DJI 30	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AAPL	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AXP	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BA	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CAT	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CSCO	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CVX	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DD	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DIS	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
GE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
GS	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
HD	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IBM	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
INTC	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
JNJ	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
JPM	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
KO	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MCD	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MMM	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MRK	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MSFT	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NKE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PFE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PG	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TRV	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
UNH	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
UTX	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
V	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
VZ	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WMT	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
XOM	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The table above reports the test decision for the null hypothesis that the RV at each frequency comes from the same population with equal mean to that of RV computed at 30 minute sampling frequency at 0.001 significance level for U.S. equities we consider. A '1' indicates rejection of the null hypothesis of equal means at 0.001 significance level, and '0' otherwise. This is consistent with the results of the signature-plots. Note s denotes second and minute as m.

Figure 2.4 reports the result for the S&P/ASX20 value-weighted index. Here, we observe that the observed 30-second sampling frequency, preferred for the US equity framework, does not hold, indicating that the microstructure noise does not disappear at this sampling interval for the Australian framework. Additionally, the standard 5-minute frequency, popularly employed for US high-frequency return data does not hold either. This suggests that researchers who blindly extend the so-called 5-minute optimal sampling frequency of the US framework to other equity markets have the potential to obtain biased estimates, which might impact their results and conclusions. We note that RV values below the 10-minute sampling frequency have an increasing trend and lie outside the confidence band, which suggests that the RVs at those frequencies are biased and do not approach our proxy for the quadratic variation (target RV).

We observe that, from 10 minutes and above, the average RV values fall within the confidence band and converge to our target RV. We infer that the preferred sampling frequency for the Australian equity framework should be less than standard 5-minute rule of thumb, and hence, we advocate a minimum bound of 10 minutes. This is in line with Bandi and Russell (2008), who show that, for illiquid stocks, 15 minutes should be the preferred sampling frequency for intra-day data.

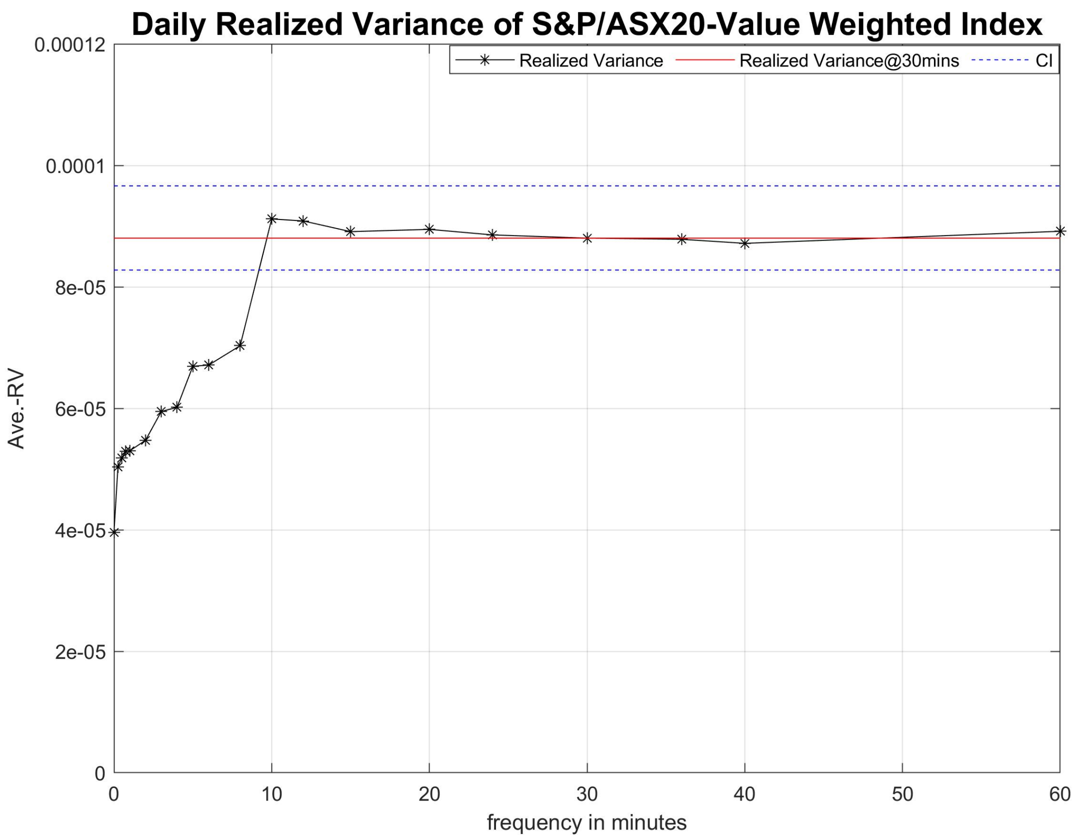

Figure 2.4: Average daily realized variance for Australia (S&P/ASX20 index)

Figure 2.5 reports the signature plot for the RV for each constituent of the S&P/ASX20 index. Similar to the index itself, the 10-minute sampling frequency holds for some stocks, such as Australia and New Zealand Banking Group (ANZ), National Australia Bank (NAB) and Macquarie Group (MQG). However, this does not hold for others, like AMP Limited (AMP), Insurance Australia Group (IAG), Transurban Group Stapled (TCL) and Scentre Group Stapled (SCG), where the preferred sampling frequency seems to be around 15 to 20 minute. This implies that, for individual stocks, the 10-minute frequency may not necessarily hold; this could be due to effects of illiquidity. Clearly, our results show that

Australian equities are different from that of the US share market.

We also note that, in some of the RV plots, such as BHP Group (BHP), Commonwealth Bank of Australia (CBA), CSL Limited (CSL), Rio Tinto Limited (RIO) and Woodside Petroleum Limited (WPL), the graph depicts an upward sloping. This suggests the presence of significant negative serial correlation at high sampling frequencies (e.g., at 1, 15, 30 and 45 seconds; see Andersen et al. (2017) for more details). In contrast, stocks with downward sloping RVs are accompanied by positive serial correlation. According to Avramov et al. (2006), high-liquid stocks do tend to exhibit negative serial correlation. Hence, we can infer that the upward slope of the RVs of BHP, CBA, CSL, MQG, RIO and WPL might be due to significant negative serial correlation, as suggested by (Andersen et al., 2017).

In Table 2.2, we report the two-sample *t*-test. As expected, the preferred sampling frequency for the index remains at 10 minutes. For individual stocks, the standard 5 minutes used in the US framework still does not hold: about 35% (7 out of 20) of the constituent stocks do not converge at the target RV. This number will increase for less liquid constituents such as the S&P/ASX200. We note that 20% (4 out of 20) of the stocks remain biased at a 10-minute sampling frequency. However, this disappears completely at a 20-minute frequency. From the results, we observe that, as the sampling frequency increases, the number of divergent stocks also decreases. This is not surprising, as the microstructure noise stabilizes when return series are sampled at a relatively finer frequency (see Figures A.4 and A.5 in Appendix A).[4]

[4]The results obtained for the S&P/ASX20 index is consistent with those of the S&P/ASX200 index reported in Figure A.3 of Appendix A.

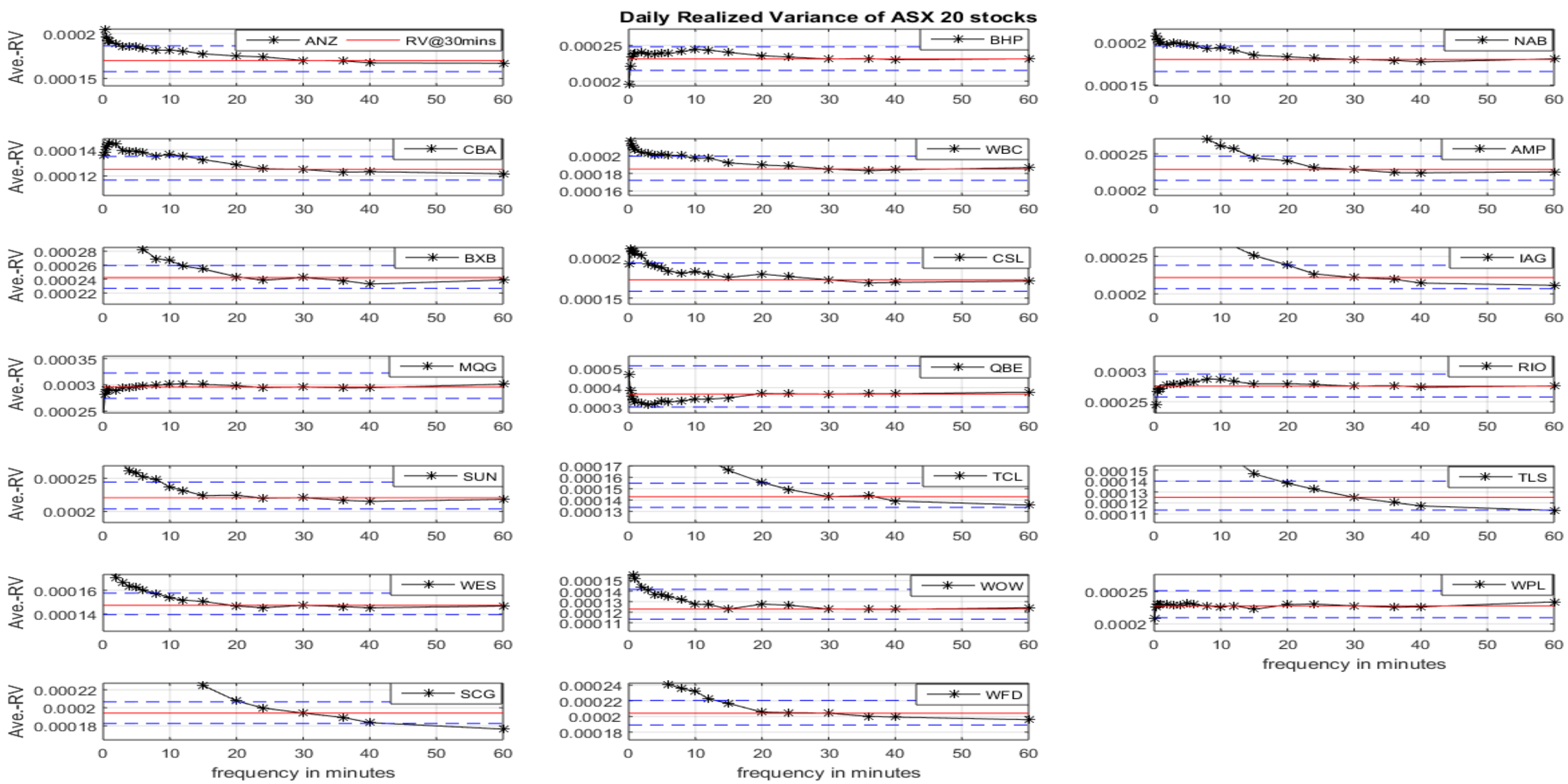

Figure 2.5: Average daily realized variance for Australia (ASX20 stocks)

Table 2.2: Two-sample t-test for equal means for S&P/ASX20 stocks-RV

Stocks	1s	15s	30s	45s	1m	2m	3m	4m	5m	6m	8m	10m	12m	15m	20m	24m	30m	36m	40m	60m
S&P/ASX20	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
ANZ	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BHP	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NAB	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CBA	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WBC	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AMP	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
BXB	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
CSL	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IAG	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
MQG	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
QBE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
RIO	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SUN	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
TCL	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
TLS	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
WES	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
WOW	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
WPL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SCG	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
WFD	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0

The table above reports the test decision for the null hypothesis that the RV at each frequency comes from the same population with equal mean to that of RV computed at 30 minute sampling frequency at 0.001 significance level for Australian equities we consider. A '1' indicates rejection of the null hypothesis of equal means at 0.001 significance level, and '0' otherwise. This is consistent with the results of the signature-plots. Note s denotes second and minute as m.

2.5 Concluding Remarks

In this study, we empirically investigate the optimal sampling frequency for high-frequency return data for the US equity framework (DJI30 index and constituent stocks) and determine whether the obtained optimal sampling frequency can be extended to that of Australian equities (S&P/ASX20 index and constituent stocks). Using 1-second raw prices from January 2010 to December 2015 downloaded from the TRTH/SIRCA database, we compute daily RVs for various sampling frequencies. The results are reported with the nonparametric time-based signature plots popularly documented in the extant literature.

We observe that, for the US framework, both the DJI30 index and stock RVs can be computed with the intra-day return series sampled as frequently as 30-seconds. Additionally, we note that the standard 5-minute rule of thumb sampling frequency also holds. In the case of the Australian framework, we note that the observed 30-second and 5-minute standard sampling frequencies cannot be extended to the S&P/ASX20 index and stocks. Here, we observe that 10 minutes is the preferred sampling frequency for the S&P/ASX20 index, and 20 minutes is preferred for computing RVs for its consistent stocks. In short, we conclude that, perhaps on an average, a 10- to 30-minute sampling window could be the preferred sampling frequency for the Australian framework.

Chapter 3

Effects of Intervaling on High-Frequency Realized Higher-Order Moments

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3.1 Introduction

Sample skewness and sample kurtosis (i.e. normalized third and fourth moments) of low-frequency financial asset returns have long been shown to display significant variations as a function of the holding-interval (i.e. daily, weekly, monthly, etc.) over which the returns are computed. This inherent variation in the computed sample moment values has been traditionally referred to in the financial literature as the 'intervaling effect' (see Hawawini, 1980; Smith, 1978; Francis, 1975; Fogler and Radcliffe, 1974). The recent advent of high-frequency data has given rise to the notion of realized skewness (RS) and realized kurtosis (RK), which have also shown to display significant variations as a function of not only the holding-intervals (as in days, week, months, etc.) but also the sampling-intervals (see

Amaya et al., 2015; Mei et al., 2017).[1] In this study, we investigate both these two intervaling effects i.e. the holding-effect and the sampling-effect. We show both theoretically and empirically, that not only the holding-interval, but also the sampling-interval directly conditions the realized higher-order moments and consequently, the normalized realized skewness and realized kurtosis estimates.

Hawawini (1980) was the first to investigate the effects of intervaling on sample skewness and kurtosis by showing analytically that 'the higher the moment's order, the more sensitive it is to the length of the holding-interval over which securities' returns are measured. However, the empirical evidence on the effects of holding-intervals on the normalized higher-order moments of asset returns was polarized in the then extant literature. For example, Fogler and Radcliffe (1974), Francis (1975) and Neuberger (2012) observed that as the holding-interval increases skewness also increases whilst Lee et al. (1985) and Fogler et al. (1977) found that increasing the holding-interval decreases the skewness of the return data. Subsequently, Lau and Wingender (1989) showed that Hawawini's formulas for computing the normalized third and fourth higher moments were incorrect and consequently contributed to unintended errors and misinterpretations. After deriving the correct formulations, Lau and Wingender (1989) showed that, for logarithmic returns, as the holding-interval increases the sample skewness and kurtosis approaches zero and three asymptotically.

Amaya et al. (2015), decades later, was the first to investigate the effect of intervaling on realized skewness and kurtosis by employing intra-day returns data. They showed that at high sampling-intervals the effects of jumps dominate in the limit and concluded that for the associated third and fourth realized moments one 'can expect very different estimates of [realized] skewness and kurtosis depending on the frequency of the data used to estimate these moments'.

[1] The holding-intervals and the sampling-intervals in high-frequency finance are generally not equivalent. Holding-intervals are commonly daily, weekly or monthly intervals; whereas sampling-intervals are the intraday-intervals at which the high-frequency data is being sampled; for example at 1-minute, 5-minute or 30-minute intervals and so on. The holding-interval effect in low-frequency finance has also been called the interval effect, the investment interval problem, the holding period problem, etc.

The extant literature provides two main explanations for the occurrence of jumps in stock markets. Firstly, jumps reflect the market reaction to unexpected information, which indicates that news announcements are the primary source of price jumps (see Lahaye et al., 2011; Lee and Mykland, 2012). Hence jumps serve as an ideal proxy for information arrival and can be utilized as tools for studying market efficiency (see Malkiel and Fama, 1970) or phenomena like information-driven trading; see for example (Cornell and Sirri (1992); Kennedy et al. (2006); Hanousek et al. (2014)). Secondly, Bouchaud et al. (2006) and Joulin et al. (2008) advocate that jumps are mainly caused by a local lack of liquidity in the market, an event they term 'relative liquidity'. In addition, an inefficient provision of liquidity can also be caused by an imbalanced market micro-structure mechanism (see Madhavan, 2000).

In this study, we link the works of Hawawini (1980), Lau and Wingender (1989) and Amaya et al. (2015) to derive the analytical relationships between realized skewness and realized kurtosis and the two types of intervals: holding-interval and sampling-interval; thus investigating the two distinct and separate effects in combination. To illustrate the effects of intervaling on the realized skewness and realized kurtosis, we motivate and justify the use of a new type of signature-plot: the count-based (as opposed to the sampling-interval based) signature-plot, to highlight any non-trivial relationship(s) between normalized realized higher-order moments and the number of infill observations (i.e. the holding-interval divided by the sampling-interval).

Our empirical findings and count-based signature-plots provide graphical evidence for the analytical findings of Amaya et al. (2015) wherein the average values of the realized skewness and realized kurtosis do not correspond nor converge to the sample skewness and sample kurtosis values for high-frequency data. This is in contrast to the correspondence and asymptotic convergence in the realized variance to the sample variance, 'where using higher-frequency returns yields more and more efficient estimates of quadratic variation'. This is consistent with equations (3.4)-(3.7), where the second moments depend on both the diffusion and jump components, on the other hand, the third and fourth moments depend exclusively on just the jump component. Hence, the realized variance converges to the total quadratic variation of the process due to the dominance of the diffusion component, whereas the realized skewness and realized kurtosis estimates do not converge to the total cubic and quartic variations correspondingly due to the dominance of the jump component (see Amaya et al., 2015).

It is also worth emphasizing here that the previous literature on this subject focuses on the effects of the holding-interval (or equivalently the sampling-interval) on the normalized higher-order moments using low-frequency return data. To the best of our knowledge, we are the first to investigate both the effects of the holding-intervals and the non-equivalent sampling-intervals on realized skewness and realized kurtosis using high-frequency return data. Using 1-minute raw index data downloaded from the Thompson Reuters Tick History provided by SIRCA database for the G7-countries, we compute and investigate the realized variance, realized skewness and realized kurtosis for several holding-intervals and sampling-intervals.[2] Our results show that both the holding-interval and the sampling-interval have distinct and dissimilar effects on the ensuring higher-order realized moments. We simulate and illustrate that the central limit theorem for skewness and kurtosis only holds when no jumps are present in the sampled price series. However, these intervaling effects are found to be analytically tractable and present some valuable insights for a deeper understanding of the implications of these effects for future studies in high-frequency finance.

The remainder of this chapter is organized as follows. Section 3.2 contains a brief review of the background theory. Section 3.3 presents a Monte Carlo simulation of an assumed jump-diffusion price generation process; with and without jumps and their corresponding count-based signature-plots. Section 3.4 describes the empirical data and the high-frequency sampling procedure used. Further, count-

[2]The G7-countries consist of Canada (GSPTSE index), France (CAC 40 index), Germany (GDAXIP index), Italy (FTMIB index), Japan (N500 index), United Kingdom (FTSE 100 Index), and United States (S&P 500 Index). Shown in parenthesis are Reuters instrument codes for the indexes.

based signature-plots and findings based on the empirical data are presented and discussed in Section 3.5. Section 3.6 concludes with a summary of the findings and some implications for future research.

3.2 Theory

Suppose the observed price follows a semi-martingale process on some filtered probability space (Ω, $\mathcal{F}$, $(\mathcal{F}_t)_{t\geq 0}$,$\mathsf{P}$) in a frictionless market where there are no arbitrage opportunities (see Back, 1991). Then in the presence of jumps, the observed price can be modeled as a continuous time semi-martingale jump-diffusion process;

$$p_t = \int_0^t \mu_D dt + \int_0^t \sigma_D dW_t + \sum_{k=1}^{N(t)} J(Q_k), \tag{3.1}$$

where μ_D is the diffusive mean, σ_D is a diffusive volatility process and dW_t is the increments to a Brownian motion W_t, $N(t)$ is a counting process and $J(Q_k)$ are the non-zero jump increments (see Fleming and Paye, 2011, for more details). The quadratic variation for the jump-diffusion process is defined as,

$$QV_t = \int_0^t \sigma_D^2 dt + \sum_{k=1}^{N(t)} J^2(Q_k), \tag{3.2}$$

where the first term is the integrated variance and the second term is the sum of the squared jumps (variance of the jump component).

Equation 3.2 reduces to a 'pure' diffusion model with continuous sample paths when there are no jumps in the price process (i.e. the jump component is set to zero). In the ensuring diffusion process, it is assumed that μ_D and σ_D are jointly independent of W_t. The integrated variance (IV) for this type of process is defined $IV_t \equiv \int_0^t \sigma_D^2 dt$ and equals to the quadratic variance (QV).

In high-frequency finance the proxy for sample variance is the realized variance

(RV); replacing the traditional use of squared returns at low frequencies (see Andersen and Bollerslev, 1998; Andersen et al., 2003; Hansen and Lunde, 2004; Barndorff-Nielsen and Shephard, 2004; Lunde et al., 2004; Andersen et al., 2007). The discrete time high-frequency returns over the holding-interval h is defined as;

$$r_{i,h} = p_{i,h} - p_{i-1,h}, \quad i = 1, 2, .., N \tag{3.3}$$

where h is the holding-interval (for instance, a trading day, week or month), $p_{i,h}$ is the i-th high-frequency log price for holding-interval of h, and N the number of infill observations for each sampling-interval, τ, partitioned into equal length such that $\tau \equiv (b-a)/N$ and $[a, b] \subset h$. The RV is defined as the sum of squared high-frequency returns as given by;

$$RV_{i,h} = RM(2)_{i,h} \equiv \sum_{i=1}^{N} r_{i,h}^2 \rightarrow \int_0^t \sigma_D^2 dt + \sum_{k=1}^{N(t)} J^2(Q_k), \quad as \quad N \rightarrow \infty \tag{3.4}$$

The RV is an efficient estimator of the quadratic variation, it converges to the QV as the number of observation (N) goes to infinity $(RV_{[a,b]}^{(N)} \rightarrow QV_{[a,b]}$ as $N \rightarrow \infty$ (see Andersen and Bollerslev, 1998; Barndorff-Nielsen and Shephard, 2002). It is also apparent from Equation 3.4 that in the absence of jumps RV converges to the IV.

In contrast to realized variance, realized skewness and realized kurtosis have to-date received minimal attention in the financial time series literature. To the best of our knowledge, Amaya et al. (2015) is the first published paper to investigate realized skewness and realized kurtosis. Following Amaya et al. (2015), the RS is formally defined as the cubic intra-day returns normalized by the square-root of RV cubed and the RK as the sum of the quartic high-frequency returns normalized by RV squared;

$$RS_{i,h} = \frac{\sqrt{N} \sum_{i=1}^{N} r_{i,h}^3}{RV_{i,h}^{3/2}} \tag{3.5}$$

$$RK_{i,h} = \frac{N \sum_{i=1}^{N} r_{i,h}^4}{RV_{i,h}^2} \tag{3.6}$$

We next derive and define the limits of realized skewness and kurtosis of a jump-diffusion process in the context of the holding- and sampling-intervals. Following Amaya et al. (2015), the third and fourth realized moments can be defined as;

$$\begin{aligned} RM(3)_{i,h} &\equiv \sum_{i=1}^{N} r_{i,h}^3 \rightarrow \sum_{k=1}^{N(t)} J^3(Q_k), \quad as \quad N \rightarrow \infty \\ RM(4)_{i,h} &\equiv \sum_{i=1}^{N} r_{i,h}^4 \rightarrow \sum_{k=1}^{N(t)} J^4(Q_k), \quad as \quad N \rightarrow \infty \end{aligned} \tag{3.7}$$

where the third realized moment converges to the sum of cubic jumps and the fourth realized moment converges to the sum of the quartic jumps. In other words, the realized third higher-order moment captures the sum of the cubic jumps and the realized fourth higher-order moment captures the sum of the quartic jumps. Consequently, for $RM(4)$, only the magnitude of the jumps are relevant and not the direction (Amaya et al., 2015, see). These jump-driven convergences conform to the findings of Kim and White (2004), who find that estimates of the higher moments of distributions of high-frequency data are heavily influenced by the presence of jumps (and outliers).

The limits of realized second moment $LRM(2)$, realized third moment $LRM(3)$ and realized fourth moment $LRM(4)$ as found in Hanson and Westman (2002), Matsuda (2004) and Amaya et al. (2015) are as follows:

$$\begin{aligned} LRM(2) &= (\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2))\tau, \\ LRM(3) &= \lambda(\mu_J^3 + 3\mu_J\sigma_J^2)\tau, \\ LRM(4) &= \lambda(\mu_J^4 + 6\mu_J^2\sigma_J^2 + 3\sigma_J^4)\tau, \end{aligned} \tag{3.8}$$

where $\tau \equiv h/N$, μ_J, and σ_J^2 are the mean and variance of the jump component,

σ_D^2 is the variance of the diffusive component, λ is the jump arrival rate (or jump rate). The limits of the second realized moments consist of both the diffusion and jump parameters while the third and fourth realized moment comprises of only the jump parameters. We can then define the limits for realized variance (LRV), realized skewness (LRS) and realized kurtosis (LRK) as follows:

$$\begin{aligned} LRV &= N \times \tau(\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2)) \\ &= N \times \frac{h}{N}(\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2)) \\ &= h(\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2)) \end{aligned} \tag{3.9}$$

$$\begin{aligned} LRS = \frac{LRM(3)}{(LRM(2))^{3/2}} &= \frac{\lambda(\mu_J^3 + 3\mu_J\sigma_J^2)\tau}{((\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2))\tau)^{3/2}} \\ &= \sqrt{\frac{N}{h}} \frac{\lambda(\mu_J^3 + 3\mu_J\sigma_J^2)}{(\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2))^{3/2}} \end{aligned} \tag{3.10}$$

$$\begin{aligned} LRK = \frac{LRM(4)}{(LRM(2))^{2}} &= \frac{\lambda(\mu_J^4 + 6\mu_J^2\sigma_J^2 + 3\sigma_J^4)\tau}{(\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2))\tau)^{2}} \\ &= \frac{N}{h} \frac{\lambda(\mu_J^4 + 6\mu_J^2\sigma_J^2 + 3\sigma_J^4)}{(\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2))^{2}} \end{aligned} \tag{3.11}$$

From equations 3.10 and 3.11, it can be seen that both the numerators only consist of the jump parameters whereas the denominators include both the diffusion and jump parameters. As such the information content embedded within realized skewness and realized kurtosis is analytically different from that embedded within the sample skewness and sample kurtosis computed using low-frequency return data.[3] This finding is corroborated by Amaya et al. (2015) who find that the sample skewness and sample kurtosis using long samples of low-frequency return

[3]The central limit theorem states that the skewness goes to zero and the kurtosis approaches three, as the number of observation approaches infinity. We observe that this only holds for stock prices generated from a pure diffusive process with no jumps.

data implicitly include both the diffusive and jump components. This subtle gloss over is seldom explicitly highlighted in the extant high-frequency finance literature.

In addition, equations 3.10 and 3.11 also delineate the relationships between the normalized realized higher-order moments and the holding- and sampling-intervals i.e. the effects of the two types of intervaling. Hence, from Equation 3.10, for any stationary jump-diffusion data generating process, there is a non-linear relationship between realized skewness and the number of infill observations for a given holding-interval. On a similar note, from Equation 3.11, a linear relationship holds for realized kurtosis and the number of infill observations for a given holding-interval.

Assuming that the jump-diffusion parameters are stationary, the above two equations basically characterize the shape (i.e. curvatures and slopes) of the ensuring realized skewness and realized kurtosis relationships with the number of infill observations in count-based signature-plots (conditioned on the holding-intervals). The effects of any non-stationary jump parameters (i.e. jump means, variances, and intensities) on the slope and curvature of the count-based signature-plots are not investigated in this study. This is left for future research.

3.3 Simulation

Any simulated price process for a typical price series should accommodate the following stylized properties: (i) jumps or large random fluctuations; both positive and negative; (ii) asymmetries or skewness; both negative and positive (iii) elongations or kurtosis. To encapsulate above the three properties, we follow Hanson and Westman (2002) and Synowiec (2008) and define an independent identically distributed (iid) price generation process p_t that satisfies a Markov, continuous-time, geometric, jump-diffusion stochastic differential equation:

$$dp_t = p_t((\mu_D - \lambda\kappa)dt + \sigma_D dW_t + J(Q)dN_t), \quad p_0 = 0 \quad p_t > 0 \tag{3.12}$$

where μ_D is the diffusive mean return rate, σ_D is the diffusive volatility, W_t is a continuous, one-dimensional Brownian motion process, $J(Q)$ is a log-return mean μ_J and variance σ_J^2 random jump amplitude and N_t is a discontinuous, one-dimensional Poisson process with arrival rate λ, $\kappa = exp(\mu_J + \frac{\sigma_J^2}{2}) - 1$. Similar to Matsuda (2004), we allow an adjustment of $\lambda\kappa$ in the drift term to ensure that the jump component is an unpredictable process. We assume μ_D, σ_D^2, μ_J, σ_J^2 and λ are constant. The source of randomness, W_t and N_t and $J(Q)$ are assumed to be independent. The discontinuous space-time jump process is defined as a Poisson process:

$$\int_{t_1}^{t_2} J(Q)dN_t = \sum_{k=1}^{N_{t_2-t_1}} J(Q_k),$$

where Q_k is a sequence of independent identically distributed random variables with the assumption that $\sum_{k=1}^{0} J(Q_k)$=0. For a filtered probability space (Ω, $\mathcal{F}$, $(\mathcal{F}_t)_{t\geq 0}$,$\mathsf{P}$) we have $\mathsf{P}(N_t = x) = \frac{(\lambda t)^x}{x} e^{-\lambda t}$. Given that $Q(J) > -1$ ensures that a single jump does not make the asset worthless, therefore, selecting Q =ln(1 $+J(Q)$) so that the constraints of $J(Q)$ above holds. With the help of Itô's lemma [6] one can simplify Equation 3.12 to obtain Equation 3.13 below (see Synowiec, 2008),

$$d(\ln(p_t)) = \left(\mu_D - \frac{1}{2}\sigma_D^2 - \lambda\kappa\right)dt + \sigma_D dW_t + QdN_t, \tag{3.13}$$

Integrating both sides of Equation 3.13 over (0,t), yields the general formula for the stock price at time t as

$$p_t = p_0 exp\left(\left(\mu_D - \frac{1}{2}\sigma_D^2 - \lambda\kappa\right)t + \sigma_D W_t + \sum_{k=1}^{N_t} Q_k\right). \tag{3.14}$$

The distribution of the price process above is depended on the distribution of the log-return jump amplitude Q, there are several distributions that could be considered, for simplicity we stick to the normal distribution, we let Q follow a normal distribution of $N(\mu_J, \sigma_J^2)$.

Using Equation 3.14, we undertake a Monte Carlo simulation of a typical price series to illustrate graphically the effects of the two types of intervaling mentioned previously. Figure 3.1 displays the signature-plot for realized higher-order moments computed from a jump-diffusion prices process with the following parameters: μ_D=6.0125e-07, the mean of the diffusion component; σ_D=2.405e-06, the volatility of the diffusion component; μ_J=6.0125e-05 the mean of the jump component; σ_J=6.0125e-05, the volatility the jump component and $\lambda = 0.1$, the arrival rate or jump intensity. We generate a total of 1,000,000 observations at a sampling-interval of τ=1 minute. The above sample size and parameters are chosen to reflect the average sizes and properties of the sampled dataset downloaded for the empirical section. We construct daily, weekly and monthly realized higher-order moment from 1-minute simulated return data, with different sampling-interval.

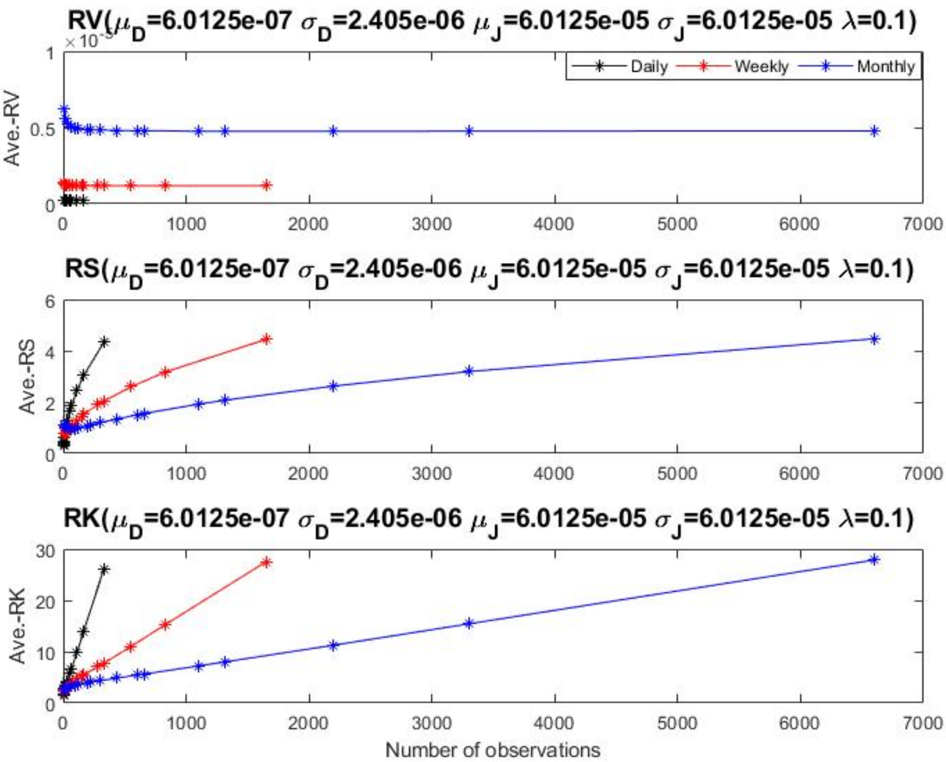

Figure 3.1: Average RV, RS and RK values (jump-diffusion simulation)

From Figure 3.1, we observe that realized skewness and kurtosis exhibit a positive (negative) relationship with the number of infill observations (sampling-frequency). For realized skewness, the relationship is non-linear while for kurtosis the relationship is linear. However, the corresponding magnitudes at a given sampling-interval for all holding-intervals are the same. This suggests that for an iid jump-diffusion process with constant parameters, holding-intervals do not influence the average realized skewness and kurtosis. In addition, increasing the number of infill observations (or increasing the sample-frequency or decreasing the sampling-intervals) results in an increase in realized skewness and kurtosis for all holding-intervals. A 1-minute sampling-interval has a higher average realized skewness and kurtosis than for a 60-minute sampling-interval. In addition, the larger (smaller) the number of infill observations (or sampling-frequency) for a given holding-interval the greater (lesser) the likelihood of jumps. Thus, the holding-intervals and the sampling-intervals have distinct effects on realized skewness and kurtosis.

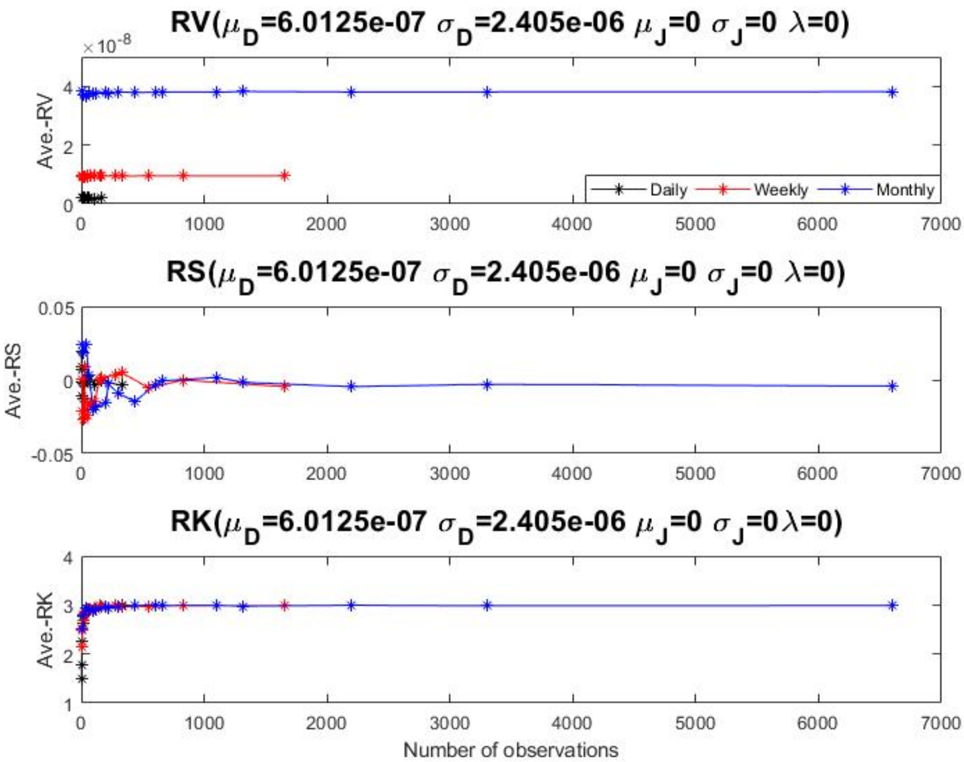

Figure 3.2: Average RV, RS and RK values (pure-diffusion simulation)

In Figure 3.2, we show the count-based signature-plot of the realized higher-order moments for a pure diffusion process, where there are no jumps and with the mean and standard deviation set to μ_D=6.0125e-07 and σ_D=2.405e-06 as before. We observe, as per the central limit theorem, that the realized skewness converges to zero and the realized kurtosis converges to three as the number of infill-observations increases or the sampling-interval decreases (see Lau and Wingender, 1989).[4]

From Figures 3.1 and 3.2 it is clear that the realized skewness and kurtosis of an iid jump-diffusion process with stationary parameters is primarily influenced by the presence of jumps, the holding-intervals, and the sampling-intervals. In addition, the sampling-intervals influence the magnitudes of the ensuring values for a given holding interval and for a given sampling-interval the magnitude of the values are the same for all holding-intervals. Another point to note is that, as the sampling-interval tends to the holding-interval i.e. at low frequencies, both the magnitudes of the realized skewness and realized kurtosis tends to the asymptotic sample skewness and kurtosis values of zero and three accordingly (as clearly shown near and at the origins of the second (RS) and third (RK) panels in Figure 3.1). In other words, as one increases the sampling-interval over which the returns are calculated i.e at low frequencies, the sample distribution looks more and more like a normal distribution. In particular, 'the shape of the distribution is not the same at different time scales' (see Cont, 2001).

3.3.1 Annualization/Projection

The annualization or projection of variance (volatility) to alternative holding-periods is easily carried out using the 'multiplicative' ('square-root') rule (see Meucci, 2010; Wong and So, 2003; Meucci, 2007). As the projected variances and volatilities are interval-invariant one can project the volatility of the estimated return series by multiplying by the square root of the ratio of projected holding period to the estimated holding period. The daily volatility is usually annualized by multiplying the volatility by $\sqrt{250}$ (i.e.$\sqrt{250}\sigma$) thus assuming there are 250 trading days per year. This is because realized variance (realized volatility) is a dimensional measure and hence the multiplicative (square-root) rule holds when

[4] In the case of realized variance, the greater the holding-interval the higher the variations and hence the realized variance increases. However, the estimates of realized variance become more efficient (and converges) as the sampling-interval decreases i.e at higher frequencies.

projected onto different horizons. As expected the annualized signature plots of the various holding period variances are bound to be horizontal with coincident estimates for all intervals. We can observe this in the first panel in Figure 3.3, which is consistent with Meucci (2010); Wong and So (2003); Meucci (2007).

The second panel in Figure 3.3 displays the signature plots of annualized realized skewness versus the number of observations. We observe that when respective holding-interval realized skewness are annualized, as per $RS_{yearly} = \frac{1}{\sqrt{250}} RS_{daily}$, $= \frac{1}{\sqrt{52}} RS_{weekly}$, $= \frac{1}{\sqrt{12}} RS_{monthly}$, the annualized/projected realized skewness measures are neither coincident nor horizontal. All the annualized RS estimates fall onto a common quadratic curve as a direct consequence of the multiplier $\sqrt{N}$. However, the individual realized skewness estimates at each of the holding-interval are not coincident (e.g. the 1-minute daily, weekly and monthly estimates all differ) unlike the realized variance estimates. Thus, there is a clear non-linear relationship between the annualized values and the number of infill observations, clearly indicating that the annualized values are directly dependent on the number of observations used i.e. the values are not holding-interval invariant.

The third panel in Figure 3.3 displays the average excess realized kurtosis, XRK. The annualization is obtained by setting $XRK_{yearly} = \frac{1}{250} XRK_{daily}$, $\frac{1}{52} XRK_{weekly}$, $\frac{1}{12} XRK_{monthly}$. This also shows that realized kurtosis relationship is not horizontal (i.e. not interval-invariant) when the daily, weekly and monthly estimates are annualized to a common horizon or holding period. The realized kurtosis when annualized have differing values and consequently are not interval-invariant i.e. the annualized values are linearly dependent on the number of infill observations (N) and conditional on the holding-interval.

From Figure 3.3, we note that the projection rule only holds for the expected values and covariances and not for the normalized higher order moments (i.e. realized skewness and realized kurtosis). Consequently, realized variance is interval invariant but the subsequent two higher order moments i.e. realized skewness and realized kurtosis, are not.[5]

[5] See the errata in the online Technical Appendices to Chapter 3 of Meucci (2007), where the

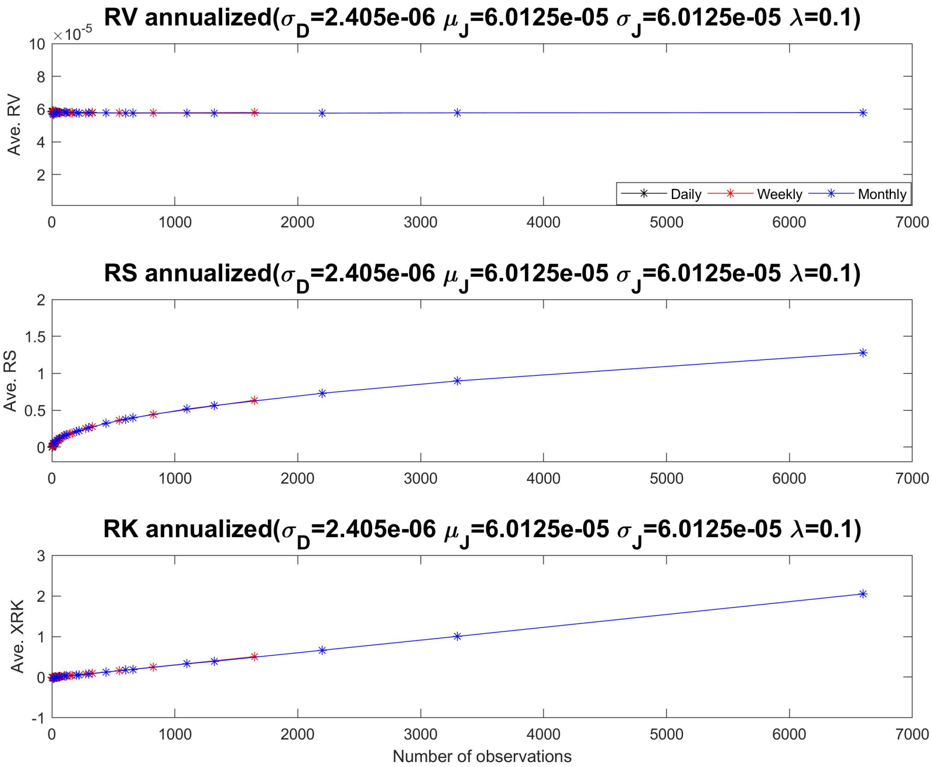

Figure 3.3: Annualized average RV, RS and RK values (jump-diffusion simulation)

3.4 Empirical Data

In the above simulation, we assumed that the supposedly 1-minute returns were stationary and identically and independently distributed (iid). In reality, the actual returns are generally not stationary nor iid and might even be autocorrelated (see Hansen and Lunde (2003, 2004, 2006) and Zhou (1996)) and consequently, the ensuring empirical signature-plots will differ in some ways from the simulated signature-plots.

Hence, in this section we undertake an empirical investigation, using high-frequency price index data from the G7-countries.[6] To minimize the effects

author has explicitly stated that 'the multiplicative relation does not hold for all raw moments and all central moments, but only holds for the projection of expected values and covariances'. The online Technical Appendices is available at https://www.arpm.co/symmys-articles/AMeucciRiskAndAssetAllocationTechnicalAppendices.pdf.

[6]The sample period for US, UK, Japan, and Canada starts from 1 May 2002 - 15 November

of market microstructure effects, the raw prices were downloaded at 1-minute intervals (and not at the 1-second intervals commonly used).[7] The data was also truncated by sampling only the raw price for each index 30-minutes after the relevant market opened and 30-minutes before it closed. This ensured that any extreme outliers due to illiquidity are removed, without any significant loss of information. We also delete overnight prices from the data before estimating the returns. We next construct the infilled price series $\{p(t)\}_{t \equiv [0,\infty)}$ artificially from the raw price data using the previous tick method that was proposed by Wasserfallen and Zimmermann (1985).[8] The sampled prices enable us to construct an equidistant (or regular) return series for computing the realized higher-order moment at any suitable sampling-interval.

We next compute the daily, weekly and monthly normalized realized higher-order moments from the high-frequency data series (i.e. realized skewness and kurtosis). The returns for each market index were obtained by taking the log differences of the high-frequency prices. Our weekly and monthly realized higher-order moments are different from the daily averaging method commonly used in estimating weekly observations in the literature (see Amaya et al., 2015).[9]

2017. For France, Germany, and Italy due to the fact that the data with the same length as those mentioned earlier was not available from the Thompson Reuters Tick History, the data sample for France starts 2 May 2002 - 15 November, 2017, Germany from 6 May 2002 - 15 November 2017 and Italy from 1 June 2009 - 15 November 2017.

[7]Using Monte Carlo techniques and 1-second data, Amaya et al. (2015) verified that estimates of the realized higher moments are reliable in finite samples and that at 1-minute return series are robust to the presence of market microstructure noise.

[8]Ideally, one should construct realized higher-order moments from high-frequency intra-day return data of the true price process, $p^*(t)$. However, $p^*(t)$ is unobservable and hence the realized higher-order moment are computed from observed prices $p(t)$ which are contaminated with market microstructure noise (Hansen and Lunde, 2004). The observed price is defined as $p(t) \equiv p^*(t) + u(t)$, where $u(t)$ is the noise component that may arise from the bid-ask bounce, price discreteness, rounding errors and price reporting error (see Bai, 2000; Andreou and Ghysels, 2002; Oomen, 2004b). In this study, we assume there are no market microstructure noise effects and consequently, for our empirical work, we download raw data at the 1-minute sampling frequency and not at the higher 1-second frequency. As such our findings are confined to 'high' frequency data and not to 'very high' frequency data; this being left for a subsequent study and paper.

[9]In order to obtain weekly observations, Amaya et al. (2015) first construct daily realized moments and then take the average in a 5-days window to obtain their weekly realized moments. These so-called weekly observations are just an average of daily realized moments. Our weekly and monthly data are computed using the relevant full holding-interval.

The number of infill-observations (or the sampling-frequency) is given by N=h/τ, where h is the holding-interval (h_{day}, h_{week}, and h_{month}) and τ is the sampling-interval as defined above. Normalizing the holding-interval by dividing with the sampling-interval enables the higher-order moments to be compared across different holding periods because N is a measure of the number of infill observations within each holding-interval relative to the sampling-interval. Suppose we want to compute the daily realized variance, skewness, and kurtosis, for example, the UK (FTSE 100 index) from 08:30 am to 4:00 pm, we set h_{day}=450 (1-minute each trading day). We divide h by τ to obtain the number of infill-observations at each sampling-interval for each τ. The values of τ are carefully selected, in order to prevent the length of the returns from being fractions. Therefore, at the τ = 10-minutes sampling-interval, the number of infill-observations will be 45 (N_τ = 450/10). For the weekly realized moments, we have h_{week} = 450 minutes×5 days = 2,250 infill-observations. The τ = 10 minutes sampling-interval has 225 weekly observations ($N_\tau = 2,250/10$). The monthly realized moments has h_{month}=450 minutes×5 days×4 weeks = 9,000 observations as such consists of 900 ($N_\tau = 9,000/10$) infill-observations. The span of the data sample for UK index yields 3,916 trading days, 784 trading weeks and 196 trading months. We repeat this process to compute realized higher-order moments at the various holding-intervals and sampling-intervals.

3.5 Empirical Results

In this section, we discuss the various count-based signature-plots generated using the raw 1-minute price data of the seven market indexes that comprise the 'G7' countries.

Figure 3.4, is the signature-plot for the normalized realized higher-order moments for the US (S&P 500 index). In Figure 3.4, the realized variance for S&P500 fluctuates at the lower infill observation numbers and then stabilizes as the number of infill increases. An increase in the holding-interval results in an increase in

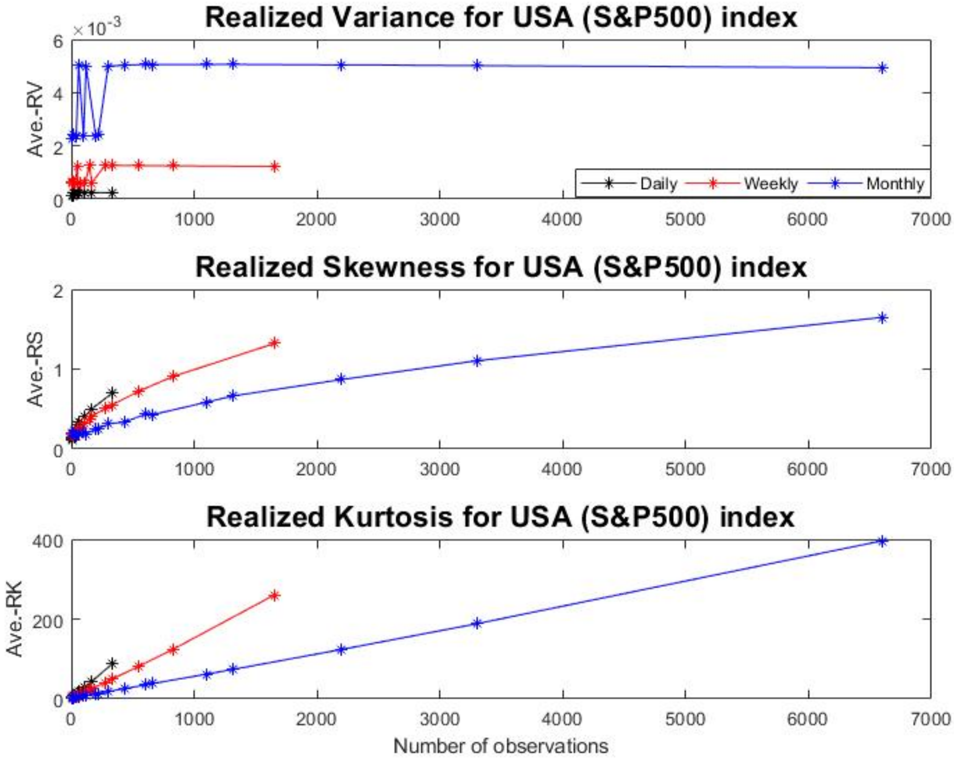

Figure 3.4: Average RV, RS and RK values for US (S&P 500 index)

the realized variance. The realized skewness and realized kurtosis signature-plots are symptomatic of the simulated jump-diffusive process (see Figure 3.1). This augurs well with our premise that high-frequency price processes are essentially jump-diffusion processes. However, an increase in holding-interval for realized skewness and realized kurtosis results in a non-equivalence in the corresponding magnitudes computed at the same sampling-interval. This is in agreement with the reported observation that the 'realized skewness of the market index actually increases with the horizon' (see Neuberger, 2012). The non-equivalence of the RS and RK values at each sampling-interval across different holding-intervals (see for instance in the bottom panel of Figure 3.4, the terminal RK values at the top end of the holding-interval lines where the sampling-interval is 1-minute) indicate either non-stationarities in the jump-diffusion parameters or auto-correlations in the observed price generation process.

In addition, as the number of infill observations increases (or sampling-interval decreases) the realized skewness and realized kurtosis increases. However, there are subtle differences between the realized skewness and realized kurtosis. Realized skewness has a non-linear relationship with the number of infill observations, whereas realized kurtosis has a linear relationship. On average, a positive realized skewness is observed across all holding periods, which is consistent with existing financial literature that asset market returns are mostly positively skewed (see Arditti, 1967, 1971; Beedles, 1979; Fielitz, 1976; Singleton and Wingender, 1986; Jurczenko and Maillet, 2006).

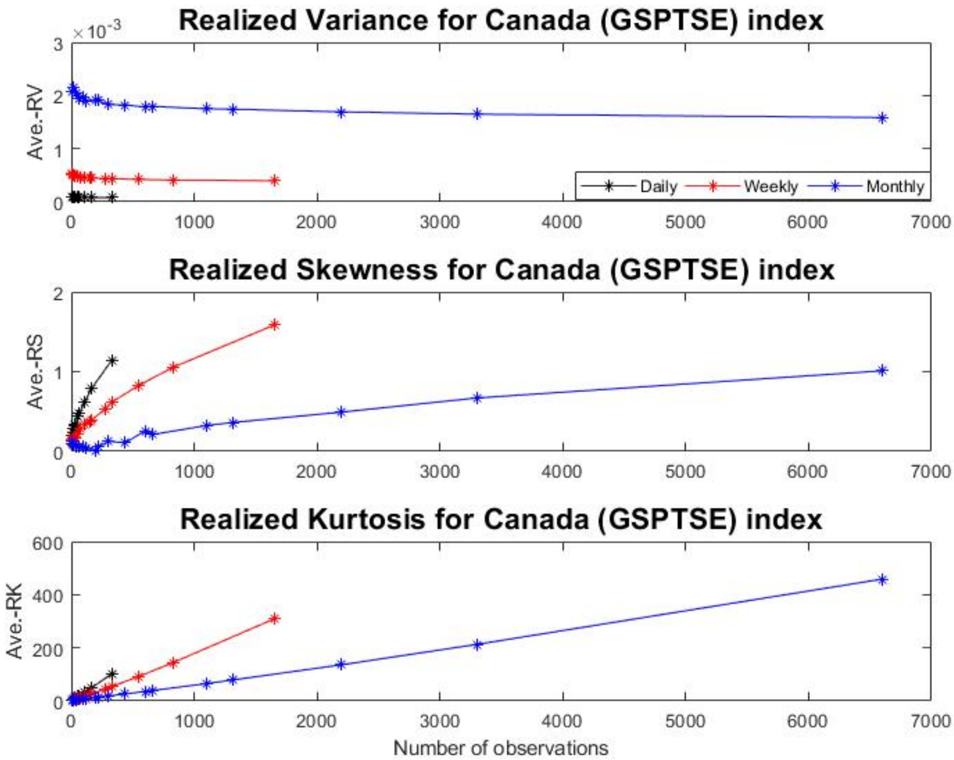

Figure 3.5: Average RV, RS and RK values for Canadian (GSPTSE index)

Figure 3.5 reports the signature-plot for the Canadian (GSPTSE index), the realized variance exhibits download sloping as the sampling-interval decreases. This pattern is associated with the realized variance being an inefficient estimate of volatility at small sampling-intervals with the noise component dying off at large sampling-intervals. In addition, as one would expect, an increase in holding-interval leads to an increase in realized variance.

The realized skewness is positive and increases for the daily, weekly and monthly moments, with few aberrant negatives values at lower sampling-intervals for the monthly holding-interval. However, the positive skewness dominates in this case as well. For the realized kurtosis, we observe that it monotonically increases as the sampling-interval decreases. Both the realized skewness and kurtosis at the 1-minute sampling frequency is relatively much higher for the weekly holding-interval than the daily-holding interval. This non-equivalence of the RS and RK values at each sampling-interval (at the terminal ends of each line) for the different holding-intervals, in Figure 3.5, indicate either non-stationarities in the jump-diffusion parameters or possible auto-correlations in the observed price generation process. In other words, intensity and magnitude of the jumps are much larger over the weekly (holding) horizon as compared to the daily or monthly (holding) horizons.

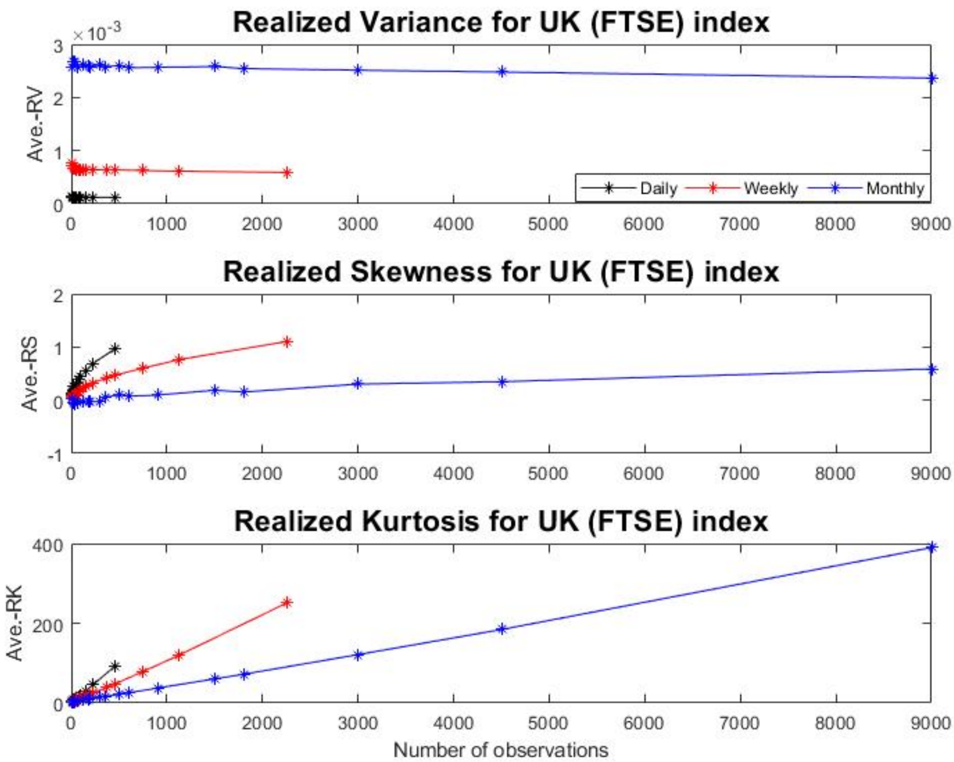

Figure 3.6: Average RV, RS and RK values for UK (FTSE 100 index)

Figure 3.6, reports the signature-plot for the UK (FTSE index) realized higher-order moments. The realized variance exhibits a decreasing pattern as the sampling-interval increases. Its size differs with each holding-interval. Interesting, the monthly realized skewness is relatively flat in comparison to its daily and weekly counterparts as the sampling-interval decreases. This subtle pattern is similarly repeated for the realized kurtosis. This indicates that jumps are highly prevalent over daily holding horizon and then quickly settle down over the weekly and monthly horizons. Possibly the FTSE market is able to immediately disseminate any new information over the short term via jumps and then stabilize over the long term.

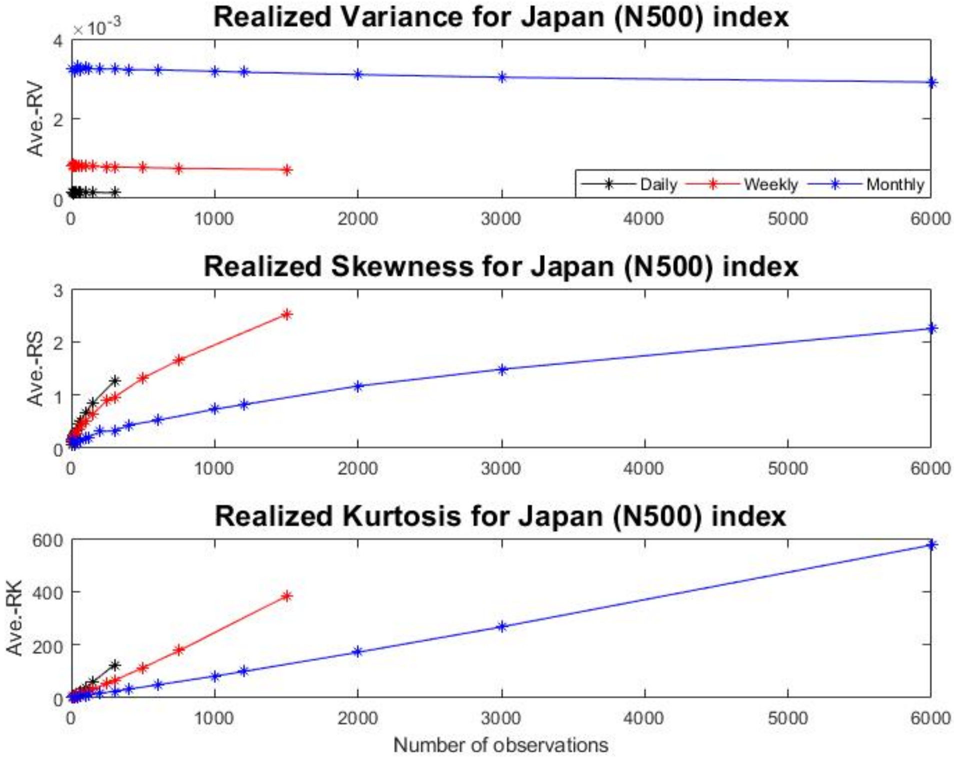

Figure 3.7: Average RV, RS and RK values for Japan (Nikkei 500 index)

Figure 3.7 represents the signature-plots for Japan (i.e. Nikkei 500 index). The realized variance seems to decrease for each holding-interval as the sampling-interval decreases. On another hand, the realized skewness is positive and upward increasing. The realized kurtosis for Nikkei index also exhibits similar patterns to that of the stock-markets discussed earlier, with the exception of the FTSE. As before, both the realized skewness and kurtosis at the 1-minute sampling frequency is relatively much higher for the weekly holding-interval than the daily-holding period. Once again, the non-equivalence of the RS and RK values at each sampling-interval (at the terminal end of each line) for the different holding-intervals, in Figure 3.6, indicate either non-stationarities in the jump-diffusion parameters or possible auto-correlations in the observed price generation process.

Lastly, we discuss the signature-plots of Germany, Italy, and France (as shown in Figure 3.8, Figure 3.9, Figure 3.10) as a sub-group since they are from the European countries and consequently have the same trading hours. Realized variance and realized kurtosis in the case of Germany and Italy indexes are very similar. However, the realized skewness differs specifically for the monthly realized skewness. The realized skewness for Italy has a distinct negative slope and negative values for the monthly interval as the sampling-interval decreases. However, the German realized skewness increases for all holding-intervals as sampling-interval decreases.

France, on the other, hand shows realized skewness to be constant for the monthly holding-interval as sampling-interval decreases. The realized kurtosis for France is similar to its counterparts realized kurtosis in the European group. France's realized variance seems to decrease for each holding-interval as the sampling-interval decreases. The signature-plots for realized kurtosis for these three European markets are very similar to that of the UK market, indicating the same dominance of jumps in the daily horizons as compared to the weekly horizons. The realized skewness signature-plot for Italy, particularly the monthly (longer term) holding horizon is able to capture the downturn in the Italian market over the recent decade.

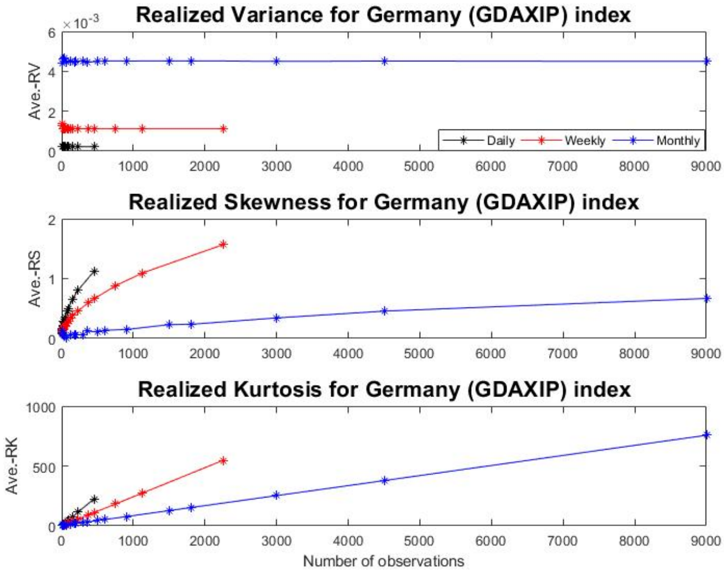

Figure 3.8: Average RV, RS and RK values for Germany (GDAXIP index)

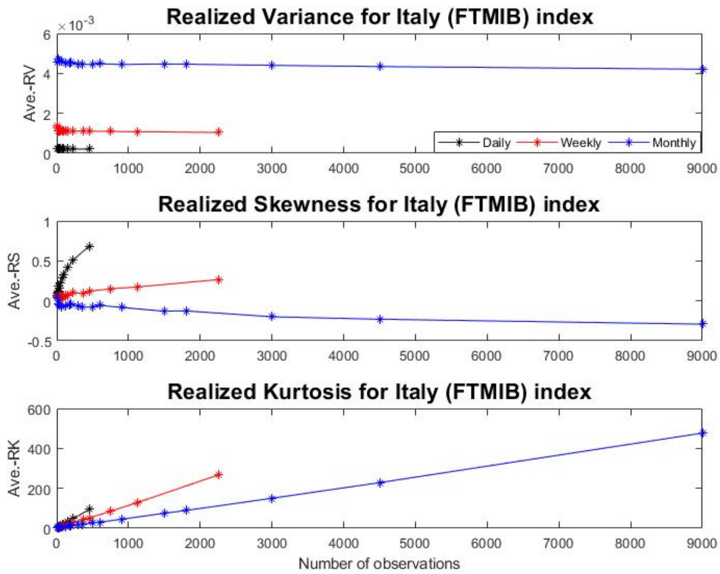

Figure 3.9: Average RV, RS and RK values for Italy (FTMIB index)

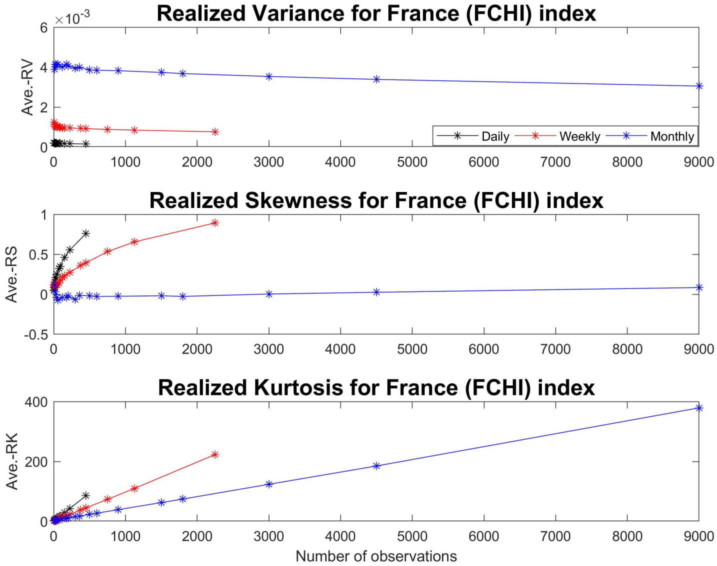

Figure 3.10: Average RV, RS and RK values for France (FCHI index)

The differences between the realized higher-order moments for various G7 equity markets considered in this study highlight a 'stylized' fact that though the equity markets are similar in many aspects of their higher-order moment characteristics, they differ subtly and substantially individually. Having presented the results of the G7 countries individually, we here discuss the findings as a group to highlight these subtle yet substantial differences, if any, in their higher-order moments i.e. realized skewness and realized kurtosis.

Figures 3.11 and 3.12 show respectively the RS and RK signature-plots with all the G7 countries overlaid in one plot with a scaled-up inset-plot. The shorter (longer) the holding-interval i.e. days, the steeper (flatter) the slopes of the plotted lines by construct. However, there is much variation in the actual realized skewness and realized kurtosis values between the different markets. Both panels display a stylized fanning-out trace with one obvious difference: the RS lines are non-linear and the RK lines are linear; thus conforming with the equations 3.10 and 3.11.

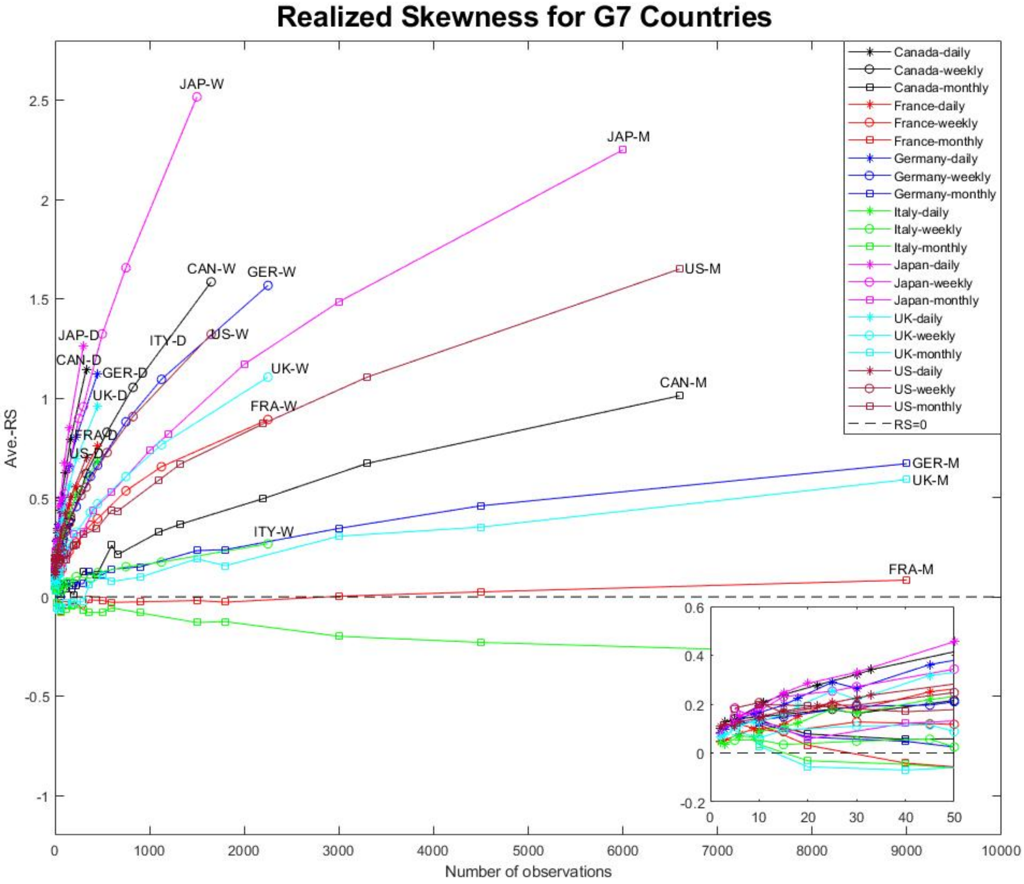

Figure 3.11: Average RS values for G7 Countries. The inset RS signature-plot highlights the asymptotic convergence of the RS values to zero near the origin.

In Figure 3.11, the daily and weekly averaged realized skewness values are all positive whereas the monthly averaged values are mixed, with Italy being clearly negative across all sampling-intervals for the monthly holding-period. There is a tendency for all holding horizons to be 'relatively more skewed' as we increase the infill observations or as we decrease the sampling-intervals and/or holding-intervals. This is in accordance with the high-frequency sampling asymptotics as implied by the equations 3.10 and 3.11 above. Individually, for the G7 countries, Japan has the highest RS values and Italy has the lowest RS values for all three holding-intervals. As a group, Japan, Canada, and the USA RS-dominate the other countries sampled and Italy, France and the UK are RS-dominated by the others. In addition, France and Italy display RS values with sign changes i.e. from positive

RS to negative in going from daily and weekly holding-horizons to monthly-holding horizons, indicating strong auto-correlations and/or non-stationarities (see Lau and Wingender, 1989). Germany's RS lines sit in the middle of the G7 sample for all three holding-periods studied.

Finally, the RS signature-plot inset at the bottom right of Figure 3.11 clearly shows that as the number of infill observations tends to zero, the RS values tend to zero for all holding-intervals concerned i.e. for low sampling-intervals the RS values are lower in absolute values than the corresponding RS values at higher sampling-frequencies for all holding-intervals.

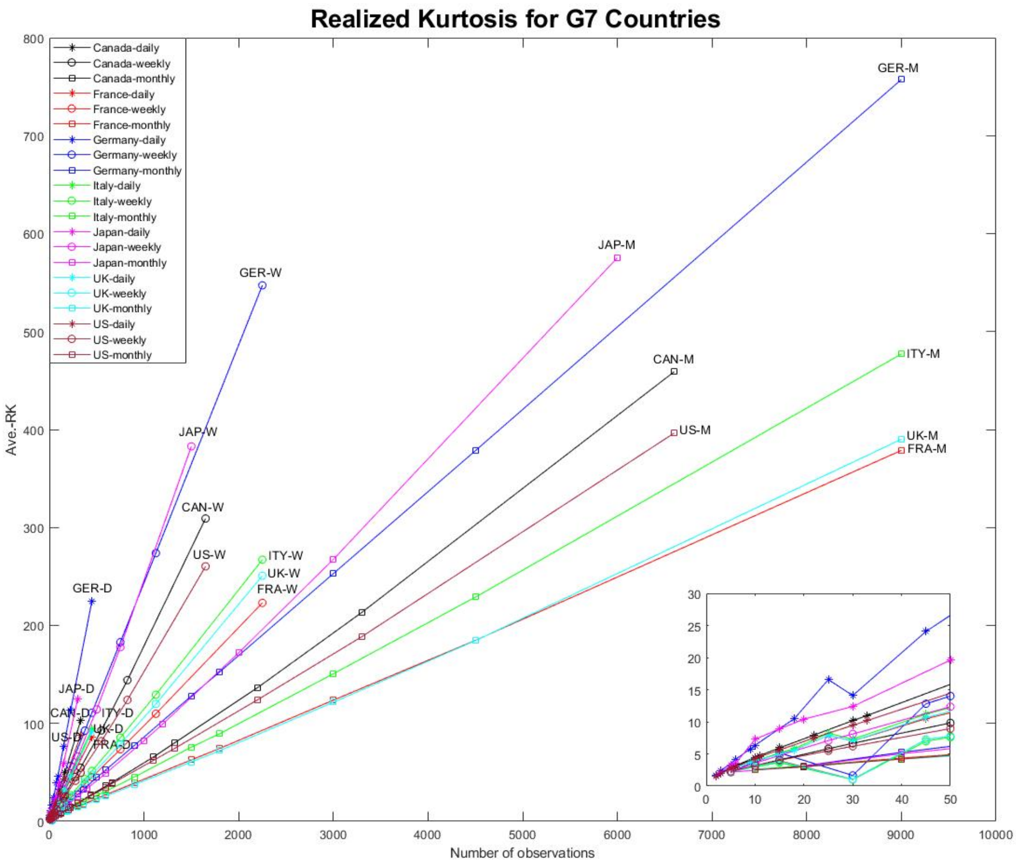

Figure 3.12: Average RK values for G7 Countries. The inset RK signature-plot highlights the fact that as the number of infill observations tends to zero, the RK values tends to three (or even lower) for all holding-intervals.

In Figure 3.12, the non-equivalent nature of the RK values at the 1-min sampling-interval (at the end of each line) for different holding-intervals further indicates non-constant or non-stationary jump-diffusion parameters within and between the G7-countries. This non-constancy in the parameters could be caused by an overall change in jump intensities over time or by an overall change in market volatility without necessarily an increase or decrease in the jump intensity (see Hanousek et al., 2014). Alternatively, it could be due to a combination of changes in all the jump-diffusion parameters: time-varying jump intensities, time-varying jump, and diffusion volatilities and drifts. In addition, it is also common knowledge that in financial markets not only volatilities but also jumps are frequently clustered i.e. the occurrence of a jump in price immediately increases the probability of observing new jumps and consequently their long-run frequency (see Hainaut and Moraux, 2017). This will further condition the jump-diffusion process and affect the characteristics of the observed signature-plots. Individually, for the G7 countries, Japan has the highest RK values and France has the lowest RK values for the same number of infill observations. As a group Japan, Germany and Canada RK-dominate the other countries and Italy, France and UK are RK-dominated by the others. US falls in the middle of the signature-plot.

Finally, the RK signature-plot inset at the bottom right of Figure 3.12 highlights the premise that as the number of infill observations tends to zero, the RK values tends to three (or even lower) for all holding-intervals i.e. at low sampling-frequencies the realized kurtosis values are lower than the corresponding RK values at higher sampling-frequencies for all holding-intervals.

3.6 Concluding Remarks

In this study, we investigate, theoretically and empirically, how the normalized realized higher-order moments of high-frequency returns are affected by these two types of intervaling. We first determine the theoretical limits of the expected values of the realized skewness and realized kurtosis for a (high-frequency) jump-diffusion process. We then simulate an iid jump-diffusion process and generate count-based signature-plots for latter comparisons with the empirical data. We note that the stylized features of these signature-plots are quite different for a diffusion process and a jump-diffusion process. Finally, using 1-minute market-index data from the G7-countries (from Thompson Reuters Tick History/SIRCA database), we compute the realized skewness and realized kurtosis values for three holding-intervals (i.e. days, weeks and months) and a number of sampling-intervals and showcase the ensuring signature-plots.

Our simulated and empirical findings are similar but not identical. For the simulated iid jump-diffusion data, we find that realized skewness and kurtosis exhibit a positive relationship with the number of infill observations (or sampling-frequencies). For realized skewness the relationship is non-linear and for kurtosis the relationship is linear. In addition, the corresponding magnitudes at a given sampling-interval for all holding-intervals are asymptotically equivalent. Also, increasing the number of infill observations (or decreasing the sampling-intervals) increases the realized skewness and kurtosis for all holding-intervals. For the empirical data, we also find that for a given holding-period, as the number of infill-observations increases, both the realized skewness and kurtosis increases. However, for a given number of infill observations, as the holding-interval increases, the realized skewness has mixed outcomes (i.e. positively and negatively sloped) whereas the realized kurtosis increases i.e is positively sloped). In other words, there is a greater degree of variability in realized skewness than in realized kurtosis over sampling-frequencies and holding intervals.

The central limit theorem for realized skewness and realized kurtosis of high-frequency data only holds at the limit when jumps are few and far in-between or are implicitly assumed away as when the size of the sampling-interval approaches the holding-interval. For such pure diffusion processes, all realized moments implicitly converge to their corresponding sample moments as asymptotically indicated by the limiting realized moment equations and in our simulations. However, for high-frequency returns data, realized moments converge to sample moments only for the ‘variance’ measure but not for the corresponding ‘skewness and kurtosis’ measures. This is directly due to the presence of ‘jumps’ in the underlying price generation process at high-frequencies, which determine the computed values of RS and RK.

Finally, the implications of the two intervaling effects on realized skewness and realized kurtosis in high-frequency finance, particularly for asset pricing, are as follows:

- high-frequency pricing models using higher-order moments will be conditioned by the holding-intervals and sampling-intervals adopted.
- high-frequency pricing models will be driven by the direction and magnitude of jumps present on the price generation process. At low sampling-frequencies the higher-order moments will not play a significant role. This clearly would explain why pricing models tend to be driven by varying number of factors across different holding-intervals.
- any asset pricing differences between short-term, medium-term and long-term trading and/or investments are likely to be due to the implicit transition from a jump-diffusion process at high frequencies to a pure diffusion process at low frequencies. In other words, sampling-intervals and holding-intervals matter.

Consequently, researchers in high-frequency quantitative finance must be observant and wary of these two intervaling effects i.e. holding-interval effect and the sampling-interval effect, on computed values of realized skewness and kurtosis. In addition, these same effects will also be useful from a practitioner’s standpoint. For example, does the presence of these intervaling effects imply any possibilities or forecasting? If it does, can this be put to use to implement an effective risk measurement and management strategy? Can one exploit the relationships depicted by the count-based signature plots to implement a skewness and/or kurtosis trading strategy? We leave these questions for future research.

Chapter 4

Trading Volume and Realized Higher-Order Moments in the Australian Stock Market

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4.1 Introduction

The relationship between volatility and trading volume has received considerable attention in the extant literature for different financial markets. In a seminal review paper, Karpoff (1987) referenced a number of papers that showed a positive relationship between volatility and trading volume, detailing the empirical and theoretical contributions in the then extant literature.[1] Further investigations of

[1] According to Karpoff (1987), the benefits of investigating the volume-volatility relationship includes: (i) it provides insight into the structure of financial markets, (ii) it is beneficiary to event studies that employ volume and volatility for inferences and (iii) have significant implications for research into futures markets. Thus price variability/volatility affects the volume of trade in futures contract.

the volume-volatility relationships by others enabled researchers to better understand the behaviour of traders (i.e. both informed and uninformed traders) (see Kyle (1985); Jones et al. (1994); Chan and Fong (2000); Giot et al. (2010); Do et al. (2014)).

To-date, three distinct theoretical hypotheses have been proposed to explain the observed contemporaneous positive volume-volatility relationship. The first being the mixture of distribution hypothesis (MDH). The formulation of the MDH hypothesis was mooted by Clark (1973) and later extended by Epps and Epps (1976); Tauchen and Pitts (1983); Harris (1986) and Andersen (1996). A key insight of the MDH hypothesis is that trading volume and price volatility are both driven by the same latent (or hidden) mixing variable i.e. the rate of information flow. This implies that trading volume reacts to changes in the arrival rate of new information to the marketplace, and to changes in the dispersion of traders' opinion based on the content of the received information; Carroll and Kearney (2015). Chan and Fong (2006) also find that the arrival rate of new information to the market is the common factor that jointly drives trading volume and volatility. In short, the MDH hypothesis suggests a contemporaneous and instantaneous positive relation between trading volume and volatility conditional on the rate of information flow into the market.[2]

The second hypothesis is the sequential information arrival hypothesis (SIAH) by Copeland (1976). The SIAH hypothesis assumes that all traders receive new information in sequential time. Although traders change their opinions instantaneously when receive new and relevant information, the sequential nature of the information arrival process means that not all traders receive the same information contemporaneously; Celik (2013). The informed traders will trade with the uninformed traders in quantities as per the time-asymmetric information at hand. As these informed investors trade based on their 'advanced' information, volatility

[2]Kalev et al. (2004) using firm-specific announcements as proxy for information flow, investigate the information-volatility relation for Australian high-frequency data. They observe a positive and significant relationship between volatility and arrival rate of information, even after controlling for trading volume and high open volatility. Their results are consistent with the positive volume-volatility relationship proposed by MDH hypothesis.

will be affected positively; Chan and Fong (2000).[3]

The third hypothesis is the difference of opinion hypothesis (DOH) by Shalen (1993) and Harris and Raviv (1993). According to the DOH hypothesis, investors tend to trade based on their subjective beliefs; this happens when they form different opinions about the valuation of their traded assets or portfolios. This often occurs when public or market-wide information switches from favourable to unfavourable or vice versa. As these diversely opined investors trade based on their subjective beliefs, volatility is bound to increase. The DOH hypothesis was then further utilized by Hong and Stein (2003) to explain the effect of short-sales constraints on stock prices and the associated asymmetric downward distribution of returns. In addition, these authors suggested and showed that negative skewness tends to be more pronounced during high trading volumes.

The SIAH and DOH hypotheses both embed the volume-volatility relationship as special cases when there are no information (time- or space-) asymmetries present i.e. information is instantaneously and symmetrically absorbed into the trading markets. What is not apparent is that, under the SIAH hypothesis, the time-asymmetric objective beliefs of informed and uninformed traders are also embedded and can be proxied by skewness (i.e. good news/positive skewness or bad news/negative skewness) and under the DOH hypothesis, space-asymmetric subjective beliefs arising from difference of opinion between traders are additionally incorporated and can be proxied by kurtosis (as kurtosis implicitly measures additional dispersion over and above the standard dispersion or volatility). Thus both the later two hypotheses i.e. the SIAH and DOH hypotheses, are not only capable of depicting the relationship between volume and volatility (or variance), but also have the capability to depict the accompanying volume-skewness (a SIAH driven time-asymmetric effect) and volume-kurtosis (a DOH driven space-asymmetric effect) relationships as implied by the second and third hypothesis. Hence, the primary motivation for this study to use the higher-order realized moments to

[3] Shen et al. (2016, 2018) investigates the volume-volatility relationship by using Baidu News as the proxy of information flow for the Chinese stock market. They find that their empirical results support the SIAH hypothesis and reject the MDH hypothesis.

empirically find support for both the two latter hypotheses.

Another approach followed by a sub-group of volume-volatility relationship investigators was to decompose the observed (realized) volatility into continuous and jump components based on the seminal high-frequency work of Andersen and Bollerslev (1998); Barndorff-Nielsen and Shephard (2002); Meddahi (2002); Andreou and Ghysels (2002). Giot et al. (2010) investigated the relationship between continuous and jump components of realized volatility with the number of trades, absolute order imbalance, and average trade size. They found that only the continuous component of the realized volatility exhibited positive and significant relationships. Shahzad et al. (2014) similarly investigated the volume-realized volatility relation by splitting the realized volatility into continuous and jump components; their volume variable was further categorized into individual and institutional volumes. Although their results and significance were mixed, the authors showed that the number of trades of individual investors has a higher explanatory power in explaining the volume-realized volatility relationship in comparison to that of the institutional traders. Unfortunately, these researchers did not investigate the accompanying volume-skewness and volume-kurtosis relationships implied by the jumps components. In the extant literature, it is well documented that jumps reflect the market reaction to unexpected information, which implies that unexpected news is the primary driver of price jumps (see Lahaye et al. (2011); Lee and Mykland (2012)). This implies that jump-related measures might act as an ideal proxy for information arrival and can be utilized as tools for studying market behaviour (see (Fama and Malkiel, 1970)) or other phenomena like information-driven trading; see for example (Cornell and Sirri (1992); Kennedy et al. (2006); Hanousek et al. (2014)).

Our second motivation, thus arises from the fact that the third realized moment (realized skewness) converges to the sum of cubic jumps, and the fourth realized moment (realized kurtosis) converges to the sum of the quartic jumps Amaya et al. (2015). Hence, realized skewness captures the normalized direction and magnitude of cubic jumps, while realized kurtosis captures the normalized magnitude of quartic jumps (the jump directions are ignored here). As such, realized skewness and realized kurtosis capture additional relevant information that is not captured by splitting realized volatility into continuous and jump components. In the extant literature, Do et al. (2014) investigated the volume-higher-order realized moment relationship and showed that directional reactions towards good or bad news are captured by realized skewness and the magnitude of such reactions are captured by realized kurtosis. However, the volume-realized skewness results reported in the literature tend to be polarized. Hong and Stein (2003) show via the DOH hypothesis, that asset returns are more negatively skewed conditional on higher trading volume. This suggests the possibility of information asymmetries; arises from investors having differing information or opinions, resulting in different asset valuations.

The results of Hong and Stein (2003) are consistent with those of Chen et al. (2001); Hutson et al. (2008) but are inconsistent with the findings of Charoenrook and Daouk (2004); Hueng and McDonald (2005).[4] These findings of mixed results for the volume-realized skewness relationship may be attributed to the type of financial markets/assets considered, the arrival rates of information to the market, and/or differing opinions of investors relative to the information available. Albuquerque (2012) further suggests that the contradictory results obtained for skewness could be due to the different nature of firm-level skewness relative to that of the market-level skewness. For kurtosis, Do et al. (2014) observe a negative volume-realized kurtosis relationship, which can be explained by the DOH hypotheses.

To the best of our knowledge Do et al. (2014), are the only published paper that investigates volume-realized higher-order moments. However, the authors' focus on the spillover effects of higher-order realized moment risks and spillover effects of

[4] Chen et al. (2001) find that negative skewness is more pronounce in stocks that have experienced an increase in trading volume relative to trend over the prior six months. Hutson et al. (2008) shows that trading volume is associated with future negative skewness. Charoenrook and Daouk (2004) show that conditional skewness of daily aggregate market returns has no predictability with trading volume. Hueng and McDonald (2005) finds that negative skewness is not pronounced under high trading volume using daily NYSE and AMEX data.

trading volume across the 18 countries considered for stock and FX markets. They use the number of trades as their only proxy for information flow and construct daily higher-order realized moments from 5-minute return data for all countries. The key point to note is that generalization of the standard U.S. 5-minute unbiased sampling frequency to the rest of the 17 countries has a potential of resulting in significant accumulation of noise, which might impact the results and conclusion obtained.[5] It is well documented that the 5-minute sampling frequency does not hold for all markets/stocks.[6] This forms the motivation for employing the 15-minutes high-frequency return data instead of the 5-minutes.[7] Alles and Murray (2017) further stress that although Australia is a developed country, it's equity market requires a separate investigation as it differs in certain aspects from the other major international equity markets.[8]

Thus in this study, we investigate the volume-higher-order realized moment relationships by employing 142 Australian stocks downloaded at a 15-minute frequency from the Thompson Reuters Tick History/SIRCA database from 2003 to 2017. We compute all the high-order realized moments (i.e. realized variance, realized skewness and realized kurtosis) for two (i.e. weekly and monthly) holding periods.

[5]Bandi and Russell (2008) show that it is paramount to compute realized volatility with unbiased intra-day return data since computing realized volatility with contaminated return data results in significant accumulation of noise.

[6]Bandi and Russell (2008) recommend that when sampling very illiquid stocks, 15-minutes could be the preferred sampling frequency for computing realized volatility, which should be lowered for very high liquid stocks. Bollerslev et al. (2008) shows that the optimal sampling frequency for 40 U.S. equities is 17.5-minutes. Oomen (2006) shows that the optimal sampling frequency for realized volatility for IBM stock to be 20-minutes while it increases to about 3 minutes with a first-order bias correction. Hansen and Lunde (2006) using DJI30 stocks show that the noise in realized volatility may be ignored when intra-day returns are sampled at low frequencies, such as 20-minutes. Andersen et al. (2003) employ a 30-minute return series to compute the realized variance.

[7]In addition, the evidence of this is in Chapter 2, which is currently under review at 'The Quarterly Review of Economics and Finance'.

[8]Alles and Murray (2017) show that relative to the U.S and U.K. market, the trading volume for the Australian equity market is less than 5% of that recorded on the New York stock exchange. The Australian equity market is concentrated in a small number of sectors, with the materials sector dominating and makes the market highly weighted in one sector. They also report that the Australian equity market is mainly represented and weighted by domestic firms, with less than 2% being overseas companies. This leads to investors not having a wide range of investment opportunities, which in turn might result in cyclical economic patterns. The unavailability of alternative investment options may impact the extent of reward available to taking up downside risk.

We then download and compute the corresponding trading volumes, the number of trades, order imbalances, average trade sizes and 'vol-trades' (defined as the product of trading volume and number of trades) as our proxies of information flow. These high-frequency data should enhance the robustness of our estimates and help us capture any volume-higher-order moment relationships and subtle differences based on the proxy for information flow being tested.

Our empirical findings confirm the contemporaneous positive (realized) volume-volatility relation suggested by the MDH hypothesis when trading volume, number of trades, and vol-trade are used as proxies of information flow. The traditional average trade size results in an inverse relationship, this contradicts predominately positive volume-volatility relationship documented in the extant literature. We next observe that realized skewness has no significant relationship with any of the information flow proxies for both weekly and monthly holding periods. As such, we investigate the volume-'negative' realized skewness relationship, which is consistent with Do et al. (2014) and also the volume-positive realized skewness relationship, which is ignored in the literature due to the fact that most researchers are biased towards a loss-oriented (negative) skewness research. In this study, we consider negative and positive realized skewness as proxies for bad and good news respectively. This suggests that the directional (negative or positive) realized skewness relationship with volume reflects investors' reaction to bad or good news. The changing directional realized skewness indirectly also reflects investors changing positive and negative opinions over time. We then observe that the number of trades has a negative and significant relationship with realized kurtosis. This phenomenon could also be explained by DOH hypothesis (see Do et al. (2014)). However, we also observe that trading volume has a positive and significant relationship with realized kurtosis, which contradicts the results of Do et al. (2014).

Additionally, we find that our new measure of information flow 'vol-trade' is also important in explaining not only the volume-realized volatility relation but also the volume-directional realized skewness and volume-realized kurtosis relation. We also observe that order imbalance has no significant relationship with the higher-order realized moments for both holding periods. However, when we consider the buyer/seller-initiated trades separately, we observe that buy/sell-initiated trades have a significant relationship with realized higher-order moments.[9] The insignificant order imbalance relationship is consistent with the findings of Chan and Fong (2006) and Giot et al. (2010), who obtained insignificant relationships between order imbalance and realized variance, and its jump component for the daily holding period. Contrary to that, Chan and Fong (2000) find that daily order imbalances drive the volume-volatility relation in comparison to that of average trade sizes. We infer that the significance of order imbalance may be dependent on the market and/or the holding period under study. Overall, we find that the significance of most of the volume-realized higher moments relationships disappear as the holding period is changed from weekly to monthly intervals. This outcome is not surprising and is consistent with a recent intervaling study by Ahadzie and Jeyasreedharan (2020) who prove theoretically and show empirically that, for a given high-frequency sampling-interval (i.e. 15-mins in our case), the estimates of higher-order moments will be conditional on the holding-interval considered (i.e. weekly and monthly in our case).

Finally, we run four separate robustness checks. Firstly, we employ the absolute residuals test of Jones et al. (1994) in investigating the volume-volatility relationship. Secondly, we investigate the relationship between trading volume and the natural logarithms of the realized higher-order moments similar to that of Shahzad et al. (2014), who studied the relationship between natural logarithms of realized variance and trading volume. Thirdly, we split the data into three sub-periods following the categorization of Dungey and Gajurel (2014). For this, only the weekly holding period is utilized since most of the monthly results tend to be insignificant for the full sample period. Fourthly, we test the significance of the volume-higher-

[9]The inclusion of buyer/seller-initiated trades to our study was carried out in response to the following a query from Associate Professor Wing Wah Tham, the discussant for our conference paper (with the same title) presented by the first author at the 2nd FIRN Ph.D. Symposium (2019), Byron Bay, New South Wales, Australia: *How can unsigned 'trading volume' be a better measure of information content and/or its arrival rate as compared to 'order-imbalance'?* Our answer: Separately, the buyer/seller-initiated trades have significant relationships with realized higher-order moments. However, when taken together, the net difference between two types of trades i.e. order imbalance, is found to have less an impact and no significance.

order realized moment relationship for the weekly post-crisis period by controlling for the market rate of information flow by including the S&P/ASX200 volume data as a control variable. We observe that in the presence of a market rate of information flow, the volume-higher-order realized moment relationship is still significant. Overall, the results are robust and consistent.

The remainder of this chapter is organized as follows: Section 4.2 gives a brief review of relevant theory for estimating higher-order moments. Section 4.3 presents the empirical data used in constructing the higher-order moments and subsequent estimates of information flow proxies. We report the descriptive statistics of the variables used in this section. The empirical results are discussed in Section 4.4, and Section 4.5 concludes.

4.2 Higher-order realized moments

We discuss a concise review of the theory relating to estimating higher-order realized moments. Suppose the observed price follows a semi-martingale process on some filtered probability space $(\Omega,\ \mathcal{F},\ (\mathcal{F}_t)_{t\geq 0}, \mathsf{P})$ in a frictionless market where there are no arbitrage opportunities (see Back (1991)). Then in the presence of jumps, the observed price can be modelled as a continuous time semi-martingale jump-diffusion process;

$$p_t = \int_0^t \mu_D dt + \int_0^t \sigma_D dW_t + \sum_{k=1}^{N(t)} J(Q_k), \tag{4.1}$$

where μ_D is the diffusive mean, σ_D is a diffusive volatility process and dW_t is the increments to a Brownian motion W_t, $N(t)$ is a counting process and $J(Q_k)$ are the non-zero jump increments (see Fleming and Paye (2011) for more details). The quadratic variation for the jump-diffusion process is defined as,

$$QV_t = \int_0^t \sigma_D^2 dt + \sum_{k=1}^{N(t)} J^2(Q_k), \tag{4.2}$$

the first term on the right-hand side of Equation 4.2 is the integrated variance and the second term is the sum of the squared jumps (variance of the jump component). We observe that Equation 4.2 reduces to a 'pure' diffusion model with continuous sample paths when there are no jumps in the price process (i.e. the jump component is set to zero). For this jump-diffusion process to hold, it is assumed that μ_D and σ_D are jointly independent of W_t. The integrated variance (IV) for this type of process is defined $IV_t \equiv \int_0^t \sigma_D^2 dt$ and equals to the quadratic variance (QV).

In high-frequency finance, the proxy for sample variance is the realized variance (RV); replacing the traditional use of squared returns at low frequencies. It is well documented that realized variance is a more robust estimate of volatility (see Andersen and Bollerslev (1998); Andersen et al. (2003); Hansen and Lunde (2004, 2003); Barndorff-Nielsen and Shephard (2004); Andersen et al. (2007)). The discrete time high-frequency returns over the holding-interval h is defined as;

$$r_{i,h} = p_{i,h} - p_{i-1,h}, \quad i = 1, 2, .., N \tag{4.3}$$

where h is the holding-interval (thus trading week or month), $p_{i,h}$ is the i-th high-frequency log price for holding-interval of h, and N the number of infill observations for each sampling-interval, τ, partitioned into equal-length such that $\tau \equiv (b-a)/N$ and $[a, b] \subset h$. The RV is defined as the sum of squared high-frequency returns as given by;

$$RV_{i,h} = RM(2)_{i,h} \equiv \sum_{i=1}^{N} r_{i,h}^2 \rightarrow \int_0^t \sigma_D^2 dt + \sum_{k=1}^{N(t)} J^2(Q_k), \quad as \quad N \rightarrow \infty \tag{4.4}$$

The RV is an efficient estimator of the quadratic variation, it converges to the QV as the number of observations (N) goes to infinity ($RV_{[a,b]}^{(N)} \rightarrow QV_{[a,b]}$ as $N \rightarrow \infty$ (see Andersen and Bollerslev (1998); Barndorff-Nielsen and Shephard (2002)). It is also apparent from Equation 4.4 that in the absence of jumps RV converges to the IV.

Following Amaya et al. (2015), the third and fourth realized moments can be defined as;

$$
\begin{aligned}
RM(3)_{i,h} &\equiv \sum_{i=1}^{N} r_{i,h}^{3} \rightarrow \sum_{k=1}^{N(t)} J^{3}(Q_k), \quad as \quad N \rightarrow \infty \\
RM(4)_{i,h} &\equiv \sum_{i=1}^{N} r_{i,h}^{4} \rightarrow \sum_{k=1}^{N(t)} J^{4}(Q_k), \quad as \quad N \rightarrow \infty
\end{aligned}
\tag{4.5}
$$

According to Amaya et al. (2015), the third realized moment converges to the sum of cubic jumps and the fourth realized moment converges to the sum of the quartic jumps. In other words, the realized third higher-order moment captures the sum of the cubic jumps and the realized fourth higher-order moment captures the sum of the quartic jumps. Consequently, for $RM(4)$, only the magnitude of the jumps are relevant and not the direction. These jump-driven convergences are consistent with the findings of Kim and White (2004), who find that estimates of the higher moments of distributions of high-frequency data are heavily influenced by the presence of jumps.

As mentioned earlier, realized skewness (RS) and realized kurtosis (RK) have received minimal attention in the financial time series literature in comparison to realized variance (RV). Following Amaya et al. (2015), the RS is formally defined as the cubic intra-day returns normalized by the square-root of RV cubed and the RK as the sum of the quartic high-frequency returns normalized by RV squared;

$$
RS_{i,h} = \frac{\sqrt{N}\sum_{i=1}^{N} r_{i,h}^{3}}{RV_{i,h}^{3/2}} \tag{4.6}
$$

$$
RK_{i,h} = \frac{N\sum_{i=1}^{N} r_{i,h}^{4}}{RV_{i,h}^{2}} \tag{4.7}
$$

Amaya et al. (2015) show that realized skewness and realized kurtosis do not converge to the sample skewness and sample kurtosis. The sample skewness and kurtosis include diffusive skewness and diffusive kurtosis component. Hence, the normalized third realized moment (realized skewness) captures the normalized direction and magnitude of the cubic jumps. The normalized fourth realized moment (realized kurtosis) captures the normalized magnitude of the quartic jumps. This means that information embedded in realized skewness and realized kurtosis is different from that of sample skewness and sample kurtosis which is normally computed from long samples of low-frequency return data (thus daily, weekly or monthly return series).

4.3 Data

In the high-frequency literature, it is typical to use returns sampled at 5-minute sampling frequency as a proxy for unbiased high-frequency return data in the U.S. framework (Andersen and Bollerslev, 1997; Andersen et al., 2007; Huang and Tauchen, 2005). The rationale is that this is a trade-off between microstructure noise and variance-bias. As mentioned earlier, we observe that the 5-minute unbiased sampling frequency does not hold for the Australian stock returns. According to Bandi and Russell (2008), it is paramount to compute realized variance with unbiased intra-day return data because computing realized variance with contaminated return data results in significant accumulation of noise, which may result in obtaining biased estimates. As such, this study uses intra-day 15-minutes last traded prices of 142 stocks listed on the ASX stock market.

The data was obtained from Thompson Reuters Tick History/SIRCA database. We initially download the 249 constituent stocks of the S&P/ASX200 index. However, 107 of the stocks were removed due to the unavailability of data for our sample period. The analysis in this study is carried out using the remaining 142 stocks. Our data sample is from 6 January 2003 - 29 December 2017 and between 10 am to 4 pm of each trading day, giving us a sample of 24 intra-day prices. We exclude weekends and overnight returns from the data. The 15-minute sampling-interval adopted results in 94,350 sampled points over the 15 years (with 754 weekly or 180 monthly holding-intervals). Our present sample is the longest data set used so far to investigate the relationship between the volume-realized high-order moments relationships and employing high-frequency data. The nearest contenders to our sample size were Shahzad et al. (2014), who used a sample of 5-years, from January 2006 to December 2010 (at 5-minute sampling-intervals and daily holding-intervals), of 216 Australian stocks to investigate volume-volatility relationship.

The intra-day returns were computed as the change in the logarithm of the closing prices of successive days. We compute weekly and monthly realized higher-order moments from the 15-minutes high-frequency returns data. We do not consider a daily holding-interval because 15-minutes intraday data spans only 24 observations in a day. The 24 data points in the daily holding-interval will not be enough for estimating higher-order moments with any statistical precision. Our selection of the alternative proxies i.e. trading volume, number of trades, order imbalance, and average trade size are consistent with the proxies for information flow found in the extant literature (Chan and Fong, 2000, 2006; Giot et al., 2010; Shahzad et al., 2014).

Table 4.1: Descriptive Statistics (Jan 2003 - Dec 2017)

	RV	RS	RK	TV	NT	ATS	OI	VT
Panel A: Weekly data								
Mean	0.0065	0.0666	18.9733	1.0145E+07	9.0954E+03	4.7957E+03	8.1898E+01	1.6886E+11
Standard Deviation	0.0142	0.6829	3.2239	3.0482E+06	6.1355E+03	2.7587E+03	9.4284E+02	1.1641E+11
Skewness	15.1843	-0.4682	0.8637	0.7033	0.1403	1.4027	0.2246	0.5556
Kurtosis	282.7418	3.5654	4.9886	3.2878	1.8468	5.5807	7.1418	3.8661
Coefficient of Variation	2.1813	10.2539	0.1699	0.3005	0.6746	0.5752	11.5123	0.6894
Panel B: Monthly data								
Mean	0.0273	-0.0158	32.5290	4.2542E+07	3.8155E+04	4.8053E+03	3.4563E+02	2.8905E+12
Standard Deviation	0.0338	0.5338	5.7365	1.1606E+07	2.5638E+04	2.7027E+03	2.6404E+03	1.9065E+12
Skewness	7.0257	0.0489	0.4527	0.4855	0.1172	1.2157	0.8654	0.3121
Kurtosis	58.8606	2.6077	2.7728	2.9560	1.8335	3.8524	6.9628	3.2162
Coefficient of Variation	1.2412	-33.7267	0.1763	0.2728	0.6720	0.5624	7.6395	0.6596

Note: Realized Variance (RV), Realized Skewness (RS), Realized Kurtosis (RK), Trading Volume (TV), Number of Trades (NT), Average Trade Size (ATS=TV/NT), Order Imbalance (OI) and Vol-Trade (VT=TV×NT).

Table 4.1 summarizes the descriptive statistics for the eight variables used in our study. Panel A presents the results for the weekly holding period for the 142 firms used while Panel B reports on the monthly holding period statistics. The average weekly and monthly volume is almost 10 and 42 million shares respectively. The number of trades has a mean of 9,095 trades per week and 38,155 trades per month, which is roughly 1,800 to 1,900 trades per day and shows enough liquidity for the S&P/ASX200 index. Following Wong and So (2003); Meucci (2010), the square root of time rule can be used to obtain the annualized realized variance. The mean weekly and monthly realized variance of 0.0065 and 0.0273 translates into an annualized realized variance of 32.76% (annualized weekly (RV)=$\frac{252}{5} \times$ 0.0065=32.76% and annualized monthly (RV)=12 $\times$ 0.0273=32.76%). This is feasible since realized variance is interval-invariant, and as such daily, weekly, or monthly realized variance would results in the same value of annualized variance (see Ahadzie and Jeyasreedharan (2020)). This variance estimate is close to 33.92% reported by Shahzad et al. (2014) for Australian data. Chan and Fong (2006) obtained 27.5% annualized realized variance for the 30 stocks on the Dow Jones Industrial Index (DJI30) and 24.6% reported by Giot et al. (2010) for the 100 largest stocks on the New York Stock Exchange (NYSE).

We observe that the average weekly and monthly realized skewness when annualized will be annualized weekly (RS)=$\frac{1}{\sqrt{\frac{252}{5}}} \times$ 0.0666=0.0094 and annualized monthly (RS)=$\frac{1}{\sqrt{12}} \times$ -0.0158=-0.0046. In the case of annualized realized kurtosis, we obtain annualized weekly (RK)=$\frac{1}{\frac{252}{5}} \times$ 18.9733=0.3765 and annualized monthly (RK)=$\frac{1}{12} \times$ 32.5290=2.7107. It clearly seen that the annualized realized skewness and kurtosis are not interval-invariant. Realized skewness and realized kurtosis do not converge to their corresponding sample skewness and kurtosis, unlike realized variance which is interval-invariant. Realized skewness and kurtosis captures the normalized cubic and normalized quartic jumps. Thus, apart from the popular jump decomposition technique in the extant literature, realized skewness and realized kurtosis as defined by Amaya et al. (2015) is an alternative formulation for indirectly but implicitly capturing the direction and magnitude of jumps respectively. Trading volume, number of trades, average trade size (numbers of shares traded divided by the number of trades), order imbalance (defined as the number of buyer-initiated trades minus the number of seller-initiated trades) and vol-trade (defined as the product of trading volume and the number of trades) are used as proxies for information flow.

4.4 Empirical Results

4.4.1 Volume-realized volatility relationship

In Table 4.2, we investigate the volume-realized volatility relationship by regressing realized variance on 12 lagged values of realized variance and each information flow proxy (thus trading volume, number of trades, and average trade size). The 12 lagged realized variances used in the regression controls for any serial dependence that might be present. The trading volume, number of trades, and average trade size captures the total information flow, frequency of information flow, and size of information flow respectively. The volume-volatility relationship is tested by running the generalized method of moments (GMM), as carried out by Huang and Masulis (2003); Giot et al. (2010). However, Giot et al. (2010) includes the robust (ROB) and median (MED) regression techniques in their analysis. The authors believe that this deal with any non-normality present in the data that might result in obtaining biased estimates. This suggests that the ROB and MED techniques produce more robust estimates. We employ GMM, ROB, and MED regression techniques for the analysis in this study. The results of the GMM are the main results discussed while ROB and MED are treated as alternative measures. Panel A reports the results for the weekly holding period, while panel B is the results for the monthly holding period. Both weekly and monthly estimates are computed from 15-minutes returns series. In the literature, the realized variance is mostly referred to as realized volatility.

In Panel A of Table 4.2, we find a positive relationship between trading volume and realized variance. The GMM shows an average coefficient of 9.2E-10, which is significant at a 5% level. The positive relationship between trading volume and realized variance is consistent with MDH hypothesis (MDH) in the extant literature (see Andersen (1996); Clark (1973); Epps and Epps (1976); Tauchen and Pitts (1983); Choi et al. (2012); Celik (2013); Do et al. (2014) just to mention a few). We also observe that the coefficient of trading volume and realized variance

Table 4.2: Trading volume, number of trades, average trade size and realized variance

	Trading Volume and RV			Number of trades and RV			Average trade size and RV		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	9.52E-10**	9.26E-11***	1.66E-10***	1.07E-07*	4.07E-07***	4.38E-07***	-1.80E-08	-9.08E-09**	1.71E-08
t-Values ≥1.96	55.63%	69.72%	64.79%	29.58%	45.07%	47.18%	7.75%	18.31%	14.79%
t-Values ≤-1.96	0.00%	2.82%	1.41%	4.23%	9.15%	10.56%	4.93%	21.83%	10.56%
Average R^2	24.65%	60.58%	-	20.57%	59.71%	-	20.11%	59.98%	-
Panel B: Monthly data									
Coefficient	7.28E-10**	1.22E-10***	1.74E-10*	9.71E-08	4.24E-07	3.17E-07	-3.30E-06	2.09E-08	1.27E-07
t-Values ≥1.96	42.96%	54.93%	44.37%	11.97%	18.31%	15.49%	6.34%	15.49%	11.27%
t-Values ≤-1.96	0.00%	0.70%	2.11%	4.93%	5.63%	5.63%	5.63%	4.93%	2.82%
Average R^2	33.99%	65.93%	-	30.68%	64.26%	-	30.79%	64.50%	-

This table reports the regression results for the relationship between realized variance and trading volume, number of trades and average trade size for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i TV_{it} + \vartheta_{it}$$

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i NT_{it} + \vartheta_{it}$$

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i ATS_{it} + \vartheta_{it}$$

where TV_{it} is the trading volume, NT_{it} is the number of trades and ATS_{it} is the average trade size for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The ‘coefficient’ and ‘Average R^2’ are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. ‘t-value≥1.96’ (‘t-value ≤-1.96’) shows the percentage of firms that have t-value statistically significant at the 5% level.

for all three regression techniques are positive and statistically significant. The ROB regression shows that, on an average 69.72% (thus 99 out 142 of the companies in our sample) are statistically significant at the 5% level. The number of trades also exhibits a positive relationship with the realized variance, with 45.07% of the firms significantly above the critical value. This is consistent with the MDH hypothesis. All things being equal, the mean trading volume (10,145,163 shares) in Panel A of Table 4.1 and the GMM coefficient of trading volume in Table 4.2 indicates that an increase of 10% of the average trading volume (thus an increase of 1,014,516 shares) will result in an increase of 9.6582E-04 in realized variance on average. According to Shahzad et al. (2014), the (realized) variance-volatility relationship is paramount to an investor, since volatility-timing trading strategies may be employed by investor in order to maximize their pay-offs and minimize their risks. Consequently, the potential for the use of skewness-kurtosis-timing strategies to further optimise investors’ risks and pay-offs should not be ignored.

In the case of the average trade size, we observe a negative relationship with realized variance, which is only significant for the ROB regression. This negative relationship between average trade size and realized variance contradicts the suggested positive relationship by MDH hypothesis. Thus, we observe subtle differences between the (realized) variance-volume relationship depending on the information proxy being employed.

In Panel B of Table 4.2, the significance of the relationship between realized variance and trading volume is persistent, although we observe a decline in the number of firms that are significantly above the critical value. For the number of trades and realized variance, the statistical significance of the relationship disappears completely. The coefficients are statistically insignificant for the average trade size. The relationship between average trade size and realized variance tends to be mixed depending on the regression technique in the monthly holding period. In conclusion, our results suggest a contemporaneous positive relationship between trading volume/number of trades and realized variance that is extensively documented in the literature and per the MDH hypothesis. We also find that the relationship between average trade size and realized variance is negative in the weekly holding period.

4.4.2 Volume-realized skewness relationship

Table 4.3 reports the results for the volume-realized skewness relationship. According to Do et al. (2014), this relationship captures investor's reactions to good or bad news. We investigate this relationship by regressing realized skewness as a dependent variable on 12 lags of realized skewness and one of the proxies of information flow. We find an insignificant relationship between realized skewness and all the proxies for information flow with mixed directions. The insignificant relationship may be attributed to positive and negative skewness cancelling out each other. Do et al. (2014) focus on the negative of realized skewness and use the number of trades as their only proxy for the information flow. Apart from the Asian Pacific emerging region, the authors find no significant relationship between negative realized skewness and the number of trades. This contradicts Hong and Stein (2003) who show that the theory of investor heterogeneity explains why the number of trades has a positive relationship with negative realized skewness. The positive relationship between negative skewness and the number of trades is therefore explained by the DOH hypothesis of Shalen (1993); Harris and Raviv (1993).

Subsequently, in Tables 4.4 and 4.5 we display the regression results for negative and positive realized skewness respectively. We observe a positive and significant coefficient for negative realized skewness and the number of trades for the weekly holding period in Table 4.4, which is consistent with Hong and Stein (2003). The investor heterogeneity theory which states that difference of opinion among investors leads to negative asymmetries that is consistent with difference of opinion. We employ this in explaining the positive and significant relationship between negative realized skewness and the number of trades. This implies the higher the degree of difference of opinion, higher level of number of trades will lead to an increase in negative realized skewness. In other words, an increase in difference of opinion coupled with a higher level of number of trades results in high levels of negative realized skewness (normalized negative jumps). However, negative realized skewness has a negative relationship with trading volume and average trade size, which is contrary to the results of Hong and Stein (2003). In short, low negative realized skewness (normalized negative jumps) is accompanied by high levels of trading volume or average trade size. The average trade size shows mixed direction for the results depending on the regression technique that is employed. In Panel B, we observe that the significance disappears during the monthly holding period. In the case of trading volume and negative realized skewness, the results show a negative and significant relationship at 10% level. The level of significance of the coefficients disappears as the holding period increases.

Table 4.5 reports the results when the regression is run on positive realized skewness and the three proxies of information flow. It is worth mentioning that the previous literature on this subject focuses on the effects of negative realized skewness on trading volume. The investor heterogeneity theory of Hong and Stein (2003) used in explaining the positive relationship between negative skewness and number of trades, could be extended to explain the negative relationship between positive realized skewness and number of trades. Thus in the presence of asymmetric-opinion, low levels of positive realized skewness (normalized positive jumps) is conditional on high levels of number of trades. When the trading volume and average trade size are used as a proxy for information flow, we observe a positive relationship with positive realized skewness, which is significant in the weekly holding period. This means that trading volume and average trade size increases positive skewness. For the monthly holding period, although the directions of the coefficients remain the same as those for the weekly holding period, we observe that the significance of the coefficients disappears.

Table 4.3: Trading volume, number of trades and average trade size and realized skewness

	Trading Volume and RS			**Number of trades and RS**			**Average trade size and RS**		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	-1.47E-08	1.23E-08	-6.12E-09	-5.20E-05	-5.57E-05	-2.04E-05	4.74E-05	5.15E-05	4.59E-05
t-Values≥1.96	4.93%	17.61%	8.45%	2.11%	4.23%	6.34%	11.27%	18.31%	15.49%
t-Values ≤-1.96	15.49%	13.38%	8.45%	22.54%	18.31%	16.90%	2.82%	1.41%	2.82%
Average R^2	2.76%	3.06%	-	2.68%	2.83%	-	2.52%	2.80%	-
Panel B: Monthly data									
Coefficient	-6.33E-09	1.59E-08	9.07E-09	-4.11E-05	-1.47E-05	-2.6E-05	9.97E-05	7.46E-05	7.39E-05
t-Values ≥1.96	7.04%	9.86%	4.93%	0.70%	4.93%	3.52%	12.68%	11.97%	6.34%
t-Values ≤-1.96	15.49%	11.27%	11.97%	12.68%	11.97%	18.31%	1.41%	4.23%	0.70%
Average R^2	9.73%	11.43%	-	8.95%	11.10%	-	8.71%	10.81%	-

This table reports the regression results for the relationship between realized skewness and trading volume, number of trades and average trade size for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$RS_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS_{it-j} + \beta_i TV_{it} + \vartheta_{it}$$

$$RS_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS_{it-j} + \beta_i NT_{it} + \vartheta_{it}$$

$$RS_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS_{it-j} + \beta_i ATS_{it} + \vartheta_{it}$$

where TV_{it} is the trading volume, NT_{it} is the number of trades and ATS_{it} is the average trade size for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The ‘coefficient’ and ‘Average R^2’ are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. ‘t-value≥1.96’ (‘t-value ≤-1.96’) shows the percentage of firms that have t-value statistically significant at the 5% level.

Table 4.4: Regression condition on negative realized skewness: Trading volume, number of trades, average trade size and negative realized skewness

	Trading Volume and RS$^-$			**Number of trades and RS$^-$**			**Average trade size and RS$^-$**		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	-1.29E-07***	-8.88E-08***	-1.07E-07**	5.18E-04***	4.84E-04***	3.62E-04***	-1.67E-05*	1.55E-05*	9.17E-06*
t-Values ≥1.96	3.52%	0.70%	2.82%	19.01%	19.72%	19.72%	14.08%	14.79%	19.72%
t-Values ≤-1.96	59.86%	53.52%	52.11%	47.89%	45.07%	44.37%	20.42%	21.83%	16.90%
Average R^2	10.29%	7.94%	-	8.08%	7.34%	-	5.97%	6.45%	-
Panel B: Monthly data									
Coefficient	-7.54E-08*	-2.40E-08	-4.42E-08	1.29E-04	8.85E-05	8.59E-05	-1.02E-04	-4.22E-05	-5.28E-05
t-Values≥1.96	3.52%	0.70%	1.41%	8.45%	7.75%	7.04%	9.15%	6.34%	3.52%
t-Values ≤-1.96	37.32%	20.42%	18.31%	23.24%	21.13%	13.38%	7.75%	13.38%	3.52%
Average R^2	25.03%	24.77%	-	21.65%	24.26%	-	20.24%	22.88%	

This table reports the regression results for the relationship between realized skewness conditioned on negative values and trading volume, the number of trades, and average trade size for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:" *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i TV_{it} + \vartheta_{it} \quad ,given \quad RS < 0$$

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i NT_{it} + \vartheta_{it} \quad ,given \quad RS < 0$$

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i ATS_{it} + \vartheta_{it} \quad ,given \quad RS < 0$$

where TV_{it} is the trading volume, NT_{it} is the number of trades and ATS_{it} is the average trade size for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The 'coefficient' and 'Average R^2' are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. 't-value≥1.96' ('t-value ≤-1.96') shows the percentage of firms that have t-value statistically significant at the 5% level.

Table 4.5: Regression condition on positive realized skewness: Trading volume, number of trades, average trade size and positive realized skewness

	Trading Volume and RS$^+$			**Number of trades and RS$^+$**			**Average trade size and RS$^+$**		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	9.83E-08***	8.19E-08***	1.00E-07**	-4.69E-04***	-3.89E-04**	-3.46E-04***	4.34E-05*	1.44E-05**	2.71E-05*
t-Values ≥1.96	59.15%	54.23%	46.48%	34.51%	34.51%	33.80%	28.87%	26.76%	20.42%
t-Values ≤-1.96	1.41%	2.11%	2.82%	23.94%	21.13%	21.83%	9.15%	16.20%	14.79%
Average R^2	7.55%	7.28%	-	6.76%	6.40%	-	5.87%	6.69%	-
Panel B: Monthly data									
Coefficient	5.13E-08	2.68E-08	3.88E-08	-1.99E-04	-1.05E-04	-1.20E-04	1.66E-04	4.47E-05	7.41E-05
t-Values ≥1.96	28.87%	24.65%	14.08%	14.79%	13.38%	9.86%	16.90%	19.72%	11.27%
t-Values ≤-1.96	1.41%	1.41%	2.11%	16.90%	10.56%	10.56%	8.45%	6.34%	5.63%
Average R^2	18.73%	20.06%	-	17.40%	19.83%	-	17.32%	20.32%	-

This table reports the regression results for the relationship between realized skewness conditioned on positive values and trading volume, the number of trades and average trade size for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:" *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i TV_{it} + \vartheta_{it} \quad ,given \quad RS > 0$$

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i NT_{it} + \vartheta_{it} \quad ,given \quad RS > 0$$

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i ATS_{it} + \vartheta_{it} \quad ,given \quad RS > 0$$

where TV_{it} is the trading volume, NT_{it} is the number of trades and ATS_{it} is the average trade size for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The 'coefficient' and 'Average R^2' are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. 't-value≥1.96' ('t-value ≤-1.96') shows the percentage of firms that have t-value statistically significant at the 5% level.

4.4.3 Volume-realized kurtosis relationship

Table 4.6 reports the results for realized kurtosis and volume. From the weekly panel, the number of trades has a negative and significant relationship with realized kurtosis. The negative relationship between the number of trades and realized kurtosis can be explained by the DOH hypothesis which is consistent with Do et al. (2014). Thus asymmetric-opinion inherent in the DOH hypothesis also enable one to explains the observed negative relationship between number of trades and realized kurtosis. As mentioned earlier, realized kurtosis measures the normalized magnitude of jumps. Hence the results obtained could also be compared with Giot et al. (2010). Giot et al. (2010) find a negative and significant relationship between the number of trades and jumps for the daily holding period. Our results show that in the presence of a high difference of opinion, a higher number of trades results in a decrease in the magnitude of jumps. For the trading volume and average trade size, we observe that high trading volume or average trade size increases realized kurtosis which can be accommodated by SIAH and DOH hypotheses. Thus information-asymmetry and asymmetric-opinions result in an increase in trading volume and/or average trade size to an increase in realized kurtosis (normalized magnitude of jumps). For the monthly period, the level of significance decreases, and the directional impact remains unchanged.

Table 4.6: Trading volume, number of trades, average trade size and realized kurtosis

	Trading Volume and RK			Number of trades and RK			Average trade size and RK		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	6.24E-07***	1.29E-07***	2.90E-07**	-2.16E-03**	-2.09E-03***	-1.85E-03***	3.47E-04	8.82E-05**	1.12E-04**
t-Values $\geq$1.96	51.41%	41.55%	41.55%	40.85%	38.03%	39.44%	19.72%	28.87%	22.54%
t-Values $\leq$-1.96	1.41%	11.27%	5.63%	17.61%	39.44%	33.80%	3.52%	19.72%	14.79%
Average R^2	15.56%	19.12%	-	13.03%	18.81%	-	12.17%	18.37%	-
Panel B: Monthly data									
Coefficient	8.24E-07*	1.25E-07**	3.02E-07	-1.93E-03	-1.91E-03**	-1.66E-03*	2.71E-03	4.25E-04**	8.05E-04
t-Values $\geq$1.96	38.73%	28.87%	26.76%	16.20%	28.17%	21.13%	14.79%	24.65%	15.49%
t-Values $\leq$-1.96	2.11%	14.08%	4.93%	10.56%	35.21%	17.61%	2.11%	14.08%	7.04%
Average R^2	17.80%	27.30%	-	14.38%	26.73%	-	14.11%	26.83%	-

This table reports the regression results for the relationship between realized kurtosis and trading volume, the number of trades, and average trade size for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01”. Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i TV_{it} + \vartheta_{it}$$

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i NT_{it} + \vartheta_{it}$$

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i ATS_{it} + \vartheta_{it}$$

where TV_{it} is the trading volume, NT_{it} is the number of trades and ATS_{it} is the average trade size for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The ‘coefficient’ and ‘Average R^2’ are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. ‘t-value$\geq$1.96’ (‘t-value $\leq$-1.96’) shows the percentage of firms that have t-value statistically significant at the 5% level.

4.4.4 Order imbalance-realized moments relationship

Following Chan and Fong (2006), we compute order imbalance as buyer-initiated trades minus the number of seller initiated trades. Table 4.7 reports the results when the realized higher-order moments are regressed on the order imbalance. We observe that for both weekly and monthly periods, the order imbalance remains statistically not significant for realized variance, skewness, and kurtosis. Panel A of Table 4.7 shows that realized variance has a negative and insignificant relationship with order imbalance, which is consistent with the findings of Giot et al. (2010). For realized kurtosis, the relationship is positive and insignificant. The relationship between realized skewness and order imbalance is positive and insignificant as well. In the case of the monthly holding period, there is no distinct direction between the realized higher-order moments and order imbalance. The insignificant order imbalance relationship is consistent with the findings of Chan and Fong (2006), however, it differs from the findings of Chan and Fong (2000), who find that for random daily 295 NYSE stocks and 231 Nasdaq stocks, the order imbalance drives the volume-volatility relation. We infer that the significance of order imbalance may be dependent on the market and/or holding period being considered.

Table 4.7: Order imbalance-realized moments relationship

	Realized Variance			Realized Skewness			Realized Kurtosis		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	-9.91E-08	-1.25E-08	-1.87E-08	6.89E-05	2.85E-05	2.66E-05	1.32E-04	6.55E-05	2.72E-05
t-Values $\geq$1.96	0.70%	1.41%	3.52%	28.87%	30.99%	27.46%	3.52%	1.41%	4.93%
t-Values $\leq$-1.96	9.15%	13.38%	16.90%	2.11%	1.41%	0.70%	4.23%	5.63%	10.56%
Average R^2	20.00%	60.38%	-	2.77%	2.99%	-	11.85%	17.51%	-
Panel B: Monthly data									
Coefficient	1.65E-07	-6.86E-08	-9.44E-08	3.56E-05	-3.17E-06	8.74E-06	-1.59E-04	4.15E-04	3.08E-05
t-Values $\geq$1.96	0.70%	0.00%	0.70%	19.72%	13.38%	15.49%	3.52%	4.93%	5.63%
t-Values $\leq$-1.96	11.97%	7.04%	6.34%	1.41%	1.41%	3.52%	4.93%	4.23%	7.75%
Average R^2	30.58%	64.38%	-	8.93%	10.79%	-	13.52%	24.92%	-

This table reports the regression results for the relationship between realized high-order moments and order imbalance for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:" *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i OI_{it} + \vartheta_{it}$$

$$RS_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS_{it-j} + \beta_i OI_{it} + \vartheta_{it}$$

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i OI_{it} + \vartheta_{it}$$

where OI_{it} is the order imbalance (buyer-initiated trades minus seller-initiated trades) for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The 'coefficient' and 'Average R^2' are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. 't-value$\geq$1.96' ('t-value $\leq$-1.96') shows the percentage of firms that have t-value statistically significant at the 5% level.

In table 4.8, we report the results for realized higher-order moments with the buyer-initiated trades, seller-initiated trades, order of imbalance, and the absolute order of imbalance for the 142 stocks. For this test, we only focus on the post-crisis period from 1 June 2009 - 29 December 2017.

Panel A shows the regression results for realized variance; in the case of the buyer/seller-initiated trades, the ROB regression shows a positive and significant relationship. Almost 50% of the firms have t- value greater than or equal to the 5% level of significance. This implies that when buyer/seller-initiated trades increase volatility is bound to increase as well. This relationship satisfies the expected symmetric relationship between realized variance and buyer/seller-initiated trades information. Order imbalance has a negative relationship with realized variance, which is insignificant. Similarly, absolute order imbalance also has an insignificant relationship with realized variance, although the direction differs for both GMM and ROB regression techniques.

Panel B reports the results for negative realized skewness, in-here both buyer/seller-initiated trades have a positive relationship with negative realized skewness. This relationship is significant at the 5% level of significance. The DOH hypothesis (i.e. investor heterogeneity) can be used to explain this relationship. This implies that in the presence of asymmetric-opinion, high buyer/seller-initiated trades increases negative realized skewness. However, the order of imbalance has a negative relationship with negative realized skewness, which is statistically insignificant. This suggests when negative realized skewness/bad news increases, the variation between buyer and seller-initiated trades decreases. In the case of absolute order imbalance, the relationship is positive and insignificant.

In Panel C, positive realized skewness has a negative relationship with buyer/seller-initiated trades; this relationship on an average is significant at the 10% level of significance. Once again, the DOH hypothesis (i.e. investor heterogeneity) can be extended to explain this relationship. In short, high buyer/seller-initiated trades decreases positive realized skewness. The result of the order of imbalance is positive, which is insignificant. This suggests when positive realized skewness/good news increases, the variation between buyer and seller-initiated trades also increases. For absolute order imbalance, the relationship is negative and insignificant.

Finally, in Panel D, we observe a negative relationship between realized kurtosis and buyer/seller-initiated trades; this relationship can be explained by DOH hypothesis, which is consistent with Do et al. (2014). This relationship suggests high buyer/seller-initiated trades decreases realized kurtosis. The variation difference of buyer and seller-initiated trades and its absolute value have an insignificant relationship with realized kurtosis.

In conclusion, we observe that individually buyer/seller-initiated trades have significant relationships with realized higher-order moments. However, the order imbalance and absolute order imbalance captures no significance, but the directional impact does reveal the asymmetry nature of the relationship.

Table 4.8: Buyer/seller-initiated trades and realized higher-order moments relationship

	Buyer-initiated trades			Seller-initiated trades			Order imbalance			Absolute order imbalance		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Realized variance												
Coefficient	6.17E-08**	3.66E-07***	3.94E-07***	-3.85E-07**	1.40E-07***	1.77E-07***	-1.86E-07	-1.20E-08	2.44E-08	-7.98E-07	1.43E-08	-2.86E-08
t-Values ≥1.96	40.14%	50.00%	47.18%	38.73%	49.30%	47.89%	2.11%	2.11%	2.82%	7.04%	9.15%	11.27%
t-Values ≤-1.96	4.93%	6.34%	7.04%	4.23%	6.34%	6.34%	9.15%	12.68%	14.08%	5.63%	3.52%	6.34%
Average R^2	16.59%	43.84%	-	16.86%	44.68%	-	15.04%	43.15%	-	15.00%	43.14%	-
Panel B: Negative realized skewness												
Coefficient	5.02E-04**	5.29E-04*	4.74E-04**	3.33E-04**	3.76E-04**	3.21E-04**	-2.29E-05	-3.89E-05	-3.45E-05	3.18E-04	3.05E-04	2.86E-04
t-Values ≥1.96	18.31%	17.61%	17.61%	14.79%	12.68%	12.68%	13.38%	10.56%	18.31%	13.38%	6.34%	10.56%
t-Values ≤-1.96	32.39%	26.06%	31.69%	42.96%	36.62%	38.73%	5.63%	1.41%	2.11%	11.27%	12.68%	13.38%
Average R^2	9.94%	9.25%	-	10.27%	9.59%	-	7.78%	7.78%	-	8.08%	8.15%	-
Panel C: Positive realized skewness												
Coefficient	-3.77E-04**	-4.58E-04*	-3.26E-04*	-4.12E-04*	-3.89E-04*	-3.48E-04	1.14E-04	1.06E-04	1.13E-04	-2.12E-04	-1.83E-04	-1.82E-04
t-Values ≥1.96	28.87%	26.76%	26.76%	24.65%	20.42%	20.42%	9.15%	7.04%	9.15%	8.45%	7.04%	8.45%
t-Values ≤-1.96	18.31%	16.90%	16.20%	14.79%	14.08%	14.79%	0.70%	1.41%	3.52%	10.56%	6.34%	6.34%
Average R^2	8.48%	8.54%	-	8.24%	8.31%	-	6.71%	7.09%	-	6.98%	7.33%	-
Panel D: Realized kurtosis												
Coefficient	-2.29E-03**	-2.08E-03***	-1.90E-03**	-1.83E-03**	-1.71E-03***	-1.53E-03**	4.21E-05	-8.13E-05	1.10E-04	-1.03E-03	-8.74E-04	-6.11E-04
t-Values ≥ 1.96	40.14%	40.85%	40.14%	43.66%	41.55%	40.14%	4.23%	2.11%	3.52%	9.86%	14.79%	8.45%
t-Values ≤ -1.96	18.31%	28.87%	26.06%	11.27%	23.24%	19.01%	3.52%	4.93%	7.04%	9.15%	4.23%	6.34%
Average R^2	10.09%	14.04%	-	10.17%	13.82%	-	8.82%	12.34%	-	8.99%	12.65%	-

This table reports the regression results for the relationship between weekly realized higher-order moment and the buyer-initiated trades, seller-initiated trades, order of imbalance, and the absolute order of imbalance for the 142 stocks. We only focus on the post-crisis period from 1 June 2009 - 29 December 2017. This is due to the fact that for the sub-period results, the post-crisis period results were more significant. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly realized variance and the three proxies of information flow (trading volume, number of trades and vol-trade), panel B reports the results for negative skewness, panel c reports the results when positive realized skewness is used, and panel D is that of realized kurtosis. The results are based on the following regressions:

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i BIT_{it} + \vartheta_{it}, \quad RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i SIT_{it} + \vartheta_{it}, \quad RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i OI_{it} + \vartheta_{it}, \quad RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i AOI_{it} + \vartheta_{it}, \quad Panel \quad A$$

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i BIT_{it} + \vartheta_{it}, \quad RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i SIT_{it} + \vartheta_{it}, \quad RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i OI_{it} + \vartheta_{it}, \quad RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i AOI_{it} + \vartheta_{it}, \quad Panel \quad B$$

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i BIT_{it} + \vartheta_{it}, \quad RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i SIT_{it} + \vartheta_{it}, \quad RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i OI_{it} + \vartheta_{it}, \quad RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i AOI_{it} + \vartheta_{it}, \quad Panel \quad C$$

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i BIT_{it} + \vartheta_{it}, \quad RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i SIT_{it} + \vartheta_{it}, \quad RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i OI_{it} + \vartheta_{it}, \quad RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i AOI_{it} + \vartheta_{it}, \quad Panel \quad D$$

where BIT_{it} is the buyer-initiated trades, SIT_{it} is the seller-initiated trades, OI_{it} is the order of imbalance (buyer-initiated trades minus seller-initiated trades) and AOI_{it} is the absolute order of imbalance for stock i on week t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares, and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The ‘coefficient’ and ‘Average R^2’ are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. ‘t-value≥1.96’ (‘t-value ≤-1.96’) shows the percentage of firms that have t-value statistically significant at the 5% level.

4.4.5 Vol-trade-realized moments relationship

Having investigated the relationship between higher-order moments and the four proxies of information flow (trading volume, number of trades, average trade size, and order imbalance). We test the relationship between higher-order moments and a new measure of information flow, which we defined as the systematic generalization of scaling the trading volume by the number of trades. The motivation for this measure stems from the derivation of the average trade size, which is defined as trading volume divided by the number of trades, in-here the new measure of the information flow is termed vol-trade (VT), computed as the product of trading volume and the number of trades.

Table 4.9 reports the regression results for the realized higher-order moments on vol-trade. In Panel A1, the realized variance has a positive and significant relationship with vol-trade. We find that approximately 93 out of our 142 stocks are significant at the 5% level for ROB regression. This positive and significant relationship is consistent with the MDH hypothesis, similar to the results obtained for trading volume and the number of trades in Table 4.2. For the weekly holding period, we find a positive and significant relation between vol-trade and negative realized skewness, and this relationship can be explained by the DOH hypothesis, this is similar to Hong and Stein (2003). This implies that a high vol-trade increases negative realized skewness. The same argument of difference of opinion is employed in explaining the results for positive realized skewness and vol-trade. We observe a negative and significant relationship between positive realized skewness and vol-trade. This suggests that low levels of positive realized skewness is dependent on a high level of vol-trade. In the case of realized kurtosis, the significant and negative relationship with vol-trade is observed, can similarly be explained by the DOH hypothesis. For the monthly holding period, we observe that the level of significance decreases.

Vol-trade (VT) does behave like the squared measure of information flow. This is in line with Campbell et al. (1993) who use volume squared to capture any nonlinearity between volume and autocorrelation. In the same token, vol-trade captures any nonlinearity between information flow and realized higher-order moments. The high significant regression coefficients of vol-trade proxy in our weekly period, when compared with other proxies of volume, highlight the nonlinear relationship between information flow and realized higher-order moment.

Table 4.9: Vol-trade-realized moments relationship

	Realized Variance			Realized Skewness < 0		
	GMM	ROB	MED	GMM	ROB	MED
Panel A1: Weekly data						
Coefficient	2.65E-13**	1.18E-13***	1.75E-13***	3.29E-11***	2.97E-11***	9.07E-02**
t-Values ≥1.96	49.30%	65.49%	64.79%	5.63%	8.45%	2.82%
t-Values ≤-1.96	0.70%	4.93%	1.41%	58.45%	56.34%	52.11%
Average R^2	24.12%	60.46%	-	10.06%	9.07%	-
Panel B1: Monthly data						
Coefficient	2.60E-14	3.30E-14**	4.55E-14	8.33E-12*	4.14E-12*	4.27E-12
t-Values ≥1.96	27.46%	40.85%	26.06%	5.63%	2.11%	1.41%
t-Values ≤-1.96	0.70%	2.82%	1.41%	32.39%	29.58%	24.65%
Average R^2	32.34%	64.70%	-	24.72%	26.87%	-
	Realized Skewness >0			Realized Kurtosis		
	GMM	ROB	MED	GMM	ROB	MED
Panel A2: Weekly data						
Coefficient	-1.58E-11***	-8.89E-12***	2.37E-11**	-1.35E-11***	-8.05E-11***	-6.05E-11**
t-Values ≥1.96	47.18%	47.18%	40.85%	55.63%	45.77%	41.55%
t-Values ≤-1.96	9.15%	6.34%	9.15%	3.52%	17.61%	6.34%
Average R^2	7.05%	7.09%	-	15.36%	20.00%	-
Panel B2: Monthly data						
Coefficient	-1.86E-12	1.15E-12	2.14E-12	-3.16E-11*	1.37E-11**	3.85E-12
t-Values ≥1.96	23.94%	18.31%	11.27%	32.39%	31.69%	25.35%
t-Values ≤-1.96	5.63%	2.82%	5.63%	2.82%	16.90%	7.04%
Average R^2	17.95%	20.43%	-	16.95%	27.25%	-

This table reports the regression results for the relationship between realized high-order moments and our new volume measure (vol-trade (VT) which is defined as the product of trading volume and the number of trades) for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A's presents the results for the weekly holding period while panel B's reports the monthly results. The results are based on the following regressions:

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i VT_{it} + \vartheta_{it}$$

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i VT_{it} + \vartheta_{it} \quad ,given \quad RS < 0$$

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i VT_{it} + \vartheta_{it} \quad ,given \quad RS > 0$$

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i VT_{it} + \vartheta_{it}$$

where VT_{it} is the vol-trade for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The ‘coefficient’ and ‘Average R^2’ are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. ‘t-value≥1.96’ (‘t-value ≤-1.96’) shows the percentage of firms that have t-value statistically significant at the 5% level.

4.4.6 Robustness Checks

4.4.6.1 An alternative measure of volatility

The first robustness test we consider in this study is to employ the absolute residual approach proposed by Jones et al. (1994). The absolute residual procedure involves a two-stage regression methodology. In the first stage, we run returns for each stock on 12 return lags to account for any movements in conditional expected returns.

$$R_{it} = \sum_{j=1}^{12} \beta_{ij} R_{it-j} + \hat{\varepsilon}_{it}$$

where R_{it} is the return of stock i on week/month t. The 12 lagged returns are used to control for any serial dependence that might be present in the return series.

The second stage regresses the absolute residuals for each stock from the first stage on 12 lags of absolute residuals, trading volume, number of trades, average trade size, and vol-trade. The models used are presented in Equations (4.8), (4.9), (4.10) and (4.11). The volatility is measured as absolute residual ($\mid \hat{\varepsilon}_{it} \mid$). To examine the relationship between the volatility and trading volume, we estimate the following regressions for each stock:

$$\mid \hat{\varepsilon}_{it} \mid = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} \mid \hat{\varepsilon}_{it-j} \mid + \beta_i TV_{it} + \vartheta_{it} \tag{4.8}$$

$$\mid \hat{\varepsilon}_{it} \mid = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} \mid \hat{\varepsilon}_{it-j} \mid + \beta_i NT_{it} + \vartheta_{it} \tag{4.9}$$

$$\mid \hat{\varepsilon}_{it} \mid = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} \mid \hat{\varepsilon}_{it-j} \mid + \beta_i NT_{it} + \gamma_i ATS_{it} + \vartheta_{it} \tag{4.10}$$

$$\mid \hat{\varepsilon}_{it} \mid = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} \mid \hat{\varepsilon}_{it-j} \mid + \beta_i VT_{it} + \vartheta_{it} \tag{4.11}$$

where TV_{it}, NT_{it}, ATS_{it} and VT_{it} are trading volume, number of trades, average trade size and vol-trade for stock i on week/month t, the lagged values of $\mid \hat{\varepsilon}_{it} \mid$ used to control for persistence in volatility. According to Jones et al. (1994), ρ_{ij} captures the persistence of volatility shocks at lag j. This procedure is extensively documented in the literature, and various researchers employ the methodology for testing volatility and trading volume relationship.

Table 4.10 Panel A1 reports the results for weekly trading volume in (Equation (4.8)), regardless of the regression technique used, we observe a positive and significant coefficient of trading volume, number of trades and vol-trade, which is consistent with the MDH hypothesis. For Equation (4.9), the number of trades has a positive coefficient, but the significance level drops to 10% level in comparison to the trading volume and the vol-trade where the significance is at 1% level for the weekly holding period. Jones et al. (1994); Chan and Fong (2006) show that for daily holding period, the number of trades dominates the trading volume in explaining the volume-volatility relationship. Our results suggest that perhaps as the holding period increase from daily to weekly, it's the trading volume/vol-trade that drives the volume-volatility relationship significantly. In model (4.10), average trade size is added to the number of trades in the regression model. It is observed that the significance level of the number of trades improves from 10% level (Equation (4.9)) to 5% level (Equation (4.10)) for the weekly period. We also find that the coefficient of average trade size in Equation (4.10) is insignificant, and as such, is not reported in this study. The regression was also run separately for average trade size. However, the results remained insignificant and not reported in this study. This is consistent with the findings of Jones et al. (1994); Chan and Fong (2000, 2006); Shahzad et al. (2014) who show that daily average trade size is not as significant as the daily number of trades in explaining the volume-volatility relationship. Vol-trade has more explanatory power than the average trade size and number of trades in the weekly holding period. The significance of the regression models disappears in the monthly period.

Table 4.10: Robustness test: An alternative measure of volatility

	Model 4.8: Trading Volume			Model 4.9: Number of trades		
	GMM	ROB	MED	GMM	ROB	MED
Panel A1: Weekly data						
Coefficient	3.21E-09***	1.52E-09***	2.12E-09***	2.87E-06*	1.35E-06*	1.96E-06
t-Values ≥1.96	73.24%	73.24%	66.20%	38.03%	32.39%	35.21%
t-Values ≤-1.96	0.00%	0.00%	0.00%	2.82%	0.70%	0.70%
Average R^2	12.94%	12.07%	-	8.83%	9.38%	-
Panel B1: Monthly data						
Coefficient	9.14E-10	6.53E-10	7.40E-10	6.63E-08	3.66E-07	4.78E-07
t-Values ≥1.96	24.65%	23.24%	16.90%	4.23%	7.0%	10.56%
t-Values ≤-1.96	2.11%	0.70%	0.70%	3.52%	0.70%	2.11%
Average R^2	11.72%	13.28%	-	10.42%	12.47%	-
	Model 4.10: Number of trades			Model 4.11: Vol-trade		
	GMM	ROB	MED	GMM	ROB	MED
Panel A2: Weekly data						
Coefficient	3.49E-06**	1.71E-06**	2.55E-06*	1.09E-12***	7.35E-13***	1.05E-12***
t-Values ≥1.96	53.52%	43.66%	39.44%	69.72%	69.72%	60.56%
t-Values ≤-1.96	2.11%	0.70%	0.70%	0.00%	0.00%	0.00%
Average R^2	9.76%	9.99%	-	11.99%	12.27%	-
Panel B2: Monthly data						
Coefficient	9.95E-08	5.34E-07	5.68E-07	9.62E-14	9.25E-14	1.67E-13
t-Values≥1.96	11.27%	11.97%	13.38%	14.79%	19.72%	14.08%
t-Values ≤-1.96	2.82%	0.70%	0.00%	1.41%	0.70%	0.70%
Average R^2	11.31%	13.26%	-	11.54%	13.77%	-

This table reports the regression results for the relationship between realized volatility and the trading volume for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results.

4.4.6.2 Natural logarithm of realized moments

In the second robustness test, we use the natural logarithm of realized volatility and realized kurtosis, the ‘sign’ times the natural logarithm of realized skewness. The ‘sign’ addresses the right direction of the realized skewness after taking its natural logarithm. In Table 4.11, we report the results for the natural logarithm of the higher-order realized moments and trading volume. This examines the relationship between trading volume and higher-order realized moments. As always, Panel A gives the results for the weekly holding period, and we observe a significant and positive relationship between trading volume and the natural logarithm of realized variance, which supports the MDH hypothesis. A positive and significant relationship is observed for log realized kurtosis and the trading volume. This implies that high trading volume increases the natural logarithm of realized kurtosis. For the monthly horizon, the significance for realized variance and kurtosis remains statistically significant. The ‘sign’ log of realized skewness has a negative but not significant relationship with trading volume for the weekly period. The monthly period remains insignificant but with mixed directions depending on the regression technique.

Table 4.11: Robustness test: using the natural logarithm of the realized moment and trading volume

	Log Realized Variance			**sign× Log Realized Skewness**			**Log Realized Kurtosis**		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	5.04E-08***	4.77E-08***	4.38E-08***	-1.85E-08	-2.25E-08	-1.74E-08	2.39E-08***	2.12E-08***	1.89E-08***
t-Values ≥1.96	73.24%	76.76%	68.31%	2.82%	4.22%	4.93%	55.63%	48.59%	42.25%
t-Values ≤-1.96	0.00%	0.70%	0.00%	10.56%	14.09%	11.27%	2.82%	3.52%	2.82%
Average R^2	43.02%	46.66%	-	2.69%	2.38%	-	20.00%	21.09%	-
Panel B: Monthly data									
Coefficient	1.03E-08**	8.73E-09***	8.15E-09**	-1.62E-10	9.85E-10	4.16E-09	1.37E-08**	9.19E-09**	1.03E-08*
t-Values ≥1.96	53.52%	54.23%	52.11%	2.82%	2.11%	4.23%	42.25%	32.39%	30.99%
t-Values ≤-1.96	2.11%	1.41%	0.70%	7.75%	8.45%	9.86%	2.82%	5.63%	4.93%
Average R^2	48.62%	53.11%	-	7.48%	8.00%	-	23.62%	26.28%	-

This table presents the regression results for the relationship between the natural logarithm of realized moments and the trading volume for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:" *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$LogRV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} LogRV_{it-j} + \beta_i TV_{it} + \vartheta_{it}$$

$$sign \times LogRS_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} sign \times LogRS_{it-j} + \beta_i TV_{it} + \vartheta_{it}$$

$$LogRK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} LogRK_{it-j} + \beta_i TV_{it} + \vartheta_{it}$$

where TV_{it} is the trading volume, $LogRV_{it}$ is the natural logarithm of realized variance, '$sign' \times LogRS_{it}$ is the natural logarithm of realized skewness taking into account the 'sign' of the skewness and $LogRK_{it}$ is the natural logarithm of realized kurtosis for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The 'coefficient' and 'Average R^2' are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. 't-value≥1.96' ('t-value ≤-1.96') shows the percentage of firms that have t-value statistically significant at the 5% level.

Table 4.12 reports the results of the log higher-order realized moments and the number of trades. The results of the weekly panel show that the coefficient of the regression between the number of trades and natural log of realized variance is positive and significant, as desired per the MDH hypothesis. In the case of log realized kurtosis, a negative and significant relationship is obtained with the number of trades, this can be explained by DOH hypothesis. This suggests a decrease in log realized kurtosis is dependent on high number of trades, which is consistent with the results discussed earlier in Table 4.6. The results for the 'sign' log realized skewness is negative and insignificant in both holding periods.

Table 4.12: Robustness test: using the natural logarithm of the realized moment and number of trades

	Log Realized Variance			**sign× Log Realized Skewness**			**Log Realized Kurtosis**		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	1.30E-04***	1.08E-04**	9.14E-05**	-3.88E-05	-2.49E-05	-4.02E-05	-5.50E-05**	-7.38E-05***	-6.80E-05**
t-Values ≥1.96	58.45%	45.77%	40.85%	1.41%	3.52%	4.93%	40.14%	40.85%	37.32%
t-Values ≤-1.96	0.70%	3.52%	4.23%	14.08%	16.90%	14.79%	12.68%	26.06%	19.72%
Average R^2	41.03%	45.21%	-	2.69%	2.38%	-	18.65%	20.25%	-
Panel B: Monthly data									
Coefficient	2.62E-05	2.34E-05	2.55E-05	-1.67E-05	-1.10E-05	-1.6E-05	-2.92E-05	-4.44E-05*	-4.20E-05*
t-Values ≥1.96	19.72%	16.20%	20.42%	1.41%	2.82%	3.52%	26.76%	23.24%	24.65%
t-Values ≤-1.96	1.41%	1.41%	2.82%	7.75%	7.04%	11.27%	7.75%	22.54%	12.68%
Average R^2	46.21%	51.57%	-	7.49%	7.88%	-	21.43%	25.32%	-

This table presents the regression results for the relationship between the natural logarithm of realized moments and the number of trades for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:" *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$LogRV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} LogRV_{it-j} + \beta_i NT_{it} + \vartheta_{it}$$

$$sign \times LogRS_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} sign \times LogRS_{it-j} + \beta_i NT_{it} + \vartheta_{it}$$

$$LogRK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} LogRK_{it-j} + \beta_i NT_{it} + \vartheta_{it}$$

where NT_{it} is the number of trades, $LogRV_{it}$ is the natural logarithm of realized variance, $sign \times LogRS_{it}$ is the natural logarithm of realized skewness taking into account the 'sign' of the skewness and $LogRK_{it}$ is the natural logarithm of realized kurtosis for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The 'coefficient' and 'Average R^2' are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. 't-value≥1.96' ('t-value ≤-1.96') shows the percentage of firms that have t-value statistically significant at the 5% level.

Table 4.13 reports the results for vol-trade, we observe a positive and significant relation between log of realized variance and vol-trade consistent with the MDH hypothesis in Panel A. It is interesting that with the vol-trade the percentage of firms that are significantly above the 5% level is 80.99% (115 out of 142 of our sample). This is higher than that of trading volume and the number of trades. The significance is persistent during the monthly period, although the number of significant firms reduces. For log realized kurtosis, a negative and significant relationship is obtained, which can be explained by difference of opinion. The monthly values are also negative and significant at the 5% level for GMM and ROB regressions. The 'sign' log of realized skewness remains insignificant in the scenario as well. We conclude that the vol-trade variable has a more explanatory power than the traditional proxy (average trade size).

Table 4.13: Robustness test: using the natural logarithm of the realized moment and vol-trade

	Log Realized Variance			sign× Log Realized Skewness			Log Realized Kurtosis		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Weekly data									
Coefficient	3.61E-11***	3.30E-11***	3.53E-11***	-1.62E-11	-4.11E-11	-3.22E-11	-3.20E-12***	-7.83E-12***	-5.82E-12***
t-Values ≥1.96	80.99%	80.28%	70.42%	2.82%	2.11%	2.82%	59.15%	57.75%	45.77%
t-Values ≤-1.96	0.00%	0.00%	0.70%	19.01%	25.35%	16.90%	2.11%	6.34%	2.82%
Average R^2	42.73%	46.16%	-	2.71%	2.47%	-	19.84%	21.18%	-
Panel B: Monthly data									
Coefficient	1.12E-12**	1.30E-12*	1.90E-12*	3.42E-13	5.74E-13	-3.34E-13	-7.59E-13**	-1.28E-12**	-3.53E-13*
t-Values ≥1.96	48.59%	44.37%	40.14%	2.11%	0.70%	3.52%	41.55%	33.80%	30.99%
t-Values ≤-1.96	0.00%	0.00%	0.70%	9.15%	10.56%	14.08%	2.82%	8.45%	6.34%
Average R^2	47.53%	52.27%	-	7.61%	8.16%	-	23.09%	26.28%	-

This table presents the regression results for the relationship between the natural logarithm of realized moments and the vol-trade for the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Significance levels:" *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly holding period while panel B reports the monthly results. The results are based on the following regressions:

$$LogRV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} LogRV_{it-j} + \beta_i VT_{it} + \vartheta_{it}$$

$$sign \times LogRS_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} sign \times LogRS_{it-j} + \beta_i VT_{it} + \vartheta_{it}$$

$$LogRK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} LogRK_{it-j} + \beta_i VT_{it} + \vartheta_{it}$$

where VT_{it} is the vol-trade, $LogRV_{it}$ is the natural logarithm of realized variance, $sign \times LogRS_{it}$ is the natural logarithm of realized skewness taking into account the 'sign' of the skewness and $LogRK_{it}$ is the natural logarithm of realized kurtosis for stock i on week/month t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The 'coefficient' and 'Average R^2' are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. 't-value≥1.96' ('t-value ≤-1.96') shows the percentage of firms that have t-value statistically significant at the 5% level.

4.4.6.3 Sub-periods

Our data sample cover extensively 15 years span (from 6 January 2003 - 29 December 2017). Following Dungey and Gajurel (2014) we split the full-sample period into pre-crisis (from 6 January 2003 - 29 June 2007), crisis-period (from 2 July 2007 - 29 May 2009) and post-crisis period (from 1 June 2009 - 29 December 2017). In this section, we investigate the volume-higher-order realized moment relation for only the weekly holding period since we observe that the significance of most of the average coefficient disappears in the monthly holding period for the results discussed earlier.

Table 4.14 presents the results for volume-volatility relation. Panel A reports the results for trading volume and realized variance. We observe positive and significant relation for pre-crisis, crisis, and post-crisis periods in accordance with the MDH hypothesis. In Panel B and C, a similar result is obtained when the number of trades and vol-trade are used as proxies for information flow. We conclude that the contemporaneous positive volume-volatility relation holds for

Table 4.14: Robustness test: realized variance, trading volume, number of trades and vol-trade using sub-periods

	Pre-Crisis			Crisis			Post-Crisis		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Trading Volume									
Coefficient	7.21E-10**	1.78E-10***	2.42E-10**	1.26E-09**	4.93E-10***	6.27E-10*	1.04E-09***	8.58E-11***	1.69E-10***
t-Values≥1.96	46.48%	54.23%	42.96%	40.85%	54.93%	35.21%	58.45%	66.20%	61.27%
t-Values ≤-1.96	1.41%	1.41%	2.11%	2.11%	2.11%	1.41%	1.41%	2.82%	2.11%
Average R^2	20.95%	30.31%	-	37.32%	59.39%	-	23.29%	45.79%	-
Panel B: Number of trades									
Coefficient	8.30E-06**	4.38E-06***	5.40E-06**	7.30E-06**	4.54E-06***	6.05E-06**	2.58E-07**	4.19E-07***	4.79E-07***
t-Values ≥1.96	59.15%	69.01%	61.97%	55.63%	64%	42.25%	46.48%	50.70%	50.00%
t-Values ≤-1.96	0.00%	0.70%	0.00%	0.70%	0.70%	0.00%	3.52%	7.04%	7.04%
Average R^2	18.49%	31.77%	-	36.20%	59.47%	-	17.98%	44.42%	-
Panel C: Vol-trade									
Coefficient	1.65E-12**	6.25E-13***	6.36E-13*	9.85E-13***	1.03E-12***	8.96E-13*	4.38E-13***	7.38E-14***	1.14E-13***
t-Values ≥1.96	50.00%	64.08%	41.55%	54.23%	61.27%	40.85%	58.45%	69.72%	59.15%
t-Values ≤-1.96	0.70%	0.70%	0.00%	0.70%	1.41%	0.00%	0.70%	2.11%	1.41%
Average R^2	23.25%	33.37%	-	39.18%	60.42%	-	25.32%	46.89%	-

This table reports the regression results for the relationship between realized variance, trading volume, number of trades and vol-trade for the 142 stocks. We split the full-sample period into pre-crisis from 6 January 2003 - 29 June 2007, crisis-period from 2 July 2007 - 29 May 2009 and the post-crisis period from 1 June 2009 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly realized variance and trading volume while panel B reports the results for the weekly realized variance and number of trades. The results are based on the following regressions:

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i TV_{it} + \vartheta_{it} \qquad Panel \quad A$$

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i NT_{it} + \vartheta_{it} \qquad Panel \quad B$$

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i VT_{it} + \vartheta_{it} \qquad Panel \quad C$$

where TV_{it} is the trading volume, NT_{it} is the number of trades, VT_{it} is the vol-trade for stock i on week t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The ‘coefficient’ and ‘Average R^2’ are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. ‘t-value≥1.96’ (‘t-value ≤-1.96’) shows the percentage of firms that have t-value statistically significant at the 5% level.

weekly holding-period during all sub-periods for trading volume, the number of trades, and vol-trade.

Table 4.15 reports the results for the relationship between negative realized skewness and information flow across the sub-periods. The results for trading volume show a negative and significant relationship in all periods. Although, the significance level increases in the post-crisis period in comparison to other sub-sample periods. In the case of the number of trades, a positive relation is obtained, which is only significant during the post-crisis. The significant positive relationship, as discussed earlier, could be explained by DOH hypothesis. For vol-trade and negative realized skewness, we observe a negative relationship in the pre-crisis period. However, this tends into a positive and significant relation during the crisis period. In the post-crisis period, the result of vol-trade is mixed. However, considering over half of the firms have significant t-value ≤ -1.96, this suggests negative relationship dominates; as such, an increase vol-trade decreases negative realized skewness.

Table 4.15: Robustness test: negative realized skewness, trading volume, number of trades and vol-trade using sub-periods

	Pre-Crisis			Crisis			Post-Crisis		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Trading Volume									
Coefficient	-1.77E-07**	-1.32E-07*	-1.84E-07	-7.72E-08*	-4.91E-08*	-5.52E-08	-1.87E-07***	-1.40E-07**	-1.77E-07**
t-Values ≥1.96	3.52%	0.70%	2.11%	4.23%	2.11%	0.70%	2.11%	1.41%	2.11%
t-Values ≤-1.96	37.32%	28.87%	24.65%	34.51%	27.46%	15.49%	64.08%	50.70%	47.89%
Average R^2	20.35%	21.26%	-	33.16%	37.80%	-	13.55%	10.95%	-
Panel B: Number of trades									
Coefficient	1.09E-03	1.01E-03	1.12E-03	1.07E-03	1.30E-03	1.01E-03	3.96E-04***	4.20E-04**	3.84E-04**
t-Values ≥1.96	9.15%	6.34%	7.75%	14.79%	7.75%	3.52%	14.79%	14.08%	11.97%
t-Values ≤-1.96	23.24%	21.13%	17.61%	21.13%	14.08%	7.04%	50.70%	41.55%	41.55%
Average R^2	18.13%	18.93%	-	31.19%	34.86%	-	11.87%	10.31%	-
Panel C: Vol-trade									
Coefficient	-1.80E-10**	-2.78E-10**	-2.93E-10	3.18E-10**	4.90E-10*	3.22E-10	2.48E-11***	-3.60E-12***	-7.56E-12**
t-Values ≥1.96	5.63%	0.70%	2.11%	7.75%	4.23%	1.41%	7.04%	2.11%	3.52%
t-Values ≤-1.96	35.92%	29.58%	23.94%	30.99%	26.06%	16.20%	64.08%	57.75%	46.48%
Average R^2	20.05%	22.31%	-	33.30%	37.11%	-	14.43%	12.86%	-

This table reports the regression results for the relationship between negative realized skewness, trading volume and number of trades for the 142 stocks. We split the full-sample period into pre-crisis from 6 January 2003 - 29 June 2007, crisis-period from 2 July 2007 - 29 May 2009 and the post-crisis period from 1 June 2009 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly negative realized skewness and trading volume while panel B reports the results for the weekly negative realized skewness and number of trades. The results are based on the following regressions:

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i TV_{it} + \vartheta_{it}, \quad given \quad RS < 0 \qquad Panel \quad A$$

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i NT_{it} + \vartheta_{it}, \quad given \quad RS < 0 \qquad Panel \quad B$$

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i VT_{it} + \vartheta_{it}, \quad given \quad RS < 0 \qquad Panel \quad C$$

where TV_{it} is the trading volume, NT_{it} is the number of trades, VT_{it} is the vol-trade for stock i on week t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The 'coefficient' and 'Average R^2' are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. 't-value≥1.96' ('t-value ≤-1.96') shows the percentage of firms that have t-value statistically significant at the 5% level.

Table 4.16 reports the results for weekly positive realized skewness and trading volume. In Panel A, we observe that positive realized skewness has a positive relationship with trading volume across all sub-periods. Although this relationship isn't highly significant, the significance level improves during the post-crisis period. This means high trading volume increases positive realized skewness and consistent with the results from the full sample period in Table (4.5). The significance level improves in the post-crisis period. In the case of the number of trades, an insignificant negative relationship is observed for pre-crisis and crisis periods. However, the coefficients during the post-crisis period are significant. The negative relationship is explained by differing of opinion of the investors as discussed earlier. For the vol-trade, a positive relationship is obtained in the pre-crisis period. Although, this relationship is only significant at the 10% level. The crisis and post-crisis period exhibit a negative relationship, and we observe that the significance of the negative relationship between positive skewness and vol-trade increases during the post-crisis period.

Table 4.16: Robustness test: positive realized skewness, trading volume, number of trades and vol-trade using sub-periods

	Pre-Crisis			Crisis			Post-Crisis		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Trading volume									
Coefficient	1.09E-07*	6.82E-08*	5.59E-08	2.72E-08*	3.21E-09	4.94E-08	1.30E-07**	9.50E-08**	1.30E-07*
t-Values ≥1.96	33.80%	32.39%	25.35%	31.69%	20.42%	11.27%	48.59%	45.77%	35.92%
t-Values ≤-1.96	2.82%	1.41%	1.41%	3.52%	3.52%	0.00%	2.82%	1.41%	2.11%
Average R^2	16.88%	16.71%	-	33.26%	39.42%	-	10.30%	10.46%	-
Panel B: Number of trades									
Coefficient	-6.81E-04	-5.94E-04	-6.79E-04	-1.72E-03	-1.53E-03	-1.56E-03	-4.12E-04**	-3.98E-04*	-4.21E-04*
t-Values ≥1.96	21.13%	19.01%	14.79%	9.15%	6.34%	6.34%	35.21%	30.99%	33.80%
t-Values ≤-1.96	11.97%	4.93%	8.45%	8.45%	7.04%	4.93%	16.20%	14.79%	14.79%
Average R^2	15.01%	15.74%	-	31.60%	37.34%	-	9.23%	8.91%	-
Panel C: Vol-trade									
Coefficient	1.09E-11*	7.48E-12*	-8.39E-11	-5.28E-10*	-4.84E-10	-2.08E-10	-2.96E-11***	-4.62E-11**	1.85E-11*
t-Values ≥1.96	34.51%	31.69%	19.72%	25.35%	19.72%	7.75%	45.77%	38.73%	35.21%
t-Values ≤-1.96	4.93%	2.11%	1.41%	5.63%	2.82%	0.00%	7.75%	4.23%	4.23%
Average R^2	16.58%	17.18%	-	32.92%	38.79%	-	10.38%	10.24%	-

This table reports the regression results for the relationship between positive realized skewness, trading volume and number of trades for the 142 stocks. We split the full-sample period into pre-crisis from 6 January 2003 - 29 June 2007, crisis-period from 2 July 2007 - 29 May 2009 and the post-crisis period from 1 June 2009 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly realized skewness conditioned on positive realized skewness and trading volume while panel B reports the results for the weekly positive realized skewness and number of trades. The results are based on the following regressions:

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i TV_{it} + \vartheta_{it}, \quad given \quad RS > 0 \qquad Panel \quad A$$

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i NT_{it} + \vartheta_{it}, \quad given \quad RS > 0 \qquad Panel \quad B$$

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i VT_{it} + \vartheta_{it}, \quad given \quad RS > 0 \qquad Panel \quad C$$

where TV_{it} is the trading volume, NT_{it} is the number of trades for stock i on week t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The ‘coefficient’ and ‘Average R^2’ are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. ‘t-value≥1.96’ (‘t-value ≤-1.96’) shows the percentage of firms that have t-value statistically significant at the 5% level.

In Table 4.17, we observe a positive relationship between realized kurtosis and trading volume. In Panel A, the coefficients are more significant during the post-crisis period. In short, high trading volume lead to an increase in realized kurtosis, which is consistent with the results discussed in Panel A of Table 4.6. The number of trades, on the other hand, has an insignificant negative relationship with realized kurtosis during the crisis period. This negative relationship can be explained by the DOH hypothesis as already discussed earlier. This suggest high number of trades decreases realized kurtosis, similar result is obtained in the case of vol-trade. Although in this case, the crisis period is significant. We conclude that the trading volume and realized kurtosis relationship is dominated by a positive relationship, while the number of trades has a negative relation with realized kurtosis. For vol-trade, the result is mixed.

Table 4.17: Robustness test: realized kurtosis, trading volume, number of trades and vol-trade using sub-periods

	Pre-Crisis			**Crisis**			**Post-Crisis**		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Trading Volume									
Coefficient	7.90E-07*	8.34E-08*	3.21E-07	3.44E-07*	2.17E-07*	5.75E-07	1.13E-06***	2.73E-07**	5.11E-07**
t-Values ≥1.96	36.62%	28.17%	25.35%	38.03%	29.58%	23.94%	60.56%	41.55%	45.77%
t-Values ≤-1.96	4.23%	7.04%	4.23%	5.63%	2.11%	0.70%	2.82%	10.56%	5.63%
Average R^2	15.70%	17.25%	-	21.66%	22.07%	-	13.69%	13.90%	-
Panel B: Number of trades									
Coefficient	-7.25E-03*	-8.13E-03**	-7.60E-03**	-7.86E-03	-6.07E-03	-6.86E-03	-2.12E-03***	-2.36E-03***	-1.71E-03***
t-Values ≥1.96	23.94%	22.54%	23.94%	20.42%	14.79%	12.68%	55.63%	48.59%	48.59%
t-Values ≤-1.96	13.38%	23.94%	25.35%	18.31%	14.79%	13.38%	12.68%	27.46%	24.65%
Average R^2	13.27%	17.47%	-	18.38%	20.15%	-	11.50%	14.70%	-
Panel C: Vol-trade									
Coefficient	-3.04E-10**	-7.17E-10**	-4.55E-10	-2.18E-09**	-1.27E-09*	-6.69E-10	8.93E-11***	-7.69E-11***	1.33E-11***
t-Values ≥1.96	34.51%	33.10%	27.46%	35.21%	28.17%	18 22/71	60.56%	52.82%	49.30%
t-Values ≤-1.96	5.63%	6.34%	5.63%	8.45%	4.93%	4.93%	3.52%	11.27%	4.93%
Average R^2	15.71%	19.39%	-	21.36%	21.86%	-	14.50%	15.99%	-

This table reports the regression results for the relationship between realized kurtosis, trading volume, number of trades and vol-trade for the 142 stocks. We split the full-sample period into pre-crisis from 6 January 2003 - 29 June 2007, crisis-period from 2 July 2007 - 29 May 2009 and the post-crisis period from 1 June 2009 - 29 December 2017. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly realized kurtosis and trading volume while panel B reports the results for the weekly realized kurtosis and number of trades. The results are based on the following regressions:

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i TV_{it} + \vartheta_{it} \qquad Panel \quad A$$

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i NT_{it} + \vartheta_{it} \qquad Panel \quad B$$

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i VT_{it} + \vartheta_{it} \qquad Panel \quad C$$

where TV_{it} is the trading volume, NT_{it} is the number of trades, VT_{it} is the vol-trade for stock i on week t. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The ‘coefficient’ and ‘Average R^2’ are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. ‘t-value≥1.96’ (‘t-value ≤-1.96’) shows the percentage of firms that have t-value statistically significant at the 5% level.

4.4.6.4 Controlling for the market rate of information flow

For the final robustness test, we use the post-crisis period data since, in this sub-period, the volume-higher-order realized moment relationship is more significant per the third robustness test in the previous section. Table 4.18 reports the weekly regression results for the volume-higher-order realized moment relationship accounting for the market rate information flow (thus the trading volume, number of trades, and vol-trade of the S&P/ASX200 index). In Panel A, we investigate the relationship of realized variance/volatility with trading volume, number of trades and vol-trade. We observe the contemporaneous and instantaneous positive volume-volatility relationship is. The relationship is statistically significant and consistent with the MDH hypothesis, even after controlling for market trading volume, market number of trades, and market vol-trade.

For Panel B, negative realized skewness has a negative relationship with trading volume conditional on market trading volume. This implies that high trading volume decreases negative realized skewness/bad news. In the case of the number of trades, the relationship is positive can be explained by the DOH hypothesis as seen in previous section. This suggests higher levels of the number of trades increases negative realized skewness/bad news (see Hong and Stein (2003)). In order words, higher liquidity increases fear and bad news. For vol-trade, the results tend to be mixed for GMM and ROB. However, considering over half of the firms have significant t-value $\leq$ -1.96, this suggests negative relationship dominates; as such, an increase in negative realized skewness is conditional on a decrease in vol-trade.

In Panel C, positive realized skewness has a positive relationship with trading volume. Hence high trading volume increases positive skewness/good news. In the case of number of trades and vol-trade, we observe a negative relationship with positive realized skewness. The asymmetric-opinion can be extended to explain this relationship. This means lower levels of number of trades or vol-trade will increase positive realized skewness/good news (thus lower liquidity will increase greed and good news).

Finally, in Panel D, we observe a positive relationship between realized kurtosis and trading volume, thus trading volume increases realized kurtosis. In the case of number of trades, we observe that number of trade decreases realized kurtosis which is consistent with the results of Do et al. (2014) and our previous results discussed. For vol-trade, the results are mixed for GMM and ROB. Since over half of the firms have significant t-value $\geq$ 1.96, this suggests positive relationship dominates, and hence an increase in realized kurtosis is conditional on an increase in vol-trade for the post-crisis period. To conclude, we observe that when we control for the rate of market information flow, the volume-higher-order realized moment relationship remains statistically significant.

Table 4.18: Robustness test: controlling for market information flow for post-crisis periods

	Trading volume			Number of trades			Vol-trade		
	GMM	ROB	MED	GMM	ROB	MED	GMM	ROB	MED
Panel A: Realized variance									
Coefficient	1.02E-09***	9.51E-11***	1.74E-10***	2.46E-07**	4.18E-07***	4.81E-07***	3.74E-13***	7.17E-14***	1.07E-13***
t-Values ≥ 1.96	58.45%	69.01%	61.27%	47.18%	50.00%	51.41%	60.56%	74.65%	62.68%
t-Values ≤-1.96	1.41%	2.82%	1.41%	3.52%	7.04%	7.04%	0.70%	2.11%	0.00%
Average R^2	23.92%	45.66%	-	18.10%	43.98%	-	26.29%	45.85%	-
Panel B: Negative realized skewness									
Coefficient	-1.84E-07***	-1.34E-07**	-1.54E-07**	4.05E-04***	4.31E-04**	3.84E-04**	1.25E-11***	-2.17E-12***	9.66E-13**
t-Values ≥1.96	2.11%	1.41%	1.41%	14.79%	14.08%	14.08%	4.93%	1.41%	3.52%
t-Values ≤-1.96	64.79%	51.41%	44.37%	51.41%	41.55%	40.14%	64.08%	54.23%	47.89%
Average R^2	15.41%	12.44%	-	12.23%	10.87%	-	16.22%	14.88%	-
Panel C: Positive realized skewness									
Coefficient	1.25E-07**	9.31E-08**	1.27E-07*	-4.14E-04**	-4.04E-04**	-4.21E-04**	-2.42E-11***	-4.59E-11**	7.79E-12*
t-Values ≥1.96	48.59%	45.07%	36.62%	35.92%	30.99%	34.51%	47.18%	42.25%	40.14%
t-Values ≤-1.96	2.11%	0.70%	1.41%	16.20%	14.79%	14.08%	3.52%	1.41%	2.11%
Average R^2	12.10%	11.81%	-	9.73%	9.45%	-	11.97%	11.97%	-
Panel D: Realized kurtosis									
Coefficient	1.19E-06***	3.36E-07**	5.72E-07**	-2.14E-03***	-2.37E-03***	-1.75E-03***	1.01E-10***	-6.22E-11***	2.40E-11**
t-Values ≥1.96	61.97%	44.37%	47.18%	55.63%	48.59%	50.00%	61.97%	54.93%	49.30%
t-Values ≤-1.96	2.82%	7.04%	3.52%	12.68%	28.17%	23.24%	2.11%	9.15%	2.82%
Average R^2	14.43%	14.98%	-	11.70%	14.96%	-	15.13%	16.64%	-

This table reports the regression results for the relationship between weekly realized higher-order moment and trading volume, the number of trades, vol-trade for the 142 stocks conditional on the market information flow. We only focus on the post-crisis period from 1 June 2009 - 29 December 2017. This is due to the fact that for the sub-period results, the post-crisis period results were more significant. Significance levels:“ *: 0.10, **: 0.05, ***: 0.01". Panel A presents the results for the weekly realized variance and the three proxies of information flow (trading volume, number of trades and vol-trade), panel B reports the results for negative skewness, panel c reports the results when positive realized skewness is used, and panel D is that of realized kurtosis. The results are based on the following regressions:

$$RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i TV_{it} + \lambda_m TV_{mt} + \vartheta_{it}, \quad RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i NT_{it} + \lambda_m NT_{mt} + \vartheta_{it}, \quad RV_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RV_{it-j} + \beta_i VT_{it} + \lambda_m VT_{mt} + \vartheta_{it}, \quad Panel \quad A$$

$$RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i TV_{it} + \lambda_m TV_{mt} + \vartheta_{it}, \quad RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i NT_{it} + \lambda_m NT_{mt} + \vartheta_{it}, \quad RS^-_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^-_{it-j} + \beta_i VT_{it} + \lambda_m VT_{mt} + \vartheta_{it}, \quad Panel \quad B$$

$$RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i TV_{it} + \lambda_m TV_{mt} + \vartheta_{it}, \quad RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i NT_{it} + \lambda_m NT_{mt} + \vartheta_{it}, \quad RS^+_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RS^+_{it-j} + \beta_i VT_{it} + \lambda_m VT_{mt} + \vartheta_{it}, \quad Panel \quad C$$

$$RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i TV_{it} + \lambda_m TV_{mt} + \vartheta_{it}, \quad RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i NT_{it} + \lambda_m NT_{mt} + \vartheta_{it}, \quad RK_{it} = \alpha_{i0} + \sum_{j=1}^{12} \rho_{ij} RK_{it-j} + \beta_i VT_{it} + \lambda_m VT_{mt} + \vartheta_{it}, \quad Panel \quad D$$

where TV_{it} is the trading volume, NT_{it} is the number of trades, VT_{it} is the vol-trade for stock i on week t. TV_{mt}, NT_{mt}, VT_{mt} are the trading volume, number of trades and vol-trade of the market index respectively. The regressions above are estimated by generalized method of moments (GMM) with Newey-west standard errors, robust regression (ROB) using iterative re-weighted least squares and the median regression (MED) with robust standard errors. The regressions are run separately for every 142 stocks being considered. The 'coefficient' and 'Average R^2' are the averages of the coefficient estimate (thus average of β_i) and the R^2 across the stocks. 't-value≥1.96' ('t-value ≤-1.96') shows the percentage of firms that have t-value statistically significant at the 5% level.

4.4.7 Summary of volume-higher-order realized moment relationships

Intuitively, the MDH hypothesis is conditional on public information, with the volume and volatility responding to the changes in the arrival rate of this publicly available information. The MDH hypothesis implicitly assumes the market is efficient, and all information is absorbed instantaneously by all market participants and is immediately reflected in the price generation process via market trading. The ensuring positive (but symmetric) volume-volatility relationship have been extensively documented in the extant literature (Epps and Epps, 1976; Clark, 1973; Tauchen and Pitts, 1983; Harris, 1986; Andersen, 1996; Chan and Fong, 2006; Carroll and Kearney, 2015). Researchers also have investigated the volume-volatility relationship via alternative models i.e. by decomposing the volatility into continuous and jump components and have reported that only the continuous component has a positive and significant relationship with trading volume (Giot et al., 2010; Shahzad et al., 2014). This is not surprising since the realized variance converges to the total quadratic variation and is dominated by the normally distributed continuous component which is implicitly information-symmetric; Amaya et al. (2015).

The SIAH hypothesis allows for asymmetric-formation via the sequential arrival of information, in which not all traders receive and absorb the same information at the same time and consequently they will hold heterogeneous objective beliefs or expectations. This means that the informed traders with 'lead-' or advanced-information would find it profitable to trade with uninformed traders with 'lag-' or retarded-information. Trade occurs because of differences of information amongst market participants i.e. investors will not trade with each other if they hold the same information and objective beliefs about an asset (Copeland, 1976; Chan and Fong, 2000; Celik, 2013). The information-asymmetry implied by the SIAH hypothesis also accommodates the observed asymmetry or realized skewness in the marketplace. If information arrives sequentially, then differences in information may actualise as asymmetric jumps especially when coupled with low number of trades i.e. low liquidity.

The DOH hypothesis goes one-step further by allowing for differences in opinions i.e. asymmetric-opinions and the formation of subjective beliefs. Trade occurs because of differences of opinion amongst market participants i.e. investors will not trade with each other if they hold the same opinions or subjective beliefs about an asset (Shalen, 1993; Harris and Raviv, 1993). The asymmetric-opinion inherent in the DOH hypothesis also enable one to explain the observed asymmetry or skewness in asset prices. As differences in opinion can also be symmetrically polarised at the tails, some researchers have also used the DOH hypothesis in explaining the observed negative relationship between volume (i.e. number of trades) and kurtosis; Do et al. (2014).

Table 4.19: Sign Matrix of Volume-Higher-order Realized Moments

	TV	NT	ATS	VT
RV	$+^{**}$	$+^{*}$	$-$	$+^{**}$
RS^-	$-^{***}$	$+^{***}$	$-^{*}$	$+^{***}$
RS^+	$+^{***}$	$-^{***}$	$+^{*}$	$-^{***}$
RK	$+^{***}$	$-^{**}$	$+$	$-^{***}$

Note: Realized Variance (RV), Negative Realized Skewness (RS^-), Positive Realized Skewness (RS^+), Realized Kurtosis (RK), Trading Volume (TV), Number of Trades (NT), Average Trade Size (ATS=TV/NT) and Vol-Trade (VT=TV×NT). The signs (+, -) and significance levels (*(0.10), **(0.05), ***(0.01)) based on Weekly data and Generalized Method of Moments (GMM).

Table 4.19 is a summary matrix of the signs of the coefficients as found between the four realized higher moments (negative realized skewness (RS^-), positive realized skewness (RS^+) are taken separately) and the four-volume proxies used in this study for volume. From Table 4.19, we note the following: trading volume (TV) and number of trades (NT) are primary proxies with average trade size (ATS=TV/NT) and vol-trade (VT=TV×NT) being secondary proxies. Considering the primary proxies first, realized variance has a positive relationship with both trading volume and number of trades. However, the other higher-moments have not only positive and negative relationships with trading volume and number of trades but trading volume and number of trades have opposite signs for negative realized skewness, positive realized skewness and realized kurtosis. Trading volume decreases negative skewness and increases positive skewness. Number of trades increases negative skewness and decreases positive skewness. Trading volume increases kurtosis and number of trades decreases kurtosis. Trading volume and number of trades have similar effects on realized variance. Trading volume and number of trades have opposite effects on realized skewness and realized kurtosis. High trading volumes means high realized variance, low negative realized skewness, high positive realized skewness and high realized kurtosis and vice versa. High trades means high realized variance, high negative realized skewness, low positive realized skewness and low realized kurtosis and vice versa. The secondary volume proxies, average trade size and vol-trade, mirror the primary proxies trading volume and number of trades respectively with the exception of the negative coefficient for average trade size when regressed with realized variance; but it is not significant.

Another point of note is that even though, trading volume and number of trades are commonly used primary proxies for volume, they are by definition related via trading volume (TV=NT×ATS) and hold opposing regression estimates with signed realized skewness and realized kurtosis. Increasing number of trades implicitly reflects more traders and more liquidity with a wider spread of trade sizes (i.e. less jumps present). Increasing trading volume embeds two possibilities; increasing number of trades or increasing average trade size or both. If it is driven by increasing average trade size then the 'effective' liquidity must be decreasing (i.e. more jumps present). This explains the equivalence of trading volume and average trade size in Table 4.19 (i.e. trading volume mirrors average trade size more than number of size). Hence trading volume increases realized kurtosis and number of trades decreases realized kurtosis. Trading volume decreases negative skewness and increases positive skewness and number of trades increases negative skewness and decreases positive skewness. In short, high trading volume (or high average trade size) coincides with the presence of more up-jumps and less down-jumps (positive skewness or good news or greed or lower liquidity) and high number of trades (or high vol-trade) coincides with more down-jumps and less up-jumps (negative skewness or bad news or fear or higher liquidity).

Thus from Table 4.19, we note that volume has a positive relationship with realized variance (with the exception of average trading size which is statistically insignificant), in that as the each of the volume proxies (with statistical significance) increases the volatility also increases. Thus the realized variance results are consistent with the variance claims of all three hypotheses i.e. MDH, SIAH and DOH hypotheses. The information-asymmetry implicit in the SIAH hypothesis and the opinion-asymmetry inherent in the DOH hypothesis both independently and jointly address the observed relationships between volume and the higher-order moments. Thus, the MDH hypothesis (symmetric-information) is implicitly nested within the SIAH hypothesis (asymmetric-information) which is then nested within the DOH hypothesis (asymmetric-opinion).

We also undertook four separate robustness checks, first of these using the absolute residuals. We observe that trading volume, number of trades, and vol-trade have positive and significant explanatory power for the weekly holding period (see Table 4.10). The second robustness test employs natural logarithms of the realized higher-order moments. We observe a positive and significant relationship between log realized variance and trading volume, number of trades, and vol-trade. In the case of log realized kurtosis, a significant negative relationship is observed for the number of trades and vol-trade, while that of trading volume exhibits a positive relationship. The 'sign' of natural logarithm of realized skewness has no explanatory power (see Tables 4.11, 4.12 and 4.13). For the third robustness test, we split the data into three sub-periods. For this test, only the weekly holding period is considered since most of the monthly results tend to be insignificant for the full sample period. The contemporaneous positive (realized) volume-volatility relation for trading volume, number of trades, and vol-trade are persistent across all sub-periods. The directional (negative/positive) realized skewness relationship with volume reflects investors' reactions to good and bad news across the sub-periods. However, the significance of realized skewness improves in the post-crisis period. The trading volume has a positive and significant relationship with realized kurtosis. The number of trades and vol-trade have a negative and significant relation with realized kurtosis across all sub-periods (see Tables 4.14, 4.15, 4.16 and 4.17). As the fourth robustness test, we test the significance of the volume-higher-order realized moment relationship for the weekly post-crisis period by controlling for market information flow. We observe that in the presence of a market rate of information flow, the volume-higher-order realized moment relationship is still significant (see Table 4.18).

4.5 Concluding Remarks

In this study, we focus on the volume-realized skewness and volume-realized kurtosis relationship by considering various proxies of information flow and capture the various volume-higher order moment relationships dependent on the proxy adopted. Accordingly, we investigate the volume-realized higher-order moments relationships for 142 constituent stocks of the S&P/ASX200 index downloaded from the TRTH/SIRCA. Using 15-minutes return data spanning from January, 2003 to December, 2017 we compute weekly and monthly realized variance, realized skewness, and realized kurtosis. We also download trading volume, the number of trades, order imbalance, and average trade size as alternative proxies for information flow. In addition to these proxies, we also test the volume-realized higher-order moments relation using a new proxy for information flow: vol-trade (i.e. trading volume$\times$number of trades).

We also observe that the order imbalance has an insignificant relationship with the higher-order realized moments for both weekly and monthly holding periods. However, when we condition the orders on buyer/seller-initiations, we observe that the buyer/seller-initiated orders had a statically significant relationships with the realized higher-order moments. Our results highlight the effectiveness of trading volume and the number of trades as the primary proxies for capturing the volume-higher-order realized moments relationships. However, the proxy used determines signs of the coefficient estimates and hence differing explanations may be drawn regarding the effects of the underlying factors i.e. the number of trades or trading volumes on the higher-order realized moments. The secondary volume proxies, average trade size and vol-trade, mirror the primary proxies trading volume and number of trades respectively with the exception of the negative coefficient for average trade size when regressed with realized variance; but it is not significant. We also note that the level of significance of the said relationships is affected by the holding periods, with the weekly holding periods having higher statistical significance than the monthly holding periods. This is not surprising, as the linkages between volume and realized higher-order moments tend to be more representative at smaller holding-intervals i.e. weekly rather than monthly. This subtle fact has not been reported and highlighted explicitly in the extant literature.

We show that whilst the MDH hypothesis is able to explain the observed volume-realized volatility relationship, the SIAH hypothesis additionally accounts for the observed volume-realized skewness relationship and the DOH hypotheses not only accounts for the realized-variance and realized-skewness relationships but also is able to account for the observed volume-realized kurtosis relationship. Thus the DOH hypothesis implicitly encompasses or nests both the SIAH and MDH hypotheses.

In addition, we also argue that, apart from volume-volatility, volume-skewness and volume-kurtosis can provide additional information that could benefit investors' trading strategies. For example, event studies that employ volume and volatility in making inferences could extend their models to account for volume-skewness and volume-kurtosis to capture relevant information that would otherwise be neglected. The relevance of realized skewness and realized kurtosis to high-frequency finance can not be ignored.

Chapter 5

Higher-Order Moments and Asset Pricing in the Australian Stock Market

5.1 Introduction

The capital asset pricing model (CAPM) of Sharpe (1964); Lintner (1975); Mossin (1966) was the first model developed for explaining the risk-return relationship. According to Black (1993); Jagannathan and Wang (1996), the CAPM is a seminal contribution to finance theory, and it forms the cornerstone of asset pricing techniques. However, in the past decade, the CAPM has received numerous theoretical and empirical criticisms. These criticisms stem from the model's simplistic nature; indeed, its stylized assumption that only co-variance risk should be considered important in the asset pricing model has been its major shortcoming.[1] Con-

[1]The CAPM model implies a linear relationship between expected returns and systematic risk (beta). For this relationship to hold, the CAPM places strong restrictions on (*i*) the asset return distribution (assuming Gaussian distribution) and (*ii*) the agent's utility function (by employing the quadratic utility function), which does not correspond to rational agent behavioural characteristics. The assumption of Gaussian distribution is normally made for reasons of convenience in theoretical models; however, it is less likely to hold in the high-frequency paradigm. This suggests that high-frequency results that depend on the normality of asset returns can be misleading.

sequently, these criticisms have inspired several researchers to propose alternative methodologies that aim to improve the model's theoretical consistency and empirical performance. For example, the multifactor CAPM models allow the standard CAPM model to be improved by incorporating additional factors (see Fama and French, 1992, 1995; Carhart, 1997). Bollerslev et al. (1988); Jagannathan and Wang (1996) employ a time-varying CAPM to account for the autoregressive component of the conditional variance. Employing financial risk measures, Shalit and Yitzhaki (1984); Okunev (1990) developed the mean Gini-CAPM, which addresses investors' behaviour when facing uncertainty for a wide category of probability distributions. The lower-moment CAPM of Price et al. (1982); Hwang and Pedersen (2002) requires fewer restrictions in comparison to the standard CAPM model. Ang et al. (2006) estimates CAPM with asymmetric betas and finds that a cross-section of stock returns reflects a downside risk premium. Chan et al. (1991); Fama and French (1992) show that firm size, book-to-market (B/M) ratio and price-to-earnings (P/E) ratio also explain the cross-sectional variation in the expected returns. Pedersen and Hwang (2007) find that downside beta outperforms the standard CAPM beta for a set of United Kingdom (UK) equity returns.

Rubinstein (1973); Ingersoll (1975); Kraus and Litzenberger (1975) were the first to consider relaxing the normality assumption of the CAPM, the authors incorporated a higher-order moment into the pricing model by considering the unconditional asymmetric characteristics of asset return distributions. Fang and Lai (1997); Dittmar (2002) presented a pricing model framework that accounted for both the asymmetric and leptokurtic characteristics of asset return distributions. Consequently, several researchers advocated employing higher-order moment pricing models (see Harvey and Siddique, 2000a; Dittmar, 2002; Lambert and Hübner, 2013; Poti and Wang, 2010; Moreno and Rodríguez, 2009; Kostakis et al., 2012, for more details). In light of their observations, they note that the higher-order moment pricing model outperforms the standard CAPM. The predictive power of the higher-order moment pricing model may be attributed to the simple notion that it accounts for the non-normality of the asset return distribution. Therefore, the higher-order moment pricing model's main objective is to estimate a linear equilibrium relationship between any risky asset's expected returns and its systematic higher-order moment risk measures (i.e. systematic co-skewness and co-kurtosis). This approach realistically deals with the investors' preferences and captures the actual shape of the asset return distribution better than that of the standard CAPM. For example, Harvey and Siddique (2000a) find that conditional systematic skewness helps explain the cross-sectional variation of expected returns across assets even when control variables are included in their model for monthly United States (US) equity returns using CRSP NYSE/AMEX and Nasdaq data.[2] Dittmar (2002) finds that the four-moment CAPM pricing model prices the cross-section of returns better than the standard CAPM, and outperforms the multifactor CAPM models for a set of 20 industry-sorted portfolios estimated with monthly return series obtained from CRSP.[3] Lambert and Hübner (2013) find that co-moment risks have significant explanatory power even when firm size and B/M factors are included in the regression for monthly stock returns from NYSE, AMEX and Nasdaq. Poti and Wang (2010) employing monthly return series from the CRSP database, find that with co-skewness and co-kurtosis risks, one could price several stocks and portfolios strategically. Moreno and Rodríguez (2009) show co-skewness to be economically and statistically significant in US mutual fund performance evaluation by using monthly return data from CRSP. Kostakis et al. (2012) find that co-skewness and co-kurtosis risk premiums are priced in the UK equity market.

In the Australian equity framework, the analysis of risk factors that yield extra returns for investors has received little documentation in the high-frequency finance domain. In contrast, using low-frequency Australian equity return data, Faff (2001); Gaunt (2004) find that the three-factor model outperforms the standard CAPM. Gharghori et al. (2007) state that the reason why Fama and French (1993) factors can explain cross-sectional variation in equity return is because the

[2]The centre for Research in Security Prices (CRSP), New York Stock Exchange (NYSE), American Express (AMEX) and National Association of Securities Dealers Automated Quotations (Nasdaq).

[3]The four-moment CAPM pricing model is the addition of systematic co-skewness and co-kurtosis risk measures to the standard CAPM pricing model.

Small-Minus-Big (SMB) and High-Minus-Low (HML) factors are proxy for default risk. However, they conclude that the default risk is not priced and that Fama and French (1993) factors do not proxy default risk. Limkriangkrai et al. (2008) find that the three-factor model captures the return of the largest company shares while that for the premium of smaller-company shares is not priced. Gharghori et al. (2009) find that the size, B/M, earnings-to-price, and cash-flow–to-price factors are priced. Galagedera and Maharaj (2008) show that with wavelength multi-scaling decomposition, normalized co-kurtosis risk is not priced in the presence of beta risk in the upmarket and downmarket conditions for daily Australian industry portfolio returns.

Similarly, Alles and Murray (2013) investigate the effects of downside beta and co-skewness exposure on the returns to investors by employing daily data from emerging Asian markets. They find that both downside beta and co-skewness are priced separately in the upmarket and downmarket state. Alles and Murray (2017) subsequently investigate the effects of downside beta and co-skewness on returns by using daily constituent data of S&P/ASX 200 index, and their findings are consistent with those of Alles and Murray (2013). Doan et al. (2014) find that a significant relationship exists between the higher-order risk preferences and systematic skewness and kurtosis of weekly stock returns and financial data for all ASX-listed firms (Australian stocks). Galagedera et al. (2003) observe that the systematic variance and skewness are priced in both upmarket and downmarket conditions, while the systematic kurtosis is not for 128 daily return series of Australian stocks. Lee et al. (2008) find that investors only require a premium for downside risk, and in the presence of co-kurtosis, downside beta loses its significance for monthly Australian-listed property trust return data. Vendrame et al. (2016) investigate why the empirical results for the four-moment CAPM reported in the extant literature are conflicting. They test whether conditional models or models that employing individual stocks rather than portfolio returns improves the standard CAPM. The authors also examine whether models that extend moment-based CAPM approach and those that incorporate Fama and French risk factors improves the standard CAPM. They note that the four-moment CAPM improves the performance of the standard CAPM in the presence of SMB factor.

To the best of our knowledge, Hurlin et al. (2009) is the only paper to investigate realized higher-order moment risk-return relationship. Employing 5-minute (high-frequency) return series for 43 French stocks, the authors estimate the realized moments for 1-month holding-period. For their analysis, they focus on extending the standard CAPM model by incorporating realized co-skewness risk measure. Their results show that realized co-skewness is priced and neglecting realized co-skewness can result in misleading conclusions. In this study, we investigate a set of realized higher-order co-moment risk-return relationships by employing 142 Australian stocks downloaded at a 15-minute frequency from the Thomson Reuters Tick History/Securities Industry Research Centre of Asia-Pacific (TRTH/SIRCA) database from 2003 to 2017 to estimate the monthly realized risk factors.[4] The primary motivation of this study is to contribute to the debate regarding whether systematic realized higher-order co-moments can explain the cross-sectional Australian stock returns. Our second motivation is based on the notion that using high-frequency return data will yield substantially more robust empirical estimates than using low-frequency return data where valuable information may be lost. Consequently, sample moments are normally computed from long samples of low-frequency return series (e.g., daily, weekly and monthly return series) while realized moments are computed from high-frequency return series (e.g., 1-second, 1-minute and 5-minute return series, etc.). Additionally, we investigate the realized higher-order co-moment risk-return relationship in the upmarket and downmarket conditions to identify the true nature of the relationship. We follow a set of sound

[4]Specifically, we compute monthly standard CAPM beta, the continuous and jump beta of Todorov and Bollerslev (2010); Bollerslev et al. (2016); Alexeev et al. (2017) and the unconditional normalized higher-order co-moment of (Homaifar and Graddy, 1988; Athayde and Flôres Junior, 1997; De Athayde and Flôres, 2000; Hwang and Satchell, 2001; Hwang and Pedersen, 2002). Additionally, we construct the upside and downside normalized realized higher-order co-moment by using the zero rate of return as the truncation point (Ang et al., 2006). We include individual firm characteristics to determine whether the obtained risk premiums are robust in the presence of such control variables. For these control variables, we consider the illiquidity factor of Amihud (2002), the size factor (log(size)) of Fama and French (1993), the B/M ratio of Fama and French (1992), the value at risk (VaR) at a 1% level of significance and past returns of Amaya et al. (2015).

methodology documented in the extant literature for investigating the realized higher-order co-moment risk-return relationship.

The first empirical test employs single sorting of excess realized return on the risk measures. We construct 10-decile equally weighted portfolios that are sorted by a set of firm characteristics. We then investigate the realized higher-order co-moment risk-return relationship across the sorted decile portfolios. The results for all market periods show that the average return on the high-beta stocks exceeds that of the low-beta stocks, although the high-minus-low spread is statistically insignificant. Regarding the downside beta, downside kappa and log(size), the average return on the lower-ranked stocks are relatively higher than that of the higher-ranked stocks. This implies that lower-ranked stocks of downside beta, downside kappa and log(size) portfolios offer relatively higher returns to the investor. Conversely, higher-ranked gamma and B/M offer positive and statistically significant excess returns to investors (for more details, see Table 5.3). We also observe that gamma yields significant gains to investors who bear gamma risk regardless of the market state. We note that the average excess returns for the conditional co-moments risk tend to be robust. The monotonic relation of Patton and Timmermann (2010) reveals some pertinent patterns regarding the possible presence of any monotonic relationship (MR) in the entire sorted portfolio.[5] This knowledge of the monotonic increasing pattern may be beneficial to investors' trading strategies.

The second empirical test employs double sorting of excess returns on both down-

[5] Over the years, financial theories have hypothesized that expected returns should either increase or decrease monotonically in firms' risk or liquidity characteristics. Patton and Timmermann (2010) states that expected returns on treasury securities should increase monotonically to maturity with time and that the pricing kernel should be monotonically decreasing in investors' ranking of future states. Consequently, the CAPM hypothesis implies that the expected return of stocks that are ranked by their market betas should increase monotonically. It is common to form portfolios of stocks that are ranked by their beta when testing for the CAPM. Most researchers test the mean spread between portfolios' highest and lowest sorted beta and report its significance by employing the t-test statistic. For example, Ang et al. (2006) employs this approach for both their single and double-sorted portfolios. Alles and Murray (2013, 2017) obtained significant t-test statistics for the high-minus-low returns sorted on downside beta and gamma risk in both the upmarket and downmarket states. This approach ignores any MR that may exist in the entire set of securities/portfolios.

side beta and gamma risk measures.[6] This process aims to capture asymmetric higher moments and downside risk. In the double-sorting procedure, the methodology measures the reward for the exposure of downside beta risk by explicitly controlling for gamma risk measure and vice versa.[7] Our results show that the downside beta and gamma risks are different and priced separately. Furthermore, we observe that investors experience significant gains for bearing gamma risk when we control for downside beta risk regardless of the market conditions.

Finally, we employ a cross-sectional Fama-MacBeth regression at the rm level to further investigate the predictive power of the risk measures. Using the Fama-MacBeth regression, researchers can undoubtedly account for more than one parameter that could explain the cross-section of the stock returns. The results show that the average excess return of the standard beta and kappa risk measures are asymmetric and diametrically opposite in upmarket and downmarket periods, which is consistent with the results obtained from the single-sorted portfolios. Kappa captures any dispersion in the return data that is ignored by beta and is sensitive to tail distributions (i.e., the magnitude of jumps). Additionally, we observe that gamma risk generates significant excess returns regardless of the market conditions. This is unsurprising since the sign of gamma depends on the sign of the skewness of market return distribution and the sign of the investor's marginal risk-preference for skewness. Therefore, gamma can either be positive or negative. Our results obtained are consistent with those of Ranaldo and Favre (2005); Liow and Chan (2005); Sharpe (1964); Jurczenko and Maillet (2006), who find that investors have a preference for odd moments (mean and skewness) and an aversion towards even moments (variance and kurtosis). Indeed, investors expect a premium for beta and kappa risks in the upmarket and a discount in downmarket conditions. The findings of this study suggests that investors will find gamma risk attractive due to the significant reward that it generates. We also note that the

[6] The double-sorting approach addresses the limitation of single-sorted portfolios. Notably, single-sorted portfolios do not control for any other known patterns in the cross-section of stock returns.

[7] Although double sorting has an advantage over single sorting, it is limited to accounting for only one parameter at a time. To overcome this limitation we employ the Fama-MacBeth regression.

upside and downside risk measures retain their direction and level of significance when we control for a set of firm characteristics.

Due to the extensive nature of our sample size, we split the data into three sub-periods following the categorization of Dungey and Gajurel (2014). For the sub-periods, only the Fama-MacBeth regression analysis is employed. We observe that the reward for gamma exposure is robust in models that incorporate gamma risk while the predictive power of kappa risk tends to vary depending on the market conditions.

The remainder of this chapter is organized as follows: Section 5.2 gives a brief review of relevant theory for estimating standard beta, continuous beta, jump beta, and (un)conditional realized higher-order co-moments. Section 5.3 presents the empirical data used in constructing the realized higher-order moments and subsequent estimates of risk measures. We report the descriptive statistics of the variables used in this Section as well. The empirical results are discussed in Section 5.4, and Section 5.5 concludes.

5.2 Methodology

5.2.1 Estimating Systematic Risks: Continuous and Jump Betas

The CAPM model of Sharpe (1964); Lintner (1975); Mossin (1966) is an important model in the finance literature for assessing the performance of portfolios and the cost of investments, as well as for choosing portfolio strategies and many more actions. The CAPM relates the expected return of individual stock returns ($r_{i,t}$) to a benchmark market return ($r_{m,t}$) and is formally defined as:[8]

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon_{i,t}, \quad i = 1, 2, .., N \quad and \quad t = 1, 2, .., T \tag{5.1}$$

[8]The stock return series $r_{i,t} = p_{i,t} - p_{i,t-1}$, where $p_{i,t}$ is the i th log price for period t.

where stock $i = 1, 2, ..., N$ and $t = 1, 2, .., T$ is time period in months. The β_i is a measure of systematic (co-variance) risk, which can also be defined as the normalized co-variance of the individual stock returns and market returns.[9] Post the inception of high-frequency data, several researchers have shown that market return can be decomposed into two main components: one that captures the continuous price movement and another that captures jumps (see Todorov and Bollerslev, 2010; Bollerslev et al., 2016; Alexeev et al., 2017). Relative to the standard beta that is estimated with low-frequency return series (daily, weekly, monthly and quarterly return series), beta that is estimated with high-frequency return series (tick-by-tick, seconds and minutes return series) leads to a statistically superior beta estimate (see Andersen et al., 2005, 2006; Bollerslev and Zhang, 2003; Barndorff-Nielsen and Shephard, 2004). Suppose that the observed price follows a semi-martingale process on some filtered probability space (Ω, $\mathcal{F}$, $(\mathcal{F}_t)_{t\geq 0}$,$\mathsf{P}$) in a frictionless market in which there are no arbitrage opportunities (see Back, 1991). Then, in the presence of jumps, the observed price can be modelled as a continuous time semi-martingale jump diffusion process:

$$p_{i,t} = \int_0^t \mu_{i,t} dt + \int_0^t \sigma_{i,t} dW_{i,t} + \sum_{k=1}^{N(t)} J(Q_{i,k}), \tag{5.2}$$

where $\mu_{i,t}$ is the diffusive mean, $\sigma_{i,t}$ is a diffusive volatility process, $dW_{i,t}$ is the increments to a Brownian motion, $W_{i,t}$, $N(t)$ is a counting process and $J(Q_{i,k})$ is the non-zero jump increments (for more details, see Fleming and Paye, 2011). We observe that Equation (5.2) reduces to a 'pure' diffusion model with continuous sample paths when no jumps are present in the price process (i.e., the jump component is set to zero). For this jump diffusion process to hold, it is assumed that $\mu_{i,t}$ and $\sigma_{i,t}$ are jointly independent of $W_{i,t}$.

Todorov and Bollerslev (2010); Bollerslev et al. (2016); Alexeev et al. (2017) have shown that the log price of any asset i, $p_{i,t}$ in Equation (5.2) follows a continuous-

[9]The standard CAPM beta, $\beta_i = \frac{Cov(r_{i,t}, r_{m,t})}{Var(r_{m,t})}$, captures the sensitivity of the expected return on the i-th asset to that of the market return.

time jump diffusion process, so the return series evolves as follows:

$$r_{i,t} \equiv dp_{i,t} = \mu_{i,t}dt + \sigma_{i,t}dW_{i,t} + JdQ_{i,t}, \tag{5.3}$$

In the case of market returns we have:

$$r_{m,t} \equiv dp_{m,t} = \mu_{m,t}dt + \sigma_{m,t}dW_{m,t} + J_{m,t}dQ_{m,t}, \tag{5.4}$$

Hence the standard quadratic variation for high-frequency return data is the realized variance and can be defined as:

$$[r_{i,t}, r_{m,t}]^2 \equiv QV_t = \int_0^t \sigma_t^2 dt + \sum_{k=1}^{N(t)} J^2(Q_k), \tag{5.5}$$

where the first term on the right-hand side of Equation (5.5) is the integrated variance (IV) and the second term is the sum of the squared jumps (i.e. the variance of the jump component). We note that Equation (5.5) reduces to a 'pure' diffusion model with continuous sample paths when no jumps are present in the price process (i.e., the jump component is set to zero). The IV for this type of process is defined as $IV_t \equiv \int_0^t \sigma_t^2 dt$ and equals to the quadratic variance (QV), since the jump component is set to zero. From Equation (5.5), the quadratic variation between $r_{i,t}$ and $r_{m,t}$ in terms of continuous and jump beta is expressed as:

$$[r_{i,t}, r_{m,t}]^2 = \beta_i^c \int_0^t \sigma_{m,t}^2 dt + \beta_i^j \sum_{k=1}^{N(t)} J^2(Q_{m,k}), \tag{5.6}$$

Following Todorov and Bollerslev (2010); Alexeev et al. (2017), the continuous beta is defined as $(\beta_i^c) = \frac{[r_{i,t}^c, r_{m,t}^c]^2}{[r_{m,t}^c, r_{m,t}^c]^2}$. Thus, the ratio of the intra-day co-variance between the asset return and the market return is normalized by the variance of the market using high-frequency return series. Generally, the market may experience jumps according to this definition, as the jump component is not restricted to zero. To account for the possible occurrence of jumps, as the number of observations approaches infinity (thus T/Δ, where T is the total number in the trading day and Δ is the sampling frequency) the sample estimate is employed. This restricts the jump component, and the continuous beta is thus expressed as:

$$\hat{\beta^c_{i,t}} = \frac{\sum_{j=1}^{T/\Delta} r_{i,j,t} r_{m,j,t} \mathbb{1}_{\{|r_{t,j}| \leq \theta\}}}{\sum_{j=1}^{T/\Delta} (r_{m,j,t})^2 \mathbb{1}_{\{|r_{t,j}| \leq \theta\}}}, \quad for \quad i = 1, 2, ..., N \tag{5.7}$$

where $\mathbb{1}$ is the indicator function and θ the truncation level for the continuous component

$$\mathbb{1}_{\{|r_{t,j}| \leq \theta\}} = \begin{cases} 1 & \text{if } |r_{t,j}| \leq \theta \\ 0 & \text{if otherwise} \end{cases}$$

For the jump beta, we have $[r^j_{i,t}, r^j_{m,t}]^{2\tau} = (\beta^j_i)^\tau \sum_{k=1}^{N(t)} J^{2\tau}(Q_k) = (\beta^j_i)^\tau [r^j_{m,t}, r^j_{m,t}]^{2\tau}$, where $\tau \geq 1$. By raising the high-frequency returns to powers, if the order is greater than two, then the continuous component can be ignored, and the jump component dominates asymptotically as the number of observations approach infinity. The parameters for the truncation threshold $\theta = \alpha_i \Delta_n^{\varpi}$, where $\varpi = 0.49, \tau = 2$, α_i varies among individual stocks and is set to $\sqrt[3]{BV_{i,t}}$. The estimate of the jump beta is:

$$\hat{\beta^j_i} = sign\left\{\sum_{j=1}^{T/\Delta} sign\{r_{i,j,t} r_{m,j,t}\} |r_{i,j,t} r_{m,j,t}|^\tau\right\} \times \left(\frac{\sum_{j=1}^{T/\Delta} sign\{r_{i,j,t} r_{m,j,t}\} |r_{i,j,t} r_{m,j,t}|^\tau}{\sum_{j=1}^{T/\Delta} r_{m,j,t}^{2\tau}}\right)^{\frac{1}{\tau}} \tag{5.8}$$

where $\tau \geq 2$, this ensures that the continuous prices movement do not matter asymptotically (for more details, see Todorov and Bollerslev, 2010; Alexeev et al., 2017). The *'sign'* in Equation (5.8) accounts for the jump beta signs that are ignored when taking the absolute values of the return series. In the presence of at least one systematic jump beta, $\hat{\beta^j_i}$ converges to β^j_i. Barndorff-Nielsen and Shephard (2006) propose a test statistic for jump detection in the market portfolio, of which detail documentation can be found in (Todorov and Bollerslev, 2010; Alexeev et al., 2017). The realized variance (RV) and bipower variation (BV) for the market return using high-frequency data is defined as:

$$\begin{aligned} RV_{m,t} &= \sum_{j=1}^{T/\Delta} r_{m,j,t}^2 \xrightarrow{P} \int_0^T \sigma_t^2 dt + \sum_{k=1}^{N(t)} J^2(Q_{m,k}), \quad as \quad \Delta \to 0. \\ BV_{m,t} &= \sum_{j=1}^{T/\Delta - 1} |r_{m,j,t}||r_{m,j+1,t}| \xrightarrow{P} \mu_1^2 \int_0^T \sigma_t^2 dt, \quad as \quad \Delta \to 0. \end{aligned} \tag{5.9}$$

The RV efficiently estimates the quadratic variation. It converges to the QV as the number of observations (N) approaches infinity ($RV_{[a,b]}^{(N)} \to QV_{[a,b]}$ as $N \to \infty$ (see Andersen and Bollerslev, 1998; Barndorff-Nielsen and Shephard, 2002)). It is also apparent from Equation (5.9) that in the absence of jumps, RV converges into the IV. Given that $\mu_1 = \sqrt{2/\pi}$, Barndorff-Nielsen and Shephard (2004) show that the BV estimates the true IV when the number of observations approaches infinity. From Equation (5.9), we have:

$$RV_{m,t} - \mu_1^{-2} BV_{m,t} \xrightarrow{P} \sum_{k=1}^{N(t)} J^2(Q_{m,k}), \quad as \quad \Delta \to 0. \tag{5.10}$$

Therefore, the difference between the RV and BV captures the contribution from the jump component. Barndorff-Nielsen and Shephard (2006); Alexeev et al. (2017) show that Equation (5.10) can be used to detect the jumps by employing the feasible test statistic as expressed by Equation (5.11) below:

$$\begin{aligned} \hat{\mathfrak{J}} = \frac{1}{\sqrt{\Delta}} \times & \frac{1}{\sqrt{(\frac{\pi^2}{4} + \pi - 5) \max(1/T, DV_{m,t}/BV_{m,t}^2)}} \\ & \times \left(\frac{\mu_1^{-2} BV_{m,t} - RV_{m,t}}{RV_{m,t}} \right) \xrightarrow{j} N(0,1). \end{aligned} \tag{5.11}$$

where $DV_{m,t}$ is the realized quad-power variation:

$$DV_{m,t} = \sum_{j=1}^{T/\Delta-3} |r_{m,j,t}||r_{m,j+1,t}||r_{m,j+2,t}|r_{m,j+3,t}| \tag{5.12}$$

In light of the discussion regarding how to compute the continuous and jump beta presented above, we decompose the high-frequency return data into both continuous and jump components. The standard CAPM beta in Equation (5.1) is then redefined as:

$$r_{i,t} = \alpha_i + \beta_i^c r_{m,t}^c + \beta_i^j . r_{m,t}^j + \varepsilon_{i,t}, \tag{5.13}$$

where $r_{m,t}$ is the market return defined as the combination of the continuous ($r_{m,t}^c$) and jump component ($r_{m,t}^j$).[10] The total effect of the systematic beta or standard beta (β_i) in Equation (5.1) can be attributed to the combination of the effects of the continuous beta (β_i^c) and jump beta (β_i^j). Intuitively, when the systematic risk that is associated with continuous and jump components differs (thus $\beta_i^c \neq \beta_i^j$), then a cross-sectional regression could be employed to determine the components' separate pricing. When the continuous beta and jump beta is different, it signifies that the asset and market move separately during the 'normal' and 'jump' market movement. If the continuous and jump beta are the same, then the model reduces to the standard beta (thus $\beta_i^c = \beta_i^j$) (Todorov and Bollerslev, 2010; Alexeev et al., 2017). Under this restriction, the asset and market co-move in the same direction in both the 'normal' and 'jump' market movement. It is extensively documented in the literature that the continuous and jump betas are not the same and that they capture different effects.

We employ high-frequency returns to estimate continuous and jump beta as discussed above and examine whether the realized higher-order co-moment risk measures will remain priced in the presence of continuous and jump betas. The predictive power of the jump beta has been extensively documented in the extant

[10]The market return is $r_{m,t} = r_{m,t}^c + r_{m,t}^j$.

high-frequency literature. Consequently, some researchers have shown that the continuous and jump betas are priced separately (e.g., see Todorov and Bollerslev, 2010; Dungey and Yao, 2013; Bollerslev et al., 2016; Chowdhury et al., 2018; Chowdhury and Jeyasreedharan, 2019).

5.2.2 Asset pricing and realized higher-order co-moments

Post the initial studies by Rubinstein (1973); Ingersoll (1975); Kraus and Litzenberger (1975), several researchers have shown that the standard CAPM beta alone cannot adequately capture the total systematic risk.[11] The CAPM's main shortcoming is that it assumes that the asset returns are normally distributed and the characteristics of the agent's utility function (by employing the quadratic utility function). However, the normality assumption can be rejected since it is well documented in the existing literature that stock returns are skewed and kurtotic (see Fama, 1965; Mandelbrot, 1997; McNeil and Frey, 2000; Bali, 2003). Therefore, one cannot ignore the systematic skewness and kurtosis when investigating the cross-sectional relationship between asset returns and market movements. The CAPM model has been extended to account for the asymmetric and elongation characteristics of the asset return distribution.[12] The main goal of this extended model is to obtain a linear relationship between excess/expected returns and systematic higher-order moment risk measures. Unlike the simple quadratic utility function of the standard CAPM, the extended higher-order moment model uses fourth-order Taylor series expansions to justify a four moments–based decision criterion (Ranaldo and Favre, 2005; Liow and Chan, 2005; Sharpe, 1964; Jurczenko and Maillet, 2006). This approach assumes explicitly that a rational investor has

[11] Banz (1981); Reinganum (1981) find that smaller firms have higher risk-adjusted returns on average than larger firms. This evidence suggests that the size effect is significant and that the CAPM is incorrectly specified. Rosenberg et al. (1985); Lakonishok et al. (1994) shows that the standard CAPM beta does not capture the significance of the B/M value. Fama and French (1993) shows that the three-factor model which is the extended CAPM beta model with an additional two risk factors (size and B/M) is priced. This suggests that the CAPM beta is not enough to explain the total systematic risk.

[12] Rubinstein (1973) was the first to account for the asymmetric characteristics of the asset return distribution.

a preference for (aversion to) the mean and positive skewness (variance, negative skewness and kurtosis) of the stock returns.

The theoretical framework that justifies the four-moment CAPM is extensively documented in (Ranaldo and Favre, 2005; Liow and Chan, 2005; Jurczenko and Maillet, 2006). They consider the pricing model for the unconditional co-variance, co-skewness and co-kurtosis of the stock return distribution. By definition, co-skewness can be interpreted as the individual stock's contribution to the market portfolio's skewness, while co-kurtosis is the individual stock's contribution to the market portfolio's kurtosis.

The four-moment CAPM model is used in the pricing model for beta (volatility), gamma (skewness) and kappa (kurtosis). Suppose that $r_{i,t}$ and $r_{m,t}$ represent the stock i return, and m is the market return at time t. The investment problem for an investor is to maximize the expected utility at the end of the period. The investor's expected utility can be represented as a Taylor expansion of order n:

$$\begin{aligned} E[U(r_{i,t})] = U[E(r_{i,t})] + \frac{1}{2!}U^{(2)}[E(r_{i,t})]E[r_{i,t} - E(r_{i,t})]^2 + \\ \frac{1}{3!}U^{(3)}[E(r_{i,t})]E[r_{i,t} - E(r_{i,t})]^3 + \frac{1}{4!}U^{(4)}[E(r_{i,t})]E[r_{i,t} - E(r_{i,t})]^4 \\ + \sum_{n}^{\infty} \frac{1}{n!}U^{(n)}[E(r_{i,t})]E[r_{i,t} - E(r_{i,t})]^n \end{aligned} \tag{5.14}$$

where U^n is the nth derivative of the utility function U and σ=$\{E[r_{i,t}-E(r_{i,t})]^2\}^{\frac{1}{2}}$, s=$\{E[r_{i,t} - E(r_{i,t})]^3\}^{\frac{1}{3}}$, k=$\{E[r_{i,t} - E(r_{i,t})]^4\}^{\frac{1}{4}}$ and m=$\{E[r_{i,t} - E(r_{i,t})]^n\}^{\frac{1}{n}}$ are, respectively, the volatility, third moment, fourth moment and nth centred higher-order moment of the investor's portfolio return distribution. The skewness and kurtosis are obtained when the third and fourth higher-order moments (s and k) are normalized with the cubic and quartic volatility (σ), respectively.

Harvey and Siddique (2000a,b); Dittmar (2002); Galagedera et al. (2003) show that conditional skewness and kurtosis help explain the cross-sectional variation of expected returns across assets and remain significant even when they control for size and B/M effect. Following Jurczenko and Maillet (2012), we define the co-skewness and co-kurtosis as:

$$
\begin{aligned}
Cos(r_{i,t}, r_{m,t}) &= E\Bigg([r_{i,t} - E(r_{i,t})][r_{m,t} - E(r_{m,t})]^2\Bigg) \\
Cok(r_{i,t}, r_{m,t}) &= E\Bigg([r_{i,t} - E(r_{i,t})][r_{m,t} - E(r_{m,t})]^3\Bigg)
\end{aligned} \tag{5.15}
$$

where co-skewness ($Cos(r_{i,t}, r_{m,t})$) can be considered a measure of the co-variance between the asset i return and the volatility of the market return m, while co-kurtosis ($Cok(r_{i,t}, r_{m,t})$) represents the co-variance between asset i return and market skewness m. Jurczenko and Maillet (2012) state that an asset that exhibits a positive (negative) co-skewness and a negative (positive) co-kurtosis with the market tends to perform the best (worst) when the market becomes more volatile and experiences substantial losses. The co-skewness and co-kurtosis between asset i return and the market return provide a measure of the asset's ability to protect the investor from unexpected shocks of the market variance and skewness (Racine, 1998).

Assuming an indirect utility function, Jurczenko and Maillet (2012) show that the financial market equilibrium for the four-moment CAPM between asset i and market portfolio m can be set by maximizing equation (5.14), which can be expressed as:

$$
r_{i,t} - r_f = \lambda_{i,1}\beta_{i,t} + \lambda_{i,2}\gamma_{i,t} + \lambda_{i,3}\kappa_{i,t} \tag{5.16}
$$

where $r_{i,t} - r_f$ represents monthly excess realized returns and $\lambda_{i,1}$, $\lambda_{i,2}$ and $\lambda_{i,3}$ are the systematic market risk premiums. Due to the investors' utility function and preference for or aversion to higher-order moment risk, we expect the $\lambda_{i,1}$ to be positive (since variance is positive), the $\lambda_{i,2}$ to have an opposite sign to that of the skewness of the market (since skewness can be positive or negative) and the $\lambda_{i,3}$ to be positive (since kurtosis is positive), which is consistent with Galagedera et al. (2003).[13] $\beta_{i,t}$ is the CAPM beta (systematic beta) that has already been defined in a previous section. We also define $\gamma_{i,t}$ as the normalized co-skewness (systematic skewness) and $\kappa_{i,t}$ as the normalized co-kurtosis (systematic kurtosis). We assume that the systematic beta, systematic skewness and systematic kurtosis in the four-moment CAPM are all priced separately, with the systematic skewness and kurtosis being defined as:

$$\begin{aligned} \gamma_{i,t} &= \frac{E([r_{i,t} - E(r_{i,t})][r_{m,t} - E(r_{m,t})]^2)}{\sqrt{Var(r_{i,t})}Var(r_{m,t})} = gamma \\ \kappa_{i,t} &= \frac{E([r_{i,t} - E(r_{i,t})][r_{m,t} - E(r_{m,t})]^3)}{\sqrt{Var(r_{i,t})}Var(r_{m,t})^{3/2}} = kappa \end{aligned} \tag{5.17}$$

According to Ranaldo and Favre (2005); Liow and Chan (2005), for an increase in beta, a decrease in gamma, and an increase in kappa, the systematic risk premia ($\lambda(s)$) in Equation (5.16) are given by:

$$\lambda_{i,1} = \frac{dE(r_{i,t})}{d\sigma^2(r_{i,t})}\sigma^2(r_{m,t}), \quad \lambda_{i,2} = \frac{dE(r_{i,t})}{dS^3(r_{i,t})}S^3(r_{m,t}), \quad \lambda_{i,3} = \frac{dE(r_{i,t})}{dK^4(r_{i,t})}K^4(r_{m,t}) \tag{5.18}$$

Hence, the four-moment CAPM in Equation (5.16) is a combination of the systematic beta, systematic skewness and systematic kurtosis with the respective market price ($\lambda(s)$). If the investor prices the co-moments of $\beta_{i,t}$, $\gamma_{i,t}$ and $\kappa_{i,t}$, then the risk premium values of $\lambda_{i,1}$, $\lambda_{i,2}$ and $\lambda_{i,3}$ should be significantly different from zero. $\lambda_{i,1}$ can be treated as the marginal investor risk-aversion to variance multiplied by the market variance ($\sigma^2(r_{m,t})$); $\lambda_{i,2}$ is the marginal investor risk-preference for skewness multiplied by the market skewness ($S^3(r_{m,t})$); and $\lambda_{i,3}$ is the marginal investor risk-aversion to kurtosis multiplied by the market kurtosis ($K^4(r_{m,t})$).

[13] The CAPM implies that stocks that covary strongly with the market have contemporaneously high average returns over the same period. Thus, CAPM suggests that an increasing relationship exists between realized average returns and market betas.

5.2.2.1 Downside and Upside systematic risk

Several studies in the financial literature have documented that investors care differently about downside losses and upside gains. Therefore, risk-averse investors will demand greater compensation for holding stocks with a high level of downside risk. This implies that downside and upside risk may be priced separately (Roy, 1952; Ang et al., 2006; Kahneman and Tversky, 2013). Ang et al. (2006) shows that aversion to downside risk (i.e., downside beta) is significantly priced in comparison to upside risk (i.e., upside beta). To construct downside and upside beta, one must define a truncation point for capturing the downside and upside states. Bawa and Lindenberg (1977); Ang et al. (2002); Lee et al. (2008); Botshekan et al. (2012); Alles and Murray (2013, 2017) use the average market return as the cut-off point for the benchmark, while Doan et al. (2014) use a lower and higher quartile of the stock market return distribution as the cut-off for downside beta and upside beta, respectively. In contrast, Ang et al. (2006) argues that their empirical results did not depend on which truncation point used in capturing the downside and upside betas. They show that their results are robust when average market returns, the risk-free rate, or zero rates of return are used as cut-off points for computing downside and upside beta.[14]

In this study, the zero rate of return is used as a cut-off point for computing the downside beta ($\beta_{i,0}^{-}$) and upside beta ($\beta_{i,0}^{+}$):

$$\beta_{i,0}^{-} = \frac{Cov(r_{i,t}, r_{m,t}|r_{m,t} < 0)}{Var(r_{m,t}|r_{m,t} < 0)} \quad \& \quad \beta_{i,0}^{+} = \frac{Cov(r_{i,t}, r_{m,t}|r_{m,t} > 0)}{Var(r_{m,t}|r_{m,t} > 0)} \tag{5.19}$$

Considering that investors generally consider all forms of downside risks, it can be inferred from the above discussion that gamma and kappa could also be computed with both downside and upside states. Hence defined as:

[14]Employing the rational disappointment aversion utility function of Gul (1991), Ang et al. (2006) shows that downside risk is priced in an equilibrium setting.

$$
\begin{aligned}
\gamma_{i,0}^{-} &= \frac{E([r_{i,t} - E(r_{i,t})][r_{m,t} - E(r_{m,t})|r_{m,t} < 0]^2)}{\sqrt{Var(r_{i,t})}Var(r_{m,t}|r_{m,t} < 0)} \\
\kappa_{i,0}^{-} &= \frac{E([r_{i,t} - E(r_{i,t})][r_{m,t} - E(r_{m,t}|r_{m,t} < 0)]^3)}{\sqrt{Var(r_{i,t})}Var(r_{m,t}|r_{m,t} < 0)^{3/2}} \\
\gamma_{i,0}^{+} &= \frac{E([r_{i,t} - E(r_{i,t})][r_{m,t} - E(r_{m,t})|r_{m,t} > 0]^2)}{\sqrt{Var(r_{i,t})}Var(r_{m,t}|r_{m,t} > 0)} \\
\kappa_{i,0}^{+} &= \frac{E([r_{i,t} - E(r_{i,t})][r_{m,t} - E(r_{m,t}|r_{m,t} > 0)]^3)}{\sqrt{Var(r_{i,t})}Var(r_{m,t}|r_{m,t} > 0)^{3/2}}
\end{aligned}
\tag{5.20}
$$

The sign expected for the coefficients of the conditional systematic risk is inferred from Galagedera et al. (2003), we expect the risk premium of upside beta to be positive, downside beta to be negative. For upside gamma and downside gamma, the sign will be conditional on the sign of the market skewness. Upside kappa to be positive and downside kappa to be negative.

Table 5.1 reports the correlation matrix for all variables that are used for analysis in this study. The standard CAPM beta is observed to capture different effects in comparison to the downside and upside beta estimates. The correlations of standard CAPM beta with downside beta and upside beta is 0.713 and 0.858, respectively. This indicates that the downside and upside betas capture different risk components. A correlation coefficient of 0.302 is observed for both downside and upside betas, which suggests that a high downside exposure does not necessarily imply a high upside exposure. These results are consistent with those of Ang et al. (2006). The correlation coefficients for standard CAPM beta with continuous and jump betas is 0.111 and 0.699, respectively. However, the correlation between continuous and jump betas is 0.079; this affirms the notion that continuous and jump betas capture different aspects of risk. The correlations between downside gamma and upside gamma is –0.358, that of downside kappa and upside kappa is 0.255. This implies that systematic risk factors capture different effects.

Table 5.1: Correlation matrix of variables, full-sample period (Jan 2003 - Dec 2017)

	β	β^-	β^+	β^c	β^j	γ	γ^-	γ^+	κ	κ^-	κ^+	B/M	Illiq	log(size)	PRet	VaR
β	1	0.713	0.858	0.111	0.699	0.009	-0.443	0.508	0.455	0.373	0.434	0.003	-0.030	0.291	0.006	-0.021
β^-		1	0.302	0.049	0.448	-0.192	-0.521	0.271	0.373	0.435	0.228	-0.001	-0.034	0.270	-0.015	0.096
β^+			1	0.116	0.613	0.188	-0.209	0.522	0.321	0.169	0.446	0.003	-0.016	0.219	0.020	-0.083
β^c				1	0.079	-0.002	-0.073	0.085	0.070	0.058	0.066	0.001	-0.004	0.094	0.004	0.004
β^j					1	0.022	-0.236	0.283	0.249	0.203	0.245	0.001	-0.008	0.081	0.003	-0.040
γ						1	0.625	0.433	-0.291	-0.652	0.447	0.002	0.003	-0.010	-0.009	0.007
γ^-							1	-0.358	-0.803	-0.960	-0.309	-0.004	0.020	-0.394	0.006	-0.088
γ^+								1	0.632	0.292	0.961	0.008	-0.017	0.436	-0.019	0.097
κ									1	0.837	0.629	0.007	-0.018	0.381	-0.013	0.080
κ^-										1	0.255	0.004	-0.017	0.320	-0.005	0.070
κ^+											1	0.007	-0.015	0.367	-0.018	0.076
B/M												1	0.004	0.064	0.077	-0.020
Illiq													1	-0.006	-0.031	-0.001
log(size)														1	0.030	0.201
PRet															1	0.009
VaR																1

This table reports the correlation of firm characteristics of the variables used in the analysis of this study. The full sample period starts 6 January 2003 - 29 December 2017. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

5.2.3 Cross-sectional regression

To investigate the relationship between asset i excess realized monthly returns and systematic risk components, we employ firm-level data and run a cross-sectional regression as proposed by Fama and MacBeth (1973). The cross-sectional regression for each month $t = 1, 2, .., T$, and all stocks $i = 1, 2, ..., N$ can be expressed in the following models:

$$
\begin{aligned}
r_{i,t} - r_f &= \lambda_{i,0} + \lambda_{i,1}\beta_{i,t} + \varepsilon_{i,t}, \quad && Model \quad 1 \\
r_{i,t} - r_f &= \lambda_{i,0} + \lambda_{i,1}\beta_{i,t} + \lambda_{i,2}\gamma_{i,t} + \varepsilon_{i,t}, \quad && Model \quad 2 \\
r_{i,t} - r_f &= \lambda_{i,0} + \lambda_{i,1}\beta_{i,t} + \lambda_{i,2}\gamma_{i,t} + \lambda_{i,3}\kappa_{i,t} + \varepsilon_{i,t}, \quad && Model \quad 3 \\
r_{i,t} - r_f &= \lambda_{i,0} + \lambda_{i,1}\beta_{i,t} + \sum_{k=2}^{p} \lambda_k Z_{i,k,t} + \varepsilon_{i,t}, \quad && Model \quad 4 \\
r_{i,t} - r_f &= \lambda_{i,0} + \lambda_{i,1}\beta_{i,t} + \lambda_{i,2}\gamma_{i,t} + \sum_{k=3}^{p} \lambda_k Z_{i,k,t} + \varepsilon_{i,t}, \quad && Model \quad 5 \\
r_{i,t} - r_f &= \lambda_{i,0} + \lambda_{i,1}\beta_{i,t} + \lambda_{i,2}\gamma_{i,t} + \lambda_{i,3}\kappa_{i,t} \\
&\quad + \sum_{k=4}^{p} \lambda_k Z_{i,k,t} + \varepsilon_{i,t}, \quad && Model \quad 6 \qquad (5.21) \\
r_{i,t} - r_f &= \lambda_{i,0} + \lambda^{c}_{i,1}\beta^{c}_{i,t} + \lambda^{j}_{i,2}\beta^{j}_{i,t} + \lambda_{i,3}\gamma_{i,t} + \lambda_{i,4}\kappa_{i,t} + \varepsilon_{i,t}, \quad && Model \quad 7
\end{aligned}
$$

$$
\begin{aligned}
r_{i,t} - r_f = \lambda_{i,0} + \lambda_{i,1}^{c}\beta_{i,t}^{c} + \lambda_{i,2}^{j}\beta_{i,t}^{j} + \lambda_{i,3}\gamma_{i,t} + \lambda_{i,4}\kappa_{i,t} \\
+ \sum_{k=5}^{p} \lambda_k Z_{i,k,t} + \varepsilon_{i,t}, \quad Model \quad 8 \\
r_{i,t} - r_f = \lambda_{i,0} + \lambda_{i,1}\beta_{i,t}^{-} + \lambda_{i,2}\beta_{i,t}^{+} + \lambda_{i,3}\gamma_{i,t}^{-} + \lambda_{i,4}\gamma_{i,t}^{+} + \lambda_{i,5}\kappa_{i,t}^{-} \\
+\lambda_{i,6}\kappa_{i,t}^{+} + \varepsilon_{i,t}, \quad Model \quad 9 \\
r_{i,t} - r_f = \lambda_{i,0} + \lambda_{i,1}\beta_{i,t}^{-} + \lambda_{i,2}\beta_{i,t}^{+} + \lambda_{i,3}\gamma_{i,t}^{-} + \lambda_{i,4}\gamma_{i,t}^{+} + \lambda_{i,5}\kappa_{i,t}^{-} + \lambda_{i,6}\kappa_{i,t}^{+} \\
+ \sum_{k=7}^{p} \lambda_k Z_{i,k,t} + \varepsilon_{i,t}, \quad Model \quad 10
\end{aligned}
$$

where $r_{i,t} - r_f$ denotes the excess realized monthly return for stock i with the explanatory variables on the right-hand side of Equation (5.21), $Z_{i,k,t}$ is a vector of characteristics and controls for the ith firm observed at month t. The illiquidity factor (Illiq) of Amihud (2002), the size factor (log(size)) of Fama and French (1993), book-to-market ratio (B/M) of Fama and French (1992), and value at risk (VaR) at 1% level of significance and past-returns (PRet) of Amaya et al. (2015) are the control variables. Pettengill et al. (1995) and Hung et al. (2004) show the effects of using realized returns as a proxy for expected returns in regard to the Fama-MacBeth two-stage regression procedure. We run the cross-sectional regression with Newey and West (1987) heteroskedastic-robust standard errors, as such the significance levels are assessed with Newey-West correct t-statistics. The average coefficient (λ) depicts the premium awarded for one unit of exposure of each explanatory variable. We focus on contemporaneous relationships between excess realized monthly returns and the explanatory variables similar to that of Ang et al. (2006); Lewellen and Nagel (2006); Alles and Murray (2013). This approach avoids any assumption that risk exposures are not time varying. Doan et al. (2014) showed that beta asymmetry is driven by time-varying higher-order risk preferences across different market states. It is worth emphasizing that the realized higher-order moment estimates employed in our analysis are time-varying. In the existing financial literature, Models 1–3 of Equation (5.21) are popularly known as the two-, three- and four-moment CAPM models, respectively.

5.3 Data

In the US framework, it is typical to use return series sampled at a 5-minute sampling frequency as a proxy for unbiased high-frequency return data (Andersen and Bollerslev, 1997; Andersen et al., 2007; Huang and Tauchen, 2005). The underlying rationale for this is that it is a trade-off between microstructure noise and variance bias. We observe that the 5-minute unbiased sampling frequency does not hold for the Australian stock return series but rather a 10 to 30-minutes sampling frequency maybe preferable.[15] Bandi and Russell (2008) affirm the importance of computing RV with unbiased intra-day return data, since computing RV with contaminated return data results in the significant accumulation of noise, which may result in obtaining biased estimates.[16] This study uses intra-day 15-minute last traded prices of 142 stocks listed on the ASX stock market. The data sample starts from January 2003 to December 2017 from 10 am to 4 pm of each trading day, giving us a sample of 24 intra-day price series. We exclude weekends and overnight return series from the data. Our sample period results in 94,350 price series over 15 years (180 months). The number of stocks that used in the analysis is solely dependent on the availability of the data for the calendar period that was considered. The 90-day accepted bill rate is used as the proxy for the risk-free rate, and the intra-day returns are computed as the change in the logarithm of the last prices of successive days. We compute the monthly normalized (un)conditional realized higher-order co-moments from high-frequency return data.

[15]The evidence for this is in Chapter 2 of this thesis, which is currently under review at 'The Quarterly Review of Economics and Finance'.

[16]Bandi and Russell (2008) recommend that when sampling very illiquid stocks, a 15-minute frequency could be the preferred sampling option for computing realized volatility (which should be lowered for very high liquid stocks). Bollerslev et al. (2008) shows that the optimal sampling frequency for 40 US equities was 17.5 minutes. Oomen (2006) shows that the optimal sampling frequency for realized volatility for IBM stock to be 20 minutes, while it changes to about 3 minutes with a first-order bias correction. Using DJI30 stocks, Hansen and Lunde (2006) shows that the noise in realized volatility might be ignored when intra-day returns are sampled at low frequencies (e.g., 20 minutes). Andersen et al. (2003) employs a 30-minute return series to compute the RV.

Alles and Murray (2017) have shown that although Australia is a developed country, its equity market requires separate investigation, as some aspects differ from those of major international equity markets. They show that relative to the US and UK markets, the trading volume for the Australian equity market is less than 5% of that recorded for the New York stock exchange. The Australian equity market is concentrated in a small number of sectors, with the materials sector dominating and rendering the market highly weighted in one sector. Alles and Murray (2017) also report that the Australian equity market is mainly represented and weighted by domestic firms, with less than 2% representing overseas companies. This subsequently entails that investors do not have a wide range of investment opportunities, which in turn might result in cyclical economic patterns. The unavailability of alternative investment options may affect the extent of reward that is available for assuming downside risk in the Australian equity market. It is worth emphasizing that the Australian equity market is unique and that its risk exposure might be different from those of major international equity markets.

In addition to the realized higher-order co-moments, we also consider explanatory variables ($Z_{i,k,t}$) from Equation (5.21), these include the book-to-market ratio (B/M) of Fama and French (1992) which is measured as the ratio of the book value of common equity to the market value of equity for the stock i. Illiquidity of stock i is measured as the average ratio of the absolute stock return to the dollar trading volume for that month (Amihud, 2002). A firm's size is measured as the natural logarithm of its market capitalization (log(size)) of Fama and French (1993). Past-return is the lagged return over previous month and value-at-risk (VaR) at 1% significance level by Amaya et al. (2015). Apart from the B/M and log(size) which was computed with data from DataStream, the remaining variables were computed with data from Thompson Reuters Tick History provided by SIRCA database.

Table 5.2 presents summary statistics for our various risk measures used for the analysis in this study. The statistics include the mean, standard deviation, skewness, kurtosis, minimum, and maximum of values of the variables for the full-sample period. The standard CAPM beta, continuous beta, jump beta, and kappa display positive returns over the studied period (6 January 2003 to 29 December 2017), while the cross-sectional average of gamma is negative. We observe that the average standard CAPM beta is low when compared with that of upside and downside betas, which is similar to Alles and Murray (2017). This could be attributed to the constituents of the S&P/ASX200 index being large and mid-size firms with relatively lower betas. In comparison, the average for the jump beta slightly increases. In the case of the normalized conditional co-moments, apart from downside gamma, the rest of the estimates depicts positive return.

Table 5.2: Summary statistics, full-sample period (Jan 2003 - Dec 2017)

	Mean	Std. Dev	Min	Max	Skewness	Kurtosis
β	0.6410	0.7565	-2.1381	3.6388	0.0358	11.4408
γ	-0.0799	0.8236	-2.3428	2.2518	0.0587	3.5009
κ	9.1531	8.9347	-16.1289	30.2714	-0.0913	3.0453
Illiq	1.58E-05	0.0001	7.15E-11	0.0014	8.5979	86.4208
log(size)	6.9847	1.9964	0.2391	11.5866	-0.1776	3.0960
PRet	0.0053	0.1333	-0.5704	0.6724	0.3481	24.1345
B/M	0.0042	0.0101	-0.0055	0.0904	5.1230	41.2669
VaR	-0.0129	0.0090	-0.0729	-0.0038	-3.0759	21.6454
β^c	0.4678	1.5678	-5.7007	7.3960	0.1344	8.2395
β^j	0.9273	1.9456	-8.0259	10.5459	-0.2959	21.7421
β^+	0.7470	1.0581	-3.0605	5.4173	0.1897	12.4047
β^-	0.7464	1.0658	-4.0060	4.4671	-0.3692	12.2085
γ^+	1.1201	1.3225	-2.7247	4.3797	-0.0672	3.2713
γ^-	-1.2853	1.4061	-4.7540	2.9670	0.1406	3.4587
κ^+	10.6166	13.1549	-29.9428	44.4020	-0.0923	3.6206
κ^-	14.2700	16.1457	-37.0612	56.0004	-0.1516	3.8540

This table reports the cross-sectional averages of the descriptive statistics of the variables used in the analysis of this study. The full sample period start 6 January 2003 - 29 December 2017. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

5.4 Empirical Results

5.4.1 Single ranking

We investigate the relationship between Australian firms' returns and each explanatory variable. Inhere, we compute 10-decile equally weighted portfolios that are sorted by each firm characteristic. We separately consider a full sample (all periods), an upmarket period (when the monthly excess realized market return exceeds the risk-free monthly rate) and the downmarket period (when the excess market return fall below the risk-free rate). The results are reported in Table 5.3.

In Panel A of Table 5.3, we report the average portfolio return for each decile. We also report the difference in average returns between portfolio 1 (lowest) and portfolio 10 (highest); its t-test statistics and p-values are used to determine the significance of the high-minus-low portfolio spread. The MR p-value of Patton and Timmermann (2010) is used to determine the presence of any monotonically increasing relationship within the entire sorted portfolio. When the p-value is less than or equal to 0.05, the test is in favour of the alternative hypothesis, which supports the presence of a monotonically increasing pattern in the entire portfolio. The average return on the high-beta stocks (i.e., 0.006) is observed to exceed that of the low-beta stocks (i.e., 0.001). The high-minus-low mean spread (i.e., 0.005) of the beta portfolio indicates a direct relationship between beta and excess returns. This suggests that high-ranked portfolios offer higher returns in comparison to lower-ranked portfolios, though the t-test and p-value on the high-minus-low spread are statistically insignificant. Alles and Murray (2017); Bilinski and Lyssimachou (2014) states that the insignificant high-minus-low beta–sorted average return can be attributed to the notion that the actual relationship between high-minus-low investments are obscured when large positive and negative returns are combined. Therefore, this study aims to capture the desired relationship when considering upmarket and downmarket periods separately. For downside beta, the average return on the low downside beta stocks is positive (i.e., 0.039), while the average return on the high-downside beta stocks is negative (i.e., -0.028). This suggests that any investor who bears lower-ranked downside-beta risk may experience relatively higher returns. This is confirmed by the negative average return (i.e., -0.067) that is observed for the difference between the high-ranked and low-ranked portfolios. We find that the high-minus-low spread for the downside beta portfolio to be statistically significant at 1% level. In regard to the upside beta, we observe that the mean spread for the highest and lowest portfolios to be positive (i.e., 0.072) and statistically significant at 1% level, which implies that higher-ranked upside beta portfolios will offer relatively higher returns than the lower-ranked upside beta portfolios. Concerning gamma, downside and upside gamma, upside kappa, B/M and illiquidity, we observe that the high-minus-low spreads are positive and statistically significant. This implies that the higher-ranked portfolios of these variables lead to a higher return to the investor. However, downside kappa and log(size) have a significant and inverse relationship with excess returns. Hence, investors investing in a higher-ranked portfolio of downside kappa and log(size) would experience relatively lower returns. We observe that gamma, downside (upside) gamma, upside kappa and illiquidity exhibit monotonically increasing patterns within the entire portfolio, according to the MR p-value results obtained.

In Panel B of Table 5.3, the results for the upmarket periods indicate that the high-minus-low beta spread is positive (i.e., 0.024) and significant; this aligns with our expectations for higher-ranked beta risk yielding higher returns to the investor in the upmarket period. For the portfolio sorted by downside beta, the difference between the high-ranked and low-ranked is negative (i.e., -0.023) and significant. This may be explained by excess exposure to the downside risk measures in the upmarket condition. In this scenario, although the market is in an up-state, investing in higher-ranked downside betas results in investors experiencing lower excess returns. Conversely, a sizeable positive (i.e., 0.057) and significant value of excess return is generated from upside beta investments. We also observe that jump beta, gamma, downside and upside gamma, kappa, upside kappa, B/M and illiquidity have positive and significant high-minus-low mean spreads which suggest that their investments also leads to higher excess realized returns while downside kappa and VaR exhibit significant losses. The monotonicity test shows that beta, upside beta, jump beta, gamma, upside gamma, upside kappa, B/M and illiquidity have a monotonically increasing relationship in the whole portfolio.

Finally, the results for the downmarket period in Panel C reveals that the high-minus-low beta spread is negative (i.e., -0.019) and significant. This is unsurprising, as higher-ranked beta investments should yield lower excess realized returns in a bear market. In regard to portfolios being sorted by downside beta, we observe that the difference between the high-ranked and low-ranked downside betas is negative (i.e., -0.045) and significant. For the upside beta portfolio, we note a positive (i.e., 0.015) and significant value for the high-minus-low spread, which implies that higher-ranked upside beta investments yields significantly higher excess realized returns. Further, we also observe that gamma, downside and upside gamma, upside kappa, B/M, illiquidity and VaR have positive and significant high-minus-low mean spreads. Such investments generate higher excess realized returns to the investors. Conversely, continuous beta, jump beta, kappa, downside kappa and log(size) exhibit significant losses or lower excess realized returns to investors. Only gamma and downside gamma are observed to have had a monotonically increasing relationship in the entire portfolio for the downmarket period.

In short, Table 5.3 outlines the single sorting of excess realized return on our risk measures. The results are also pertinent and robust in the upmarket and downmarket periods. The findings in this section are consistent with the results of Table C1, which reports the Fama-MacBeth regression run separately for each risk measure we consider. It is worth mentioning that the single-sorting approach does not control for any other known patterns in the cross-section of stock returns. The double-sorting approach and Fama-MacBeth regression are employed to address this limitation.

Table 5.3: Estimates of realized returns for equally-weighted decile portfolio sorted on firm characteristics, full-sample period (Jan 2003-Dec 2017)

Portfolio	1(L)	2	3	4	5	6	7	8	9	10(H)	H-L	(H-L) t-stat	(H-L) p-value	MR p-value
Panel A: All periods														
β	0.001	0.002	-0.001	0.003	-0.002	0.004	0.004	0.001	0.002	0.006	0.005	0.529	0.298	0.698
β^-	0.039	0.014	0.002	0.007	0.001	0.002	-0.003	-0.003	-0.011	-0.028	-0.067***	-8.907	0.000	0.850
β^+	-0.029	-0.008	-0.007	0.000	-0.002	0.001	0.004	0.008	0.010	0.043	0.072***	9.460	0.000	0.065
β^c	0.005	0.001	-0.002	0.000	0.001	-0.003	-0.001	0.001	0.003	0.000	-0.005	-1.173	0.120	0.178
β^j	-0.002	-0.002	0.002	0.002	0.004	0.001	0.004	0.001	0.001	0.007	0.009	0.985	0.162	0.509
γ	-0.051	-0.025	-0.017	-0.012	0.003	0.005	0.012	0.020	0.030	0.055	0.106***	19.736	0.000	0.000
γ^-	-0.025	-0.015	-0.002	-0.004	-0.002	0.000	0.005	0.005	0.021	0.037	0.062***	14.359	0.000	0.024
γ^+	-0.040	-0.010	-0.003	0.001	0.002	0.005	0.006	0.013	0.016	0.030	0.070***	14.707	0.000	0.000
κ	-0.003	0.005	0.006	-0.001	0.002	0.001	0.002	0.004	0.001	0.003	0.006	1.234	0.109	0.600
κ^-	0.032	0.015	0.009	0.001	0.005	-0.002	-0.002	-0.003	-0.011	-0.024	-0.056***	-13.505	0.000	0.218
κ^+	-0.033	-0.011	0.001	-0.001	0.001	0.006	0.005	0.010	0.013	0.030	0.063***	14.140	0.000	0.002
B/M	-0.004	-0.013	-0.009	-0.010	0.000	-0.001	0.001	0.014	0.017	0.023	0.027***	4.779	0.000	0.517
Illiq	-0.040	-0.010	-0.003	0.001	0.002	0.005	0.006	0.013	0.016	0.030	0.070***	14.707	0.000	0.000
log(size)	0.011	0.010	0.002	0.000	0.002	-0.002	0.002	-0.001	-0.002	-0.003	-0.014***	-2.755	0.003	0.366
Pret	0.005	0.002	0.001	-0.002	0.004	0.006	-0.001	0.002	0.003	0.001	-0.004	-0.782	0.217	0.894
VaR	0.012	-0.004	-0.002	0.002	0.001	0.004	0.001	0.001	0.004	0.001	-0.011	-0.961	0.168	0.860
Panel B: Upmarket periods														
β	0.010	0.009	0.009	0.012	0.012	0.016	0.018	0.018	0.023	0.033	0.024***	4.200	0.000	0.002
β^-	0.035	0.017	0.011	0.015	0.013	0.015	0.013	0.015	0.012	0.013	-0.023***	-3.627	0.000	0.885
β^+	-0.006	0.004	0.005	0.011	0.013	0.016	0.019	0.023	0.025	0.051	0.057***	8.522	0.000	0.000
β^c	0.019	0.013	0.001	0.003	0.009	0.013	0.013	0.017	0.018	0.022	0.003	1.226	0.110	0.993
β^j	0.008	0.007	0.008	0.012	0.015	0.016	0.018	0.020	0.022	0.034	0.025***	3.998	0.000	0.026
γ	-0.013	0.001	0.007	0.010	0.017	0.019	0.025	0.026	0.029	0.045	0.058***	11.798	0.000	0.000
γ^-	0.005	0.010	0.015	0.013	0.016	0.015	0.020	0.015	0.025	0.033	0.028***	6.486	0.000	0.632
γ^+	-0.011	0.006	0.012	0.016	0.017	0.019	0.020	0.023	0.028	0.036	0.047***	11.368	0.000	0.000
κ	0.007	0.013	0.017	0.014	0.014	0.016	0.019	0.021	0.022	0.023	0.016***	4.820	0.000	0.077
κ^-	0.031	0.021	0.017	0.018	0.017	0.015	0.014	0.015	0.012	0.005	-0.026***	-6.120	0.000	0.075
κ^+	-0.006	0.005	0.015	0.016	0.015	0.019	0.019	0.022	0.026	0.035	0.041***	11.437	0.000	0.014
B/M	0.005	0.009	0.011	0.010	0.016	0.014	0.019	0.022	0.025	0.025	0.020***	4.355	0.000	0.029
Illiq	-0.011	0.006	0.012	0.016	0.017	0.019	0.020	0.023	0.028	0.036	0.047***	11.368	0.000	0.000
log(size)	0.014	0.019	0.017	0.017	0.017	0.015	0.016	0.016	0.014	0.013	-0.001	-0.371	0.355	0.488
PRet	0.021	0.017	0.016	0.014	0.015	0.018	0.013	0.017	0.014	0.016	-0.005	-1.253	0.105	0.909
VaR	0.030	0.017	0.016	0.017	0.016	0.018	0.014	0.013	0.012	0.006	-0.024***	-3.066	0.001	0.498
Panel C: Downmarket periods														
β	-0.008	-0.007	-0.010	-0.009	-0.014	-0.012	-0.014	-0.018	-0.021	-0.027	-0.019***	-3.305	0.000	0.520
β^-	0.004	-0.004	-0.009	-0.008	-0.012	-0.013	-0.017	-0.018	-0.023	-0.041	-0.045***	-6.750	0.000	0.117
β^+	-0.023	-0.012	-0.012	-0.011	-0.015	-0.015	-0.016	-0.015	-0.016	-0.008	0.015***	4.001	0.000	0.899
β^c	-0.014	-0.012	-0.004	-0.003	-0.008	-0.016	-0.014	-0.016	-0.015	-0.022	-0.008***	-2.851	0.002	0.969
β^j	-0.010	-0.009	-0.006	-0.010	-0.011	-0.016	-0.014	-0.019	-0.021	-0.026	-0.016***	-2.954	0.002	0.940
γ	-0.038	-0.026	-0.023	-0.022	-0.014	-0.014	-0.013	-0.006	0.000	0.010	0.048***	8.041	0.000	0.001
γ^-	-0.030	-0.025	-0.017	-0.017	-0.018	-0.014	-0.014	-0.009	-0.004	0.004	0.033***	8.421	0.000	0.006
γ^+	-0.029	-0.017	-0.015	-0.015	-0.015	-0.013	-0.014	-0.010	-0.012	-0.005	0.024***	5.937	0.000	0.147
κ	-0.010	-0.008	-0.011	-0.015	-0.012	-0.014	-0.016	-0.017	-0.021	-0.020	-0.010***	-2.855	0.002	0.711
κ^-	0.001	-0.006	-0.008	-0.017	-0.012	-0.017	-0.016	-0.018	-0.023	-0.029	-0.030***	-7.799	0.000	0.503
κ^+	-0.027	-0.016	-0.014	-0.017	-0.014	-0.013	-0.014	-0.012	-0.013	-0.005	0.022***	5.817	0.000	0.352
B/M	-0.009	-0.022	-0.020	-0.020	-0.016	-0.016	-0.017	-0.008	-0.008	-0.002	0.007**	2.177	0.015	0.996
Illiq	-0.029	-0.017	-0.015	-0.015	-0.015	-0.013	-0.014	-0.010	-0.012	-0.005	0.024***	5.937	0.000	0.147
log(size)	-0.003	-0.009	-0.016	-0.017	-0.015	-0.017	-0.015	-0.017	-0.016	-0.015	-0.012***	-3.168	0.001	0.265
PRet	-0.016	-0.014	-0.015	-0.016	-0.011	-0.012	-0.014	-0.015	-0.012	-0.015	0.001	0.387	0.349	0.467
VaR	-0.018	-0.021	-0.018	-0.015	-0.015	-0.014	-0.013	-0.012	-0.009	-0.005	0.013*	1.651	0.049	0.084

This table reports the estimates of expected returns for equally-weighted 10-decile portfolio sorted on firm characteristics for the 142 Australian stocks from our sample starting 6 January 2003 - 29 December 2017. Panel A presents the full sample (all periods), Panel B is the upmarket period which is defined as when the monthly excess realized market return exceeds the risk-free rate and Panel C is the downmarket period is when the excess market return falls below the risk-free rate. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR) and H-L is high-minus-low portfolio.

5.4.2 Double ranking

In the literature, it is well documented that downside beta and gamma both capture asymmetric higher moments and downside risk. Following Ang et al. (2006); Alles and Murray (2017), we measure the magnitude of the reward for exposure to downside beta, while explicitly controlling for the effect of gamma. The results are reported in Table 5.4.

Table 5.4: Realized returns of stocks sorted by realized downside beta and realized gamma

Panel A1: Gamma (All period)							
	Portfolio	1 Low	2	3	4	5 High	Average
Downside Beta	1 Low	-1.76%	2.22%	2.23%	3.48%	5.93%	2.42%
	2	-2.32%	-0.20%	0.85%	1.76%	3.65%	0.75%
	3	-2.18%	-0.71%	0.26%	0.91%	3.05%	0.27%
	4	-2.47%	-1.65%	0.07%	0.66%	2.73%	-0.13%
	5 High	-5.79%	-2.99%	-1.67%	-0.42%	1.60%	-1.86%
	High-Low	-4.03%	-5.21%	-3.90%	-3.91%	-4.33%	-4.28%
	t-stat	-6.55	-7.86	-6.25	-6.30	-5.25	-6.44

Panel A2: Downside Beta (All period)							
	Portfolio	1 Low	2	3	4	5 High	Average
Gamma	1 Low	-2.29%	-2.29%	-2.81%	-3.75%	-6.14%	-3.46%
	2	-0.31%	-1.31%	-0.84%	-1.65%	-3.36%	-1.50%
	3	1.97%	1.05%	0.19%	0.09%	-1.41%	0.38%
	4	3.06%	1.29%	0.96%	1.12%	1.06%	1.50%
	5 High	5.30%	3.67%	3.25%	2.87%	4.49%	3.92%
	High-Low	7.59%	5.96%	6.06%	6.62%	10.63%	7.37%
	t-stat	13.01	14.91	15.20	15.55	12.34	14.20

Panel B1: Gamma (upmarket period)							
	Portfolio	1 Low	2	3	4	5 High	Average
Downside Beta	1 Low	-3.34%	4.08%	5.25%	5.71%	8.62%	4.06%
	2	-0.38%	1.73%	3.17%	3.83%	5.98%	2.86%
	3	-0.41%	1.73%	2.61%	3.60%	5.49%	2.61%
	4	0.02%	1.41%	2.48%	3.83%	5.63%	2.67%
	5 High	-1.63%	0.90%	2.19%	3.68%	6.10%	2.25%
	High-Low	1.71%	-3.18%	-3.06%	-2.04%	-2.52%	-1.82%
	t-stat	3.21	-3.55	-3.94	-3.28	-2.66	-2.04

Panel B2: Downside Beta (upmarket period)							
	Portfolio	1 Low	2	3	4	5 High	Average
Gamma	1 Low	-3.38%	-0.05%	-0.18%	-0.59%	-1.39%	-1.12%
	2	2.43%	1.40%	1.55%	1.08%	0.25%	1.34%
	3	4.79%	3.25%	2.73%	2.58%	2.78%	3.22%
	4	6.23%	3.71%	3.38%	3.64%	4.61%	4.31%
	5 High	7.95%	6.15%	5.25%	5.25%	6.63%	6.25%
	High-Low	11.32%	6.20%	5.44%	5.85%	8.02%	7.37%
	t-stat	21.25	13.16	11.98	12.40	7.52	13.26

Panel C1: Gamma (downmarket period)							
	Portfolio	1 Low	2	3	4	5 High	Average
Downside Beta	1 Low	2.70%	-0.31%	-1.90%	0.44%	2.24%	0.63%
	2	-4.98%	-2.85%	-2.29%	-1.07%	0.45%	-2.15%
	3	-4.60%	-4.06%	-2.96%	-2.78%	-0.28%	-2.94%
	4	-5.89%	-5.83%	-3.25%	-3.65%	-1.24%	-3.97%
	5 High	-11.48%	-8.30%	-6.97%	-5.99%	-4.60%	-7.47%
	High-Low	-14.19%	-7.99%	-5.07%	-6.43%	-6.83%	-8.10%
	t-stat	-25.20	-8.72	-5.41	-5.68	-4.96	-9.99

Panel C2: Downside Beta (downmarket period)							
	Portfolio	1 Low	2	3	4	5 High	Average
Gamma	1 Low	2.69%	-5.35%	-6.42%	-8.05%	-12.67%	-5.96%
	2	-4.06%	-5.03%	-4.13%	-5.40%	-8.32%	-5.39%
	3	-1.88%	-1.96%	-3.27%	-3.30%	-7.16%	-3.51%
	4	-1.29%	-2.03%	-2.32%	-2.37%	-3.79%	-2.36%
	5 High	1.67%	0.30%	0.49%	-0.39%	1.57%	0.73%
	High-Low	-1.02%	5.66%	6.91%	7.66%	14.24%	6.69%
	t-stat	-1.82	8.90	10.68	10.94	10.61	7.86

This table examines the relation between downside beta and gamma. In Panel A1, we first rank stocks into quintiles (1-5) based-on gamma. Then, we rank stocks within each first-sort quintile into additional quintiles according to downside beta. For each 5×5 grouping, we form an equal-weighted portfolio. In Panel A2, we reverse the order so that we first sort on downside beta and then on gamma. The sample period is 6 January 2003 - 29 December 2017 for 142 Australian stocks. The row labelled "High-Low" reports the difference between the returns of portfolio 5 and portfolio 1. The entry labelled t-stat is the t-statistic of the High-Low value. For the column labelled "Average," we report the average return of stocks in each second sort quintile. This controls for gamma (downside beta) in Panel A1 (A2). The procedure is repeated for upmarket period in Panels B1 and B2. Panel C1 and C2 report the results for downmarket period.

We control for the effect of gamma before assessing the return to downside beta. First, all realized stock returns are ranked into quintile portfolios using realized gamma. Each gamma quintile stock is then sorted into five equally weighted portfolios based on downside beta and then average the excess returns of each downside beta quintile over the five gamma portfolios. This procedure ensures that the stocks that are allocated to each quintile have relatively similar levels of gamma. Hence, we effectively control for the differences in gamma for each portfolio. In controlling for gamma exposure, the investor's reward for downside beta can be obtained.

Panel A1 of Table 5.4 reports the average realized excess returns of 25 gamma × downside beta portfolios. The column labelled 'Average' outlines the average realized excess returns of the downside beta quintiles while controlling for the gamma risk for the full-sample period (all periods). We also highlight the differences in the average returns between the highest and lowest-ranked portfolios (high-minus-low) and the significance of the high-minus-low portfolio reported at 't-stat'. We note that controlling for gamma enables the relationship between investors' excess returns and downside beta to be investigated explicitly. The average realized excess return of –4.28% in the bottom right of Panel A1 represents the difference in average excess returns between the fifth and first downside beta quintile portfolios that control for gamma risk. This difference has a t-statistic of –6.44 and suggests that gamma risk cannot account for the losses of bearing downside beta risk. Additionally, the high-minus-low values for each quintile indicate that investors who accept higher levels of downside beta risk experience significantly poorer performance, although the relationship is not always monotonic across the gamma quintile. This implies that realized excess returns and losses to investors who accept downside beta are not explained by a premium for gamma. The average realized excess returns of the downside beta quintiles that control for gamma have a monotonically decreasing pattern across the portfolio. We also observe that lower-ranked downside beta portfolios in positive (higher) gamma quintiles offers excess positive returns and vice versa. This suggests that all portfolios in the higher gamma quintiles experience gains while those in the lower gamma quintile portfolios experience losses. Ang et al. (2006) shows that gamma is effectively the co-variance of a stock's returns with the volatility of the market. They also show that stocks with negative (low) gamma result in low returns when market volatility is high.

In Panel A2 of Table 5.4, we repeat the same procedure as Panel A1 to investigate the reward for the gamma while controlling for downside beta in the all-period category. In this panel, we control for downside beta before accessing the return to gamma investment. The average realized excess return of 7.37% in the bottom right of Panel A2 represents the difference in the average excess returns between the fifth and first gamma quintile portfolios that controls for downside beta risk. This difference has a t-statistic of 14.20, which suggests that downside beta risk cannot account for the gains obtained for bearing gamma risk. The high-minus-low portfolio values for each quintile indicate that investors who accept higher levels of gamma experience significant and better performance, though the relationship is not always monotonic across the downside beta quintile. Additionally, we observe the pattern within the average column to exhibit a monotonically increasing pattern. We also note that higher-ranked gamma portfolios in higher downside beta quintiles offer excess positive returns and vice versa and that gamma risk yields significant positive excess returns when downside beta is controlled for.

Panel B1 of Table 5.4 repeats the process in Panel A1, with the estimation performed for the upmarket condition rather than the full-sample period. Similarly, the average realized excess return of –1.82% in the bottom-right entry of Panel B1 represents the difference in the average excess returns between the fifth and first downside beta quintile portfolios when we control for gamma risk. This difference has a t-statistic of –2.04, which suggests that downside beta risk is priced separately. The high-minus-low values for each quintile indicate that investors who accept higher levels of downside beta will experience significant losses. Although the low-ranked portfolio is an exception with an excess return of 1.71% (thus the 'high-minus-low of column 1 low' of Panel B1 of Table 5.4), this suggests that, on average, the realized excess returns and losses to investors who accept downside beta risk are not explained by a premium for gamma. We observe a decreasing pattern for the average column, which is not always monotonic. We also note that lower-ranked downside beta portfolios in higher gamma quintiles offer excess positive returns and vice versa.

Panel B2 of Table 5.4 outlines the results we obtain from investigating the reward for gamma by controlling for downside beta in the upmarket condition. The average realized excess return of 7.37% in the bottom right of Panel B2 represents the difference in the average excess returns between the fifth and first gamma quintile portfolios that control for downside beta risk and is statistically significant, with a t-stat of 13.26. The high-minus-low portfolio values for each quintile indicate that investors who accept higher levels of gamma risk experience significant and better performance. However, the relationship is not always monotonic across the downside beta quintile. The average column displays a monotonically increasing pattern, which can be beneficial to the investors' trading strategy. We also note that higher-ranked gamma portfolios in lower downside beta quintiles offer excess positive returns and vice versa.

Panel C1 of Table 5.4 repeats the process that is performed in Panel A1, but for the downmarket period. In this panel, the average realized excess return is –8.10% (as shown in the bottom-right entry of Panel C1). In comparison to the losses in all periods and the upmarket periods, the downmarket losses had the highest magnitude and significance. This implies that downside beta losses cannot be ignored regardless of controlling for gamma risk. The high-minus-low values for each quintile indicate that investors who accept higher levels of downside beta experience significant losses across the sorted portfolios.

Finally, Panel C2 of Table 5.4 reports the results of the downmarket period when we investigate the reward for gamma by controlling for downside beta. We control for downside beta before the return to gamma investment is accessed. The average realized excess return of 6.69% is shown in the bottom-right entry of Panel C2. This value is significant at 7.89 and suggests that downside beta risk cannot account for the gains that are obtained from bearing gamma risk. The high-minus-low portfolio values for each quintile indicate that investors who accept higher levels of gamma risk experience significant and better performance, although the low-ranked portfolio is an exception, as it has an excess return of –1.02% (thus the high-minus-low of column 1 low). The average column displays a monotonically increasing pattern. We note that gamma risk results in a significant and positive excess return to the investor when we control for the downside beta. We also observe a small –1.02% spread for the gamma quintiles for the lowest downside beta, while the difference in average returns between the high gamma and high downside beta is 14.24%. This shows the presence of a large spread in the average excess returns across the gamma quintiles for stocks with high gamma.

In summary, downside beta and gamma risks are different and priced separately under various market conditions. We observe high significant returns for gamma risk when controlling for downside beta risk and vice versa. We also note significant differences in the magnitude of the gains and losses relative to the market conditions (all periods, the upmarket period and the downmarket period). The reward for taking gamma risk and the losses that are obtained from downside beta risk are consistent with the results obtained for the single-sorted portfolios on gamma and downside beta in Table 5.3.

5.4.3 Fama-MacBeth regressions (full period)

We run a cross-sectional Fama-MacBeth regression at the firm level to further investigate the predictive power of the risk measures (explanatory variable) considered in this study. The realized monthly excess return for each firm is the dependent variable, λ's from Equation (5.21) are the risk premiums. Table (5.5) reports the regression results for the full sample period (all period) starting 6 January 2003 - 29 December 2017, where tables (5.6) and (5.7) are the results for the upmarket period and downmarket period of the full sample period, respectively. The Fama-Macbeth regression aims to capture any contemporaneous relationship between realized excess returns and the related risk measures.

In Table (5.5), Model 1, which is commonly referred to as the traditional two-moment CAPM shows that the standard realized beta has a negative relationship with realized excess return, this implies losses on standard beta investments. Nevertheless, these losses are not statistically significant for this model. Model 2 is the three-moment CAPM; for this model, gamma is added to standard beta. We observe that the beta coefficient remains negative and insignificant (-0.0010), while gamma has a significant and positive relationship with realized excess returns (0.0345). This suggests that gamma investments yield significant gains to the investor and is consistent with the excess realized return gamma relationships that we observed in previous section. The four-moment CAPM of Model 3, shows that in the presence of standard beta and kappa, only gamma investment leads to significant gains to the investor.

In Model 4, we assess the two-moment CAPM by controlling for a set of firm characteristics. We observe that the beta becomes positive, but still insignificant in this scenario. However, we expect stocks with high book-to-market (B/M) value to have high levels of excess return. As such, we observe a substantial and highly significant positive coefficient (1.0169) on the firm's B/M value, which is consistent with the results of (Fama and French, 1992; Ang et al., 2006). Model 5 shows that when the three-moment CAPM is combined with control variables, both gamma and B/M value have a highly significant and positive coefficient of 0.0361 and 0.8861, respectively. This implies high gamma and high B/M stock generate high excess returns for the investor. Model 6 is when the four-moment CAPM accounts for the firm characteristics, in which we note that gamma and B/M investments yield gains. For the other control variables, the coefficient of illiquidity is positive (0.006) while that of firm size is (-0.013) are significant at a 10% level. The negative relationship between firm size and excess return suggests that larger companies are associated with lower excess returns.

Following the methodology of Todorov and Bollerslev (2010); Bollerslev et al. (2016); Alexeev et al. (2017), we decompose the standard beta into continuous beta and jump beta. In Model 7, we note that both continuous beta and jump beta have a negative relationship with excess returns and that the jump beta is insignificant, while the continuous beta is significant at a 10% level. We find that gamma generates significant excess returns, while kappa has an insignificant negative relationship with excess returns. For Model 8, accounting for additional measures of risk does not eliminate the direct significance of gamma and B/M. In short, we observe a robust gain for holding stocks with gamma risk and controlling for a set of firm characteristics. Continuous and jump beta remain insignificant with positive and negative relationships, respectively. The coefficient of -0.0013 for the firm size and excess return still suggests larger companies are associated with lower excess returns at 10% significance level.

In Model 9, we replace the standard beta with upside and downside beta, gamma with upside and downside gamma, kappa with upside and downside kappa. We note that both upside and downside risk measures are priced. This is unsurprising as we expect conditional high-order co-moments to have predictive power. Therefore, upside beta, upside gamma, downside gamma, and downside kappa have a highly significant positive relationship with excess returns. We also note that downside beta and upside kappa result in substantial losses for investors' holding such risks; this is consistent with the portfolio sort in Table (5.3). Model 10 shows that both upside and downside beta, gamma, and kappa risk are robust when we control for a set of firm characteristics. The level of significance and directional impact remains almost the same as that of Model 9. In Model 10, we note that stocks with high B/M value tend to have high excess returns, and stocks with high past returns experience significant low returns.

Table 5.5: Fama-MacBeth cross-sectional regressions, full sample (All period)

Model	1	2	3	4	5	6	7	8	9	10
β	-0.0013	-0.0010	-0.0079	0.0035	0.0035	-0.0015				
γ		0.0345***	0.0608***		0.0361***	0.0555***	0.0573***	0.0535***		
κ			-0.0007			-0.0005	-0.0006	-0.0003		
Illiq				0.0042	0.0053	0.0060*		0.0050		0.0060*
log(size)				-0.0006	-0.0011	-0.0013*		-0.0013*		-0.0011
Pret				-0.0127	-0.0128	-0.0119		-0.0123		-0.0180**
B/M				1.0169***	0.8861***	0.8777***		0.9082***		0.8540***
VaR				0.2243	0.2931	0.2062		0.1616		0.2739
β^c							-0.0021*	0.0004		
β^j							-0.0037	-0.0011		
β^+									0.0258***	0.0226***
β^-									-0.0351***	-0.0272***
γ^+									0.0797***	0.0803***
γ^-									0.0704***	0.0631***
κ^+									-0.0079***	-0.0078***
κ^-									0.0070***	0.0060***
Intercept	0.0024	0.0046	0.0024	0.0055	0.0117	0.0102	0.0019	0.0097	0.0021	0.0090
Avg. R-squared	0.0738	0.143	0.186	0.241	0.300	0.319	0.230	0.343	0.287	0.392

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks, the full-sample period; starting 6 January 2003 - 29 December 2017 (all period). Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

We follow Ang et al. (2006) approach in interpreting the economic magnitude of the risk premia obtained in the Fama-MacBeth regression. Since the time-series averages reported in Table 5.2 are for the full sample period, we focus on the interpretation of Table 5.5. The cross-sectional standard deviation of gamma is 0.8236, which implies that for gamma risk-reward of 6.08% in Model 3, given a two standard deviation move across stocks in terms of gamma will correspond to a change in expected excess return of 0.4461% per month.[17] In the case of downside beta in Model 9, the cross-sectional standard deviation is 1.0658, hence a two standard deviation move across downside beta will change expected excess returns by -0.0148% per month.[18] This confirms our results that significant gains accompany gamma investments, while downside beta is associated with losses. From figure 5.1, we report the economic magnitudes for the regression result in Table 5.5. We observe that the reward for bearing gamma risk in the models (2, 3, 5, 6, 7, and 8) are quite significant and cannot be ignored, even in the presence of a set of firm characteristics.

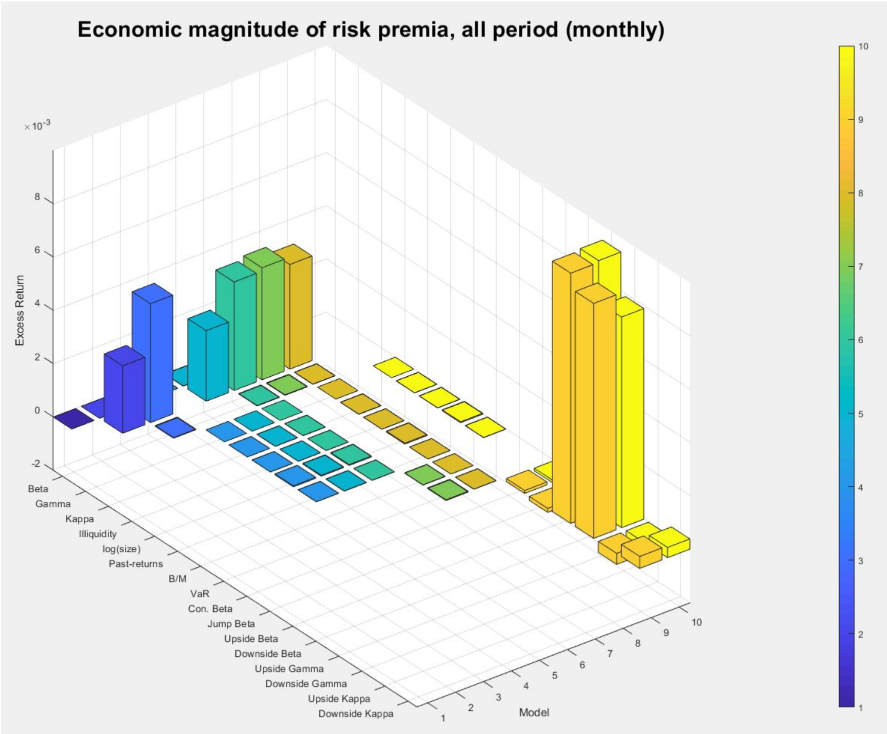

Figure 5.1: The economic magnitude of the monthly risk premia for the full-sample period (all period)

Consequently, the conditional gamma exhibits an even higher excess returns in models 9 and 10. This suggests that for a two standard deviation move across stocks in terms of upside (downside) gamma will result in about 1% increase in

[17] Ahadzie and Jeyasreedharan (2020) show that estimating realized higher-order moments from high-frequency returns to various holding periods, the magnitude of the moment is conditional on the number of observations. Therefore, the change is $2\times0.0608\times0.8236 = 10.01\%$ divided by $\sqrt{504} = 0.4461\%$ (thus adjusting for the scaling effect from estimating monthly gamma from 15-minutes return series.)

[18] Similarly, for downside beta, $2\times$-0.0351×1.0658 = -7.48%, thus -0.0148%= -7.48%/504.

the excess returns while downside beta results in almost -0.015% losses for the monthly holding. Perhaps the significant high reward for gamma risk measures may be associated with investors' preference for bearing gamma risks.

Table (5.6) reports the Fama-MacBeth regression for the upmarket period (58% of the full sample period is the upmarket period, while 42% of the full sample span is downmarket period). For this table, we observe that in Model 1, the traditional two-moment CAPM shows that standard beta has a highly significant positive (0.0064) relationship with realized excess return. This suggests that when the market is in the upmarket period bearing standard beta risk leads to significant gains to the investor, which is consistent with the findings of (Lambert and Hübner, 2013; Alles and Murray, 2017). In Model 2, we observe that the beta-excess return relationship remains positive, but the significance disappears entirely in the presence of gamma risk measure. The coefficient of gamma is 0.0189 and significant at 1% level, this implies that gamma investments yield significant gains to the investor. For the four-moment CAPM of Model 3, we note that the standard beta is positive and significant at 10% level, while gamma has a high significant direct relationship with excess returns. However, kappa has a significant negative relationship with excess returns (with a coefficient of -0.0021). The results suggest that standard beta and gamma investments generate significant gain to investors, while kappa investments lead to substantial losses.

Similarly, in Model 4, we assess the robustness of the two-moment CAPM when combined with a set of firm characteristics. We note that the standard beta remains significant and positive in the presence of the control variables. Additionally, B/M value has a significant positive coefficient of 0.5126, which is consistent with the results in Table (5.5). We also observe a highly significant negative relationship between firm's past returns and excess returns (with a coefficient of -0.0193). For Model 5, the standard beta, gamma, and B/M value all have a significant and positive coefficients of 0.0051, 0.0204, and 0.4450 (respectively) at 1% level of significance. This suggests high levels of standard beta, gamma, and B/M investments generate high excess returns for the investor. In this Model, exposure to stock's past return remains negative and significant. Model 6 results show that standard beta, gamma, and B/M value have a positive and significant relationship with excess returns at 1% level, while kappa and past return risk have a negative and significant relationship with excess returns at 1% significance level.

Model 7 shows that both gamma and kappa are significantly priced in the presence of continuous beta and jump beta. However, the coefficients of the continuous and jump beta are insignificant. In Model 8, accounting for additional measures of risk does not eliminate the directional impact and significance of gamma and kappa. We note that the significance of the continuous and jump beta improves to 5% and 10%, respectively. Additionally, B/M value and past returns retain their signs and significance as in previous models.

For Model 9, we note that both upside and downside risks are priced, but with a lesser magnitude in comparison to the results of Table (5.5). We also observe that upside beta, upside gamma, downside gamma, and downside kappa have a significant and positive relationship with excess returns. In the case of downside beta and upside kappa, a significant and negative relationship is obtained. Model 10 shows that both upside and downside beta, gamma, and kappa risk are robust to the inclusion of firm size, illiquidity, VaR, past returns, and B/M value. Although the level of significance and directional impact is robust, the magnitude of the coefficient is almost half of what was observed for Table (5.5). Similarly, stocks with high B/M value tend to have high excess returns than those with high past returns.

Table 5.6: Fama-MacBeth cross-sectional regressions, full sample (Upmarket period)

Model	1	2	3	4	5	6	7	8	9	10
β	0.0064***	0.0030	0.0057*	0.0097***	0.0051***	0.0076***				
γ		0.0189***	0.0361***		0.0204***	0.0326***	0.0341***	0.0312***		
κ			-0.0021***			-0.0016***	-0.0016***	-0.0013***		
Illiq				0.0023	0.0034	0.0037		0.0022		0.0040
log(size)				0.0002	-0.0005	-0.0001		0.0003		0.0000
Pret				-0.0193***	-0.0202***	-0.0193***		-0.0212***		-0.0235***
B/M				0.5126***	0.4387***	0.4450***		0.4507***		0.4366***
VaR				-0.4809	-0.5411	-0.4744		-0.5850*		-0.3310
β^c							0.0004	0.0014**		
β^j							0.0020	0.0019*		
β^+									0.0186***	0.0156***
β^-									-0.0129***	-0.0104***
γ^+									0.0443***	0.0451***
γ^-									0.0451***	0.0356***
κ^+									-0.0045***	-0.0044***
κ^-									0.0047***	0.0035***
Intercept	-0.0026	-0.0035	-0.0040	-0.0141*	-0.0117	-0.0126*	-0.0040	-0.0163**	-0.0044	-0.0128*
Avg. R-squared	0.0419	0.0822	0.103	0.141	0.177	0.187	0.133	0.201	0.166	0.231

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks, for full-sample period starting 6 January 2003 - 29 December 2017 (upmarket period). The upmarket period which is defined as when the monthly excess realized market return exceeds the risk-free rate. Significance levels:‘ *: 0.10, **: 0.05, ***: 0.01’. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms’ market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Table (5.7) reports the regression results for the downmarket period. In Model 1, we observe that the standard beta has a significant and negative (-0.0077) relationship with realized excess return. This suggests that investors experience substantial losses for holding standard beta risk during the downmarket period, which is consistent with the findings of (Lambert and Hübner, 2013; Alles and Murray, 2017). This confirms the notion that the insignificant standard beta observed in Model 1 of Table 5.5 is indeed attributed to the significant positive (0.0064) and negative (-0.0077) coefficients of the upmarket and downmarket periods cancelling out each other. For Model 2, we note that beta remains negative, but is insignificant when gamma risk is included in the model. The coefficient of gamma is 0.0156 and significant at 1% level, which implies that gamma investments yield significant excess returns. In the four-moment CAPM of Model 3, we observe that the standard beta is still negative and significant at 5% level, while gamma and kappa have a significant and positive relationship with the excess returns at 1% level. One may argue that kappa risk in the downmarket yield gains for investors since they require a reward for taking up additional dispersion over and above the standard dispersion (beta).

Model 4 shows that beta remains significant and negative (-0.0062) in the presence of the control variables. We also note that high B/M value and VaR have high excess return with significant coefficients of 0.5043 and 0.7052, respectively. For Model 5, the significance of the beta disappears but remains negative, while gamma, B/M value, and VaR have a significant and positive coefficient of 0.0156, 0.4474, and 0.8341, respectively, at 1% level. This suggests that beta, gamma, and B/M investment generate high excess returns for the investor. In Model 6, we note that gamma, kappa, B/M, and VaR have a positive and significant relationship with excess returns. However, beta and past returns have a negative and significant relationship with excess returns. In short, standard beta, gamma, and kappa do not lose their explanatory power after accounting for individual firm characteristics.

In Model 7, we note that gamma and kappa have a positive and significant relationship with excess returns. The coefficients of the continuous and jump are (-0.0025) and (-0.0057), respectively, which are significant at 5% level. The magnitude of the jump beta is twice that of the continuous beta, suggesting investors are worse off holding jump beta risk during market downmarket periods. In Model 8, accounting for additional measures of risk has no directional and significant impact on gamma and kappa investment. Although the significance of the continuous beta does disappear, the coefficient remains negative while jump beta remains still significant at 5% level. As discussed earlier, B/M value and VaR have a positive and significant relationship with the average excess returns. The highly significant negative coefficient (-0.0015) for the firm's size of excess returns confirms that smaller companies have high excess returns.

Model 9 show that the magnitude of the gains for upside beta, upside and downside gamma, downside kappa are far lower than the level of gains observed in the upmarket period. For downside beta and upside kappa, the magnitude of the losses in the downmarket period is far greater than the losses observed in the upmarket period, which is unsurprising. In Model 10, upside and downside beta, gamma and kappa risk are robust to the inclusion of firm characteristics and the level of significance and magnitude did not change that much relative to those of

Model 9. We observe that stocks with high B/M value and VaR tend to have high excess returns. The high significant negative coefficient (-0.0012) of firm size implies that large companies tend to have low excess returns in the downmarket period and vice visa.

Table 5.7: Fama-MacBeth cross-sectional regressions, full sample (Downmarket period)

Model	1	2	3	4	5	6	7	8	9	10
β	-0.0077***	-0.0041	-0.0135**	-0.0062***	-0.0016	-0.0091***				
γ		0.0156***	0.0246***		0.0156***	0.0228***	0.0232***	0.0223***		
κ			0.0013***			0.0011***	0.0010***	0.0010***		
Illiq				0.0018	0.0020	0.0023		0.0028		0.0020
log(size)				-0.0008	-0.0006	-0.0012**		-0.0015***		-0.0012***
Pret				0.0066	0.0074	0.0075		0.0088		0.0055
B/M				0.5043***	0.4474***	0.4328***		0.4575***		0.4174***
VaR				0.7052**	0.8341***	0.6806***		0.7466***		0.6049**
β^c							-0.0025**	-0.0011		
β^j							-0.0057**	-0.0029**		
β^+									0.0071***	0.0070***
β^-									-0.0222***	-0.0168***
γ^+									0.0354***	0.0352***
γ^-									0.0253***	0.0275***
κ^+									-0.0034***	-0.0034***
κ^-									0.0023***	0.0025***
Intercept	0.0069**	0.0100***	0.0083**	0.0215***	0.0252***	0.0247***	0.0078**	0.0279***	0.0084**	0.0237***
Avg. R-squared	0.0319	0.0604	0.0826	0.0998	0.123	0.132	0.0967	0.141	0.120	0.161

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks, full-sample starting 6 January 2003 - 29 December 2017 (downmarket period). The downmarket period is when the excess market return falls below the risk-free rate. Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Overall, the results of Tables (5.5), (5.6) and (5.7) have shown that the excess return of beta and kappa are asymmetric in upmarket and downmarket periods. Perhaps the directional (positive or negative) impact can be explained by investor's general dislike for bearing second- and fourth-moment risk measures (see Jurczenko and Maillet, 2012). It is worth noting that regardless of the market state, gamma risk always leads to positive and significant returns to the investors. We also observe that all upside and downside risk premium retain their direction and level of significance when the control variables are included in the models. To conclude, market does price upside and downside normalized realized higher-order co-moments risk measures for all market conditions.

5.4.4 Fama-MacBeth cross-sectional regressions (sub-periods)

The extant financial literature has associated the period 2007-2009 with a drastic decline in stock prices worldwide. This historical period commonly known as the global financial crisis, which began as a result of the US real estate bubble bursting in 2007. The phenomena significantly affected advance, emerging, and developing countries and destabilized the world's financial markets. This suggests that any results obtained during the crisis period could be biased or entirely driven by the crisis's destabilising patterns. As such, since this study's full-sample period extensively covers 15 years span (6 January 2003 - 29 December 2017), which includes the financial crisis period. We follow Dungey and Gajurel (2014) and we split the full-sample period into three sections: the pre-crisis (6 January 2003 - 29 June 2007), the crisis-period (2 July 2007 - 29 May 2009) and the post-crisis period (1 June 2009 - 29 December 2017). For each sub-sample period, we run the Fama-MacBeth regression aimed at investigating the dynamic behaviour of the risk measures across cycles.

5.4.4.1 Pre-crisis period (6 January 2003 - 29 June 2007)

Table (5.8) reports the regression results for all periods in the pre-crisis period. In Model 1, beta has a negative relationship with excess return, which is consistent with the results of Table 5.5. Similarly, we observe that beta investment yield insignificant losses. For Model 2, we note that gamma has a significant positive relationship with excess returns (0.0359). We observe that gamma retains its significance and positive relationship at a coefficient of (0.0628) for Model 3. Contrary to the results shown in Table 5.5, the negative relationship between kappa risk measure and excess return is significant at 5% level.

In Model 4, controlling for a set of firm characteristics for the two-moment CAPM shows that the standard beta becomes positive (0.098) and significant at 5% level.

The B/M value has a positive and significant relationship with excess returns. For Model 5, the three-moment CAPM shows that gamma and B/M value have a significant and positive coefficient of 0.0358 and 0.4680, respectively. This signifies that high levels of gamma and B/M stocks increase excess returns for an investor. Model 6 shows that for the four-moment CAPM, when combined with firm characteristics; gamma, B/M value, beta have a positive relationship with excess returns, although beta and illiquidity are only significant at 10% level. In contrast, kappa has a significant negative relationship with excess return at 5% level.

In relation to Model 7, we note that gamma (kappa) have a positive (negative) relationship with excess returns. Model 8, controls for the firm characteristics of Model and continuous and jump betas. We note that accounting for the firm characteristics does not eliminate the direct significance of gamma and B/M value (thus 0.0587 and 0.4242, respectively) while the coefficient of -0.0016 for kappa indicates a significant loss of excess returns at 5% level.

In Model 9, we note that the upside beta, upside gamma, downside gamma, and downside kappa have a highly significant positive relationship with excess returns, while downside beta and upside kappa have a negative and significant relationship with excess returns. Model 10 the results for the conditional co-moments are robust and stocks with high B/M value tend to have high excess return and vice versa.

Table 5.8: Fama-MacBeth cross-sectional regressions, pre-crisis period (All period)

Model	1	2	3	4	5	6	7	8	9	10
β	-0.0015	-0.0072	-0.0040	0.0098**	0.0053	0.0077*				
γ		0.0359***	0.0628***		0.0358***	0.0589***	0.0620***	0.0587***		
κ			-0.0019**			-0.0020**	-0.0019**	-0.0016**		
Illiq				0.0105	0.0125*	0.0121*		0.0100		0.0131**
log(size)				-0.0006	-0.0011	-0.0008		-0.0006		-0.0000
Pret				-0.0047	-0.0108	-0.0144		-0.0121		-0.0254**
B/M				0.5617***	0.4680***	0.4237***		0.4242***		0.3901***
VaR				-0.9732	-0.9286	-0.8451		-0.8165		-0.5844
β^c							-0.0005	0.0006		
β^j							-0.0031	0.0009		
β^+									0.0373***	0.0346***
β^-									-0.0356***	-0.0312***
γ^+									0.0425***	0.0483***
γ^-									0.0482***	0.0393***
κ^+									-0.0052***	-0.0056***
κ^-									0.0055***	0.0044***
Intercept	0.0143**	0.0136**	0.0122**	-0.0009	0.0027	0.0026	0.0112**	0.0027	0.0102**	-0.0001
Avg. R-squared	0.0982	0.170	0.213	0.259	0.323	0.346	0.256	0.367	0.348	0.444

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the pre-crisis sample period starting 6 January 2003 - 29 June 2007 (all period). Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Table (5.9) reports the results for the upmarket period during the pre-crisis period. We note that the beta has an insignificant and positive (0.0030) relationship with excess return, which is contrary to that of Table (5.6). For Model 2, we observe that the beta-excess return relationship becomes negative and still insignificant. As in previous cases, the gamma coefficient remains positive and significant at 1% level. Model 3 also shows that gamma yields significant gains, while kappa yields significant losses at 1% level.

Model 4 shows that beta in the presence of firm characteristics has a positive and significant relationship with excess returns. We also observe that high B/M value investment has a high rate of significant excess return. In Model 5, gamma and B/M value have a significant and positive coefficient of 0.0276 and 0.3273 respectively at 1% level of significance implying that high gamma and B/M stocks generate a high excess return for the investor while exposure to stock's VaR results in losses. For Model 6, beta, gamma, and B/M value have a positive relationship with excess return. Whereas, kappa, past returns, and VaR have a negative and significant relationship with excess returns, although the only kappa is significant at 1% level.

Model 7 shows that gamma and kappa are significantly priced when beta is replaced with continuous beta and jump beta. The robustness of the significant positive gamma relationship with excess returns and the significant negative kappa relationship with excess returns is quite interesting. Accounting for the additional measures of risk does not eliminate the significant positive (negative) relationship of gamma (kappa) with excess returns in Model 8. Also, B/M value, past returns, and VaR retain their directional impact and significance at 5% level.

Both upside and downside risks are robust for Model 9. Thus upside beta, upside gamma, downside gamma, and downside kappa have significant and positive relationships with excess return while downside beta and upside kappa have a significant and negative relationship with excess return. This suggests that investors' holding downside beta and upside kappa risks experience substantial losses and vice versa. In Model 10, we note that both upside and downside beta risk measures are robust when a set firm characteristics are included in the model. The B/M value and illiquidity have a positive relationship with excess return, while past returns exhibit a negative and significant relationship with excess return.

Table 5.9: Fama-MacBeth cross-sectional regressions, pre-crisis period (Upmarket period)

Model	1	2	3	4	5	6	7	8	9	10
β	0.0030	-0.0035	0.0010	0.0115***	0.0052	0.0091**				
γ		0.0279***	0.0469***		0.0276***	0.0439***	0.0465***	0.0440***		
κ			-0.0027***			-0.0026***	-0.0023***	-0.0021***		
Illiq				0.0097	0.0104	0.0101		0.0080		0.0123*
log(size)				0.0003	-0.0004	0.0003		0.0003		0.0010
Pret				-0.0136	-0.0185	-0.0191*		-0.0216**		-0.0256**
B/M				0.4104***	0.3273***	0.2849**		0.2824**		0.2503***
VaR				-1.2475*	-1.2862*	-1.2017*		-1.2123**		-0.8952
β^c							-0.0006	0.0006		
β^j							-0.0018	0.0010		
β^+									0.0295***	0.0267***
β^-									-0.0227***	-0.0198***
γ^+									0.0336***	0.0383***
γ^-									0.0382***	0.0311***
κ^+									-0.0041***	-0.0044***
κ^-									0.0044***	0.0036***
Intercept	0.0117*	0.0095	0.0091	-0.0109	-0.0087	-0.0104	0.0084	-0.0101	0.0073	-0.0121
Avg. R-squared	0.0712	0.132	0.159	0.191	0.243	0.257	0.199	0.273	0.263	0.331

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the pre-crisis sample period starting 6 January 2003 - 29 June 2007 (upmarket period). The upmarket period which is defined as when the monthly excess realized market return exceeds the risk-free rate. Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Table (5.10) reports the regression results for the downmarket period in pre-crisis period. In Model 1, we observe that the beta has a negative (-0.0045) relationship with excess return at 10% level of significance. Model 2, shows that the relationship between beta and excess returns remains negative but insignificant when gamma risk is included in the model. For gamma, we observe that gamma generate significant rewards to investors who bear gamma risk. Model 3 shows that gamma and kappa have a direct relationship with the excess returns at 1% and 10% level of significance, respectively.

In Model 4, shows that the beta remains insignificant and negative (-0.0017) when a set of firm characteristics are included in the model. We also note that the significance of the B/M value decreases to 10% level of significance in comparison to those of Model 4 in Tables (5.8) and (5.9). Model 5 shows that gamma and B/M have a positive relationship with excess return, although the B/M coefficient is not that significant. Model 6 shows that gamma and B/M risk measures continue to yield significant excess returns to investors.

Model 7 shows that only gamma is priced when combined with continuous and jump beta. In the case of Model 8, the results obtained did not change much from that of Model 7. Accounting for additional measures of risk has no directional or significant effect on gamma investment.

For Model 9, we note that upside and downside gamma lose their significance in comparison to previous results discussed. The level of significance of kappa also reduces from 1% level to 5% level. Upside beta investments yield gains at 1% level of significance, while downside beta leads to highly substantial losses. The results obtained in Model 10 show that the direction and significance of the upside and downside risk measures are robust akin to Model 9.

In light of these points, the results of Tables (5.8), (5.9) and (5.10) have shown that the risk measure-excess return relationship has a relatively lower level of significance in the pre-crisis period in comparison to that of the full-sample period. Generally, the reward for gamma risk exposure is robust in all models that incorporate gamma risk, while kappa risk significance tends to vary depending on the sub-period or market condition.

Table 5.10: Fama-MacBeth cross-sectional regressions, pre-crisis period (Downmarket period)

Model	1	2	3	4	5	6	7	8	9	10
β	-0.0045*	-0.0037	-0.0049	-0.0017	0.0001	-0.0014				
γ		0.0080***	0.0159***		0.0082***	0.0150***	0.0155***	0.0147***		
κ			0.0008*			0.0006	0.0005	0.0005		
Illiq				0.0008	0.0021	0.0020		0.0020		0.0008
log(size)				-0.0009	-0.0007	-0.0010		-0.0009		-0.0010
Pret				0.0090	0.0077	0.0047		0.0096		0.0003
B/M				0.1513*	0.1407*	0.1387*		0.1419*		0.1398*
VaR				0.2743	0.3576	0.3566		0.3958		0.3107
β^c							0.0001	0.0000		
β^j							-0.0013	-0.0002		
β^+									0.0078***	0.0079***
β^-									-0.0129***	-0.0115***
γ^+									0.0089*	0.0100**
γ^-									0.0100	0.0081
κ^+									-0.0010**	-0.0011**
κ^-									0.0010**	0.0008*
Intercept	0.0168***	0.0183***	0.0173***	0.0242***	0.0256***	0.0272***	0.0170***	0.0270***	0.0171***	0.0262***
Avg. R-squared	0.0270	0.0387	0.0546	0.0678	0.0796	0.0886	0.0567	0.0935	0.0843	0.113

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the pre-crisis sample period starting 6 January 2003 - 29 June 2007 (downmarket period). The downmarket period which is defined as when the monthly excess realized market return falls below the risk-free rate. Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

5.4.4.2 Crisis-period (2 July 2007 - 29 May 2009)

Table (5.11) reports the Fama-MacBeth regression results for the crisis-period (2 July 2007 - 29 May 2009). In Model 1, the two-moment CAPM shows that the standard beta has a negative and insignificant relationship with realized excess return, which is consistent with the results of Table 5.5 and Table 5.8. Model 2 shows that gamma has a highly significant coefficient of (0.0473) in the three-moment CAPM. We also note that gamma remains positive (0.0735) and significant for the four-moment CAPM of Model 3, as already discussed, this significant positive coefficient of gamma suggests that investors bearing gamma risks experience significant gains.

Model 4 shows beta remains negative (-0.0052) and insignificant when combined with a set of firm characteristics. Additionally, the coefficient of the VaR shows that stocks with high VaR value have a direct relationship with excess returns, which is significant at 10% level. Model 5 shows that gamma and VaR have a positive relationship with excess returns, which is significant at 1% and 10% level (respectively), hence high levels of gamma and VaR stocks increase excess return. Model 6 shows that only gamma and kappa have a significant positive relationship with excess returns, which highlights the relevance of realized higher-order co-moment pricing.

In Model 7, we note that both gamma and kappa retain their predictive power in the presence of continuous and jump beta. Inhere, gamma, and kappa have a significant and positive coefficient of 0.0730 and 0.0030, respectively. In contrast, continuous and jump betas have a negative and insignificant relationship with excess returns. Additionally, Model 8 access the robustness of Model 7 when we control for a set of firm characteristics, we note that the results for both gamma and kappa are stable and robust. The coefficient of -0.0105 for VaR, indicates a significant loss of excess returns at a 10% level.

For Model 9, we observe that the significance level of the conditional normalized co-moments for the financial crisis period decrease relative to that of the full-sample period results. The directional impact of the upside and downside risks remain the same. In Model 10, we note that the significance of downside gamma improves. Both upside and downside risks are priced, though none of the control variables displayed any predictive power.

Table 5.11: Fama-MacBeth cross-sectional regressions, crisis period (All period)

Model	1	2	3	4	5	6	7	8	9	10
β	-0.0148	-0.0088	-0.0439	-0.0052	0.0033	-0.0205				
γ		0.0473***	0.0735***		0.0515***	0.0686***	0.0730***	0.0706***		
κ			0.0034*			0.0030**	0.0030**	0.0033**		
Illiq				-0.0058	-0.0049	-0.0039		-0.0079		-0.0040
log(size)				-0.0006	-0.0005	-0.0015		-0.0026		-0.0012
Pret				-0.0196	-0.0267	-0.0221		-0.0122		-0.0217
B/M				0.4912	0.3574	0.3009		0.3110		0.3553
VaR				2.0277*	2.1378*	1.7157		1.7319		1.2375
β^c							-0.0039	0.0019		
β^j							-0.0223	-0.0105*		
β^+									0.0317**	0.0250***
β^-									-0.0811***	-0.0522***
γ^+									0.0991***	0.0923***
γ^-									0.0472*	0.0694***
κ^+									-0.0088***	-0.0076***
κ^-									0.0045**	0.0059***
Intercept	-0.0236*	-0.0183*	-0.0249*	0.0157	0.0237	0.0174	-0.0244*	0.0274	-0.0246	0.0077
Avg. R-squared	0.0458	0.122	0.168	0.199	0.267	0.288	0.197	0.304	0.247	0.345

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the crisis sample period starting 2 July 2007 - 29 May 2009 (all period). Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Table (5.12) reports the Fama-MacBeth regression for the upmarket period (crisis period). We observe that the beta has a significant positive (0.0157) relationship with excess return, which is similar to that of Table (5.6). In Model 2, the relationship between beta and excess return remain positive but insignificant when gamma risk is included in the model. The coefficient of gamma is 0.0168, which is highly significant at 1% level. Model 3 shows that the beta and gamma coefficients are positive and significant at 10% level, while kappa has an insignificant negative relationship with excess returns.

In Model 4, we note that the significance level of beta improves, with beta exhibiting a positive relationship with excess returns when the two-moment CAPM model is combined with a set of firm characteristics. We observe that past returns has an inverse relationship with excess returns, which is significant at 10% level. For Model 5, the significance of the beta disappears, while gamma has a positive and significant coefficient (0.0177) at the 5% level. The results show that stocks with high past returns experience losses at 5% significance level. While accounting for a set of firm characteristics for the four-moment CAPM of Model 6, we note that only gamma and kappa are priced. The positive (0.0218) relationship between gamma and excess returns suggest significant gains, while the negative (-0.009) coefficient of kappa implies substantial losses.

For Model 7, we note that only gamma is significantly priced when combined with continuous beta and jump beta. In the presence of a set of firm characteristics, Model 8 shows that gamma remains positive (0.0219) and significant. Apart from the past return that is significant at 10% level, none of the control variables have predictive power.

In Model 9, we observe that both upside and downside risk measures; the statistical significance decreases significantly in comparison to that of Table 5.6 and Table 5.9 which capture upmarket periods for full-sample and pre-crisis periods, respectively. However, the directional sign of the upside and downside risks remains the same. Model 10 shows that accounting for a set of firm characteristics does not improve the significance of the upside and downside risk measures. Similarly, for the control variables, only past return has a significant relationship with excess returns.

Table 5.12: Fama-MacBeth cross-sectional regressions, crisis period (Upmarket period)

Model	1	2	3	4	5	6	7	8	9	10
β	0.0157**	0.0090	0.0187*	0.0127**	0.0048	0.0127				
γ		0.0168***	0.0249*		0.0177**	0.0218**	0.0252**	0.0219**		
κ			-0.0020			-0.0009**	-0.0012	-0.0004		
Illiq				-0.0036	-0.0035	-0.0027		-0.0042		-0.0035
log(size)				0.0005	0.0004	0.0009		0.0014		0.0011
Pret				-0.0414*	-0.0495**	-0.0476**		-0.0409*		-0.0458**
B/M				0.2553	0.1572	0.1589		0.1423		0.1753
VaR				-0.1531	-0.2852	-0.1497		-0.3372		-0.3256
β^c							-0.0014	0.0006		
β^j							0.0047	0.0015		
β^+									0.0215*	0.0121
β^-									-0.0103*	-0.0053
γ^+									0.0302**	0.0322**
γ^-									0.0356*	0.0263**
κ^+									-0.0027**	-0.0025*
κ^-									0.0027	0.0017**
Intercept	-0.0438**	-0.0432**	-0.0410**	-0.0522**	-0.0536**	-0.0529**	-0.0400**	-0.0583***	-0.0385**	-0.0564**
Avg. R-squared	0.0183	0.0467	0.0583	0.0943	0.119	0.125	0.0801	0.132	0.0947	0.147

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the crisis sample period starting 2 July 2007 - 29 May 2009 (upmarket period). The upmarket period which is defined as when the monthly excess realized market return exceeds the risk-free rate. Significance levels:‘ *: 0.10, **: 0.05, ***: 0.01’. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms’ market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Table (5.13) reports the regression results for the downmarket period (financial crisis period). Model 1 shows that the beta has a significant negative (-0.0036) relationship with excess returns, which is unsurprising. Model 2 shows that gamma has a significant positive (0.0305) relationship with excess returns. In Model 3, we observe that the risk measures for the four-moment CAPM are priced. Inhere, gamma and kappa have a highly significant positive coefficient of 0.0485 and 0.0054, respectively. The coefficient of the beta remains negative (-0.0626) and significant at 5% level.

Model 4 shows that accounting for firm characteristics, the two-moment CAPM is robust. The beta has a significant negative (-0.0180) relationship with excess returns. We also note that VaR has a positive and significant relationship with excess returns. This implies that stocks with high value at risk offer a significant reward to investors’ bearing such risk during market downmarket period. From Model 5, we note that in the presence of gamma risk, the significance of the beta disappears. The results show that gamma and VaR yield significant positive excess returns, while the rest of the explanatory variables lack any form of predictability. In Model 6, we note that gamma, kappa, and VaR have a positive relationship with excess return. For this model, investing in beta risk leads to substantial losses.

For Model 7, we observe that both gamma and kappa have a positive and significant relationship with excess returns, which suggests the presence of significant gains from gamma and kappa investments. However, the jump beta has a significant negative (-0.0270) relationship with excess returns suggesting substantial losses for jump beta investments. Model 8 shows that the results of gamma, kappa, and jump beta are robust in the presence of firm characteristics. For the control variables, we note that VaR and past returns have a positive and significant relationship with excess returns, while substantial losses accompany larger firm size.

Model 9 also shows that downside beta and upside kappa exhibit a significant and negative coefficient of -0.0708 and -0.0061, respectively. However, upside gamma investments yield significant gains to the investor. In Model 10, the significance of upside beta, downside gamma, and downside kappa improves and maintains their directional impact.

Table 5.13: Fama-MacBeth cross-sectional regressions, crisis period (Downmarket period)

Model	1	2	3	4	5	6	7	8	9	10
β	-0.0306**	-0.0178	-0.0626**	-0.0180**	-0.0015	-0.0332**				
γ		0.0305***	0.0485***		0.0338***	0.0468***	0.0478***	0.0488***		
κ			0.0054***			0.0039***	0.0042***	0.0036**		
Illiq				-0.0022	-0.0013	-0.0012		-0.0037		-0.0006
log(size)				-0.0011	-0.0009	-0.0024		-0.0040**		-0.0023
Pret				0.0217	0.0228	0.0255		0.0287*		0.0241
B/M				0.2359	0.2002	0.1421		0.1687		0.1800
VaR				2.1808**	2.4230***	1.8654**		2.0691**		1.5630*
β^c							-0.0025	0.0013		
β^j							-0.0270**	-0.0121***		
β^+									0.0101	0.0129*
β^-									-0.0708***	-0.0469**
γ^+									0.0689***	0.0601**
γ^-									0.0116	0.0431**
κ^+									-0.0061**	-0.0051*
κ^-									0.0018	0.0042**
Intercept	-0.0141	-0.0094	-0.0182	0.0337	0.0430*	0.0361	-0.0187	0.0515**	-0.0204	0.0299
Avg. R-squared	0.0275	0.0754	0.110	0.104	0.148	0.163	0.117	0.172	0.152	0.198

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the crisis sample period starting 2 July 2007 - 29 May 2009 (downmarket period). The downmarket period is when the excess market return falls below the risk-free rate. Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

In summary, we notice that in comparison to the full-sample and pre-crisis period the significance of most of the coefficients decreased significantly during the crisis period. In addition, we also observe that the reward for bearing gamma risk remain priced, and robust across different market periods, with the significance varying across sub-periods.

5.4.4.3 Post-crisis period (1 June 2009 - 29 December 2017)

In Table (5.14), we report the Fama-MacBeth regression results for the post-crisis period (all period), starting 1 June 2009 - 29 December 2017. Here, Model 1 shows that beta for the two-moment CAPM has a positive and insignificant relationship with realized excess return. Similarly, Model 2 shows that beta remain positive and insignificant when combined with gamma risk, while the coefficient of gamma is positive (0.0310) and highly significant. In Model 3, we note a negative and insignificant relationship between standard beta, kappa, and excess returns. However, the gamma risk measure exhibits a positive coefficient of 0.0568 which is significant at 1% level. Undoubtedly, this implies that in the presence of realized higher-order moment risk measures (in the four-moment CAPM), gamma yields significant gains for investor.

Furthermore, Model 4 shows that when the two-moment CAPM is combined with a set of firm characteristics, beta remains positive (0.0022) and insignificant. Despite this, the coefficient (1.3730) of the B/M value implies that stocks with high B/M value yield significant gains for investors. Model 5 also shows that gamma has a positive and significant relationship with excess returns when we account for control variables. In this model, the B/M value is still positive and significant. Additionally, Model 6 combines a set of firm characteristics to the four-moment CAPM. We observe that gamma and B/M value stocks have a positive and significant relationship with excess returns. Conversely, beta and kappa have no explanatory power.

Model 7 shows that only gamma retains its explanatory power in the presence of continuous beta and jump beta risks. Consequently, Model 8 shows that not only gamma is robust metric when we control for a set of firm characteristics, but also the B/M value is also robust.

However, unlike the financial crisis period, the level of significance of the coefficients obtained for upside and downside risk measures in Model 9 is significant at 1% level. Furthermore, Model 10 shows that the upside and downside risk measures retain their directional signs and significance when we account for the control variables, which is consistent with the results of the full-sample period of table 5.5. Overall, the B/M value remains positive and highly significant.

Table 5.14: Fama-MacBeth cross-sectional regressions, post-crisis period (All period)

Model	1	2	3	4	5	6	7	8	9	10
β	0.0018	0.0040	-0.0019	0.0022	0.0026	-0.0020				
γ		0.0310***	0.0568***		0.0328***	0.0507***	0.0513***	0.0470***		
κ			-0.0010			-0.0004	-0.0007	-0.0004		
Illiq				0.0031	0.0039	0.0050		0.0053		0.0045
log(size)				-0.0005	-0.0012	-0.0016		-0.0013		-0.0017*
Pret				-0.0153	-0.0107	-0.0082		-0.0125		-0.0134
B/M				1.3730***	1.2234***	1.2446***		1.2953***		1.2085***
VaR				0.4494	0.5216	0.4203		0.3237		0.5088
β^c							-0.0026	-0.0001		
β^j							0.0002	0.0001		
β^+									0.0184***	0.0157***
β^-									-0.0245***	-0.0196***
γ^+									0.0949***	0.0945***
γ^-									0.0872***	0.0741***
κ^+									-0.0092***	-0.0090***
κ^-									0.0084***	0.0069***
Intercept	0.0019	0.0050	0.0034	0.0066	0.0137	0.0126	0.0030	0.0094	0.0038	0.0141
Avg. R-squared	0.0673	0.133	0.176	0.240	0.295	0.312	0.224	0.339	0.264	0.375

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the post-crisis period starting 1 June 2009 - 29 December 2017 (all period). Significance levels:‘ *: 0.10, **: 0.05, ***: 0.01’. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms’ market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Table (5.15) reports the Fama-MacBeth regression for the upmarket period during the post-crisis period. We note that the traditional two-moment CAPM shows that beta has a positive (0.0062) and significant relationship with excess return, which is consistent with the results of Table (5.6). Similarly, Model 2 shows that both beta and gamma have a positive and significant relationship with excess returns. Interestingly, in Model 3, we find that the significance of the beta disappears when kappa is included in the regression. Additionally, gamma exhibits a significant positive (0.0330) relationship with the excess return. Contrarily, kappa shows a significantly negative (-0.0017) relationship with excess returns.

In Model 4, we note that beta has a positive (0.0081) and significant relationship with excess returns when a set of firm characteristics are combined with the beta. We also observe that the B/M value has a direct and significant relationship with excess returns, with a coefficient of 0.6236. This signifies the generation of gains to the investor who bear beta and B/M risks. Furthermore, Model 5 shows that the beta, gamma, and B/M value have a positive and significant relationship with the excess returns. Similarly, the significant gains of beta, gamma, and B/M value investments prevail in model 6, while kappa investment leads to substantial losses.

Model 7 investigates when the beta in the four-moment CAPM, is replaced with continuous and jump betas. Consequently, we note that gamma and jump beta investments lead to significant gains, while kappa investment leads to substantial losses. When a set of firm characteristics are added to model 7, we note that the relationship of excess returns and gamma, kappa, jump beta remains the same in Model 8. However, the significance of jump beta reduces to 10% significance level. Meanwhile, B/M value (past returns) have a significant direct (inverse) relationship with excess returns.

In Model 9, we note that both upside and downside risk measures are significantly priced, which is consistent with the results of Table 5.6. Finally, Model 10 shows that accounting for a set of firm characteristics does not change the significance of the upside and downside risk measures. For the additional explanatory variables, B/M value investments generate significant gains, while investments based on past returns result in substantial losses.

Table 5.15: Fama-MacBeth cross-sectional regressions, post-crisis period (Upmarket period)

Model	1	2	3	4	5	6	7	8	9	10
β	0.0062***	0.0052**	0.0052	0.0081***	0.0051**	0.0057**				
γ		0.0147***	0.0330***		0.0173***	0.0291***	0.0295***	0.0266***		
κ			-0.0017**			-0.0012***	-0.0014**	-0.0011**		
Illiq				-0.0002	0.0012	0.0017		0.0006		0.0012
log(size)				0.0001	-0.0007	-0.0006		0.0000		-0.0007
Pret				-0.0174*	-0.0145	-0.0131		-0.0165**		-0.0174**
B/M				0.6236***	0.5600***	0.5928***		0.6077***		0.5926***
VaR				-0.1521	-0.2076	-0.1656		-0.3115		-0.0365
β^c							0.0013	0.0020**		
β^j							0.0034**	0.0023*		
β^+									0.0123***	0.0105***
β^-									-0.0083***	-0.0067***
γ^+									0.0531***	0.0516***
γ^-									0.0508***	0.0400***
κ^+									-0.0051***	-0.0048***
κ^-									0.0052***	0.0039***
Intercept	-0.0009	-0.0014	-0.0026	-0.0072	-0.0039	-0.0047	-0.0024	-0.0102	-0.0029	-0.0034
Avg. R-squared	0.0319	0.0642	0.0846	0.125	0.155	0.164	0.110	0.179	0.132	0.196

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the post-crisis period starting 1 June 2009 - 29 December 2017 (upmarket period). The upmarket period is defined as when the monthly excess realized market return exceeds the risk-free rate. Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Table (5.16) reports the regression results for the downmarket period (post-crisis period). Model 1 shows that beta has a negative (-0.0043) insignificant relationship with excess returns. Model 2 shows that gamma has a significant positive (0.0163) relationship with excess returns, while the standard beta has an insignificant negative (-0.0012) relationship with the excess returns. For the four-moment CAPM of Model 3, the results suggest gamma and kappa investments generate positive excess returns to the investor.

Model 4 shows that accounting for a set of firm characteristics, we note that beta has a significant negative (-0.0059) relationship with excess returns. In addition, high B/M stocks have a significant positive relationship with excess returns. From Model 5, we note that gamma, B/M value, and value-at-risk stocks have positive relationships with excess returns. The coefficients of gamma (0.0155) and B/M value (0.6634) are significant at 1% level, while the VaR is significant at 10% level. Model 6 shows that the four-moment CAPM risk measures are priced with the inclusion of firm characteristics. The results show that gamma, kappa, and B/M value stocks yield significant gains to the investor while beta investments result in substantial losses.

Regarding Model 7, we note that gamma and kappa have a positive and significant relationship with excess returns. However, the continuous beta has a significant negative (-0.0039) relationship with excess returns. This suggest that investing continuous beta during the downmarket period results in the investor experiencing substantial losses, which contradicts the results obtained in Table 5.14. Model 8 shows that the results obtained for gamma and kappa are robust when we control for the firm characteristics. We observe that high B/M value yields significant gains.

For Model 9, we observe that upside and downside risk measures are priced and statistically significant. In Model 10, the significance of upside and downside risk measures are robust in the presence of the firm characteristics. Additionally, B/M value has a positive and significant relationship with excess return, while firm size has an inverse relationship with excess returns.

Table 5.16: Fama-MacBeth cross-sectional regressions, post-crisis period (Downmarket period)

Model	1	2	3	4	5	6	7	8	9	10
β	-0.0043	-0.0012	-0.0071	-0.0059***	-0.0025	-0.0077**				
γ		0.0163***	0.0239***		0.0155***	0.0216***	0.0218***	0.0204***		
κ			0.0007*			0.0008***	0.0006**	0.0007***		
Illiq				0.0033	0.0027	0.0033		0.0047*		0.0032
log(size)				-0.0006	-0.0005	-0.0010		-0.0013*		-0.0011**
Pret				0.0020	0.0038	0.0049		0.0040		0.0040
B/M				0.7493***	0.6634***	0.6518***		0.6875***		0.6160***
VaR				0.6016	0.7292*	0.5858		0.6352*		0.5453
β^c							-0.0039**	-0.0022		
β^j							-0.0032	-0.0023		
β^+									0.0061**	0.0052***
β^-									-0.0163***	-0.0129***
γ^+									0.0418***	0.0429***
γ^-									0.0365***	0.0341***
κ^+									-0.0041***	-0.0042***
κ^-									0.0032***	0.0030***
Intercept	0.0064	0.0100***	0.0096***	0.0173*	0.0211**	0.0209**	0.0090***	0.0231***	0.0102***	0.0210***
Avg. R-squared	0.0355	0.0685	0.0913	0.116	0.140	0.148	0.113	0.159	0.132	0.178

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the post-crisis period starting 1 June 2009 - 29 December 2017 (downmarket period). The downmarket period is when the excess market return falls below the risk-free rate. Significance levels:' *: 0.10, **: 0.05, ***: 0.01'. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR).

Overall, the results discussed for the post-crisis period are robust in both the upmarket and downmarket conditions. The significance and directional signs of the risk measures suggest that the post-crisis results predominately drives the results obtained for the full-sample period. In general, we note that the beta and kappa risk measures are asymmetric in upmarket and downmarket period, while gamma yields significant gains to the investor regardless of the market conditions.

Similarly, investors who hold stocks with high B/M value experience significant excess returns, which is consistent with the findings in the existing literature.

5.5 Concluding Remarks

In this study, we investigate some realized higher-order co-moment risk-return relationships for 142 stocks that are constituents of the S&P/ASX200 index downloaded from the TRTH/SIRCA database. Using 15-minute return data spanning from January 2003 to December 2017, we compute monthly realized returns and risk measures. We follow a set of rigorous methodologies (i.e., the single sorting of excess return on risk measures, the double sorting of excess returns on risk measures, and the Fama-MacBeth cross-sectional regression), which are well documented in the extant literature. This study contributes to the discussion regarding whether systematic realized co-skewness (gamma) and systematic realized co-kurtosis (kappa) can explain Australian stock returns.

The empirical results from the three methods as employed are consistent and robust. We find that the average returns for standard beta and kappa risks are both asymmetric and diametrically opposite in upmarket and downmarket periods. Specifically, standard beta and kappa have a positive (negative) relationship with realized excess returns in the upmarket (downmarket) condition. This suggests that the average gain or loss for standard beta and kappa risks depends on bullish or bearish market states. Furthermore, we observe that gamma risk yields significant gains for investors regardless of the market conditions. This can be explained by Equation (5.18), which highlights that the directional impact of gamma coefficient is conditional on both the marginal investor risk-preference for skewness and the skewness of the market, while that of beta and kappa coefficients is conditional on just the marginal investor risk-aversion to variance and kurtosis respectively. The gains obtained from gamma risk remain robust even with models that incorporate other risk measures. However, the significance of the reward or loss for bearing kappa risk tends to vary depending on the market condition or the sample period that is being considered. This insight highlights the attractive nature of gamma risk to investors, relative to that of kappa risk. In short, this finding is consistent with the investor's preference for (or aversion to) gamma (kappa) risk that is predominately documented in the existing literature. Additionally, it is critical to note that gamma and kappa risk remain priced even in the presence of continuous and jump betas, presumably because of the orthogonal differences in their explicit asymmetries. We also observe that the upside and downside beta, gamma, and kappa retain their direction and level of significance. This remains the case even when we control for a set of firm characteristics for the full-sample and post-crisis periods by employing Fama-MacBeth cross-sectional regression. Our findings indicate that realized higher-order co-moment risks do matter.

Chapter 6

Conclusion

The presence of skewness and kurtosis in asset returns has been extensively documented in the literature on empirical asset pricing, and the relevance of these factors to investors' portfolio allocation and asset pricing strategies has been demonstrated. It has been shown that not only the systematic co-variance risk factor but also the systematic co-skewness and co-kurtosis risk factors can explain the cross-section of asset returns. As such the higher-order co-moment risk is priced, and skewness and kurtosis help improve the theoretical consistency and empirical performance of financial models such as the standard CAPM. The present thesis adopts various theoretical methodologies, simulation techniques and empirical procedures to provide new perspectives on the following questions using high-frequency return data: (*i*) Does the optimal sampling frequency differ between the US and Australian equity markets? Could the 5-minute rule of thumb applied in US markets be extended to Australian markets? (*ii*) How do the sampling-interval and holding-interval affect the estimated realized variance, realized skewness and realized kurtosis, and what are the implications on investors' trading strategies? (*iii*) Can the signals from information flow (trading volume) be explained by realized high-order moments as a means of observing the dynamics of this relationship across holding periods and sample periods? (*iv*) How should realized higher-order co-moment risk be measured and priced to capture the differences between systematic co-variance, co-skewness and co-kurtosis risks? Further, are the risk premia the same across different market conditions and sample periods? The high-frequency return data utilized in the present thesis enables us to obtain robust empirical estimates. We observe that the answers to these questions have a direct or indirect impact on both investors' decision-making criterion and trading strategies.

Chapter 2 examines the optimal sampling frequencies for the realized variance of American and Australian stocks and indices. We determine whether the 5-minute optimal unbiased sampling frequency rule of thumb employed in the US framework for realized variance holds for all DJI30 stocks and DJI30 index (US equity framework). We note that the observed preferred sampling frequency for computing realized variance in the US framework cannot be extended to S&P/ASX20 stocks and S&P/ASX20 index (Australian equity framework). We infer that perhaps a '10-' to '30-minute' window could be the preferred sampling frequency for the Australian equity framework. We argue that researchers in the high-frequency finance paradigm should consider the preferred sampling frequency for the country under investigation, rather than generalizing the 5-minute US rule to other equity markets. Therefore, this study contributes to the body of existing literature as the preferred sampling frequency for the Australian equity market is predominately ignored.

In Chapter 3, we determine analytically, theoretically and empirically, the relationship between realized skewness and realized kurtosis and the sampling-interval and holding-interval. We employ a count-based signature-plot in the presentation and discussion of the results. Our results show that both the holding-interval and the sampling-interval have distinct and dissimilar effects on the realized skewness and realized kurtosis. The central limit theorem for skewness and kurtosis is simulated and shown to hold when no jumps are present in the sampled price series. However, these intervaling effects are found to be analytically tractable, which has implications for future research in the area of high-frequency finance. This chapter contributes to the literature on high-frequency finance, as previous literature on the subject focuses on the effects of the holding-interval on skewness and kurtosis using low-frequency return data.

Chapter 4 investigates the relationship between information flow (trading volume) and realized higher-order moments. The results of the present thesis go beyond the number of trade proxy documented in prior research by considering various proxies of volume. We find that the type of volume proxy influences the signs of the volume higher-order realized moment regression coefficients. We attempt to relate our empirical findings to the MDH, SIAH and DOH hypotheses, noting that the DOH hypothesis implicitly encompasses or nests both the SIAH and MDH hypotheses. The dynamic and significant link between volume and higher-order moments is shown by highlighting the significance of the regression coefficient across holding periods and various market conditions. In addition, we show that, apart from volume-volatility, volume-skewness and volume-kurtosis can provide additional information that could benefit investors' trading strategies. For example, event studies that employ volume and volatility in making inferences could extend their models to account for volume-skewness and volume-kurtosis to capture relevant information that would otherwise be neglected. We note that, whether asset returns that are conditional on volume have superior trading strategy benefits is worthy of future exploration. If this is shown to be the case, it raises the question of whether an improved trading/portfolio allocation performance could be achieved with conditional returns? We also wonder if the ability of risk models to predict return distribution using volume-related variables (for volume-based VaR models) could be improved? A further question that remains to be explored is whether the co-dependence of volume is related to co-variance, co-skewness and co-kurtosis. We acknowledge that the sub-sampling technique proposed by Zhang et al. (2005) can be employed to estimate higher-order moments, rather than the single 15-minute grid. However, as to whether the sub-sampling approach significantly outperforms the 15-minute sampling frequency is left for future research.

In Chapter 5, we examines the relationship between monthly realized returns and monthly realized higher-order co-moment risk factors to determine how investors price systematic co-skewness and systematic co-kurtosis under different market conditions and sample periods. We estimate the various risk exposures of the different risk factors and their respective risk premiums and identify the most important systematic risk components (systematic co-variance, co-skewness, co-kurtosis) associated with Australian stock returns. We find that realized higher-order moment risk factors are superior to both the standard CAPM model and the jump-diffusive two-beta CAPM model. According to Tibiletti (2012), higher-order moments fall short in preserving the marginal asset properties under a portfolio. The authors argue that higher moments are not 'coherent measures of risk' and, therefore, could be the cause of the above-mentioned shortcoming. Instead, they propose one-sided higher-order moments as an alternative approach to overcome this problem. We admit that it will be fruitful to test the validity of this claim in an empirical setting across other financial assets and markets in future studies. Prior literature has also documented models and empirical results that show the significance of higher-order moments as they affect stochastic discount factors. Conrad et al. (2013) finds that stochastic discount factors produce similar results to those of higher-order moments. We question whether a similar approach could be applied to our empirical setup. However, we leave this for future research.

In summary, this thesis provides an insight into the dynamic behaviour of higher-order moments in high-frequency finance. This research contributes to the body of literature on high-frequency finance in its finding that the preferred 5-minute sampling frequency popularly used in sampling US high-frequency data should not be extended to all markets, particularly the Australian equity market. We show that estimates of realized skewness and realized kurtosis are subject to the effects of the holding-interval and sampling-interval, which has a variety of important implications for high-frequency finance. It is shown that the direction and magnitude of volume-realized skewness and volume-realized kurtosis relationship are conditional on the type of volume proxy, holding-period and sample-period under consideration. Our higher-order pricing model reveals the relevance of realized systematic co-skewness and co-kurtosis in explaining the cross-section of asset returns even in the presence of the diffusive and jump risk factors.

Bibliography

Acharya, V. V. and L. H. Pedersen (2005). Asset pricing with liquidity risk. *Journal of Financial Economics 77*(2), 375–410.

Adcock, C. J. (2010). Asset pricing and portfolio selection based on the multivariate extended skew-student-t distribution. *Annals of Operations Research 176*(1), 221–234.

Ahadzie, R. M. and N. Jeyasreedharan (2020). Effects of intervaling on high-frequency realized higher-order moments. *Quantitative Finance 20*(7), 1169–1184.

Aït-Sahalia, Y., P. A. Mykland, and L. Zhang (2005). How often to sample a continuous-time process in the presence of market microstructure noise. *The Review of Financial Studies 18*(2), 351–416.

Albuquerque, R. (2012). Skewness in stock returns: Reconciling the evidence on firm versus aggregate returns. *The Review of Financial Studies 25*(5), 1630–1673.

Alexeev, V., M. Dungey, and W. Yao (2017). Time-varying continuous and jump betas: The role of firm characteristics and periods of stress. *Journal of Empirical Finance 40*, 1–19.

Alles, L. and L. Murray (2013). Rewards for downside risk in Asian markets. *Journal of Banking and Finance 7*(37), 2501–2509.

Alles, L. and L. Murray (2017). Asset pricing and downside risk in the Australian share market. *Applied Economics 49*(43), 4336–4350.

Amaya, D., P. Christoffersen, K. Jacobs, and A. Vasquez (2015). Does realized skewness predict the cross-section of equity returns? *Journal of Financial Economics 118*(1), 135–167.

Amihud, Y. (2002). Illiquidity and stock returns: Cross-section and time-series effects. *Journal of Financial Markets 5*(1), 31–56.

Amihud, Y., B. J. Christensen, and H. Mendelson (1992). *Further evidence on the risk-return relationship*, Volume 11. Citeseer. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.203.1898&rep=rep1&type=pdf.

Andersen, T., T. Bollerslev, F. Diebold, and P. Labys (2000). Great realisations. *Risk-London-Risk Magazine Limited- 13*(3), 105–109.

Andersen, T. G. (1996). Return volatility and trading volume: An information flow interpretation of stochastic volatility. *The Journal of Finance 51*(1), 169–204.

Andersen, T. G. and T. Bollerslev (1997). Intraday periodicity and volatility persistence in financial markets. *Journal of Empirical Finance 4*(2-3), 115–158.

Andersen, T. G. and T. Bollerslev (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review*, 885–905.

Andersen, T. G., T. Bollerslev, and F. X. Diebold (2007). Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *The Review of Economics and Statistics 89*(4), 701–720.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens (2001). The distribution of realized stock return volatility. *Journal of Financial Economics 61*(1), 43–76.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys (2003). Modeling and forecasting realized volatility. *Econometrica 71*(2), 579–625.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and G. Wu (2006). Realized beta: Persistence and predictability. In *Econometric Analysis of Financial and Economic Time Series*, pp. 1–39. Emerald Group Publishing Limited.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and J. Wu (2005). A framework for exploring the macroeconomic determinants of systematic risk. *American Economic Review 95*(2), 398–404.

Andersen, T. G., T. Bollerslev, and N. Meddahi (2004). Analytical evaluation of volatility forecasts. *International Economic Review 45*(4), 1079–1110.

Andersen, T. G., G. Cebiroglu, and N. Hautsch (2017). Volatility, information feedback and market microstructure noise: A tale of two regimes. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2921097.

Andreou, E. and E. Ghysels (2002). Rolling-sample volatility estimators: Some new theoretical, simulation and empirical results. *Journal of Business & Economic Statistics 20*(3), 363–376.

Ang, A., J. Chen, and Y. Xing (2002). Downside correlation and expected stock returns. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=282986.

Ang, A., J. Chen, and Y. Xing (2006). Downside risk. *The Review of Financial Studies 19*(4), 1191–1239.

Ang, J. S. and J. H. Chua (1979). Composite measures for the evaluation of investment performance. *Journal of Financial and Quantitative Analysis*, 361–384.

Arditti, F. D. (1967). Risk and the required return on equity. *The Journal of Finance 22*(1), 19–36.

Arditti, F. D. (1971). Another look at mutual fund performance. *Journal of Financial and Quantitative Analysis 6*(3), 909–912.

Athayde, G. M. d. and R. G. Flôres Junior (1997). A CAPM with higher moments: Theory and econometrics. Available at: https://bibliotecadigital.fgv.br/dspace/handle/10438/515.

Avramov, D., T. Chordia, and A. Goyal (2006). Liquidity and autocorrelations in individual stock returns. *The Journal of Finance 61*(5), 2365–2394.

Back, K. (1991). Asset pricing for general processes. *Journal of Mathematical Economics 20*(4), 371–395.

Bai, X. (2000). *Beyond Merton's utopia: Effects of non-normality and dependence on the precision of variance estimaters using high-frequency financial data*. Ph. D. thesis, University of Chicago, Graduate School of Business. Available at: http://faculty.chicagobooth.edu/jeffrey.russell/research/merton1.pdf.

Bai, X., J. Russell, and G. Tiao (2001). Beyond Merton's utopia (I): Effects of non-normality and dependence on the precision of variance using high-frequency financial data. *University of Chicago, GSB Working Paper July*. Available at: https://faculty.chicagobooth.edu/jeffrey.russell/research/merton1.pdf.

Bali, T. G. (2003). An extreme value approach to estimating volatility and value at risk. *The Journal of Business 76*(1), 83–108.

Bandi, F. M. and J. R. Russell (2008). Microstructure noise, realized variance, and optimal sampling. *The Review of Economic Studies 75*(2), 339–369.

Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of Financial Economics 9*(1), 3–18.

Barberis, N. and M. Huang (2008). Stocks as lotteries: The implications of probability weighting for security prices. *American Economic Review 98*(5), 2066–2100.

Barndorff-Nielsen, O. E. and N. Shephard (2002). Estimating quadratic variation using realized variance. *Journal of Applied Econometrics 17*(5), 457–477.

Barndorff-Nielsen, O. E. and N. Shephard (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of Financial Econometrics 2*(1), 1–37.

Barndorff-Nielsen, O. E. and N. Shephard (2006). Econometrics of testing for jumps in financial economics using bipower variation. *Journal of Financial Econometrics 4*(1), 1–30.

Barone Adesi, G., P. Gagliardini, and G. Urga (2004). Testing asset pricing models with coskewness. *Journal of Business & Economic Statistics 22*(4), 474–485.

Bawa, V. S. and E. B. Lindenberg (1977). Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics 5*(2), 189–200.

Beedles, W. L. (1979). On the asymmetry of market returns. *Journal of Financial and Quantitative Analysis 14*(3), 653–660.

Bilinski, P. and D. Lyssimachou (2014). Risk Interpretation of the CAPM's Beta: Evidence from a New Research Method. *Abacus 50*(2), 203–226.

Black, F. (1993). Beta and return. *The Journal of Portfolio Management 20*(1), 8–18.

Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *Journal of Political Economy 81*(3), 637–654.

Bollerslev, T., R. F. Engle, and J. M. Wooldridge (1988). A capital asset pricing model with time-varying covariances. *Journal of Political Economy 96*(1), 116–131.

Bollerslev, T., T. H. Law, and G. Tauchen (2008). Risk, jumps, and diversification. *Journal of Econometrics 144*(1), 234–256.

Bollerslev, T., S. Z. Li, and V. Todorov (2016). Roughing up beta: Continuous versus discontinuous betas and the cross section of expected stock returns. *Journal of Financial Economics 120*(3), 464–490.

Bollerslev, T. and B. Y. Zhang (2003). Measuring and modeling systematic risk in factor pricing models using high-frequency data. *Journal of Empirical Finance 10*(5), 533–558.

Botshekan, M., R. Kraeussl, and A. Lucas (2012). Cash flow and discount rate risk in up and down markets: What is actually priced? *Journal of Financial and Quantitative Analysis 47*(6), 1279–1301.

Bouchaud, J.-P., J. Kockelkoren, and M. Potters (2006). Random walks, liquidity molasses and critical response in financial markets. *Quantitative Finance 6*(02), 115–123.

Boyer, B., T. Mitton, and K. Vorkink (2010). Expected idiosyncratic skewness. *The Review of Financial Studies 23*(1), 169–202.

Breen, W. J. and R. A. Korajczyk (1993). On selection biases in book-to-market based tests of asset pricing models. *Northwestern Universi& Workingpaper 167*. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.199.782&rep=rep1&type=pdf.

Brown, C. A. and D. M. Robinson (2002). Skewness and kurtosis implied by option prices: A correction. *Journal of Financial Research 25*(2), 279–282.

Brunnermeier, M. K., C. Gollier, and J. A. Parker (2007). Optimal beliefs, asset prices, and the preference for skewed returns. *American Economic Review 97*(2), 159–165.

Campbell, J. Y., S. J. Grossman, and J. Wang (1993). Trading volume and serial correlation in stock returns. *The Quarterly Journal of Economics 108*(4), 905–939.

Campbell, J. Y., L. M. Viceira, L. M. Viceira, et al. (2002). *Strategic asset allocation: Portfolio choice for long-term investors*. Clarendon Lectures in Economic.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance 52*(1), 57–82.

Carroll, R. and C. Kearney (2015). Testing the mixture of distributions hypothesis on target stocks. *Journal of International Financial Markets, Institutions and Money 39*, 1–14.

Celik, S. (2013). New evidence on the relation between trading volume and volatility. *Business and Economic Research 3*(1), 176–186.

Chan, C. C. and W. M. Fong (2006). Realized volatility and transactions. *Journal of Banking & Finance 30*(7), 2063–2085.

Chan, K. and W.-M. Fong (2000). Trade size, order imbalance, and the volatility–volume relation. *Journal of Financial Economics 57*(2), 247–273.

Chan, L. K., Y. Hamao, and J. Lakonishok (1991). Fundamentals and stock returns in Japan. *The Journal of Finance 46*(5), 1739–1764.

Chang, B. Y., P. Christoffersen, and K. Jacobs (2013). Market skewness risk and the cross section of stock returns. *Journal of Financial Economics 107*(1), 46–68.

Charoenrook, A. A. and H. Daouk (2004). Conditional skewness of aggregate market returns. *Available at SSRN 562163*. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=562163.

Chen, G.-m., M. Firth, and O. M. Rui (2001). The dynamic relation between stock returns, trading volume, and volatility. *Financial Review 36*(3), 153–174.

Chen, J., H. Hong, and J. C. Stein (2001). Forecasting crashes: Trading volume, past returns, and conditional skewness in stock prices. *Journal of Financial Economics 61*(3), 345–381.

Choi, K.-H., Z.-H. Jiang, S. H. Kang, and S.-M. Yoon (2012). Relationship between trading volume and asymmetric volatility in the Korean stock market. *Modern Economy 3*(05), 584.

Chowdhury, B. and N. Jeyasreedharan (2019). An empirical examination of the jump and diffusion aspects of asset pricing: Japanese evidence. Available at: https://eprints.utas.edu.au/29545/1/2019-02_Chowdhury_Jeyasreedharan.pdf.

Chowdhury, B., N. Jeyasreedharan, and M. Dungey (2018). Quantile relationships between standard, diffusion and jump betas across Japanese banks. *Journal of Asian Economics 59*, 29–47.

Christie-David, R. and M. Chaudhry (2001). Coskewness and cokurtosis in futures markets. *Journal of Empirical Finance 8*(1), 55–81.

Chung, Y. P., H. Johnson, and M. J. Schill (2006). Asset pricing when returns are nonnormal: Fama-french factors versus higher-order systematic comoments. *The Journal of Business 79*(2), 923–940.

Clark, P. K. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica: Journal of the Econometric Society*, 135–155.

Conrad, J., R. F. Dittmar, and E. Ghysels (2013). Ex ante skewness and expected stock returns. *The Journal of Finance 68*(1), 85–124.

Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance 1*(2), 223–236.

Copeland, T. E. (1976). A model of asset trading under the assumption of sequential information arrival. *The Journal of Finance 31*(4), 1149–1168.

Cornell, B. and E. R. Sirri (1992). The reaction of investors and stock prices to insider trading. *The Journal of Finance 47*(3), 1031–1059.

Corrado, C. J. and T. Su (1996). Skewness and kurtosis in s&p 500 index returns implied by option prices. *Journal of Financial Research 19*(2), 175–192.

Corrado, C. J. and T. Su (1997). Implied volatility skews and stock return skewness and kurtosis implied by stock option prices. *The European Journal of Finance 3*(1), 73–85.

Cowan, A. R. and A. M. Sergeant (2001). Interacting biases, non-normal return distributions and the performance of tests for long-horizon event studies. *Journal of Banking & Finance 25*(4), 741–765.

Damodaran, A. (1985). Economic events, information structure, and the return-generating process. *Journal of Financial and Quantitative Analysis 20*(4), 423–434.

De Athayde, G. M. and R. G. Flôres (2000). Introducing higher moments in the CAPM: Some basic ideas. In *Advances in Quantitative Asset Management*, pp. 3–15. Springer.

De Athayde, G. M. and R. G. Flôres Jr (2004). Finding a maximum skewness portfolio—a general solution to three-moments portfolio choice. *Journal of Economic Dynamics and Control 28*(7), 1335–1352.

Desmoulins-Lebeault, F. (2012). Gram–Charlier expansions portfolio selection in Non-Gaussian universes. *Multi-moment Asset Allocation and Pricing Models*, 79–112.

Dittmar, R. F. (2002). Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns. *The Journal of Finance 57*(1), 369–403.

Do, H. X., R. Brooks, S. Treepongkaruna, and E. Wu (2014). How does trading volume affect financial return distributions? *International Review of Financial Analysis 35*, 190–206.

Doan, M. P., C.-T. Lin, and M. Chng (2014). Higher moments and beta asymmetry: evidence from Australia. *Accounting & Finance 54*(3), 779–807.

Dungey, M. and D. Gajurel (2014). Equity market contagion during the global financial crisis: Evidence from the world's eight largest economies. *Economic Systems 38*(2), 161–177.

Dungey, M. and W. Yao (2013). Continuous and jump betas: Firm and industry level evidence. Available at: https://acfr.aut.ac.nz/__data/assets/pdf_file/0005/29975/403275.pdf.

Ebens, H. et al. (1999). Realized stock volatility. *Department of Economics, Johns Hopkins University*. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.26.7641&rep=rep1&type=pdf.

Epps, T. W. and M. L. Epps (1976). The stochastic dependence of security price changes and transaction volumes: Implications for the mixture-of-distributions hypothesis. *Econometrica: Journal of the Econometric Society*, 305–321.

Faff, R. (2001). An examination of the Fama and French three-factor model using commercially available factors. *Australian Journal of Management 26*(1), 1–17.

Fama, E. F. (1965). The behavior of stock-market prices. *The Journal of Business 38*(1), 34–105.

Fama, E. F. and K. R. French (1992). The cross-section of expected stock returns. *The Journal of Finance 47*(2), 427–465.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Fama, E. F. and K. R. French (1995). Size and book-to-market factors in earnings and returns. *The Journal of Finance 50*(1), 131–155.

Fama, E. F. and J. D. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy 81*(3), 607–636.

Fama, E. F. and B. G. Malkiel (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance 25*(2), 383–417.

Fang, H. and T.-Y. Lai (1997). Co-kurtosis and capital asset pricing. *Financial Review 32*(2), 293–307.

Fielitz, B. D. (1976). Further results on asymmetric stable distributions of stock price chances. *Journal of Financial and Quantitative Analysis 11*(1), 39–55.

Fleming, J. and B. S. Paye (2011). High-frequency returns, jumps and the mixture of normals hypothesis. *Journal of Econometrics 160*(1), 119–128.

Fogler, H. R., W. A. Groves, and J. G. Richardson (1977). Bond portfolio strategies, returns, and skewness: A note. *Journal of Financial and Quantitative Analysis 12*(1), 127–140.

Fogler, H. R. and R. C. Radcliffe (1974). A note on measurement of skewness. *Journal of Financial and Quantitative Analysis 9*(3), 485–489.

Francis, J. C. (1975). Skewness and investors' decisions. *Journal of Financial and Quantitative Analysis 10*(1), 163–172.

Fung, H.-G. and G. A. Patterson (1999). The dynamic relationship of volatility, volume, and market depth in currency futures markets. *Journal of International Financial Markets, Institutions and Money 9*(1), 33–59.

Galagedera, D. T. U. and E. A. Maharaj (2008). Wavelet timescales and conditional relationship between higher-order systematic co-moments and portfolio returns. *Quantitative Finance 8*(2), 201–215.

Galagedera, D. U., D. Henry, and P. Silvapulle (2003). Empirical evidence on the conditional relation between higher-order systematic co-moments and security returns. *Quarterly Journal of Business and Economics*, 121–137.

Gaunt, C. (2004). Size and book to market effects and the Fama French three factor asset pricing model: Evidence from the Australian stockmarket. *Accounting & Finance 44*(1), 27–44.

Gharghori, P., H. Chan, and R. Faff (2007). Are the Fama-French factors proxying default risk? *Australian Journal of Management 32*(2), 223–249.

Gharghori, P., R. Lee, and M. Veeraraghavan (2009). Anomalies and stock returns: Australian evidence. *Accounting & Finance 49*(3), 555–576.

Giot, P., S. Laurent, and M. Petitjean (2010). Trading activity, realized volatility and jumps. *Journal of Empirical Finance 17*(1), 168–175.

Greene, M. T. and B. D. Fielitz (1977). Long-term dependence in common stock returns. *Journal of Financial Economics 4*(3), 339–349.

Gul, F. (1991). A theory of disappointment aversion. *Econometrica: Journal of the Econometric Society*, 667–686.

Hainaut, D. and F. Moraux (2017). Modelling the clustering of jumps in equity returns: A bifactor self-exciting jump diffusion approach. Available at: https://2999ffa7-a-62cb3a1a-s-sites.googlegroups.com/site/donatienhainaut/HainautMorauxClustering.pdf.

Hanousek, J., E. Kočenda, and J. Novotnỳ (2014). Price jumps on European stock markets. *Borsa Istanbul Review 14*(1), 10–22.

Hansen, P. R. and A. Lunde (2003). An optimal and unbiased measure of realized variance based on intermittent high-frequency data. *Unpublished paper, Department of Economics, Stanford University*. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.199.5381&rep=rep1&type=pdf.

Hansen, P. R. and A. Lunde (2004). An unbiased measure of realized variance. *Available at SSRN 524602*. Available at: http://www.ims.nus.edu.sg/Programs/econometrics/files/al_paper1.pdf.

Hansen, P. R. and A. Lunde (2006). Realized variance and market microstructure noise. *Journal of Business & Economic Statistics 24*(2), 127–161.

Hanson, F. B. and J. J. Westman (2002). Jump-diffusion stock return models in finance: Stochastic process density with uniform-jump amplitude. In *Proc.*

15th International Symposium on Mathematical Theory of Networks and Systems, Volume 7. Available at: https://pdfs.semanticscholar.org/44ff/eaf8e2dca6609f6202b93b9d3869ee8e8fbf.pdf.

Harris, L. (1986). Cross-security tests of the mixture of distributions hypothesis. *Journal of Financial and Quantitative Analysis 21*(1), 39–46.

Harris, M. and A. Raviv (1993). Differences of opinion make a horse race. *The Review of Financial Studies 6*(3), 473–506.

Harvey, C. R. and A. Siddique (2000a). Conditional skewness in asset pricing tests. *The Journal of Finance 55*(3), 1263–1295.

Harvey, C. R. and A. Siddique (2000b). Time-varying conditional skewness and the market risk premium. *Research in Banking and Finance 1*(1), 27–60.

Hawawini, G. A. (1980). An analytical examination of the intervaling effect on skewness and other moments. *Journal of Financial and Quantitative Analysis 15*(5), 1121–1127.

Homaifar, G. and D. B. Graddy (1988). Equity yields in models considering higher moments of the return distribution. *Applied Economics 20*(3), 325–334.

Hong, H. and J. C. Stein (2003). Differences of opinion, short-sales constraints, and market crashes. *The Review of Financial Studies 16*(2), 487–525.

Huang, R. D. and R. W. Masulis (2003). Trading activity and stock price volatility: Evidence from the London Stock Exchange. *Journal of Empirical Finance 10*(3), 249–269.

Huang, X. and G. Tauchen (2005). The relative contribution of jumps to total price variance. *Journal of Financial Econometrics 3*(4), 456–499.

Hueng, C. J. and J. B. McDonald (2005). Forecasting asymmetries in aggregate stock market returns: Evidence from conditional skewness. *Journal of Empirical Finance 12*(5), 666–685.

Hung, D. C.-H., M. Shackleton, and X. Xu (2004). CAPM, Higher Co-moment and Factor Models of UK Stock Returns. *Journal of Business Finance & Accounting 31*(1-2), 87–112.

Hurlin, C., P. Kouontchou, and B. B. Maillet (2009). a robust conditional realized extended 4-capm. *Available at SSRN 1343884*. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1343884.

Hutson, E., C. Kearney, and M. Lynch (2008). Volume and skewness in international equity markets. *Journal of Banking & Finance 32*(7), 1255–1268.

Hwang, S. and C. S. Pedersen (2002). Best practice risk measurement in emerging markets: Empirical test of asymmetric alternatives to CAPM. In *Working Paper, Cass Business School, UK*. Citeseer. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.11.7565&rep=rep1&type=pdf.

Hwang, S. and S. E. Satchell (1999). Modelling emerging market risk premia using higher moments. *International Journal of Finance & Economics 4*(4), 271–296.

Hwang, S. and S. E. Satchell (2001). Modelling emerging market risk premia using higher moments. In *Return Distributions in Finance*, pp. 75–117. Elsevier.

Ingersoll, J. (1975). Multidimensional security pricing. *Journal of Financial and Quantitative Analysis 10*(5), 785–798.

Jagannathan, R. and Z. Wang (1996). The conditional CAPM and the cross-section of expected returns. *The Journal of Finance 51*(1), 3–53.

Jondeau, E. and M. Rockinger (2003). How higher moments affect the allocation of assets. *Finance Letters 1*(2), 1–5.

Jones, C. M., G. Kaul, and M. L. Lipson (1994). Transactions, volume, and volatility. *The Review of Financial Studies 7*(4), 631–651.

Joulin, A., A. Lefevre, D. Grunberg, and J.-P. Bouchaud (2008). Stock price jumps: News and volume play a minor role. *arXiv preprint arXiv:0803.1769*. Available at: https://arxiv.org/pdf/0803.1769.pdf.

Jurczenko, E. and B. Maillet (2006). Theoretical foundations of asset allocation and pricing models with higher-order moments. *Multi-moment Asset Allocation and Pricing Models 399*, 1.

Jurczenko, E. and B. Maillet (2012). The four-moment capital asset pricing model: Between asset pricing and asset allocation. *Multi-moment Asset Allocation and Pricing Models*, 113–163.

Jurczenko, E., B. B. Maillet, and P. Merlin (2005). Hedge funds portfolio selection with higher-order moments: A non-parametric mean-variance-skewness-kurtosis efficient frontier. *Available at SSRN 676904*. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=676904.

Kahneman, D. and A. Tversky (2013). Prospect theory: An analysis of decision under risk. In *Handbook of the fundamentals of financial decision making: Part I*, pp. 99–127. World Scientific. Available at: https://www.worldscientific.com/doi/abs/10.1142/9789814417358_0006.

Kalev, P. S., W.-M. Liu, P. K. Pham, and E. Jarnecic (2004). Public information arrival and volatility of intraday stock returns. *Journal of Banking & Finance 28*(6), 1441–1467.

Kandel, S., R. McCulloch, and R. F. Stambaugh (1995). Bayesian inference and portfolio efficiency. *The Review of Financial Studies 8*(1), 1–53.

Karpoff, J. M. (1987). The relation between price changes and trading volume: A survey. *Journal of Financial and Quantitative Analysis 22*(1), 109–126.

Kennedy, D. B., R. Sivakumar, and K. R. Vetzal (2006). The implications of IPO underpricing for the firm and insiders: Tests of asymmetric information theories. *Journal of Empirical Finance 13*(1), 49–78.

Kim, D. (1995). The errors in the variables problem in the cross-section of expected stock returns. *The Journal of Finance 50*(5), 1605–1634.

Kim, T.-H. and H. White (2004). On more robust estimation of skewness and kurtosis. *Finance Research Letters 1*(1), 56–73.

Kirchler, M. and J. Huber (2007). Fat tails and volatility clustering in experimental asset markets. *Journal of Economic Dynamics and Control 31*(6), 1844–1874.

Klemkosky, R. C. (1973). The bias in composite performance measures. *Journal of Financial and Quantitative Analysis*, 505–514.

Knif, J., D. Koutmos, and G. Koutmos (2020). Higher co-moment capm and hedge fund returns. *Atlantic Economic Journal*, 1–15.

Kostakis, A., K. Muhammad, and A. Siganos (2012). Higher co-moments and asset pricing on London Stock Exchange. *Journal of Banking & Finance 36*(3), 913–922.

Kothari, S. P., J. Shanken, and R. G. Sloan (1995). Another look at the cross-section of expected stock returns. *The Journal of Finance 50*(1), 185–224.

Kraus, A. and R. H. Litzenberger (1975). Market equilibrium in a multiperiod state preference model with logarithmic utility. *The Journal of Finance 30*(5), 1213–1227.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society*, 1315–1335.

Lahaye, J., S. Laurent, and C. J. Neely (2011). Jumps, cojumps and macro announcements. *Journal of Applied Econometrics 26*(6), 893–921.

Lakonishok, J., A. Shleifer, and R. W. Vishny (1994). Contrarian investment, extrapolation, and risk. *The Journal of Finance 49*(5), 1541–1578.

Lambert, M. and G. Hübner (2013). Comoment risk and stock returns. *Journal of Empirical Finance 23*, 191–205.

Lau, H.-S. and J. R. Wingender (1989). The analytics of the intervaling effect on skewness and kurtosis of stock returns. *Financial Review 24*(2), 215–233.

Lee, C., J. Robinson, and R. Reed (2008). Downside beta and the cross-sectional determinants of listed property trust returns. *Journal of Real Estate Portfolio Management 14*(1), 49–62.

Lee, C. F., R. M. Leuthold, and J. E. Cordier (1985). The stock market and the commodity futures market: Diversification and arbitrage potential. *Financial Analysts Journal 41*(4), 53–60.

Lee, S. S. and P. A. Mykland (2012). Jumps in equilibrium prices and market microstructure noise. *Journal of Econometrics 168*(2), 396–406.

Leland, H. E. (1999). Beyond Mean–Variance: Performance Measurement in a Nonsymmetrical World (corrected). *Financial Analysts Journal 55*(1), 27–36.

Lewellen, J. and S. Nagel (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics 82*(2), 289–314.

Limkriangkrai, M., R. B. Durand, and I. Watson (2008). Is liquidity the missing link? *Accounting & Finance 48*(5), 829–845.

Lintner, J. (1975). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. In *Stochastic Optimization Models in Finance*, pp. 131–155. Elsevier.

Liow, K. H. and L. C. Chan (2005). Co-skewness and co-kurtosis in global real estate securities. *Journal of Property Research 22*(2-3), 163–203.

Liu, C. H., D. J. Hartzell, and T. V. Grissom (1992). The role of co-skewness in the pricing of real estate. *The Journal of Real Estate Finance and Economics 5*(3), 299–319.

Liu, L. Y., A. J. Patton, and K. Sheppard (2015). Does anything beat 5-minute RV? A comparison of realized measures across multiple asset classes. *Journal of Econometrics 187*(1), 293–311.

Lunde, A., P. R. Hansen, et al. (2004). Realized variance and iid market microstructure noise. In *Econometric Society 2004 North American Summer Meetings*, Number 526. Econometric Society. Available at: http://repec.org/esNASM04/up.24351.1075581838.pdf.

Madhavan, A. (2000). Market microstructure: A survey. *Journal of Financial Markets 3*(3), 205–258.

Malkiel, B. G. and E. F. Fama (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance 25*(2), 383–417.

Mandelbrot, B. B. (1997). The variation of certain speculative prices. In *Fractals and Scaling in Finance*, pp. 371–418. Springer.

Markowitz, H. M. and G. P. Todd (2000). *Mean-variance analysis in portfolio choice and capital markets*, Volume 66. John Wiley & Sons.

Matsuda, K. (2004). Introduction to Merton jump diffusion model. *Department of Economics. The Graduate Center, The City University of New York*. Available at: http://maxmatsuda.com/Papers/Intro/Intro%20to%20MJD%20Matsuda.pdf.

McNeil, A. J. and R. Frey (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach. *Journal of Empirical Finance 7*(3-4), 271–300.

Meddahi, N. (2002). A theoretical comparison between integrated and realized volatility. *Journal of Applied Econometrics 17*(5), 479–508.

Mei, D., J. Liu, F. Ma, and W. Chen (2017). Forecasting stock market volatility: Do realized skewness and kurtosis help? *Physica A: Statistical Mechanics and its Applications 481*, 153–159.

Meucci, A. (2007). *Risk and Asset Allocation*. Springer Science & Business Media.

Meucci, A. (2010). Quant nugget 4: Annualization and general projection of skewness, kurtosis and all summary statistics. *GARP Risk Professional-" The Quant*

Classroom, 59–63. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1635484.

Mitton, T. and K. Vorkink (2007). Equilibrium underdiversification and the preference for skewness. *The Review of Financial Studies 20*(4), 1255–1288.

Moreno, D. and R. Rodríguez (2006). Performance evaluation considering the coskewness. *Managerial Finance 32*(4), 375–392.

Moreno, D. and R. Rodríguez (2009). The value of coskewness in mutual fund performance evaluation. *Journal of Banking & Finance 33*(9), 1664–1676.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the Econometric Society*, 768–783.

Mujtaba Mian, G. and C. M. Adam (2001). Volatility dynamics in high frequency financial data: An empirical investigation of the Australian equity returns. *Applied Financial Economics 11*(3), 341–352.

Neuberger, A. (2012). Realized skewness. *The Review of Financial Studies 25*(11), 3423–3455.

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica (1986-1998) 55*(3), 703.

Okunev, J. (1990). An alternative measure of mutual fund performance. *Journal of Business Finance & Accounting 17*(2), 247–264.

Oomen, R. (2004a). Properties of bias corrected realized variance in calendar time and business time. *manuscript, Warwick Business School, The University of Warwick*. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.640.2104&rep=rep1&type=pdf.

Oomen, R. C. (2004b). Modelling realized variance when returns are serially correlated. Technical report, WZB Discussion Paper. Available at: https://www.econstor.eu/bitstream/10419/51076/1/393956164.pdf.

Oomen, R. C. A. (2006). Properties of realized variance under alternative sampling schemes. *Journal of Business & Economic Statistics 24*(2), 219–237.

Oomen Roel, C. (2002). Modelling realized variance when returns are serially correlated. Available at: https://www.econstor.eu/bitstream/10419/51076/1/393956164.pdf.

Pástor, L. and R. F. Stambaugh (2003). Liquidity risk and expected stock returns. *Journal of Political Economy 111*(3), 642–685.

Patton, A. J. and A. Timmermann (2010). Monotonicity in asset returns: New tests with applications to the term structure, the capm, and portfolio sorts. *Journal of Financial Economics 98*(3), 605–625.

Pedersen, C. S. and S. Hwang (2007). Does downside beta matter in asset pricing? *Applied Financial Economics 17*(12), 961–978.

Pettengill, G. N., S. Sundaram, and I. Mathur (1995). The conditional relation between beta and returns. *Journal of Financial and Quantitative Analysis 30*(1), 101–116.

Poti, V. and D. Wang (2010). The coskewness puzzle. *Journal of Banking & Finance 34*(8), 1827–1838.

Prakash, A. J. and R. M. Bear (1986). A simplifying performance measure recognizing skewness. *Financial Review 21*(1), 135–144.

Price, K., B. Price, and T. J. Nantell (1982). Variance and lower partial moment measures of systematic risk: Some analytical and empirical results. *The Journal of Finance 37*(3), 843–855.

Racine, M. D. (1998). Hedging volatility shocks to the Canadian investment opportunity set. *Quarterly Journal of Business and Economics*, 59–79.

Ranaldo, A. and L. Favre (2005). How to price hedge funds: From two-to four-moment CAPM. *UBS Research Paper*. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=474561.

Reinganum, M. R. (1981). Misspecification of capital asset pricing: Empirical anomalies based on earnings' yields and market values. *Journal of Financial Economics 9*(1), 19–46.

Roll, R. and S. A. Ross (1994). On the cross-sectional relation between expected returns and betas. *The Journal of Finance 49*(1), 101–121.

Rosenberg, B., K. Reid, and R. Lanstein (1985). Persuasive evidence of market inefficiency. *The Journal of Portfolio Management 11*(3), 9–16.

Roy, A. D. (1952). Safety first and the holding of assets. *Econometrica: Journal of the Econometric Society*, 431–449.

Rubinstein, M. E. (1973). The fundamental theorem of parameter-preference security valuation. *Journal of Financial and Quantitative Analysis 8*(1), 61–69.

Samuelson, P. A. (1975). The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. In *Stochastic Optimization Models in Finance*, pp. 215–220. Elsevier.

Scott, R. C. and P. A. Horvath (1980). On the direction of preference for moments of higher order than the variance. *The Journal of Finance 35*(4), 915–919.

Sears, R. S. and K. J. Wei (1988). The structure of skewness preferences in asset pricing models with higher moments: An empirical test. *Financial Review 23*(1), 25–38.

Shahzad, H., H. N. Duong, P. S. Kalev, and H. Singh (2014). Trading volume, realized volatility and jumps in the Australian stock market. *Journal of International Financial Markets, Institutions and Money 31*, 414–430.

Shalen, C. T. (1993). Volume, volatility, and the dispersion of beliefs. *The Review of Financial Studies 6*(2), 405–434.

Shalit, H. and S. Yitzhaki (1984). Mean-Gini, portfolio theory, and the pricing of risky assets. *The Journal of Finance 39*(5), 1449–1468.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance 19*(3), 425–442.

Shen, D., X. Li, and W. Zhang (2018). Baidu news information flow and return volatility: Evidence for the sequential information arrival hypothesis. *Economic Modelling 69*, 127–133.

Shen, D., W. Zhang, X. Xiong, X. Li, and Y. Zhang (2016). Trading and non-trading period internet information flow and intraday return volatility. *Physica A: Statistical Mechanics and its Applications 451*, 519–524.

Singleton, J. C. and J. Wingender (1986). Skewness persistence in common stock returns. *Journal of Financial and Quantitative Analysis 21*(3), 335–341.

Smith, D. R. (2007). Conditional coskewness and asset pricing. *Journal of Empirical Finance 14*(1), 91–119.

Smith, K. V. (1978). The effect of intervaling on estimating parameters of the capital asset pricing model. *Journal of Financial and Quantitative Analysis 13*(2), 313–332.

Stephens, A. and D. Proffitt (1991). Performance measurement when return distributions are nonsymmetric. *Quarterly Journal of Business and Economics*, 23–41.

Synowiec, D. (2008). Jump-diffusion models with constant parameters for financial log-return processes. *Computers & Mathematics with Applications 56*(8), 2120–2127.

Tauchen, G. E. and M. Pitts (1983). The price variability-volume relationship on speculative markets. *Econometrica: Journal of the Econometric Society*, 485–505.

Tibiletti, L. (2012). Higher-order moments and beyond. *Multi-moment Asset Allocation and Pricing Models*, 67–78.

Todorov, V. and T. Bollerslev (2010). Jumps and betas: A new framework for disentangling and estimating systematic risks. *Journal of Econometrics 157*(2), 220–235.

Tse, Y. (1999). Price discovery and volatility spillovers in the DJIA index and futures markets. *Journal of Futures Markets 19*(8), 911–930.

Vanden, J. M. (2006). Option coskewness and capital asset pricing. *The Review of Financial Studies 19*(4), 1279–1320.

Vendrame, V., J. Tucker, and C. Guermat (2016). Some extensions of the capm for individual assets. *International Review of Financial Analysis 44*, 78–85.

Vines, T., C.-H. Hsieh, and J. Hatem (1994). The role of systematic covariance and coskewness in the pricing of real estate: Evidence from equity REITs. *Journal of Real Estate Research 9*(4), 421–429.

Wasserfallen, W. and H. Zimmermann (1985). The behavior of intra-daily exchange rates. *Journal of Banking & Finance 9*(1), 55–72.

Wong, C.-M. and M. K. So (2003). On conditional moments of GARCH models, with applications to multiple period value at risk estimation. *Statistica Sinica*, 1015–1044.

Xing, Y., X. Zhang, and R. Zhao (2010). What does the individual option volatility smirk tell us about future equity returns? *Journal of Financial and Quantitative Analysis*, 641–662.

Zhang, L., P. A. Mykland, and Y. Aït-Sahalia (2005). A tale of two time scales: Determining integrated volatility with noisy high-frequency data. *Journal of the American Statistical Association 100*(472), 1394–1411.

Zhou, B. (1996). High-frequency data and volatility in foreign-exchange rates. *Journal of Business & Economic Statistics 14*(1), 45–52.

Appendix A

Appendix for chapter 2

A.1 Appendix A: Data set

Table A.1: List of stocks used in the analysis of Chapter 2

No.	Symbol	DJI 30 stocks	Symbol	S&P/ASX20 stocks
1	MMM	3M	AMP	AMP Limited
2	AXP	American Express	ANZ	Australia And New Zealand Banking
3	AAPL	Apple	BHP	BHP Billiton Limited
4	BA	Boeing	BXB	Brambles Limited
5	CAT	Caterpillar	CBA	Commonwealth Bank of Australia
6	CVX	Chevron	CSL	SL Limited
7	CSCO	Cisco	IAG	Insurance Australia Group Limited
8	KO	Coca-Cola	MQG	Macquarie Group Limited
9	DIS	Disney	NAB	National Australia Bank Limited
10	DD	EI du Pont.	QBE	QBE Insurance Group Limited
11	XOM	Exxon Mobil	RIO	RIO Tinto Limited
12	GE	General Electric	SCG	Scentre Group Stapled
13	GS	Goldman Sachs	SUN	Suncorp Group Limited
14	HD	Home Depot	TLS	Telstra Corporation Limited
15	IBM	IBM	TCL	Transurban Group Stapled
16	INTC	Intel	WES	Wesfarmers Limited
17	JNJ	Johnson & Johnson	WFD	Westfield Corporation Stapled
18	JPM	JPMorgan Chase	WBC	Westpac Banking Corporation
19	MCD	McDonald's	WPL	Woodside Petroleum Limited
20	MRK	Merck	WOW	Woolworths Limited
21	MSFT	Microsoft		
22	NKE	Nike		
23	PFE	Pfizer		
24	PG	Procter & Gamble		
25	TRV	Travelers Companies Inc.		
26	UTX	United Technologies		
27	UNH	United Health		
28	VZ	Verizon		
29	V	Visa		
30	WMT	Wal-Mart		

The data was obtained from Thompson Reuters Tick History provided by SIRCA database. Our data sample starts from 4 January 2010 - 31 December 2015 between 10 am to 4 pm of each trading day, giving us a sample of 21,600 intra-day price series. We use 1-second high frequency price series which results in 32.62 million price series over 5 years (1,510 trading days).

A.2 Brief history of realized variance/volatility

Table A.2: This table gives a summary of the history of realized variance/volatility in the extant literature

Author	Data	Sampling Frequency	Methodology	RV	Findings
Andersen and Bollerslev (1997)	Deutschemark- U.S exchange rate and S&P 500 composite stock index futures contract	5-minute returns and daily returns	Analysis of intraday volatility patterns using MA(1)-GARCH(1,1)	Daily and multiple-day frequencies	When traditional time series methods are applied to 5-minute high frequency data, high inferences about the return volatility dynamics can be made
Hansen and Lunde (2003, 2004)	DJI 30 stocks and S&P 500 index Transaction prices from Trade and Quote(TAQ)	Tick-by-tick high frequ-ency data	Newey-West type modification of RV	Daily	Observe the Newey-West RV to be unbiased at even 1-minute
Oomen (2004a)	IBM stock, S\&P cash index 500 and simulation	Tick-by-tick high frequ-ency data and simulation	First order bias correction of realized variance	Daily	The optimal sampling frequency without bias correct-ion is about 20-minutes while it is around 2 minute with a first order bias correction.
Bandi and Russell (2008)	IBM stock and simulation	Quote-to-quote IBM price change and simulation	Signature plots	Daily	Find 5-minute RV to approach the target volatility for the day and concluded that for very illiquid stock 15-minute could be preferred sampling frequency
Zhang et al. (2005)	Monte Carlo simulations	Several subsamples	Estimator which is based on subsample techniques	Daily	Showed why and how volatility estimator fails when the returns are sampled at highest frequencies. Found that the subsampling technique to be a good proxy for estimating the integrated variance
Aït-Sahalia et al. (2005)	Monte Carlo simulations	Simulation	Modeling the noise component of RV	Daily, weekly and monthly	Find the optimal sampling interval to be 22 minute for the day
Hansen and Lunde (2006)	DJIA 30 stocks	Tick-by-tick high frequ-ency data	Study of the dynamic effects of transaction prices and quotes caused by changes in the efficie-nt price	Daily	The microstructure noise maybe ignored when intra-day returns are sampled at low freqeuencies, such as 20 minute
Bollerslev et al. (2008)	U.S. 40 stock	Tick-by-tick high frequ-ency data	They use signature plots	Daily	Find 17.5 minute to be optimal sampling frequency
Lahaye et al. (2011)	Three stock index futures(Nasdaq, Dow Jones, S\&P 500) abd other data considered	Tick-by-tick high frequ-ency data	They use signature plots	Daily	Find volatility signature plots show that realized volatility starts to starbilize at about 15 minutes

A.3 Daily realized variance for S&P/ASX200 index from 2010 to 2015

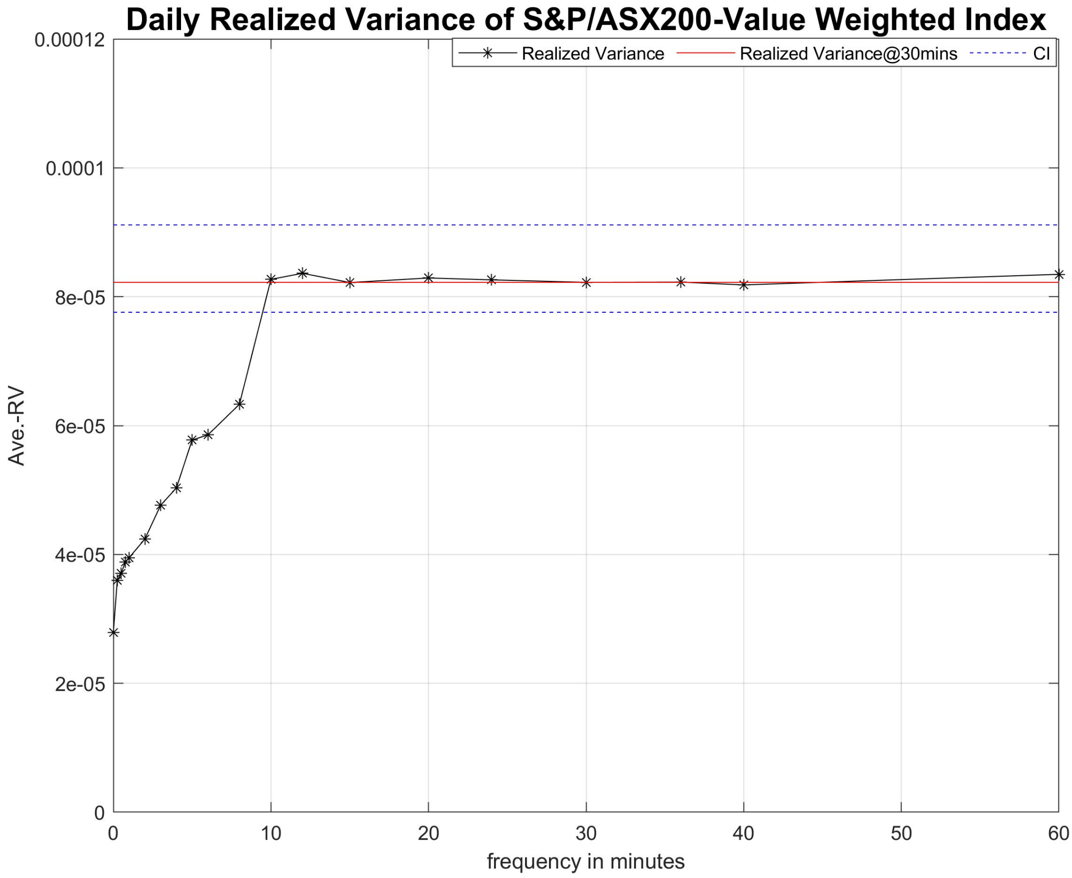

Figure A.3: Average daily realized variance for Australia (S&P/ASX200 index)

A.4 Autocorrelation plots of DJI30 and S&P/ASX20 indices

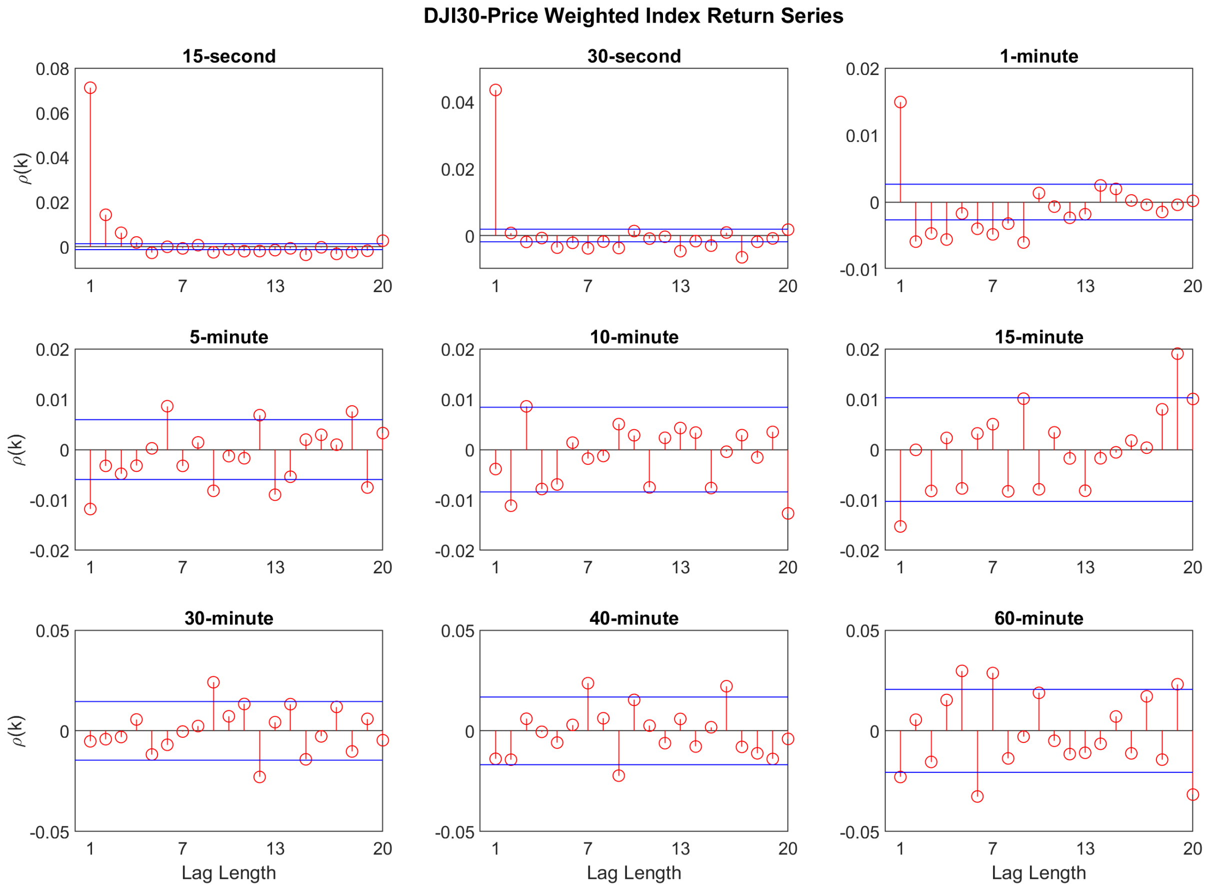

Figure A.4: This figure reports the autocorrelation for U.S. (DJI30 index) return series.

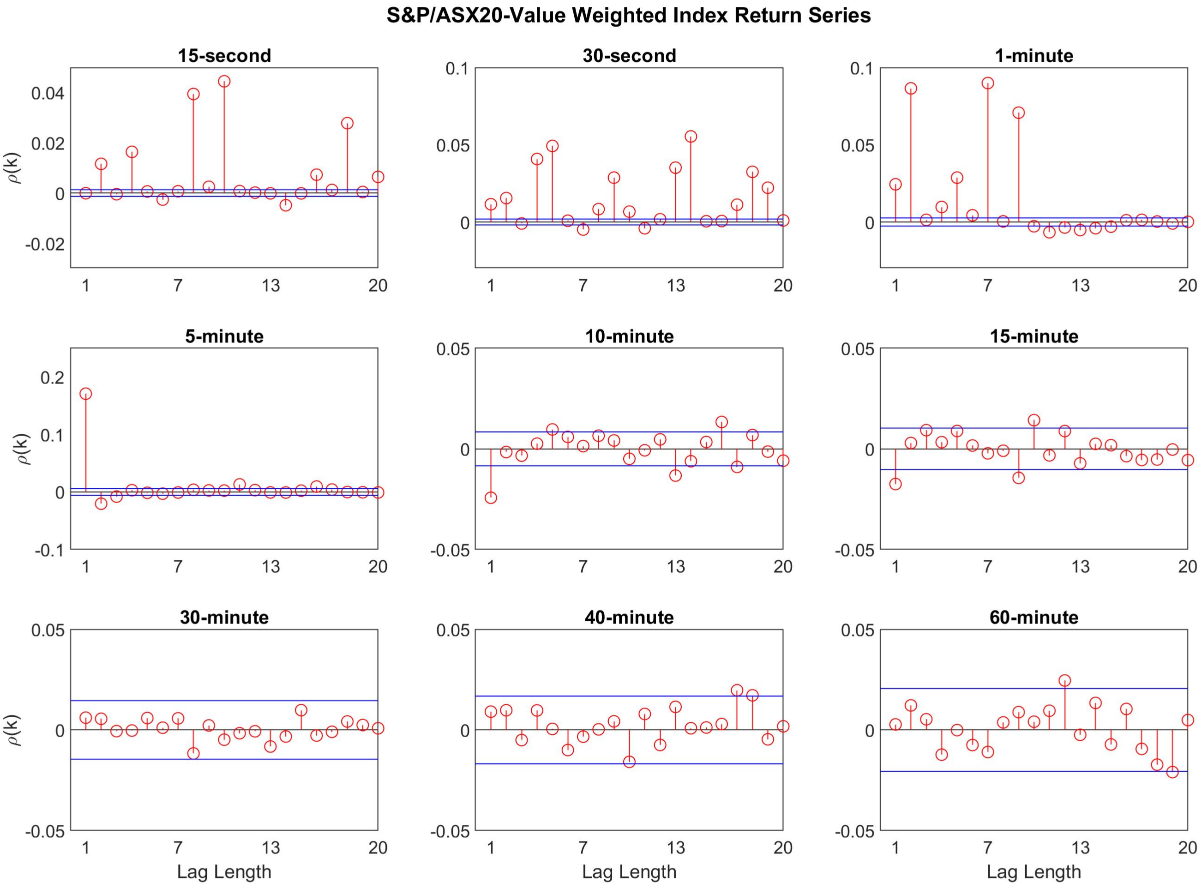

Figure A.5: This figure reports the autocorrelation for Australia (S&P/ASX200 index) return series.

Appendix B

Appendix for chapter 4

B.1 Data-set

Table B1: List of stocks used in the analysis of Chapters 4 and 5

No.	RIC code	Firm Name	No.	RIC code	Firm Name	No.	RIC code	Firm Name
1	AAC.AX	Australian Agricultural Company Ltd	51	DOW.AX	Downer EDI Ltd	101	RKN.AX	Reckon Ltd
2	AAD.AX	Ardent Leisure group Ltd	52	ELX.AX	Ellex Medical Lasers Ltd	102	RMD.AX	Resmed Inc
3	ABC.AX	Adelaide Brighton Ltd	53	ERA.AX	Energy Resources Of Australia Ltd	103	SBM.AX	St Barbara Ltd
4	ABP.AX	Abacus Property Group	54	FAR.AX	FAR Ltd	104	WPP.AX	WPP Aunz Ltd
5	AGG.AX	AngloGold Ashanti Ltd	55	FBU.AX	Fletcher Building Ltd	105	SGP.AX	Stockland Corporation Ltd
6	AGI.AX	Ainsworth Game Technology Ltd	56	FKP.AX	FKP Property Group	106	SHL.AX	Sonic Healthcare Ltd
7	AGL.AX	AGL Energy Ltd	57	FLK.AX	Folkestone	107	SHV.AX	Select Harvests Ltd
8	AHD.AX	Amalgamated holdings Ltd	58	FLT.AX	Flight Centre Travel Group Ltd	108	SOL.AX	Washington H Soul Pattinson and Company Ltd
9	ALK.AX	Alkane Resource	59	FXJ.AX	FAIRFAX Media	109	SRV.AX	Servcorp Ltd
10	ALL.AX	Aristocrat Leisure Ltd	60	GNC.AX	Graincorp Ltd	110	SRX.AX	Sirtex Medical Ltd
11	AMC.AX	Amcor PLC	61	GPT.AX	GPT Group	111	STO.AX	Santos Ltd
12	AMP.AX	AMP Ltd	62	HVN.AX	Harvey Norman Holdings Ltd	112	SUN.AX	Suncorp Group Ltd
13	ANN.AX	Ansell Ltd	63	IAG.AX	Insurance Australia Group Ltd	113	TAH.AX	Tabcorp Holdings Ltd
14	AOG.AX	Aveo GROUP	64	IFM.AX	Infomedia Ltd	114	TCL.AX	Transurban Group
15	APA.AX	APA Group	65	IGO.AX	Independence Group NL	115	TLS.AX	Telstra Corporation Ltd
16	APE.AX	AP Eagers Ltd	66	ILU.AX	Iluka Resources Ltd	116	TNE.AX	TechnologyOne Ltd
17	API.AX	Australian Pharmaceutical Industries Ltd	67	IMF.AX	IMF Bentham Ltd	117	TOX.AX	Tox Free Solutions
18	APN.AX	APN Property Group	68	IOF.AX	Investa Office Fund	118	SXY.AX	Senex Energy Ltd
19	ASB.AX	Austal Ltd	69	IRE.AX	Iress Ltd	119	VRL.AX	Village Roadshow Ltd
20	ASX.AX	ASX Ltd	70	IRI.AX	Integrated Research Ltd	120	SWM.AX	Seven West Media Ltd
21	AVG.AX	Australian Vintage Ltd	71	HLO.AX	Helloworld Travel Ltd	121	WBC.AX	Westpac Banking Corp
22	AVJ.AX	Avjennings Ltd	72	JHX.AX	James Hardie Industries PLC	122	WEB.AX	Webjet Ltd
23	AWC.AX	Alumina Ltd	73	KSC.AX	K&S Corporation Ltd	123	WES.AX	Wesfarmers Ltd
24	AWE.AX	AWE Ltd	74	LLC.AX	LendLease Group	124	WOW.AX	Woolworths Group Ltd
25	BBG.AX	Billabong International Ltd	75	MAH.AX	Macmahon Holdings Ltd	125	WPL.AX	Woodside Petroleum Ltd
26	BEN.AX	Bendigo and Adelaide Bank Ltd	76	SYD.AX	Sydney Airport Holdings Pty Ltd	126	WTP.AX	Watpac Ltd
27	BHP.AX	BHP Group Ltd	77	MAQ.AX	Macquarie Telecom Group Ltd	127	AIA.AX	Auckland International Airport Ltd
28	BKW.AX	Brickworks Ltd	78	MGR.AX	Mirvac Group	128	ALU.AX	Altium Ltd
29	BLD.AX	Boral Ltd	79	MGX.AX	Mount Gibson Iron Ltd	129	ASL.AX	Ausdrill Ltd
30	BNO.AX	Bionomics Ltd	80	MND.AX	Monadelphous Group Ltd	130	BKL.AX	Blackmores Ltd
31	BOQ.AX	Bank of Queensland Ltd	81	NAB.AX	National Australia Bank Ltd	131	COE.AX	Cooper Energy Ltd
32	BPT.AX	Beach Energy Ltd	82	NBL.AX	Noni B Ltd	132	EWC.AX	Energy World Corporation Ltd
33	BSL.AX	BlueScope Steel Ltd	83	NCM.AX	Newcrest Mining Ltd	133	EZL.AX	Euroz Ltd
34	BWP.AX	BWP Trust	84	NUF.AX	Nufarm Ltd	134	FPH.AX	Fisher & Paykel Healthcare Corporation Ltd
35	CAB.AX	Cabcharge Australia Ltd	85	ORG.AX	Origin Energy Ltd	135	HTA.AX	Hutchison Telecommunications (Australia) Ltd
36	CBA.AX	Commonwealth Bank of Australia	86	ORI.AX	Orica Ltd	136	LYC.AX	Lynas Corporation Ltd
37	CCL.AX	Coca-Cola Amatil Ltd	87	OSH.AX	Oil Search Ltd	137	MLB.AX	Melbourne IT
38	CCP.AX	Credit Corp Group Ltd	88	PME.AX	Pro Medicus Ltd	138	RHL.AX	Ruralco Holdings
39	CCV.AX	Cash Converters International Ltd	89	PMP.AX	PMP Ltd	139	SDG.AX	Sunland Group Ltd
40	CIM.AX	CIMIC Group Ltd	90	PPT.AX	Perpetual Ltd	140	SKC.AX	Skycity Entertainment Group Ltd
41	CLH.AX	Collection House Ltd	91	PRY.AX	Primary Health Care	141	SPL.AX	Starpharma Holdings Ltd
42	CNI.AX	Centuria Capital Group	92	QAN.AX	Qantas Airways Ltd	142	WOR.AX	Worley Ltd
43	COH.AX	Cochlear Ltd	93	QBE.AX	QBE Insurance Group Ltd			
44	ALQ.AX	ALS Ltd	94	RCR.AX	RCR Tomlinson			
45	CPH.AX	Creso Pharma Ltd	95	RCT.AX	Reef Casino Trust			
46	CPU.AX	Computershare Ltd	96	REA.AX	REA Group Ltd			
47	CSL.AX	CSL Ltd	97	REH.AX	Reece Ltd			
48	CSR.AX	CSR Ltd	98	RHC.AX	Ramsay Health Care Ltd			
49	CTX.AX	Caltex Australia Ltd	99	RIC.AX	Ridley Corporation Ltd			
50	CWP.AX	Cedar Woods Properties Ltd	100	RIO.AX	Rio Tinto Ltd			

The data was obtained from Thompson Reuters Tick History provided by SIRCA database. Our data sample starts from 6 January 2003 - 29 December 2017 between 10 am to 4 pm of each trading day, giving us a sample of 24 intra-day price series. We use 15-minute high frequency price series which results in 94,350 price series over 15 years. This data-set is used for the analysis in Chapters 4 and 5 of this thesis.

B.2 Granger causality test for realized higher-order moments

Table B2 reports the Granger causality test for realized higher-order moments and trading volume. Panel A reports the results for the weekly holding period and Panel B the monthly holding period for our full sample period. The VAR(p) model can be found in equation B2.1 and B2.2. We use five lags based on the Schwarz criterion. The null hypothesis in equation B2.1 tests if the realized higher-order moment does not Granger cause trading volume. While equation B2.2, tests the null hypothesis that trading volume does not Granger cause realized higher-order moments. From Panel A, we observe a uni-direction relationship for realized variance, realized skewness, positive realized skewness, and trading volume. The uni-directional causality result of realized variance and trading volume is consistent with Fung and Patterson (1999); Tse (1999). Do et al. (2014) find a bi-directional relationship between trading volume and realized volatility in FX markets and for the stock markets in nearly most cases apart from the Western European region, which they observe a uni-directional relationship. We observe a bi-directional causality effect exists for negative realized skewness, realized kurtosis, and trading volume. For the monthly period, we observe that realized variance, realized skewness, negative realized skewness have a uni-directional causality relationship with trading volume. In this period, the positive realized skewness and realized kurtosis tends to have a bi-directional causality. The bi-directional effect of realized kurtosis is persistent from weekly to monthly period.

In Table B3, we report the Granger causality test between the number of trades and realized higher-order moments. Similar to the results above, five lags were used in estimating the VAR(p) model in equations B3.1 and B3.2. The results suggest uni-directional Granger causality between realized variance and the number of trades in both weekly and monthly holding period. For realized skewness, negative and positive realized skewness, realized kurtosis, and the number of trades, we observe a bi-directional Granger causality. This bi-directional causality is persistent across holding periods.

Table B2: Granger causality test between trading volume and realized higher-order moments

Granger causality test	F-Statistic	P-Value
Panel A: Weekly data		
Realized Variance does not Granger cause trading volume	3.5100	0.0036
Trading Volume does not Granger cause realized variance	0.0000	1.0000
Realized Skewness does not Granger cause trading volume	5.0963	0.0001
Trading Volume does not Granger cause realized skewness	0.4826	0.7895
Negative realized Skewness does not Granger cause trading volume	7.1372	0.0000
Trading Volume does not Granger cause negative realized skewness	46.8560	0.0000
Positive realized Skewness does not Granger cause trading volume	0.8978	0.4814
Trading Volume does not Granger cause positive realized skewness	60.5190	0.0000
Realized kurtosis does not Granger cause trading volume	28.5400	0.0000
Trading Volume does not Granger cause realized kurtosis	43.2060	0.0000
Panel B: Monthly data		
Realized Variance does not Granger cause trading volume	5.0464	0.0001
Trading Volume does not Granger cause realized variance	0.0000	1.0000
Realized Skewness does not Granger cause trading volume	9.5297	0.0000
Trading Volume does not Granger cause realized skewness	2.3165	0.0548
Negative realized Skewness does not Granger cause trading volume	8.2920	0.0000
Trading Volume does not Granger cause negative realized skewness	2.0588	0.0834
Positive realized Skewness does not Granger cause trading volume	4.1991	0.0008
Trading Volume does not Granger cause positive realized skewness	2.3835	0.0491
Realized kurtosis does not Granger cause trading volume	12.0700	0.0000
Trading Volume does not Granger cause realized kurtosis	7.7029	0.0000

This table reports the Granger causality test between trading volume and realized higher-order moments. We use the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Panel A presents the results for the weekly holding period while panel B reports the monthly results. The following VAR (p) model is estimated:

$$RM_t = \sum_{k=1}^{p} \psi_{1k} RM_{t-k} + \sum_{k=1}^{p} \varphi_{1k} TV_{t-k} + \varepsilon_{1t} \tag{B2.1}$$

$$TV_t = \sum_{k=1}^{p} \psi_{2k} RM_{t-k} + \sum_{k=1}^{p} \varphi_{2k} TV_{t-k} + \varepsilon_{2t} \tag{B2.2}$$

where TV_t is the trading volume, RM_{it} is realized higher-order moments (realized variance, realized skewness and realized kurtosis). Using the Schwarz criterion, we obtain the optimal lag of p=5. In Equation ($B2.1$), we test the null hypothesis that the realized higher-order moments does not Granger cause trading volume against the alternative that the realized higher-order moments does Granger cause trading volume. Equation ($B2.2$) also tests the null hypothesis that the trading volume does not Granger cause realized higher-order moments against the alternative hypothesis that the trading volume does Granger cause realized higher-order moments. This is to capture the feedback effect between realized higher-order moments and trading volume.

Table B3: Granger causality test between the number of trades and realized higher-order moments

Granger causality test	F-Statistic	P-Value
Panel A: Weekly data		
Realized Variance does not Granger cause number of trades	2.3356	0.0395
number of trades does not Granger cause realized variance	0.8092	0.5428
Realized Skewness does not Granger cause number of trades	25.1250	0.0000
number of trades does not Granger cause realized skewness	9.6495	0.0000
Negative realized Skewness does not Granger cause number of trades	13.3640	0.0000
number of trades does not Granger cause negative realized skewness	4.5538	0.0004
Positive realized Skewness does not Granger cause number of trades	18.6500	0.0000
number of trades does not Granger cause positive realized skewness	4.4325	0.0005
Realized kurtosis does not Granger cause number of trades	23.1010	0.0000
number of trades does not Granger cause realized kurtosis	19.1390	0.0000
Panel B: Monthly data		
Realized Variance does not Granger cause number of trades	3.3000	0.0056
number of trades does not Granger cause realized variance	0.8749	0.4968
Realized Skewness does not Granger cause number of trades	19.0460	0.0000
number of trades does not Granger cause realized skewness	8.2392	0.0000
Negative realized Skewness does not Granger cause number of trades	6.2753	0.0000
number of trades does not Granger cause negative realized skewness	7.2213	0.0000
Positive realized Skewness does not Granger cause number of trades	15.8740	0.0000
number of trades does not Granger cause positive realized skewness	4.1837	0.0008
Realized kurtosis does not Granger cause number of trades	25.4000	0.0000
number of trades does not Granger cause realized kurtosis	7.4429	0.0000

This table reports the Granger causality test between the number of trades and realized higher-order moments. We use the 142 stocks from our sample starting from 6 January 2003 - 29 December 2017. Panel A presents the results for the weekly holding period while panel B reports the monthly results. The following VAR (p) model is estimated:

$$RM_t = \sum_{k=1}^{p} \psi_{1k} RM_{t-k} + \sum_{k=1}^{p} \varphi_{1k} NT_{t-k} + \varepsilon_{1t} \tag{B3.1}$$

$$NT_t = \sum_{k=1}^{p} \psi_{2k} RM_{t-k} + \sum_{k=1}^{p} \varphi_{2k} NT_{t-k} + \varepsilon_{2t} \tag{B3.2}$$

where NT_t is the number of trades, RM_{it} is realized higher-order moments (realized variance, realized skewness and realized kurtosis). Using Schwarz criterion, we obtain the optimal lag of p=5, in Equation ($B3.1$) we test the null hypothesis that the realized higher-order moments does not Granger cause the number of trades against the alternative that the realized higher-order moments does Granger cause the number of trades. Equation ($B3.2$) also tests the null hypothesis that the number of trades does not Granger cause realized higher-order moments against the alternative hypothesis that the number of trades does Granger cause realized higher-order moments. This is to capture the feedback effect between realized higher-order moments and the number of trades.

Appendix C

Appendix for chapter 5

C.1 Fama-MacBeth cross-sectional regression run separately for each risk measure

Table C1: Fama-MacBeth cross-sectional regression, full-sample period

Risk measure	**All period**	**Upmarket period**	**Downmarket period**
β	-0.0013	0.0064***	-0.0077***
γ	0.0372***	0.0204***	0.0168***
κ	0.0002	0.0006***	-0.0004***
Illiq	0.0031	-0.0005	0.0036
log(size)	-0.0020***	-0.0012**	-0.0008**
PRet	0.0074	-0.0081	0.0155*
B/M	1.2187***	0.4862***	0.7325***
VaR	-0.1056	-0.5671*	0.4615*
β^c	-0.0017	0.0012	-0.0029***
β^j	-0.0013	0.0032***	-0.0045**
β^+	0.0178***	0.0145***	0.0032**
β^-	-0.0207***	-0.0068***	-0.0138***
γ^+	0.0134***	0.0089***	0.0045***
γ^-	0.0131***	0.0070***	0.0062***
κ^+	0.0014***	0.0009***	0.0005***
κ^-	-0.0013***	-0.0007***	-0.0006***

This table reports the monthly Fama-MacBeth cross-sectional regression results for the 142 Australian stocks for the full sample period starting from 6 January 2003 - 29 December 2017. For this table, we run the regression separately on each risk measure. The upmarket (downmarket) period are defined as when the monthly excess realized market return exceeds (less than) the risk-free rate. The standard CAPM beta (β), downside beta (β^-), upside beta (β^+), continuous beta (β^c), jump beta (β^j), gamma (γ), downside gamma (γ^-), upside gamma (γ^+), kappa (κ), downside kappa (κ^-), upside kappa (κ^+), book-to-market value (B/M), illiquidity (Illiq), natural logarithm of firms' market capitalization (log(size)), lagged return over previous month (PRet), value-at-risk (VaR). Significance levels:' *: 0.10, **: 0.05, ***: 0.01'.